НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
ТОМСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

САРАТОВСКИЙ НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ
ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМ. Н. Г. ЧЕРНЫШЕВСКОГО

РОССИЙСКИЙ УНИВЕРСИТЕТ ДРУЖБЫ НАРОДОВ

ИНСТИТУТ ПРОБЛЕМ УПРАВЛЕНИЯ
им. В.А. ТРАПЕЗНИКОВА РАН

ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ (ИТММ-2019)

МАТЕРИАЛЫ
XVIII Международной конференции
имени А. Ф. Терпугова
26–30 июня 2019 г.

Часть 2

ТОМСК
«Издательство НТЛ»
2019

ТЕОРИЯ МАССОВОГО ОБСЛУЖИВАНИЯ И ТЕЛЕТРАФИКА

On the Total Amount of the Occupied Resources in the Multi-resource QS with Renewal Arrival Process

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The methods of queuing theory are widely used to describe different economic problems, to process large data in technical systems, as well as cloud computing. Modern computer networks are characterized by the integration of heterogeneous streams, including phone calls, text messages, video sources, etc., which require the use of a more complex flow model [1–3]. To study such models, it is necessary to take into account different kinds of resources needed for the transmission and processing of the transmitted information. Thus, the development of computer and mobile communication networks has led to the need of developping new "*resource*" models that would allow us to estimate the amount of the occupied resource [6, 7].

Most often the analisys is limited to systems with incoming stationary Poisson flow and exponential service time. But the fact is that the Poisson flow does not always accurately describe real flows and the service time is not always exponential [8, 9]. Therefore, it is very relevant in practice to consider a system with an incoming non-Poisson (for example, renewal arrival process) flow and an arbitrary service time.

Mathematical Model

Consider a queuing system with infinite number of servers and arbitrary service time. Renewal arrival process is determined by the distribution function $A(z)$ of the interarrival times. Each arriving customer instantly occupies the first free server, with service time distribution $B(\tau)$ and different resources ($i = 1, \ldots, n$) with distribution $G_i(y)$ depending on the type i of the resource. When the service is completed, the customer leaves the system. Resource amounts and service times are mutually independent and do not depend on the epochs of customer arrivals.

Denote by $V_i(t)$ the total amount of i-th type resources ($i = 1, \ldots, n$) occupied at time t. Our goal is to derive the probabilistic characterization of the n-dimensional process $V(t) = [V_1(t), \ldots, V_n(t)]$. This process is, in general, not Markovian and, therefore, we use the dynamic screening method for its investigation. Consider two time axes that are numbered as 0 and 1 (see Fig.1). Let axis 0 show the epochs of customers' arrivals, while axis 1 corresponds to the screened process.

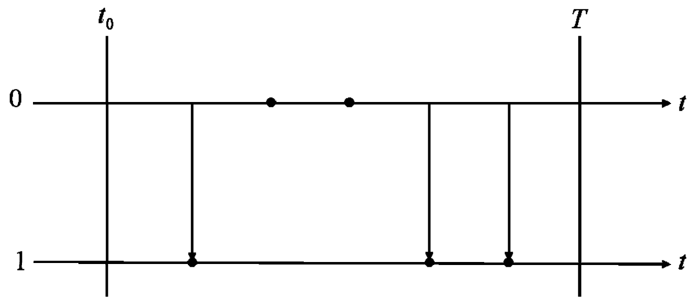

Fig. 1. Screening of the customers arrivals

We introduce the function (dynamic probability) $S(t)$ that satisfies the condition $0 \le S(t) \le 1$. The incoming flow event can be screened on the axis 1 with probability $S(t)$ and not screened with probability $1 - S(t)$. Let the system be empty at moment t_0, and let us fix some arbitrary moment T in the future. $S(t)$ represents the probability that a customer arriving at the time t will be serviced in the system by moment T. It is easy to show that $S(t) = 1 - B(T - t)$ for $t_0 \le t \le T$.

Denote by $W_i(t)$ the total amount of i-th type resource screened on axis i. It easy to prove that

$$P\{\mathbf{V}(t) < \mathbf{x}\} = P\{\mathbf{W}(t) < \mathbf{x}\}, \qquad (1)$$

for all $\mathbf{x} = \{x_1, \ldots, x_n\}$, where the inequalities $\mathbf{V}(T) < \mathbf{x}$ and $\mathbf{W}(T) < \mathbf{x}$ mean that $V_1(T) < x_1, \ldots, V_n(T) < x_n$ and $W_1(T) < x_1, \ldots, W_n(T) < x_n$, respectively. We use (1) to investigate the process $\{\mathbf{V}(t)\}$ via the analysis of the process $\{\mathbf{W}(t)\}$.

Integro-Differential Equations

Let us consider the $n+1$ – dimensional Markovian process $\{z(t), \mathbf{W}(t)\}$, where $z(t)$ is the residual time from t to the next arrival. Denoting the probability distribution of this process by

$$P\{z(t) < z, \mathbf{W}(t) < \mathbf{w}\} = P(z, \mathbf{w}, t)$$

and taking into account the formula of total probability, we can write the following system of Kolmogorov integro-differential equations

$$\frac{\partial P(z,\mathbf{w},t)}{\partial t} = \frac{\partial P(z,\mathbf{w},t)}{\partial z} + \frac{\partial P(0,\mathbf{w},t)}{\partial z}\left(A(z)-1\right) +$$

$$+ A(z)S(t)\left[\int_0^{w_1}\ldots\int_0^{w_n}\frac{\partial P(0,\mathbf{w}-\mathbf{y},t)}{\partial z}dG_n(y_n)\ldots dG_1(y_1) - \frac{\partial P(0,\mathbf{w},t)}{\partial z}\right],$$

where $\mathbf{y} = \{y_1, \ldots y_n\}$, with the initial conditions

$$P(z,\mathbf{w},t_0) = \begin{cases} R(z), & \mathbf{w} = \mathbf{0}, \\ 0, & \text{otherwise,} \end{cases}$$

where $R(z)$ denotes the stationary probability distribution of the random variable, which is determined by equality

$$R(z) = \lambda\int_0^z (1 - A(x))dx,$$

where
$$\lambda = \frac{1}{\int_0^\infty (1 - A(x))dx}.$$

We introduce the partial characteristic function

$$h(z,\mathbf{v},t) = \int_0^\infty e^{jv_1w_1}\ldots\int_0^\infty e^{jv_nw_n}P(z,d\mathbf{w},t),$$

where $j = \sqrt{-1}$ is the imaginary unit. Then, we can write

$$\frac{\partial h(z,\mathbf{v},t)}{\partial t} = \frac{\partial h(0,\mathbf{v},t)}{\partial z}\left[A(z) - 1 + A(z)S(t)(G^*(\mathbf{v}) - 1)\right] + \frac{\partial h(z,\mathbf{v},t)}{\partial z}, \quad (2)$$

where

$$G^*(\mathbf{v}) = \int_0^\infty e^{jv_1 y_1} dG_1(y_1) \dots \int_0^\infty e^{jv_n y_n} dG_n(y_n),$$

with the initial condition

$$h(z, \mathbf{v}, t_0) = R(z). \tag{3}$$

Gaussian Approximation

In general, the exact solution of equation (2) is not available, but it may be found under asymptotic conditions. In this paper, we consider the case of infinitely growing arrival rate. Let us write the distribution function of the interarrival times as $A(Nz)$, where N is some parameter used for the asymptotic analysis ($N \to \infty$ in theoretical analysis [4, 5]).

Then, the equation (2) takes the form

$$\frac{1}{N}\frac{\partial h(z,\mathbf{v},t)}{\partial t} = \frac{\partial h(z,\mathbf{v},t)}{\partial z} + \frac{\partial h(0,\mathbf{v},t)}{\partial z}\Big[A(z) - 1 + A(z)S(t)\big(G^*(\mathbf{v}) - 1\big)\Big].$$

Theorem. Asymptotic joint stationary n-dimensional probability distribution of the total resource amount in the system GI$^{(v)}$/GI/∞ is asymptotically n-dimensional Gaussian with mean:

$$\mathbf{a} = N\lambda\Big[a_1^{(1)} \quad a_1^{(2)} \quad \cdots \quad a_1^{(n)}\Big]b, \; b = \int_0^\infty (1 - B(\tau))d\tau,$$

and covariance matrix:

$$\mathbf{K} = N\big(\lambda\mathbf{K}^{(1)}b + \kappa\mathbf{K}^{(2)}\beta\big),$$

where

$$\mathbf{K}^{(1)} = \begin{bmatrix} a_2^{(1)} & 0 & \dots & 0 \\ 0 & a_2^{(2)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_2^{(n)} \end{bmatrix}, \quad \mathbf{K}^{(2)} = \begin{bmatrix} \big(a_1^{(1)}\big)^2 & a_1^{(1)}a_1^{(2)} & \dots & a_1^{(1)}a_1^{(n)} \\ a_1^{(2)}a_1^{(1)} & \big(a_1^{(2)}\big)^2 & \dots & a_1^{(2)}a_1^{(n)} \\ \dots & \dots & \dots & \dots \\ a_1^{(n)}a_1^{(1)} & a_1^{(n)}a_1^{(2)} & \dots & \big(a_1^{(n)}\big)^2 \end{bmatrix},$$

$$\beta = \int_0^\infty (1 - B(\tau))^2 d\tau.$$

Numerical Example

We assume that the input renewal process is characterized by the following distribution function

$$A(z)=\begin{cases}0, & z<0.5,\\ z-0.5, & z\in[0.5,1.5],\\ 1, & z>1.5.\end{cases}$$

Hence, the fundamental rate of arrivals is $\lambda=1$ customers per time unit. Moreover, each arriving customer occupies 2 types of resources and the corresponding customer capacities have uniform distribution in the range [0; 1] and [0; 2], respectively. Service time has gamma distribution with parameters $\alpha=\beta=0.5$ and so the fundamental rate of arrivals is *N* times the service rate.

In the Table 1 report the values of the Kolmogorov distance for bidimensional distributions for the two types of resource, highlighting that the goodness of the approximation depends not only on *N*, but also on the different statistical features of the considered types of customer.

Table 1

N	1	3	5	7	10	20	50	100
Δ	0.304	0.085	0.04	0.026	0.019	0.013	0.008	0.006

By Fig. 2 compared the asymptotic approximations with the empirical results for the total resource amount of each type for two different values of *N*.

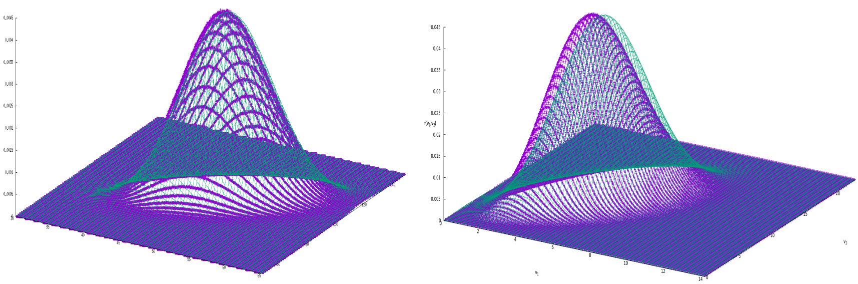

Fig. 2. Distributions of the total resource amount first and second type

Conclusions

In this paper we presented the analysis of Multi-resource $GI^{(v)}/GI/\infty$ queueing system with renewal arrival process and arbirtary service time. We applied dinamic screening method to obtain asymptotic expression for the stationary probability distribution of the process describing the total volume of the occupied resource in the system. We showed that the *n*-dimensional probability distribution of the total resource amount is asymptotically *n*-dimensional Gaussian. Numerical experiments and simulations allow us to determine the applicability area of the asymptotic result for different classes of users.

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