

A predictive model for fracture in human ribs based on [in-vitro](#) Acoustic Emission Data

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Abstract

Purpose. The aim of this paper is to propose a [fracture model](#) for human ribs based on Acoustic Emission (AE) data. [The accumulation of micro-cracking until a macroscopic crack is produced can be monitored by AE. This macro-crack propagation causes the loss of the structural integrity of the rib.](#)

Methods. The AE technique was used in [in-vitro](#) bending tests of human ribs. The AE data obtained were used to construct a quantitative model that allows an estimation of the failure stress from the signals detected. [The model predicts the ultimate stress with an error of less than 3.5% \(even at stresses 15% lower than failure stress\), which makes it possible to safely anticipate the failure of the rib.](#)

Results. [The Percolation Theory was used to model crack propagation. Moreover, a quantitative probability-based model for the expected number of AE signals has been constructed, incorporating some ideas of percolation theory. The model predicts that AE signals associated with micro-failures should exhibit a vertical asymptote when stress increases. The occurrence of this vertical asymptote](#)

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27 was attested in our experimental observations. The total number of
28 micro-failures detected prior to the failure is $N \approx 100$ and the ultimate
29 stress is $\sigma_{\infty} = 197 \pm 62$ MPa. A significant correlation ($p < 0.0001$) be-
30 tween σ_{∞} and the predicted value is found, using only the first $N = 30$
31 micro-failures (correlation improves for N higher).

32 *Conclusions.* The measurements and the shape of the curves pre-
33 dicted by the model fit well. In addition, the model parameters seem
34 to explain quantitatively and qualitatively the distribution of the AE
35 signals as the material approaches the macroscopic fracture. More-
36 over, some of these parameters correlate with anthropometric vari-
37 ables, such as age or BMI. The proposed model could be used to
38 predict the structural failure of ribs subjected to bending.

39
40 KEYWORDS: Acoustic Emission, Biomechanics, Human Rib, Prob-
41 abilistic models, Percolation.

42 1 Introduction

43 The prediction of bone fracture under stress is a problem of practical interest.
44 Recently, some medical imaging techniques have been developed to estimate
45 the degree of deformation of a bone based on medical images [1]. This, cou-
46 pled with computationally adequate constitutive models, provides a potential
47 method for predicting the strength capacity of bone. However, there is still
48 considerable uncertainty about the accuracy of such methods. For example,
49 some studies have attempted to evaluate the accuracy of computational mod-
50 els for predicting fracture value, finding that the value predicted by Finite
51 Element Model (FEM)-based simulations can contain errors that range from
52 5 to 46% [2], a considerable magnitude. Therefore, given the current state
53 of the art in medical imaging technology and the reliability of bone compu-
54 tational models, it is interesting to explore other approaches of work. Other
55 empirical techniques such as Acoustic Emission (AE) can be used to detect
56 the growth of incipient micro-failures, which could potentially be applied to
57 *in vivo* monitoring of bone under stress [3, 4, 5].

58
59 The AE technique has been previously used in the biomedical field, show-
60 ing its wide range of applications. In the field of orthopedics, for example,
61 AE has been used for implant design, failure prediction or even orthopedic
62 diagnosis [6, 7]. In the latter approach, it has been shown that AE can even

determine the appropriate time for the removal of external fixations in bone healing before radiographic diagnosis [8].

The AE technique has been extensively used in different materials for real-time monitoring [9, 10], including biological materials [11, 12]. However, the literature on AE in human bones is less extensive and has been limited to the description of the emission release pattern and the study of the signal increase with load [5, 13, 14]. Some other studies analyzed the loading process of specific bones such as the femur or the tibia [14, 15, 16, 17], but most of these studies focused on the monitoring of damage under increasing load [18], and also on the influence of age and the distribution of energies and amplitudes of the recorded signals [19, 20]. In most cases, no specific models have been outlined to describe the appearance of AE signals as stress increases. The model presented in this study is an attempt to fill this gap. In the framework of biomechanical applications, some research has used the AE technique for the analysis of fracture of bone and other biological materials; however, there are few quantitative models which use AE data for the prediction of fracture [21, 22, 23]. Moreover, these predictive models were developed for soft collagenous tissues, which makes them unsuitable for application in hard tissue (bone).

Using the AE technique and an appropriate quantitative model, it may be possible to estimate failure stress with some anticipation of its occurrence. However, a large part of the available studies of AE in the context of biomechanics are excessively empirical and are limited to descriptive comparisons. In fact, many of these studies focus either on the comparison of AE signals between specimens [11], or on the qualitative description of the region where AE signals are predominant in the force-deflection curve of bone [5, 16], or on the number of signals and their amplitude variation as a function of the strain rate [19]. Therefore, it seems necessary to develop quantitative fracture models for an estimation of the failure stress prior to fracture. Based on the aforementioned ideas, this work proposes such a model based on *in vitro* data with whole human ribs subjected to bending.

The main ideas of the model of this study are based on Percolation Theory. Even though the Percolation Theory has already been used in theoretical biomechanical models of bone fracture [24, 25, 26], no study has used the percolation theory statements along with AE data in the biomechanical field. Other studies, however, were developed in soil mechanics and fracture prop-

101 agation in rock materials, with models based on the principles of Percolation
102 Theory to model crack propagation with AE data [27].

103 2 Data and Methods

104 The materials used in this study consisted of fresh human rib specimens har-
105 vested from forensic autopsies conducted at the Forensic Pathology Service of
106 the Legal Medicine and Forensic Science Institute of Catalonia (FPS/IMLCFC).
107 All the specimens were initially removed for complementary medical-legal in-
108 vestigation. This study was approved by the Research and Ethics Committee
109 of the IMLCFC. Fifteen healthy 4th ribs were obtained from autopsies of ten
110 *post mortem* human subjects (PMHS). Each complete rib was subjected to a
111 three-point bending test and was loaded to complete fracture. In the bending
112 tests, the AE technique described in section 2.1 was incorporated. Prior to
113 the experimental tests, the soft tissue and cartilage were removed.

114 2.1 Acoustic Emission

115 The AE is a Non-Destructive Testing (NDT) technique used for the detec-
116 tion of elastic waves spreading in the material. The waves are generated by
117 the release of elastic energy during the micro-cracking process: “There is a
118 sudden drop in stress from the original level to zero on the new crack surface
119 area when the crack jumps; this causes the radiation of elastic waves” [28].
120 The AE measurements consists in the placement of small sensors on the spec-
121 imen that detect the elastic waves released in the material, during the suc-
122 cession of jumps spreading micro-cracks [28], usually grouped in avalanches,
123 until macroscopic failure occurs (Figure 1) [20]. The signals exceeding the
124 energy threshold are considered micro-cracking signals or *hits*, as it has been
125 verified in collagenous tissues when fibers fail [21].

126 In this study, it is assumed that the progression of micro-cracks in the
127 inter-osteon space produces most of the detected AE signals: when the micro-
128 cracks produced near the maximum stress converge, they produce a continu-
129 ous macro-crack that eventually nucleates a macroscopic crack. In addition,
130 it is assumed that when a bond (or set of bonds) within the osteon net-
131 work goes from intact state to broken state, an AE signal is produced, and
132 the breaking process could be modeled by Percolation Theory with non-
133 independent bond breaking probability associated to the stress field (see sec-

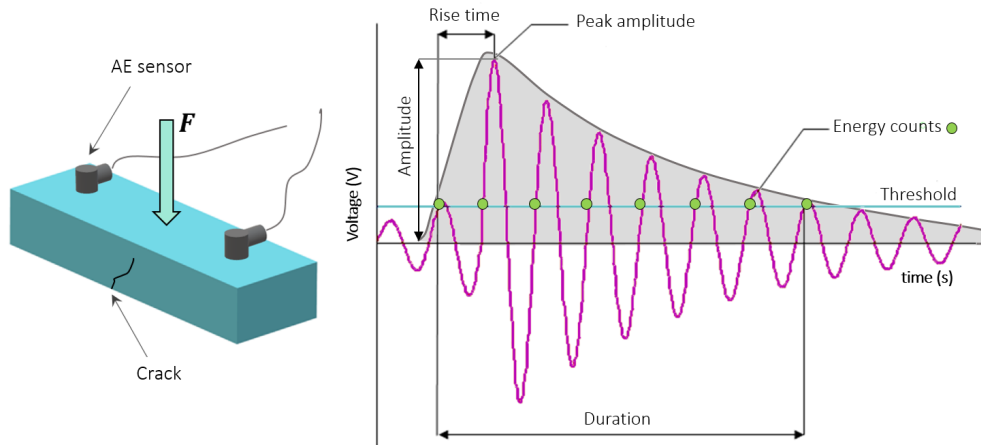


Figure 1: (a) AE technique, (b) Main AE wave features. Threshold discriminates the noise or disturbing signals and energy (gray area) is computed by means of the amplitude, count and duration parameters.

tion 2.3).

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2.2 Experimental Setting

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The human ribs were tested with a specific methodology that allows for frictionless sliding of the ends of the rib. This setting can be used to determine the deflections and displacements at any point of the rib. The entire ribs were subjected to three-point bending tests, as can be seen in Figure 2. This configuration for the tests allows a sliding and opening of ends of the rib, then the use of Digital Image Correlation (DIC) makes it possible to compute the displacements along the rib from which to calculate the strain. In addition, the stress can be computed from the measured force and the geometrical cross-sectional properties along the rib.

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Bending tests were performed with a ZwickRoell® Proline 7.1 and a load cell HBM®. A U-shaped guide was placed on the upper platform into which the rib extremes were inserted (the rib was contained in the machine plane). The guide was covered with lubricant and the rib ends were wrapped with polytetrafluoroethylene band to ensure a sliding and to minimize the friction. On the lower platform, a base was fixed in which the impactor was attached (see Figure 2). The impactor exerted the force on the central external region of the rib. The base supported four safety bars to prevent possible slippery

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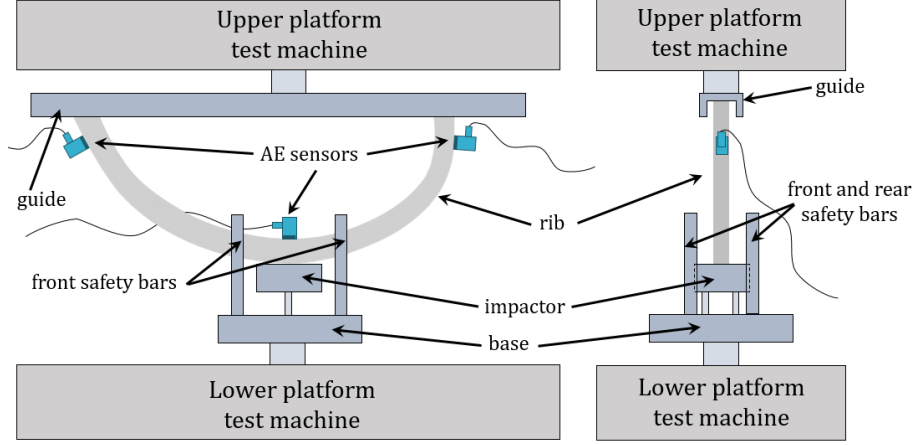


Figure 2: Experimental setting: the rib is placed with its extremes inside the guide and the outer central region in contact with the impactor. An AE sensor is placed in the central region where the main fractures occur along with two sensors placed near the ends to discriminate friction signals.

153 of the rib during the test (the bars were never in contact with the rib). The
 154 whole test was recorded with a high-speed camera (PCO 1200s) and from the
 155 sequence of video frames, the displacements on the rib during the test were
 156 determined following a DIC procedure with MATLAB[®]. From displacements
 157 and force, strain and stress on the fracture region of each rib were obtained
 158 during the whole test.

159 For the computations of the stress, the axial force N_x and the bending mo-
 160 ment M_z in the fracture cross-section was determined for each time during
 161 the test and with these values, the stress was calculated by means of the
 162 Navier's formula [29]:

$$\sigma(t) = \frac{N_x(t)}{A} - \frac{yI_{yy} - zI_{yz}}{I_{yy}I_{zz} - I_{yz}^2} M_z(t) \quad (1)$$

163 where I_{yy}, I_{zz} are the second moments of area, I_{yz} the product moment of
 164 area and y, z the vertical and horizontal distances from the centroid of the
 165 fractured section to the bottom point of the rib where the maximum stresses
 166 occur. These magnitudes were determined from rib CT images, from which
 167 the geometric area magnitudes necessary for the application of the formula
 168 (1) could be calculated.

169 Prior to the test, three resonant AE sensors (VS700-D, Vallen System

Gmbh) were placed along the rib; two sensors at the ends to discriminate friction signals and another sensor in the inner central region of the rib, where greater stresses and macroscopic fracture occur. Three AE amplifiers (AEP4) were used, together with a four channel system (AMSY-5) and a band-pass filter between 25–1100 kHz to discriminate noise or possible friction signals.

A lighting system was installed in the camera and its voltage was connected to both force and AE acquisition systems. Prior to the force increase, the light was interrupted for a very short time, and the acquisition systems recorded null voltages during this time. Thus, based on the recorded voltage drop and the video light switch-off, forces, displacements and AE signals were synchronized, with a precision better than 10 ms.

2.3 Model based in Percolation Theory

The cortical bone is constituted by an aggregate of quasi-cylindrical units called osteons, consisting of mineral lamellae and collagen fibers that surround the Havers canal. Osteons are mainly aligned along the longitudinal axis of long bones and the boundary between osteons is composed by the cement lines and interstitial bone [30]. Thus, in a cross-section to the axis of the bone, the cortical bone approaches to a set of circle-like structures corresponding to the osteons, embedded in the matrix (interstitial bone and cement lines).

The process of micro-cracking in the bone osteon network has been addressed in the literature. Previous studies suggest that osteons act as a barrier in the propagation of micro-cracks, which would preferably occur along the cement lines and interstitial tissue, promoting the separation of osteons [31, 32, 33, 34]. Thus, it can be assumed that cortical bone is formed by a net of basic cells that represent the osteons and whose limits or bonds correspond to the cement lines and interstitial tissue. The micro-cracks preferably propagate between osteons (see Figure 3), which implies the cracking of the mesh bonds, and this propagation implies an energy release as an AE signal. Therefore, an increase in the load applied to the bone means an increase in the number of cracked bonds in the mesh, and macroscopic fracture occurs when a set of bonded cracked bonds crosses the net completely.

The prediction of the propagation and bonding of cracked bonds up to macroscopic fracture can be studied by the Percolation Theory of bonds

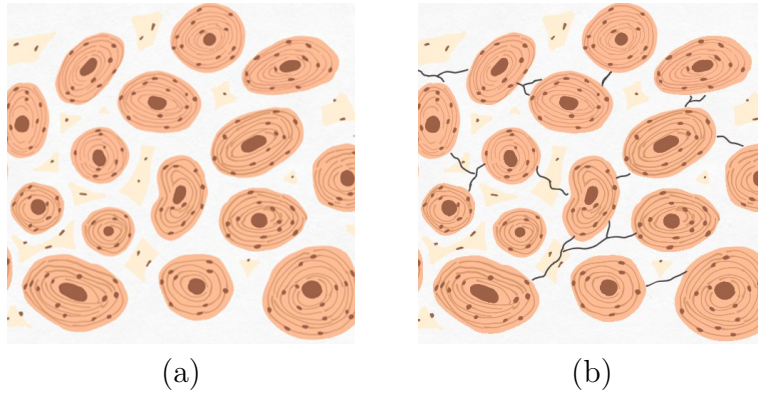


Figure 3: (a) Osteons in an intact state, (b) Osteon system with micro-cracks.

206 [24, 25, 26]. Applied to this context, the Percolation Theory deals with the
 207 probability p of cracking of the net bonds and the critical probability p_c
 208 of percolation, meaning the complete propagation of the crack from end to end
 209 of the net.

210 The most common percolation model is a model where the bonds are indepen-
 211 dently percolated according to a Bernoulli stochastic process: the probability
 212 p of a bond to be cracked does not depend on the cracking of neighboring
 213 bonds, and is homogeneous across the net of bonds (so p does not depend on
 214 the level of stress or other factors at each point). For modeling mechanical
 215 fracture, the Bernoulli independent percolation model is not very realistic and
 216 needs an adequate generalization: either associating the percolation proba-
 217 bility p to the non-homogeneous stress field or including some other type of
 218 dependence among the percolation of neighboring bonds. For this reason, a
 219 new percolation model is proposed.

220 Because bone can be treated as an almost brittle material, the cracking
 221 of a bond can be described commonly by a Weibull probability distribution
 222 [35, 36]. Due to this, we propose a non-independent percolation model where
 223 the percolation probability of a bonding is a function of the local stress:

$$p(\sigma; \sigma_0, \alpha) = 1 - e^{-(\sigma/\sigma_0)^\alpha} \quad (2)$$

224 The above formula defines a Weibull distribution, being σ the bond stress, σ_0
 225 the scale parameter of the Weibull distribution, and α the shape parameter.
 226 In previous studies of the literature of ceramics and biomaterials [36, 37, 38],
 227 most authors provided values of $\alpha \approx 4$ and being $\alpha = 4$ the limit value between

both types of materials, for this reason, the proposed model uses this value. 228
This value is verified in the results section, showing that small deviations 229
from $\alpha = 4$ exhibit no significant changes in the behavior of the model. On 230
the other hand, σ_0 is a parameter that can be fitted for each rib analyzed from 231
the vertical asymptote $\sigma = \sigma_\infty$ determined experimentally from the hits-stress 232
curve of each rib, as: 233

$$\sigma_\infty = \sigma_0 \ln^{1/\alpha} \left(\frac{1}{1 - p_c} \right) \quad (3)$$

where p_c is the critical percolation probability that in a basic orthogonal 234
mesh is $p_c = 1/2$ [39]. 235

When the net tends to the complete percolation (completely cracked state), 236
the probability of percolation p of the bonds tends to the critical probability 237
 p_c , and the size of the greatest cluster of cracked bonds $\#\mathcal{C}_0$ grows according 238
to a law independent of the shape of the network. It is assumed that for each 239
cracking of a bond, there is an AE signal, so the total number of AE detected 240
(N_{AE}) grows proportionally to the predicted size $\#\mathcal{C}_0$. The Percolation 241
theory estimates this number as: 242

$$N_{AE} \propto \#\mathcal{C}_0 \propto \frac{1}{|p - p_c|^\gamma} \quad (4)$$

being γ the universal critical exponent (which is independent of the net 243
shape). By introducing equations (2) and (3) in equation (4), this leads to: 244

$$N_{AE} = \frac{n_0}{\left| (1 - p_c) - \exp \left[\left(\frac{\sigma}{\sigma_\infty} \right)^\alpha \ln(1 - p_c) \right] \right|^\gamma} \quad (5)$$

where n_0 is a proportionality parameter whose value is different for each rib. 245
Thus, equation (5) predicts the stress behavior of the rib based on the hits 246
or AE signals detected on the specimen and the fracture propagation, where 247
 N_{AE} and σ are the experimental values, $p_c = 1/2$ and $\alpha = 4$ are prescribed 248
values and n_0, γ and σ_∞ are the parameters to be fitted. 249

250 **3 Results**

251 A set of 15 complete human ribs were tested under three-point bending to
 252 complete fracture, incorporating the AE technique. The stress-strain curves
 of the fracture section of some ribs are shown in Figure 4.

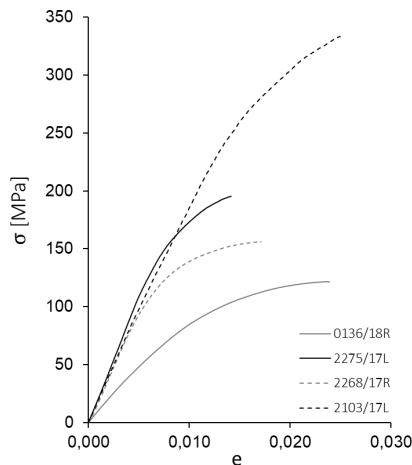


Figure 4: Stress-strain curves of rib bending tests.

253 As shown in Figure 5, hits are mainly detected at high stress levels where
 254 irreversible damage appears. From the set of tests, it is observed that the
 255 signals occur from the 45% of the maximum stress, where in most ribs, the
 256 signals occur from the 45% of the maximum stress, where in most ribs, the
 257 80% of the hits appear in the range of 75–100% of the stress fracture (see
 258 Figure 5). This concentration of hits at high stress levels results in a clear
 259 vertical asymptote of hits at maximum stress values.

260
 261 The model proposed in equation (5) was fitted to the stress-hits data
 262 obtained from the tests, as shown in Figure 5. For the fittings, values of
 263 $\alpha = 4$ and $p_c = 1/2$ were assumed, while the parameters n_0, γ and σ_∞ were
 264 obtained for each rib by a fitting procedure. By means of a second fitting
 265 based on the parameters obtained, it was confirmed that $\alpha \approx 4$ is a reasonable
 266 guess.

267 Once the parameters were obtained, a more detailed analysis was carried out
 268 to determine the influence of the anthropometric factors in the parameters
 269 and the mechanical properties by means of Linear Regression Analysis (LRA)
 270 and Analysis of Variance (ANOVA).

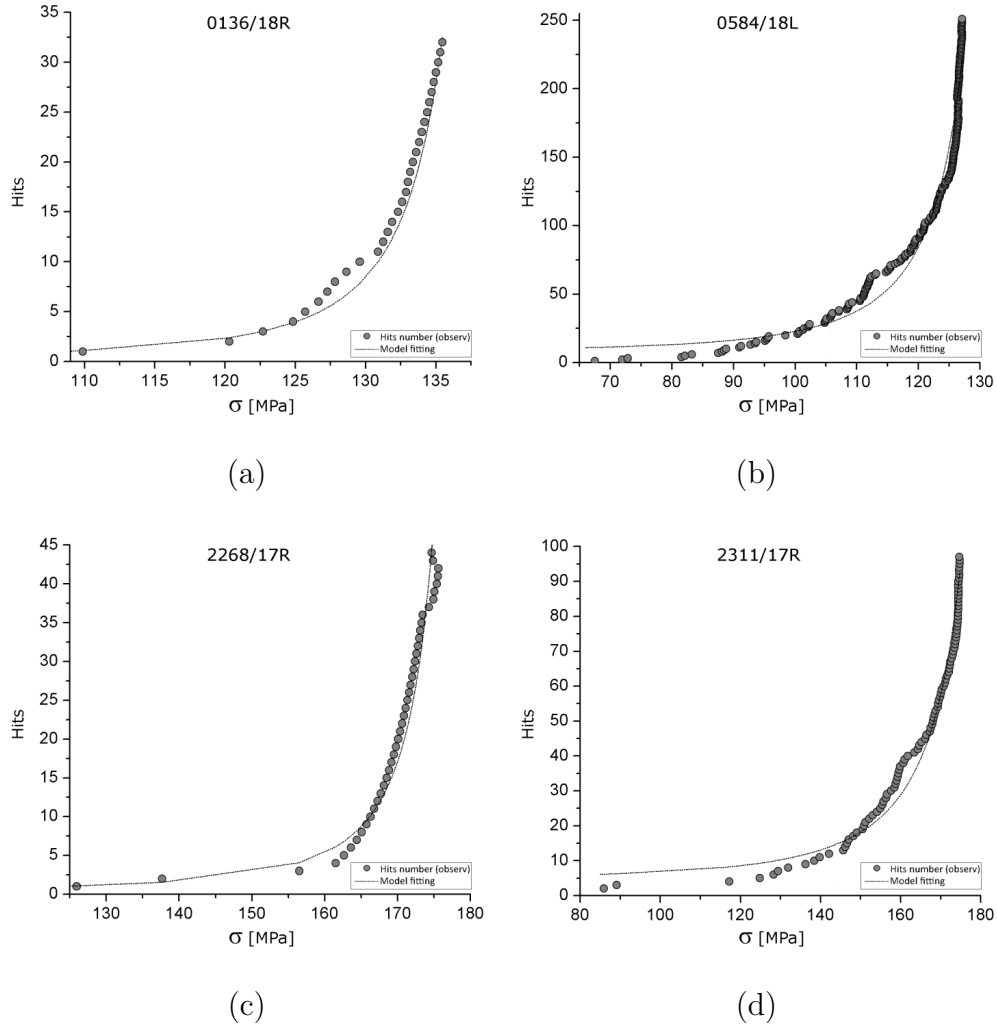


Figure 5: *Hits*-stress plot (dots) of rib bending tests detected by the AE technique and model fitting (line). Hits are concentrated at high stress levels in a clear asymptote.

As expected, the vertical asymptote stress σ_∞ showed to be statistically 271
 correlated with age ($p = 0.002$), showing a decrease in older subjects. 272
 A decrease in the scale parameter n_0 with increasing age ($p = 0.0160$) was also 273
 attested. Nevertheless, the critical percolation exponent γ showed to be relatively 274
 constant between the specimens and in a range of $1.80 \leq \gamma \leq 2.40$. No 275
 influence of any variable was seen in this parameter. 276

277

Table 1: Parameters obtained from fitting the model to the experimental data of hits and stress.

Rib	σ_∞	n_0	γ	λ	Rib	σ_∞	n_0	γ	λ
0136/18L	165	0.338	2.32	0.329	2268/17L	185	0.712	1.89	0.370
0136/18R	142	0.061	2.28	—	2268/17R	183	0.128	2.07	0.354
0182/18L	156	0.110	2.10	0.341	2273/17L	168	1.045	1.95	0.332
0182/18R	180	0.400	2.10	0.328	2273/17R	180	0.600	1.89	—
0520/18L	310	1.508	1.89	—	2275/17L	189	0.530	1.80	0.268
0584/18L	139	2.156	2.12	0.325	2311/17L	191	1.472	1.91	0.286
2102/17L	200	0.480	2.43	0.380	2311/17R	198	2.400	1.80	0.336
2103/17L	325	1.900	1.80	—					

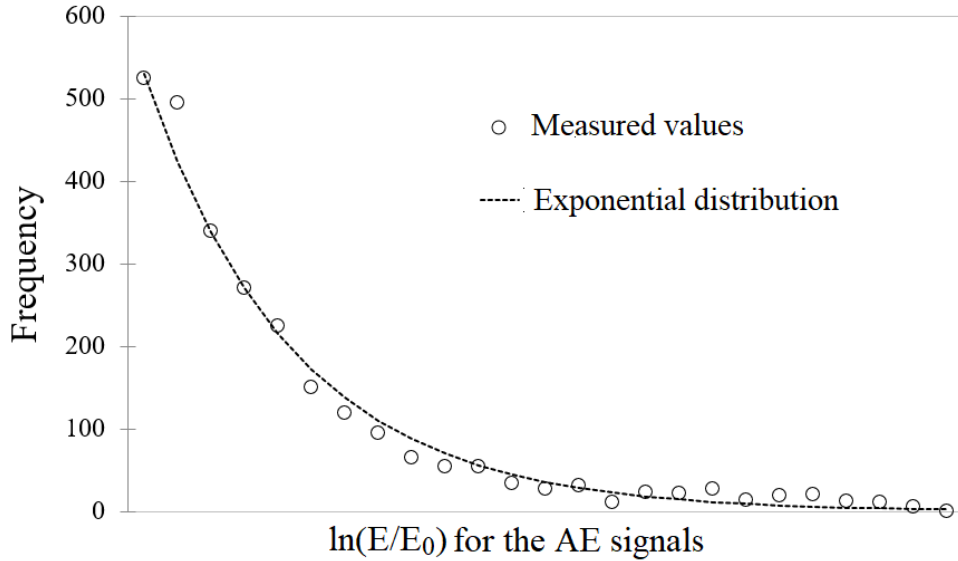


Figure 6: Comparison of the total observed number of hits (circles) and an exponential distribution (dotted line).

278 Finally, the experiments were also used to examine other hypotheses that
 279 fell outside the proposed model. For example, it is a well-known fact that
 280 intensities and energies of the AE signals follow well-known distributions
 281 [21, 20]. In particular, many authors have found that the energy of AE

signals are distributed according to a power law of this type:

$$\mathbb{P}(\text{energy an AE signal} \leq E) = 1 - \left(\frac{E_0}{E}\right)^\lambda \quad (6)$$

where \mathbb{P} refers to a probability, $\lambda > 0$ is the exponent of the power law, and E_0 is the threshold energy (lower energy signals are ignored since they correspond to phenomena other than micro-cracking, so $E \geq E_0$). By defining the logarithmic variable $U = \ln(E/E_0)$, one has that the new variable U is distributed according to an exponential distribution with probability density function given by:

$$p_U(u) = \lambda e^{-\lambda u} \quad (7)$$

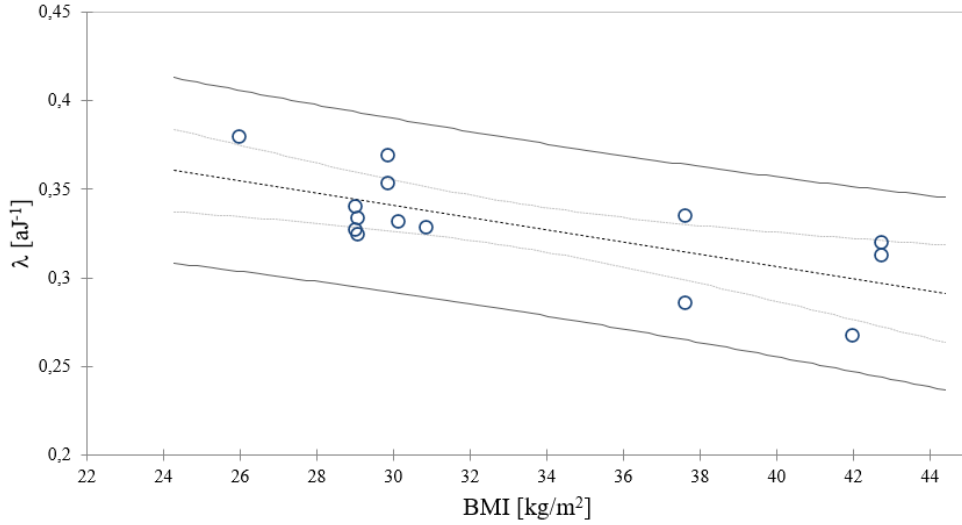


Figure 7: Scatter plot of the power-law parameter λ for the distribution of AE energies and BMI.

This is precisely what is observed in the measured energy of the AE signals whose distribution is shown in Figure 6. In Table 1, the values of λ for all the samples with a sufficiently large number of hits N are given (when N is not large, the error in λ is significant, and there is not a good fitting to an exponential distribution). It is interesting to note that, unexpectedly, λ seems to be very significantly correlated with the [Body Mass Index \(BMI\)](#)

296 of the individuals (p -value < 0.005), see Figure 7. Previous analysis showed
297 that there was not a significant relation between age and BMI (p -value =
298 0.162).

299 4 Discussion

300 In this paper, it has been proposed a model for the prediction of the onset of
301 micro-cracking related to the beginning of irreversible damage and prior to
302 the macroscopic fracture, based on the Percolation Theory. For this purpose,
303 whole human rib bending tests were performed, incorporating the AE sensors
304 during the tests. Experimental data were used to fit the model parameters
305 and to show the model behavior.

306
307 Some existing models describe the succession of AE signals by assuming a
308 pre-imposed power law, trying to fit the sudden increase of hits or AE signals
309 once achieved certain pressure level near the maximum pressure [40]; the
310 same power law is found for biological tissues [20, 21]. In any case, different
311 explanations have been proposed for the occurrence of power laws, from
312 models based on viscoplasticity [41] to explanations based on Percolation
313 Theory.

314 Micro-cracking is a stochastic process controlled by different factors and,
315 for this reason, it involves some kind of randomness. This randomness seems
316 to be associated with microscopic details in micro-structure both in biological
317 materials and in non-biological materials. In biological tissue the microstruc-
318 ture is influenced by anthropometric factors [42]. However, as the AE demon-
319 strates, the fracture process occurs progressively, where micro-cracks propa-
320 gate through the microstructure until the main fracture completely crosses
321 the material. This process can be studied based on the Percolation Theory of
322 bonds (analogous to the cement lines and interstitial bone between osteons),
323 which progressively break.

324
325 As shown in Figure 5, the ideas of the Percolation Theory predict that
326 there will be an asymptotic behavior in the number of detected AE signals.
327 Furthermore, the approximation to the asymptote is given by a universal ex-
328 ponent γ whose values are close to the values predicted by the independent
329 Percolation Theory. In 2D percolation, the expected value of the exponent
330 is $\gamma \approx 2.38$, while 3D percolation, it is $\gamma \approx 1.80$ [43, 44]. Although we have a

plane stress state, when the crack appears, the symmetry is broken, and [this is reason](#) why we would expect values [in the range](#) $1.80 \leq \gamma \leq 2.38$ as [it indeed](#) happens: $\gamma = 2.023 \pm 0.22$. The expected values correspond to independent Bernoulli percolation, but in the present case, the bond breakages are not independent nor the probability is homogeneous, [which](#) would also make us expect some deviation with respect to the theoretical value for the exponents γ . Indeed, this is a limitation of the study since the mathematical problem of finding the values for a non-independent percolation process depending on a stress field is still an open problem [45, 46].

On the other hand, the parameter n_0 shows a great variability, but it exhibits a significant descent with age (p -value = 0.016). Also, the parameter σ_∞ shows a significant decrease with age (p -value = 0.0002), [which](#) is not a surprise, since the parameter is essentially the failure stress, and it is well known that this [magnitude](#) decreases with age [47].

Another virtue of the proposed model is the prediction/anticipation of structural failure. As the hits occur, it would be possible with each new hit to re-evaluate the expected value σ_∞ for the failure stress that appears in the equation (3). If the quotient between the current stress and that σ/σ_∞ value is close enough to 1, we could conjecture that we are close to [complete failure](#) even without getting too close to it, which could have several applications as a potentially usable method to anticipate mechanical failure from the evidence of accumulated micro-cracks.

5 Conclusion

[Using some](#) ideas of the Percolation Theory, [we have proposed](#) a model that is able to quantitatively describe the distribution of AE signals as stress on cortical bone increases. This model could be used in practical situations to anticipate structural failure of the human rib subjected to bending. The model provides a reasonable fit, [and](#) the results show that the model parameters exhibit consistency with the two exponents of the Percolation Theory. Moreover, it has been found that some parameters appear to correlate with anthropometric variables (age and BMI), suggesting that this model is able to capture some influence of these factors on bone microstructure.

366 Besides, the energy distribution of the AE signals has been examined,
367 and it has been found that an exponent determining its distribution appears
368 to be significantly correlated with the subject's BMI, which is a promising
369 finding because it suggests that there is a correlation between BMI and mi-
370 crostructure of human cortical bone.

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