

Large-Eddy Simulations of the flow on an airfoil with leading-edge imperfections

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ABSTRACT

We performed large-eddy simulations of the flow over an airfoil to understand the effects of leading-edge roughness designed to mimic ice accretion. The roughness elements protrude outside the boundary layer, which, near the leading edge, is very thin; thus, the configuration does not represent a classical rough-wall boundary layer, but rather the flow over macroscopic obstacles. A grid convergence study is conducted and results are validated by comparison to numerical and experimental studies in the literature. The main effect of the obstacles is to accelerate transition to turbulence. Significant variations in structure generation are observed for different roughness shapes. The three-dimensionality of the irregularities has a strong impact on the flow: it creates alternating regions of high-speed (“peaks”) and low-speed (“valleys”) regions, a phenomenon termed “channelling”. The valley regions resemble a decelerating boundary layer: they exhibit considerable wake and higher levels of Reynolds stresses. The peak regions, on the other hand, are more similar to an accelerating one. Implications of the channelling phenomenon on turbulence modelling are discussed.

KEYWORDS

Large-eddy simulations; airfoil; roughness; icing

1 Introduction

Maintaining laminar flow on aircraft can decrease the drag and fuel consumption significantly. Thus, much effort has been expended to design airfoils and engine nacelles in which the transition to turbulence is delayed. This has been particularly successful in the design of Natural Laminar Flow (NLF) nacelles [1, 2]; maintaining laminar flow, for instance, results in a 1-2% reduction in total drag, which in turn amounts to about 1-2% reduction in cruise fuel burn. One of the main causes of disruption of the laminar flow on an engine nacelle or a wing is the presence of surface imperfections; they can be due to manufacturing (gaps between metal plates, rivets etc.), or to natural phenomena such as ice formation.

Ice depositions have particularly negative effects in aeronautical applications, since they affect the lift as well as the drag. Thus, much research has been carried out in this area, focusing onto three questions: (1) what are the geometrical ice shapes encountered in practice? (2) what are the effects of these ice-shapes on the flow field?

15 and, (3) what are the ways to control or prevent ice accretion? A complete review of
16 icing research can be found in papers by Gent *et al.* [3] and Cebeci and Kafyeke [4].
17 The literature regarding the first two questions will be briefly reviewed here; (3) is
18 outside the scope of the present study.

19 Past research into ice accumulation dealt mainly with the physics of accretion and
20 the prediction of different ice-shapes. Early efforts were carried out, for example by
21 NASA [5, 6], ONERA [7, 8] and the National Research Council (NRC) in Canada
22 [9, 10]. Over the years, researchers have documented many different types of ice-shapes
23 using both experimental measurements and numerical ice-accretion prediction models
24 such as LEWICE [11, 12, 13, 14] and FENSAP-ICE [15, 16, 17].

25 The leading edge of an airfoil has been established to be the most sensitive region
26 for ice formation [18, 19]. Cebeci [20, 21] found that the accumulation can resemble
27 sand-grain roughness, or can be large-scale, and change the shape of the airfoil entirely.
28 Lynch and Khodadoust [22] observed that both of these types of accumulations are
29 highly dangerous and can cause performance degradation, in terms of maximum lift
30 capability, up to 40% (for sandgrain-type accumulations) or 80% (for large-scale ac-
31 cretion). They classified these shapes based on the physical processes governing their
32 formation and their topology. Bragg *et al.* [23] classified ice accretions into four major
33 categories: ice roughness, horn ice, streamwise ice and spanwise-ridge ice, based on the
34 difference in the flow-field observed. They also observed that many ice shapes do not
35 belong to a single category, but may have features representative of two or more.

36 Ice roughness (which supplies the motivation for this study) occurs in the initial
37 stages of the ice-accretion process, where each roughness element acts, more-or-less,
38 as an isolated entity. Shin and co-workers [24, 25, 26, 27] performed experiments on
39 a NACA 0012 airfoil, and found ice roughness to have a height greater than the lo-
40 cal boundary layer thickness, even in the early stages of the ice-accretion process.
41 They further described the geometrical features of ice roughness, which has an effec-
42 tively smooth zone sandwiched between two rough patches (one on each side of the
43 airfoil). McClain and co-workers [28, 29, 30, 31] made detailed measurements of the
44 ice-roughness topology on a NACA 0012 airfoil at 0° angle of attack and showed that
45 the surface topology varies significantly, from a primarily 3D accumulation to a more
46 2D character, with accumulation time.

47 Most experimental work on the effect of ice roughness on airfoils and wings have tried
48 to quantify the degradation of the aerodynamic performance. The general consensus
49 is that these accretions result in an increased drag [22] and reduction in lift [32, 33].
50 Some studies, however, have observed an increase [34, 35, 36] in lift where, they note,
51 the roughness effectively acts as leading edge slat. More detailed studies have focused
52 on other aspects, such as the changes in the transition process [37, 38, 39], the effect of
53 size [40, 41], shape [42, 43, 21], location [44, 45] and density [21, 46] of the roughness
54 elements.

55 Kerho and Bragg [37] observed that roughness triggers the transition process at
56 (or very near) the roughness trailing-edge through a markedly different route than
57 on smooth airfoils. Plogmann *et al.* [39] conducted experiments on a single hemi-
58 spherical roughness element on the leading edge and found that flow transitions only
59 when $h/\delta > 0.75$, where h is the roughness height and δ the local boundary layer
60 thickness; for $h/\delta < 0.2$ they did not observe any noticeable mean velocity distortion.
61 Zhang *et al.* [40, 41] found roughness height to be a more significant factor in low- Re
62 flow degradation than the distribution pattern (aligned *vs.* staggered elements). More
63 recently, Vinnes *et al.* [47] conducted experiments to demonstrate the feasibility of
64 reduced-order modelling for flow over iced airfoils.

65 Numerical studies on the flow over rough airfoils have been carried out mostly using
66 the Reynolds-Averaged Navier-Stokes (RANS) approach and, more recently, hybrid
67 methods that solve the Reynolds-Averaged Navier-Stokes (RANS) equations near the
68 wall, switching to large-eddy simulations (LES) away from the solid surfaces. Hybrid
69 RANS-LES methods will be referred to as HRL. Due to the scale and simplicity of
70 the geometry and its strong effects on aerodynamics, most studies have focused on the
71 horn type of roughness. Brown *et al.* [48] performed resolved-calculations using the
72 implicit LES (ILES) approach and found the results to be accurate until the pre-stall
73 regime; ILES fails to predict the unsteadiness of the flow at higher angles of attack.
74 They also found a strong effect (with a variance of 95%) of the spanwise variation in
75 the geometry on lift coefficients for angle of attack in the range $5^\circ - 15^\circ$.

76 In a recent review paper, Stebbins *et al.* [49] made a comprehensive assessment of
77 RANS [50, 51, 52] and HRL methods [53, 54, 55] in the study of horn geometry and
78 concluded that the results of HRL methods are more accurate than RANS solutions,
79 particularly at high angles of attack, owing to the ability of LES-based methods to
80 capture the unsteadiness of the flow. Still, they displayed discrepancies with the ex-
81 perimental data. Xiao *et al.* [56] used wall-modelled LES (WMLES) and observed
82 better predictions of the separated shear-layer dynamics.

83 Studies related to ice-roughness geometry are fewer. In their review, Stebbins *et al.*
84 [49] mention no study of ice roughness that uses eddy-resolving methods. Konig *et al.*
85 [57] and Ribeiro *et al.* [58] performed Very Large Eddy Simulations (VLES) combined
86 with a Lattice-Boltzmann method, on ice-roughness geometries with height $\sim 0.2\%$
87 of the chord. While Konig *et al.* [57] noted an over-prediction of the maximum lift,
88 Ribeiro *et al.* [58] observed that the use of simplified shapes that are spanwise constant
89 is not realistic because of the three-dimensional flow field due to an uneven separation
90 pattern behind the roughness. Recently, Ribeiro *et al.* [59, 60] reported a DNS study of
91 a NACA 0012 airfoil, at $Re = 657,000$ and angle of attack of 0° , with sand-grain variety
92 of roughness on the leading edge ($x/c < 20\%$) using a Lattice-Boltzman method, within
93 the context of noise prediction; a detailed flow analysis was not performed. Moreover,
94 although they mentioned the grid to have a uniform distribution of $y^+ = 0.5$ on the
95 airfoil surface, a proper grid-convergence study was not reported.

96 To understand better how ice formations affect the transition to turbulence on
97 airfoils and nacelles, and how the turbulent flow is modified, we carried out a study
98 of the flow over a NACA 4412 airfoil at small angle of attack ($\alpha = 5^\circ$) and moderate
99 Reynolds number ($Re = 200,000$). The leading edge is modified by adding either two-
100 dimensional trip wires, or three-dimensional protuberances similar to those observed
101 in the ice-accretion experiments of McClain *et al.* [29]. We use Wall-Resolved LES
102 (WRLES), which allow us to study the unsteadiness and three-dimensionality of the
103 flow in great detail. In our simulations, most of the turbulent scales are resolved
104 everywhere (even near the roughness) which is an advantage over RANS and HRL
105 methods, since the model for the unresolved scales has a less significant effect on the
106 results, and the flow near the roughness elements can be studied. It should be remarked
107 that, although we sometimes refer to the irregularities as “roughness”, they are much
108 larger than is usual in rough-wall studies. Since they are placed at the leading edge,
109 where the boundary layer is very thin, they protrude out of the boundary layer itself.
110 Thus, we are considering the boundary-layer flow over large obstacles, rather than a
111 classical rough-wall boundary layer.

112 In the following we will first formulate the problem, in terms of numerical method,
113 physical model and geometric configuration. Then we will validate the simulations
114 and present the results. Finally, comments and recommendations for future work will

115 conclude the paper.

116 2. Problem formulation and methodology

117 2.1. Governing equations

118 The flow over a NACA 4412 airfoil at $Re = U_o c / \nu = 200,000$ (based on freestream
119 velocity U_o and chord c) and angle-of-attack $\alpha = 5^\circ$ was studied using Wall-Resolved
120 Large-Eddy Simulations (WRLES). The filtered Navier-Stokes equations for incom-
121 pressible flow were solved:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0. \quad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}. \quad (2)$$

122 where $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ are the unresolved Sub-Filter Scale (SFS) stresses, x , y , z
123 (or x_1 , x_2 , x_3) are the streamwise, vertical and spanwise directions, respectively. The
124 corresponding filtered instantaneous pressure and velocity fields are \bar{p} and \bar{u} , \bar{v} , \bar{w}
125 (or \bar{u}_i). We will also be using subscripts t and n to indicate the velocity components
126 tangential and normal to the airfoil surface. In the following, the overline will be
127 dropped; u_i and p will be implicitly assumed to be filtered quantities.

128 The unresolved, subfilter-scale, stresses are modelled using the ILSA model [61], in
129 its local formulation [62]. The model parameter, s_τ , which measures the SFS contri-
130 bution to the dissipation, is set to 0.1 based on the recommendation of Lehmkuhl *et*
131 *al.* [63] for prismatic elements.

132 The simulations in this study are performed using Alya, a multi-physics, massively
133 parallelized, unstructured finite-element simulation code developed at Barcelona Su-
134 percomputing Center [64, 65]. It has been widely validated in many turbulent-flow
135 configurations: [66, 67, 68, 69, 63, 70]. The governing equations (1) and (2) are dis-
136 cretised on a collocated unstructured grid by means of low dissipation second-order
137 conservative schemes [71]. The same interpolation scheme for velocity (u_i) and pressure
138 (p) is used in space. A third order Runge-Kutta explicit time discretization is used to
139 advance the solution in time, combined with an eigenvalue-based time-step estimator
140 [72]. A fractional time-step algorithm is used to solve the resulting system of linear
141 equations [73]. The Poisson equation is solved using a Deflated Conjugate Gradient [74];
142 convergence and stopping criteria are based on the lagged algebraic residual.

143 2.2. Airfoil geometries

144 Four surface geometries (shown in Figure 1) were studied. They are all based on the
145 NACA 4412 airfoil, with various leading-edge modifications. In addition to the unmodi-
146 fied airfoil with a smooth leading-edge (SLE), we considered a Tripped Boundary-Layer
147 (TBL) case with three cylindrical elements near the leading edge on the suction side,
148 and two Rough Leading-Edge cases (RLE1 and RLE2) with randomly distributed
149 roughness elements.

150 The three semi-cylindrical trips in the TBL geometry were located at 0.1%, 2.1% and
151 4.2% of the chord c ; their heights were $0.003 c$ (see Figure 1(b)). The size and location
152 of the trips was chosen to achieve early establishment of the turbulent flow regime. The

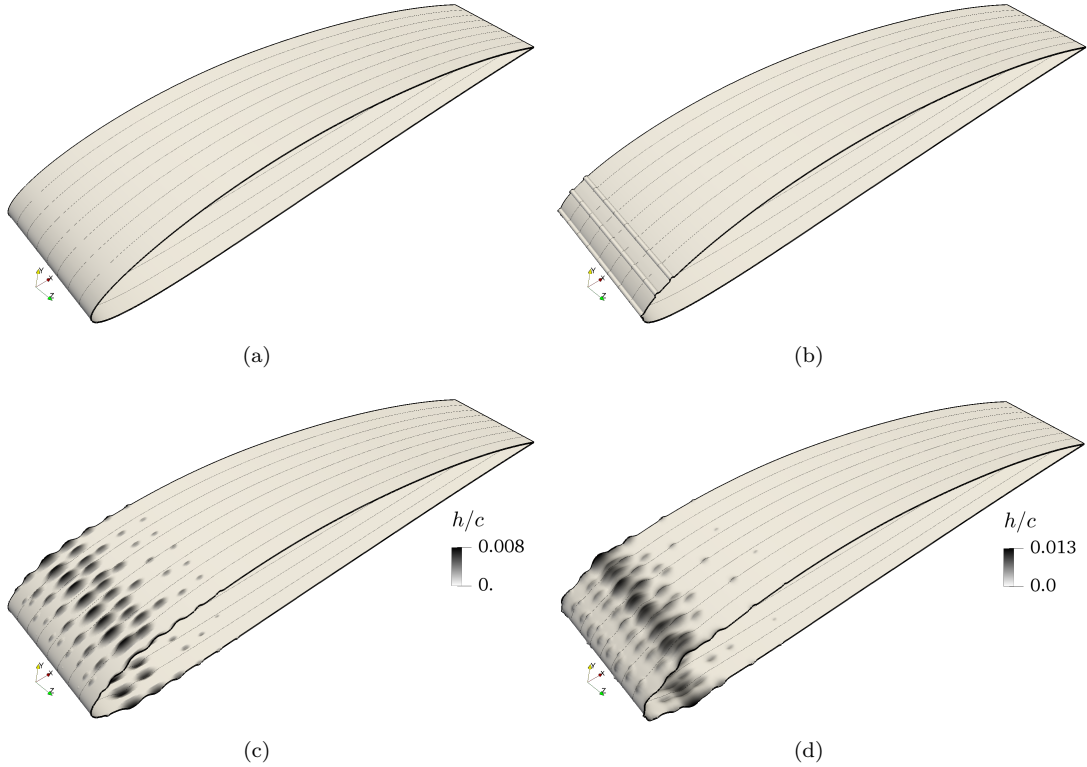


Figure 1. NACA4412 airfoil geometries used in the present study: (a) Smooth Leading Edge (SLE); (b) Tripped Boundary Layer (TBL); (c) Rough Leading Edge 1 (RLE1); (d) Rough Leading Edge 2 (RLE2)

153 RLE1 and RLE2 cases try to reproduce the ice accretions in the study by McClain *et*
 154 *al.* [31] (see Figure 3 in that paper). The roughness elements were ellipsoids; their
 155 height, h , depended on their x location, to have a similar distribution as in the study by
 156 McClain *et al.* [31]. The maximum peak-height, $h_{p,\max}$, was $0.008c$ and $0.013c$ for the
 157 RLE1 and RLE2 cases, respectively. The corresponding mean peak-roughness height,
 158 \bar{h}_p , was $0.004c$ and $0.009c$. Since the roughness elements were located close to the
 159 leading edge, where the boundary layer is very thin, they extend outside the boundary
 160 layer; for example, in TBL the ratio of roughness height to the local boundary-layer
 161 thickness are 23, 2 and 1 respectively for the three semi-cylindrical trips used.

162 Although care was taken for these geometries to resemble those found in the ex-
 163 periments of McClain *et al.* [31], some simplification was required to maintain grid
 164 quality. The spacing between the elements, for instance, is nearly constant, with a
 165 small random variation, and their shape is ellipsoidal. The ice deposition, as shown by
 166 [31], has a more random distribution. By matching the height distribution and length
 167 scale of the real ice roughness, however, we retained key elements of a real geometry
 168 to be able to improve the understanding of the flow physics involved.

169 **2.3. Computational domain and boundary conditions**

170 All computations were carried out on a $40c \times 40c \times 0.2c$ domain (shown in Figure
 171 2), with the leading edge placed at the origin. This is consistent with the previous
 172 simulations carried out by [75, 76] who used the same methodology to study the flow
 173 over a NACA 0012 airfoil. The two-point correlation of the velocity components in the

Case	N_z	Δs^+	Δn^+	Δz^+	$N_{dof} \times 10^{-6}$	C_l	C_d
SLE	97	2.8	2.8	18	14.1	0.95	0.0176
TBL - C	97	4.3	4.3	20	8.1	0.84	0.0239
TBL - M	145	1.52	1.52	13	28.8	0.81	0.0244
TBL - F	197	0.85	0.85	10	80.5	0.80	0.0248
RLE1	145	1.8	1.8	14.5	28.3	0.77	0.0267
RLE2	145	1.8	1.8	14.5	28.3	0.77	0.0327
Smooth, Ref. [78]	-	18	0.64	9	336	0.885	0.0185

Table 1. Summary of computational parameters. The grid spacing in viscous units (denoted by +) is calculated at $x/c = 0.6$ on the suction side of the airfoil. C, M and F refer to Coarse, Medium, and Fine meshes. N_z : number of points in the spanwise direction; Δs , Δn : grid spacing in wall-parallel and wall-normal directions; N_{dof} : number of degree of freedom per unknown. C_l , C_d are the lift and drag coefficients.

174 spanwise direction (not shown) confirmed that the domain is wide enough to contain all
175 the important structures. An open-source grid generation tool, Gmsh [77], was used to
176 generate the mesh. A body-fitted grid with triangular-prism elements was used. Mesh
177 smoothing was used in all directions to maintain a good quality of the final grid used.
178 A sample grid used in this study is shown in Figure 2; an uniform element-size is used
179 on the airfoil surface and grid is stretched as we move away from the body.

180 The boundaries used the following conditions: at the inlet a uniform velocity
181 $(u, v, w) \equiv (U_o \cos \alpha, U_o \sin \alpha, 0)$ was assigned; at the outlet, a pressure-based con-
182 dition was applied, where the total pressure (p_d) and outlet normal velocity (u_n) are
183 related as $p_d = 1/2\rho u_n^2$. No-slip conditions were imposed on the airfoil surface, and the
184 flow was assumed to be periodic in the spanwise direction. Since no disturbances were
185 introduced artificially, the flow transitions naturally from the amplification of small
186 perturbations (due to round-off or truncation error). As will be shown, the presence
187 of leading-edge imperfections plays a critical role in the transition process.

188 Parameters related to the mesh are listed in Table 1, along with the lift (C_l) and
189 drag (C_d) coefficients. They are defined as:

$$C_l = \frac{2\mathcal{L}}{\rho U_o^2 z_D c}; \quad C_d = \frac{2\mathcal{D}}{\rho U_o^2 z_D c}; \quad (3)$$

190 where \mathcal{L} and \mathcal{D} are the lift and drag forces calculated as the sum of pressure and
191 viscous contributions at each computational cell on the airfoil surface.

192 *2.4. Averaging operators*

193 In studies of the flow over airfoils it is customary to average the turbulent quantities in
194 both time and the spanwise direction, in which the flow is statistically homogeneous.
195 Here, in the RLE1 and RLE2 cases, the three-dimensional nature of the roughness
196 introduces spanwise inhomogeneities that persist along the airfoil (as will be shown
197 momentarily). It is convenient, then, to introduce two averaging operators, similar to
198 the triple decomposition commonly used to study flows over rough surfaces [79, 80].

199 In addition to the standard time-averaging (indicated by an overbar or by a capital
200 letter: $\bar{f} = F$), we can also average quantities both in time and in the spanwise direction

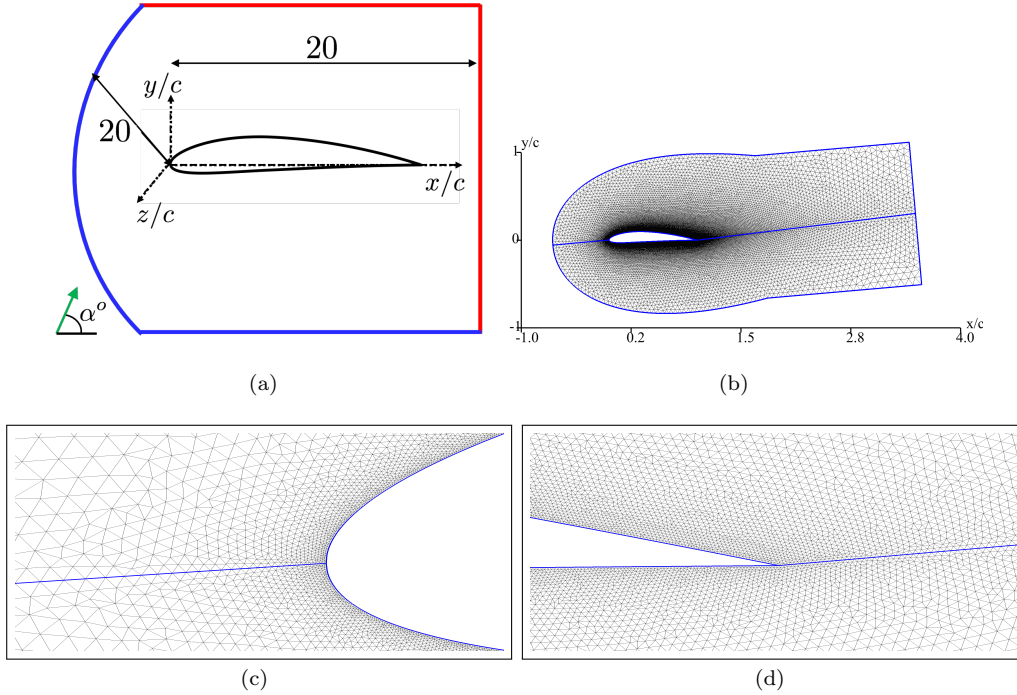


Figure 2. A sample grid distribution; actual grid densities for different cases are reported in Table 1. (a) Sketch (not to scale) showing the computational domain, blue represents inflow and red represents outflow conditions; (b) grid distribution near the airfoil; (c) enlargement of the leading edge, and (d) of the trailing edge.

201 (“double-averaged”, or “DA”, quantities); the spanwise averaging is indicated by angle
 202 brackets, so that a DA quantity would be written as $\langle F \rangle = \langle f \rangle$. Note that if the flow
 203 is statistically homogeneous, time-averaging, spanwise averaging and double averaging
 204 give the same result. A turbulent quantity can be decomposed in various ways:

$$f = F + f' = \langle F \rangle + \tilde{f} + f' = \langle F \rangle + f'' \quad (4)$$

205 \tilde{f} is known as the “wake field,” or “form-induced perturbation.” It represents the devi-
 206 ation of the time-averaged field from the time-and-space averaged one, and highlights
 207 the geometry-induced effects. The wake field is stationary, and its spanwise average is
 208 zero. f' is the stochastic fluctuations. f'' is the deviation from the DA quantity, which
 209 contains a steady component, \tilde{f} , as well as the fluctuation f' .

When we apply the triple decomposition to the velocity and calculate the DA second-order moments, we obtain:

$$\langle \tilde{u}_i \tilde{u}_j \rangle = \langle U_i \rangle \langle U_j \rangle + \langle \tilde{u}_i \tilde{u}_j \rangle + \langle \overline{u'_i u'_j} \rangle \quad (5)$$

210 $\tilde{u}_i \tilde{u}_j$ are the “dispersive stresses”, $\overline{u'_i u'_j}$ the stochastic ones. If the flow is homogeneous
 211 in z , as well as far from the roughness elements, the dispersive stresses vanish.

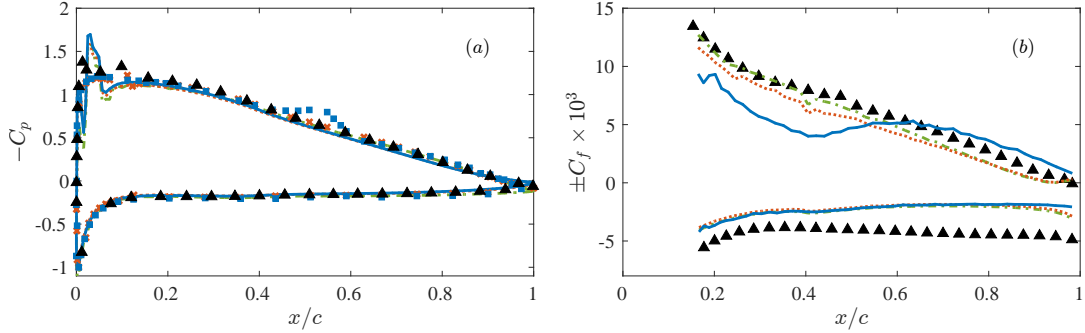


Figure 3. Grid convergence study for the Tripped Boundary Layer (TBL) case: (a) Pressure (C_p); and (b) Friction (C_f) coefficients. — Coarse; - - - Medium; ··· Fine; ▲ LES [78]; × Exp., tripped [81]; ■ Exp., smooth [81]. For clarity, C_f is plotted on the suction side, $-C_f$ on the pressure side.

212 3. Results

213 3.1. Grid-refinement study

214 We performed a grid-refinement study, using the TBL configuration. The three grid
 215 resolutions used were reported in Table 1, where they are denoted by TBL-C, TBL-M
 216 and TBL-F for the coarse, medium and fine meshes, respectively.

217 The pressure (C_p) and skin-friction (C_f) coefficients, are defined as

$$C_p = \frac{2(\langle P \rangle - p_\infty)}{\rho U_o^2}; \quad C_f = \frac{2\langle \bar{\tau}_w \rangle}{\rho U_o^2} \quad (6)$$

218 where p_∞ is the reference pressure, and τ_w is the wall shear-stress respectively. The
 219 pressure coefficient is fairly insensitive to grid resolution in the smooth part of the
 220 airfoil and all grids give similar results. C_f is more sensitive to the grid size; the
 221 medium and fine meshes are in good agreement with each other. Both C_f and C_p
 222 agree well with the reference data.

223 Figure 4 shows profiles of the mean wall-parallel velocity U_t and turbulent kinetic
 224 energy (TKE), $\mathcal{K} = \overline{u'_i u'_i} / 2$. Both U_t and \mathcal{K} are also spanwise-averaged, and normal-
 225 ized by the edge velocity U_e (which will be defined momentarily). The coloured area
 226 represents error bars for the medium grid. The error is calculated as

$$\epsilon = \frac{f_2 - f_1}{f_1(r^p - 1)} \quad (7)$$

227 where f_1 represents the quantity considered (velocity or TKE) on the fine grid, f_2
 228 the same on the medium grid, $r = 3^{1/3}$ is the grid refinement ratio, and $p = 2$ is the
 229 order of accuracy of the spatial scheme used. The factor $(r^p - 1)$ in the denominator
 230 serves to avoid underestimation of the error when the grid is refined by small amounts
 231 (i.e., $r \simeq 1$). While the mean velocity is grid-converged, the difference in the TKE
 232 between medium and fine grid is significant, especially near the trailing edge. Similar
 233 observations on the grid convergence were made by Vinuesa *et al.* [78] in their study
 234 of Reynolds number effects on flow over NACA 4412 airfoil at $Re = 1 \times 10^6$ with a
 235 coarser grid resolution. Since the mean velocity is grid-converged on the medium grid,
 236 while the TKE matches at least the general behaviour, the medium grid is considered
 237 sufficient to resolve the main phenomena discussed hereafter, with the *caveat* that any

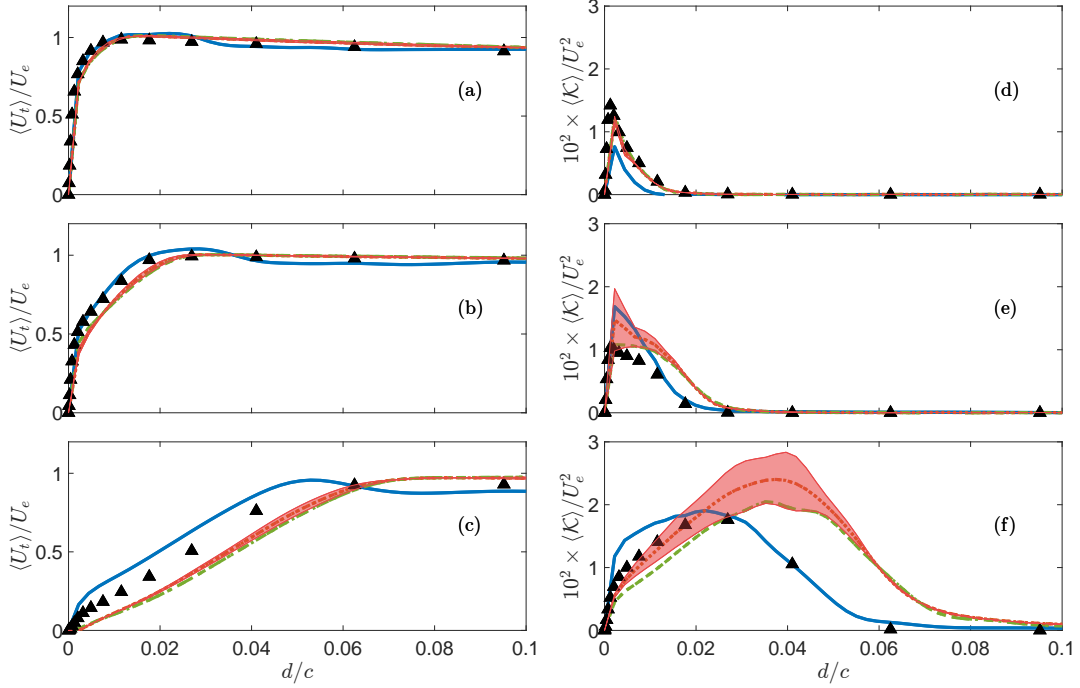


Figure 4. Grid convergence study for the Tripped Boundary Layer (TBL) case: (a,b,c) wall-parallel velocity ($\langle U_t \rangle / U_e$); (d,e,f) Turbulent Kinetic Energy ($10 \times \langle \mathcal{K} \rangle / U_e^2$) at specific streamwise locations, (a,d) $x/c = 0.2$; (b,e) $x/c = 0.58$ and (c,f) $x/c = 0.98$ respectively; d is the wall-normal distance; U_e is the velocity at the boundary layer edge. — Coarse; - - - Medium; ··· Fine; ▲ LES [78].

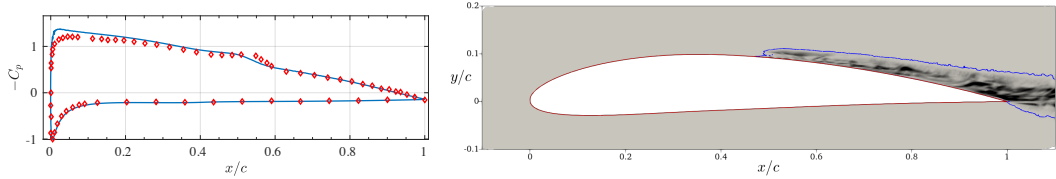


Figure 5. (a) Pressure coefficient for the SLE case, medium grid. Line: present results; symbols [81]. (b) Instantaneous SFS eddy-viscosity ν_t (normalized by the molecular viscosity) in an xy - plane. Colours range from 0 (white) to 10 (black); the solid blue line is the $\nu_t / \nu = 0.1$ contour.

238 conclusion regarding the TKE can only be qualitative. Although the fine resolution
 239 would be optimal, its computational requirements did not make it possible to use it for
 240 all cases. A typical simulation on the medium grid required around 7.2 million CPU
 241 hours on the Compute/Calcul Canada systems.

242 Finally, in Figure 5(a) we compare the pressure coefficient for the SLE case with
 243 the experimental data of Mallor [81], who carried out measurements for this airfoil
 244 at the same flow configuration. Measurements were taken for a case in which the
 245 boundary layer was tripped, and for an untripped one. Only the C_p is available for
 246 the untripped case corresponding to SLE. The agreement of the simulation with the
 247 experimental data is very good. In particular, we note that the onset of transition
 248 results in a kink in the pressure-coefficient profile at $x/c \simeq 0.55$, which is captured
 249 well by the simulation. We also note that the ILSA model has the property that ν_t
 250 vanishes in laminar regions of the flow; this behaviour is observed here, Figure 5(b).
 251 Thus, the prediction of transition is only affected by the grid resolution and not by
 252 the SFS model.

253 A point that requires further discussion regards the differences observed in Figure 4
 254 between the present results and the data by Vinuesa *et al.* [78], who studied the
 255 same airfoil, at the same Reynolds number and angle of attack. The flow in this
 256 configuration has been shown to be very sensitive to the laminar/turbulent transition
 257 process, i.e., to the tripping device used [82, 83]. Vinuesa *et al.* [78] tripped the
 258 boundary layer, on both sides of the airfoil, using volumetric forcing at $x/c = 0.1$ (see
 259 [83] for details). In our study, on the other hand, we use localized perturbations (the
 260 trip wires for case TBL, the roughness for RLE1 and RLE2), which cause significantly
 261 different routes to turbulence. Recently, in fact, Tanarro *et al.* [84] extended the work
 262 of Vinuesa *et al.* [78] to enable Adaptive Mesh Refinement (AMR) for the simulations
 263 of the NACA 4412 airfoil. Although they used the same code and numerical setup as
 264 [78], with a minimal change in the tripping method, they still observed differences in
 265 the flow field, both on the suction and on the pressure sides. Given this sensitivity
 266 of boundary layer to the tripping methodology used, perfect quantitative agreement
 267 between the current study and the data of [78] could not be achieved, and should not
 268 be expected. For a case in which the tripping is not present, however, we obtain better
 269 agreement. We rely, instead, on the level of grid convergence achieved, on the fact that
 270 the computational methodology has been thoroughly validated for different flows of
 271 this type [75, 85, 63, 70], in terms of both numerical scheme and SFS model, and on
 272 the good agreement with a case in which the flow was untripped, discussed earlier.

273 3.2. Instantaneous flow structures

274 Figures 6 and 7 show the iso-surfaces of the second invariant of the velocity-gradient
 275 tensor,

$$Q \equiv \frac{1}{2} (|\mathbf{\Omega}|^2 - |\mathbf{S}|^2) \quad (8)$$

276 where $\mathbf{\Omega}$ and \mathbf{S} are the rotation and rate-of-strain tensors. All quantities are normalized
 277 by U_o and c .

278 In the SLE case the boundary layer on the suction side, is initially laminar; the
 279 flow separates at $x/c = 0.41$ and reattaches at $x/c = 0.53$ creating a thin, closed
 280 laminar separation bubble with a maximum height of $0.005c$. The flow then undergoes
 281 a transition process that appears to be associated with undulations of the spanwise
 282 vortices formed in the separated shear-layer. The flow breaks down shortly after the
 283 formation of these secondary instabilities, re-attaches and finally develops into Λ -
 284 shaped vortices.

285 In the TBL case the flow is significantly different; spanwise vortices are formed in
 286 the shear layer emanating from the top of the first semi-cylindrical element (marked
 287 as *T1* in Figure 6(b)), which are advected downstream. The flow remains coherent for
 288 the first 20-30% of the chord (region **A** in the Figure), but then the 3D perturbations
 289 break the coherence of the spanwise vortices, and horseshoe vortices appear (region **B**).
 290 The flow becomes turbulent much earlier than in the SLE case due to the formation
 291 and breakdown of the rollers.

292 In the RLE1 case, we observe an early formation of 3D structures, which coalesce
 293 into hairpin-like vortices downstream of the roughness, in the region marked **B** in
 294 Figure 6(c). An interesting feature of this flow is the alignment of the hairpins (in
 295 the region marked as **A**, for instance). The alignment is associated with a channelling
 296 phenomenon that will be discussed momentarily. Eventually, the structures become

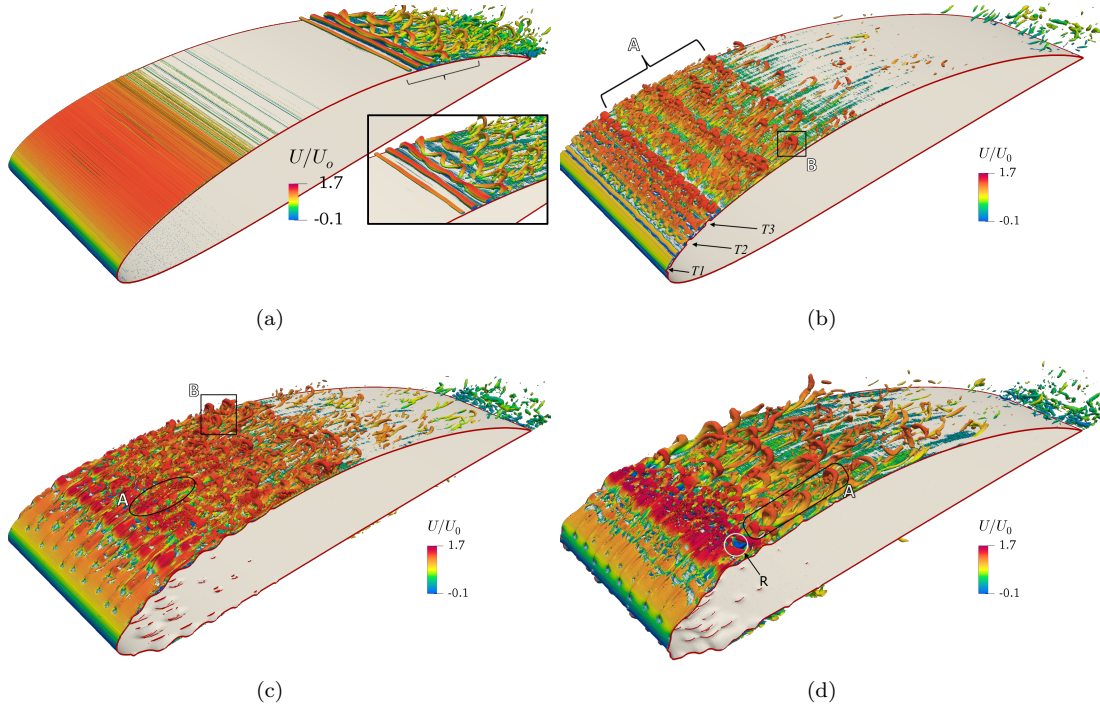


Figure 6. Iso-surfaces of $Q = 750U_o^2/c^2$, coloured by the time-averaged streamwise velocity U/U_o , for (a) SLE case; (b) TBL case; (c) RLE1 case; (d) RLE2 case. A top view of the region marked by curly braces in the SLE case is shown in the inset. The locations of the trips in TBL case are shown by arrows and marked with $T1$, $T2$ and $T3$.

297 larger and lose their alignment, and a more random distribution is observed. In the
 298 adverse pressure-gradient region the vortices are less coherent and less frequent.

299 For the RLE2 case, quasi-2D vortex-shedding from the obstacles is visible (for ex-
 300 ample, in region **A** in Figure 6(d)); the shed vortices become elongated downstream
 301 due to the shear. The turbulent structures are significantly larger than in the other
 302 cases, reflecting the greater height of the imperfections.

303 Since in the SLE and TBL cases the surface on the lower side is smooth and the
 304 pressure gradient is favourable the boundary layer there remains laminar. The in-
 305 stantaneous structures for the RLE1 and RLE2 cases are shown in Figure 7. The
 306 mechanism of flow breakdown is quite different for the two cases; while the flow in
 307 the RLE2 case undergoes transition soon after encountering the roughness elements,
 308 quasi-laminar regions can be observed in the wake of the smaller elements (regions
 309 that will be termed “valley regions”).

310 The instantaneous fields are useful to understand the general features of the flow.
 311 In the following, a quantitative analysis will be carried out for the boundary layer
 312 parameters and the three-dimensional effect of the leading-edge obstacles on the mean
 313 flow.

314 **3.3. Boundary-layer behaviour**

315 There are various criteria in the literature for finding the edge of the boundary layer in
 316 turbulent flows; among them is the use of composite profiles [86, 87], or of an intermit-
 317 tency factor [88], the spanwise-vorticity approach ([89]), the modified diagnostic-plot

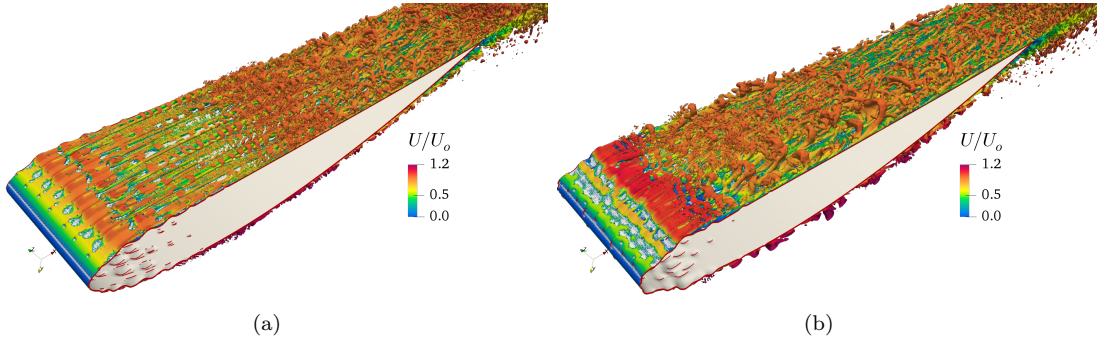


Figure 7. Iso-surfaces of $Q = 50U_o^2/c^2$, coloured by the time-averaged streamwise velocity U/U_o , for (a) RLE1 case; (b) RLE2 case on the pressure side.

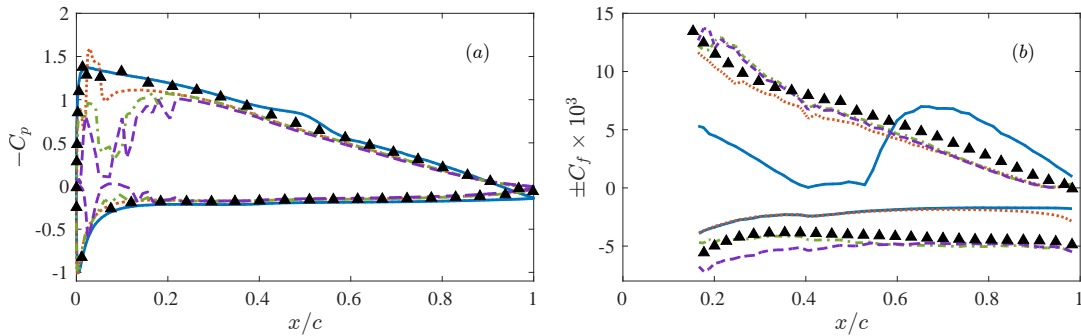


Figure 8. (a) Pressure coefficient; (b) Friction coefficient. — SLE; ··· TBL; - - - RLE1; - · - RLE2; ▲ LES [78]. For clarity, C_f is plotted on the suction side, $-C_f$ on the pressure side.

318 concept [90] and the method proposed by Griffin *et al.* [91] in which they fit a theoretical
 319 inviscid velocity profile (U_I) to the actual viscous one, and the edge of the boundary
 320 layer (i.e., the location where the velocity is $n/100$ of the edge velocity) is defined as
 321 the wall-normal location where $\langle U \rangle / U_I = n/100$. We have applied both Griffin's and
 322 diagnostic-plot criteria. On the suction side they give nearly identical results; on the
 323 pressure side the diagnostic-plot is harder to apply since the flow is laminar in the
 324 SLE and TBL cases. Therefore, only the results obtained using Griffin's criterion will
 325 be shown.

326 Figure 8 shows the pressure (C_p) and friction coefficients (C_f) on the airfoil. Once
 327 turbulence is established the surface-pressure distribution shows minimal variations
 328 among all cases, indicating the insensitivity of the outer, inviscid, flow to the boundary-
 329 layer behaviour at moderate and high Reynolds numbers; this is confirmed by the lift
 330 coefficient, Table 1; all the rough cases are within 1% of each other.

331 The C_f distribution, on the other hand, is quite sensitive to upstream conditions
 332 and to the tripping mechanism. On the suction side, SLE shows a typical transitional
 333 boundary-layer behaviour, with an overshoot of C_f above the turbulent values. Very
 334 small differences can be observed between the three rough cases: the TBL has a slightly
 335 lower friction; all cases agree reasonably well with the results of [78]. On the pressure
 336 side, the SLE and TBL cases have lower friction, since the flow is laminar. RLE2 has
 337 higher skin-friction than RLE1 immediately after the roughness; only for $x/c \geq 0.7$
 338 the results of the two rough cases collapse (and also agree with the data by [78]).
 339 This difference can be due to several factors: one is the larger roughness elements in

340 the RLE2 case, which cause a stronger disruption of the flow. The generation, in case
 341 RLE1, of streamwise vortices that alter the mean flow (which will be discussed later)
 342 may also play a role.

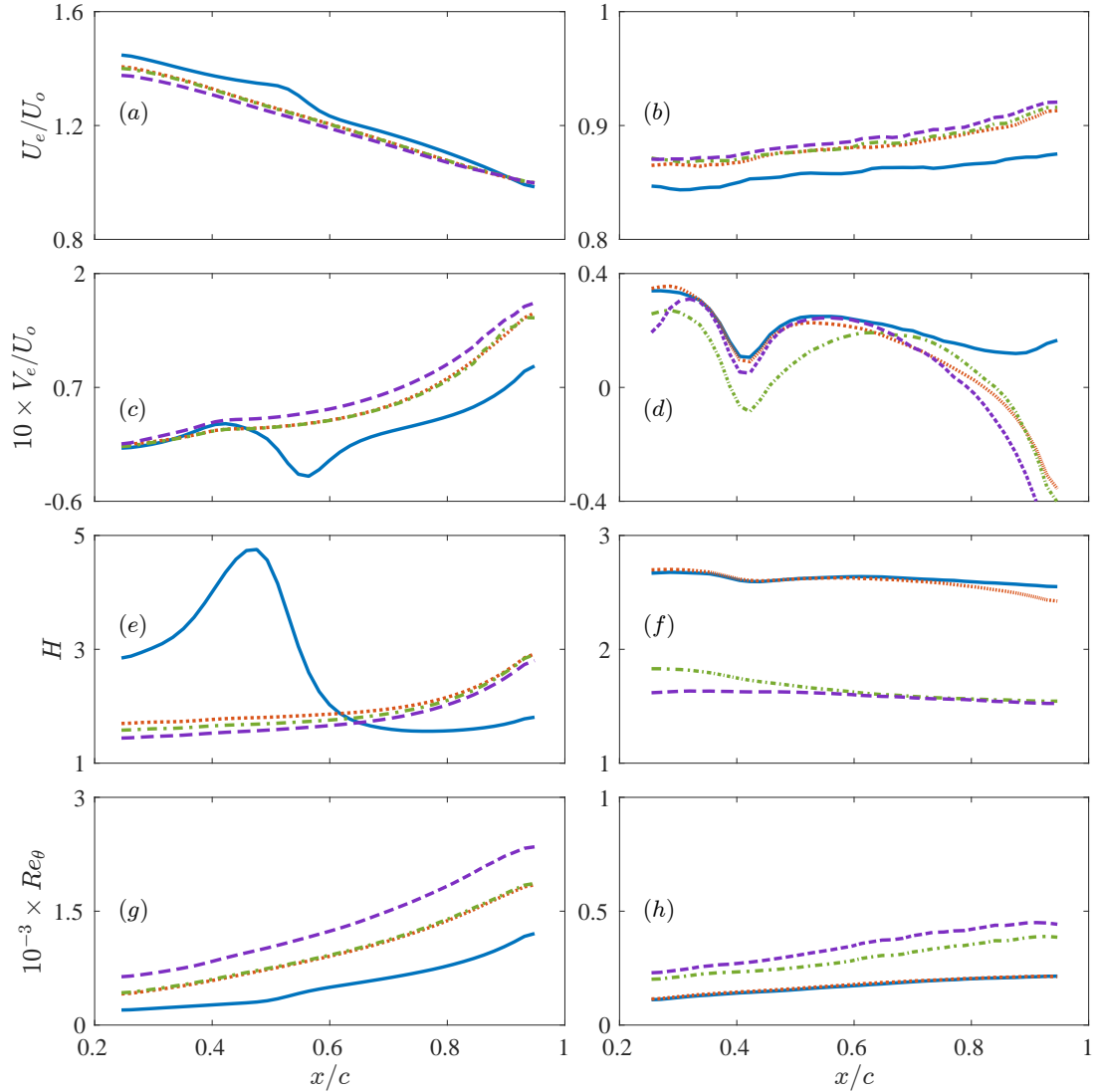


Figure 9. Boundary-layer parameters: (a,b) Wall-parallel edge velocity, U_e/U_o ; (c,d) wall-normal edge velocity, V_e/U_o ; (e,f) shape factor, H ; (g,h) momentum-thickness Reynolds number, Re_θ . Suction side: (a,c,e,g); pressure side: (b,d,f,h). — SLE; ··· TBL; - - - RLE1; - · - RLE2.

343 Figure 9 shows the streamwise distribution of various boundary-layer parameters:
 344 only the flow downstream of the roughness is plotted. All quantities are calculated
 345 using double-averaged data. The streamwise and wall-normal edge velocities (U_e and
 346 V_e), the shape factor $H = \delta^*/\theta$, and momentum-thickness Reynolds number ($Re_\theta =$
 347 $U_e\theta/\nu$) are shown. The momentum and displacement thicknesses are defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{\langle U \rangle}{U_e}\right) dy \quad \theta = \int_0^\infty \frac{\langle U \rangle}{U_e} \left(1 - \frac{\langle U \rangle}{U_e}\right) dy. \quad (9)$$

348 The velocity at the edge of the boundary layer shows very little difference in the

rough cases; the small bulge at $x/c \approx 0.5$ on the suction side for the SLE case is due to the laminar separation bubble. The boundary layer is considerably thicker when the flow is turbulent (cases TBL, RLE1 and RLE2 on the suction side, RLE1 and RLE2 only on the pressure side). This is reflected both in the larger magnitude of wall-normal velocity at the boundary-layer edge and in the momentum-thickness Reynolds number. It also confirms the visual impression from Figures 6 and 7. The boundary layer in case RLE2, in which the roughness elements are taller and the turbulent structures appeared larger, is the thickest. On the suction side the shape factor is affected by the recirculation bubble in case SLE; it is very similar in the other cases. Note that immediately after the roughness $H \simeq 1.6$ (close to the flat-plate turbulent boundary-layer value) in all cases in which the flow is turbulent; H increases in the adverse pressure-gradient (APG) region on the suction side, while remains nearly constant in the favourable pressure-gradient (FPG) region on the pressure side. There, $H \simeq 2.6$ (the laminar flat-plate value) when the flow remains laminar.

3.4. Double-averaged velocity

Figures 10 and 11 show the double-averaged velocity-component tangent to the wall, $\langle U_t \rangle$. Double-averaged statistics were collected for more than 100 LETOTs δ^*/u_τ (where δ^* and u_τ are calculated at $x/c = 0.6$ on the suction side) and then averaged in the spanwise direction.

On the suction side all the cases except SLE are turbulent from the first location onward. In fact, between $x/c \simeq 0.3$ and 0.7 they are very close to the standard logarithmic law, shown in Figure 11(a,b). Here wall units, denoted by a +, are calculated using the local value of u_τ . Further downstream APG effects become significant and the velocity profile goes above the logarithmic law. In the rough cases the momentum deficit is larger than in the smooth one: the leading-edge protrusions act as flow obstacles and, in some cases, cause local flow separation. Although their roughness height, location and topology are different, the TBL and RLE1 cases have quite similar profiles at all locations. The SLE case has an altogether different behaviour: the flow is initially laminar (and the boundary layer is significantly thinner than in the other cases, Figure 10); an inflectional velocity then develops, and a recirculation region is formed. The flow then reattaches and a fuller velocity profile is established that tends towards an equilibrium turbulent boundary layer.

On the pressure side the SLE and TBL cases have laminar profiles at all locations. RLE1 and RLE2 cases, however, are turbulent and have similar profiles, following the logarithmic law. The dimensionless pressure gradient $K = (\nu/U_e^2)(dU_e/dx)$ is of order 10^{-7} , low enough that the slope of the logarithmic region is not altered very much. Only close to the roughness one can observe some difference between the two cases, RLE2 being closer to an equilibrium turbulent boundary layer. Possible causes for this difference will be discussed in the following.

3.5. Time-averaged statistics

The roughness elements near the leading edge in the TBL, RLE1 and RLE2 cases act as flow obstacles. Depending upon their placement, shape and distribution, they cause flow three-dimensionality and transition to turbulence, as discussed in Section 3.2. In the TBL case, on the suction side an initially 2D structure characterized by large spanwise-oriented vortices is followed by vortex breakdown and formation of hairpin-

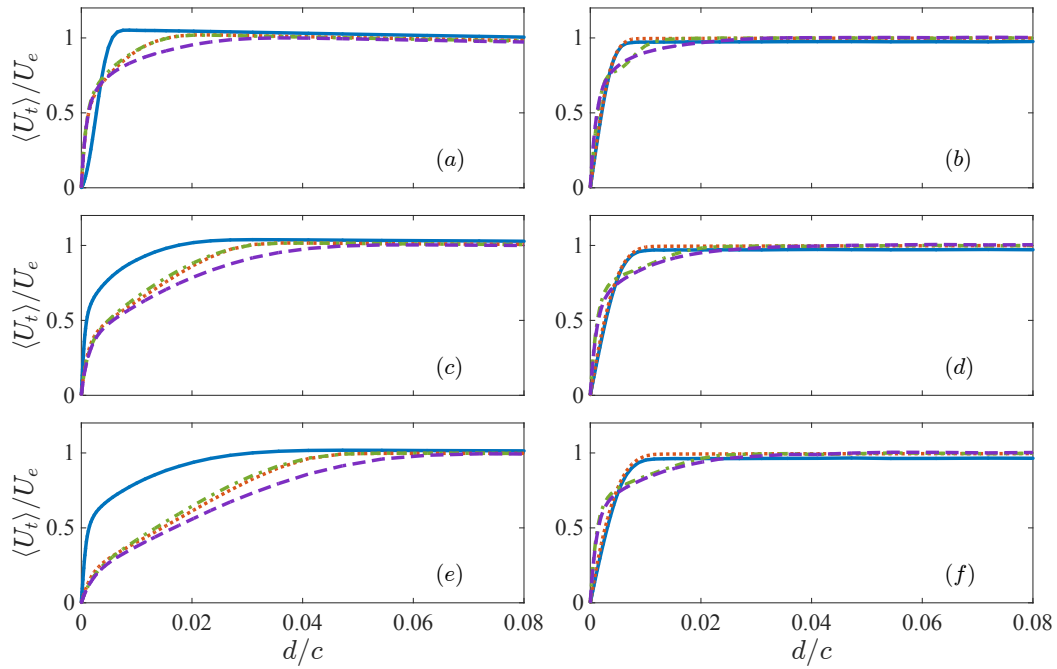


Figure 10. Wall-parallel velocity profiles. (a, b) : $x/c = 0.35$, (c, d) : $x/c = 0.67$, (e, f) : $x/c = 0.82$. (a, c, e) Suction side; (b, d, f) pressure side. — SLE; ··· TBL; - - RLE1; - · - RLE2.

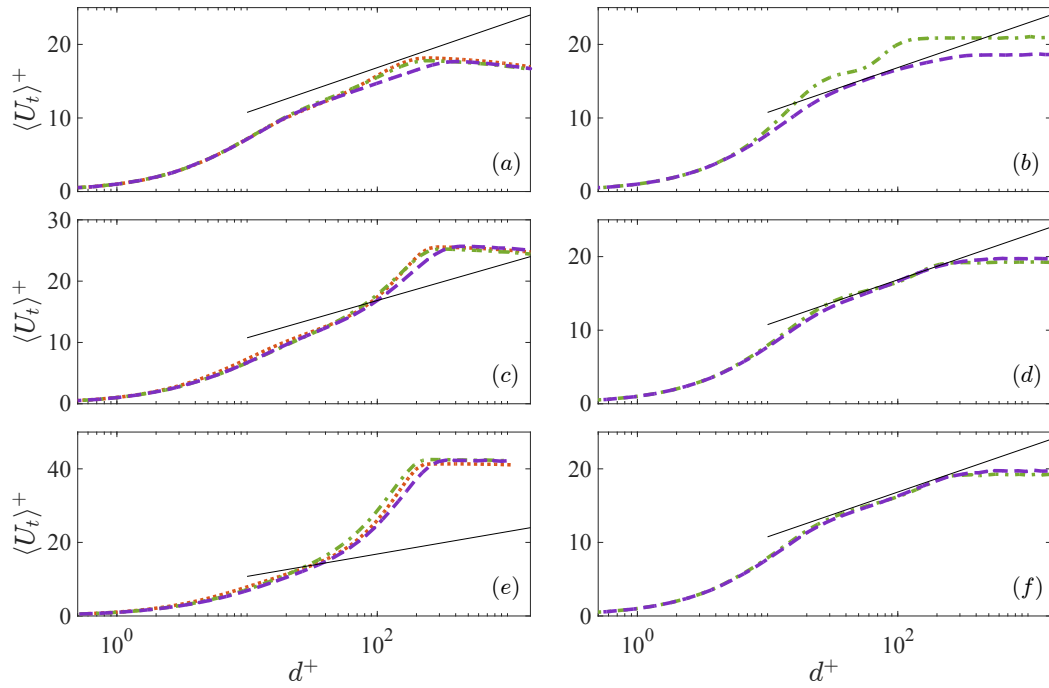


Figure 11. Wall-parallel velocity profiles in wall units for the turbulent cases. (a, b) : $x/c = 0.35$, (c, d) : $x/c = 0.67$, (e, f) : $x/c = 0.82$. (a, c, e) Suction side; (b, d, f) pressure side. ··· TBL; - - RLE1; - · - RLE2; Thin line: $2.5 \log d^+ + 5$.

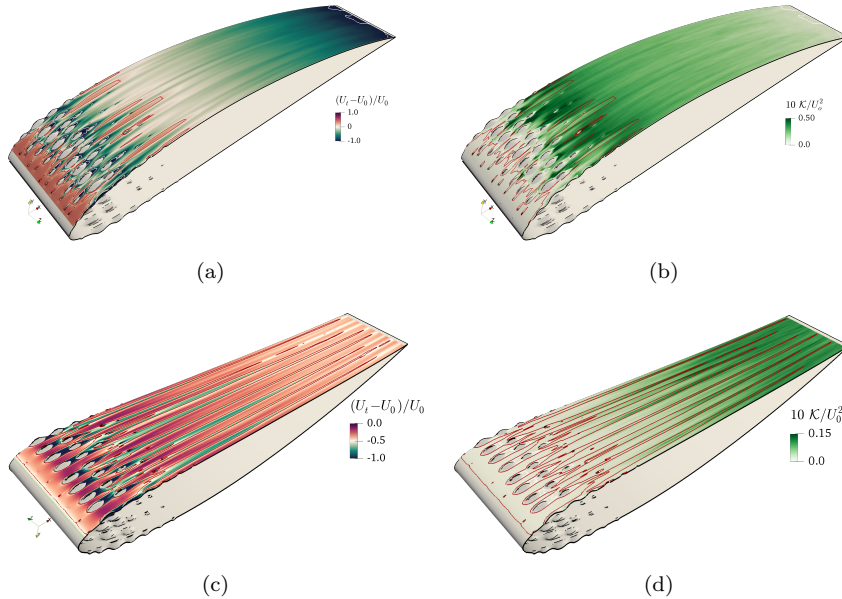


Figure 12. RLE1 case. (a,c) Time-averaged wall-parallel velocity-deficit contours; (b,d) TKE contours. The contours are plotted on a surface at a distance $d/c = 0.002$ from the airfoil. (a,b) Suction side; (c,d) pressure side. The red contour lines represent regions where $U_t = 1.1U_o$ on suction side and $U_t = 0.5U_o$ on pressure side.

394 shaped structures around 30% of the chord; on the pressure side the flow remains
 395 laminar. In the RLE1 case the 3D character of the roughness results in immediate
 396 formation of trains of aligned hairpin vortices, on both sides. In the RLE2 case a
 397 massive separation behind the roughness is followed by an immediate formation of
 398 large hairpin vortices, whose distribution is more random, although some tendency
 399 towards the formation of aligned trains can still be observed, especially on the pressure
 400 side. These features are reflected in the time-averaged fields for the RLE1 and RLE2
 401 cases (for the TBL case the time-averaged quantities are statistically equal to the
 402 double-averaged ones).

403 Figures 12 & 13 show the normalized mean wall-parallel velocity-deficit, $(U_t -$
 404 $U_o)/U_o$, and the turbulent kinetic energy, \mathcal{K}/U_o^2 , for the RLE cases. In the RLE1
 405 case the three-dimensionality of the roughness results in the formation of streamwise-
 406 oriented, alternating regions of high- and low velocity-fluid, Figure 12(a, c), associated
 407 with corresponding regions of low and high TKE. We will term this phenomenon “chan-
 408 nelling” and refer to the high velocity streaks as “peak regions”, the low-velocity ones
 409 as “valley regions”. The flow channelling is strongly affected by the pressure gradient;
 410 on the pressure side the stabilizing effect of the FPG (which tends to align the vortical
 411 structures [92]) the streaks maintain their coherence for longer distances, while the
 412 APG on the suction side tends to mix the flow more effectively and break down the
 413 streaky structures. For the RLE2 case on the other hand flow channelling, although
 414 clearly visible in Figure 13, doesn’t look as prominent as RLE1. In fact, due to a closer
 415 positioning of roughness elements we see a spanwise ridge like behaviour.

416 The valley regions are not in the wake of the larger roughness elements. Rather,
 417 Figure 14(a), they are formed when the wakes of adjoining elements merge together,
 418 and low-speed fluid from the recirculation regions is entrained into the mean flow. The
 419 time-averaged data shows pairs of counter-rotating longitudinal vortices, centered on

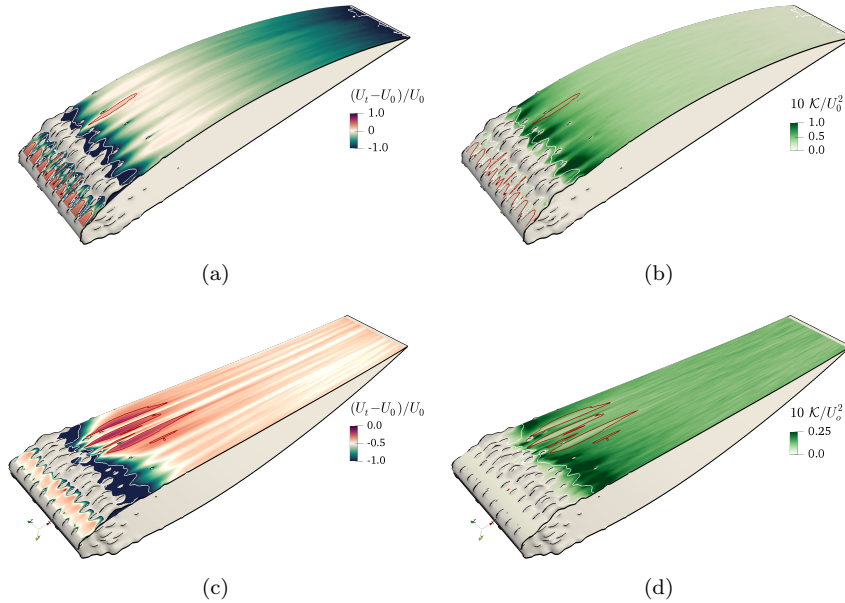


Figure 13. RLE2 case. (a,c) Time-averaged wall-parallel velocity-deficit contours; (b,d) TKE contours. The contours are plotted on a surface at a distance $d/c = 0.002$ from the airfoil. (a,b) Suction side; (c,d) pressure side. The red contour lines represent regions where $U_t = 1.1U_o$ on suction side and $U_t = 0.7U_o$ on the pressure side.

420 the valley regions and trailing downstream, Figure 14(b); on the pressure side they
 421 are remain coherent due to the FPG, but they are also visible on the suction side.

422 Near the leading edge horseshoe vortices are formed as the flow encounters the first
 423 row of roughness elements, which act like isolated obstacles. The legs of the horseshoes
 424 trail downstream, meandering in between roughness elements. They remain coherent
 425 for the first 20% of the chord on the suction side (longer on the pressure side) and the
 426 valleys are the upwash region between the vortices. Towards the end of the roughness
 427 region the vortices begin to break down, forming hairpins with lifted heads and shorter,
 428 trailing legs. The streamwise vorticity that is present in the time-averaged contours,
 429 from this point on, is not due to coherent vortices, but rather to the footprint of
 430 the legs of the hairpins that are advected. The alignment of the hairpins observed in
 431 Figures 6 and 7 is due to the fact that they originate from the streamwise horseshoe
 432 vortices associated with the valleys.

433 Figure 15 shows the streamwise vorticity in cross planes at $x/c = 0.25$ (note that
 434 in the planes shown, the mean flow is almost aligned with the x direction). In the
 435 RLE1 case the streamwise vortex pairs are very clear on both sides. In the upwash
 436 region between two counter-rotating vortices, the valley regions, the boundary layer is
 437 thickened, and the wall stress is decreased.

438 A similar behaviour can be observed for the RLE2 case, Figure 16. Horseshoe vor-
 439 tices are formed around each roughness element; their trailing legs interact with the
 440 horseshoe vortices formed by the next row; this interaction tends to cause the for-
 441 mation of hairpins (*e.g.*, at $x/c \simeq 0.12$ in Figure 16(a)). As in the RLE1 case, the
 442 hairpin vortices tend to be aligned with low-speed valley regions, but they are larger
 443 and extend further from the wall, probably because of the larger size of the roughness
 444 elements. Once the streamwise vortices break down, streamwise vorticity can still be
 445 observed; in this case it is the footprint on the time average of the trailing legs of the

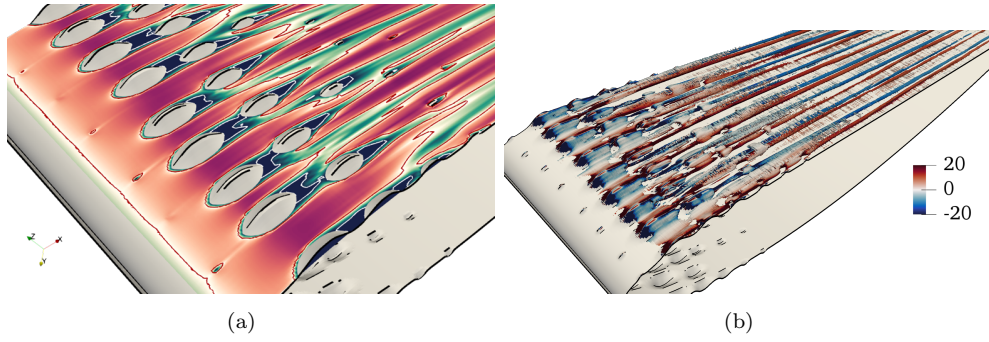


Figure 14. RLE1 case, pressure side. (a) Time-averaged wall-parallel velocity-deficit contours; (green: -1.0, red: 0.0) (b) Iso-surfaces of $\bar{Q} = 20U_o^2/c^2$, coloured by streamwise vorticity, $\Omega_x c/U_o$.

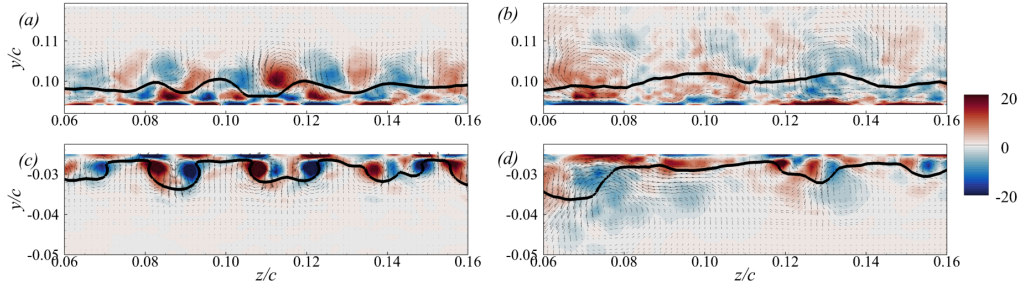


Figure 15. Contours of streamwise vorticity, $\Omega_x c/U_o$, in the $x/c = 0.25$ plane. (a,b) Suction side; (c,d) pressure side. (a,c) RLE1; (b,d) RLE2. The solid black lines represent $U_t/U_e = 0.8$.

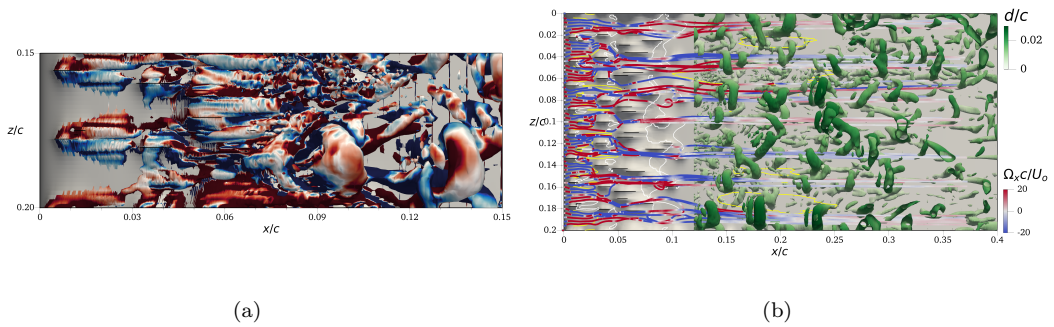


Figure 16. (a) Isosurfaces of Q coloured by instantaneous streamwise vorticity (blue: $-50U_o/c$, red: $50U_o/c$) near the leading edge. (b) Time-averaged streamlines, coloured by streamwise vorticity, $\Omega_x c/U_o$, on the suction side for RLE2 case. Instantaneous structures are also shown after the roughness zone, coloured by distance from the wall d/c . White lines denote separated region and yellow lines denote fast regions on a surface parallel plane at $x/c = 0.002$.

446 hairpins.

447 The presence of similar elongated motion in the instantaneous flow-field has been
 448 observed by many different studies in standard turbulent boundary layers, both for
 449 smooth [93, 94] and rough-wall cases [95, 96]. These regions contribute significantly to
 450 TKE and Reynolds shear stresses [97, 98, 99, 100]. Later studies [101, 102, 103, 104] also
 451 identified a high degree of spanwise heterogeneity in the mean flow and termed these
 452 regions "low-" and "high-momentum pathways" (LMPs, HMPs). The flow channelling
 453 observed in this study has a similar vortical signature as HMPs and LMPs, but the
 454 size of the roughness elements ($h/\delta \gg 1$) is much larger than in typical boundary
 455 layer studies ($h/\delta \sim 1/20 - 1/40$). The channelling observed here and the vortical
 456 structure associated with it have a larger scale, more akin to that of flows in urban
 457 environments, although their effects on the mean flow are in many ways similar.

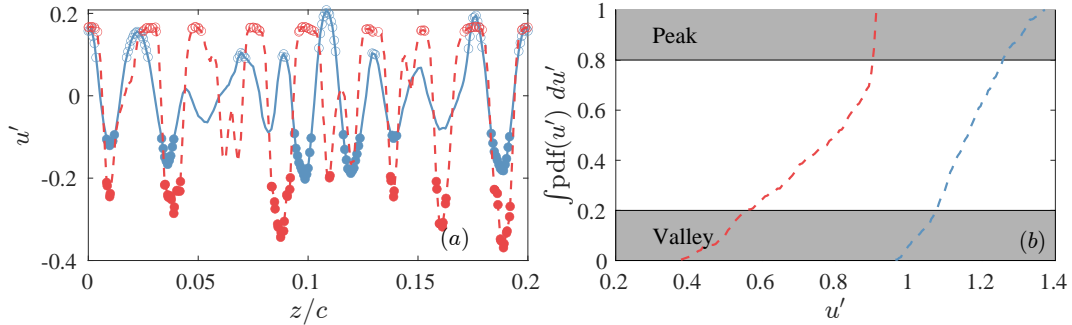


Figure 17. Conditional average threshold choice. (a) Profile of u' at $x/c = 0.2$, $d/c = 0.005$; the circles indicate the events identified as "peaks" (empty) and "valleys" (filled). (b) Cumulative pdf of u' . — Suction side; - - - pressure side.

458 3.6. Conditional and phase averaging

459 Figures 12 and 13 show that both in RLE1 and RLE2 cases the high-speed peak
 460 regions are accompanied by lower levels of TKE. To understand better this behaviour,
 461 and its possible implications on flow physics and modelling, we performed conditional
 462 averages of the time-averaged quantities in peak and valley regions. The peaks and
 463 valleys were identified by considering a spanwise profile of the $\Delta U = U - \langle U \rangle$, at
 464 $x/c = 0.2$ and $d/c = 0.005$, shown in Figure 17(a). The cumulative probability-density
 465 function (CPDF), shown in Figure 17(b), was calculated, and the events responsible
 466 for the top and bottom 20% of the CPDF were classified as "peaks" and "valleys",
 467 respectively. They are shown as empty and full circles in the Figure. The average over
 468 all peak or valley events will be denoted by a hat: \hat{U} .

469 The conditionally averaged result on the suction side, for RLE1 and RLE2 cases
 470 are shown in Figure 18, and compared with the double-averaged velocity. Significant
 471 differences between the flow statistics are observed. First, the friction velocity is ap-
 472 proximately 12% lower than the DA one in the valley regions, and higher in the peak
 473 regions by 3% (for the RLE2 case) or 7% (for RLE1). Furthermore, in the valley re-
 474 gions, the wake region is more pronounced and the velocity profiles resemble those in
 475 boundary layers with adverse pressure gradient (APG), Figures 18(a,b). Conversely,
 476 in the peak regions the behaviour tends towards that of a boundary layer in favourable
 477 pressure gradient (FPG), with a slight increase of the von Kàrmàn constant and the
 478 disappearance of the wake region. In outer units, the velocity difference between peak

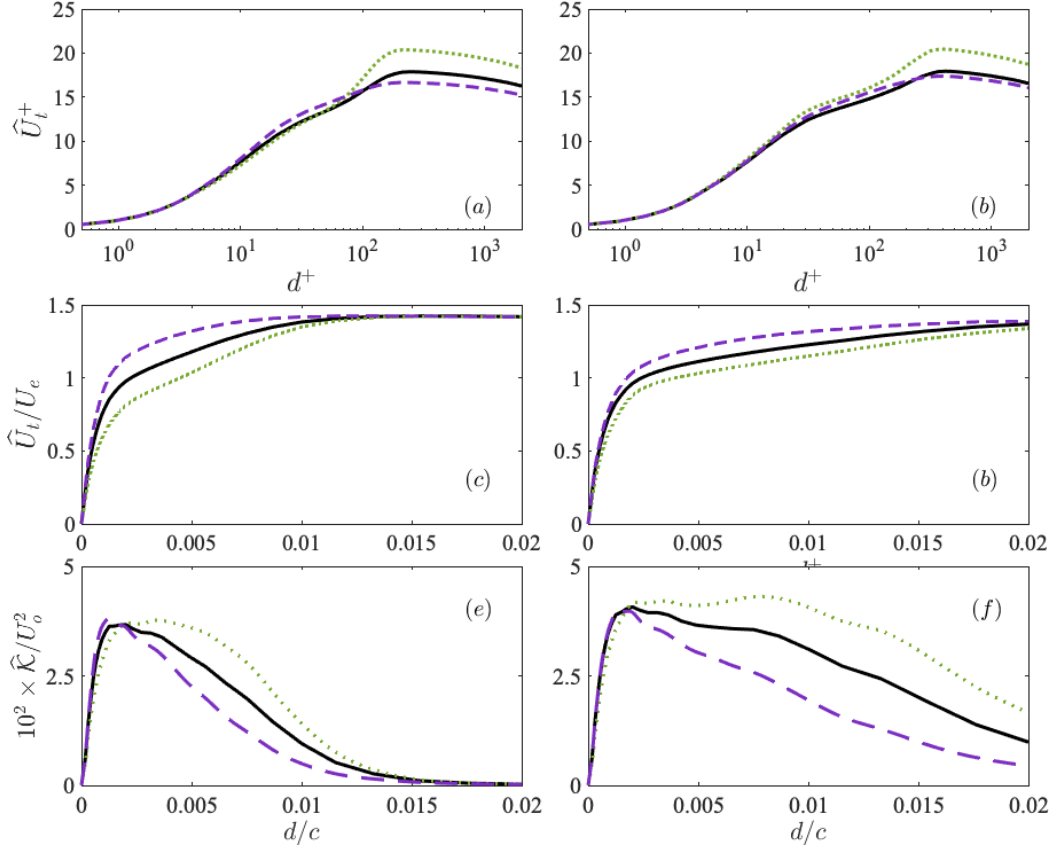


Figure 18. Conditionally averaged tangential velocity and turbulent kinetic energy (in outer coordinates) in peak and valley regions. $x/c = 0.25$. (a,c,e) RLE1; (b,d,f) RLE2; (a,b) tangential velocity in wall units; (c,d) tangential velocity in outer coordinates; (e,f) turbulent kinetic energy in outer coordinates. — Double average; ··· Valley; - - - Peak.

479 and valley regions can be as high as 30% of the edge velocity in the RLE1 case, 18%
480 in the RLE2 case, Figures 18(c,d). In the valley regions the velocity profile has in-
481 flection points in both cases, although less marked in the RLE2 case. These inflection
482 points may be responsible for the significant increase of the TKE in the valley regions,
483 Figure 18(d,e). The increased TKE in the valleys and, conversely, the increased one
484 in the peak regions, are also consistent with the APG-FPG behaviour of these re-
485 gions. By $x/c = 0.4$ the differences between valley and peak regions are not significant
486 any longer. The peak/valley behaviour is similar on the pressure side (not shown),
487 although the differences are more marked (especially the inflectional behaviour of the
488 tangential velocity) and persist farther downstream.

489 4. Conclusions

490 We performed large-eddy simulations of the flow over the NACA4412 airfoil at $Re =$
491 200,000 and 5° angle of attack to study the effect of leading-edge ice deposits on
492 the turbulent flow. Four cases were considered: an airfoil with a smooth leading edge
493 (SLE), one with a tripped leading edge (TLE) with 3 semi-cylindrical bars aligned in
494 the z -direction, and two cases with Rough Leading-Edges (RLE1 and RLE2) to mimic
495 the glaze icing described in the study of McClain *et al.* [29]. A grid-convergence

496 study was performed for TBL case; three grids were chosen: coarse, medium and fine.
497 The first-order statistics converge using the medium resolution; the convergence of
498 second-order moments is marginal, but within acceptable bounds.

499 Statistical quantities and instantaneous flow visualizations were computed. The
500 triple decomposition introduced for rough-wall flows [79, 80] was used to distinguish
501 the double-averaged quantities (averaged over time and the spanwise direction) from
502 the time-averaged ones. Conditional averages were also performed to isolate the flow
503 in the peak and valley regions.

504 In the SLE case the laminar flow separates, and transition is caused by the instability
505 of the separated shear layer. Two-dimensional vortices are formed after reattachment,
506 which develop three-dimensionalities; eventually, the flow becomes turbulent around
507 the middle of the airfoil.

508 The main effect of the leading-edge roughness is to accelerate the transition to tur-
509 bulence. In the TBL cases transition occurs much earlier. The separated flow over the
510 semi-cylindrical trips causes the formation of quasi two-dimensional spanwise vortices.
511 The remain 2D for 20-30% of the chord, then 3D structures are formed and the flow
512 becomes more chaotic.

513 A very three-dimensional flow is established immediately in the RLE1 and RLE2
514 cases. In the first, in which the roughness elements are separated, their wakes merge
515 and low- and high-speed streaks are formed; they are referred to “valley” and “peak”
516 regions, respectively. We refer to this phenomenon as “channelling”. Horseshoe vortices
517 are initially formed after each roughness element; the trailing legs interact with those
518 of successive roughness elements and eventually break down, giving rise to trains
519 of hairpin vortices, aligned with the low-speed streaks. In the mean field, both the
520 horseshoe legs and the hairpin legs result in coherent and fairly stationary streamwise-
521 vorticity regions. In the upwash region between the hairpin legs the boundary layer is
522 thickened, the wall stress is lower than the average, and low-speed streaks are formed.
523 The flow in the valley regions resembles that in an APG boundary layer: the con-
524 ditionally averaged velocity profiles exhibits a more considerable wake and inflection
525 points. As a consequence, turbulent kinetic energy and Reynolds stresses are signifi-
526 cantly larger in the valley regions. In the peak regions, conversely, the flow appears
527 more similar to an FPG boundary layer. The channelling is amplified by the favourable
528 pressure gradient on the lower side of the airfoil, where the streaks last much longer,
529 and the streamwise vortices remain more coherent.

530 In the RLE2 case the largest roughness elements merge together, forming a ridge.
531 The channelling phenomenon is less clear, but still present. Peaks and valleys are
532 still observed, and show similar characteristics. the hairpin vortices are larger, extend
533 further from the surface and meander more.

534 The channelling is similar to that observed in other rough wall flows [103], but the
535 particular geometry considered, with more isolated roughness elements than in more
536 conventional rough-wall boundary layers, affects its features significantly. It may be the
537 most interesting finding of this work, also because of its implications for modelling.
538 In industrial applications, calculations of the flow over an entire wing are typically
539 performed solving the Reynolds-Averaged Navier-Stokes (RANS) equations with tur-
540 bulence models. They use grid spacings of order $0.01 - 0.02c$; in rare cases finer grids
541 are used. In our calculations the roughness size was less than $0.01c$, and the spacing
542 between consecutive high- or low-speed regions is of order $0.025c$. None of these geo-
543 metric characteristics would be resolved on a typical grid. The roughness, therefore,
544 would have to be included through a modification of the turbulence model. Several
545 such modifications have been developed (see, for instance, the discussion in [105]).

546 They are all based on statistical considerations, and a single parameter (generally
547 the equivalent sandgrain roughness) is used to describe the roughness. Such approach
548 cannot distinguish between the various mechanisms associated with the geometries
549 considered here, however, yielding only the equivalent of the double-averaged statis-
550 tics. Moreover, it would not be able to account for the channelling phenomenon, which
551 may play a role at high angles of attack, causing three-dimensional separation. In fully
552 turbulent flows, roughness generally causes the flow to separate earlier; the APG in
553 the valley regions might further accelerate separation, resulting in a very 3D structure
554 of the separated-flow region. It is unclear if this phenomenon could be captured at all
555 by a Reynolds-Averaged Navier-Stokes solution.

556 The present simulations were carried out at a moderate Reynolds number, and
557 it is possible that some of the problems described lose importance if the Reynolds
558 number is increased. It should be remarked, however, that the dimensions of the three-
559 dimensional structures observed here scale with geometric parameters (the roughness
560 size) and not in viscous units. It is, therefore, unlikely that the phenomena observed
561 would entirely disappear in flight conditions. It would be, however, desirable to verify
562 this conjecture by performing simulations at higher Reynolds number and angle of
563 attack. These calculations would also help determine the error bars for the turbulence
564 models used in these configurations.

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