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OPTIMAL POPULATION POLICY

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Whatever their immediate goals (pills taken, IUDs in place, pregnancies aborted) the ultimate objective of birth control schemes is an improvement in welfare. The question naturally arises, then, of whether a best policy can be found, i.e., one that maximizes the improvement in welfare, and whether a corresponding optimal time-path for the population exists.

Any attempt to optimize population policy must face a difficult question of ethics: who is to be benefited by the policy? Potential beneficiaries (called "clients" in Churchman's (1971) term) of population policies include the presently living generation, future generations, potential immigrants, and "unborn people"--who are in a sense candidates to be born. As we shall see, depending on whose welfare is to be uppermost, many possible optimal policies may exist, each quite different from the rest. The choice of policy instruments is also an ethical problem. For example, a scheme relying on economic sanctions to influence fertility has a different ethical basis from one that attempts to manipulate attitudes to family size. The optimal population trajectories in the two cases would also be quite different.

The few papers that have appeared in the area of optimal population policy have bound themselves at the outset to a rigid set of assumptions about welfare, social preferences, and birth

control schemes.<sup>1</sup> They have made little attempt to evaluate the implications for optimal policy of the choice of assumptions. What is needed to study these implications is a theoretical framework in which initial assumptions are arbitrary and the resulting optimal policies can be compared and analyzed. Such a framework would have to recognize the important dynamic aspect of population problems. Over time, preferences, institutions and population structures change, the economy grows, and the spectrum of possible policy instruments widens. Any static analysis would severely distort the problem.

The framework we propose is based on modern optimal control theory, which is well suited to dealing with dynamic processes such as population growth. A control theoretic formulation can take account of the changing character of societies' needs and institutions, as well as the intricate interactions among population, the environment and the economy. Technical and political limitations on control programs are easily incorporated, as is the available knowledge of the fertility response to various forms of expenditures on population control.

The first part of this paper will develop a control theoretic framework as a broad general setting for the policy problem. The second part examines the implications for optimal

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<sup>1</sup>See, for example, Dasgupta (1969), Pitchford (1968) and Votey (1971).

policy of different ethical positions, welfare preferences, and birth control schemes. We do this in the context of illustrative numerical examples, since closed form solutions are not available when the problem passes an elementary level. The framework, however, still allows some analytical insight into the tradeoffs which determine the optimal path.

# I

## GENERAL THEORETICAL FORMAT

We shall first set up the policy problem in fairly general, abstract terms and afterwards give specific examples of the format in practical use.

Population policy is to be optimized within the context of an interacting and changing system, which includes the population, the economy and the environment. The system may be represented by the set  $\{x(t), y(t), t\}$  where  $x(t)$  is an  $n$ -dimensional vector that describes the state of the system at time  $t$ , and  $y(t)$  is an  $m$ -dimensional vector of policy instrument variables at time  $t$ . In a practical application, state variables,  $x(t)$ , might include population size and distribution, the birth rate, and national income; policy variables,  $y(t)$ , might include the interest rate, public expenditures on birth control programs, the rate of immigration, and so on. The proper choice of variables will depend upon the nature of the particular problem under study.

The behavioral dynamics of the system are given by the set of  $n$  differential equations

$$\dot{x} = f[x(t), y(t), t] \quad . \quad (1)$$

These equations might represent population growth, capital accumulation, etc. The state and control variables are constrained by

$$C[x(t),y(t),t] \leq 0 ; \quad (2)$$

for example, budgetary limitations or political restraints on control.

### Objective Criterion

For social policy problems, a convenient objective criterion would be the "amount of welfare" received in the arbitrary time period  $(t_0, t_f)$ .<sup>2</sup> If  $U$  is a measure of the per-capita welfare rate, and  $w(t)$  weights welfare at time  $t$ , the objective function is:

$$J = \int_{t_0}^{t_f} w(t)U[x(t),y(t),t] dt \quad (3)$$

In most of this paper, for convenience we shall assume  $w(t)$  is  $\exp[-\rho(t-t_0)]$ .

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<sup>2</sup>Here, we assume that intertemporal welfare is comparable, i.e., that welfare can be integrated over a time period.

<sup>3</sup>Under certain assumptions on the rationality of social choice it is impossible to find a welfare function that represents society as a whole. (See Sen (1970), Arrow (1951)). We are interested in the policy implications of a given welfare criterion; hence, we do not require that the criterion represent society. (A sufficient assumption for  $U$  to represent society is that each person have the same preference ordering and that the components of welfare be homogeneously distributed over society.)



The above approach assumes that the welfare rate  $U(x,y,t)$  is well defined in terms of the system variables--population, capital stock, immigration, etc. However, it is difficult to formulate a cardinal utility function directly in terms of these variables. For convenience in constructing a welfare criterion we shall assume that welfare is a function of several factors which together are important to "quality of life." We need include only those components of "quality of life" that can be influenced by changes in the policy variables. Examples might be standard of living, crowding, quality of the environment, etc. Denoting these welfare factors by  $F_1, \dots, F_k$ ,  $U$  becomes

$$U = U[F_1(t), \dots, F_k(t)] .$$

These factors are vague entities, however. We will, therefore, suppose that each can be represented by an index function defined in terms of the system variables (see Arthur, 1972). Corresponding to the underlying factor  $F_i(t)$  would be the index function  $A_i(x,y,t)$ . The indices,  $A_i$ , are chosen carefully to capture what is intuitively understood by  $F_i$ . If population, GNP, and capital investment are among the system variables, for example, the welfare factor "standard of living" could be represented roughly by the index per capita GNP; a somewhat better index would be per capita consumption.

The welfare rate in the "index space" is now

$$U = U(A_1, \dots, A_k) \quad .$$

Providing the indices are independent, first order welfare changes due to a small change in a state variable,  $x_i(t)$ , or control variable,  $y_j(t)$ , are given respectively by<sup>4</sup>

$$\frac{\partial U}{\partial x_i} = \frac{\partial U}{\partial A_1} \frac{\partial A_1}{\partial x_i} + \frac{\partial U}{\partial A_2} \frac{\partial A_2}{\partial x_i} + \dots + \frac{\partial U}{\partial A_k} \frac{\partial A_k}{\partial x_i} \quad , \quad (4)$$

$$\frac{\partial U}{\partial y_j} = \frac{\partial U}{\partial A_1} \frac{\partial A_1}{\partial y_j} + \frac{\partial U}{\partial A_2} \frac{\partial A_2}{\partial y_j} + \dots + \frac{\partial U}{\partial A_k} \frac{\partial A_k}{\partial y_j} \quad . \quad (5)$$

That is, a marginal additional unit of a system variable may affect each welfare index. This in turn changes the welfare rate,  $U$ , according to equations (4) or (5). The partials  $\partial U / \partial A_i$  may be regarded as the relative preferences for a marginal unit of each welfare factor.

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<sup>4</sup>The preferences may change over time and also depend on the value of the indices. Hence, we should write  $\partial U[(A_1, \dots, A_k), t] / \partial A_i$ . For notational simplicity we shall simply write  $\partial U / \partial A_i$ , and assume the argument is understood. Throughout this paper we shall drop time arguments, unless they are necessary for clarity. E.g., we shall write  $U(x, y, t)$  for  $U[x(t), y(t), t]$ .

### The Policy Problem

In summary, the framework we intend to use to study optimal population policy is as follows. We must choose a policy trajectory  $y(t)$  to maximize

$$\int_{t_0}^{t_f} w(t)U(A_1, \dots, A_k)dt ,$$

where  $\dot{x} = f(x, y, t)$  - system dynamics,  
 $C(x, y, t) \leq 0$  - constraints,  
and  $A_i = A_i(x, y, t)$  - welfare indices.

### Necessary Conditions for Optimality

If there are no constraints present, the above control problem becomes a classical Bolza problem. By introducing an  $n$ -dimensional vector of adjoint variables,  $\lambda(t)$  (where  $\lambda_i$  corresponds to the system equation  $\dot{x}_i = f_i$ ), it is easy to show that an optimal policy,  $y^*(t)$ , will satisfy the following necessary conditions (see, for example, Bryson and Ho, 1969):

$$\dot{x} = f(x, y^*, t) , \tag{A}$$

$$-\dot{\lambda}(t) = \sum_i \frac{\partial U}{\partial A_i} \frac{\partial A_i}{\partial x} + \sum_j \lambda_j(t) \frac{\partial f_j}{\partial x}, \quad (B)$$

where  $\lambda(t_f) = 0$  ;

$$0 = \sum_i \frac{\partial U}{\partial A_i} \frac{\partial A_i}{\partial y^*} + \sum_j \lambda_j(t) \frac{\partial f_j}{\partial y^*} \quad (C)$$

The vector notation here is concise. There are  $n$  dynamic equations (A),  $n$  corresponding adjoint equations (B), and  $m$  optimality equations (C). These relations can give some insight into what constitutes optimality in population policy.<sup>5</sup>

Dynamic equations. The conditions (A) state that the dynamic equations must hold. This, of course, is true whether the policy is optimal or not.

Adjoint equations. The theory shows that we can interpret the adjoint variable  $\lambda_i(t)$  as the value of a unit marginal increase in the state variable  $x_i$  at time  $t$ , if control is kept unchanged. For example, if the variable  $x_i$  is capital stock,  $\lambda_i(t)$  will be the value of an extra unit of capital stock at time  $t$ . In order that the adjoint variables may be interpreted as values or prices, they must, by the theory, be determined according to (B).

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<sup>5</sup>For more details on interpreting the necessary conditions, especially the adjoint equations, see Arthur (1972).

Optimality conditions. If a policy  $y(t)$  is optimal, adjustments in the policy trajectory can cause no further improvement in total welfare in  $(t_0, t_f)$ . Adjustments in policy have two effects. First, they have an immediate impact on the welfare rate, represented by  $\frac{\partial U}{\partial y} = \sum_i \frac{\partial U}{\partial A_i} \frac{\partial A_i}{\partial y}$ . For example, they may cost money which would lower consumption. Secondly, acting through the dynamic equations, policy adjustments cause changes  $\frac{\partial f}{\partial y}$  in the state variables, valued at  $\sum_j \lambda_j \frac{\partial f_j}{\partial y}$ . For example, a policy which reduces the population growth rate causes fewer people to be present in the system at a later date. An optimal policy balances the net short-term welfare effects against the net long-term effects. Or, from another point of view, it balances private against social costs and benefits.

#### Example

Let us examine optimality in a simple system. Suppose that the government of an underdeveloped country is pursuing an optimal birth control policy by means of a family-planning program. The policy variable is, say, annual expenditure on birth control. An increase in family-planning expenditures has several effects: in the short run it reduces consumption and also the dependency burden of children; in the long run it causes fewer people to be in the system, and a lower stock of capital than otherwise. Because the expenditure schedule is optimal, a marginal expenditure

increase would not be worth making. Benefits due to the decreased dependency burden and fewer future people would be offset by losses in consumption and industrial growth. Again, since the expenditure trajectory has been assumed optimal, decreases in family planning expenditures would not be worth making. Thus, along an optimal trajectory, these private and social welfare effects must balance each other. These optimality tradeoffs can also be shown analytically, by means of the optimality conditions (C).<sup>6</sup>

#### Optimal Policy and Overpopulation

Suppose we define a society to be overpopulated if, at time  $t$ , the per-capita welfare rate could be increased by a marginal reduction in population. (Obviously our definition of overpopulation is relative to the chosen welfare function  $U$ .) A society may be overpopulated in this sense, and yet the optimal policy might well be to increase the population level. The optimal policy must consider not only present welfare, but future welfare as well. For example, reducing population in the present may cause a fall in investment, so that future economic losses might outweigh present welfare gains. Population should be increased. Similarly, it is easy to imagine circumstances where an underpopulated country

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<sup>6</sup>For more detailed examples, see Arthur (1972) and the appendix of McNicoll (1971).

should decrease its population. In the control theory formulation such intertemporal tradeoffs stand out clearly in the optimality conditions.

We shall now choose variables, dynamic relationships, and welfare assumptions in a simplified system to illustrate the control-theoretic format in use.

#### A SIMPLE ILLUSTRATIVE EXAMPLE

We will consider an extremely simple model with a homogeneous population and a one-good economy. The state variables of the system are the population size ( $L$ ) and capital stock ( $K$ ); the single policy variable is the growth rate of population,  $n$ . The average propensity to save ( $s$ ) is constant.

##### Objective Function

Suppose that the psychic components of welfare that have relevance to population policy are the following:

1. material standard of living;
2. perceived degree of crowdedness;
3. state of the physical environment;
4. extent of realization of family size goals.

Others could be listed, but these are probably the most important in an industrialized country.

Index functions,  $A_i$ , must be chosen for each of the four welfare components listed above. Since the policy situation is sensitive to the functional form of each index, care must be exercised in their selection.<sup>7</sup>

Standard of living. Per capita consumption, defined by

$$C = (1-s)F/L$$

where  $F$  is total output, is a conventional but fairly adequate index of the average standard of living. We assume that  $\partial U/\partial C > 0$  and  $\partial^2 U/\partial C^2 < 0$ .

Indices for the other factors are less obvious.

Perceived crowding. This welfare component is highly subjective, with a substantial cultural element involved. For the purpose of our example, however, the psychic welfare due to the presence of other people in the system will be linked to the simple index:

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<sup>7</sup>Details of the index functions and parameter values used in this example are given in the appendix.



average population density (D).<sup>8</sup> We assume that humans have some desire for the company of others, but welfare falls beyond a certain level of density ( $\partial^2 U / \partial D^2 < 0$ ).

State of the environment. By this index we mean to reflect the psychic welfare costs due to environmental degradation that is irreversible within a realistic time horizon. Irreversible degradation may cover, for example, changes in atmospheric composition, increase in background radiation, pollution of the oceans, and alienation of unique natural areas. We will assume that, to a great extent, this degradation is an inevitable concomitant of

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<sup>8</sup>Even ignoring culture as being exogenous to the system, the problem of defining the welfare impact of crowding is complex. The level and speed of urbanization, the geographical mobility of the population, the provision of space-intensive recreation facilities, and urban and suburban planning, are among the significant factors that should be considered. In support of the index chosen we would note that if city sizes over a nation are approximately distributed according to a Pareto distribution (the so-called "rank-size rule"), which is generally the case, then the expected size of the city in which a randomly chosen person lives increases in direct proportion to the average population density in the nation. Moreover, there is some evidence of stress responses to density in human populations and to the reduced accessibility of open spaces, whether or not they were ever utilized. As city size grows, urban density also generally rises and accessible open space per capita diminishes.

economic growth, and therefore take the size of the economy, i.e., GNP, as the index of the environmental contribution to welfare (E).<sup>9</sup> Naturally,  $\partial U/\partial E$  is negative.

Realization of family-size goals. Some population policies may cause a deviation between desired and actual family size. To measure the degree to which family size goals are realized, the most straightforward index would be the average number of births foregone (B), i.e., the difference between desired and actual average births per family. The growth rate corresponding to the desired family size is denoted  $\bar{n}$ . We assume that welfare falls as parents are less able to realize their family-size goals:  $\partial U/\partial B$  is negative.

To obtain sample values for the relative preferences between welfare components,  $\partial U/\partial C$ ,  $\partial U/\partial D$ ,  $\partial U/\partial B$ , we have questioned colleagues as to their tradeoffs between marginal consumption, crowding, family-size, etc. The results, together with their implications for the function  $U$ , are listed in the appendix.

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<sup>9</sup>If we roughly separate environmental changes into "stock effects" and "flow effects" (which is admittedly to dichotomize a continuum), then  $E$  refers only to the former. The flow component, meaning the currently produced effluent of industry--the water, air and noise pollution of everyday concern--we assume could be largely eliminated by the appropriate diversion of resources (impelled by legislation), albeit at an ultimate cost to the consumer.

### System Dynamics

The dynamics of this system example are simple. We assume for convenience a Cobb-Douglas production function with neutral technological progress at a rate  $r$  :

$$F = Ae^{rt} K^\alpha L^\beta, \quad A = \text{constant},$$

where  $\alpha$ ,  $\beta$  are the output elasticities of capital and labor. Investment funds  $sF$  are distributed to capital growth  $\dot{K}$  and depreciation  $\delta K$ , where  $\delta$  is the constant rate of decay of capital. Population,  $L(t)$ , grows according to the simple law

$$\dot{L} = nL,$$

where  $n$  is the population growth rate.

Population policy will be exercised by direct manipulation of  $n$ , involving no costs other than the "psychic costs" that enter the welfare function. Such a policy need not be dictatorial: Hardin's (1968) "mutual coercion mutually agreed upon," in some relatively mild form, would suffice. But clearly other mechanisms are possible--this question will be taken up in a later section.

### The Problem

The optimal policy problem for this simple example can now be stated:

$$\text{Maximize}_{n(t)} \int_{t_0}^{t_F} e^{-\rho(t-t_0)} U(C,D,E,B) dt, \quad (6)$$

where  $C, D, E, B$  are the indices specified above,  
subject to

$$\dot{L}(t) = n(t)L(t), \quad L(0) = L_0; \quad (7)$$

and

$$\dot{K}(t) = sAe^{rt}K^\alpha(t)L^\beta(t) - \delta K(t), \quad K(0) = K_0. \quad (8)$$

As parameters and initial values we take the following,  
which would be similar to those of the U.S. circa 1970 if it were  
to conform to this crude model.

$L(0) = 2 \times 10^8$	$\alpha = .25$
$K(0) = \$3 \times 10^{12}$	$\beta = .75$
$\bar{n} = .01$	$r = .02$
$A = 450$	$\delta = .04$
$s = .15$	$\rho = .03$

(A is expressed in the appropriate normalizing units.)

#### Tradeoffs on the Optimal Trajectory

What are the optimality tradeoffs in this example? The  
optimality condition (C), when applied to this model, gives

$$e^{-\rho t} \left[ \frac{\partial U}{\partial C} \frac{\partial C}{\partial n} + \frac{\partial U}{\partial D} \frac{\partial D}{\partial n} + \frac{\partial U}{\partial E} \frac{\partial E}{\partial n} + \frac{\partial U}{\partial B} \frac{\partial B}{\partial n} \right] + \lambda_L(t) \frac{\partial \dot{L}}{\partial n} + \lambda_K(t) \frac{\partial \dot{K}}{\partial n} = 0 . \quad (9)$$

The growth rate,  $n$ , is not an argument of  $C$ ,  $D$ ,  $E$  and  $\dot{K}$ . Hence, the partials  $\partial C/\partial n$ ,  $\partial D/\partial n$ ,  $\partial E/\partial n$  and  $\partial \dot{K}/\partial n$  are zero. ( $C$ ,  $D$ ,  $E$  and  $\dot{K}$  do, however, help determine the value of an additional person,  $\lambda_L$ , and of marginal capital,  $\lambda_K$ .) Equation (9) reduces to

$$e^{-\rho t} \frac{\partial U}{\partial B} \frac{\partial B}{\partial n} + \lambda_L(t) \frac{\partial \dot{L}}{\partial n} = 0 . \quad (10)$$

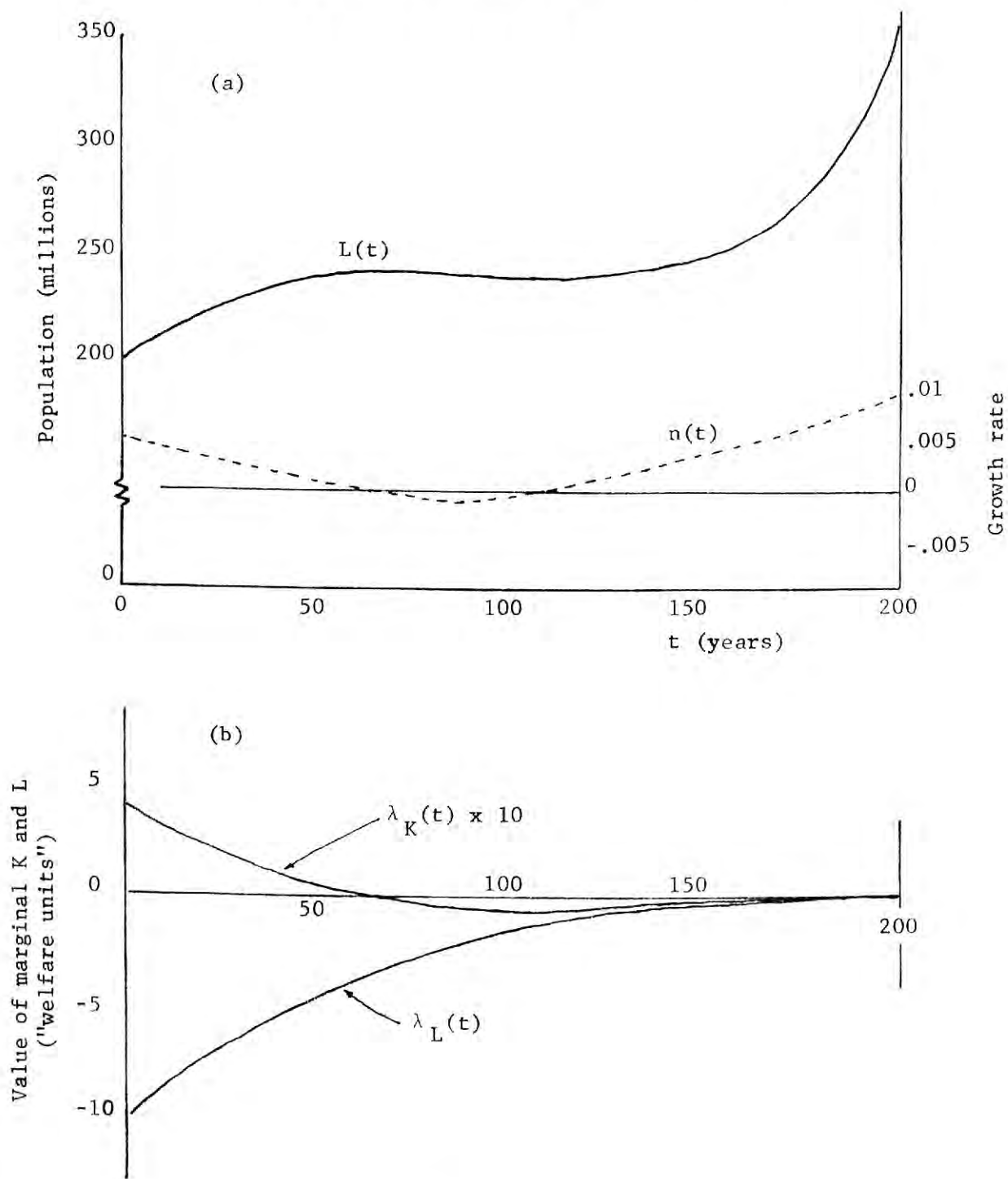
Here, the population growth rate is optimized when, for a marginal decrease in the growth rate, private parental losses are offset by the benefits of having fewer people in the system, as valued by  $\lambda_L$ .

We have developed a gradient algorithm to calculate optimal trajectories of the system variables in this model over a 200 year period. Figure 1 shows the optimal time-paths of population and the growth rate, and the "value" of marginal capital,  $\lambda_K$ , and marginal people,  $\lambda_L$ .<sup>10</sup>

<sup>10</sup>In this simple example,  $L$  is taken to be a scalar. In a more realistic model, the population age structure would be recognized, and the broad fluctuations that a non-stable age distribution cause in the total population trajectory would significantly influence the results. In particular, the local maximum population after 60 years, shown in Fig. 1a, could be much greater if the initial net reproduction rate is greater than one, due to the "momentum" of the age distribution. An immediate drop in U.S. fertility rates to replacement level, for example, would imply an ultimate stationary population of over 270 million (Keyfitz, 1971).

FIGURE 1

Optimal time paths in basic model: (a) population,  $L$ , and population growth rate,  $n$ ; (b) value of marginal capital,  $\lambda_K$ , and value of a marginal person,  $\lambda_L$ .



In this example, the value of a marginal unit of capital,  $\lambda_K$ , is positive at the beginning of the period. For consumption purposes it is worthwhile for the system to accumulate capital. However, as the economy grows, adverse environmental effects begin to dominate and the value of extra capital to the system becomes negative. Additional people, on the other hand, are detrimental to this system throughout the whole period. An additional person lowers per-capita consumption, increases crowding and, through his impact on economic growth, produces further environmental deterioration. Near the end of the time period there is little time left for the marginal person to contribute adversely to welfare; thus his "value" is near zero. Going back towards  $t_0$ , the person has longer to contribute and his "value,"  $\lambda_L$ , will become progressively more negative. One mitigating effect on the value of an additional person is his potential to speed the accumulation of capital, provided that capital is valued positively--as it is in the early part of the program.

We can now see how the optimal trajectory develops in this example. Although people are negatively valued, parental desires for children keep the population growing near its normal rate. As time goes on the parental desires of future generations are weighted progressively less by  $\exp - \rho t$  (see equation (10)) and large populations make small changes in population growth very effective. The population trajectory turns down. Near the end

of the period, since this model ignores welfare after time  $t_f$ , there is no point in keeping the population down--parental wishes again predominate.

Naturally, this is not the only possible model, but it does illustrate some of the complex tradeoffs which occur even in a highly simplified situation. Extensions can be incorporated fairly easily. For example, the savings rate  $s$  can be taken as a policy instrument in addition to  $n$ , and a terminal constraint imposed in the form of a minimum permissible level of capital,  $K(t_f)$ .

In the next part of the paper we shall use the theoretical framework to study the sensitivity of optimal policy to the choice of welfare ethic, to social preferences, and to various population control schemes. Again we intend to use simplified examples similar to that presented above to provide insight into some major issues involved in population policy. The discussion falls naturally into three sections: the policy implications of broad ethical assumptions; of social preferences; and of specific birth control programs.



## ETHICAL ASSUMPTIONS

Several of the basic issues raised by population policy are ethical in nature, although often debated in political terms.<sup>11</sup> These problems come to the fore when we try to choose a suitable welfare criterion.

The Bentham debate

Most ethical problems arise in the task of specifying who the beneficiaries of the policy should be. The much discussed Benthamite question can be phrased in terms of the striking metaphor used by Malthus in the third edition of the Essay: to what extent, if any, should the welfare of those now at "Nature's feast" be sacrificed to admit more participants? A per-capita welfare criterion maximizes the welfare of only those already present in the system. New entrants (babies or immigrants) would be admitted only if their presence added to the welfare of those already present. On the other hand, a total welfare criterion, where per-capita welfare is weighted by the population size, also considers

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<sup>11</sup>A more complete discussion of these questions is given in McNicoll (1971).

the welfare of the potential entrant. This is often called the Benthamite criterion.<sup>12</sup>

Using the simple system of the previous section, Figure 2 compares the optimal trajectories under the two ethics--per capita and Benthamite.<sup>13</sup> The difference between the two trajectories is quite striking. We have also included the uncontrolled "natural" trajectory of this system. As one might guess, when we consider the welfare of potential entrants, a considerably higher optimal population trajectory results. Under the per-capita ethic the value of a marginal person,  $\lambda_L$ , was negative throughout. The same system with a Benthamite ethic values the marginal person positively at the beginning. This is because the new entrant's welfare gain is not offset by the present and future losses to the rest of society. Now parents must undergo sacrifices so that the welfare of the potential entrant is realized--they must have more children than they desire. Eventually, crowding and environmental deterioration diminish the welfare gain of the new entrant; societal

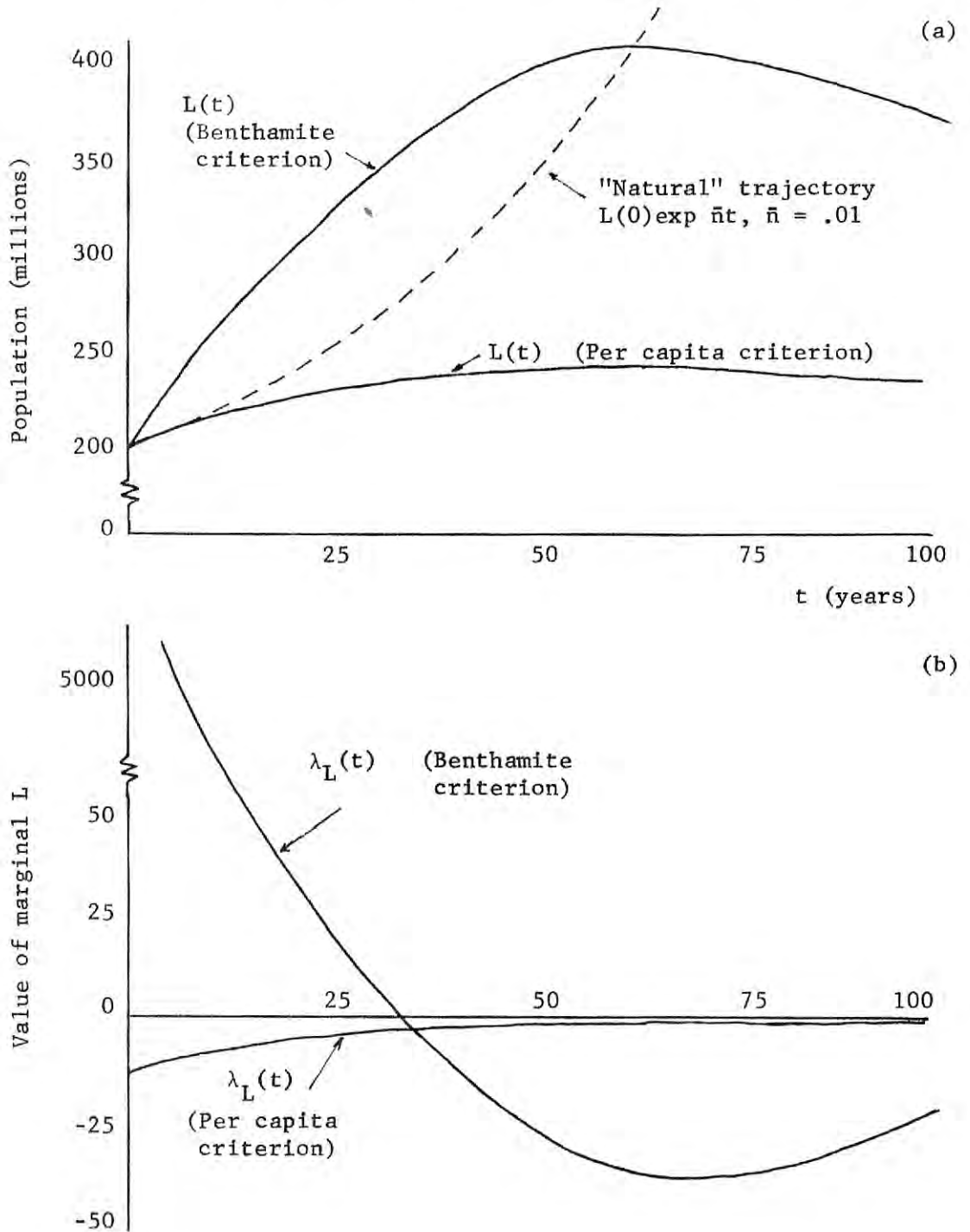
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<sup>12</sup>Technically, we can write the total welfare or Benthamite criterion as  $\int_{t_0}^{t_f} L(t)U(t)dt$ , where  $\int_{t_0}^{t_f} U(t)dt$  is the per-capita criterion.

<sup>13</sup>In this and subsequent diagrams, only the first 100 of the 200 year computed trajectories are shown.

FIGURE 2

Optimal time paths under per capita and Benthamite welfare criteria: (a) population,  $L$ ; (b) value of a marginal person,  $\lambda_L$ .



losses predominate and parents must forego children. The trajectory turns down, but at a much higher (and less well-off) population than before.<sup>14</sup>

#### Intertemporal Equity

A different aspect of the client problem is the question of assigning relative weights to the welfare of future generations. This issue of intertemporal equity can enter our formulation very simply through the choice of  $w(t)$  in (3), that is, through the time discount rate. In the previous numerical examples, we arbitrarily chose a discount rate of three percent. This would imply that the welfare of the "next generation"--the population in, say, 24 years' time--is worth only half the welfare of the present generation.<sup>15</sup>

To give an indication of the significance of the intertemporal equity problem, we have taken two neighboring values, one percent and five percent, and computed the optimal population

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<sup>14</sup>It can be shown that the (decreasing) value of the marginal person passes through zero at the point in the trajectory when parents have exactly the family-size they desire.

<sup>15</sup>It is an open question whether any discounting of future welfare should be made. Note that we approach this problem from a welfare perspective, in which even a 3% rate is a strong assumption about intergenerational equity. Discount rates of the order of 10%, as found in cost-benefit analyses of public projects, are justified as being the opportunity costs of the resources utilized. From an equity standpoint, however, acceptance of such rates would be to weight all future generations out of the problem.

trajectories under the same assumptions as in the original example (with the per capita criterion). Figure 3 shows the results, together with the intermediate case of three percent discounting from Figure 1a. From an initial population of 200 million, the difference between the one percent and five percent assumption is 84 million after 50 years, 170 million after 100 years. Five percent is still a very modest discount rate, yet under it the population never decreases: the generation presently living can virtually ignore adverse effects of future generations.

#### Further Ethical Issues

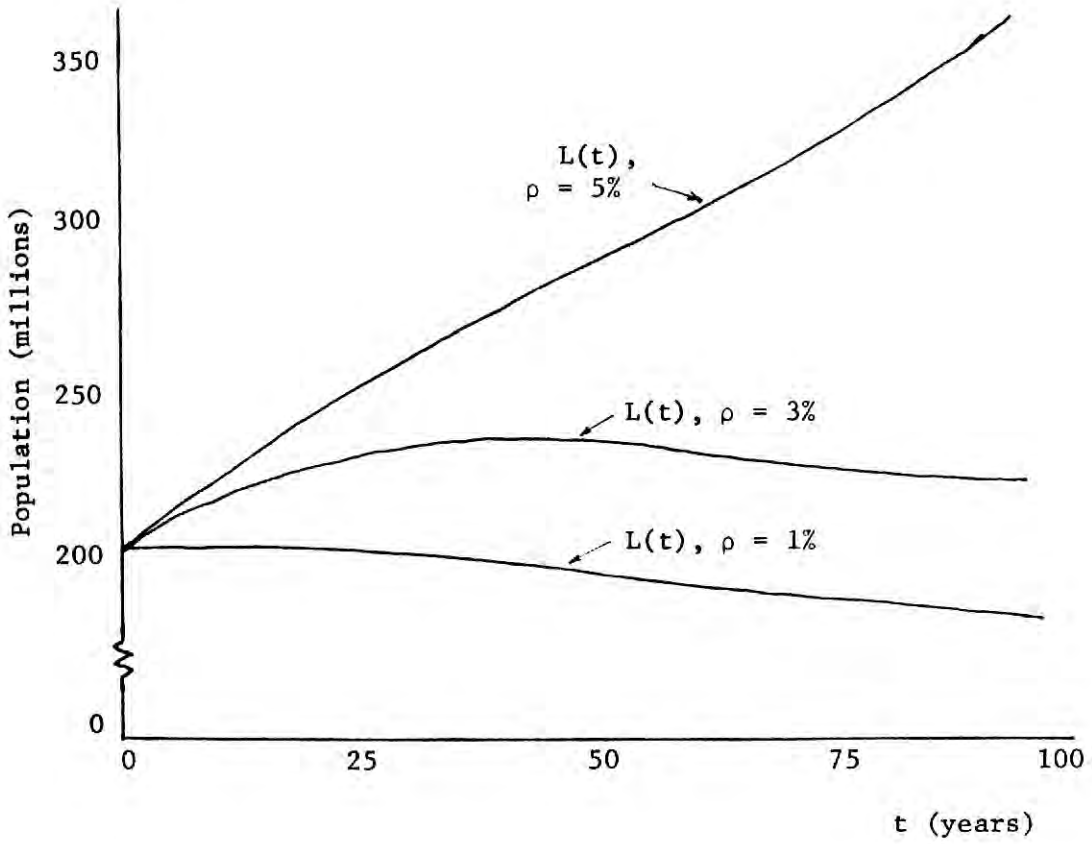
A different ethical problem is that of whether the government or planning body has the right to attempt any manipulation of tastes or attitudes in achieving birth control. Most governments, as well as the U.N., emphasize the parents' right to determine their family size. The parent is accorded "consumer sovereignty."<sup>16</sup> However, fertility norms depend on a social and institutional environment that itself can be varied as a policy option--in other policy areas institutional reform is accepted as a legitimate function of government. In the discussion of birth control schemes below we examine policy in the case where fertility norms can be adjusted.

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<sup>16</sup>Equally reasonable might be an ethic which grants "consumer sovereignty" to the entire family--children as well as parents. When preferences of the children are considered the desired family-size norm is changed. The resulting optimal trajectory would also be different from the parents-sovereign case.

FIGURE 3

Optimal population trajectories under discount rates of 1%, 3%, and 5% (per capita welfare criterion).

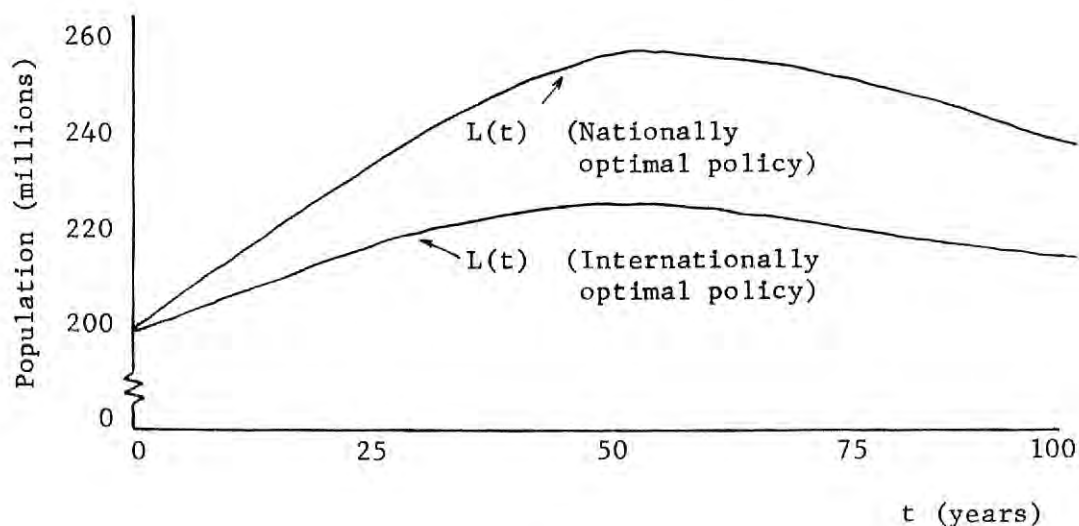


Finally, we should mention the problem of how to delimit geographically the population whose welfare is to be taken into account in policy decisions. A policy which was optimal for the inhabitants of a country may not be optimal from an international point of view. An international policy would have to consider migration and other linkage effects. Ethical problems are compounded when several nations are involved.

The other side of this problem is that an internationally imposed policy may be far from optimal from the viewpoint of single nations involved. This is similar to the question of states' rights. In Figure 4 below we suppose that a policy optimal for a group of states is imposed involuntarily on one member of the group with lower average density. The imposed group-optimal policy differs substantially from the state-optimal policy.

FIGURE 4

Nationally and internationally optimal population trajectories for a country with a population density low compared to the world average.



## SOCIAL PREFERENCES

Population policy is also of course sensitive to variations in the tradeoffs among welfare components. In a society used to relatively high-density living, for instance, crowding will have a lower disutility relative to, say, foregone births, and the optimal trajectories will reflect the difference. We should caution that numerical results must inevitably be influenced to some extent by the analytical form chosen to indicate welfare. An investigation of the significance of assumptions on social preferences is nevertheless of interest. Further research is needed to determine the degree to which findings hold across a range of different welfare functions and system models.

### Example

For illustration we will study the effect of varying social preferences in the welfare function (6). First, consider the tradeoff between material standard of living and quality of the environment. Figure 5 shows the impact of varying this tradeoff by a factor of 2 in both directions.<sup>17</sup> After 50 years, economic growth doubled the environmental degradation index, forcing an absolute decline in population in the high disutility case as the

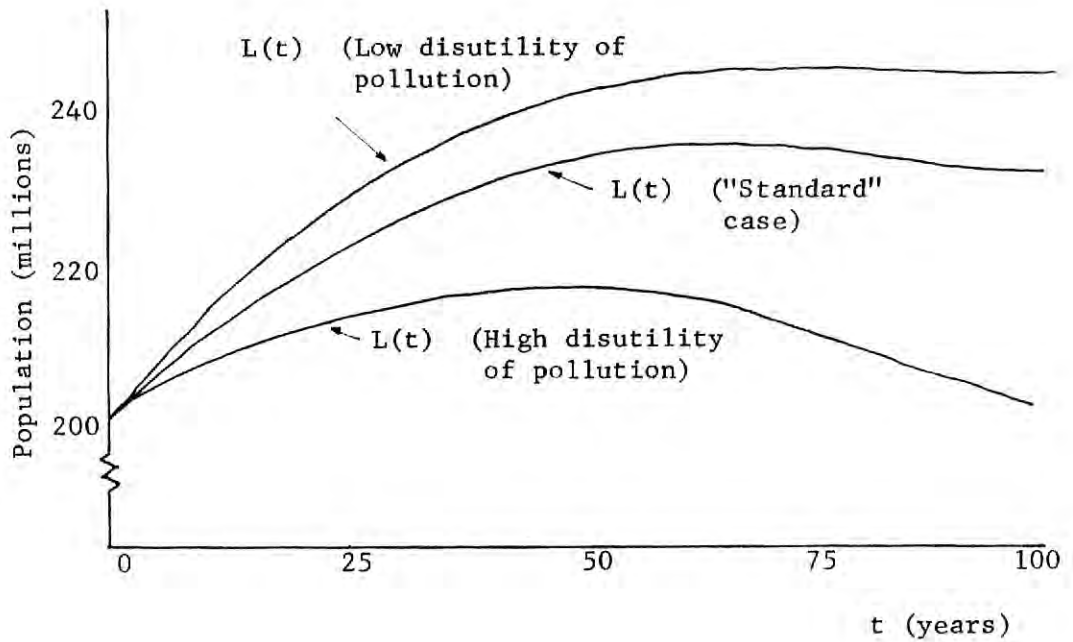
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<sup>17</sup>In the "standard" case the welfare impact of the environmental deterioration caused by a 25% real increase in GNP could be "compensated for" by a 10% rise in per capita consumption. We vary this necessary compensation to 5% and to 20%, reflecting a low and a high disutility of pollution, respectively.



FIGURE 5

Effect on optimal population of varying tradeoff between consumption and environmental quality by factor of 2 around "standard" level (used in Figure 1a).



only available means of continuing to raise per capita consumption while holding back economic growth. The difference between the two extreme population trajectories after 50 years, however, is a very modest 22 million.

As a second example, suppose we vary the tradeoff between consumption and foregone births by a factor of two around the originally specified level.<sup>18</sup> This has a larger impact on population size, giving a spread of 40 million after 50 years, as seen in Figure 6. Such a result is predictable from the strong assumption made on the adverse welfare effect of deviating from desired family size.

What is more surprising, however, is that compared with the consequences of varying the ethical bases of welfare, changes in social preferences seem relatively less important. (The disutility of crowding had even less effect on the optimal trajectory than did the environmental index--see Appendix Table A1.)

#### BIRTH CONTROL SCHEMES

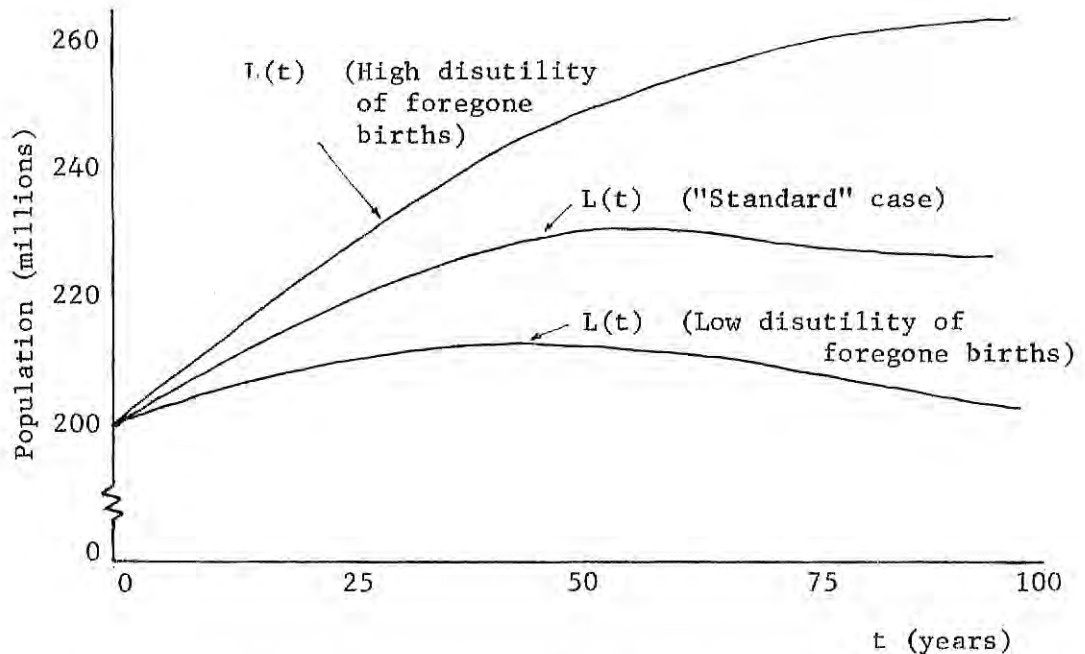
Until now the examples have assumed that the birth rate was directly controllable as a policy instrument. Most realistic population programs would require a more complicated control mechanism. This section discusses some of the major types of

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<sup>18</sup>I.e., we suppose the average person could be compensated for having one less child than actually desired by a permanent increase in income of 25% (standard case), 12½% and 50%.

FIGURE 6

Effect on optimal population of varying tradeoff between consumption and foregone births by factor of 2 around "standard" level.



program among those aimed directly at population control. Policies in other areas, such as housing or social security, which might have an important impact on population, are not examined. First, we shall describe the programs and then use the theoretical framework to compare and analyze them. For convenience, we classify control schemes into three broad categories (although recognizing that any particular scheme would usually have elements of each):<sup>19</sup>

- financial inducement;
- manipulation of attitudes and values;
- coercive schemes.

<sup>19</sup>For example, most family-planning programs are a combination of inducement and manipulation.

### Financial Inducement

Many suggested control programs rely on influencing fertility by positive or negative economic sanctions. Adjusting tax deductions allowed for dependents, paying bonuses for delaying childbearing, and making pension rates vary inversely with family size are examples. Schemes such as these would require continued annual expenditures to keep the birth rate depressed, unless a side-effect of the program was a shift in attitudes toward lower fertility.

Example. Suppose a financial inducement scheme gives a graduated tax rebate to women for each person-year without a birth. Family-size preferences are not shifted and the number of prevented births increases with the program expenditures in a given year, although the effectiveness of the scheme shows diminishing returns.

The "cost" of this program arises as follows. All payments made are transfer payments, and ignoring distribution effects there is no net consumption cost to society.<sup>20</sup> However, there still remain the psychic parental costs to those who were bribed to forego children. This is the net cost of the scheme (though borne by taxpayers at large, since the parents themselves accepted

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<sup>20</sup> Except to the extent that such transfer payments affect the savings rate in the economy (see the Enke-Demeny exchange in Economic Development and Cultural Change, 1961). We shall assume here that they do not.

compensation for their sacrifice). If on the margin people are indifferent between accepting a bribe and foregoing a child, this net cost is roughly equal to the number of births prevented multiplied by the average payment for foregoing a birth.<sup>21</sup>

Naturally, a substantial amount of empirical work would be required to determine even very roughly the birth-rate response of a particular incentive scheme. Among other complications, both economic conditions and age distribution effects would have to be allowed for. For the purposes of the example, we have chosen a simple response function, with parameter values to make it comparable with the preceding examples. The details are set out in the appendix.

Optimality, under this scheme, requires a tradeoff between the benefits of having fewer people and the cost of preventing births. As such, tradeoffs and trajectories are very similar to those in the basic model (equation (10) and Figure 1) where the growth rate could be directly manipulated under a psychic cost of foregoing children. Two typical trajectories are drawn in Figure 7.

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<sup>21</sup>The amount that must be budgeted for such a program would substantially exceed this net cost, however, since there is no means of excluding parents who accepted compensation but had not intended to have a child or who required a smaller bribe to reach that decision.

### Manipulation of Attitudes and Values

A different category of population control schemes aims to change either the social institutions related to fertility or individual attitudes and values. Like the incentive schemes discussed above they do not impose direct parental welfare costs, since such costs are by definition incurred in deviating from social norms. Monetary expenditures, however, would usually be involved--as, for example, in a propaganda campaign.<sup>22</sup> These schemes raise the ethical question mentioned above of the "sovereignty" of individual preferences. Moreover, attitudes toward fertility are not the only ones which could be changed. It is equally defensible in ethical and welfare terms to seek to make people happier at a lower rate of consumption or at higher densities and pollution levels.

Example. As an illustration we consider a case analogous to the preceding example, except that the expenditure on population control now will influence the rate of change of family-size norms. Attitudes, unlike bonuses and tax rebates, tend to persist in influencing behavior and we will assume that the new family-size norm is maintained when expenditure stops.

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<sup>22</sup>Some institutional rearrangements might be essentially costless. The manipulation of social supports to fertility to remove pro-natalist pressures, as advocated in the writings of Judith Blake (see especially 1965) would be examples.

Parameters for this example were chosen so that the scheme could be directly compared to the financial inducement example.<sup>23</sup> Details of the relationships and parameters assumed are to be found in the appendix. The resulting population trajectories for two different assumptions on program costs are seen in Figure 7. For comparison, the corresponding trajectories under the financial inducement scheme are also shown.

In this example of manipulating attitudes, the trajectory must balance tradeoffs between the cost of the program and the costs or benefits of population growth. If there were no cost of manipulating attitudes, the static optimal population would be immediately realizable. This trajectory is also shown in Figure 7. Although a larger population would increase economic growth and fulfill the desire for company in this model (the ideal density is taken to correspond to a population of 200 million), the zero-cost optimal population must fall in order to attenuate the growing environmental deterioration.

When the control scheme has a cost, the optimal policy cannot adjust attitudes instantaneously at an acceptable cost, and for this reason the manipulation scheme trajectory tends to "lag"

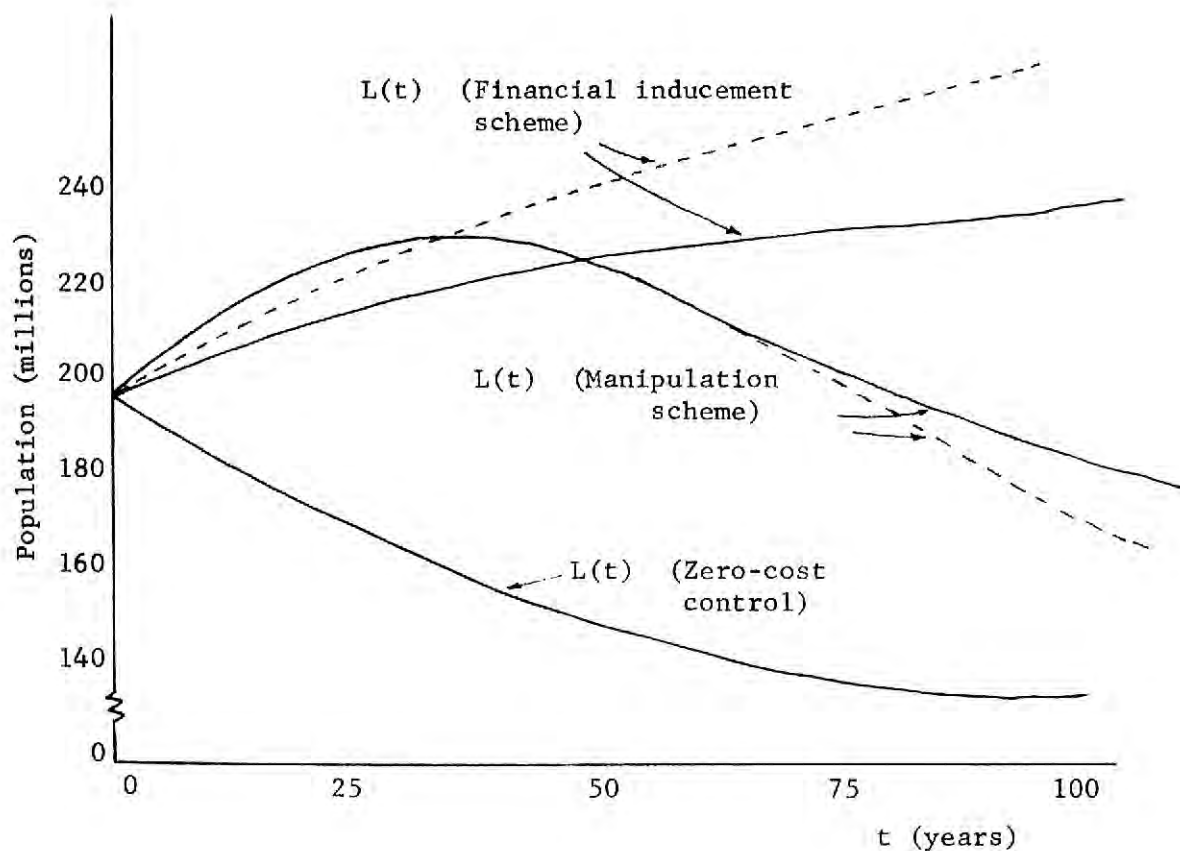
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<sup>23</sup>In the sense that a given proportion of resources allocated to reducing  $n$  from its initial value, spent in one year, would have the same impact on  $n$  in both schemes (the difference being that the impact is permanent in the present example).

the zero-cost trajectory. Manipulating attitudes could be a highly effective means of control, provided that attitudes could be changed significantly in a time period comparable to significant changes in the system.

FIGURE 7

Comparison of optimal population trajectories under alternative control assumptions: financial inducement, manipulation of attitudes, and zero-cost control.





### Coercive Programs

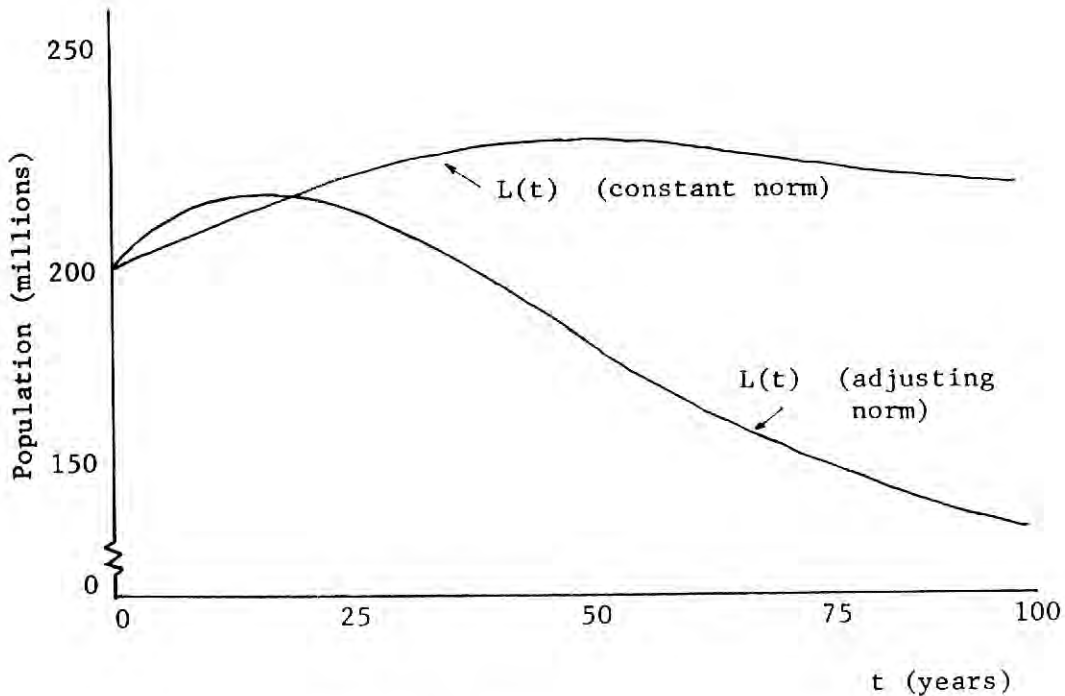
Many drastic proposals for coercive control programs are to be found in the extensive literature on population problems. Often these originate from biologists who would take an expansive view of the clients of the population-environment system. Provided that in optimizing the policy an appropriate accounting is made of the relative aversion to involuntary fertility control, the question of whether a coercive policy should ever be chosen is simply an ethical one.

Example. Suppose that a coercive policy is adopted through legislation. The cost of this type of control is incurred by parents in the form of foregone births. The basic model specified in equations (6)-(8), where the growth rate can be manipulated at a parental welfare cost, covers this situation.

However, family-size norms may adjust to the actual prevailing family size, with a suitable time lag. Figure 8 shows that the optimal trajectory under this assumption of an adjusting norm is quite different from the non-adjusting trajectory. The reason is that the fertility rate in the non-adjusting case cannot fall too far below its static norm without attributing very large psychic costs to parents. But if parents' attitudes slowly readjusted, lower optimal growth rates would be achievable.

FIGURE 8

Comparison of optimal population trajectories with constant fertility norm and with norm that adjusts (with lag) to actual fertility.



We may conclude this section by briefly reviewing the findings. Inducement and coercive schemes differ from a scheme which attempts to manipulate preferences in that an attitudinal change does not require expenditure to be maintained. Thus an attitude-manipulative control scheme can be more relaxed about population growth early in the program because growth can be fairly rapidly reduced later when effects such as pollution and crowding become dominant.

If fertility behavior itself can be directly controlled, through legislative coercion, the resulting optimal policy is quite similar to that under financial inducement. The reason is that in both of these programs similar costs are involved--those of foregoing children. However, in the coercive scheme, would-be parents have to bear these psychic costs themselves. In an inducement scheme, the parental costs are neutralized and diffused over all of society which must pay for the scheme.

Each of the trajectories we have computed is the optimal strategy under the particular control scheme specified. But the only sense in which we can say that one trajectory is "better" than another is that ethically the control scheme underlying it is preferable.

#### DISCUSSION

We have investigated the notion of optimality in population policy briefly through the use of simple analytical and numerical examples. This has perhaps obscured the generality and flexibility of the control-theoretic format in which the optimal population problem is embedded. The framework does not restrict us to a particular criterion of welfare, nor to particular assumptions about the economy or population. It is designed instead so that, given an arbitrarily specified index of societal welfare and given a set of assumptions on underlying economic and population

dynamics, the corresponding policies can be deduced that maximize over time this measure of welfare. The necessary conditions for the solution of the problem provide insights into the tradeoffs inherent in maintaining optimality and permit the complex interactions that enter even a simple specification to be analyzed relatively easily. In addition, the analysis allows us to isolate the factors and parameters to which the optimal policy in a given situation is most sensitive. We feel the absence of an explicit framework has detracted from much of the previous work on policy, in that there has been no clear means of linking welfare assumptions, demographic mechanisms, economic factors, and population policies. Even where little is known on a subject, such as population-environment interactions, it is helpful to know the impact that various assumed relationships would have on optimal policies.

#### Possible use in influencing policy formation.

Any attempt to use this approach to develop or influence actual policy obviously would call for much larger and more precisely formulated models than any used here. This would involve a very substantial research effort. There are intrinsic difficulties in the way of such a program, however.<sup>24</sup> Rather, we foresee the main use

<sup>24</sup>Basically, these are the objections, lodged by writers such as Churchman (1971), Braybrooke and Lindblom (1963), Vickers (1968), and Wildavsky (1966) to the analysis of social policy problems in quantitative terms. They stress, among other weaknesses of the approach, the likely biases in the formulation of the problem, the tendency to exclude intangibles, the inability to take account of innovative or revolutionary solutions, and the abstraction from the political process.

for analyses such as ours in the development of ad hoc theory and fairly small-scale models designed to clarify particular aspects of policy.

We reject the notion that there is any one model or welfare criterion toward which efforts such as this should converge. Analysis of particular situations by the approach we suggest may nonetheless be a powerful aid to clear-thinking on the policy issues involved. A study such as the above can demonstrate which issues really matter in policy formation, and thereby highlight areas where deeper analysis and further research would be useful.

The extremely simple models we have presented above, for example, have served to focus attention on the ethical assumptions of population policy. Whether a society has a high or a low aversion to crowding has relatively little impact on its optimal population policy compared to its valuation on the welfare of future generations and its ethical position on the Benthamite-per capita welfare spectrum.

## APPENDIX

### Welfare assumptions in basic models

Using the notation given in the text, the welfare rate is

$$U = U(C, D, E, B) .$$

The marginal utilities were assumed to take the following forms:

1. 
$$\partial U / \partial C = a_1 C^{-a_2} , \quad a_1, a_2 > 0 .$$

The elasticity parameter,  $a_2$ , reflects the degree to which people can be satiated (or alternatively, it can reflect an intertemporal egalitarian bias). For any elasticity greater than one,  $\partial U / \partial C$  approaches zero as  $C$  increases--the upper bound on the utility due to consumption corresponding to some "bliss level" of  $C$ .

2. 
$$\partial U / \partial D = b_1 - b_2 D , \quad b_1, b_2 > 0 .$$

The marginal utility of density is assumed to decrease linearly with density. Its zero value then corresponds to a static "optimum density," taken (crudely) to be invariant to levels of per capita consumption and pollution.

3. 
$$\partial U / \partial E = -c_1 , \quad c_1 > 0 .$$

The disutility of environmental degradation is assumed to vary directly with  $E$ , i.e., with GNP. Its marginal value is therefore a constant.

4.  $\partial U / \partial B = - d_1 B$  ,  $d_1 > 0$  .

The marginal disutility of deviating from desired family size increases with the deviation. (We assume that there are no reasons other than the policy measures applied for a deviation between desired and actual family size.) For analytical convenience, it is easier to choose the difference between the logarithms of desired and actual births, since this can be transformed into a difference in population growth rates.<sup>25</sup> Denoting the growth rates corresponding to the average desired and actual births by  $\bar{n}$  and  $n$  , respectively, we will measure the foregone births index  $B$  as  $n - \bar{n}$  . Then

$$\partial U / \partial n = - d_1 (n - \bar{n})$$
 ,  $d_1 > 0$  .

The utility function  $U(\cdot)$  is now specified up to a constant, which is all this analysis requires. In a more elaborate development, the various parameters that enter it, namely  $a_1$  ,  $a_2$  ,  $b_1$  ,  $b_2$  ,  $c_1$  ,  $d_1$  , would themselves be made functions of the system variables. (The various welfare components enter  $U(\cdot)$  additively; the corresponding multiplicative form could be obtained by a logarithmic transform.)

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<sup>25</sup> If mortality is low and fertility rates are not changing rapidly, the population growth rate varies with the logarithm of average family size.



The parameters of the utility function could in theory be estimated from sample interviews, with respondents being asked directly or indirectly to state their relative preferences among welfare factors. Techniques would have to be developed to eliminate the inconsistencies that would certainly arise. Alternatively, a careful study might be able to impute tradeoffs to individuals on the basis of their revealed preferences: for example, do people in fact sacrifice income to live in a less crowded area? The parameters used here are not defended on the grounds of their realism, although they do result from a very informal sampling of colleagues.

We assume that at the initial time (with a population of 200 million, natural increase of .01 -- corresponding roughly to a total fertility rate of 2.8, GNP of  $\$10^{12}$ , and per capita consumption of \$4250), the following average tradeoffs are accepted:

1. The welfare impact of a 25% increase in average density could be compensated for by a 12.5% rise in per capita consumption.
2. The welfare impact of the irremediable environmental deterioration equivalent to that caused by a 25% real increase in GNP could be compensated for by a 10% rise in per capita consumption.



3. The average person could be compensated for having one less child than actually desired by a permanent increase in income of 25%.<sup>26</sup> (This would presumably result from a substantially larger compensation for a drop from two children to one, and smaller compensation for dropping one from a desired level of 3 or higher.)

In addition, the elasticity of the marginal utility of consumption ( $a_2$ ) was put at 0.75. This would imply that a \$1000 increase in per capita consumption over a level of \$4250 has the same effect on an individual's welfare as a \$1900 increase at \$10,000.<sup>27</sup> Finally, the fertility norm was assumed to correspond to a growth rate norm ( $\bar{n}$ ) of .01.

The parameter values taken to represent these assumed tradeoffs were as follows:  $a_1 = 6.25$ ;  $a_2 = 0.75$ ;  $b_1 = 1.0 \times 10^{-6}$ ;  $b_2 = 5.0 \times 10^{-15}$ ;  $c_1 = 1.9 \times 10^{-11}$ ; and  $d_1 = 2.2 \times 10^5$ .

The optimization was carried out using a 200-year period by means of a first-order gradient algorithm.

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<sup>26</sup>With current U. S. proportions never married and married but involuntarily childless, this decrease in family size would correspond approximately to a drop in the net reproduction rate from 1.3 to 0.9.

<sup>27</sup>However, the results were found to be relatively insensitive to the value of  $a_2$  over a realistic range (0.5 - 1.5).

### Modification for a Benthamite criterion

The per capita welfare rate  $U(C,D,E,B)$  is now replaced by  $L.U(C,D,E,B)$ . The adjoint equation determining  $\lambda_L$  becomes

$$-\dot{\lambda}_L = U + L \frac{\partial U}{\partial L} + \lambda_L \frac{\partial \dot{L}}{\partial L} + \lambda_K \frac{\partial \dot{K}}{\partial K},$$

and a base level of  $U$  thus has to be specified. We have taken this value,  $U_0$ , as the utility rate implied by the initial level of consumption, \$4250, which gives approximately  $U_0 = 200$ .

The resulting optimal population trajectory and associated value of a marginal birth is plotted in Figure 2 for the first half of the 200 year period.

### States' rights case

A simple means of illustrating the significance of choice of geographical unit is the following. We take the optimal population trajectory implied by the "standard" level of aversion to crowding as representing the internationally optimal policy in the country in question. Then the corresponding nationally optimal policy is given by the same model with the same aversion to crowding in relation to the static "optimum density," but a lower level of this optimum.

The static optimum density is determined by  $\partial U / \partial D = 0$ , i.e.,  $D = b_1 / b_2$ . The values chosen were:

Internationally optimal case -  $b_1 / b_2 = 200$  million

Nationally optimal case -  $b_1 / b_2 = 150$  million.

### Policy sensitivity to welfare tradeoffs

Using the original model with the per capita welfare rate  $U(C,D,E,B)$  and a time horizon of 200 years, the sensitivity of the optimal population trajectory to variations in the tradeoffs among the arguments of  $U$  was examined. The results are summarized in Table A1 below and in Figures 5 and 6.

Table A1

Percentage effect on optimal population size of varying welfare tradeoffs by factor of two in standard model\*

Tradeoff	Percent effect on L after		
	25 years	50 years	100 years
Crowding vs. per capita consumption	2	4	5
Environmental deterioration vs. per capita consumption	3	5	11
Foregone births vs. per capita consumption	4	9	15

\* Average of percentage changes in L resulting from doubling and halving standard tradeoffs, disregarding signs.

### Control by financial inducement

The relationship between program cost and effect on births assumed for the example was:

$$g = k_1 \log \frac{\bar{n} + k_2}{n + k_2}, \quad (A1)$$

where  $g$  is the "net cost" of the program (as defined in the text) as a proportion of GNP, and  $k_1, k_2$  are positive constants. In the feasible range of  $g$ , (A1) has the desired properties that  $dn/dg < 0$  and  $d^2n/dg^2 < 0$ .

Since the disutility of foregone births is fully compensated for, the argument  $B$  drops out of the welfare function. However, the total compensation paid,  $gF$ , must be deducted from the consumption stream in computing the utility due to consumption. We would otherwise be double counting and effectively getting a free control program. Per capita consumption is therefore

$$C = (1 - s - g)F/L .$$

It is of course artificial to assume that  $g$  is a policy instrument (through  $n$ ) but  $s$  is not. However the purpose here is to contrast various different birth control mechanisms and a more realistic model would cloud these comparisons.

To make this case formally similar to the coercive control example described earlier, the value of  $k_1$  was chosen so that, in the initial state, the consumption cost of an incentive program sufficient to reduce  $n$  from .01 to 0 equaled in utility terms the psychic cost of foregone births of the same reduction in  $n$  in the coercion case. The parameter  $k_2$  was taken to be the assumed death rate of the population (.008). The two trajectories drawn in Figure 7 are based on "standard" and "low" propensities to forego births, corresponding to the "standard" and "high" disutilities of foregone births in the coercion case of Figure 6. The values of  $k_1$  were 0.39 and 0.62, respectively.

### Control by manipulation of attitudes

This control policy was modeled by assuming that program expenditures influenced the population growth rate,  $\dot{n}$ , instead of  $n$  as in the inducement case. The relation specified was

$$g = -k_1 \frac{\dot{n}}{n + k_2} \quad (A2)$$

where  $k_1$  and  $k_2$  are again positive constants. As before,  $k_2$  is interpreted as the population death rate, so that  $n + k_2$  is always positive. There are diminishing returns to raising  $g$ : as  $n$  gets smaller, the level of expenditure required to maintain a given rate of decline in  $n$  increases. The population growth rate here corresponds to the growth rate norm  $\bar{n}$  in the inducement case, although it is now of course not a constant.

This system requires the state variable  $n$  in addition to  $L$  and  $K$ . The dynamic equations are

$$\begin{aligned}\dot{L} &= nL \\ \dot{K} &= sF(K,L) - \delta K \\ \dot{n} &= n(g,n) ,\end{aligned}$$

where the last of these is the inverse of (A2).

Two values of  $k_1$  were specified so that the impact on  $n$  of an expenditure of 5% of GNP in the first year was the same under this program as in the corresponding financial inducement case. In the second and subsequent years, of course, the costs of the two programs for given changes in  $n$  would necessarily diverge. The levels used were  $k_1 = 0.43$  for the standard case and  $k_1 = 0.60$  for the attitude manipulation program initially equivalent to an inducement scheme with a low propensity to forego births.

In addition to these two trajectories, the system was solved for the special case  $k_1 = 0$ , i.e., where there is no cost of population control. This trajectory is also plotted in Figure 7.

Coercion with an adjusting family-size norm

Suppose we modify the original "coercion" model by letting the population growth rate norm  $\bar{n}$  track the actual rate  $n$  according to the distributed lag formula

$$\bar{n}(t) = \bar{n}(t - \tau) + v[n(t - \tau) - \bar{n}(t - \tau)] .$$

This assumes that the norm follows  $n$  with a geometric lag of period  $\tau$ , the weighting coefficients decreasing in a geometric series with ratio  $(1 - v)$ .<sup>28</sup>

Taking  $\tau = 2$  years and  $v = 0.1$ , the resulting optimal trajectory of  $L$  is given in Figure 8.

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<sup>28</sup>Written in full, the expression for  $\bar{n}(t)$  is

$$\bar{n}(t) = v[n(t - \tau) + (1 - v)n(t - 2\tau) + (1 - v)^2 n(t - 3\tau) + \dots] .$$

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## ABSTRACT

This paper investigates the notion of optimality in population policy through the use of simple analytical and numerical examples. A control theoretic framework is adopted which does not restrict the analysis to a particular criterion of welfare nor to particular assumptions about the economy or population. Given an arbitrarily specified index of societal welfare and a set of assumptions on underlying economic and population dynamics, the corresponding policies can be deduced that maximize over time this measure of welfare. The necessary conditions for the solution of the problem provide insights into the tradeoffs inherent in maintaining optimality. In addition, the analysis allows the isolation of the factors and parameters to which the optimal policy in a given situation is most sensitive.

The very simple models discussed in the paper serve to stress the importance of the ethical assumptions of population policy. They suggest that whether a society has a high or a low aversion to crowding or environmental degradation has relatively little impact on its optimal population policy compared to its valuation on the welfare of future generations and of potential entrants to the society.

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