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A new crack spacing model for reinforced concrete specimens with multiple bars subjected to axial tension using 3D nonlinear FEM simulations

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Abstract

Crack spacing is a governing parameter in widely used crack width calculation models. Axial tensile experiments are conducted to examine the crack spacing behavior of reinforced concrete specimens with multiple reinforcement bars. To reduce the time, cost, and labor of the experiments, nonlinear finite element simulations are widely used. In this study, 3D non-linear finite element simulation models have been developed with the smeared cracking approach to predict the average crack spacings. These models are calibrated and validated using both the experiment conducted by the authors and an experiment given in the literature. The governing crack spacing parameters have been identified as concrete cover thickness and clear distance between tensile bars. After conducting a series of 3D nonlinear finite element method simulations with the calibrated model, an equation is developed to predict the average crack spacings using multiple linear regression analysis. The validity of the proposed crack spacing equation has been checked with 18 recent experimental results in the literature. The proposed crack spacing equation gives a good agreement with the results of these experiments.

KEYWORDS

axial tension, bar spacing, concrete cover thickness, crack spacing, nonlinear FEM

1 | INTRODUCTION

Crack spacing is an important parameter in predicting crack widths. Axial tensile tests are conducted to study

the cracking behavior of reinforced concrete (RC) specimens with several reinforcement bars, because the behavior is much similar to actual RC members in practice.¹ In order to conduct such experiments, additional effort is required in designing the load application method.² Conducting such experiments on relatively large RC specimens requires a special type of test rigs,³ which are not commonly available. Due to several benefits, the finite element method (FEM) can be used to analyze the

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cracking behavior of RC structures. Reducing the time, cost and labor in laboratory experiments is one of the main benefits, while the ability to observe the internal behavior of an RC specimen is another benefit of FEM. Nonlinear FEM has been used to study crack spacing behavior and has been reported in several pieces of research.^{4–7} Crack analysis of reinforced concrete structures in FEM can mainly be carried out by using two crack models: namely, “discrete crack” and “smeared crack” models. The discrete crack model was developed, as a crack would create a complete discontinuity between element edges.^{8,9} In the discrete modeling, it is necessary to pre-define the location and path of the crack along the finite element edges.¹⁰ As the crack location is needed to pre-define, this method is not suitable for studying crack spacings. Ingraffea and Saouma¹¹ proposed re-meshing the concrete element after the cracks appear. However, that would make the model much more complicated and require computational time and cost. In order to overcome these issues, the “smeared crack model” can be used to study crack spacing behavior.¹² This model was first introduced by Rashid.¹³ When a crack occurs in an element, this method proposes changing the constitutive properties (material stiffness, tensile strength perpendicular to crack direction, etc.) of the cracked concrete elements.

When considering existing crack spacing prediction models, they are mainly based on three theories, namely: “bond-slip,”¹⁴ “no-slip,”¹⁵ and “combined theories.”¹⁶ The “bond-slip” theory considers that a slip occurs between the reinforcement and concrete. The “no-slip” theory considers that there is a perfect bonding between reinforcement and concrete and therefore no slip would occur. According to this theory, within the concrete, the stress would transfer according to St. Venant’s principle,¹⁷ which means that the crack spacing is affected by the thickness of the surrounding concrete of the tensile reinforcement. The combined theory considers that crack spacing behavior is affected by the aforementioned two theories. However, the findings in Beeby¹⁸ and McLeod¹⁹ emphasized that the no-slip theory has a dominant effect on the cracking behavior of RC specimens with deformed bars. Further, the axial tensile tests conducted on RC ties mentioned in Yannopoulos²⁰ and Tammo et al. in^{21,22} identified that a negligible amount of slip occurs at the end faces of RC ties (faces perpendicular to reinforcement).

The bond-slip behavior adapted to Model Code 2010 (MC 2010) is from the Ciampi–Eligehausen model,^{23,24} which is based on the Rilem-type pull-out tests²⁵ (the same Ciampi–Eligehausen model is used to develop the crack width calculation model by Balazs,²⁶ which is also considered in *fib* bulletin 52.²⁷ According to this MC 2010 bond-slip model, the slip value can be up to 2 mm

for the good-bond condition. On the other hand, the bond-slip behavior of RC specimens subjected to axial tension was identified in Beconcini et al.²⁸ and Doerr.²⁹ The maximum slip values identified in these experiments are 0.055 mm and 0.1 mm, respectively. Therefore, when comparing the bond-slip behavior in MC 2010 and in axial tension, it could identify that the “slip” in axial tension is almost negligible. These differences in the bond-slip behaviors between axial tension and pull-out tests have been thoroughly discussed in Naotunna et al.³⁰ For these reasons, for this study, the cracking behavior of an RC member is considered to be more related to the “no-slip” theory.

From the existing literature, many crack spacing models based on the “no-slip” theory can be identified. Such models are found in.^{15,31–33} Further, several other crack spacing models can be identified without the “bond parameter (φ/ρ),”¹⁸ which is the dominant parameter of crack spacing models developed from “bond-slip” theory. The models mentioned in^{34–37} are such models; since the background of these models are not easy to find, it cannot be stated that these models are directly based on “no-slip” theory. When considering the models in the aforementioned literature, the governing crack spacing parameters can be identified as concrete cover thickness,^{15,31,32,36} spacing between tensile reinforcement,^{32,34–36} concrete quality,³⁶ reinforcement ratio,³⁷ and number of tensile reinforcement layers in concrete.³⁶ Since 1965, concrete cover thickness has been identified as a dominant crack spacing governing parameter.¹⁵ Existing crack width calculation models in^{36,38–41} have considered, and many recent experiments^{1,42–44} have proved, that concrete cover thickness is a crack-spacing parameter. Spacing between tensile bars has been considered a parameter of crack spacing in the past (Model Code 1978,⁴⁵ Bazant et al.’s model in,⁴⁶ etc.). Eurocode 2⁴¹ recommends a different calculation model for members with large tensile bar spacings. Further, the recent literature of Gribniak et al.⁴² and Hossin and Marzouk⁴⁷ experimentally proved that spacing between tensile bars is a governing parameter of crack spacing. Many studies have identified that, although the crack initiation load is related to the tensile strength of concrete, the crack widths and crack spacings have a negligible effect from concrete tensile strength.^{48–51} Other than that, Broms and Lutz³² and Theriault and Benmokrane⁵¹ identified that the reinforcement ratio is not a governing parameter of crack spacing. Therefore, in accordance with these facts, concrete cover thickness and tensile bar spacing have been considered governing crack spacing parameters for this study.

ATENA by Cervenka Consulting⁵² is nonlinear FEM analysis software that is widely used to simulate the cracking behavior of RC members. As mentioned, this study focuses on cracking behavior based on “no-slip”

theory (or perfect bond criteria). This condition can be represented by modeling with smeared reinforcement which leads to smeared cracking behavior.⁵³ Recent studies mentioned in Mang et al.⁴ and Rimkus et al.⁵⁴ have identified that the perfect bonding models give good agreement with the experimental cracking behavior.

The authors of this article have conducted an axial tensile experiment with $2\text{ m} \times 0.2\text{ m} \times 0.2\text{ m}$ (length, width, height) specimens and four 32-mm-diameter bars. The details of this experiment are mentioned in Naotunna et al.³ As previously stated, such experiments consume a considerable amount of time, cost, and labor. Therefore, a three-dimensional finite element simulation model has been developed to simulate this experiment. The crack spacing behavior has been calibrated with the results of this experiment. By using the same FEM parameters, the model has been verified to predict the mean crack spacing of a $3.2\text{ m} \times 0.35\text{ m} \times 0.35\text{ m}$ specimen with four 20-mm-diameter bars, mentioned in Barre et al.⁵⁵ After the calibration and validation of the 3D FEM simulation model, several virtual experiments could be conducted to study the effect of “concrete cover thickness” and “clear distance between tensile bars” on crack spacing. The “mean crack spacing” parameter is a good representative of the overall crack spacing behavior of an RC specimen. Therefore, widely used crack spacing models in Eurocode 2 (EC2) and MC 2010 first developed their model for mean crack spacing and multiplied it by a factor (factor of 1.7) to predict the “maximum crack spacing.”^{56,57} Therefore, the obtained mean crack spacing values from the conducted virtual experiments have been used to develop a new crack spacing equation, which can be used in the axial tensile specimens with multiple reinforcement bars. The applicability of this developed equation has been checked with the results of recent experiments reported in the literature.

2 | USE OF AXIAL TENSILE EXPERIMENTS TO CALIBRATE AND VALIDATE 3D FEM SIMULATION MODEL

Axial tensile experiments can represent the tensile region of a bending member,^{58,59} and several rebar layers can be placed in specimens, similar to in practice. In this study, the results of the experiment published by Naotunna et al.³ have been used to calibrate the 3D FEM simulation model. In this experiment, three identical specimens with $2\text{ m} \times 0.2\text{ m} \times 0.2\text{ m}$ (length \times width \times height) were cast with four 32-mm-diameter reinforcement bars (Figure 1). The concrete cover thickness of the tested specimens was 35 mm. The mechanical properties of the concrete and the specimen details are shown in Table 1. The characteristic yield strength of the reinforcement was 500 MPa, and Young’s modulus was 200GPa. After unmolding the specimens, they were stored in a 20°C room temperature, with the necessities for concrete curing. During the test, the axial tensile load was applied to the reinforcement, and the test rig was connected to the specimen through a nut and bolt mechanism, as shown in Figures 1 and 2. Crack spacing measurements were taken, respective to the position of the reinforcement. Moreover, to validate the developed 3D FEM simulation model, the results of Barre et al.⁵⁵ were considered; a detailed discussion is presented in Section 3.

3 | 3D NONLINEAR FEM SIMULATION TO STUDY CRACK SPACING

The crack simulation of the aforementioned experiment was carried out with the finite element software called

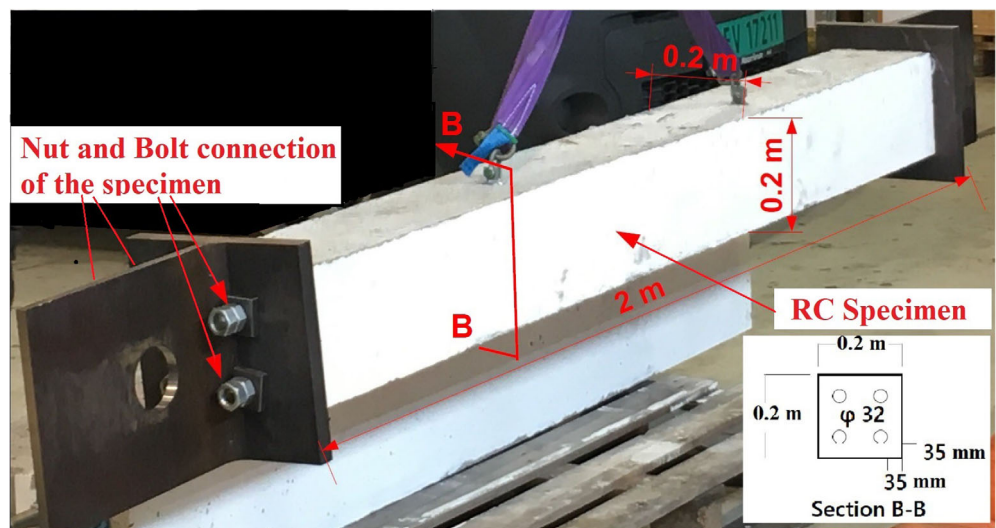


FIGURE 1 Details of the tested RC specimen in Naotunna et al.³

TABLE 1 Parameters of the test specimens

Specimen	Cross section (m × m)	Length (m)	Concrete strength (MPa)		Concrete cover (mm)	Bar profile no × diameter (mm)
			Compressive	Tensile		
Naotunna et al. ³	0.2 × 0.2	2	35	3.2	35	4 × 32
Barre et al. ⁵⁵	0.355 × 0.355	3.2	43.6	3.7	65	4 × 25

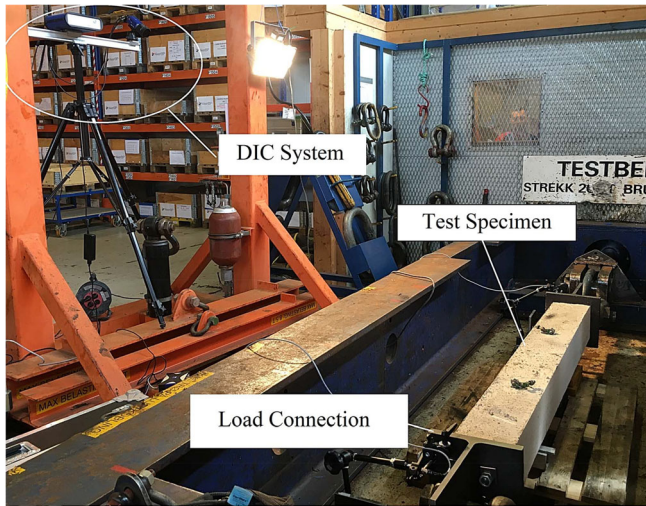


FIGURE 2 Tensile load application on the RC specimen mentioned in Naotunna et al.^{3,30}

“ATENA” by “Cervenka Consulting,” Version 5.7.0 with “GiD 14.0.2” interface. The modeling was conducted with the “GiD 14.0.2” interface. When developing the 3D nonlinear FEM simulation model, eight-noded hexahedral elements were used to model concrete. The reinforcement was modeled as smeared reinforcement, where reinforcement and concrete are discretized into element with the same geometrical boundaries.⁶⁰ Since the reinforcement is modeled as smeared reinforcement, material properties of reinforcement superimpose the concrete material properties within the defined space of reinforcement. The material stiffness matrix of smeared reinforcement is a function of the direction of the reinforcement axis, elastic modulus of reinforcement, and the reinforcement ratio. Since the reinforcement is modeled as smeared reinforcement, the cracks have idealized as smeared cracks.^{61,62} This method does not allow “slip” to occur between the reinforcement and concrete. The recent study by Rimkus et al.,⁵⁴ on the uncertainty of the smeared crack model, identified that the perfect bond model gives the best fit with the experimental results. When modeling the axial tensile experiment, the “fixed crack model” was decided on, where the crack direction freezes to the principal stress direction of the crack occurrence.⁶³ Since the axial tensile experiments were performed by keeping one end fixed and loading in the other

end, a similar type of boundary condition was applied in the FEM model. In this model, the external load is applied by a displacement-controlled method, with the displacement given as 0.1 mm incremental steps. Further, as the FEM is loaded in a horizontal direction, vertical constraints were added to the bottom edges of the specimen (required at the initial displacement steps). The solution of the numerical model is based on the Newton–Raphson method.

3.1 | Material models

The constitutive model selected for concrete consists of a “fracture” part to model the tensile failure, while the “plastic” part is to model the compressive failure.⁶⁴ The material model follows the De-Borst⁶⁵ strain decomposition rule, where the strain in concrete decomposes into elastic, plastic, and fracture. However, as the developed FEM simulation model is only subjected to tension, concrete strain will not decompose as plastic strain. When a crack occurs in concrete, to match the continuous algorithm of FE analysis, it is considered that an amount of tensile stress would transfer across the crack. This amount of stress is decided in accordance with Hordijk’s law.⁶⁶ According to Hordijk’s law, the tensile stress of concrete perpendicular to the crack is a function of crack width. Detailed information about the fracture model, which is based on Rankine failure criterion, is mentioned in Cervenka and Papanikolaou.⁶⁴

3.2 | Calibration and validation of the 3D FEM simulation model

The axial tensile test published by Naotunna et al.,³ discussed in Section 2, is simulated by using nonlinear FEM and the model is calibrated with the experimental mean crack spacing value. Hereafter, this 3D FEM simulation model is referred to as the “calibration model.” This is a symmetric model and, therefore, in order to save computational time and cost, only a symmetric half of it can be modeled. However, since this is a crack spacing study, when a crack occurs near the edge of the symmetric half, obtaining the crack spacing value can be problematic.

Therefore, the “complete specimen” has been modeled in three dimensions, to obtain the crack spacings. The smeared crack model is sensitive to the size of the mesh.⁵³ Mesh size must be larger than the maximum aggregate size (16 mm) to represent the inhomogeneous behavior of concrete⁶⁷ and less than the reinforcement diameter (32 mm) to affect the bond conditions.⁵³ In the aforementioned concrete model, there are several factors which can be used to influence the tensile behavior of concrete. Such parameters are “*crack spacing min*,” “*tension stiffening*,” “*shear factor*” and so on. In heavily reinforced members, the cracks may not fully propagate and in such cases, it may need to adjust the “limiting value” of tensile strength of concrete in the tension softening diagram. The “*tension stiffening*” parameter is used to do these changes. In the software, a value can be assigned to the “*tension stiffening factor*” from 0 (the limiting tensile strength of concrete reaches to zero after

cracking) to 1 (the limiting tensile strength of concrete does not drop after cracking). The “*shear factor*” parameter is used to define a relationship between normal stiffness and shear stiffness of cracked concrete (shear stiffness is “*shear factor*” × normal stiffness obtained from the tension softening curve⁶⁸). However, since the specimens are subjected to axial tension, this parameter is not dominant for the conducted experiment. Therefore, “*shear factor*” has not been activated in this model. The “*crack spacing min*” parameter is used to correct the fracture energy parameter based on the element size. Since the concrete softening curve is modeled with laboratory specimens of 100–150 mm size, the value of this parameter can be decided accordingly. According to the flow chart illustrated in Figure 3, after several trial-and-error iterations, the mesh size was decided as 25 mm, the value of “*tension stiffening*” parameter was set at zero and the “*crack spacing min*” parameter was set at 100 mm.

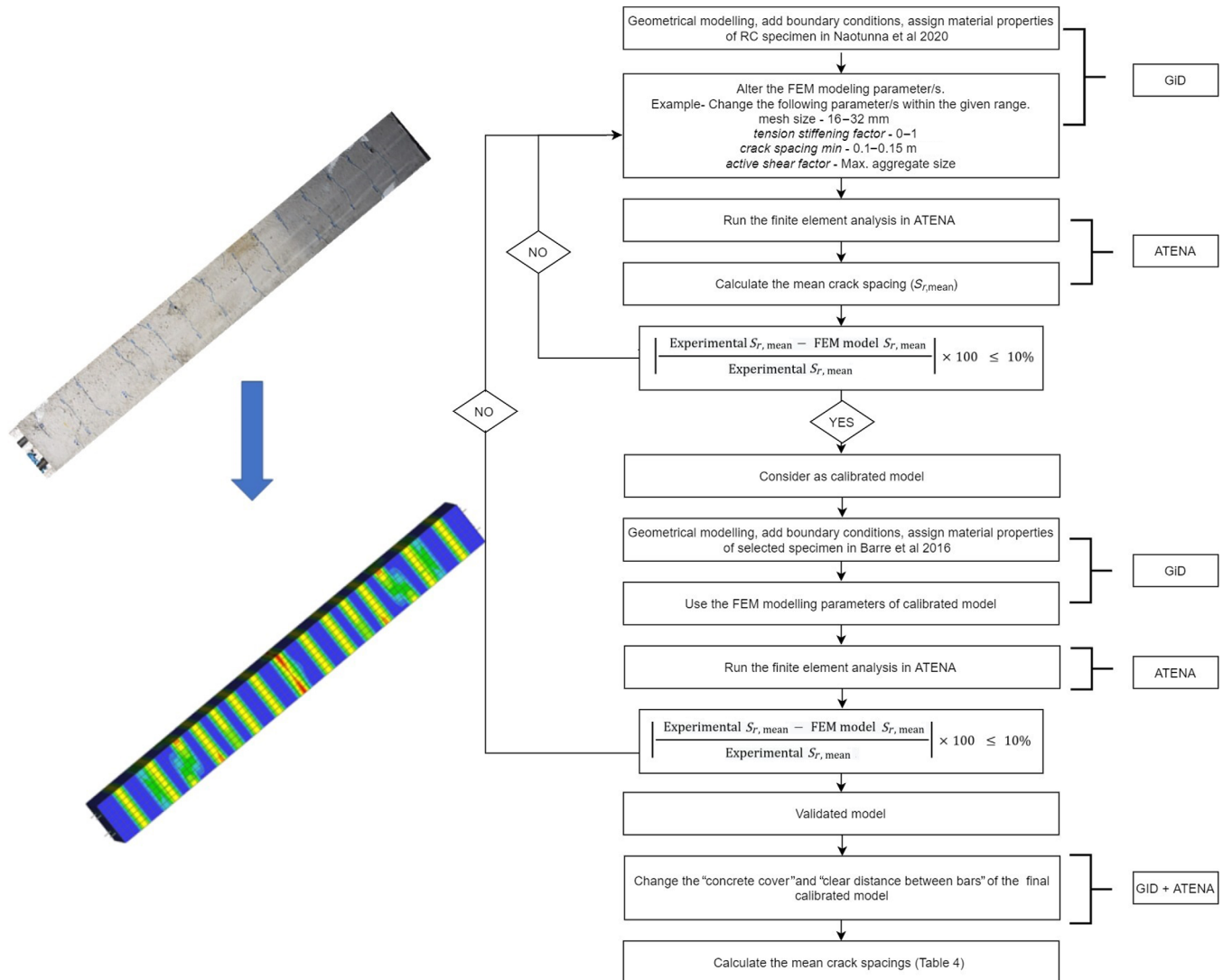


FIGURE 3 Flow chart of the calibration, validation and the parametric analysis conducted during this study

Specimen	Mean crack spacing		
	FEM simulation models (mm)	Experimental (mm)	Error ^a %
Calibration ^{3,30}	125	133	6
Validation ⁵⁵	188	200	5.8

^aError = (Experimental value – FEM simulation model value)/Experimental Value.

A 3D FEM calibration model is developed by using the mean concrete compressive and tensile strengths measured at the laboratory (refer Table 2). Young's modulus and Poisson's ratio are considered according to Eurocode 2⁴¹ provisions, the values being 34 GPa and 0.2, respectively. The fracture energy is automatically generated according to the equation proposed in Vos.⁶⁹ Since the reinforcement is subjected only to axial tension, the smeared reinforcement is represented in a one-dimensional element. Characteristic yield strength and the Young's modulus of the reinforcement are assigned as 500 MPa and 200 GPa, respectively. Figure 4a,d show the cracking behavior of the calibrated FEM model at two loading steps: at 500 kN and 800 kN, respectively. Further, the steel stresses of the reinforcement can be observed, as given in Figure 4b,d. It is important to mention that, at the crack, the steel stress becomes higher than at the uncracked locations. The crack spacing of the numerical model is considered as the center-to-center distance between the cracked elements. As shown in the Table 2, the mean crack spacing values of the calibration model gives a good agreement with the experimental results, where the error value is less 6%.

After calibration, the next task was to check whether the developed model can represent the crack spacing values for similar types of experiments (validate the model). For this task, the experimental results of a similar axial tensile experiment with four tensile reinforcement bars were required. An experimental program mentioned in Barre et al.⁵⁵ was considered for this case, and the FEM simulation model is named the "validation model." The number of reinforcements and the tensile load application method are similar to the experiment considered in the calibration model. Details of the selected specimen are mentioned in Table 1, and the comparison of crack spacing values is shown in Table 2. The flow chart in Figure 3 clarifies the steps conducted to develop this validated model. The crack spacing values of the specimen are given in Figure 5. According to the comparison in Table 2, the developed 3D FEM simulation model shows the ability to predict the mean crack spacing behavior of specimens subjected to axial tension. Table 2 shows that the mean crack spacing values give very good agreement with the experimental results. In both cases, the experimental mean crack spacing value is 6% higher than in the FEM simulation models.

TABLE 2 Comparison of the experimental crack spacing values with the numerical model predictions

4 | DEVELOPMENT OF NEW CRACK SPACING MODEL

As mentioned in Section 1, the governing crack spacing parameters for this study have been identified as concrete cover thickness and the clear distance between tensile bars. Therefore, the developed calibration model has been used to study the effect of these two parameters on crack spacing. Since the FEM simulation model is calibrated with a mesh size of 25 mm, the specimen size can be increased in 25-mm steps. With the increase in model size, the number of elements in the FEM models also increases. This leads to a further increase in the computational time and space. When designing a structure for a longer service life, a large concrete cover thickness is required. The current requirement for concrete cover thickness can be as large as 120 mm, according to the Norwegian Public Road Administration guidelines.⁷⁰ Therefore, in the parametric analysis, the concrete cover has been changed from 35 to 122.5 mm, and the clear distance between tensile bars has been changed from 66 to 216 mm, in several steps. Table 3 shows the mean crack spacing values of the developed 3D FEM simulation models, with changed concrete cover thickness and bar spacing. When developing these models, not only the mesh size, material properties and the boundary conditions but also the other FEM modeling parameters are kept the same as in the calibration model.

According to Table 3, the experimental mean crack spacing values are 6% larger than the FEM simulation model predictions. Therefore, the obtained results from the aforementioned FEM simulation models have been increased by 6%. Several models can be identified which do not have a satisfactory convergence criterion. The model is developed to stop the iterations, if the relative error in displacement or force equilibrium is larger than 10% in several steps in a row, since the results tend to be unreliable.⁷¹ In such cases, there are several methods that can be used to improve the model, that is, reducing the stiffness of the overall specimen, reducing the displacement step, and so on.⁷¹ Since it is not possible to change the calibrated FEM modeling parameters, such models were not improved, and the results of these models were not considered for this analysis (blank spaces in Table 3).

Broms and Lutz³² identified that the crack spacing behavior would change when the concrete cover

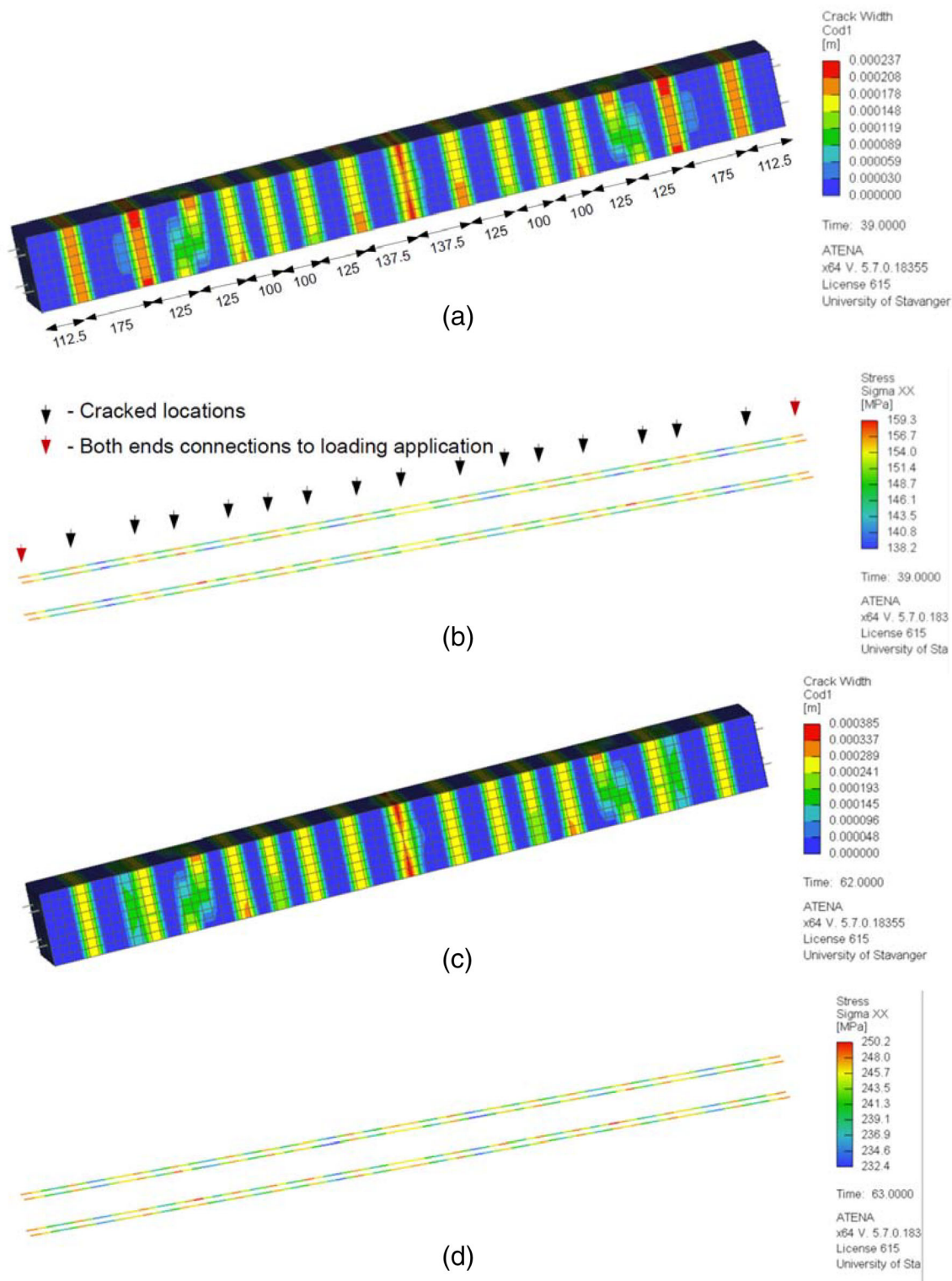


FIGURE 4 Cracking behavior of the 3D FEM calibrated model, (a) cracking behavior of the specimen at the applied tensile load of 500 KN (crack spacing measurements are in mm), (b) stress distribution of reinforcement at the applied tensile load of 500 KN, (c) cracking behavior of the specimen at the applied tensile load of 800 KN, (d) stress distribution of reinforcement at the applied tensile load of 800 KN

thickness is larger than the clear distance between the tensile bars. The highlighted data in Table 3 are from such cases. Therefore, the first focus is on cases where the concrete cover thickness is less than the clear

distance between the tensile bars. In this study, the independent variables are concrete cover thickness and the clear distance between tensile reinforcements, and the dependent variable is the mean crack spacing. The

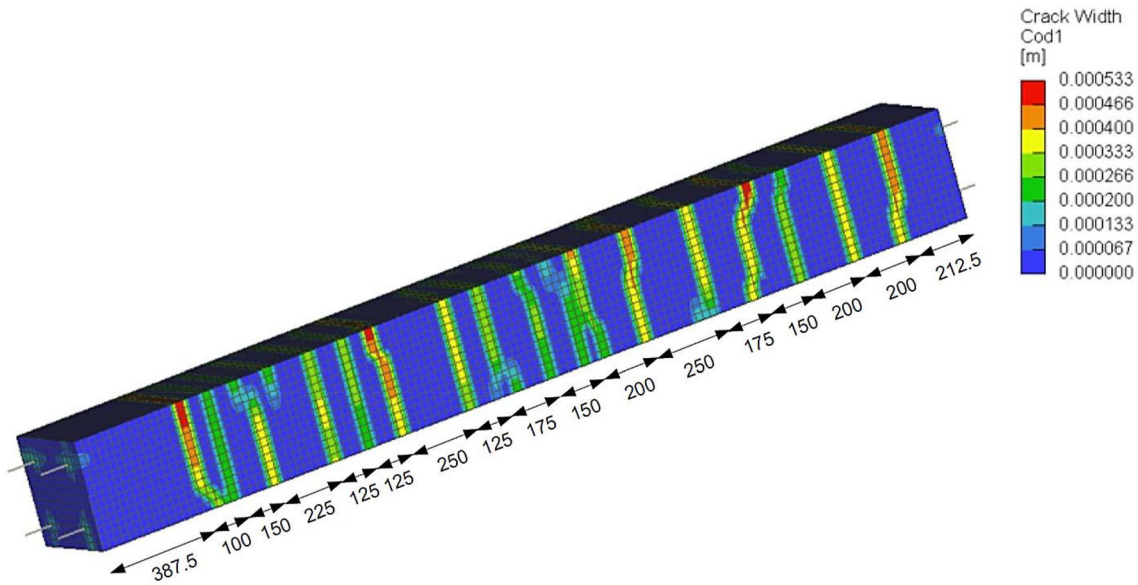


FIGURE 5 Cracking behavior of the FEM simulation model developed to simulate the mentioned experiment in Barre et al.,⁵⁵ the validation model (crack spacing measurements are in mm)

TABLE 3 Obtained mean crack spacing values from the FE analysis

	Concrete cover thickness (mm)						
	35	60	85	97.5	110	122.5	
Clear distance between tensile reinforcement (mm)	66	133	151	146	177	184	173
	91	125	141	151	180	163	173
	116	163	163	177	-	163	177
	141	170	177	212	-	-	-
	166	177	-	236	-	193	-
	191	184	212	-	-	-	-
	216	193	236	265	-	-	-

Note: “-” convergence limit is larger than 10% in several steps. Mean crack spacing values are highlighted for those specimens in which the concrete cover thickness is larger than the clear distance between bars. 133 This mark shows the locations of calibrated model. - This mark shows the approximate location of the validated model (cover is 65 mm and clear distance between bars is 175 mm).

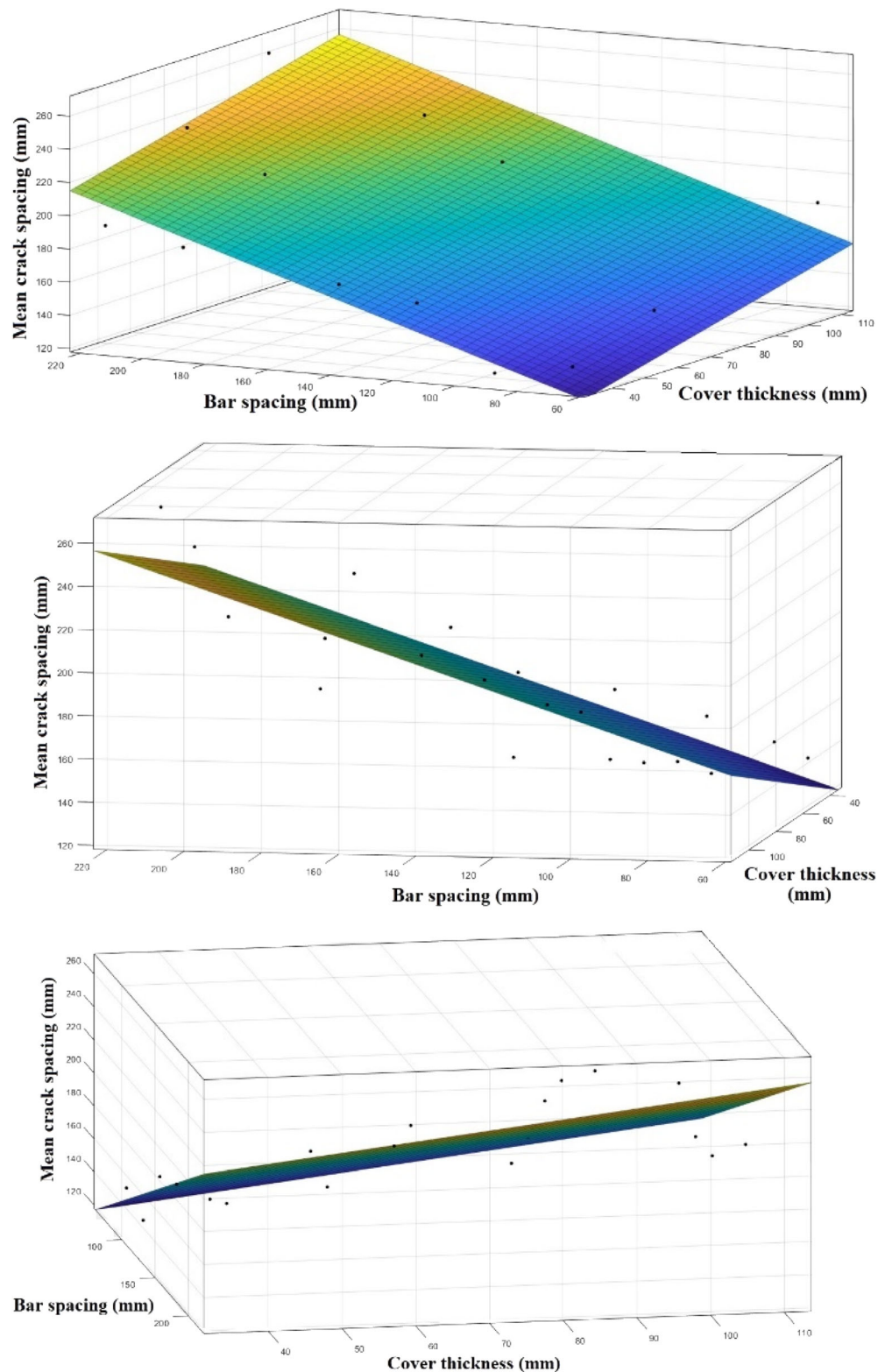
purpose was to obtain a simple equation to predict the mean crack spacing, which can be more practically used in industry. A linear regression analysis can be conducted for the mentioned data in Table 3, to find the best-fitting equation. Since this is a study with two independent variables, the solution of the linear regression analysis will be a plane (surface). As the best fitting surface, a first-degree polynomial equation has been obtained as given in Equation (1). This is the equation for the surface given in Figure 6. Even though a relatively large amount of data points has been used for the analysis (Table 3), the R^2 value is 0.78.

$$S_{r,\text{mean}} = 66 + 0.51 c + 0.6 s \quad (\text{all values in mm}) \quad (1)$$

where $S_{r,\text{mean}}$ is the mean crack spacing, “ c ” is the concrete cover thickness, and “ s ” is the clear distance between tensile reinforcements.

The size effect or the geometric parameters of an RC specimen have been identified as a factor influencing crack spacing.⁶ The virtual experiments conducted using nonlinear 3D FEM simulation models are only for those cases where RC specimens have four reinforcements (two tensile bars are close to the surface of each face). When the number of bars close to the specimen surface (n) is two, concrete cover thickness and clear distance between bars gives a good representation of size effect. However, in general, the value of “ n ” can vary, and in such cases, the concrete cover and bar spacing do not represent the

FIGURE 6 Behavior of the mean crack spacing for specimens with different cover thicknesses and bar spacings in three different rotation angles

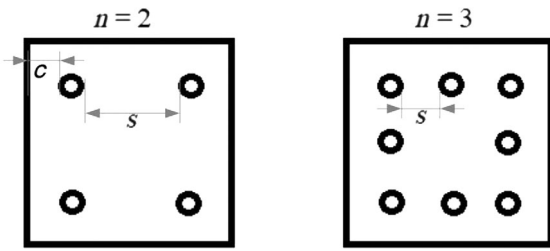


geometric parameters of a specimen (refer to Figure 7, when $n = 3$). Equation (1) is developed for those specimens with two tensile bars close to the surface of each face ($n = 2$). Therefore, to represent the size effect of specimens with different numbers of “ n ” values, the term with bar spacing in Equation (1) has been divided by two and multiplied with “ n .” Equation (2) represents the

modified crack spacing model with several reinforcement bars close to the surface.

$$S_{r,\text{mean}} = 66 + 0.51 c + 0.6 s n_{/2} \quad (\text{all values in mm}). \quad (2)$$

As mentioned, Equations (1) and (2) have been developed for cases where the value of “ s ” is larger than “ c .” The highlighted cases in Table 3 are those where “ c ” is



“*c*”– Concrete cover thickness
 “*s*”– Clear spacing between bars
 “*n*”– Number of bars respective to specimen surface

FIGURE 7 Notations of the proposed equation

larger than “*s*.” For these cases, the effect of parameter “*s*” can be identified as becoming less important to the crack spacing. Then, it can be seen that, when parameter “*s*” in Equation (2) is replaced with “*c*,” it gives better predictions with the highlighted “green” data in Table 3. Equation (3) shows the developed mean crack spacing model for the RC beams, when the concrete cover is larger than the clear distance between the tensile bars.

$$S_{r,\text{mean}} = 66 + c (0.51 + 0.6 n/2) \quad (\text{all values in mm}). \quad (3)$$

5 | VERIFICATION OF THE PROPOSED EQUATIONS

The equations developed to predict the mean crack spacing of RC beams with multiple reinforcements subjected to axial tension are given in Equations (2) and (3). In order to validate the equations, they have been checked with the results of recent experimental studies. Recent literature has recorded a limited number of axial tensile tests conducted on RC specimens with multiple bars. Cases were selected from Tan et al.,⁴⁴ Naotunna et al.,³ Barre et al.,⁵⁵ Rimkus and Gribniak¹ and Garcia and Caldentey.⁷² Details of the RC specimens of these selected specimens are listed in Table 4. Moreover, Tan et al.⁴⁴ did not apply the tensile load directly to the reinforcement. However, in every other experiment mentioned, the tensile load is directly applied to the reinforcement. Other than the experiment considered to validate the numerical model, another axial tensile test was conducted in Barre et al.⁵⁵ for a specimen with eight reinforcements, and this is mentioned as case 3 in Table 4. Rimkus and Gribniak¹ studied the effect on crack spacing of different reinforcement layouts. Tested specimens are around 0.5 m long, and it is important to mention that cases 13, 14, and 15 in Table 4 have two reinforcement layers, respective to each concrete surface. Garcia and Caldentey⁷² focused on the effect of the casting position on the cracking behavior. The “good

bond” and “poor bond” faces are considered as the bottom and top faces of the specimen, respectively, when it is being cast. Since the developed equations are based on the perfect bond criteria, only the results of “good bond” cases have been considered in this study. Another important fact is that, from these cases in Table 4, the mean compressive strength of concrete varies from 32 to 74 MPa, and different reinforcement layouts have been used.

The experimental mean crack spacing values of the aforementioned experiments have been compared with the proposed models in Broms¹⁵ and Beeby and Scott,³¹ since they are based on the “no-slip” theory. EC2 and MC 2010 have models for calculating the “maximum crack spacing” values. Both the EC2 and MC 2010 models consider the maximum crack spacing to be 1.7 times the mean crack spacing, based on Braam⁷³ and CEB design manual,⁷⁴ respectively. Therefore, Table 4 consists of the mean crack spacing predictions of both EC2 and MC 2010, by dividing their maximum crack spacing predictions by the 1.7 factor. The predictions from the proposed models are based on Equations (2) and (3). In cases 6, 7, 11, 12, 14, and 15, the concrete cover thickness is larger than the clear distance between tensile reinforcements. Therefore, Equation (3) is used for those cases, while Equation (2) is used for the other cases.

When considering the error values in Table 4, the EC2 model predictions are considered to be more on the conservative side. This relatively high overestimation of the EC2 crack spacing model is noted in several studies.^{3,44,57} Except for four cases, the MC 2010 predictions are on the conservative side. However, compared to the EC2 predictions, the percentage of overestimations is lower in the MC 2010 estimations. Of all 18 cases, the predictions from Broms¹⁵ and Beeby and Scott³¹ have underestimated the experimental values in 15 and 13 cases, respectively. These two models are mainly developed for the axial tensile RC specimens consisting of a single embedded reinforcement. When considering the proposed model, four cases underestimate the experimental values. However, these underestimated percentages are still lower than 20%. Of the 18 cases mentioned in Tables 4, 12 cases have an “absolute error” value less than 20%. From the cases mentioned in Table 4, five cases have a concrete cover thickness larger than 60 mm (up to 90 mm). For these cases, the predictions of the proposed model give good agreement, where the absolute error values lie below 20%. Furthermore, as highlighted (light green) in Table 4, the proposed model gives the best fit for the majority of cases. When comparing the predictions of the aforementioned models, the relative best agreement for the experimental mean crack spacing values are obtained from the proposed Equations (2) and (3).

TABLE 4 Comparison of the experimental mean crack spacing values with the prediction models

Study	Case no.	Width × height × length (m × m × m)	Concrete cover (mm)	Clear distance between bars (mm)	No. of bars close to each surface	Exp. mean crack spacing (mm)	Predicted mean crack spacing (mm)					Error ^a %				
							Proposed equation ^b	Broms ¹⁵	Beeby and Scott ³¹	Proposed equation	Broms ¹⁵		Beeby and Scott ³¹	EC2 ⁴¹	MC2010 ³⁸	
Nautunna et al. ³	1	0.2 × 0.2 × 2	35	66	2	133	124	102	107	149	106	6.8	23.3	19.7	-12.0	20.3
Barre et al. ⁵⁵	2	0.355 × 0.355 × 3.2	65	175	2	200	204	155	198	391	347	-2.0	22.5	0.9	-95.5	-73.5
	3	0.355 × 0.355 × 3.2	45	108.5	3	174	187	106	137	314	257	-7.5	39.1	21.1	-80.5	-47.7
Tan et al. ⁴⁴	4	0.4 × 0.4 × 3	40	130	3	163	203	122	122	335	255	-24.5	25.2	25.2	-105.5	-56.4
	5		40	112	3	178	187	122	122	239	177	-5.1	31.5	31.5	-34.3	0.6
	6		90	80	3	217	193	274	275	435	314	11.1	-26.3	-26.5	-100.5	-44.7
	7		90	62	3	266	216	274	275	339	236	18.8	-3.0	-3.2	-27.4	11.3
Gracia and Caldentey ⁷²	8	0.35 × 0.45 × 5.22	32	79.3	4	162	178	76	98	240	182	-9.9	53.1	39.8	-48.1	-12.3
	9		32	74	4	147	171	80	98	203	151	-16.3	45.6	33.6	-38.1	-2.7
	10		32	62	4	115	157	89	98	163	119	-36.5	22.6	15.1	-41.7	-3.5
	11		82	40.6	4	220	206	180	250	477	352	6.4	18.2	-13.7	-116.8	-60.0
	12		82	28.6	4	184	206	189	250	365	260	-12.0	-2.7	-35.9	-98.4	-41.3
Rimkus and Gribniak ¹	13 ^d	0.15 × 0.15 × 0.5 ^e	30	33	3	100	111	68	92	119	84	-11.0	32.0	8.5	-19.0	16.0
	14 ^d		30	22	4	92	117	66	92	119	84	-27.2	28.3	0.5	-29.3	8.7
	15 ^d		30	28	4	73	117	65	92	132	94	-60.3	11.0	-25.3	-80.8	-28.8
	16		30	62	2	100	119	74	92	162	118	-19.0	26.0	8.5	-62.0	-18.0
	17		30	66	2	86	121	72	92	179	133	-40.7	16.3	-6.4	-108.1	-54.7
	18		30	70	2	113	123	70	92	204	152	-8.8	38.1	19.0	-80.5	-34.5

Note: In the pink colored cases, the concrete cover is larger than the clear distance between bars. Therefore, Equation (3) is used to predict the mean crack spacing values. The light-green colored cases show the closest value to the experimental mean crack spacing.

^aError = (Experimental Value - Predicted Value)/Experimental Value.

^bBroms¹⁵ Equation $S_{r,mean} = 2(c + \phi/2)$; where ϕ is the diameter of tensile reinforcement.

^cBeeby and Scott³¹ Equation $S_{r,mean} = 3.05c$.

^dSpecimens consist of several layers of reinforcement.

^eRimkus and Gribniak's specimens vary in length between 379 and 505 mm.

6 | CONCLUSION

This paper studies the crack spacing behavior of RC specimens with multiple bars, subjected to axial tension. The main identified crack spacing governing parameters for this study are concrete cover thickness and the clear distance between tensile reinforcements. To reduce the time, labor, and cost of laboratory experiments, finite element modeling has been used to study the cracking behavior. From the previous literature, it can be identified that a negligible amount of slip occurs between the reinforcement and concrete in axial tension. Therefore, a finite element model has been developed, considering a perfect bond between reinforcement and concrete. After calibrating and validating the FEM simulation models, they have been used to study the effect of concrete cover thickness and tensile bar spacing on crack spacing. The following are the most important findings identified from this study,

The smeared crack model with perfect bonding criteria can predict the crack spacing behavior of an RC specimen with multiple bars subjected to axial tension.

The developed FEM simulation models can predict the crack spacing behavior with different concrete cover thicknesses and changes in the spacing between tensile bars.

With the results of several virtual experiments, a “mean crack spacing model” could be developed for use in axial tensile specimens with several reinforcements.

This proposed model is developed from the results of 3D nonlinear FEM models, up to a concrete cover thickness of 122.5 mm. It has been proved that this model gives good predictions with the experimental results of RC specimens up to 90-mm cover thickness.

7 | FUTURE WORK

The proposed model is developed from the data of FEMs with four reinforcement bars. Then it has been improved for specimens with several bars. However, the effect of several tensile reinforcement layers is not considered for this model. This is a parameter of the Japanese code model, and when the highest overestimated case of the proposed model is considered (case 15 in Table 4 has two layers of tensile reinforcements respective to each face), the necessity of studying this parameter is apparent.

The parameter “ n ,” which is the number of reinforcements respective to the specimen face, is introduced to make the proposed model more applicable to general beams. However, this parameter makes it difficult to use this model for RC slabs. Therefore, future research is required to make this model more applicable to RC slabs.

Developing a mean crack spacing model is the first step to develop the maximum crack spacing model.

Widely used “maximum crack spacing models” have been developed by multiplying the mean “crack spacing model” with a factor that represents the ratio of maximum to mean crack spacing. EC2 and MC2010 consider this ratio to be 1.7. By considering the results of several recent axial tensile experiments, Naotunna et al.³ identified that this ratio varies between 1.2 and 1.7. However, Broms and Lutz³² and Beeby and Scott³¹ considered this parameter to be 2. Therefore, the next step is to decide on a suitable ratio for the maximum to mean crack spacing and develop an equation for “maximum crack spacing.”

This study has been conducted by using FEM simulations. Aghajanzadeh et al.⁷⁵, has modeled the concrete fracture process by combining both smeared crack approach with extended finite element (XFEM) method. Therefore, improved simulation models can be developed using XFEM to predict the crack spacings in RC specimens.

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CONFLICT OF INTEREST


The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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