
An Environment Test for Risk Tolerance Assessment Verification in Lifelong Financial Planning for Households

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Abstract:

Purpose: The aim of this study is to propose and discuss a theoretical framework of a testing environment for verification of household risk tolerance assessment methods, based on a financial plan optimization model. It is intended to be suited to the specificity of life-long financial planning for households.

Design/Methodology/Approach: The assumption of the proposed testing environment is that a risk tolerance measure should be used as part of the input to a household life-long financial plan optimization procedure. The risk of the received plan should therefore be consistent with the risk tolerance estimated at the beginning.

Findings: After the analysis of the structure of the existing financial plan optimization model (developed by the Authors as part of some former research project) it has been concluded that, after some modifications, it is possible to use it as a test environment for verification of household risk tolerance assessment methods.

Practical Implications: The possibility of verification of a risk tolerance assessment method for a household life-long financial plan is necessary to be able to construct plans that are really suited to household needs. So far, there do not exist proper household risk aversion measures, nor methods of household risk aversion estimation, that would consider the nature of risk that is present in the household life-long financial planning. But as soon as such methods are developed, it will be necessary to be able to verify whether the risk tolerance estimates obtained from them are at all reliable and if they are in line with how households understand their risk tolerance.

Originality/Value: This study is a step towards verifying household risk tolerance models for lifetime financial plans, which is a new research area, not yet undertaken in the literature. It is very important both from the point of view of the further development of the theory of personal finance and for the practice of financial planning.

Keywords: Household, life-long financial planning, risk tolerance assessment verification.

Paper type: Research article.

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1. Introduction

Risk tolerance assessment is a necessary task in financial planning for households, as well as in portfolio management and wealth management. In life-long financial planning, however, the very definition of risk is much more complex. Even portfolio management for an individual investor, which is close to the problems of personal finance, does not require so multi-faceted approach to risk. Moreover, there is no state-of-the-art consensus or any industry standard saying how to actually define risk of a household life-long financial plan. Therefore, even the problem of adequately defining risk tolerance is highly non-trivial, not to mention risk aversion estimation or verifying methods of this assessment.

The above is a consequence of the different nature of life-long financial planning objectives, as compared to the objectives of, for example, portfolio management. The aim of life-long financial planning is to accomplish all financial goals, which reflect life objectives of household members. At the same time, the financial plan must make it possible to maintain regular consumption at an acceptable and safe level. The life objectives of the household are diverse and can be very heterogeneous, like: bringing up children, buying or building a house, buying an apartment or being able to rent one, education (for children and for oneself), endowment, bequest, luxury goods, some special, qualified forms of recreation, hobby, and being able to leave a decent bequest for one's offspring, or even leaving something behind for the broadly understood humanity.

From the point of view of the household members, the plan will be considered as successful if they are able to accomplish all their goals at the pre-planned moments and to the full extent, while maintaining financial liquidity and remaining solvent. Thus, the risk of a plan must be understood as the possibility of such a failure that the household is not able to achieve all of that. The risk realizes if whichever of the pre-set goals is not accomplished, fully and on time, and for whatever reason. This is something different and more complex than portfolio risk, which is usually understood as either volatility of returns, or the threat of incurring a loss, or the possibility of not achieving the desired profit. A different risk definition entails a different approach to risk aversion and risk tolerance.

If household risk tolerance is understood as the household's tolerance of the possibility that the life-long financial plan fails somewhere along the line because of any of many reasons, then it can be said that no research on this kind of risk tolerance is presented in the existing literature. There are some attempts to model the risk as understood in this way, and some measures of risk have been proposed (Pietrzyk and Rokita, 2017). These are the so-called integrated measures of household financial plan risk. But no attempts have been made so far to define a measure of risk tolerance for this kind of risk. Let alone estimating risk tolerance. Even if there existed a measure and a method of its estimation, there are no ready

instruments that would allow to track the relationship between risk tolerance of a household, financial plan optimized with the risk tolerance information taken into account, the risk of the financial plan, and the acceptance of this level of risk by the household. If there existed such a testing environment, the method of risk tolerance assessment might be verified by a straightforward comparison of (1) the risk tolerance measure estimated on the basis of the information obtained from the household in the initial phase of the financial plan preparation and (2) the reaction of the household members to the information about how risky the plan they received, presented in a way that is understandable to them. This is, of course, under the assumption that the risk tolerance measure is used as an input to the plan optimization procedure and that the plan is optimized with the risk tolerance information considered.

The aim of this research is to propose a construction scheme of the test environment for household risk tolerance assessment verification and to show the position of risk tolerance assessment task in the whole financial planning model used for this purpose. The research question is of a qualitative nature and consists in attempting to resolve the issue whether the existing financial plan optimization model, proposed by the authors of this study (Jajuga *et al.*, 2015; Pietrzyk and Rokita, 2016b) within one of the previously conducted research projects, has a structure suitable to adapt it to the task of verifying the risk tolerance estimation method.

The methodology of the research is a theoretical study of the structure of the existing model by Jajuga, Feldman, Pietrzyk and Rokita (2015), theoretical study of the desired properties of the test environment that would be helpful in verification of a risk tolerance assessment method, and comparison between these two. Building a theoretical model, and then, perhaps as part of next research, developing it to a form of a practically applicable IT solution, that would allow for verification of risk tolerance assessment methods, would be of great importance for both the theory and practice of personal finance.

The presented approach is new in several respects. Firstly, it refers to the risk of a life-long financial plan of a household. Secondly, the risk tolerance is understood as tolerance of integrated risk of the whole plan, like the risk quantified with the measures proposed by Rokita and Pietrzyk (2017), albeit these measures will not be used here. Thirdly, the approach to verification of a risk assessment method is based on a financial plan optimization model, which potentially allows to grasp a direct relationship between risk aversion declared and the resulting financial plan.

2. Research Outline – A Broader View

The research is one of important steps in preparation of a conceptual framework of a further, more thorough research whose main goal will be to elaborate a risk tolerance model that will, in turn, allow for developing a fully automated risk tolerance

assessment procedure. That bigger project will be further referred to as the *second-phase project*. As it has already been explained, the risk tolerance assessment, addressed by the second-phase model, is intended for needs of financial planning in personal finance.

Here, the main subject of financial decisions is a household, further referred to as a *client*. Sometimes the term “client” will be also used to denote the household member who represents the household and is formally the client of a personal financial advisor. The household may be run by a single individual, a couple, a couple with dependent persons or it may have some other structure, but it is assumed that there are one or two *main household members* in it. The term “main household members” means here the persons for whom the financial plan, including retirement, is indeed prepared. They have two features that distinguish them from other members. The first is that, at the moment when preparation of the plan starts, they are expected to intend to be members of the household until they die. Thus, although children of the main household members are household members, they are not main household members in this sense. The second is that they have not reached their retirement age, neither before nor at the moment when the plan starts (otherwise they would not be included in retirement planning). Thus, elderly parents who are not able to run their own household anymore and have been taken to their son’s or daughter’s home are not main household members in this sense. Main household members are the ones for whom the retirement plan is constructed. The retirement plan is the core (and, in many cases, even the only) element of the whole financial plan of a household. This is why the main household members are defined as those members of the household (1) for whom retirement planning is a relevant task and (2) their retirement plans are part of the household financial plan and so will be in the future.

This particular piece of research is part of a project that is focused on a review, discussion, and preparation of methodology for the second-phase project, as defined above. They goals are as follows:

1. Preparation of general methodology that will be then used for identification and assessment of clients’ risk tolerance;
 2. Preparation of the methodology for risk aversion assessment verification.
 3. Preparation of a technical feasibility study of an IT solution that will be used for realization of the second-phase project.
 4. Preparation of a technical feasibility study of a practically applicable test environment based on the theoretical framework mentioned in point 2.
- Out of these five points, this article tackles the task described in the point 2.

The first goal (point 1) includes preparation of a general concept of questionnaire-based research, maybe also supported by in-depth interviews and focus groups.

Realization of the second goal (point 2), that is the topic of this article, is underlain by the connections between the following seven elements:

- 1) input from the client,
- 2) assessment of client risk tolerance (based on automatized procedure),
- 3) risk tolerance parameters (to be used as part of the input to the financial plan optimization procedure),
- 4) financial plan after optimization,
- 5) risk of the optimized financial plan,
- 6) visualization of financial plan risk in a form that is understandable for the client,
- 7) feedback from the client whether she accepts or does not accept the risk.

As it has already been mentioned in the Introduction, the idea is that, if risk tolerance assessment is correct, well translated into a risk aversion parameter or parameters, and the financial plan is optimized with these parameters taken account of, then the optimal plan should be not more risky than it would be acceptable by the client. It is assumed that the client is unable to specify her risk tolerance precisely, let alone quantifying it in a form of one parameter or a vector of several parameters.

The multi-fold feedback from the client may, however, reveal the necessary information if collected, processed, and interpreted using a proper risk tolerance assessment model. Such a model is aimed at interpreting the information from the client so that the risk tolerance parameter (or parameters) can be then used in the financial plan optimization procedure. Given that the optimization procedure does what it is expected to do (it is not the subject matter of this project), the relationship between the input and the resulting optimal plan should be such that, amongst other conditions, the plan meets client's preferences pertaining to risk. To let the client, know how risky the plan is, a scenario-based presentation of plan performance is used.

Realization of the third goal (point 3) consists in presentation of a first, still very general, sketch of an IT solution design and its technical feasibility study. The IT solution will be developed in later phases of the research, on the basis of the model that is expected to be obtained from the second-phase project.

Realization of the fourth goal (point 4) will be similar to the realization of the third goal. Most likely, it will be a plan of how some parts of the system designed as part of the third goal realization may be used in risk tolerance assessment verification.

In the life-long financial planning for a household, risk is inevitable. This is because a long planning horizon and also due to a big number of risk factors that influence the ability of a household to accomplish all its goals. Whereas we could speak of a risk-free investment, there are no risk-free life-long financial plans. Nevertheless,

risk of a financial plan may be reduced or increased in many ways. A riskier plan may allow for more ambitious goals, but under the threat that they will not be achieved in a bigger number (or in more probable) scenarios than it would be for a less risky plan. The level of regular “every-day” consumption may be also higher in a riskier plan. This is, however, also at the cost of being more vulnerable to some less advantageous scenarios. A less risky plan offers the household members more security. The financial situation of the household is more robust, and thus not so easily struck by the adverse scenarios of the future. But the cost of this safety must be paid, and it is usually twofold. The first element is higher regular contribution to retirement investments and alike. The second is higher permanent financial cushion. The last is maintained in a form of cash or “cash-like”, risk-free, highly liquid financial instruments. If the plan requires that these two elements are of a large value, there is less cash remaining for consumption and also more modesty is needed when planning one’s financial goals, especially those less necessary.

Put it all together, it is very important to know how much risk can the household accept, because different plans would be optimal for the clients with high risk tolerance and completely different plans would be recommended to those with low risk tolerance. This article is not devoted to the issue of risk tolerance definition, nor to risk tolerance assessment, but tackles a yet another problem, namely the theoretical concept of the structure of a calculation environment framework (in the future – maybe also an IT solution based on it) that would allow to verify if the risk assessment is in compliance with the actual attitude of the household to the financial plan risk.

Let us say that, after a series of studies, based on in-depth interviews, focus groups and CAWI or CATI questionnaires, preceded by a comprehensive desk research and followed by a thorough and skilful analysis of obtained results, a method of risk tolerance assessment was developed. Then, the method was applied to a client (a household or a member of the household that represents it in contacts with a financial planner). Finally, a measure of risk tolerance is obtained as a result of applying the method to this household. The measure may be a scalar or a vector. For instance, each element of the vector may correspond to a different aspect of risk tolerance (risk propensity, risk capacity, etc.).

Having assumed all of that, several questions arise. What is (what should be) the relationship between the risk tolerance measure and the life-long financial plan for the household? How risky the plan can be? How aversions against different types of risk contribute to the overall risk tolerance (where the types of risk may be of so different nature as, e.g., risk of investments and financing, risk of health deterioration, and, let us say, risk of a damage to the physical property). These are only examples of the questions, but even these three seem to require some expert analysis, based on an individual approach and maybe even direct personal contact between the financial planner and the household. If the model of financial planning

is to serve as a theoretical background for a practical analytical tool, we would like, of course, to avoid the need of any kind of expert analysis, and even any kind of human intervention in the system. The theoretical framework should be useful for the purposes of developing an algorithmizable and automatable tool that would support financial planning for a large number of households. For a really large number of clients, the system must be unmanned, or else it would be not less expensive than traditional financial advising and wealth management services, which are traditionally offered to rather high-net-worth clients.

Instead of answering all these questions, it may be much easier to compare the risk tolerance measure obtained at the beginning with the reaction of the household to the information how risky the whole resulting plan is. This procedure may be first supported by a personal contact between a financial advisor (being a human at this stage of the system development) and a representative of a household. But then, after the model has been calibrated, all new clients may be onboarded to the system without any human financial advisor present in it.

The risk aversion assessment verification may be, of course, performed in practice only if one does not need to wait until the end of the planning period. The planning period means here the expected further lifetime of the one of the household members who is expected to live longer, plus some safety margin (e.g., 10 years). We want to know if the plan is indeed as risky as the household finds acceptable (after all, the plan was constructed with the risk tolerance information taken into account). If the only way to check it was to wait the whole life, the method would be hardly verifiable.

Moreover, even if we really waited so long and realized that the plan worked, it would still be no evidence that the plan was no more and no less risky than the household wanted. This would only prove that the plan worked well under the condition of the scenario that actually came true. The proposal presented in this article allows to check if a financial plan is in compliance with the decision-maker's risk tolerance, based on visualisation of risk of the whole plan, presented in a way that should be understandable for households.

It requires, however, to go through almost the whole procedure of the financial plan construction, from onboarding, until the analysis of the result of optimization. In the last stage, simulations based on the already-optimized plan are presented to the household and a feedback information is collected. And this procedure must be repeated with the whole sample of households whose risk tolerance assessments are to be verified. Not earlier than at this point, a statistical analysis of the risk tolerance assessment quality is possible.

2.1 Risk Tolerance

For the purpose of this study, the terms “risk aversion” and “risk tolerance” are distinguished. Risk aversion is understood in two ways: in its classical meaning, following Arrow (1971) and Pratt (1964), and in the meaning in which it is used by Jajuga, Feldman, Pietrzyk and Rokita (2015) when expressing the life-length risk aversion in their household financial plan optimization model. These are not equivalent approaches, as the first one is based on the shape of the utility function, whereas the second – on the width of the range of possible scenarios a decision maker takes into account when performing plan optimization. As far as risk tolerance is concerned, in turn, we are trying to go beyond both these narrow models.

2.2 Risk Aversion

In the classical meaning of risk aversion, a decision maker is considered risk averse if her certainty equivalent of an uncertain future outcome with known expected value (e.g., a risky investment with known distribution of return) is lower than the expected value of this outcome. Let us say, for instance, that the case is about investments, and that an investor has to choose between two variants: one risky and that other risk-free. Let, in addition, the certain return on that second (risk-free) investment be exactly equal to the expected value of return on the first (risky) investment. From the point of view of the expected return, both investments are identical, but, of course, they differ in terms of risk.

The investor shows risk aversion if she prefers the risk-free investment, instead of being indifferent whether to choose the first or the second one. Risk aversion may be defined for wealth, return, monetary value of a good, etc. Generally speaking, these are such kinds of quantities for which a utility function may be defined. Risk aversion, as defined above, is indeed a property of the utility function being concave. This concept underlies the absolute and relative measures of risk aversion proposed by Arrow (1971) and Pratt (1964). The classical measures of risk aversion describe thus the shape of the utility function.

The second of the aforementioned approaches is described in more details in subsection 4.5. It is suited to the household financial plan optimization. The criterion function of the optimization procedure is based on expected discounted utility of consumption and expected discounted utility of bequest. The available consumption level is dependent on the shape of the all-life term structure of cumulated net cash flows in the decision maker’s household. The shape of the term structure and its length depends on random variables called risk factors.

The optimization of the plan is performed over a range of values of the risk factors. A set of risk factor values is called a scenario. A risk-averse decision maker would optimize her plan for a broad range of risk factor values, that is – would take more scenarios into account. A less risk averse client will accept a plan that is optimized

for a narrower range, letting the plan be not optimized for scenarios that do not fall within the range. A risk-indifferent client would, in turn, optimize her plan for the expected values of the risk factors only, ignoring the fact that other scenarios are possible.

This approach does not analyse the shape of the utility functions that are discounted, weighted with the probabilities of scenarios, and summed. Only the range of the considered scenarios is controlled. Of course, the utilities may be, after all, more or less concave. And the extent to which they are concave may be also expressed with a parameter, independently of the discussion about the range of considered scenarios. This means that the classical risk aversion measures may be also used at the same time. Therefore, it may be concluded that these two approaches are not competing but rather complementary.

2.3 Risk Tolerance Aspects

An often more difficult issue than defining risk aversion and its measure is its estimation. As regards the estimation of risk aversion, it is necessary to analyse two characteristics of the investor. The first is her ability to take risk, or otherwise, the ability to absorb potential losses (*risk capacity*). The second is the desire to take risk (*risk propensity*). Today, even a bigger number of personal traits related to a decision maker's attitude towards risk are distinguished. They contribute to the so-called *risk tolerance*. Risk tolerance is a term sometimes equated with classical risk aversion. Some authors, however, understand risk tolerance as a broader concept than just risk aversion. For example, Cordell (2001) proposed to include such characteristics describing risk tolerance as: risk knowledge, risk propensity, risk attitude, and risk capacity. Basing on Cordell's concept, the following four components of risk tolerance have been adopted for this study:

- *risk propensity* – opposite to the classical risk aversion; that is – how much of the additional expected payoff a decision maker requires to feel comfortable with an additional "unit" of risk;
- *risk awareness* – knowledge and understanding of risk, including also any kind of decision maker's cognitive biases pertaining to risky situations;
- *risk attitude* – emotional traits related to risk acceptance and risk avoidance and to the situations when risk realizes indeed, including any kind of emotional biases; in addition to that, here belong such features as the emotional ability to actively take risk and the emotional ability to act and make decisions in situations of uncertainty and risk;
- *risk capacity* – ability to cope with situations when risk realizes indeed, especially – the ability to cover losses in case they happen; for the types of risk that are associated with a threat of financial losses, the risk capacity depends on the financial situation of the decision maker and can be simply

put as a question: “How much risk may the decision maker accept so that she will not go bankrupt if this risk realizes?”

Previous studies (Linciano and Soccorso, 2012) indicate that surveys used in financial institutions to analyse their clients' risk tolerance do not even allow for the correct identification of the above-mentioned aspects. All the more so, they do not give precise estimates of risk tolerance in all these four dimensions.

In research on risk tolerance, the tools of classical finance, assuming the rationality of the decision-maker, are not sufficient. This applies both to static asset allocation decisions – maximizing expected utility (Markowitz, 1959; Tobin, 1958), and to inter-temporal choice in the decision maker's life cycle – maximizing expected discounted utility (Yaari, 1965; Ando and Modigliani, 1957; 1963). There may be a need to use observations and methods from the area of behavioural finance, neurofinance (Motterlini, 2010), as well as from other sciences such as psychology and sociology.

Behavioural finance provides a number of observations that are in odds with the rational investor hypothesis. This includes the perception and understanding of risk. These deviations may be emotional in nature (Loewenstein *et al.*, 2001; MacGregor *et al.*, 2000; Nosić and Weber, 2010), or result from cognitive limitations (Olsen, 1997; Shefrin, 2007; Thaler, 1985; 1990). A properly constructed questionnaire should allow, among other things, to distinguish between emotional and cognitive determinants of risk tolerance. It would be especially useful to be able to make this distinction because the decision-maker is often unaware of the decision biases and of their nature.

The problem of proper assessment of decision maker's risk tolerance has not yet been fully resolved. Also, the task of identifying all important elements of risk tolerance is still a challenge that the current literature of the subject does not yet provide a complete answer. Similarly, it has not been fully identified which elements can be omitted without compromising the accuracy of this assessment.

3. Considered Types of Risk

In personal planning, many risk types need to be taken into consideration. They may be of very different nature. The most important are life-length-related risks, risk pertaining to household's investments, threats resulting from financing with debt, risk of loss of a job or other threats to household incomes, risk related to the health condition of household members, risk of damage to the personal property, especially high-value, durable components, and risk of public liability.

Jajuga, Feldman, Pietrzyk and Rokita (2015) proposed the following systematization of household risk in the context of life-long financial planning (systematization no.

VI among those proposed there – the one that is further used by them for the needs of their model):

- 1) Life-length risk;
- 2) Risk of investment and financing;
- 3) Income risk;
- 4) Risk of events (insurance-like events);
- 5) Risk of goal realization;
- 6) Operational risk of plan management (particularly risk of plan implementation);
- 7) Model risk.

When discussing life-length risk, two sub-types need to be considered. The first one is the longevity risk. This is the risk that the client will live longer than expected (or longer than expected plus some safety margin). If the financial plan is calculated so that some elements of the retirement-age financial security are made sufficient for some certain life expectancy only, the cumulated financial surplus may become depleted after the person exceeds the assumed life length. Of course, the easiest way to protect one's retirement-age income against longevity risk is annuitization, which has, however, both its advantages and disadvantages. The second sub-type of life-length risk is early-death risk, also called premature-death risk.

From financial point of view, this risk exists only in households that are composed of two or more members. Two most severe instances of premature-death risk realization are when the person who earns more dies either very early, leaving the family without sufficient income, or when this household member lives until his or her retirement age, buys an individual life annuity and dies immediately after this moment. Then, the whole retirement capital vanishes. A way to deal with the threat of the first case, that is – a very early death of the main breadwinner, is death insurance. A remedy for the second case, that is – total loss of life annuity due to the death of the insured person, may be buying a life annuity for a couple, which pays insurance benefits until the death of the person who survives longer.

Risk of investment and financing is mainly the market risk, that is – the risk resulting from volatility of prices, indices, and main financial indicators in the markets. This refers to financial markets, commodity markets, as well as real-estate market. Market risk is further divided into:

- Stock price risk;
- Interest rate risk;
- Commodity price risk;
- Foreign exchange rate risk;
- Real-estate price risk.

Besides market risk, an important type of risk related to investments is credit risk. This is the risk that a counterparty in a contract will not fulfil their contractual obligations. A sub-type of credit risk which plays a significant role in investments is the issuer risk. The issuer risk is the risk that an issuer of a financial instrument, typically – a debt instrument (like a bond, or a short-term or medium-term note), will not pay interests or that they will not redeem the instrument. Also, the situations when the payments made by the issuer are delayed or when the issuer pays only some part of the par value when it comes to redemption of the instruments are instances of issuer risk realization.

The market value of bonds depends both on interest rates prevailing in the market and the creditworthiness of the issuer. Whereas changing interest rates belong to the domain of the market risk, creditworthiness of the issuer belongs to the domain of the credit risk.

Interest rates and foreign exchange rates play an important role not only for households' investments, but they also influence households' financing. Interest rates have an obvious impact on the cost of loans taken by the household. To that, if the household has taken credits in foreign currencies (like Swiss franc mortgages, that were extremely popular in the Central and Eastern Europe in the years 2004-2008), foreign exchange rates are also very important factors of risk. Thus, these two subtypes of market risk (interest rate risk and foreign exchange risk) are of a very big importance to financing.

Income risk is the risk related to the threat that incomes from other sources than investments, mainly salaried income, will be lower than expected or that some source of income will be lost. Part of income risk is the risk of losing one's job.

Risk of events is the risk pertaining to the possibility of such adverse events that households usually buy insurance policies against. Here belong physical damages to the property, injuries of persons, and alike. Thus: theft, fire, flood, accidents, serious medical condition, etc. will be examples of realization of this type of risk.

Risk of goal realization should not be mistaken with the risk of investing and financing, which is of course also closely related to clients' ability to accomplish financial goals. Risk of goal realization is, however, not connected with the threats to the client's ability of gathering necessary funds, but rather with random characteristics of the goals themselves. For instance, let us say that a couple plans to achieve the financial preparedness for covering all additional costs connected with bringing up children. Let us assume that they want to be prepared for the increased consumption not later than, for example, in two years since now. The time that will elapse indeed since the present moment until the birth of their first child is a risk factor. The couple has control only over the moment when they start to try to

conceive, but the actual moment when the child is born is a random variable, of course conditional on the moment the efforts begin.

Another example of the goal realization risk is a threat that the price of a property planned to be bought will increase so much that the planned purchase proves impossible. This is something else than the risk of funding. Here, the source of risk is not the uncertainty whether the client will be able to accumulate as much capital as planned, or whether she will be able to take a loan of the planned amount, but whether the object of her interest itself will not change its critical characteristics in another way than forecasted (e.g., if the price will not grow faster than implied by the assumed growth rate).

Operational risk of plan management is the type of risk that would normally not be included in the plan by the clients on their own. Here belongs the risk that the plan will fail to be successful because of failures in plan realization, so – neither because of an inaccuracy of the plan itself, nor because of an unpredicted adverse scenario occurrence. One of the reasons for such failures may be insufficiently careful adherence to the plan by household members.

This subtype of the operational risk may be called risk of clients' insufficient self-discipline. It makes a lot of sense to take it into account when constructing the plan, but it is questionable if the system should inform the client about the safety margins assumed for this type of risk. If the clients knew that safety margins were added to each pre-planned investment contribution and that all anticipated expenses were increased by a mark-up resulting from their anticipated extravagance, then the predictions about their lack of financial discipline would quickly become a self-fulfilling prophecy.

For this reason, it is unlikely that the financial planning system will analyse clients' attitude towards this kind of risk at onboarding. The system should gauge clients' financial self-discipline by indirect questions in the onboarding survey and take the correction to it in the plan, but not necessarily informing the client about this correction.

Model risk is the risk that the models underlying the plan are wrong. They may be fundamentally wrong in the sense that they may be wrongly specified, or they may be well-specified but not well-calibrated. It is hard to control the first subtype of the model risk. The second, that is – miscalibration risk, often boils down to the risk of estimation errors. If distributions of the parameter estimators are known and the samples on the basis of which the parameters are estimated are in compliance with the model assumptions, then the measurement of estimation risk is relatively easy and consists indeed in estimation of the volatility of estimators.

3.1 Assessment

The risk tolerance assessment can be made on the basis of the type of risk criterion and the risk aversion aspect criterion. An additional piece of research may be needed to determine if dividing risk aversion assessment into separate threads, each concerning another type of risk, is at all necessary. If decision makers who are, generally speaking, more risk averse show lower tolerance of any type of risk than those who are generally less risk averse, then the detailed information about clients' attitude towards each type of risk is not necessary.

As far as risk tolerance aspects are concerned, it seems necessary to split the assessment into separate parts pertaining to different aspects of risk tolerance. This is obvious that a client may show really unconnected and dissimilar levels of different risk tolerance aspects as defined in subsection 3.2. For instance, client's risk propensity may have not much in common with her risk awareness or her risk capacity. Therefore, these aspects must be investigated separately. However, a completely different issue is whether information about all aspects of risk tolerance is really needed in the financial plan optimization model.

3.2 Risk Tolerance in Household Financial Plan Optimization

Although the parameter or parameters transmitting risk aversion information as an input to the financial plan optimization procedure must be consistent with the result of the risk tolerance assessment, they do not have to take the same form. Not only do they not have to, but they should not. The result of risk aversion assessment will be arranged according to risk-aversion-aspects scheme. Let us assume that general risk tolerance of a household pertains to all types of risk. Let us moreover assume that risk tolerance of the person who represents the household during the system onboarding is the risk tolerance of the whole household. Having said that, some kind of operationalization of the risk tolerance assessment outcome is needed to make it compatible with the plan optimization procedure. The plan optimization procedure receives the information about client's attitude towards risk in the following form:

- 1) The width of the range of considered scenarios, where the expected scenario is the centre (the lower the risk tolerance, the wider the range);
- 2) Curvature of consumption utility functions (the same functions are used to describe utility of bequest but with different weights);
- 3) Loan-to-value ratio, when planning realization of goals that can be financed by debt (perhaps, the lower the risk tolerance the bigger the proportion of own contribution and lower the proportion of debt).

The construction of the financial plan optimization model used for purposes of this research is such that any increasing and concave function may be used as utility of consumption and bequest. Therefore, the results of risk tolerance assessment will not

influence its parametrization. Point 2) may be thus omitted. Points 1) and 3), in turn, remain and are relevant, but their interpretation and their role in the plan optimization model is different.

The proportion of own contribution to debt (or the proportion of debt to the value of a goal) is one of kinds of decision variables in the plan optimization procedure. So, it is not a parameter that might be made dependent on the results of risk tolerance assessment. Some constraints might be, however, imposed on this decision variable, and these constraints might be dependent on risk tolerance. Point 3) may be used to reflect risk tolerance, but only to some extent.

What remains is indeed point 1). Scenarios are actually some values of a random vector containing risk factors. The risk factors are discretized in the model, so the number of possible scenarios is finite on any finite range. On the level of generality on which the very concept is presented, it does not really matter if the distributions of the risk factors are discrete or continuous. It is, however, necessary to discretize them to overcome obstacles resulting from computational complexity of the model. It does not seem that any closed-form analytical solution exists, and a way to speed up numerical computations is to reduce the number of considered scenarios under which the solution is searched for. It needs emphasizing that the discretization pertains to scenarios of the risk factors, not to the decision variables of the optimization function. The decision variables are, for instance, such quantities as: the initial consumption rate, the proportion in which main household members contribute to private-retirement-dedicated investments of the household, or the proportion of own contributions in loan-financed goals. These decision variables may be treated as continuous or discrete, but it is more natural to take them as continuous. This, however, has nothing to do with the discrete or continuous distributions of risk factors, nor with the way in which scenarios are constructed, anyway.

The risk factors naturally correspond to types of risk. For example, the time that will elapse since the moment when the financial plan is prepared until the date when a given person dies is a risk factor that corresponds to life-length risk. Rate of return on a well-diversified portfolio of stocks (usually represented by a market index) naturally corresponds to market risk and thus to risk of investments, as defined in subsection 3.3. Interest rate will also correspond to the investment risk and, in addition, to the risk of financing.

Even if we maintain the assumption that the estimated level of risk tolerance applies equally to all types of risk, we still need to find a way to translate it into the widths of the scenario ranges for each of the risk factors. It cannot be ruled out that the limits of these scenario ranges must be included (directly or indirectly) in the questions of the onboarding survey.

The remaining issue that needs clarifying is how to include the information about different aspects of risk tolerance in the parameters of the plan optimization procedure. This task should be approached from two directions. The first is to analyse the existing parameters of the model for the possibility of including some information about other aspects of risk tolerance than just straightforward risk propensity (or risk aversion). The second is to modify the model itself to include different aspects of risk tolerance into account.

4. General Scheme of the Financial Plan Optimization Model

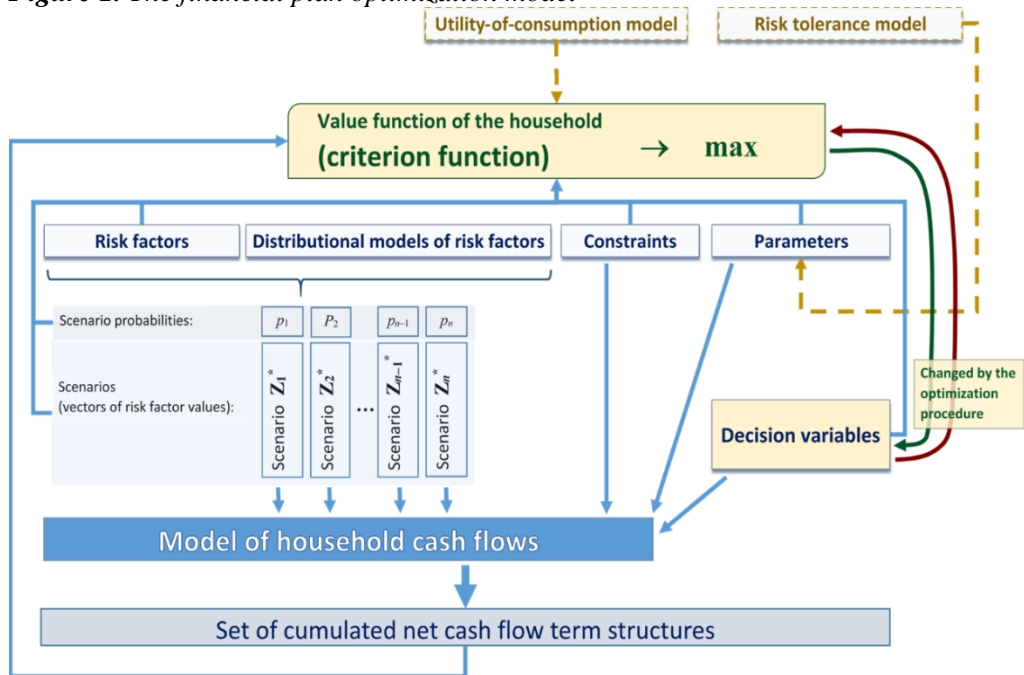
The risk tolerance assessment procedure is part of a broader financial planning procedure, which is based on the household financial plan optimization model. And also vice versa – the financial planning procedure is used for verification of the risk tolerance assessment, as it has been outlined in the previous sections. Therefore, when discussing risk tolerance assessment, the general structure of the household financial plan optimization model is an important piece of information.

4.1 Main Components of the Whole Financial Planning Model

The financial plan optimization model is composed of the following 5 components:

- Model of household cashflows, taking as an input the following data:
 - Risk factors,
 - Constraints,
 - Parameters,
 - Decision variables;
- Distributional model(s) of risk factors – model or set of models that describe statistical properties of risk factors;
- Value function of the household – the criterion function of the plan optimization procedure;
- Model of consumption utility;
- Risk tolerance model.

The general scheme of the whole financial plan optimization model is presented in Figure 1. Distributional model(s) of risk factors may be based on the concept of a random variable or stochastic process. The models describe risk factors. Theoretically, it might be one multivariate model. Different types of risk factors (like, for example, further lifetime of a person, real estate prices, etc.) may, however, be really far apart in their nature, therefore they are usually treated as independent. Their independence is usually not even verified. Some other risk factors may be dependent. This may refer, for example, to further lifetimes of two members of a married couple). But it does not mean that each survival model will treat their lifetimes as a bivariate random variable or a bivariate stochastic process. This depends on the survival model that is applied by the analyst (two independent univariate survival models are also used).

Figure 1. The financial plan optimization model

Source: Own elaboration by Pawel Rokita

Distributional model(s) of risk factors are used to generate scenarios and attach probabilities to them. One scenario is a vector of some particular values taken by the risk factors. A set of scenarios is used to generate a bunch of possible future term structures of the household's cumulated net cash flow (cumulated surplus). One term structure corresponds to one scenario. The term structures are generated by the model of household cashflows, which takes the scenarios, parameters, and other data as an input, and returns the term structures as the output. The term structures of the cumulated net cash flows, together with the information about the probabilities of their corresponding scenarios, are used, in turn, by the value function of the household. The value function is a criterion function of the plan optimization procedure. This function is based on expected discounted utility. Both utility of consumption and bequest is taken into account.

As the value function of the household builds on the concept of expected discounted utility, an underlying utility model must be also assumed. The utility model is thus a component of the whole financial plan model, too. In the construction of the value function, the information about risk aversion, and more generally – risk tolerance, is embedded. It is reflected in a subset of the parameter set. This subset is referred to as

risk-tolerance parameters. It is also possible that some constraints on the household plan risk are imposed. The risk tolerance parameters are estimated using the risk tolerance model.

4.2 Main Elements – More Detailed Picture

More details on risk factors, constraints, parameters, and decision variables are given in Figure 2.

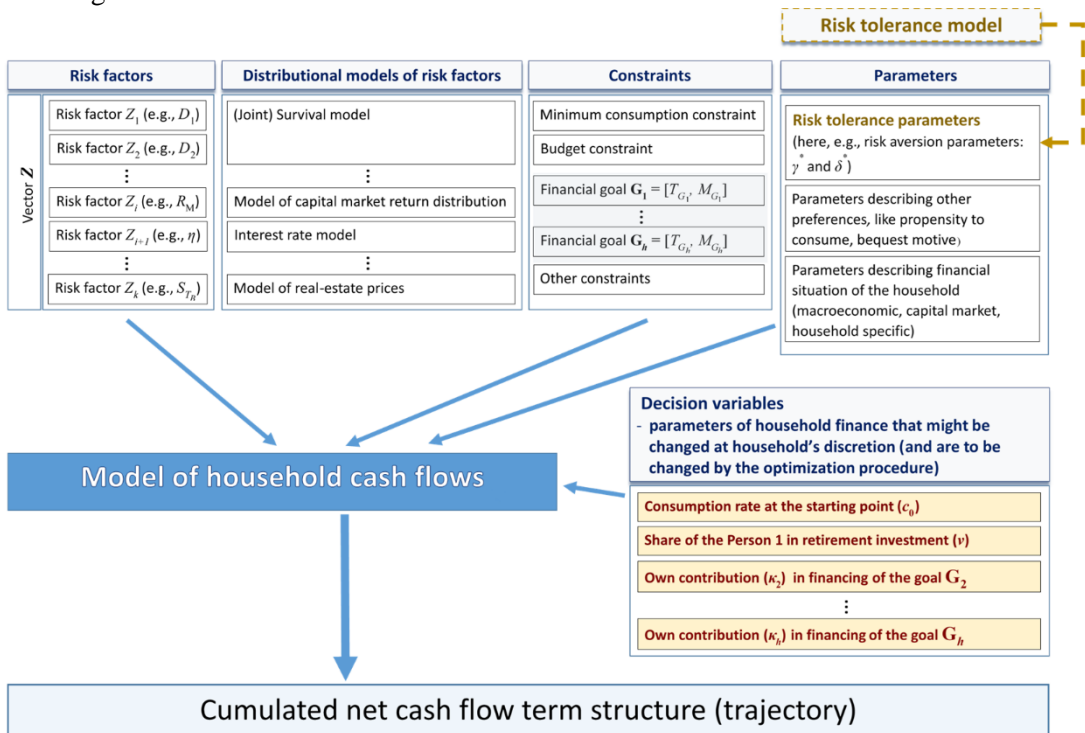


Figure 1. Household cashflow model – details on the input

Source: Own elaboration by Pawel Rokita

In Figure 2, the vector of risk factors (Z) may contain, for example, such random variables as:

- unconditional lifetime of main household members, that means – further lifetimes conditional on survival until the day when the plan starts – variables denoted as D_1 and D_2 (or, when referring to them as to components of the vector Z , denoted with the symbols Z_1 and Z_2 , respectively),
- return on the market portfolio (or relative change of value of a stock market index) – the variable denoted as R_M (or as Z_3 , when referring to it as to a component of Z),

- reference interest rate – the variable denoted as η (also more nodes of a yield curve may be used; or a yield curve model may be assumed and the reference rate η may be used as a benchmark point, just to determine the general level of the whole curve),
- benchmark real estate prices; these may be, for instance: (1) the price of a house or an apartment at the moment when the household plans to buy it, and (2) the price of investment real estate at the moment when household members retire (the risk works in two different directions in these cases: in the first one – the lower price the better, in the second – the higher price the better),

The set of constraints includes the following elements:

- minimum consumption constraint – the minimum acceptable level of household consumption,
- budget constraint – the constraint that the total expenses during a period cannot exceed the total sum of income during this period (including incomes financed by debt),
- financial goals – each financial goal of the household is defined as a pair of numbers: (1) the time since the start of the plan until the pre-planned moment when the goal is intended to be accomplished, and (2) value of the goal in nominal prices; for example, the goal of buying an apartment may be defined as [10 years, \$400 000], which means that, in 10 years since the plan start, the household plans to buy an apartment of such standard and in such location that apartments of this kind are now worth 400 thousand dollars (at today's prices – the model takes nominal values for input of this kind); the goals are constraints in the model – all goals must be accomplished fully and on time (if any of them is infeasible, less ambitious goals should be set by the household during the phase of plan revision, which is repeated cyclically, usually once a year, unless some abrupt change in client's life situation enforces an earlier revision),
- other constraints – for example, there may be a boundary condition that a measure of risk never exceeds some level.

Main types of parameters in the models are:

- risk tolerance parameters – for example, parameters of life-length risk aversion, determining leftward and rightward width of the so called range of concern (respectively, γ - early-death risk aversion parameter and δ - longevity risk aversion parameter),
- parameters describing other preferences than the attitude towards risk; here, the main two are: propensity to consume (denoted with α in the model) and bequest motive (denoted as β),
- parameters describing financial situation of the household:
 - macroeconomic, like: forecasted long-term average growth rate of labour incomes, forecasted long-term average inflation rate, etc.,

- capital market – expected return on the market portfolio (or a broad market index); remark: the return of the market portfolio belongs to risk factor, but its long-term mean value is a parameter.
- household-specific parameters, like income, income-growth-rate group, age, sex, fixed expenses, etc.

Decision variables:

- consumption rate at the moment when financial plan starts (c_0),
- share of Person 1 in retirement investment of the household (v),
- household's own contributions in financing of the goals that may be (partially) financed by debt (κ_i , where i stands for the ordinal number assigned to the goal).

The name “decision variables” would have no interpretation if considered within the household cashflow model only, but it is relevant for the financial plan optimization procedure. The decision variables are just parameters in the household cashflow model. These are, however, such parameters that might be changed at household's discretion. Yet, they are changed by the plan optimization procedure in the process of plan optimization. And, from this point of view, they are variables.

4.3 Scenario Generating Mechanism

As it has already been already mentioned, the vector of risk factors is a multivariate random variable which may take many values. Each such value is called a scenario. The model assumes discretization of all risk factors. Discrete distributions with finite numbers of states are obtained. In some cases, it is very natural, in some other ones – it is not so intuitive. Further lifetimes belong to the group of risk factors whose discretization is easy and natural. The granularity of the discretization is a direct consequence of the assumed standard period length. For instance, let us assume that annual periods are used in the model. Let the younger of the main household members is 35 years old, her expected further lifetime is 43 years, the longevity risk margin has been set as 22 years. Then the number of all possible future years in which she may die is $43 + 22 = 65$. If, in turn, her spouse is 40, his expected further lifetime is 35, and a 22-year safety margin is added, too, the number of all future years in which he may die is $35 + 22 = 57$. Thus, the number of all values of the bivariate random variable is $65 \times 57 = 3,705$ for this variant of discretization. Then, using a survival model, one could obtain probabilities for each of these pairs of years. In this way, a discrete distribution of the bivariate random variable would be constructed. Each such pair is a survival scenario.

Depending on how extensive the model is, there may be more or less risk factors considered. Examples have just been listed above. Knowing the risk factors and the models describing their statistical properties, a discrete set of scenarios with attached probabilities may be generated.

4.3.1 Cumulated net cash flows term structures (trajectories)

The main output of the whole plan optimization process is a set of term structures of cumulated net cash flow (sometimes also referred to as cumulated surplus trajectories). Each term structure corresponds to a scenario. As scenarios are generated from discretized distributions with finite numbers of states, the number of term structures is also finite. Further analytics of the cumulated financial surplus gives the view of plan riskiness. Figure 3 shows an example visualizing the idea.

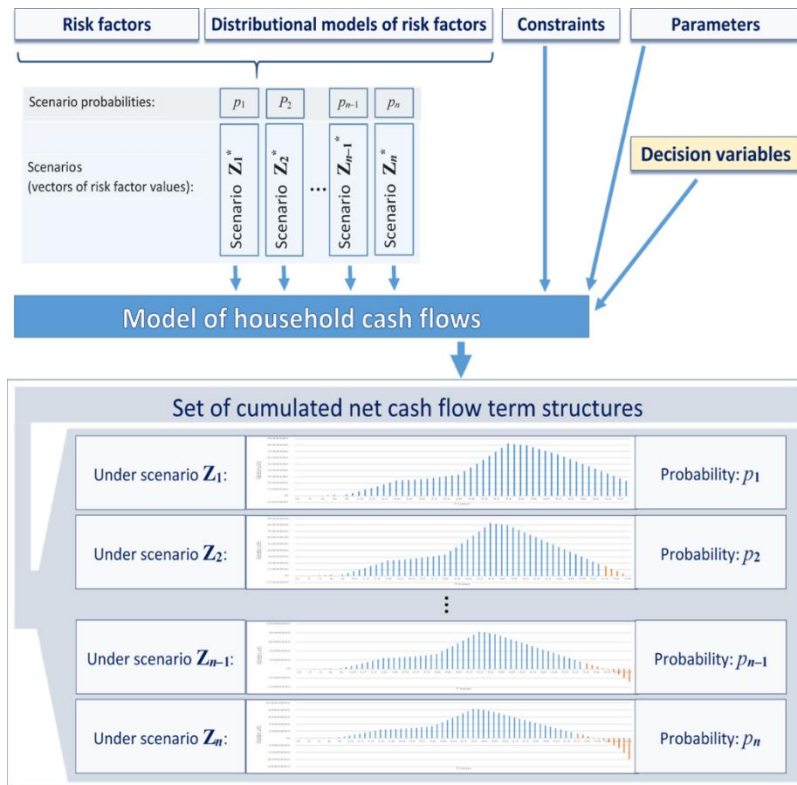


Figure 2. Generation of cumulated net cash flow term structures (cumulated surplus trajectories)

Source: Own elaboration by Pawel Rokita

4.3.2 Range of concern and risk aversion

The concept of the *range of concern* comes from the model of household plan optimization proposed by Jajuga, Feldman, Pietrzyk and Rokita (2015). It was originally intended to include aversion to life-length risk in the financial plan optimization model. Theoretically, it could be generalized to include risk aversion to other types of risk. For the life-length risk, the attitude of client towards risk is measured using two parameters: parameter of aversion against premature death risk (early death risk) and aversion to longevity risk.

The risk factor is defined here as the order number of years, starting from the financial plan start, in which the person dies. As two main household members are considered, this is rather a pair two moments of death (D_1 , D_2). The risk aversion parameters are, in turn, defined as the number of years before (γ) and after (δ) the expected moment of death that the household (client) is concerned about and want to be protected against in their financial plan. Put it differently, if the household had no risk aversion at all, they would optimize their financial plan for the expected scenario ($E(D_1)$, $E(D_2)$) only, not caring about any other scenarios. And if the household were extremely risk averse, they would try to take all possible scenarios into account.

Thus, they would try to optimize their financial plan for all possible years of death and for all possible cross-sections of D_1 add D_2 . Such an attempt would mean including, for example, such an extremely adverse scenario when the household member who has higher income dies immediately after the plan is constructed and the spouse who earns less lives with the same financial plan until her maximum biologically possible age (let us say, until the age of 125). This scenario is not only of a low probability, but it is also not very realistic from the point of view of human psychology and real-life practice. In the real life, a young, recently married, person who has lost her spouse will not continue the same financial plan until her death. It is also hardly plausible that, after some time, she will not enter into a new relationship with somebody, and that she will never launch a new household.

A new household means a new financial plan. This, in turn, makes the old plan invalid. Including that scenario into financial plan optimization procedure would not seem rational. It is the more so that the plan optimization boundary conditions, like all-goals-full-accomplishment constraint, are not so easy to meet even for more realistic but adverse scenarios. Building a plan that is possible to be optimized under all scenarios would force the household to exclude any more ambitious goals and relax most other constraints. In this way, the plan would be optimized for all scenarios, but it would be satisfactory under neither of them. If the household members chose to optimize the plan only for the expected scenario, then they would be able to enforce much more ambitious goals and set more restrictive other constraints (like minimum acceptable level of consumption), but, in turn, they would be completely unprepared for any potential deviations from the expected scenario.

Thus, the plan optimized with no risk aversion assumed would be also useless. Therefore, clients will rather be somewhere between these two extremes.

The idea of using the information contained in risk-aversion parameters, γ and δ , is that the plan optimization procedure takes only the scenarios that belong to the so-called range of concern. The range of concern is determined in the following way:

1. One-person range of concern for survival scenarios:

- Lower bound of the range of concern:

$$LB_1 = E(D_1) - \gamma$$

- Upper bound of the range of concern:

$$UB_1 = E(D_1) + \delta$$

- The range of concern:

$$[LB_1, UB_1]$$

Where:

$E(D_1)$ – expected year of death,

γ – premature-death risk aversion parameter,

δ – longevity risk aversion parameter.

Graphically, a one-person range of concern for life-length risk may be visualized as in Figure 4.

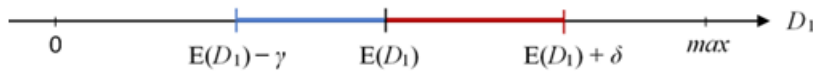


Figure 3. Range of concern (for life-length risk) – one person

Source: Jajuga, Feldman, Pietrzyk and Rokita (2015)

The range of concern for one person (Person i) may be, thus, defined as:

$$Q_{cont_i} = [E(D_i) - \gamma; E(D_i) + \delta] \tag{1}$$

As the model assumes discretization of all risk factors, the same applies to the range of concern. The discrete range of concern (eq. 2) is an intersection of the range Q_{cont_i} and the set that defines a grid corresponding to the pre-assumed granularity (here: annual periods).

$$Q_i = [E(D_i) - \gamma; E(D_i) + \delta] \cap Grid_D \tag{2}$$

2. Two-person range of concern for survival scenarios:

For a two-person household (or with a bigger number of persons, but with two main household members), the range of concern pertaining to life-length risk is a cross section of two single-person ranges of concern. In the model, it is assumed that the risk aversion parameters are declared by the household, treated as one entity (the client). Thus, there is only one gamma and one delta parameter. These

parameters are common to the whole household. Expected further lifetimes are of course individual and they may be different for each person.

Formally, the range of concern for two persons is the following Cartesian product of two individual ranges (eq. 3):

$$Q_H = [E(D_1) - \gamma; E(D_1) + \delta] \times [E(D_2) - \gamma; E(D_2) + \delta] \cap (Grid_D \times Grid_D) \quad 3$$

or, equivalently (eq. 4):

$$Q_H = Q_{D_1} \times Q_{D_2} \quad 4$$

A two-person range of concern is visualized in Figure 5.

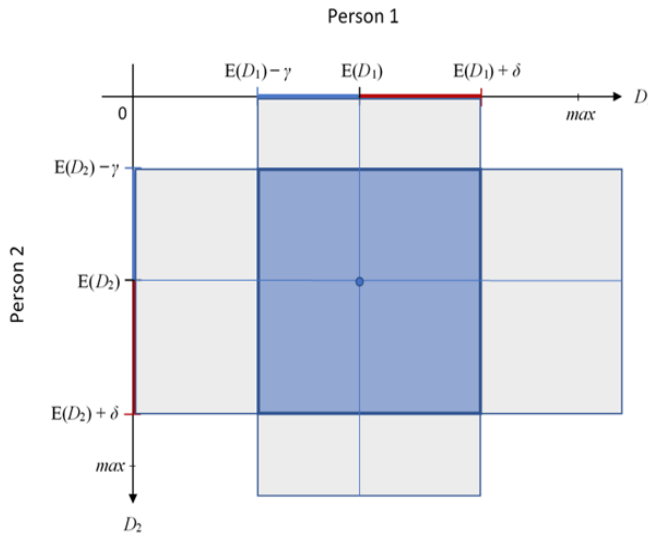


Figure 4. Range of concern (for life-length risk) – two persons

Source: Jajuga, Feldman, Pietrzyk and Rokita (2015)

Generally speaking, the stronger the risk aversion, the wider the range concern. And vice versa: the weaker the risk aversion, the narrower the range of concern. For example, a client with delta and gamma parameters of, for instance, 5 years optimizes her financial plan for the scenarios in which the household members die five years earlier than expected, up to the scenario in which they live five years longer than expected, taking all combination of further life times that are between these two bounds into accounts. A household whose gamma is, let us say, 7 and delta is 9 years is more risk averse. This client needs a broader scope of scenarios to be included in the plan. Put it differently, the second household wants to be prepared for a bigger number of potential contingencies.

4.3.3 Optimization in the range of concern

The optimization procedure takes only such scenarios into account that belong to the range of concern. Formally, this concept is embedded in the construction of the criterion function (value function of the household).

Let the model with only life-length risk be discussed. Then, for a household with two main household members, only two risk factors are considered (D_1 and D_2). Using the notation introduced in the subsection 4.2, the risk factor may be denoted as $\mathbf{Z} = [D_1, D_2]$. Let $\mathbf{Z}^* = [D_1^*, D_2^*]$ be a single scenario (a particular value of the random vector \mathbf{Z}). Since each scenario is a pair of order numbers of years in which the main household members will die, counting from the starting moment of the plan (year zero), the bigger of these two numbers is the order number of the year in which the household will cease to exist under this particular scenario. Thus, the moment of the household end for a given scenario \mathbf{Z}^* is:

$$T_B^* = \max \{D_1^*, D_2^*\}$$

The value function incorporates utility of consumption and utility of bequest. For each scenario, the sum of discounted values of the utility of consumption is calculated. The utilities are taken for each year, from the year zero until the end of the household. Of course, the moment of the household end depends on the scenario (scenarios are of different “lengths”). Sum of discounted utilities for one scenario is given in the Equation 5:

$$U_c(\mathbf{X}, \mathbf{Z}^*, \mathbf{\Gamma}, \mathbf{\Theta}) = \sum_{t=0}^{T_B^*} \frac{1}{(1+r_c)^t} u(c(t; \mathbf{v}, \mathbf{Z}^*, \mathbf{\Gamma}, \mathbf{\Theta}); \vartheta(t, \gamma, \delta)) \quad 5$$

where:

\mathbf{X} is the vector of decision variables (e.g., $\mathbf{X} = [c_0, v, \kappa_1, \dots, \kappa_h]$, where c_0 is consumption rate at the beginning, v is the proportion of Person 1 contribution in the retirement investment of the household, and κ_i is household’s own contribution in financing of an i -th goal);

\mathbf{Z}^* is the scenario;

$\mathbf{\Gamma}$ is the vector of risk aversion parameters (in the baseline version of the model, like discussed above, these are just γ and δ parameters);

$\mathbf{\Theta}$ is the vector of all other parameters of the model;

$\vartheta(t, \gamma, \delta)$ is a utility modifier, used in some versions of the model to reflect nonlinear relationship between clients’ risk aversion and time (the modifier is a function of time, taking risk aversion measures γ and δ as parameters; the value of this function at a given moment t is, in turn, a parameter of the utility function).

The utility of bequest is calculated only at the end of each scenario. It is the utility of the net wealth of the household at the moment when the last household member dies. For one particular scenario, it is described by the Equation 6:

$$U_B(\mathbf{X}, \mathbf{Z}^*, \mathbf{\Gamma}, \Theta) = \frac{1}{(1 + r_B)^{T_B^*}} u(W(T_B^*; \mathbf{V}, \mathbf{Z}^*, \mathbf{\Gamma}, \Theta)) \quad 6$$

Both consumption ($C(\cdot)$) and residual net wealth ($W(\cdot)$) are calculated within the model of household cashflows (mentioned in Figures 1-3). In the model, the utility function used to express the utility of consumption and utility of bequest is the same. These functions are, however, given different weights in the value function of the household. The value function of the household is, in turn, constructed in the following way (equation 7):

$$V(\mathbf{X}, \mathbf{\Gamma}, \Theta) = \sum_{D_1^* = E(D_1) - \gamma}^{E(D_1) + \delta} \sum_{D_2^* = E(D_2) - \gamma}^{E(D_2) + \delta} p_{D_1^* D_2^*} [\alpha U_C(\mathbf{X}, \mathbf{Z}^*, \mathbf{\Gamma}, \Theta) + \beta U_B(\mathbf{X}, \mathbf{Z}^*, \mathbf{\Gamma}, \Theta)] \quad 7$$

where:

$V(\cdot)$ – household value function (criterion function of the optimization procedure),

$p_{D_1^* D_2^*}$ – probability of a scenario $Z^* = [D_1^*, D_2^*]$,

α – propensity to consume (one of parameters),

β – bequest motive ($\beta = 1 - \alpha$; $\alpha, \beta \in [0, 1]$).

Of course, not any scenario \mathbf{Z}^* is present on the list of the value function ($V(\cdot)$) arguments. Unlike the functions $U_C(\cdot)$ and $U_B(\cdot)$, the value function of the household is not defined for a specific scenario only. It takes the information on the range of concern, in the form of the argument $\mathbf{\Gamma}$, and then performs calculations with all scenarios belonging to the range of concern taken into account.

The result is thus a sum of discounted utilities of consumption and bequest obtained in all scenarios belonging to the range of concern. The lower and upper limits imposed on the summation indices D_1^* and D_2^* in the Equation 7 delineate, at the same time, the range of concern (compare eq. 3). The utility of consumption and bequest is calculated only for the risk factor values that are fitted within it.

Put it more generally, the value function of the household may be given as (equation 8):

$$V(\mathbf{X}, \mathbf{\Gamma}, \Theta) = \sum_{Q_H} p_{Z^*} [\alpha U_C(\mathbf{X}, \mathbf{Z}^*, \mathbf{\Gamma}, \Theta) + \beta U_B(\mathbf{X}, \mathbf{Z}^*, \mathbf{\Gamma}, \Theta)] \quad 8$$

where:

Q_H is a range of concern determined by the parameters describing client's attitude towards risk, contained in Γ ,

p_{Z^*} is the probability of a scenario Z^* ($Z^* \in Q_H$).

The financial plan optimization procedure consists in maximization of the function $V(\mathbf{X}, \Gamma, \Theta)$, by changing the vector of decision variables \mathbf{X} , under a set of constraints Ξ . That is:

$$\left\{ \begin{array}{l} V(\mathbf{X}; \Gamma, \Theta) \\ \Xi \end{array} \right. \xrightarrow{\mathbf{X}} \max \quad 9$$

The plan optimization is one of the most important steps in the cyclical financial plan management process.

4.3.4 Financial plan management process

The process of financial plan management is a cyclical process, assuming regular revision of the financial plan. It is composed of the following stages:

1. Household overview.
2. Household data input.
3. External data input.
4. Preparation of the plan optimization task, which is composed of two or three sub-stages:
 - 4.1. Optimization model preparation:
 - a) preparation of the plan optimization model – if it is the first run of the procedure,
 - b) or revision of the plan optimization procedure – if this is any of the following iterations of the procedure.
 - 4.2. Implementation review:
 - a) this step is not taken in the first run;
 - b) review of the plan implementation to date – if this is any of the following iterations of the procedure.
 - 4.3. Constraints setting
 - a) setting of the boundary conditions, which also includes setting of household financial goals (like: buying or building a house, being prepared for expenses related to bringing up children, being prepared for expenses related to education for the children or for oneself, being ready for making a pre-planned endowment, having built ones private retirement capital not later than until a pre-assumed retirement date, etc.) – if it is the first run of the procedure,

- b) or revision of the boundary conditions, including goals – if this is any of the following iterations of the procedure.
5. Financial plan optimization.
6. Measurement of risk, using an integrated household financial plan measure of risk (Pietrzyk and Rokita, 2016a; 2017), for a plan being an outcome of the optimization procedure).
7. Subjective risk assessment of the risk by the main household members (the client) to decide whether it is acceptable to them.
8. Plan implementation if it is accepted.
9. The procedure for transferring the succession if the last member of the household is dead.

The process may be illustrated using a flowchart presented in Figure 6.

The process is cyclical and recursive. It assumes regular revisions, at least once a year. The revisions are necessary because both household life situation and general economic situation changes over time. Due to the long planning period, this plan cannot be prepared once for the whole life of the household. Although the plan is based on long-term projections, both when it is first developed and after each revision, these projections are nevertheless updated every time the plan is revised. Besides the regular revisions, there may emerge a need of an unplanned revision. This may be the case if any unforeseen event significantly changes the health or income situation of the household, or if the biological structure of the household changes (in a way other than provided for in the plan). In addition, the plan management process must also consider that the household may not be able to immediately set such set of constraints, including financial goals, that the optimization task is certainly solvable. This is why an iterative process must be assumed, in which the household sets some constraints, then checks if any solution is feasible, and then is able to come back to the phase of constraint setting to modify them if necessary. In most cases, constraint modification will consist in reduction of the value of some financial goals, rescheduling the goals, or even abandoning some of them.

4.3.5 Making use of some chosen elements of the plan management process in risk tolerance model verification

Whereas the cyclical process of financial plan management is not the subject matter of this research project, it may be used to improve risk tolerance assessment methods. This time, the plan management framework will have to be used for many households, but without any attempt to implement the plan in real life and without regular revisions after that. The goal is different. A sample of households (clients) is chosen and then risk tolerance of these households is assessed. It must be then expressed by means of a risk tolerance parameter or a set of risk tolerance parameters that are compatible with the financial plan optimization procedure. These parameters may take some chosen or all aspects of risk tolerance into account (compare section 3).

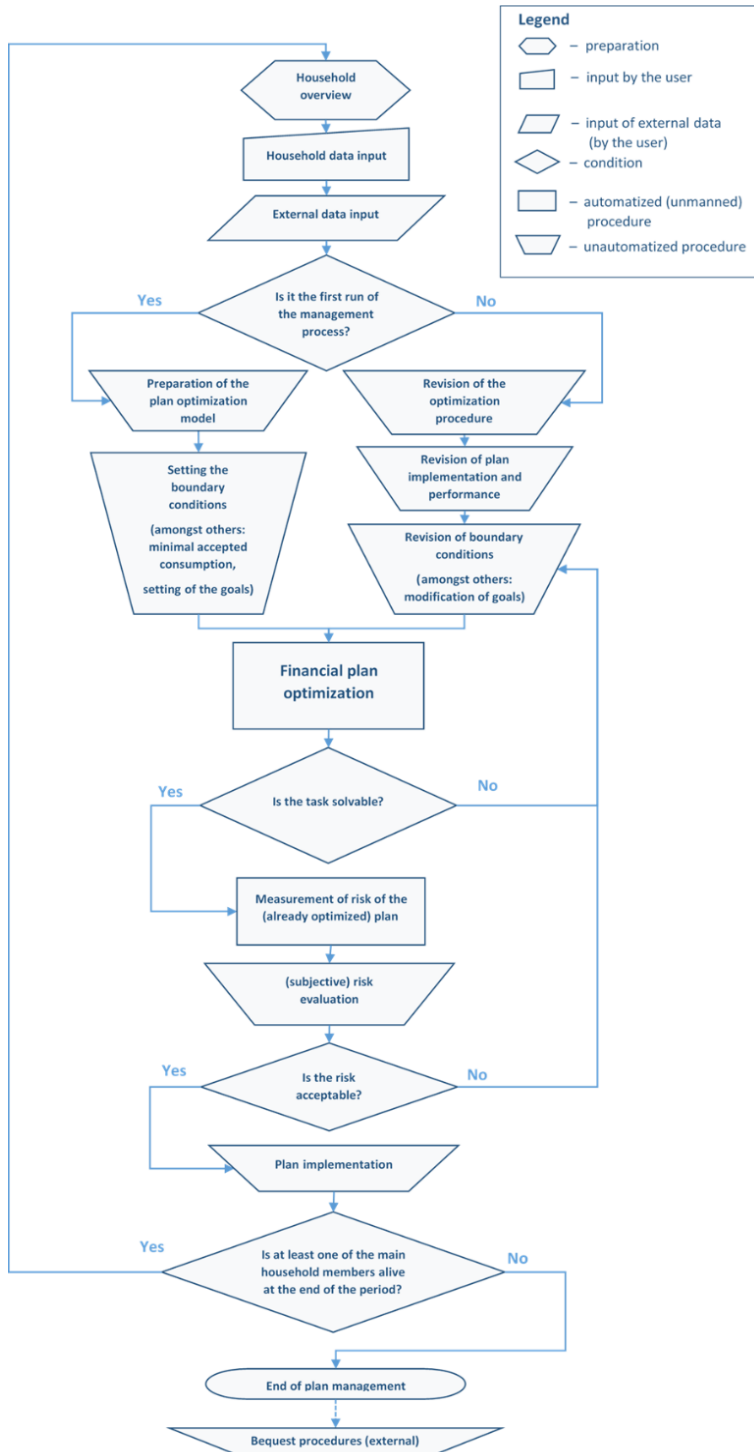


Figure 5. Household financial plan management flowchart

Source: Own elaboration by Pawel Rokita

The decision how many risk tolerance parameters will be used and what they will be like is yet not part of this research. At this stage of the project, a general framework is to be prepared. Then, with the risk tolerance parameters estimated, the plan optimization procedure is used. Risk tolerance parameters, together with other parameters mentioned in the “Parameters” panel of Figure 2, are part of the input to the household cashflow model. The household cashflow model is, in turn, used to calculate household consumption and bequest values used by the criterion function of the plan optimization procedure.

As the plan optimization procedure takes risk tolerance parameters into account, the resulting optimal plan should be in line with the household's risk tolerance. To check it, the risk of the optimal plan must be measured and then expressed in a way that is understandable for the household members. If the result of optimization really complies with household risk tolerance, the household will accept the plan. The verification of the way in which risk tolerance is assessed may be performed on the basis of the positive and negative answers given by the respondents from the analysed sample. The procedure of risk tolerance assessment method verification is visualized in Figure 7.

The research based on the scheme described by the flowchart in the Figure 7 may be conducted using statistical surveys, requiring multiple in-depth interviews with many respondents. As mentioned above, this will involve the simulation of the full process of building a long-term financial plan for each household, including the preparatory phase, estimating the risk tolerance, presentation of the simulation illustrating the risk of this plan to the respondent, and gathering information from the respondent whether the risk of the financial plan received is in line with his expectations and whether it is acceptable. A mix-mode study, with decision makers representing households, could be performed in 4 stages (Pachnowska, 2020):

Stage 1: the preparatory phase: the analysis of needs, risk tolerance assessment – self-assessment in CAWI surveys.

Stage 2: main phase: preparation of the plan risk simulation, using the tool set described in Section 4 and in Figure 7.

Stage. 3: own evaluation of the simulations generated for the plan prepared in stage 2, in self-completed CAWI surveys.

Stage 4: in-depth CATI or online interviews with households assessing plans critically or very high (as very adequate or very inadequate) to determine the reasons for the discrepancy / consistency (after top 10 extreme opinions).

5. Summary

The presented study prepares the ground for testing risk tolerance assessment methods that may be then used as part of client onboarding to an automated financial planning system.

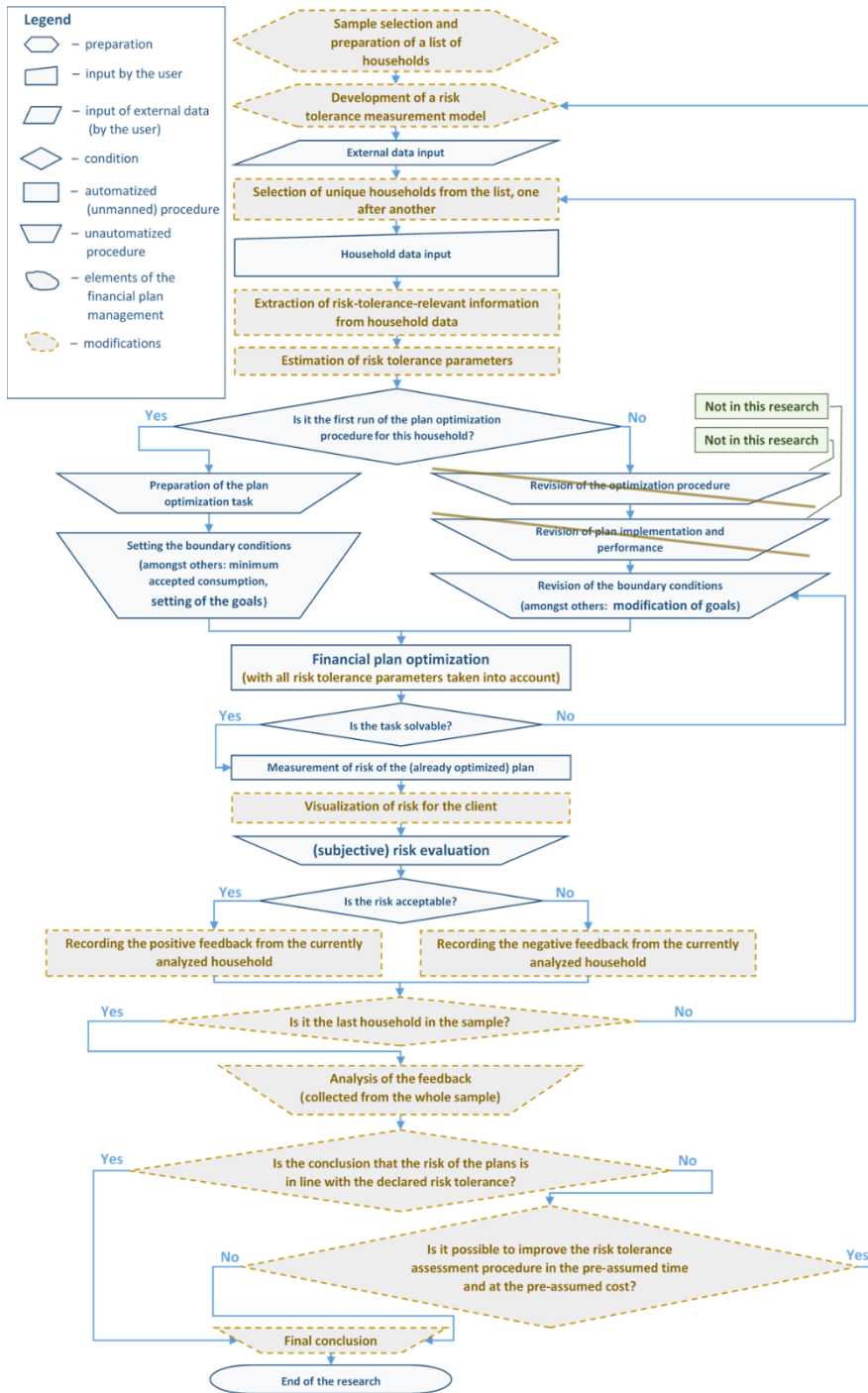


Figure 6. Household financial plan management flowchart

Source: Own elaboration by Paweł Rokita

The aim of this research was to propose a construction scheme of the test environment for household risk tolerance assessment verification and to show the position of risk tolerance assessment task in the whole financial planning model used for this purpose.

There was also an intention of the research to consider if a financial plan optimization model proposed by Jajuga, Feldman, Pietrzyk and Rokita (2015) and Pietrzyk and Rokita (2016b) is adaptable to the task of verifying the risk tolerance estimation method. It is shown that the model plan management process constructed for the purposes of that model may be almost directly used. At the same time, risk tolerance parameters used by the model, and the way in which the parameters are included in the criterion function of the plan optimization procedure, must be, of course, adjusted to the definition of risk tolerance that will be used and to the risk tolerance measure that will be constructed.

The proposal described in this article may find application in preparation of a risk tolerance model for the financial plan of a household. That one, in turn may provide a theoretical basis for yet another, already more technical, IT-based project to automate the household risk tolerance assessment process.

The main idea underlying this project is to define risk tolerance (section 3), place its estimation in the more general household financial plan management process (subsections 4.1-4.7), and then use previously developed and already working elements of the financial plan optimization model in the process of verifying the risk tolerance estimation method (section 5). The very estimation of risk tolerance will be the subject of research in a next-stage, larger research project.

In the longer term, this solution should be developed to support the entire automated system for managing a household's financial plan, including optimizing the plan. The decision-making members of the household will of course set goals and some other constraints that are naturally at their discretion. They will also be the only ones authorized to change the value of the goals, postpone goals accomplishment in time, give up some goals, or add new ones. In the target model, all other operations will be performed automatically. Customers will react in case the system informs about the necessity of intervention. Such a need may occur, for example, when it proves impossible to achieve all the intended financial goals with the given client's income, or when an unforeseen event requires a decision changing the previous assumptions of the financial plan.

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