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A transmission expansion model for dynamic operation of flexible demand

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ABSTRACT

This paper proposes a model to include investments in demand flexibility into traditional transmission expansion problems under uncertainty. To do so, a dynamic power flow model is proposed. The model is solved via applying a value function approximation in form of a neural network on the operational problem, allowing to yield a result for the non-convex investment problem. Additionally, robust sets are applied and linearized to deal with uncertainty and decrease computational complexity. In similar manner, Karush Kuhn Tucker conditions are used to transform a tri-level into a bi-level problem. Case studies for systems of varying complexity show the convergence of the algorithm as well as that flexible resources can be used as a cost-effective substitute for transmission lines in grid expansion.

1. Introduction

In its essence, transmission expansion planning problems provide an overlap between long-term investment problems and short-term operational problems. This is a result of that such decisions have to accommodate not only for long-term financial impact and potential future system changes, but also have to consider short-term goals such as n-1 criteria and current day network topologies. Transmission expansion in electrical power systems has been established as a distinct family of problems approaching this balance [1]. To ensure operational stability whilst considering potential detrimental developments in the systems, focus on methods to prepare for the worst possible scenarios has led to robust optimization techniques providing the state-of-the-art [2].

Advances in real-time operation and renewable forms of generation impose additional challenge on current day transmission expansion. Those challenges include consideration of increases in wind power penetration [3], implementation of intelligent transmission systems by modeling line switching [4] and balancing of short-term uncertainty such as renewable generation with long-term uncertainty such as future demand peaks [5].

Further, approaches to accommodate for electrical power markets [6] and electricity storage [7] can be found in literature.

Traditionally, transmission expansion problems deal with an upper-level problem consisting of binary investment decisions and the lower-level problem of operational feasibility of each investment. Such Expansion models and the resulting real world implementation projects often favor line investments to non-transmission alternatives [8,9]. As a result of this, modern electrical power markets show a similar bias towards grid capacity enhancements [10].

Nonetheless, elastic demand has been considered in computationally demanding transmission expansion problems such as AC power flow problems [11] or problems under renewable generation and storage [12].

Demand flexibility is defined as shifting eligible loads across the hours of a day to off-peak periods, reshaping the daily load curve in order to reduce generation cost [13]. In other words, demand flexibility is an effect similar to price elasticity, that is connected dynamically, i.e. over subsequent time periods [14].

As such, demand flexibility can be considered a form of demand response referred to as 'deferrable demand response' [15]. The difference to traditional formulation of demand response which can often be seen as increased elasticity of the demand curves is that flexible demand is consumed instead of shed, but can be postponed to a later time period. This dynamic, flexible operation is often provided by household and end consumer devices such as water boilers, heating and lighting. On a small scale, such effects of dynamically postponing such consumption through flexible devices or loads through storage has proven itself a valid tool to approach traditional grid problems [16]. Even under competition, installation of such devices has shown beneficial financial effects on both consumers and retailers in the grid [17].

Furthermore, entire national systems have been analyzed for the potential prospects of a system-wide introduction of 'smart' appliances allowing for such demand flexibility. For example analyzed Ref. [18] future projections of device installation in Great Britain, showing a significant potential impact of these devices on the national grid. The paper concludes in that regulation, market design and the need of additional control schemes present the largest hurdles in future utilization of such flexibility resources.

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Nomenclature

Index

Z	transmission line indicator
Y	demand flexibility indicator
S	load shedding indicator
g	generation unit
n, n_2	system nodes
ref	reference node
t, t_1, t_2	time period
k	cut

Variables

z	transmission investment decision [binary]
x	generated quantity [MWh]
x^S	shed load [MWh]
δ	voltage angle [°]
y	demand flexibility investment decision [binary]
s	demand shift into a subsequent period [MWh]
η	operational problem lower bound [€]

Dual Values

γ^b, γ^f, γ^g, γ^h	equality constraint
$\underline{\mu}^e$	inequality constraint lower bound
$\bar{\mu}^c$, $\bar{\mu}^d$, $\bar{\mu}^e$, $\bar{\mu}^i$	inequality constraint upper bound

Parameters

ξ	uncertainty
$d(\xi)$	nodal demand [MWh]
B	transmission line susceptance [siemens]
$\bar{F}$	line flow capacity [MW]
$\underline{x}(\xi)$, $\bar{x}(\xi)$	generation capacities [MW]
$\bar{t}$	demand flexibility period capacity [h]
$\bar{s}(\xi)$	demand flexibility quantity capacity [MWh]
Γ	uncertainty budget [%]
θ	approximation parameters (neural network weights) $\mathbb{R}$

Functions

C^I	investment cost function [€]
C^O	operations cost function [€]
c	generation cost function [€]
p^S	price of unfulfilled demand [€/MWh]
ϕ	value function approximation [€]

Sets/Vectors

N^Z	transmission line starting nodes
N_2^Z	transmission line ending nodes
N^Y	demand flexible nodes
$N^{Z,I} \subseteq N^Z$, $N_2^{Z,I} \subseteq N_2^Z$	transmission line investment nodes
$N^{Y,I} \subseteq N^Y$, $N_2^{Y,I} \subseteq N_2^Y$	demand flexibility investment nodes
$N = N^Z \cup N^Y$	system nodes
G	system generation units
T	considered time periods
N^{zone}	nodes gathered in a zone
K	cuts

Robust Set

d, $\underline{x}$, $\bar{x}$, $\bar{s}$	replacement variables for uncertain parameters [MWh], [MW]
d^{ref}, $\underline{x}^{\text{ref}}$, $\bar{x}^{\text{ref}}$, $\bar{s}^{\text{ref}}$	reference points [MWh],[MW]
d^{Δ}, $\underline{x}^{\Delta}$, $\bar{x}^{\Delta}$, $\bar{s}^{\Delta}$	fluctuation ranges [MWh],[MW]

As shown, most of the studies focus on a holistic perspective, mainly based on market structure and the technical pitfalls of large scale implementation, whilst leaving small scale operational effects out of the analysis [19]. An example is given in the analysis provided by the two-part paper shown in Refs. [20,21], where the first part formulates a market clearing approach for the non-convex problem of adequately remunerating flexible demand and the second part demonstrates how to apply such on electric vehicles. Even though the papers present a well-performing market model in itself, the issue is not only the legal hurdles that individual countries would provide in practical implementation of such concepts but also the lack of grid-specific implications. Another example is provided by Ref. [14] which formulates a dynamic demand elasticity model to test various market models, focusing on the pricing scheme aspect.

The market-centric view itself is inherited from the implementation hurdles related to classical demand response systems which mostly analyze the control-problem of large scale control of the individual end consumer devices [22] and is similarly applied on studies related to demand flexibility such as optimal charging of electric vehicles [23].

Expansion planning of demand response in order to capture uncertainty from wind power generation has been analyzed in Ref. [24]. The paper utilizes a linearized AC power flow approach to model the grid. A similar focus on modeling uncertainty under renewable generation is provided by Ref. [25] which presents a multi-objective, probabilistic transmission expansion planning problem.

Comparison studies of demand flexibility and transmission line investments have been conducted and showed indication that investments in flexible demand could support dealing with congestion issues. For examples uses Ref. [26] a deterministic investment model in its evaluation, concluding that flexible in the power system might support resolving congestion issues. In addition, demand flexibility as a tool to reduce transmission via capacity market participation has been introduced in Ref. [27].

Investments in flexible demand as an alternative for transmission investments has been analyzed in Ref. [28]. The paper analyzes the long term effects of shiftable loads. However, only long-term effects of installing such shiftable loads are included in the model. However, similar to other literature the paper does not display the operational/short-term effects of the dynamic nature of shiftable loads (i.e. optimally utilizing shifts between smaller periods such as hours). Analysis of such decisions is in real-world examples implemented via combining several model types [29], which weakens the direct comparison of such solutions as short-term effects are a core factor in profitability of flexible demand [30,31].

The reason for approximating these short-term operations in order to make long-term investment decisions can be found in the computational complexity of short-term dynamic models. Ref. [32] approaches this investment in flexible demand from a profit-making generators perspective. Nonetheless, operational decisions as part of the investment analysis of a grid-expanding agent is not included in the proposed model.

This is the issue that the here presented work attempts to solve.

2. Contributions of this paper

The concept proposed in this paper extends the traditional robust optimization problem as presented in e.g. Refs. [2,28] to multiple, dynamic periods. The novelty of the proposed model lies in that the value of investment is considered from a purely operational perspective instead of utilizing market-based concepts such as e.g. flexibility contracts [33]. Focusing purely on operations allows for a leaner problem formulation that reduces modeling complexity compared to market-based alternatives presented in literature [14,34,21,35]. In turn, this allows considering transmission grid-topology and dynamic power flows, allowing to compare long-term grid investments directly to the dynamic short-term operation of flexibility investments, a first in literature.

However, the resulting dynamic operations and transmission expansion problem creates computational hurdles [36]. To deal with increasing search spaces, heuristics based on non-linear function approximations was proposed [37] and successfully applied in literature [38].

Recent advances in computational resources and associated efficiencies has led to an increase of application of such tools in fields such as image and speech recognition [39]. In similar manner, this paper proposes an algorithm that utilizes neural networks in order to detect the 'most beneficial investments' in order to allow heuristics as an alternative for application in dynamic problems that exceed the limits of computational resources under traditional robust optimization techniques [40]. Thus, the contributions can be summarized as:

- establish a dynamic power flow model for flexible resources that combines short-term dynamic operational decisions with long-term investments. This is a first in literature, allowing to trace the long-term effects of investing in flexible demand and storage directly with transmission line investments under uncertainty.
- propose a value function approximation of the operations problem in order to efficiently solve the investment problem. This allows for efficiency gains over traditional solution methods for robust optimization problems such as the later discussed 'Column-and-Constraint-Generation'.
- demonstrate the importance of such a model framework by utilizing dynamic extensions of well-established case-studies to show that investments in flexible demand can be a cost-effective alternative to grid investments. This demonstrates the capabilities of the proposed solution method in such and similar dynamic current day problems (e.g. problems under storage).

The proposed concept of the value function approximation builds on a suggestion raised by Ref. [40]: enhancing solution capabilities in robust problems via detecting 'significant contributors' (i.e. investments contributing most to cost reduction). In comparison to previous 'dynamic transmission expansion problems' such as Ref. [36], the here presented focus of the dynamic problem is in the operations level, i.e. the inner problem. This allows for inclusion of demand flexibility in form of load-shifts but also other temporal factors such as storage, that previous literature did not allow for.

3. Transmission expansion under uncertainty

Assumed be a number of transmission lines connecting the busses/nodes in vector N^Z with the busses/nodes in vector N_2^Z. A number of those transmission lines between $N^{Z,I} \subseteq N^Z$ and $N_2^{Z,I} \subseteq N_2^Z$ can only be utilized after a one-time investment with cost c^Z.

The optimal (i.e. minimal) cost of such an expansion investment problem is thus as a solution to a Mixed Integer Problem (MIP) in the form of:

$$\begin{aligned} C^I(\xi) = \min_z C^O\left(\xi, z\right) + \sum_{n\in N^{Z,I}} \sum_{n_2\in N_2^{Z,I}} c^Z_{n,n_2}(z_{n,n_2}) \\ \text{s. t. } [z_{n,n_2} = z_{n_2,n}] \in [0,1] \quad \forall\, n \in N^{Z,I},\, n_2 \in N_2^{Z,I} \\ z_{n,n_2} = 1 \quad \forall\, n \in N^Z \backslash N^{Z,I},\, n_2 \in N_2^Z \backslash N_2^{Z,I} \end{aligned} \tag{1}$$

Solving this upper-level problem requires a solution to the lower-level problem defining the optimal operational cost C^O considering uncertainty.

It has to be noted that this uncertainty still implicates the optimal investment decision and dealing with such will be attempted in a later section of this paper. Until then, it will be assumed that uncertainty ξ comes in form of a single known scenario.

The lower-level problem is the DC optimal power flow representation of Ref. [2]:

$$C^O\left(\xi, z\right) = \min_{x,\delta} \sum_{g\in G} c_g(x_g) + \sum_{n\in N} p^S(x_n^S) \tag{2a}$$

$$\text{s. t.} \sum_{g\in G_n} x_g - d_n(\xi) - \sum_{n_2\in N_2^Z} [z_{n,n_2} B_{n,n_2}(\delta_n - \delta_{n_2})] + x_n^S = 0 \;\forall\, n \in N \tag{2b}$$

$$0 \leqslant x_n^S \leqslant d_n(\xi) \;\forall\, n \in N \tag{2c}$$

$$z_{n,n_2} B_{n,n_2}(\delta_n - \delta_{n_2}) \leqslant \overline{F}_{n,n_2} \;\forall\, n \in N^Z,\, n_2 \in N_2^Z \tag{2d}$$

$$\underline{x}_{t,g}(\xi) \leqslant x_g \leqslant \bar{x}_{t,g}(\xi) \;\forall\, g \in G \tag{2e}$$

$$\delta_{n_{\text{ref}}} = 0 \tag{2f}$$

$$x_n^S \geqslant 0 \;\forall\, n \in N$$

Objective function (2a) is the minimization of the cost of fulfilling the demand. It contains the generation cost functions of all generation units in the system and the price of not meeting the demand and thus shedding the load. Decisions are generation, shed load and the voltage angles. Shed load is included to ensure full recourse for the upper-level problem. In order to have generation preferred over load shedding, constant p^S should have a high value assigned.

Equality Constraint (2b) is the implementation of Kirchhoff's law, that requires the power balance (i.e. generation - demand + nodal balance) to be equal to 0 at every bus/node. The constraint is a DC-approximation of the physical (AC-) reality as it only considers the line susceptance whilst ignoring line resistances. Formulating this operations problem as such an approximation is not unique to the work presented here. Instead, it presents the norm in traditional transmission expansion problems [41,42]. The reason is that the demanding non-convex investment problem requires a simplification in the operations problem, as the nonlinearities of the AC power flow would otherwise not allow for the optimal search in the space of (i.e. the comparison of) potential investments [43].

In addition, demand is assumed to be subject to uncertainty.

Capacity constraint (2c) defines that the range of the maximum load shed has to be within the range of the demand.

Capacity constraint (2d) requires the flows on the transmission lines to stay within the given limits. It has to be noted that the set N considers both candidate lines as well as already existing transmission lines.

Capacity constraints (2e) define the generation limits within which the units operate. These limits are considered dependent on uncertainty in order to accommodate for renewable generation capacities.

Equality constraint (2f) defines the reference bus of the system (which could e.g. be the node with the largest generator).

4. Flexibility expansion under uncertainty

The investment problem for demand flexibility expansion is formulated similar to transmission flexibility problem (1). In this problem, a number of busses/nodes $N^{Y,I} \subseteq N^{Y}$ offer potential to expand their existing capacities for flexible demand:

$$
\begin{aligned}
C^{I}(\xi) = & \min_{y} C^{O}\left(\xi, y\right) + \sum_{n \in N^{Y,I}} c_n^{Y}(y_n) \\
\text{s. t.} \quad & y_n \in [0, 1] && \forall\, n \in N^{Y,I} \\
& y_n = 1 && \forall\, n \in N^{Y} \backslash N^{Y,I}
\end{aligned} \tag{3}
$$

It can be observed that the investment problem for transmission expansion and expansion in flexible demand show near similarity, with the exception of transmission lines being established between two busses/nodes n and n_2 and demand flexibility connecting a single such node n with itself inter-temporal. The investment cost in flexible demand c^{Y} is in general of smaller magnitude than the investment in transmission lines c^{Z} [13]. This is also represented in the case study presented below. Due to this, it can be reasonably assumed, that the number of potential investments will thus also be higher than in traditional transmission expansion models. Such lower cost and more potential locations (i.e. buses with connected consumers such as households) will thus increase the complexity for the investment problem, requiring efficient solution methods (such as proposed below) in order for the investment problem on this 'deferrable demand response'.

This mechanism is displayed in Fig. 1, which compares the two expansion problems. The connection over time periods that shifting flexible demands represents leads to different operational problems as well. This is due to the possibility of decreasing generation (cost) in one period by increasing the generation (cost) in an other period:

$$
C^{O}\left(\xi, y\right) = \min_{x, x^{S}, \delta, s} \sum_{t \in T} \sum_{g \in G} c_g(x_{t,g}) + \sum_{t \in T} \sum_{n \in N} p^{S}(x^{S}_{t,n}) \tag{4a}
$$

$$
\text{s. t.} \quad \begin{aligned}
& \sum_{g \in G_n} x_{t,g} - d_{t,n}(\xi) - \sum_{n_2 \in N_2^{Z}} [B_{n,n_2}(\delta_{t,n} - \delta_{t,n_2})] - \\
& \sum_{t_1 \in T} s_{t_1,t,n} + \sum_{t_2 \in T} s_{t,t_2,n} + x^{S}_{t,n} = 0
\end{aligned} \quad \begin{aligned} & \forall\, t \in T, \\ & n \in N \end{aligned} \tag{4b}
$$

$$
x^{S}_{t,n} + \sum_{t_2 \in T} s_{t,t_2,n} \leqslant d_{t,n}(\xi) \;\forall\, t \in T,\, n \in N \tag{4c}
$$

$$
B_{n,n_2}(\delta_{t,n} - \delta_{t,n_2}) \leqslant \overline{F}_{n,n_2} \;\forall\, t \in T,\, n \in N^{Z},\, n_2 \in N_2^{Z} \tag{4d}
$$

$$
\underline{x}_{t,g}(\xi) \leqslant x_{t,g} \leqslant \bar{x}_{t,g}(\xi) \;\forall\, t \in T,\, g \in G \tag{4e}
$$

$$
\delta_{t,n_{\mathrm{ref}}} = 0 \;\forall\, t \in T \tag{4f}
$$

$$
s_{t_1,t_2,n} = 0 \;\forall\, t_1 \geqslant t_2; t_1,\, t_2 \in T,\, n \in N \tag{4g}
$$

$$
s_{t_1,t_2,n} = 0 \;\forall\, (t_2 - t_1) > \bar{t}_n; t_1,\, t_2 \in T,\, n \in N \tag{4h}
$$

$$
\sum_{t_2 \in T} s_{t_1,t_2,n} \leqslant y_n \bar{s}_{t_1,n}(\xi) \;\forall\, t_1 \in T,\, n \in N \tag{4i}
$$

$$
x^{S}_{t_1,n},\, s_{t_1,t_2,n} \geqslant 0 \;\forall\, t_1,\, t_2 \in T,\, n \in N
$$

As annotated, objective function (4a) under demand flexibility has to consider the cost minimization of several consecutive periods.

Implementing demand shift from a period to the next is included in power balance equality constraint (4b). In this formulation, the power balance can be breached in a single period t. The shift of flexible demand is decomposed into its' origin periods t_1 and its' target periods t_2, similar to electricity storage as introduced in Ref. [44]. As such, the demand shift decision $s_{t_1,t_2,n}$ denotes the quantity moved from a period t_1 to a period t_2 within a node n.

Such a shift of demand can only happen if there is demand

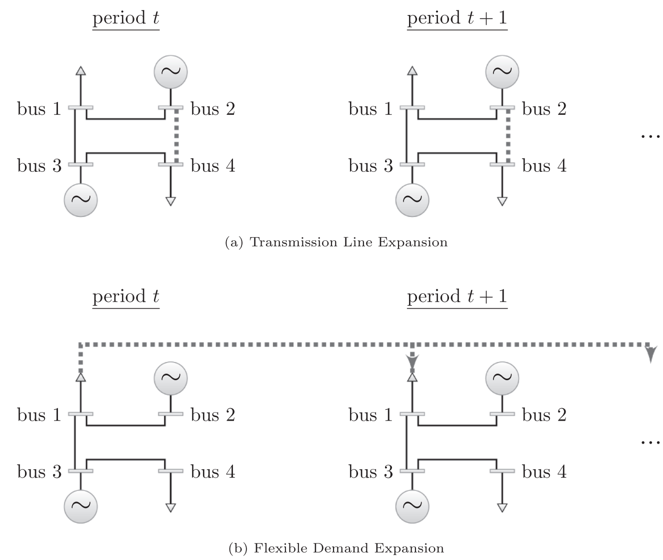


Fig. 1. Comparison of proposed Expansion Models.

available. This is implemented by the changes in constraint (4c).

Transmission capacity constraint (4d), generation capacity constraints (4e) and reference bus balance (4f) are implemented similar to the previous case. Only the additional dimension of time is considered and the transmission line investment decision z was removed in this case.

The remaining constraints (4g)–(4i) are the results of the decomposition into 'from' and 'to' periods:

Inter-temporality constraint (4g) ensures that flexible demand can only be shifted into future periods.

Period capacity constraint (4h) ensures that demand can only be shifted within a limited number of time periods instead of the entire model duration $t \in T$.

Quantity capacity constraint (4i) ensures that the total demand shift from a single period stays within a given capacity limit.

Assumed Quantity and time limits of demand shifts such as postponing running appliances or charging electric vehicles are considered subject to uncertainty. The reason is that those demand shifts are a result of uncertain consumer behavior.

5. Transmission and flexibility expansion model under uncertainty

Considering the two introduced models and their comparison in Fig. 1 allows to establish a trade-off model between both potential types of investment decision.

The formulation of the upper- and lower-level problems of this combined transmission and flexibility expansion model is a fusion of the previously established models and can be found in A.

On initial inspection, this model might resemble a transmission expansion problem under storage as solved in e.g. Ref. [7]. However, there exist fundamental differences in the utilization of flexible demand compared to storage units.

In addition to flexible demand not having to implement (often non-linear) charging efficiency curves and the order of load consumption/load provision being reversed[1], Fig. 2 highlights an additional difference in the dynamic decision making of flexible demand. This difference is that the periods where decisions on using the utility providing the flexible demand can be made are not continuous over an infinite time frame.

[1] With demand flexibility, the load is provided and later consumed, in storage, the load is consumed and provided later.

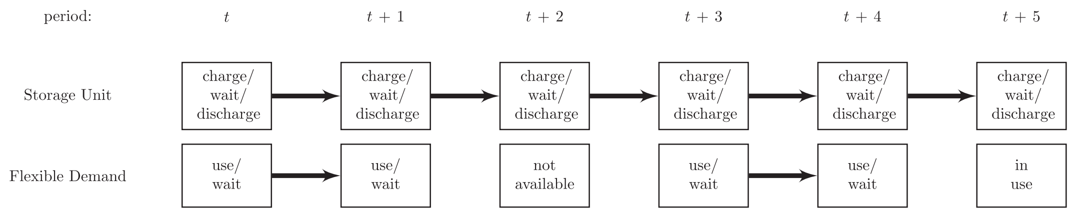


Fig. 2. Comparison: Energy Storage and Flexible Demand.

The reason is that storage units are considered constantly available in the system (except for maintenance) and accessible for receiving scheduling decisions. In contrast, flexible demand such as household appliances and electric vehicles cannot be considered consistently available, as there are mandatory down-times (e.g. a consumer not wanting to run their washing machine during nights) or times with mandatory usage (e.g. a consumer unplugging and using their electric vehicle).

In traditional systems under storage scheduling this, potentially infinite, connection in time leads to planning horizons of up to several years, where accurate modeling of the uncertainties become a crucial focus of the scheduling models [45].

In the here considered problem setup, though, the potential time frame is limited to the consumers willingness to shift demand to other time periods, which restricts demand flexibility models to time frames of a limited number of time frames such as several hours, with special focus on the transition between peak- and base-load periods [14].

6. Handling uncertainty

As hinted above and as described in Appendix A, accurate as well as compact representation of the uncertainty is crucial for solving the lower-level operational problem and thus the upper-level investment problem it is nested in.

Appendix A describes how separation into an upper- and lower-level problem allows to transform the decision problem such that the lower-level operational problem disregarding uncertainty comes in form of a convex problem.

In addition to this, the split in a two-level problem also enables another favorable trait of the expansion problem. The combined investment problem (A.1) shows how uncertainty only affects the lower-level problem.

Here be assumed that there exists a representative approximation of the operational problem:

$$\phi^O(y, z) \approx C^O(\xi, y, z) \tag{5}$$

How to yield this approximation will be explored in the subsequent section. For now it shall be assumed that such a value function approximation ϕ^O exists, making investment problem (A.1) an MIP. As discussed in Appendix A, such a problem scales moderately well regarding to system size as it does not consider the length of the considered time frame.

The remaining uncertainties can thus be found in the lower-level operational problem. To transform this problem into a robust formulation, uncertain parameters are transformed into decision variables of the lower-level problem:

$$d(\xi) \equiv d,\ \underline{x}(\xi) \equiv \underline{x},\ \bar{x}(\xi) \equiv \bar{x},\ \bar{s}(\xi) \equiv \bar{s} \tag{6}$$

In this formulation, the uncertain parameters are not considered to be disconnected to uncertainty. This approach aims to maximize regrets (i.e. the operational cost) by choosing an equivalent, robust solution [46]. To provide an example, the objective function $\min_x c(\xi, x)$ would be transformed into $\max_\xi \min_x c(x)$. This is the reason for that such robust optimization problems are also referred to as 'maximin' or 'max regret'-formulations.

The uncertainty set is described as proposed in Ref. [47]. In this formulation, a single uncertain variable (e.g. demand $d(\xi)$) is described by a reference point (e.g. d^{ref}) and a maximum range of deviation from this reference point (e.g. $+/-d^{\Delta}$). Instead of considering a distribution of the potential outcomes of the uncertain parameters the values of the parameters are selected to specifically maximize the regret, i.e. the operational cost C^O.

Extending the time-independent description provided in Ref. [47], in a single time period t a 'stack' of all variable representations of the uncertain parameters in the respective time period is considered in the uncertainty set representation.

The formulation of such a linear uncertainty set based on Ref. [2,48] is provided in Appendix C. Ref. [49] provides an adaptive reformulation of such uncertainty sets that increase performance through reduction of uncertainty space. Nonetheless, the here presented work will be focused on the traditional formulation, with this newer formulation offering a starting point for future research.

It allows to transfer uncertainty budget to different system nodes within a zone N^{zone} in order to create bottlenecks in critical parts of the zone. This allows to establish the worst case situation in parts of the system if there is considerable impact on the total cost of the system.

Having introduced the constraints of the problem allows to formulate the robust equivalent of the lower-level operations problem:

$$\begin{aligned}
\phi^O(y, z) \approx \max_{d,\underline{x},\bar{x},\bar{s}} & \sum_{t\in T} \sum_{g\in G} c_g(x_{t,g}) + \sum_{t\in T} \sum_{n\in N} p^S(x^S_{t,n}) \\
\text{s. t.} \quad & \text{Robust Set Definition (6)} \\
& \text{Robust Set Limits (C. 1)} \\
& \text{Stationary Conditions (B. 1)} \\
& \text{Primal Feasibility Conditions (A. 2b) – (A. 2h)} \\
& \text{Dual Feasibility Conditions (B. 2)} \\
& \text{Complementary Slackness Conditions (B. 3)}
\end{aligned} \tag{7}$$

As discussed above, applying the approximation of the robust operational cost (5) similarly on the upper-level expansion planning problem (A.1) allows to solve this problem as a MIP with a limited number of decisions considerably less than the range of potential solutions to the lower-level problem. Nonetheless, even for a limited range of investment decisions, problems in scalability might arise, that do not allow to solve for all possible permutations of the binary decisions. These complications are the reason for the value function ϕ^O being an approximation, i.e. the usage of the symbol ≈ instead of = in Eq. (7). However, in case study #1 the technique shows its capabilities to converge towards the global solution (which would in turn mean = instead of ≈ in Eq. (7)). As the goal of conducting this approximation, though, is to not cover the uncertainty space entirely however and the upper-level problem being non-convex, it cannot be entirely disregarded that the solution does not provide the global optimum. Nonetheless, the approximation will converge towards this solution, as Appendix D discusses in detail.

7. Handling the 'Curse of Dimensionality'

As presented above and compared to the storage problem, the here described flexible demand problem does not cover a potentially infinite time frame. Nonetheless, the here presented model still deals with a problem typically associated with the Bellman equation: the 'curse of dimensionality' introduced by the intertemporality of decisions.

In addition to this obstacle and as analyzed in Appendix B, the number of binary decisions increases beyond the size of a typical transmission expansion problem. This is amplified by the investment decisions in flexible demand being cheaper compared to transmission expansion, making the potential target nodes in the system more numerous and thus drastically increasing the problem size of the upper-level problem.

As a result, the current problem, even though showing no potentially infinite size as the case with typical Bellman equations, shows a larger number of binary permutations of the upper-level investment problem where each lower-level operations problem associated with a single permutation has a larger size itself.

In literature addressing robust optimization, and specifically in power system applications, a decomposition approach titled 'Column-and-Constraint-Generation' (CnCG) [40] provides a popular algorithm to yield upper-level solutions. A comparison to the proposed technique is provided in Appendix E. In addition, several other decomposition methods have been proposed [1].

Similar to other models from literature [2], the here proposed demand flexibility model relaxes the incompleteness of recourse by giving the possibility to alleviate high demands by allowing for load shedding in form of variable x^S. Disregarding the possibility of too high minimum generation limits allowing for incomplete recourse, the presented problem without feasibility cuts would still face a problem due to optimality cuts.

As described in Ref. [40], this problem is that of the CnCG algorithm increasing the problem dimension. As discussed, the problem size encountered with the here presented problem might already outweigh traditional examples. Thus, instead of using traditional decomposition algorithms, this paper proposes an alternative in form of a search heuristic. This heuristic is based on two assumptions, whereas assumption 1 has been discussed above:

1. The size of the problem does not allow for efficient decomposition alternatives.
2. There is an, albeit potentially non-linear or non-existent, influence of the individual expansion decisions on the operational cost.

Assumption 2 expresses that certain investment decisions might be on a spectrum between favorable or indifferent for the operational problem. In other words, adding additional transmission line or flexible demand capacities whose investment cost are not considered in the operational problem, will, for at least some of the investments, lead to a more or less beneficial outcome as without those capacities. This means, that a continuous function accordingly approximating ϕ^O could be utilized to determine the most beneficial investment decisions and thus transform the discontinuous upper-level investment problem into a continuous problem.

The proposed solution heuristic is the following:

0) **Initialize** $\phi^O(y, z)$, $Y^{\text{solved}} = \{\emptyset\}$, $Z^{\text{solved}} = \{\emptyset\}$
1) **Calculate**

$$\phi^I\left(y, z\right) = \phi^O\left(y, z\right) + \sum_{n \in N^{Y,I}} c_n^Y(y_n) + \sum_{n \in N^{Z,I}} \sum_{n_2 \in N_2^{Z,I}} c_{n,n_2}^Z(z_{n,n_2})\ \forall\, y, z$$

2) **Sample** $y^* \notin Y^{\text{solved}}$, $z^* \notin Z^{\text{solved}}$ that solves $\min_{y^*, z^*} \phi^I(y^*, z^*)$
3) **Solve** lower-level problem (7) for y^*, z^*
4) **Update** approximation $\phi^O(y, z)$ with the result of $\phi^O(y^*, z^*)$
5) **If** not converged and permutations left: back to 1)
else: finished

The algorithm can be considered a variation of value space approximation with one-step lookahead [50]. A proof of convergence is provided in Appendix D.

Compared with traditional CnCG techniques, this algorithm allows for a parallelization in step 2) and 3), where several lower-level problems can be sampled and solved at the same time and the results used to train the same function approximation.

The function approximation applied in the here presented case studies is introduced in Appendix F. Next and in order to assess the quality of the convergence algorithm, a small scale case study will be presented.

8. Case Study #1: 42 nodes

In order to validate the proposed search algorithm, an extended version of the Matpower IEEE 14 bus system over 3 time periods, i.e. 42 nodes (as shown in Fig. 1), is utilized [51]. The test case is extended by two line investment options, five demand flexibility options and considers three total time periods with variations in available generation, flexibility and demand subject to uncertainty. Thus, the total number of permutations for the investment decisions is $2^{2+5} = 128$. The lower-level problem for a single such permutation solved on average within 80 s on an *Intel i7-8850H @2.6Ghz* core[2] utilizing the non-commercial solver 'IPOPT' [52], allowing to yield all potential operational solutions in order to validate the capabilities of the search algorithm to converge towards and find the global solution. This is done by brute-forcing, i.e. calculating all possible iterations for z and y. The lower-level problem being convex and every outcome of the objective of the upper-level problem being known thus ensures that the algorithm is validated against the global optimum in this 14-bus case.

This is shown in Fig. 3, which displays the sum of operational and investment cost. In addition, it shows the trajectory of two example runs of the algorithm with two different starting points, that both find the global optimum within 6 and 11 iterations respectively. The figure also shows how investments in flexible capacity outperform investment in transmission lines, a topic that will be analyzed in the larger case study presented below.

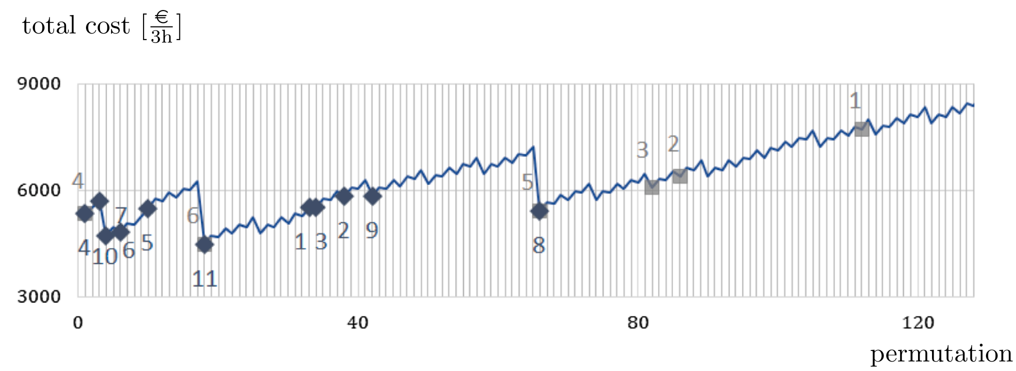
Fig. 4 shows the convergence of the upper-level investment problem of the case study for the network shown in Appendix F. As the algorithm continues searching for the next best solution after discovering a minimum, it can be used to discover the second-best, third-best and subsequent optimal investment decisions. Each of the here presented trial runs found the globally optimal investment decision within 10% of the potential permutations calculated, decreasing the total solution time from 2:48 h to 0:16 h when applying the value function search algorithm as proposed. In addition, the method converged with 2% fewer iterations as CnCG and also required 25% less time per iteration compared to CnCG.

The presented technique thus attempts to find and quantify the most beneficial investments in order to solve those first. Similar data analysis in transmission problems has been established e.g. in order to detect transmission line outliers in big data sets [53]. Nonetheless, no techniques have been proposed on using such techniques in scenario selection to increase the speed of robust optimization.

In addition, it has to be noted that the here presented method can be considered a compliment to established techniques instead of a replacement. For example, and as described in Ref. [40], a detection algorithm as the here presented method could be utilized combined with traditional methods in order to enhance the convergence speed of CnCG itself, but was omitted from this paper in order to demonstrate the capabilities of the presented algorithm on its own. Nonetheless, utilizing similar value-function approximations in such methods designed to solve robust optimization problems can be suggested as an important starting point for future research.

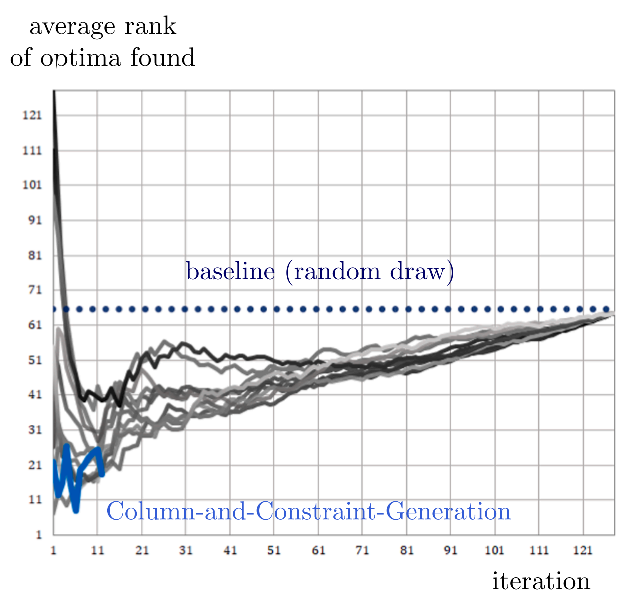
Below, the method will be proven on a larger test case.

[2] With similar results on another househould computer with an Intel i5-9300H @2.40 GHz core.



Rank	Line 1	Line 2	Flex 1	Flex 2	Flex 3	Flex 4	Flex 5
1	0	0	1	0	0	0	0
2	0	0	1	0	0	1	0
3	0	0	0	0	0	1	0
4	0	0	1	0	0	0	1
5.5	0	0	1	0	1	0	0
5.5	0	0	1	1	0	0	0
Inv. Cost	54.7 M€	32.8 M€	8.5 M€	4 M€	4 M€	2.6 M€	3.3 M€
Inv. Period	25 yrs	20 yrs	15 yrs	15 yrs	15 yrs	15 yrs	15 yrs

Fig. 3. IEEE 14 Total Cost Curve.



Minima found	Average number of iterations
1.	9.45
1-2.	23.54
1-3.	27.36
1-4.	31.45
1-6.	63.55

Fig. 4. IEEE 14 Bus Convergence (various test runs).

9. Case Study #2: 360 nodes

In order to test the limitations of the proposed solution approach, a medium size problem as presented in Fig. 5 and based on the Matpower IEEE 30 bus system is utilized [51]. As presented in Fig. 1 and described above, the considered time periods of the problem are interconnected, thus leading to a 30 bus system over 12 periods (i.e. $30 \times 12 = 360$ connected buses) showing a greater computational challenge than a traditional medium size problem in form of the Matpower IEEE 300 bus system solved non-dynamically (i.e. $300 \times 1 = 300$ connected buses) with additional complexity in form of e.g. dual variables for state equations. The investment cost and the supply/demand patterns with the potential ranges of the uncertainty are shown in Fig. 6. This figure also highlights the potential lack of generation capacity within the first periods.

The amount of investment permutations is $2^9 = 512$ with an average runtime of 1 h each. As in the previous case study the algorithm showed to find the global minimum with 7.5% of the permutations, the chosen number of permutations were 52 (i.e. 10%). It has to be noted here, that this in no way suggests that the yielded solution is the global optimum, as the non-convex approximation algorithm only converges towards it. The resulting optimal investment decisions are displayed in Table 1. The results show that investments in flexible capacity in bus 5,7,8 and 21 are considered preferable to line investments (with the most optimal transmission line investment still requiring a combination with investments in flexibility capacities). This reinforces the underlying assumption stated in the introduction of this paper that investments in flexible demand capacities could provide cost-effective alternatives to transmission line investments. In addition it demonstrates the necessity of models as the one introduced in this paper.

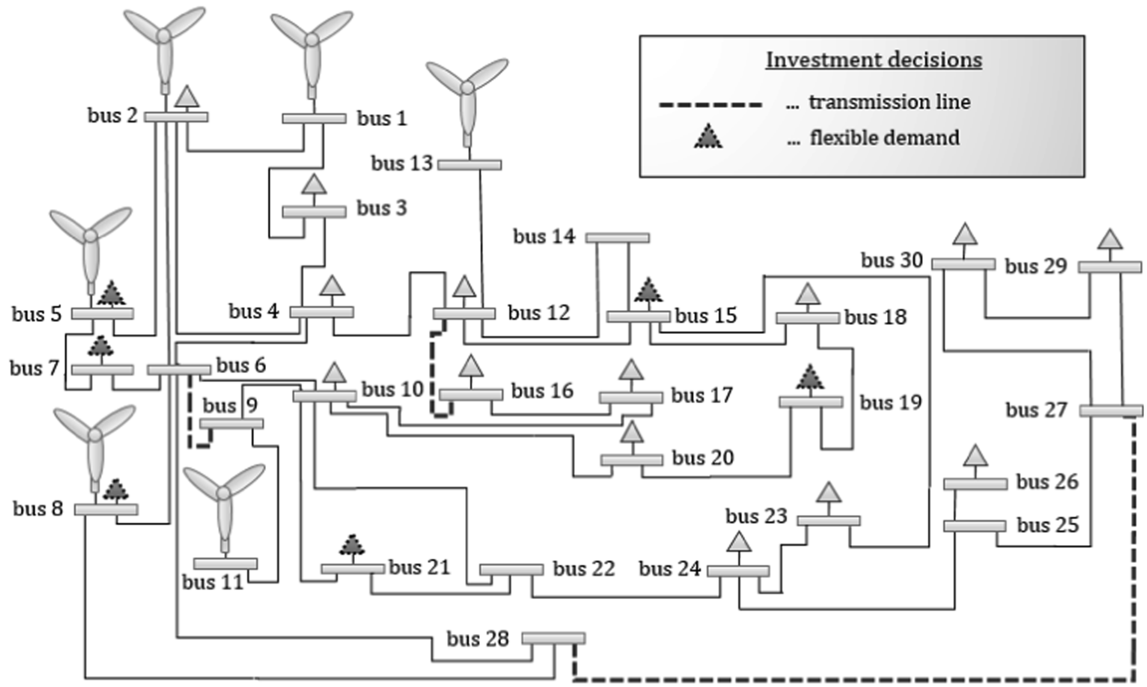


Fig. 5. Case Study #2 Network.

In contrast to this, and as illustrated in Fig. 7, adding such a transmission line reduces the loads on various transmission lines in the system. The figure compares the first of Table 1, i.e investing in flexibility in buses 5, 7 and 8 with the last line of Table 1, i.e. investing in a transmission line from bus 28 to 27 and flexibility in buses 19 and 21. Certain drastic changes in terms of average line utilization can be observed, for example is the transmission line from bus 22 to 24 nearly not utilized when choosing the transmission line investment, where in the optimal investment case it and its neighboring lines are highly utilized. This also shows that instead of adding another overly utilized line (being represented as the bottom right purple square in the right figure, with a utilization of 1 or -1 being congestion) installing flexible resources allows for distributing the utilization over the network.

This is also supported by the results for the uncertainty sets displayed in Fig. 8. The figure illustrates the amplitudes as a result of the lower-level optimization problem. This clearly shows how the robust optimization identifies bus 5 as a weakness, with the aim of creating a bottleneck there. As a result, the algorithm finds the optimal counter-strategy to this bottleneck in form of installing capacity at this bus or, as described in the second line in Table 1, in form of installing flexible capacity in its neighboring bus 7. Utilizing these installments do not only resolve the congestion problem but moreover also lead to a cheaper investment compared to reducing the strain on lines by installing the transmission line closest to bus 5.

As a result, this case study on this standard system demonstrates the viability of investments in flexible demand as a feasible and cost-effective alternative to transmission lines.

10. Conclusion

This paper proposes a novel dynamic power flow formulation in order to extend the traditional transmission expansion problem to flexible demand and thus to several time periods. The resulting master/subproblem consisting of an upper-level investment and a lower-level operations problem introduces a general issue of dynamic optimization to the model: the curse of dimensionality [50].

In order to approach this issue of limited scalability, uncertainty sets are utilized to enhance solution speed. In addition, various problem reformulations such as quadratic reformulation of complementarity constraints and linear reformulations of absolute values are applied. The result is a non-linear binary problem, where each of the permutations solves well on available non-commercial software. To increase solution speed and allow for approaching larger problems, a heuristic search based on a value-function approximation of the non-linear total

Investments		
Line 1 (6-9)	2 (12-16)	3 (28-27)
22 M€, 30 yrs	10 M€, 30 yrs	12.5 M€, 24 yrs
Flex. 1 (5)	2 (7)	3 (8)
949 k€, 10 yrs	163.5 k€, 7 yrs	255.5 k€, 7 yrs
Flex. 4 (15)	5 (19)	6 (21)
93.44 k€, 8 yrs	700.8 k€, 8 yrs	210.2 k€, 9 yrs

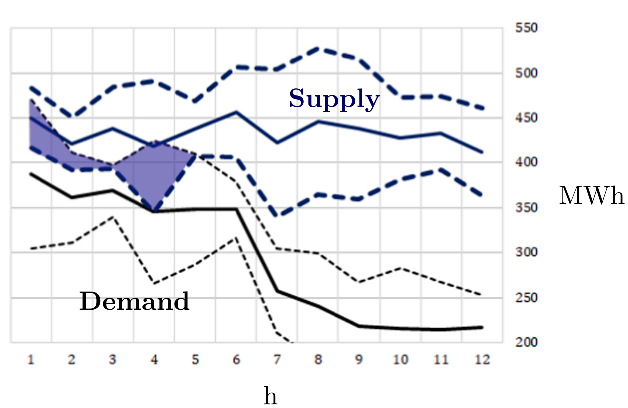


Fig. 6. Case Study #2 Data.

Table 1
Case Study # Total Cost 12 h period.

Line1	Line2	Line3	Flex1	Flex2	Flex3	Flex4	Flex5	Flex6	total cost [k€]
bus (6–9)	(12–16)	(28–27)	(5)	(7)	(8)	(15)	(19)	(21)	
0	0	0	1	1	1	0	0	0	9763.247
0	0	0	0	1	0	0	0	0	9905.228
0	0	0	0	1	0	0	0	1	9937.228
					…				
0	0	1	0	0	0	0	1	1	10710.16

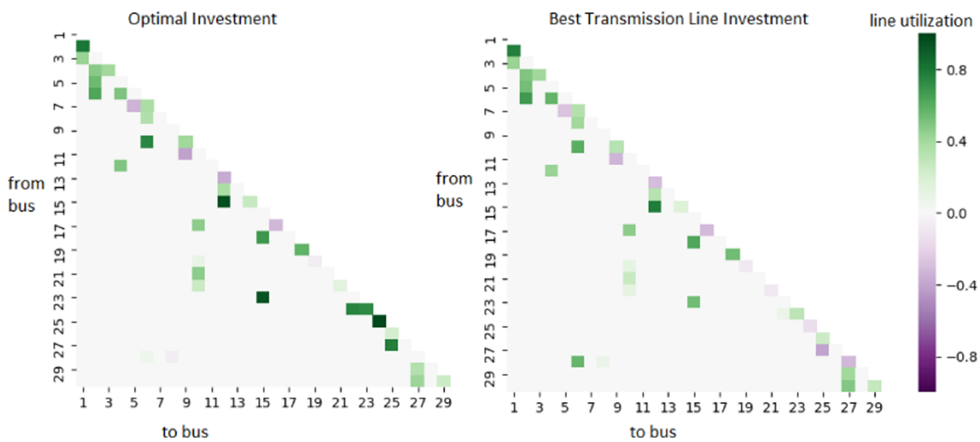


Fig. 7. Average Transmission Line Utilization Results.

cost curve containing operational and investment expenses is conducted. This method was applied on two standard test systems of small and medium size and showed a significant increase in solution speed.

The analyzed test cases not only highlight the efficiency gains of applying the framework, they also show the importance of utilizing such dynamic models. This comes as a result of that the investments in flexible demand capacities was considered financially advantageous to the investment in transmission lines. This might open up new possibilities for transmission system operators that can utilize this insight to lower system investment and operational cost simultaneously. In

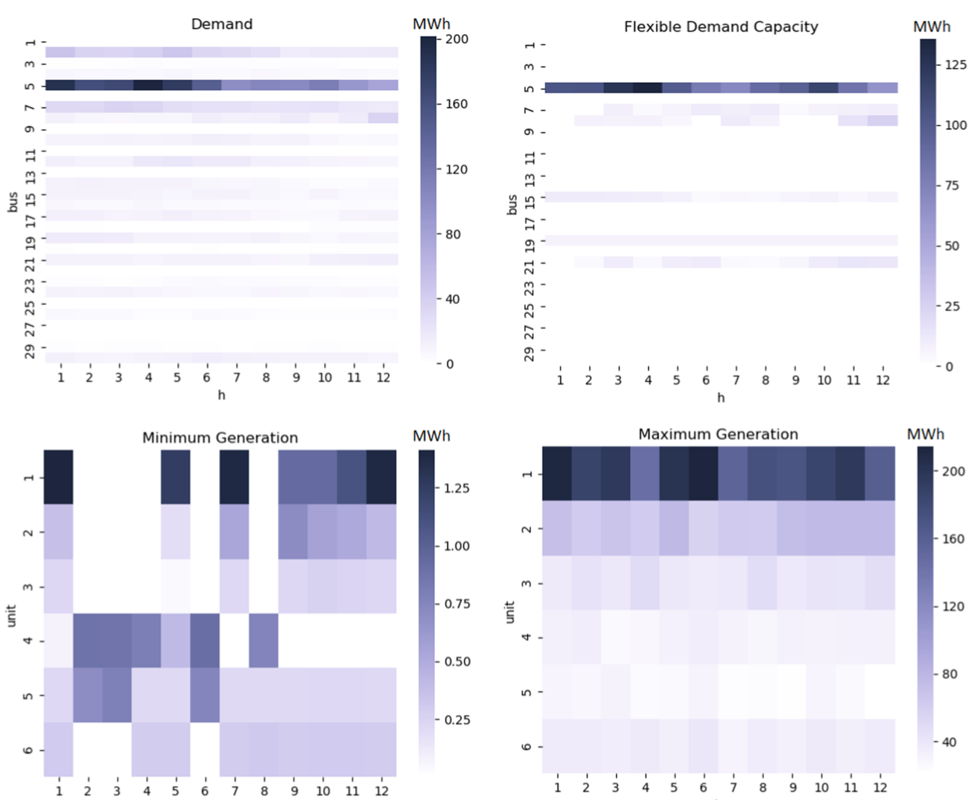


Fig. 8. Uncertainty Set Results (Optimal Investment Decisions).

traditional literature, this aspect was separated into two, potentially opposing, objectives.

In addition to this, the paper also highlights various starting points for future research such as extending the proposed model with different uncertainty set formulations or combining traditional solution techniques such as 'Column-and-Constraint-Generation' with the proposed approximation technique. Current advances in deriving such approximations via neural networks suggest the potential of future techniques to solve the here presented problem even more efficiently in the future, highlighting the potential of future techniques to solve similar robust optimization problems via methods built on the core structure of the heuristic outlined in this paper.

Furthermore, the proposed model provides applications beyond analysis of flexible demand. By reformulating the lower-level problem as Markov decision processes and applying related techniques such as e.g. discounting future values and adding required technical specifications such as charging efficiencies, the problem could similarly be applied on investment decisions in storage facilities. In addition, the problem could be reformulated to cope with various remuneration schemes for demand response via formulating different cost curves in the lower-level problem. This would connect the investment problem to tariff- and market models which provide the largest share of existing literature on flexible demand.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Combined optimization model

The upper-level problem of the transmission and demand flexibility expansion model can be formulated as:

$$\begin{aligned} C^I(\xi) = \min_z C^O\left(\xi, y, z\right) + \sum_{n \in N^{Y,I}} c_n^Y(y_n) + \sum_{n \in N^{Z,I}} \sum_{n_2 \in N_2^{Z,I}} c_{n,n_2}^Z(z_{n,n_2}) \\ \text{s. t.} \quad y_n \in [0,1] \quad \forall\, n \in N^{Y,I} \\ [z_{n,n_2} = z_{n_2,n}] \in [0,1] \quad \forall\, n \in N^{Z,I},\, n_2 \in N_2^{Z,I} \\ y_n = 1 \quad \forall\, n \in N^Y \backslash N^{Y,I} \\ z_{n,n_2} = 1 \quad \forall\, n \in N^Z \backslash N^{Z,I},\, n_2 \in N_2^Z \backslash N_2^{Z,I} \end{aligned} \tag{A.1}$$

Assumed the uncertainty is non-existent, i.e. the outcome of demand, capacity and available flexibility are fully known, makes this model a non-linear MIP model. Nonetheless, moderate scalability of this formulation can be assumed, as the investment decisions are not affected by the length of the chosen time period T. In addition, the maximum number of investment decision permutations are the case where no transmission lines or demand flexibility are already available: in this case the number of considered decisions is $2^{\text{card}(N^Y)+\text{card}(N^Z)\times\text{card}(N_2^Z)}$, where $\text{card}(\cdot)$ denotes the cardinality.

$$C^O\left(\xi, y, z\right) = \min_{x, x^S, \delta, s} \sum_{t \in T} \sum_{g \in G} c_g(x_{t,g}) + \sum_{t \in T} \sum_{n \in N} p^S(x_{t,n}^S) \tag{A.2a}$$

$$\text{s. t.} \quad \begin{aligned} \sum_{g \in G_n} x_{t,g} - d_{t,n}(\xi) - \sum_{n_2 \in N_2^Z} [z_{n,n_2} B_{n,n_2}(\delta_{t,n} - \delta_{t,n_2})] - \\ \sum_{t_1 \in T} s_{t_1,t,n} + \sum_{t_2 \in T} s_{t,t_2,n} + x_{t,n}^S = 0 \end{aligned} \quad \begin{aligned} \forall\, t \in T, \\ n \in N \end{aligned} (\gamma_{t,n}^b) \tag{A.2b}$$

$$x_{t,n}^S + \sum_{t_2 \in T} s_{t,t_2,n} \leqslant d_{t,n}(\xi) \ \forall\, t \in T,\, n \in N\, (\bar{\mu}_{t,n}^c) \tag{A.2c}$$

$$z_{n,n_2} B_{n,n_2}(\delta_{t,n} - \delta_{t,n_2}) \leqslant \overline{F}_{n,n_2} \ \forall\, t \in T,\, n \in N^Z,\, n_2 \in N_2^Z\, (\bar{\mu}_{t,n,n_2}^d) \tag{A.2d}$$

$$\underline{x}_{t,g}(\xi) \leqslant x_{t,g} \leqslant \bar{x}_{t,g}(\xi) \ \forall\, t \in T,\, g \in G\left(\underline{\mu}_{t,g}^e, \bar{\mu}_{t,g}^e\right) \tag{A.2e}$$

$$\delta_{t,n_{\text{ref}}} = 0 \ \forall\, t \in T\, (\gamma_t^f) \tag{A.2f}$$

$$s_{t_1,t_2,n} = 0 \ \forall\, t_1 \geqslant t_2; t_1,\, t_2 \in T,\, n \in N\, (\gamma_{t_1,t_2,n}^g) \tag{A.2g}$$

$$s_{t_1,t_2,n} = 0 \ \forall\, (t_2 - t_1) > \bar{t}_n; t_1,\, t_2 \in T,\, n \in N\, (\gamma_{t_1,t_2,n}^h) \tag{A.2h}$$

$$\sum_{t_2 \in T} s_{t_1,t_2,n} \leqslant y_n \bar{s}_{t_1,n}(\xi) \ \forall\, t_1 \in T,\, n \in N\, (\bar{\mu}_{t_1,n}^i) \tag{A.2i}$$

$$x_{t_1,n}^S,\, s_{t_1,t_2,n} \geqslant 0 \ \forall\, t_1,\, t_2 \in T,\, n \in N \tag{A.2j}$$

Even though Constraints (A.2b) and (A.2d) show investment decisions z being multiplied with the voltage angles δ, the separation of investment and operations problem makes this lower-level operations problem a convex problem.

Despite the computational simplicity, the scalability of this problem might be more limited due to the potential size of the problem. For an example, the susceptance matrix B, even though sparsly populated, has a size of $\text{card}(N^Z) \times \text{card}(N_2^Z)$ and is applied in every period t. Depending on the size of the considered system, a longer considered time frame might thus scale the size of this problem dramatically and affect scalability severely.

Appendix B. Karush Kuhn Tucker Conditions

Assuming convex cost functions means that Slater's constraint qualifications and thus strong duality holds for the lower-level problem (A.2) [54]. This makes the Karush Kuhn Tucker conditions necessary and sufficient.

Given the dual values assigned above, the stationary conditions of the lower-level problem can therefore be formulated the following:

$$\frac{\partial \mathscr{L}}{\partial x_{t,g}} = \frac{\partial c_g(x_{t,g})}{\partial x_{t,g}} + \gamma^b_{t,n} - \underline{\mu}^e_{t,g} + \bar{\mu}^e_{t,g} = 0 \forall\, t \in T,\, g \in G_n,\, n \in N \tag{B.1a}$$

$$\frac{\partial \mathscr{L}}{\partial x^S_{t,n}} = \frac{\partial p^S(x^S_{t,n})}{\partial x^S_{t,n}} + \gamma^b_{t,n} + \bar{\mu}^c_{t,n} = 0 \forall\, t \in T,\, n \in N \tag{B.1b}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial \delta_{t,n}} = &- \sum_{n_2 \in N_2^Z \setminus n} [z_{n,n_2} B_{n,n_2} \gamma^b_{t,n}] + \sum_{n_2 \in N^Z \setminus n} [z_{n_2,n} B_{n_2,n} \gamma^b_{t,n_2}] + \\ &\sum_{n_2 \in N_2^Z \setminus n} [z_{n,n_2} B_{n,n_2} \bar{\mu}^d_{t,n,n_2}] - \sum_{n_2 \in N^Z \setminus n} [z_{n_2,n} B_{n_2,n} \bar{\mu}^d_{t,n_2,n}] + \quad \forall\, t \in T,\, n \in N \\ &\begin{cases} \gamma^f_t & \text{if } n = n_{\text{ref}} \\ 0 & \text{else} \end{cases} = 0\end{aligned} \tag{B.1c}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial s_{t_1,t_2,n}} = &\gamma^b_{t_1,n} - \gamma^b_{t_2,n} + \bar{\mu}^c_{t_1,n} + \begin{cases} \gamma^g_{t_1,t_2,n} & \text{if } t_1 \geqslant t_2 \\ 0 & \text{else} \end{cases} + \\ &\begin{cases} \gamma^h_{t_1,t_2,n} & \text{if } (t_2 - t_1) > \bar{t}_n \\ 0 & \text{else} \end{cases} + \bar{\mu}^i_{t_1,n} = 0 \qquad \forall\, t_1,\, t_2 \in T,\, n \in N\end{aligned} \tag{B.1d}$$

Primal feasibility is given by the constraints of the lower-level problem provided by Eqs. (A.2b)–(A.2j). Dual feasibility is given by the bounds of the dual variables:

$$\begin{aligned} \bar{\mu}^c_{t,n},\, \bar{\mu}^i_{t,n} &\in \mathbb{R}^+ \forall\, t \in T,\, n \in N \\ \bar{\mu}^d_{t,n,n_2} &\in \mathbb{R}^+ \forall\, t \in T,\, n \in N,\, n \in N_2 \\ \underline{\mu}^e_{t,g},\, \bar{\mu}^e_{t,g} &\in \mathbb{R}^+ \forall\, t \in T,\, g \in G \\ \gamma^b_{t,n} &\in \mathbb{R} \forall\, t \in T,\, n \in N \\ \gamma^f_t &\in \mathbb{R} \forall\, t \in T \\ \gamma^g_{t_1,t_2,n},\, \gamma^h_{t_1,t_2,n} &\in \mathbb{R} \forall\, t_1,\, t_2 \in T,\, n \in N \end{aligned} \tag{B.2}$$

In addition, the complementary slackness of the inequality conditions has to be considered:

$$0 \leqslant \bar{\mu}^c_{t,n} \perp x^S_{t,n} + \sum_{t_2 \in T} s_{t,t_2,n} - d_{t,n}(\xi) \leqslant 0 \;\forall\, t \in T,\, n \in N \tag{B.3a}$$

$$0 \leqslant \bar{\mu}^d_{t,n,n_2} \perp z_{n,n_2} B_{n,n_2} (\delta_{t,n} - \delta_{t,n_2}) - \overline{F}_{n,n_2} \leqslant 0 \;\forall\, t \in T,\, n \in N^Z,\, n_2 \in N_2^Z \tag{B.3b}$$

$$0 \leqslant \underline{\mu}^e_{t,g} \perp \underline{x}_{t,g}(\xi) - x_{t,g} \leqslant 0 \;\forall\, t \in T,\, g \in G \tag{B.3c}$$

$$0 \leqslant \bar{\mu}^e_{t,g} \perp x_{t,g} - \bar{x}_{t,g}(\xi) \leqslant 0 \;\forall\, t \in T,\, g \in G \tag{B.3d}$$

$$0 \leqslant \bar{\mu}^i_{t_1,n} \perp \sum_{t_2 \in T} s_{t_1,t_2,n} - y_n \bar{s}_{t_1,n}(\xi) \leqslant 0 \;\forall\, t_1 \in T,\, n \in N \tag{B.3e}$$

The approach here chosen to deal with the complementarity constraint denoted by $\perp$ is that of a quadratic transformation [55]. Assumed be a complementarity constraint in the following form:

$$0 \leqslant \mu \perp g(x) \leqslant 0 \tag{B.4}$$

The transformation is a reformulation of this constraint that adds a variable u:

$$\begin{aligned} &g(x)u \leqslant 0 \\ &\mu g(x) \leqslant 0 \\ &\mu,\, u \in \mathbb{R}^+ \end{aligned} \tag{B.5}$$

In literature such as Ref. [40], typically the 'Fortuny-Amat notation' [56] also referred to as the 'Big M notation' is chosen. However, in the here provided example, a large constant might be used in the price of unfullfilled generation p^S. This ensures total recourse whilst making not fulfilling demand the least attractive option. This in turn would however interfere with the large constant required by the 'Fortuny-Amat notation'.

Appendix C. Uncertainty Set Limits

In the here presented model, the approach chosen to represent uncertainty is that of a 'cardinality constrained robust set' [57]. The limits of such a set are defined the following:

$$\begin{aligned} &d_{t,n}^{\text{ref}} - d_{t,n}^{\Delta} \leqslant d_{t,n} \leqslant d_{t,n}^{\text{ref}} + d_{t,n}^{\Delta}\ \forall\, t \in T, n \in N \\ &\underline{x}_{t,g}^{\text{ref}} - \underline{x}_{t,g}^{\Delta} \leqslant \underline{x}_{t,g} \leqslant \underline{x}_{t,g}^{\text{ref}} + \underline{x}_{t,g}^{\Delta}\ \forall\, t \in T, g \in G \\ &\bar{x}_{t,g}^{\text{ref}} - \bar{x}_{t,g}^{\Delta} \leqslant \bar{x}_{t,g} \leqslant \bar{x}_{t,g}^{\text{ref}} + \bar{x}_{t,g}^{\Delta}\ \forall\, t \in T, g \in G \\ &\bar{s}_{t,n}^{\text{ref}} - \bar{s}_{t,n}^{\Delta} \leqslant \bar{s}_{t,n} \leqslant \bar{s}_{t,n}^{\text{ref}} + \bar{s}_{t,n}^{\Delta}\ \forall\, t \in T, n \in N \end{aligned} \tag{C.1a}$$

$$\begin{aligned} &\sum_{n\in N^{\text{zone}}}\left[\frac{|d_{t,n}-d_{t,n}^{\text{ref}}|}{d_{t,n}^{\Delta}}\right] \leqslant \frac{\Gamma_t^{d}}{100}\text{card}(N^{\text{zone}})\ \forall\, t\in T, N^{\text{zone}}\subseteq N \\ &\sum_{n\in N^{\text{zone}}}\sum_{g\in G_n}\left[\frac{|\underline{x}_{t,g}-\underline{x}_{t,g}^{\text{ref}}|}{\underline{x}_{t,g}^{\Delta}}\right] \leqslant \frac{\Gamma_t^{\underline{x}}}{100}\text{card}(N^{\text{zone}})\ \forall\, t\in T, N^{\text{zone}}\subseteq N \\ &\sum_{n\in N^{\text{zone}}}\sum_{g\in G_n}\left[\frac{|\bar{x}_{t,g}-\bar{x}_{t,g}^{\text{ref}}|}{\bar{x}_{t,g}^{\Delta}}\right] \leqslant \frac{\Gamma_t^{\bar{x}}}{100}\text{card}(N^{\text{zone}})\ \forall\, t\in T, N^{\text{zone}}\subseteq N \\ &\sum_{n\in N^{\text{zone}}}\left[\frac{|\bar{s}_{t,n}-\bar{s}_{t,n}^{\text{ref}}|}{\bar{s}_{t,n}^{\Delta}}\right] \leqslant \frac{\Gamma_t^{\bar{s}}}{100}\text{card}(N^{\text{zone}})\ \forall\, t\in T, N^{\text{zone}}\subseteq N \end{aligned} \tag{C.1b}$$

Constraints (C.1a) define the maximum and minimum ranges of deviation from the given reference points.

Constraints (C.1b) define the uncertainty budget that every uncertain parameter is assigned. This budget is dependent on the period to incorporate the increase of uncertainty over extended periods.

Ref. [58] proposes a unidirectional reformulation of those constraints and proves its validity. Ref. [54] also utilizes this reformulation. In the here presented example, such a formulation would mean approximating the bounds in Constraints (C.1a) the following: $d_{t,n}^{\text{ref}} - d_{t,n}^{\Delta} \approx d_{t,n}^{\text{ref}}$, $\underline{x}_{t,g}^{\text{ref}} - \underline{x}_{t,g}^{\Delta} \approx \underline{x}_{t,g}^{\text{ref}}$, $\bar{x}_{t,g}^{\text{ref}} + \bar{x}_{t,g}^{\Delta} \approx \bar{x}_{t,g}^{\text{ref}}$ and $\bar{s}_{t,n}^{\text{ref}} + \bar{s}_{t,n}^{\Delta} \approx \bar{s}_{t,n}^{\text{ref}}$. This would allow removing the absolute operator in Constraints (C.1b), i.e. $|\cdot| \to \cdot$. This reformulation increases solution speed due to reducing the variable space for each of the robust variable by 50% and thus decreasing the potential search space of the applied solver.

In addition and similar to recent publications on the topic [7], zonal uncertainty sets are utilized, that allow shifting the ranges of the uncertain variables in a predefined zone in favor of the worst-case solution of such a zone.

Appendix D. Proof of convergence

Convergence of the approximation algorithm in Section 7 follows the principles of traditional value function approximation as described in e.g. Ref. [62] with the difference of an observed state being supplied by the solution to the convex lower-level problem.

Assumed be that lower-level problem ϕ^{O*} is a perfect approximation, i.e.:

$$\begin{aligned} \phi^{O*}\left(y, z\right) = &\max_{d,\underline{x},\bar{x},\bar{s}} \sum_{t\in T}\sum_{g\in G} c_g(x_{t,g}) + \sum_{t\in T}\sum_{n\in N} p^S(x_{t,n}^S) \\ &\text{s. t. constraintsofEq. (7)} \end{aligned} \tag{D.1}$$

As a result, the approximation of the upper-level problem becomes perfect as well:

$$\phi^{I*}\left(y, z\right) = \phi^{O*}\left(y, z\right) + \sum_{n\in N^{Y,I}} c_n^Y(y_n) + \sum_{n\in N^{Z,I}}\sum_{n_2\in N_2^{Z,I}} c_{n,n_2}^Z(z_{n,n_2})\ \forall\, y, z \tag{D.2}$$

This means that finding the optimal, global result of the entire problem becomes solving $\min_{y^*,z^*}\phi^{I*}(y^*, z^*)$ which is computationally tractable via brute-forcing every iteration (due to solving the lower-level problem now only requires a function-call to the perfect approximator ϕ^{O*}). This means that finding the global optimum to the investment problem requires finding the best fit for the lower-level problem approximation ϕ^{O}.

Formulating this as in the value function formulation of Ref. [62] leads to the following approximation problem:

$$\phi^{O}(y, z) = \mathbb{E}[\phi^{O*}(y, z)|y, z] \tag{D.3}$$

In other words, the approximation is the expectation of the perfect fit to the lower-level problem considering any given investment decisions. Traditionally, such value function approximation problems show noise. However, by instead selecting the robust solution instead of sampling a random solution from the uncertainty set, no noise is encountered here. This means the expectation is an expectation on missing but deterministic data (i.e. the lower-level problems not solved for yet) instead of an expectation of a stochastic outcome. These, albeit computationally demanding, results $\phi^{O*}(y, z)$ can be calculated for specific values of y, z. This in turn, allows to compare the fit of the calculated outcome to the approximation of the expectation. Utilizing an error function such as the Mean Squared Error (MSE) this can be formulated the following:

$$\begin{aligned} \text{MSE} &= (\phi^{O*}(y, z) - \mathbb{E}[\phi^{O*}(y, z)|y, z])^2 \\ &= (\phi^{O*}(y, z) - \phi^{O}(y, z))^2 \end{aligned} \tag{D.4}$$

The goal is reaching the minimum of $MSE = 0$ in order to yield the perfect fit of (D.1) and therefore (D.2). Denoting the parameters of the approximator as θ (in the here presented study this would correspond to the weights and biases of the neural network) leads to a parameterized form of the approximator denoted as $\phi^{O}(y, z|\theta)$. The update rule for an update $\Delta^{\theta}(y, z)$ based on the calculated solution is therefore the following:

$$\Delta^{\theta}\left(y,z\right)=2\left(\phi^{O*}\left(y,z\right)-\phi^{O}\left(y,z\middle|\theta\right)\right)\frac{\delta\phi^{O}(y,z|\theta)}{\delta\theta} \tag{D.5}$$

This requires the approximator to be able to be updated via either gradient or quasi-gradient methods, a condition the here applied neural networks fulfill. Thus, training this approximator via those updating steps will result in a convergence towards the global optimum of the problem.

Appendix E. Comparison to State-of-the-Art

Due to its importance in the field of robust optimization, the technique proposed in this paper was compared to 'Column-and-Constraint-Generation' (CnCG) as described in Ref. [40].

For the lower-level problem, this technique solves similar as the method proposed in this paper. The difference can be found in the upper-level problem, which is in turn formulated the following:

$$\phi^{I}\left(y,z\right)=\min_{y,z,x,x^S,\delta,s}\eta+\sum_{n\in N^{Y,I}}c_n^Y(y_n)+\sum_{n\in N^{Z,I}}\sum_{n_2\in N_2^{Z,I}}c_{n,n_2}^Z(z_{n,n_2}) \tag{E.1a}$$

$$\eta\geqslant\sum_{t\in T}\sum_{g\in G}c_g(x_{k,t,g})+\sum_{t\in T}\sum_{n\in N}p^S(x_{k,t,n}^S)\ \forall\ k\in K \tag{E.1b}$$

$$\text{s. t.}\ \begin{array}{l}\sum_{g\in G_n}x_{k,t,g}-d_{t,n}(\xi_k)-\sum_{n_2\in N_2^Z}[B_{n,n_2}(\delta_{k,t,n}-\delta_{k,t,n_2})]-\\ \sum_{t_1\in T}s_{k,t_1,t,n}+\sum_{t_2\in T}s_{k,t,t_2,n}+x_{k,t,n}^S=0\end{array}\ \begin{array}{l}\forall\ t\in T,\\ n\in N\\ k\in K\end{array} \tag{E.1c}$$

$$x_{k,t,n}^S+\sum_{t_2\in T}s_{k,t,t_2,n}\leqslant d_{t,n}(\xi_k)\ \begin{array}{l}\forall\ t\in T,\ n\in N\\ k\in K\end{array} \tag{E.1d}$$

$$B_{n,n_2}\left(\delta_{k,t,n}-\delta_{k,t,n_2}\right)\leqslant\overline{F}_{n,n_2}\ \begin{array}{l}\forall\ t\in T,\ n\in N^Z,\\ n_2\in N_2^Z,\ k\in K\end{array} \tag{E.1e}$$

$$\underline{x}_{t,g}(\xi_k)\leqslant x_{k,t,g}\leqslant\bar{x}_{t,g}(\xi_k)\ \forall\ t\in T,\ g\in G,\ k\in K \tag{E.1f}$$

$$\delta_{k,t,n_{\text{ref}}}=0\ \forall\ t\in T,\ k\in K \tag{E.1g}$$

$$s_{k,t_1,t_2,n}=0\ \forall\ t_1\geqslant t_2;t_1,\ t_2\in T,\ n\in N,\ k\in K \tag{E.1h}$$

$$s_{k,t_1,t_2,n}=0\ \forall\ (t_2-t_1)>\bar{t}_n;t_1,\ t_2\in T,\ n\in N,\ k\in K \tag{E.1i}$$

$$\sum_{t_2\in T}s_{k,t_1,t_2,n}\leqslant y_n\bar{s}_{t_1,n}(\xi_k)\ \forall\ t_1\in T,\ n\in N,\ k\in K \tag{E.1j}$$

$$x_{k,t_1,n}^S,\ s_{k,t_1,t_2,n}\geqslant 0\ \forall\ t_1,\ t_2\in T,\ n\in N,\ k\in K$$

The difference is that instead of an approximation for the operations problem, CnCG uses a lower bound η on this lower-level problem. The bound is updated with iteratively added cuts k. The heuristic to find the optimal investment is thus the following:

0) **Initialize** $K=\{\emptyset\}$
1) **Solve** upper-level problem (E.1) to receive y^*, z^*
2) **Solve** lower-level problem (7) to receive robust scenario ξ_{k^*}
3) **Add** cut k^* to set of cuts K
4) **If** not converged and permutations left: back to 1)

This illustrates a crucial factor in applying such a robust solution, the increasing upper-level problem size due to an increasing cardinality of K. This is already an issue in traditional transmission expansion problems [2], but amplifies in the application presented here. There are two reasons for this. First, the problem extends this traditional expansion problem by additional binary variables representing investments in flexible demands and loads. This means a higher number of potential investment iterations and thus a larger search space for the upper-level problem. Second, in CnCG the dynamic operations of shifting demand and storage have to be considered in the upper-level as well. This means that the increase in complexity caused by connecting the time periods as discussed in Ref. [50] or displayed in Fig. 1 affects the problem size twice in CnCG and only once in the method proposed in this paper. Latter issue is also discussed as a downside in Ref. [40], an issue that is eliminated by using the proposed approximation technique instead of the bounding technique that is CnCG.

Appendix F. Value function approximation

The chosen function approximation of the case studies presented in this paper is that of a traditional feed-forward neural network [39] which has been used to train similar value function approximations [59,50]. Fig. 9 shows the topology of the network. It consists of two parts: hidden layers that contain linear and non-linear (rectified linear unit) layers which are normalized after before each linear layer and a dense linear layer that sums the network outputs to a single continuous output representing $\phi^O(y,z)$. The applied optimizer to tune the weights of the network was 'Rprop' [60] and the chosen error function traditional 'Mean Squared Error'.

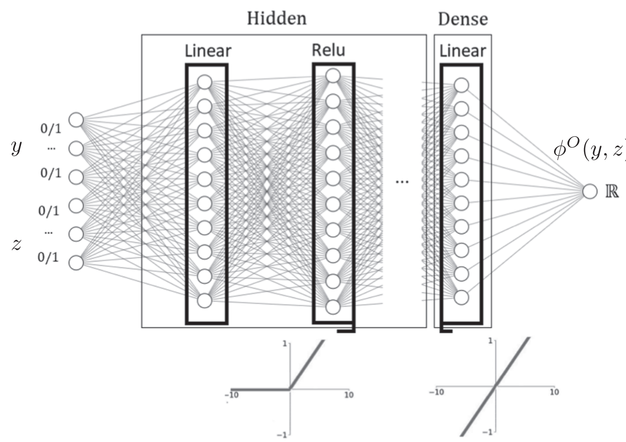

Fig. 9. Function Approximation Network.

In addition, rescaling of outputs (by dividing the cost by a certain factor to have the operational cost approximate lower digit numbers) and 'Simulated Annealing' [61] performed on the learning rate were applied to support convergence and ensure a better fit of the function approximation.

In the given application, this network will often not be supplied large data sets (as finding a result for a single permutation can be time-consuming) and could thus encounter issues training large networks. As a result, it might be advisable to keep the number of hidden layers in the network low. The number used in the given case studies was two sets of linear/non-linear hidden layers with a layer size of 50 nodes each.

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