Journal of Qualitative Research in Education

How Do Pre-service Elementary Teachers Notice Students' Algebraic Way of Thinking in Written Works?

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To cite this article:

Dogan Coskun, S. (2021). How do pre-service elementary teachers notice students' algebraic way of thinking in written works? *Journal of Qualitative Research in Education*, 27, 103-124. doi:10.14689/enad.27.6

Abstract: The purpose of this embedded-single case study was to examine pre-service elementary teachers' noticing expertise of students' algebraic thinking in written works considering three skills: attention to students' solutions, interpretation of students' solutions, and deciding how to respond to students' solutions. The participants in this study involved 32 pre-service teachers who were enrolled at an Elementary Teacher Education Program in a public university in Turkey. The data were utilized by pre-service elementary teachers' responses to four students' solutions to a figural pattern task and were analyzed using the framework developed by Jacobs et al. (2010). The analysis indicated although the pre-service teachers could not provide robust evidence of attention and interpretation, they could be able to provide robust evidence of deciding how to respond. Specifically, the percentage of pre-service teachers demonstrating robust evidence was greatest in the skill of deciding how to respond, then interpreting, with attending having the lowest percentage of pre-service teachers demonstrating robust evidence.

Keywords: Algebraic thinking, noticing expertise, pre-service elementary teachers

Article Info

Received: 30 Apr. 2020
Revised: 10 Oct. 2020
Accepted: 17 Feb. 2021

Article Type

Research

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Declaration of Conflicts of Interests: None

Introduction

The National Council of Teachers of Mathematics [NCTM] (2014) states that "Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (p. 53). In the same way, Barnhart and van Es (2015) emphasize that effective teachers are able to use students' thinking "to make subsequent pedagogical decisions" (p. 84). Actually, this expertise is called professional noticing and defined as "making sense of how individuals process complex situations" (Jacobs et al., 2010, p. 171). This expertise is a collection of three specific skills which are *attending, interpreting*, and *deciding how to respond*. That is, teachers not only need to attend and interpret students' ideas but also need to decide how to respond to help students improve their mathematical understanding. However, considering the characteristics of a mathematics classroom, it is nearly certain that there would be several ideas students suggest. Without knowing what to attend among the suggested ideas, it is difficult to contribute to students' future success in mathematics (Erickson, 2011; Kilpatrick et al., 2001). To be able to recognize noteworthy or important ideas and use these ideas for the remaining part of instruction, teachers need to have noticing expertise (Jacobs et al., 2010; Star & Strickland, 2008). Furthermore, what teachers notice is foundational for how they act throughout instruction (Schoenfeld, 2011). From this point of view, the purpose of this case study is to examine pre-service elementary teachers' noticing expertise of students' thinking associated with algebra.

Noticing Expertise

Teachers need to actively look and see to be able to notice things that they are not normally aware of (Sherin & Star, 2011). In the same way, Mason (2011) defines noticing as "a collection of practices designed to sensitize oneself so as to notice opportunities in the future in which to act freshly rather than automatically out of habit" (p. 35). These explanations indicate that noticing is not a passive process, but rather an intentional and active process. With this emphasis, researchers have defined teacher noticing in different ways. According to Miller (2011), teacher noticing refers to not only monitoring students while they are working on a task or presenting their ideas but also understanding what pops into these students' minds. Focusing on mathematics teaching, noticing is the ability of a teacher to recognize the mathematical details that emerge while students are engaged in problem-solving activities (Sánchez-Matamoros et al., 2019). That is, teacher noticing is not just determining whether or not the students' answers to problems are correct (Wilson et al., 2013). Instead, it is a specific ability for teachers to notice details that other professions may not notice (Stevens & Hall, 1998). Additionally, points that are noticed by a teacher may be different from the ones noticed by another teacher. To explain a possible reason for this difference, van Es and Sherin (2002) state that noticing expertise is more than identifying noteworthy aspects of instruction. It is also about using their knowledge about the identified aspects to reason about and make connections. Considering its importance, Sherin et al. (2011) accept noticing expertise as one of the core activities of mathematics teaching.

Deciding how to respond was added to the noticing expertise in addition to the skills of attending to and interpreting of students' understandings by Jacobs et al. (2010) in their framework of professional noticing of children's mathematical thinking. The first of these skills, attending, is the extent to which teachers attend to specifics or mathematical essence of children's thinking. Interpreting, the second skill, is the extent to which teachers understand and grasp what children did and why they did (Jacobs et al., 2010). The final skill, deciding how to respond, is about both the extent to which teachers use and benefit from the children's thinking throughout instruction and how teachers' reasoning is consistent with the related literature (Jacobs et al., 2010). There are other researchers who include the skill of responding in their noticing frameworks as well (Erickson, 2011; Santagata et al., 2007). However, these researchers explain that a teacher decides how to respond after they attend to and interpret a student's understanding. Contrary to this statement, to emphasize the relationship among these skills, Jacobs et al. (2010) state that teachers constantly and almost simultaneously consider what their next moves would be. Therefore, for this study, Jacobs et al.'s framework (2010) was used to investigate pre-service elementary teachers' noticing expertise of students' algebraic thinking in written works.

Algebraic Thinking

One of the important and core concepts of mathematics, algebra, is necessary to conceptually and deeply learn and understand mathematics (Blanton & Kaput, 2005). Specifically, Mason (2008) conceptualizes algebra as "a succinct and manipulable language in which to express generality and constraints on that generality" (p. 77). In the same way, algebra is accepted as one of the ways to represent relationships among quantities or formalize these relationships (Chazan, 2000; Kaput, 1999; NCTM, 2000). Usiskin (1988) accepts "algebra as generalized arithmetic" to emphasize the relationship between algebra and arithmetic (p. 11). Furthermore, contrary to the researchers who conceive algebra as the use and manipulation of symbols or letters, Kieran (2004) mentions that students can engage in algebraic activities without using any symbols or letters, which promotes students' algebraic thinking. Actually, Kieran (1992) asserts that using symbols or letters may be really difficult for most of the students. To help students succeed in abstract form of algebra, mathematics programs include algebra starting from early grades (Ministry of National Education [MoNE], 2018; NCTM, 2000). As it can be expected, including algebra in programs is not enough to develop students' abilities to think algebraically. Teachers need to ask their students to use different forms of representations such as drawings and models (Brizuela & Schliemann, 2004). Teachers also need to encourage their students to express their thinking related to figural patterns or communicate mathematically for number sequences (Mason, 2008; Radford & Sabena, 2015). To be able to succeed and hence to make the transition from arithmetic to algebra easier, teachers need to know how to attend to, interpret, and decide how to respond to students' thinking.

By recognizing the importance of teacher noticing in student learning, there have been a number of studies that focused on examining or improving pre/in-service teachers' noticing of students' mathematical thinking in recent years. One of the tools that were used to examine teachers' noticing expertise of students' mathematical thinking was students' written works (Fernández et al., 2013; Goldsmith & Seago, 2011; Ivars et al., 2018; Sánchez-Matamoros et al., 2019). Using students' written works in noticing studies is important as it helps teachers focus on students' mathematical thinking rather than teachers or their pedagogy. Furthermore, students' written works allow teachers to see instruction from students' eyes and consequently support teachers in learning to notice students' mathematical thinking. Throughout this process, teachers carefully consider students' written works and try to interpret how they arrived at their answers, and thereby decide how to respond to them (Stockero, 2014).

There are also studies examining pre-service teachers' noticing expertise of students' algebraic ways of thinking using students' written works. (Callejo & Zapatera, 2017; Magiera et al., 2013; Mouhayar, 2019; Mouhayar & Jurdak, 2013; Simpson & Haltiwanger, 2016). In these studies, the students' written works were supplied by the instructor of the course from the related literature or were hypothetically written by the authors. Contrarily, all the written works used in this study were the real solutions of students to a figural pattern task. Furthermore, students' solutions including different reasoning for algebraic thinking were included in the study to see if these differences affect teachers' noticing expertise. The above-mentioned studies focused on pre-service teachers' attending and interpreting skills of noticing expertise. In addition to these skills, the present study also considered pre-service teachers' deciding how to respond skill of noticing expertise. Since there are not any studies including all three skills of noticing expertise of students' algebraic thinking, the findings reported in this study would expand the noticing literature. Furthermore, the findings of this study would allow researchers or teacher educators to see in which skill(s) pre-service elementary teachers' levels are lower. This study also provides pre-service teachers who will teach in the future with opportunities to be familiar with the real solutions of students. Additionally, the findings would help in-service teachers gain insight into their algebraic thinking and offer significant insights for teachers to be aware of how they can respond to students to improve their algebraic thinking. In parallel with these aims and contributions, the research questions guiding this study were given below:

1. To what extent do pre-service elementary school teachers attend to and interpret students' algebraic thinking in written works?
2. What is the nature of decisions pre-service elementary teachers make to respond to students' algebraic thinking in written works?

Methodology

Research Design

A case study is an in-depth understanding of an issue, a person, or a group of people in a specific context (Creswell, 2007). Besides, a case study does not aim to generalize results, rather it aims to gain insight or improve knowledge about the case by studying it in its own context (Yin, 2009). In this regard, to gain insight into pre-service elementary teachers' noticing expertise of students' algebraic thinking, a case study was used in this study. The case in this study was 32 pre-service elementary teachers, and the context was the Elementary Teacher Education program in the Central Anatolia Region of Turkey. As noted by Yin (2009), a single case study may involve more than one unit of analysis. Although the main unit of analysis was the pre-service elementary teachers' noticing expertise of students' algebraic thinking, the skills of attending to, interpreting, and deciding how to respond were the sub-units of analysis. Therefore, the design employed in this study was an embedded-single case study.

Context and Participants

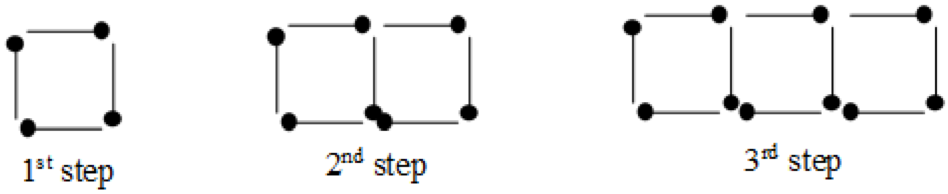
Since this study examined pre-service elementary teachers' noticing expertise of students' algebraic thinking, selecting the cases that met some criteria was important (Creswell, 2007). The criteria for being included in this study were being enrolled in the author's section of Methods of Teaching Mathematics II course and volunteering to be a part of the study. The Methods of Teaching Mathematics II course was the last of three required courses related to mathematics for the pre-service elementary teachers in the Elementary Teacher Education program. This is important as it can be assumed that the pre-service teachers were familiar with instructional methods and strategies, and were aware of importance of students' thinking in mathematics. Therefore, within the Methods of Teaching Mathematics II course, 32 pre-service elementary teachers who agreed to participate were purposefully selected for this study.

Data Collection and Analysis

To examine pre-service elementary teachers' noticing expertise of students' algebraic thinking, students' solutions to a figural pattern task, taken from an existing study (Rivera & Becker, 2003), were used. Students can use different ways to correctly solve the figural task without necessarily finding a general rule. Semi-structured interviews had been designed to gather information about students' solutions to this figural pattern task. The task had been provided to 10 third grade students during the interviews, and the students had been asked to explain their thinking after they solved the task. Throughout the process, to allow the students to clarify how they solved or what they thought, the author had asked some questions such as "Why did you multiply 3 by 22?" "Why did you subtract 24 from 100?" or "How did you conclude that the 24th step would be 80?" These questions were also important for preventing the author from misinterpreting the students' solutions. Furthermore, before conducting the interviews, the task had been shared with two mathematics educators to decide whether or not the task had been appropriate for the purpose. After applying the task to the elementary students, four of the students' solutions that included important details for algebraic thinking were selected for the pre-service teachers. The students' solutions also differed in considering their accuracy and ways of reasoning. This issue is important as Jacobs et al. (2010) emphasize that teachers' attention, interpretation, and responses, which constitute the noticing expertise, can differ according to correctness or incorrectness of solutions. In the same way, students' understandings of a concept are mostly reflected in their strategies used to solve a task (Jacobs et al., 2010). That is the task and the students' solutions were strategically selected for the current study. The task and the students' solutions were given below:

Figure 1.

The Task and The Students' Solutions to The Task

<table>
<tr><th colspan="4">The Task</th></tr>
<tr><td colspan="4">In the figures below, the first three of the steps of a pattern formed by using matchsticks were given.


According to the steps, find the total number of matchsticks in the 25th step.</td></tr>
<tr><th colspan="4">The Solutions Given to the Task</th></tr>
<tr><th>Student A</th><th>Student B</th><th>Student C</th><th>Student D</th></tr>
<tr><td>4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70, 73, 76

76 matchsticks will be used in the 25th step.</td><td>22×3=66

10+66=76

In the 25th step, there will be 76 matchsticks.</td><td>25×3=100

100 – 24=76

76 matchsticks in the 25th step.</td><td>3rd step: 10

24th step: 80

1st step: 4

84 matchsticks in the 25th step.</td></tr>
</table>

The pre-service teachers were each given four student solutions to analyze and were asked to explore how each student solved the task through the questions. Specifically, the questions included "(1) Determine whether each of the student's way of thinking is true or false. Explain how you have decided the correctness of each student's way of thinking by providing evidence from the student's solution. (2) Explain how each student has solved the task considering the student's solution. Explain what each student knows and does not know about algebraic thinking considering the student's solution. (3) If you were the teacher, how would you respond to each student?" This process was also piloted with ten pre-service elementary teachers who were enrolled in the other section of the Methods of Teaching Mathematics II course and not the participants of the current study.

To make meaning out of the pre-service elementary teachers' responses to the above-given questions, first of all, a coding schema described in Jacobs et al.'s framework of professional noticing of children's mathematical thinking (2010) was adapted for this study. Merriam (2009) states that data analysis is the process of "consolidating, reducing, and interpreting what people have said and making meaning out of it" (p. 176). By means of the schema, a level for each pre-service elementary teacher's response for each skill of attending, interpreting, and deciding how to respond was examined. Specifically, the noticing levels of each of the skills of the noticing expertise and the description of these levels were detailed in Table 1 below.

Table 1.

The Coding Schema for Analysis of Noticing Level of Attending, Interpreting, and Deciding How to Respond to Skill

Skill of Noticing Expertise	Noticing Level	Description of Level
Attending	Robust Evidence	• Identifying whether or not the solutions are correct by providing mathematical details for the solutions. • Providing specific evidence from the student's solution.
	Limited Evidence	• Identifying whether or not the solutions are correct by providing general statements for the solutions.
	Lack of Evidence	• Identifying the solutions incorrectly.
Interpreting	Robust Evidence	• Interpreting mathematical details of the solution. • Providing specific evidence about how the student thinks. • Recognizing what the student knows or does not know.
	Limited Evidence	• Interpreting the student's solution correctly but with less depth or with general statements. • Making relevant connections to the student's solution but without going beyond the solution provided.
	Lack of Evidence	• Interpreting the student's solution incorrectly. • Providing wrong evidence about how the student thinks. • Not providing any evidence of the student's understanding. (Not providing specifics about how the student was thinking.) • Making irrelevant connections to the student's solution.
Deciding how to respond to	Robust Evidence	• Asking another problem to invite the student to use another strategy in addition to the used strategy. • Asking a question to extend the student's understanding. • Providing another solution to help the student discover that the task can be solved in a different way.
	Limited Evidence	• Asking questions to understand the student's thinking.

		• Responding to the student with building on the student's understanding but with less depth or with general statements. • Offering similar future steps for the students whose solutions are different from each other.
	Lack of Evidence	• Appreciating the student. • Responding to the student without building on the student's understanding. • Asking a similar problem with different wording to make the student practice. • Changing the numbers given in the problem to make it more difficult. • Providing unrelated responses as if the student's solution was not examined.

Based on the coding schema given above, the pre-service elementary teachers' responses for each noticing expertise skill were coded under three levels of noticing: robust, limited, and lack of evidence similar to that of LaRochelle et al. (2019). To be able to easily and correctly code these levels, some key terms for each level of the skill of the noticing expertise were identified. Specifically, for the skill of attending, the mathematical details within each student's solution were identified in addition to the correctness of each student's solution. That is, identifying the student's solutions as correct or not correct was not enough to be coded under robust evidence. The pre-service teachers also needed to provide mathematical details within and specific evidence from the student's solution to be coded under robust evidence. If the pre-service teachers identified the correctness of each student's solution but described each of them in a short way, their responses, then, were coded under limited evidence. Contrary to the above-mentioned two levels, the pre-service teachers whose responses were coded under lack of evidence could not identify the correctness of each student's solution.

For the interpreting skill, making sense of the student's solution to be able to explain why the student solved the task in that way or what the student did not know was necessary to be coded as robust evidence. The responses that include connections without going beyond or redescription of the students' solutions are coded as limited evidence. If the pre-service teachers could not interpret students' solutions correctly, could not provide correct evidence about how the students think, or did make irrelevant connections to the students' solutions, then their responses were coded under lack of evidence.

Lastly, for the skill of deciding how to respond, the responses trying to extend the student's understanding were coded as robust evidence. In the responses with limited evidence, the pre-service teachers asked questions to understand what the students think or responded to the students without aiming to extend their understanding. Finally, the responses in which the pre-service teachers just appreciated the students, provided unrelated responses to the students, or asked similar problems with different wording were placed under lack of evidence. Throughout this process, the evidence from the pre-service elementary teachers' statements were also noted to make the question of why their responses were coded under this level clear.

To ensure the trustworthiness of the study, thick description of the pre-service teachers, the task, the students' solutions to the task, and the data collection and analysis process were provided. Additionally, the author and a mathematics educator separately coded the pre-service teachers' responses to three questions provided above in terms of the noticings levels. Throughout this process, if any discrepancy exists among the coders, it was discussed until reaching a consensus. Having a mathematics educator having expertise and experience in teacher noticing to code the pre-service teachers' responses ensured the reliability of the study. The inter-rater reliability was calculated at 90% or higher. Furthermore, direct quotations from the pre-service teachers' responses to the students' solutions were included. Throughout this process, their explanations were reviewed repeatedly to reduce any inaccuracies or misinterpretations.

Findings

Since this study examined the pre-service elementary teachers' noticing expertise of students' algebraic thinking considering the specific skills: attending, interpreting, and deciding how to respond, the findings were presented in three parts. Throughout these three parts, the frequency of the noticing levels of the skills, as well as the excerpts from the pre-service teachers' papers, were provided.

Skill of Attending to Students' Algebraic Thinking

Table 2 shows the overall frequency of each pre-service elementary teacher's response coded as robust, limited, or lack of evidence among the four students' solutions.

Table 2.

The Frequency and Percentage of Pre-service Elementary Teachers' Responses for Each Level of Attending Skill for Each Student

Students / Level	Student A	Student B	Student C	Student D	Total
Robust Evidence	7 (21.88%)	6 (18.75%)	7 (21.88%)	7 (21.88%)	27 (21.09%)
Limited Evidence	25 (78.12%)	16 (50%)	22 (68.75%)	25 (78.12%)	88 (68.75%)
Lack of Evidence	0	10 (31.25%)	3 (9.37%)	0	13 (10.16%)

N=32
*The solutions of Student A, Student B, and Student C are correct; the solution of Student D is not correct.

As shown by Table 2, all pre-service elementary teachers were able to demonstrate at least limited evidence of attending to the solutions of Student A and Student D. In addition, the analysis of the pre-service elementary teachers' responses revealed that

21.88% of them demonstrated robust evidence of attending to these two students. This finding is important because Student A's solution is correct, while Student D's solution is not correct. A closer look at these responses showed that the pre-service elementary teachers did not just describe the solutions of Student A and Student D; they also provided mathematical details and specific evidence from their solutions regardless of the solutions' correctness. Below is an excerpt from a pre-service elementary teacher's response about Student A's solution:

> The student correctly found the number of matchsticks that would be used in the twenty-fifth step by adding three to the number of matchsticks in the previous step one by one. I mean, s/he wrote it as 13, 16, 19, … 70, 73, 76 for the fourth, fifth, sixth, … twenty-third, twenty-fourth, and twenty-fifth step, respectively. That is, the student just listed the numbers by increasing the number of matchsticks by three in each step without making a generalization. (PST12)

Similarly, the following response given by PST1 for Student D is an example of robust evidence of attention:

> The student counted the number of matchsticks in the third step and wrote it as 10. Then, the question asks the number of matchsticks in the twenty-fifth step. So, the student thought that if s/he multiplies the third step by 8 to find the twenty-fourth step, s/he has to multiply the number of matchsticks in the third step by 8 which is equal to 80 matchsticks. Then, s/he added the number of matchsticks in the first step, 4, to 80, and found the number of matchsticks in the twenty-fifth step as 84. However, the student did not consider that some of the matchsticks in this solution were counted more than once. Therefore, the student's solution is wrong.

Table 2 also shows that there were six pre-service teachers who demonstrated robust evidence of attention to Student B, and seven pre-service teachers for Student C. One of the responses provided for Student B is presented below:

> Instead of adding 3 each time to find the next step and writing 13, 16, 19, … until the twenty-fifth step similar to Student A, s/he multiplied 22 by 3 and found the number of matchsticks as 66. Actually, the student found how many matchsticks would increase until the twenty-fifth step. However, there were 10 more matchsticks in the third step. So, the student added these two numbers and found 76. The student's reasoning is correct. (PST20)

Another example of robust evidence of attention was provided by PST7 to Student C's solution. The following excerpt demonstrates how PST7 attended to the solution.

> Student C identified that the step number was equal to the number of squares. Therefore, s/he multiplied 25 by 4 as each square has four equal sides, matchsticks in this task. However, s/he also noticed that there were common matchsticks. That is, for example, there was 1 common matchstick in the second step; were two matchsticks in the third step, and would be 24 common matchsticks in the twenty-fifth step. Therefore, s/he subtracted 24 from 100 and concluded that for the twenty-fifth step, there would be 76 matchsticks which is correct.

As can be seen from the above responses demonstrating robust evidence of attention, these pre-service teachers were able to explain in detail how Student A, Student B, and Student C correctly found the total number of matchsticks in the twenty-fifth step, and why Student D's solution was not correct.

On the other hand, there were also some others who could not correctly attend to the solutions of Student B and Student C. There were differences in the percentage of pre-service elementary teachers who demonstrated robust, limited, and lack of evidence of attending to the solutions of Student B and Student C. Specifically, the analysis of the pre-service elementary teachers' responses revealed that they were able to attend to Student B's solution 18.75% with a robust, 50% with a limited, and with 31.25% with a lack of evidence. Similarly, 21.88% of the pre-service elementary teachers demonstrated robust evidence of attending to Student C's solution compared to 78.12% of the pre-service elementary teachers who demonstrated limited or lack of evidence of attending. The percentage of pre-service elementary teachers who demonstrated lack of evidence of attending was higher for Student B compared to the other students. To be more specific, these pre-service elementary teachers could not even find out Student B's solution's correctness. Therefore, they could not correctly attend to and report what Student B did to find the number of matchsticks in the twenty-fifth step. As an example, PST16's response to Student B is given below:

> I could not understand how Student B solved the task.

In the same way, the following excerpts taken from the pre-service elementary teachers' responses are the examples of lack of evidence of attending to Student B's solution:

> The student might have counted the total number of matchsticks in the first, second, and third steps, and found as 22 instead of 21. Since the student discovered that the number of matchsticks increases by 3 between the consecutive steps, s/he multiplied 22 by 3 and found 66. Then, s/he added the number of matchsticks in the third step, 10, to 66, and found 76. However, the student accidentally counted the total number of matchsticks in the first three steps, the solution is not correct. (PST5)

> The first, second, and third steps were given in the task. The student identified the difference between the consecutive steps was 3 and concluded that it would increase by 3. So, s/he thought that s/he had to multiply 22 (the difference between the twenty-fifth and the third steps) by 3. However, I could not understand where the number 10 came from or why the student added 10 to 66. I think the student added these numbers just to find 76. So, the student's solution is wrong. (PST14)

Just as the previous excerpts indicated, all of the pre-service elementary teachers who demonstrated lack of evidence of attention stated that the solution of Student B was incorrect, although Student B correctly solved the task in a different way compared to Student A and Student C. Additionally, three pre-service elementary teachers whose responses were coded as lack of evidence could not provide any explanation for the solution of Student C.

Skill of Interpreting Students' Algebraic Thinking

The overall frequency of each pre-service elementary teacher's response coded as robust, limited, or lack of evidence of interpreting among the four students' solutions is provided in Table 3 below.

Table 3.

The Frequency and Percentage of Pre-service Elementary Teachers' Responses for Each Level of Interpreting Skill for Each Student

Students / Level	Student A	Student B	Student C	Student D	Total
Robust Evidence	12 (37.5%)	5 (15.62%)	6 (18.75%)	10 (31.25%)	33 (25.78%)
Limited Evidence	17 (53.13%)	14 (43.75%)	18 (56.25%)	17 (53.13%)	66 (51.56%)
Lack of Evidence	3 (9.37%)	13 (40.62%)	8 (25%)	5 (15.62%)	29 (22.66%)

N=32
*The solutions of Student A, Student B, and Student C are correct; the solution of Student D is not correct.

As displayed above in Table 3, more than 30% of the pre-service elementary teachers provided robust evidence of interpretation for Student A and Student D. As mentioned in the skill of attending to students' algebraic thinking part of this paper, while the solution of Student A is correct, the one of Student D is not correct. Although there is a slight difference between the percentage of pre-service elementary teachers' responses for Student A and Student D, it can be stated that these pre-service teachers interpreted both correct and incorrect solutions with robust evidence. One of the responses demonstrating robust evidence of interpretation of Student A's solution is presented below:

> The student discovered that the pattern in the task can be written as a number pattern that increases by three starting from four. However, the student may not know how to find a rule relating the number of matchsticks to the step number. I mean, when we ask him/her to explain the pattern, s/he would probably be able to explain that the number of matchsticks increases three by three in each step and count in threes until the twenty-fifth step. Although the student could correctly count, s/he could not make a generalization. (PST4)

Another example of robust evidence of interpretation provided by PST19 for Student D is as follows:

> The student's solution is interesting compared to the other students' solutions. The student accepted the number of matchsticks in the third step as a unit and calculated the multiples of this unit to find the number of matchsticks in the twenty-fifth step. Although the student started with his/her solution with correct reasoning, s/he did forget or did not consider that some of the matchsticks in the figures were common. Therefore, when the unit, 10 in his/her solution, was multiplied by 8, some of the matchsticks were counted more than once. If s/he had found the number of common matchsticks and subtracted this number in the final step of his/her solution, then the solution would have been correct.

As provided in Table 3, less than 20% of the pre-service elementary teachers provided robust evidence of interpretation for Student B and Student C as well. Below the excerpts are examples of the responses including robust evidence of interpretation for Student B and Student C, respectively.

> The student recognized that the number of matchsticks starts from 4 and grows by adding 3 step by step. S/he also discovered that s/he needs to add 3 for 22 times. Therefore, instead

> of adding three by three, s/he thought that the task can be solved by multiplication as s/he knew that multiplication could be used for repeated addition. (PST11)
>
> The student saw the task as a figural pattern task and recognized that the matchsticks form squares in each step. Therefore, s/he accepted each step of the pattern as if it consisted of squares. She found the total number of matchsticks in the twenty-fifth step using these squares. Furthermore, s/he identified that there would be 24 common matchsticks in the twenty-fifth step. I mean, she did not forget to subtract the number of common matchsticks. (PST6)

As understood from these excerpts, the pre-service teachers whose responses coded as having robust evidence of interpretation were able to make sense of students' solutions with reasoning by providing specific evidence from their solutions.

Contrary to the responses demonstrating robust evidence of interpretation of the solutions of Student A and Student D, there were also responses coded as lack of evidence of interpretation. Specifically, these responses did not include any evidence of how Student A or Student D thought in the solution process or did make irrelevant connections to Student A's or Student D's solutions. Two of the examples of interpretation with lack of evidence for Student A and Student D, respectively, are given below:

> The student needed to write to correctly find the number of matchsticks in the twenty-fifth step. (PST9)
>
> The student could not solve the task correctly. (PST30)

Moreover, the number and percentage of the responses demonstrating lack of evidence of interpretation were higher for the solutions of Student B and Student C than the ones of Student A and Student D. Specifically, while the percentage of pre-service teachers' interpretation with lack of evidence for Student B is 40.62%, the percentage is 25% for Student C. These pre-service teachers did not or could not correctly interpret Student B's and Student C's thinking reflected in their solutions. Furthermore, some of the pre-service teachers' interpretations were not relevant to these students' solutions. For a more specific example of how these pre-service teachers interpreted Student B's solution, PST13 stated:

> The student multiplied 22 by 3 and added 10 to the result to find 76 as an answer. The total number of the matchsticks in the twenty-fifth step is 76 right; however, the student accidentally found this number. I mean, there is not logic underlying the solution. Instead, that is, if s/he could not solve the task by finding a rule, step by step, s/he could have written the total number of matchsticks in each step similar to Student A.

The below excerpt is an example of the response demonstrating lack of evidence of interpretation of Student C's solution.

> At first, I was surprised by the student's solution as s/he wanted to look at the pattern different than the other students; however, s/he could not succeed. After the student found 100 as a result for the multiplication of 25 by 4, s/he should have subtracted the total number of matchsticks in the first three steps which was 21 from 100 to correctly find the result. (PST10)

Clearly seen from the above excerpts, either because of the pre-service elementary teachers provided little to no evidence from the students' solutions or their lack of making relevant connections, their responses were coded as lack of evidence of interpretation.

Skill of Deciding How to Respond to Students' Algebraic Thinking

Table 4 provides the overall frequency of each pre-service elementary teacher's response coded as robust, limited, or lack of evidence of deciding how to respond to the four students' solutions.

Table 4.

The Frequency and Percentage of Pre-service Elementary Teachers' Responses for Each Level of Deciding How to Respond Skill for Each Student

Level / Students	Student A	Student B	Student C	Student D	Total
Robust Evidence	12 (37.5%)	4 (12.5%)	3 (9.37%)	16 (50%)	35 (27.34%)
Limited Evidence	13 (40.62%)	9 (28.13%)	8 (25%)	10 (31.25%)	40 (31.25%)
Lack of Evidence	7 (21.88%)	19 (59.37%)	21 (65.63%)	6 (18.75%)	53 (41.41%)

N=32
*The solutions of Student A, Student B, and Student C are correct; the solution of Student D is not correct.

As indicated in Table 4, all of the pre-service teachers provided a response to each student; however, they did it at different levels. Specifically, the percentage of pre-service teachers who demonstrated robust evidence is higher for Student A and Student D rather than for Student B and Student C. More specifically, among the four student solutions, the percentage of the responses at the robust level is the highest for Student D's solution. This finding means that the pre-service teachers decided to respond to the student with an incorrect solution at a more robust level compared to the students with correct solutions. To better exemplify the details of the responses at the robust level provided for Student D, an example of them is given below:

> The student has difficulty in recognizing that the adjacent edges of the squares are common. Therefore, instead of directly making this explanation to the student, I would ask questions such as "Can you find the number of matchsticks in the third step by counting?" "How many matchsticks are there in the third step using your rule?" "How many matchsticks will be in the fourth step using your rule?" "Can you draw the figure for the fourth step and check it if your answer is correct? "Can you compare the results that you found by counting and by your rule?" "What may be the reason for this difference?" By means of these questions, I can help the student recognize why his/her solution is not correct. Furthermore, I can understand how s/he solved the task or what s/he thought while solving the task. According to his/her answers, I would ask other questions to direct the student to discover other ways to find the number of matchsticks in the twenty-fifth step. (PST15)

Another example of a response at the robust evidence was provided by PST8 for Student A. PST8 wrote the following in response to how would you respond to Student A to help him to notice the points he did not know:

> I would tell him "you found the number of the matchsticks in the twenty-fifth step by adding threes like 4, 7, 10,… Your answer is correct". Then, I would ask the question of "How many matchsticks will be in the hundredth step?" or "Can you find the number of matchsticks in the thousandth step by counting?" I try to help him feel that it would be a waste of time if he counted similar to the way that he did. I would tell him to examine a relationship between the step number and the number of matchsticks in that step to easily find the number of matchsticks in any step. I do not know the student's level, but I may try to show the student how s/he can use the t-table to solve pattern tasks.

While 12.5% of the pre-service teachers responded at the robust level to Student B, 9.37% of them responded at the robust level to Student C. One of the four responses provided for Student B is given below:

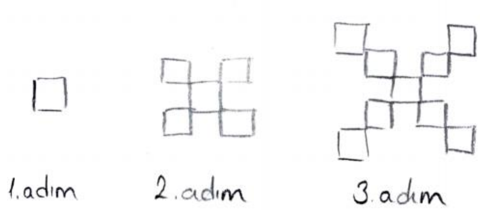
> I would ask the student why s/he specifically takes the third step into consideration. If I correctly understood the solution, s/he thought that since there were twenty-two steps until the twenty-fifth step, s/he had to multiply twenty-two by three and the number of matchsticks in the third step to find the number of matchsticks in the twenty-fifth step. I would ask him/her if he can take the second step into consideration and find the same result. If my understanding is correct, the student will say that there were twenty-three steps between the second and twenty-fifth steps. Then, s/he will multiply twenty-three by three and find 69, and finally, s/he will add 7, the number of matchsticks in the second step to 69 to find 76, the number of matchsticks in the twenty-fifth step. By means of this solution, s/he will recognize that s/he does not have to take the last step given in the task. Furthermore, I do not know how s/he noticed the pattern increases by three. I mean, "Did the student write as a number pattern like Student A?" or "Did he focus on the pieces of the figures in each step?" If he focused on the pieces, I would ask some questions to lead him to consider other pieces so that s/he can solve the task in a different way. (PST19)

Below is another example in the same level, robust evidence, excerpted from PST13's response for Student C:

> I would tell the student that "You saw that each step consisted of squares and the number of squares in each step is the same as the number of steps. Therefore, you multiplied 25 by 4 and found 100. Can you explain why you subtracted 24 from 100?" Actually, I know that the student subtracted 24 as he counted 24 matchsticks twice; however, I would like to listen his/her reasoning. If s/he indicates that s/he subtracted the common matchsticks, then I can ask the question of "How would the figures in each step be like, there would not be any common matchsticks, or you would not need to subtract the common matchsticks?" I really wonder if s/he can draw such a pattern. But, if s/he cannot draw, I would draw the below figure and help him to see the difference between the pattern given in the task and the pattern given below.

Figure 2.

The figural pattern task asked by PST13



As indicated in the above excerpts, the pre-service elementary teachers who demonstrated robust evidence first referred to the students' solutions and understanding by providing direct evidence from their solutions. Then, they asked questions to extend their understanding or to help them use other strategies to solve the same task.

Contrary to the pre-service teachers whose responses were at the robust level, there were also pre-service teachers who responded to the students with the lack of evidence level. As evidenced in Table 4, nearly 20% of the responses for Student A and Student D were at the lack of evidence level. Specifically, the responses provided for Student D were similar to the excerpt below in that the pre-service teachers explained that although they knew the solution of Student D was not correct, they did not know where the error was. One of these responses is as follows:

> I could not identify the step in which the student made an error. Therefore, I may ask the student to check his/her steps. (PST2)

The percentage of the responses at the lack of evidence level was much higher for Student B and Student C with 59.37% and 65.63%, respectively. The reason for these high percentages is that the pre-service teachers just appreciated the students whose solutions were correct. The followings are three sample responses provided for Student A, Student B, and Student C, respectively.

> Good job. (PST1)
>
> Your solution is correct. (PST17)
>
> You correctly found the result. I appreciate you. (PST22)

As can be understood from these excerpts, the pre-service teachers whose responses at the lack of evidence level did not consider the students' algebraic thinking; and hence, did not provide different responses to them despite the differences in their reasoning.

Discussion, Conclusion, and Suggestions

This study examined the nature of pre-service elementary teachers' noticing expertise of students' algebraic thinking in written works. Considering the purposes of this study, noticing expertise included three skills described by Jacobs et al. (2010): (1) attending, (2), interpreting, and (3) deciding how to respond. Focusing on these skills separately enabled the author to determine the skills in which pre-service elementary teachers could or not be able to demonstrate robust evidence.

With respect to the first skill of noticing expertise, the findings indicated that more than half of the pre-service teachers attending at a limited level made correct judgments for all of the students' solutions. This finding is similar to those of other researchers who explained that pre/in-service teachers were good at attending to students' solutions (Callejo & Zapatera, 2017; LaRochelle et al., 2019; Sánchez-Matamoros et al., 2019). However, nearly 20% of the pre-service teachers were able to attend to the students' solutions at robust level. That is, only these teachers justified their reasons and supported their justifications with mathematical details from the students' solutions. Most of the pre-service teachers in this study had difficulties in providing evidence from the students' solutions similar to the related studies (Jacobs et al., 2010; Goldsmith & Seago, 2011). The reason for this difficulty might result from the pre-service teachers not knowing what was mathematically important in the students' solutions (Schoenfeld, 2011; Sherin, 2007). In the same way, Star and Strickland (2008) emphasize that teachers may not be able to attend to students' solutions because of their lack of mathematical knowledge. Therefore, it can be concluded that the pre-service teachers demonstrating limited evidence of attention may have some limitations in their mathematical knowledge which has an impact on their noticing skills.

In addition, the percentage of pre-service teachers who provided robust evidence was nearly equal for all of the students which means that they were able to attend regardless of the correctness of these students' solutions. On the other hand, there was a difference among the percentage of pre-service teachers who demonstrated lack of evidence for the students whose solutions were correct. The responses demonstrating lack of evidence of attention were actually only provided for two out of three correct solutions: the solutions of Student B and Student C. This finding is not entirely surprising despite the fact that pre-service teachers attend to students' solutions regardless of these solutions' correctness (Jacobs et al., 2010). To be more specific, Mouhayar (2019) explains that teachers' level of attention differs according to the students' strategies and solutions. Furthermore, being familiar with these strategies or solutions has an effect on the level of teachers' attention (Goldsmith & Seago, 2011). Considering these explanations, it might be stated that the pre-service teachers who provided lack of evidence of attention were not familiar with the solutions provided by Student B and Student C.

Similarly, most of the responses demonstrating lack of evidence of interpretation were provided for the same two students. Additionally, the percentages of these responses were greater than the percentages of the responses demonstrating lack of evidence of attention. Consistent with previous studies (Barnhart & van Es, 2015; Goldsmith & Seago, 2011; Sánchez-Matamoros et al., 2019), this means that although some of the pre-service teachers identified the correctness, they could not correctly interpret or could make irrelevant interpretations of Student B's and Student C's solutions. Barnhart and van Es (2015) explain that pre-service teachers with lack of teaching experience have difficulties in interpreting students' mathematical thinking. Moreover, such kind of interpretations might even be challenging for experienced teachers (Little & Curry, 2008). Considering that the pre-service teachers in this study had no experience in a real classroom environment, it might be concluded that the increase in the number of responses demonstrating lack of evidence of interpretation is not surprising.

Strikingly, in contrast to this finding, there were also pre-service teachers who provided robust evidence of interpretation of the solutions of Student A and Student D, although they could not attend to their solutions with robust evidence. More specifically, these pre-service teachers were able to interpret mathematical details of their solutions and recognize what Student A knew or what Student D did not know. Although the attending skill is accepted as foundational for the interpreting skill, Barnhart and van Es (2015) state that robust evidence of attention does not assure robust evidence of interpretation. Considering the findings of this study, it can also be stated that the limited level of attention does not inhibit the robust level of interpretation. The reason for this difference may result from encouraging teachers to interpret these students' solutions by asking questions such as "Explain how each student has solved the task considering the student's solution." and "Explain what each student knows and does not know about algebraic thinking considering the student's solution." That is, when the pre-service teachers were given these questions, they might have considered how Student A or Student D thought throughout the solution or what the reasoning underlying their solutions could be. Therefore, this finding extends the related literature by showing that the level of a teacher's interpretation can be greater than that of a teacher's attention.

Considering the skill of deciding how to respond, there were differences in the percentage of pre-service teachers who provided robust evidence for the students. Specifically, the percentage of these pre-service teachers was the greatest for Student D whose solution was not correct. Researchers emphasize that teachers have more difficulties in effectively responding to students with incorrect solutions (Son & Crespo, 2009). Although responding with robust evidence may be difficult for pre-service teachers as it requires understanding the child's way of thinking (Ginsburg, 1997), they can succeed this process by asking questions to help the child explain his/her strategy or thinking. By means of these questions, they can discover the child's error(s) and hence extend the child's understanding (Jacobs & Ambrose, 2008). Like the case in the previous study, the pre-service teachers responding with robust evidence tried to extend the understanding of Student D after identifying why the solution of Student D was wrong. Wager (2014) emphasizes that not providing opportunities for students to explain their strategies or simply replying as wrong answer may result in students shut down. Since the pre-service teachers in this study did not directly provide Student D with the correct answer, it can be concluded that the pre-service teachers' responses would not result in shutdown of Student D.

Contrary to the responses extending the student's understanding, most of the responses provided for the students with correct solutions were in the level of lack of evidence of deciding how to respond. These responses did not differ for the students despite the differences in their solutions. Specifically, the pre-service teachers who demonstrated lack of evidence just appreciated the students for their correct solutions. Actually, this finding is similar to that of Crespo (2002) who explains that appreciating or praising a student is usually accepted sufficient for a correct solution. In the same way, Milewski and Strickland (2016) state that teachers reflexively respond to these students "move on, praise, or affirm the answer" (p. 128). However, NCTM (2000) emphasizes that teachers need to provide responses more than "right or wrong" (p. 24). Jacobs and Ambrose (2008) explain the answer to why teachers need to go beyond by stating that "important learning can occur after a correct answer is given when a child is asked to articulate, reflect on, and build on initial strategies" (p. 266). With respect to this emphasis and explanation, it can be stated that appreciation of a student's solution is absolutely important, however, it needs to be followed by specific and different responses to the student to extend not only understanding of this student but also that of other students.

In short, when the percentages of pre-service teachers providing robust evidence were compared considering the skill of noticing expertise, it was found that the percentage was the greatest for the skill of deciding how to respond. Then, it was greater for the interpreting skill, and followed by the attending skill with the lowest percentage. The distribution of these percentages shows that although the pre-service teachers could not provide robust evidence of attention or interpretation, they could be able to provide robust evidence of deciding how to respond. Despite the researchers who emphasize that attending is a foundational skill for the two other skills of noticing expertise (Jacobs et al., 2010; Jacobs et al., 2011; LaRochelle et al., 2019; Mouhayar, 2019), this study revealed that the pre-service teachers' skills of interpretation and deciding how to respond were not affected by their skills of attending. A similar study might be conducted to see if the findings would be similar to those of this study. If not, the possible reasons for this difference might be further explored.

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