THE UNIVERSITY OF SYDNEY

SCHOOL OF CIVIL ENGINEERING

THE SHIFT OF THE EFFECTIVE CENTROID IN PLAIN CHANNEL SECTION COLUMNS

RESEARCH REPORT R966

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November 2020

ISSN 1833-2781

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School of Civil Engineering, Research Report R966
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November 2020

ISSN 1833-2781

Published by:
School of Civil Engineering
The University of Sydney
Sydney NSW 2006
Australia

ABSTRACT

The report derives an equation for the shift of the effective centroid that occurs in plain channel section columns as a result of local buckling. The shift causes bending when the column is compressed between pinned ends and may need to be accounted for in design. The equation is based on Stowell's work on simply supported plate elements with a free longitudinal edge. It is shown to be in reasonable agreement with eccentricities calculated using results of geometric nonlinear shell finite element analyses of plain channel columns. It is also shown to provide more accurate values of eccentricity than the effective width method, in which the effective width of each plate element is calculated to produce an effective cross-section and an associated effective centroid.

KEYWORDS

Plain channel section; column; local buckling; effective centroid

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THE SHIFT OF THE EFFECTIVE CENTROID IN PLAIN CHANNEL SECTION COLUMNS

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1. Introduction

This report presents the derivation and assessment of a simplified equation to estimate the shift in the centroidal axis *e* due to local buckling in plain channel sections (C-sections) when subjected to compression. This expression is necessary in the design of C-section columns when the Direct Strength Method approach is adopted, since the DSM does not provide a direct estimation of the load eccentricity to be considered in design (as opposed to the Effective Width Method approach, which inherently provides a measure of the shift in the centroidal axis).

Local buckling causes a redistribution of the internal stress which leads to a shift of the line of action of the resultant force unless the cross-section is doubly-symmetric. In singly-symmetric sections, the shift occurs along the principal axis of symmetry. The effect of the shift of the effective centroid may be severe if the cross-section derives most of its flexural strength from unstiffened plate elements as in plain channel section columns and plain angle section columns, and may need to be considered in the design of these section, whereas for other sections with lip-stiffened flange elements, the shift is considerably less significant.

If the column is compressed between pinned ends, the shift in the effective centroid causes overall bending, whereas if the ends are fixed against rotation, restraining moments develop at the ends which negate the effect of the shift of the effective centroid [1]. In the intermediate situation where the ends are elastically restrained against rotation, it has been shown that only a modest rotational stiffness is required to effectively negate the effect of the shift in the effective centroid [1].

If follows that for design purposes, the shift in the effective centroid needs only be considered for plain channel and plain angle section columns that are pin-ended at both ends. This situation may occur when the column is bolted through the centroid and restraining moments can be assumed not to develop at the ends.

An equation for the shift in the effective centroid of equal and unequal plain angle section columns is derived in [2]. A similar derivation is presented in this report for plain channel section columns.

A summary of the derived equation is first presented in a form useful for design guidelines and specifications. This is followed by a detailed explanation of the assumptions adopted in the derivation. Then, finite element models are developed for the validation of the derived equation and finally comparisons are made between the finite element predictions and the shift of the effective centroid calculated from the derived equation and determined using the Effective Width Method.

2. Summary of derivation

The shift in the centroidal axis *e* due to local buckling for plain channel sections to be adopted in C-section column checks when the Direct Strength Method approach is adopted can be estimated from:

$$\text{For } B > D/3 \text{ and } \lambda_{py} > 1.0 \quad e = \frac{5}{32}\left(1 - \frac{1}{\lambda_{py}}\right)B \qquad \text{Eq. 1}$$

$$\text{Otherwise} \quad e = 0$$

where,

B: overall flange width of the plain channel section
D: overall web height of the plain channel section
λ_{py}: slenderness of the flange element, calculated from Eq. 2 and Eq. 3,

$$\lambda_{py} = \sqrt{\frac{f_y}{\sigma_{cr}}} \qquad \text{Eq. 2}$$

$$\sigma_{cr} = \frac{k_f \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{B}\right)^2 \qquad \text{Eq. 3}$$

where,

f_y: yield stress
E: Young's modulus
ν: Poisson's ratio
t: thickness of the plain channel section
σ_{cr}: critical buckling stress of flange element
k_f: buckling factor, $k_f = 0.425$ for outstand elements in pure compression

3. Shift of effective centroid

This Section presents the detailed explanation of the assumptions adopted in the derivation of an equation for the shift in the centroidal axis *e* due to local buckling in plain channel sections. The idealization of the plain C-section and the definition of the cross-section parameters are given in Figure 1.

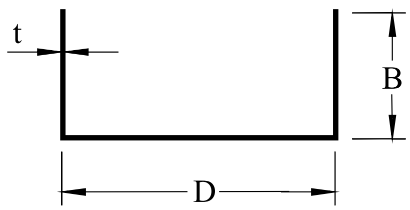

Figure 1. Idealization of plain channel cross-sections and parameter definition.

From Appendix II in Research Report R830 [2], the shift *s* of the effective centroid (Fig. 2) of a flange element at the ultimate (average) stress ($\sigma_u = \sigma_{av}$) can be determined as,

$$s = \frac{5}{16}\left(1 - \frac{\sigma_{cr}}{f_y}\frac{f_y}{\sigma_u}\right)B \qquad \text{Eq. 4}$$

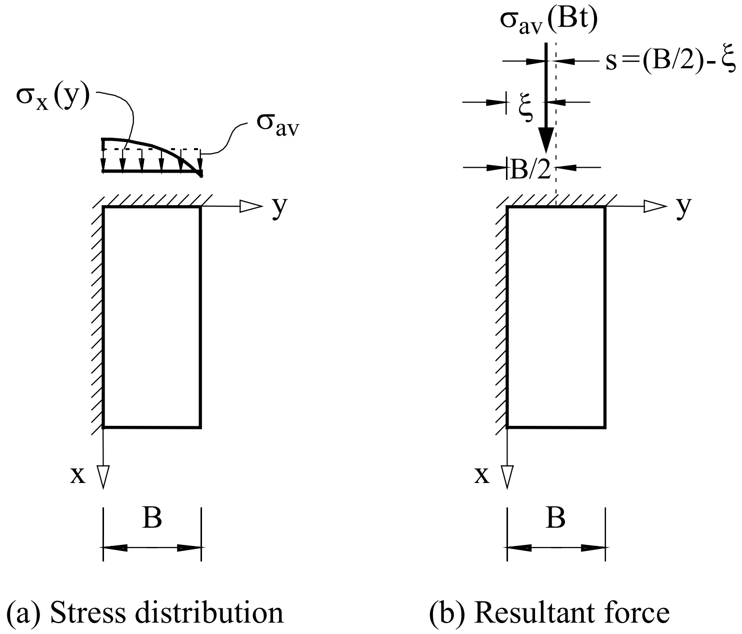

Figure 2. Shift of the effective centroid of a flange element [2].

If the von Karman's strength equation given in Eq. 5 is used to determine the ultimate stress (σ_u), and noting that the relation for the critical stress is given by Eq. 6,

$$\frac{\sigma_{av}}{f_y} = \frac{\sigma_u}{f_y} = \frac{1}{\lambda_{py}} \qquad \text{Eq. 5}$$

$$\frac{\sigma_{cr}}{f_y} = \frac{1}{\lambda_{py}^2} \qquad \text{Eq. 6}$$

then the shift *s* of the effective centroid of a flange element can be obtained as,

$$s = \frac{5}{16}\left(1 - \frac{\sigma_{cr}}{\sigma_y}\frac{1}{\frac{\sigma_{av}}{\sigma_y}}\right)B = \frac{5}{16}\left(1 - \frac{1}{\lambda_{py}^2}\frac{1}{\frac{1}{\lambda_{py}}}\right)B = \frac{5}{16}\left(1 - \frac{1}{\lambda_{py}}\right)B \quad \text{for} \quad \lambda_{py} > 1.0 \qquad \text{Eq. 7}$$

where

$$\lambda_{py} = \sqrt{\frac{f_y}{\sigma_{cr}}} \qquad \text{Eq. 8}$$

$$\sigma_{cr} = \frac{k_f \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{B}\right)^2 \qquad \text{Eq. 9}$$

We proceed to consider the full cross-section of a plain channel.

The assumption is made that the largest shift of the effective centroid will happen when local buckling does not occur in the web (since local buckling of the web would push the effective centroid towards the tip of the flanges, rather than towards the web). This implies that $B > D/3$, as per Eq. 10 to Eq. 12.

$$\sigma_{cr,f} = \frac{k_f \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{B}\right)^2 < \sigma_{cr,w} = \frac{k_w \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{D}\right)^2 \qquad \text{Eq. 10}$$

$$\left(\frac{D}{B}\right)^2 < \frac{k_w}{k_f} = \frac{4}{0.425} = 9.41 \qquad \text{Eq. 11}$$

$$\frac{D}{B} < 3.07 \approx 3.0 \text{ or } B > D/3 \qquad \text{Eq. 12}$$

If the web does not locally buckle, the stress distribution can be assumed to be as shown in Figure 3, where C represents the centroid of the gross-section, C_e is the centroid of the effective section and σ_{av} is the average compression stress in the cross-section:

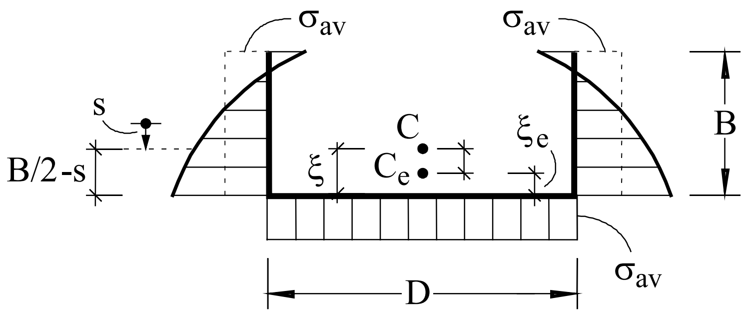

Figure 3. Stress distribution assumed for the estimation of the shift of the effective centroid.

The implied discontinuity in stress at the flange-web junctions is accepted, which would not occur in reality and is a consequence of ignoring local buckling deformations in the web combined with assuming a pinned condition for the flange at the web junction.

The distances from the web to the gross section centroid (ξ) and effective centroid (ξ_e), as defined in Figure 3, are calculated as follows:

$$\xi(2B + D)t = 2B\frac{B}{2}t \quad \rightarrow \quad \xi = \frac{B^2}{2B+D} \qquad \text{Eq. 13}$$

$$\xi_e(2B + D)t = 2B\left(\frac{B}{2} - s\right)t \quad \rightarrow \quad \xi_e = \frac{B^2 - 2Bs}{2B+D} \qquad \text{Eq. 14}$$

The shift of the effective centroid *e* can now be calculated as:

$$e = \xi - \xi_e = \frac{1}{2B + D}[B^2 - (B^2 - 2Bs)] = \frac{s}{1 + \frac{D}{2B}} \qquad \text{Eq. 15}$$

which shows that the shift of the effective centroid *e* decreases with increasing D.

Most commercial sections have D/B ratios in the range between 2 and 3. If D=2B is used as a reasonable lower bound, then the maximum expected eccentricity is:

$$e = \frac{s}{1 + \frac{2B}{2B}} = \frac{s}{2} \qquad \text{Eq. 16}$$

Using Eq. 7, the shift of the effective centroid is obtained as,

$$e = \frac{5}{32}\left(1 - \frac{1}{\lambda_{py}}\right)B \quad \text{for} \quad \lambda_{py} > 1.0 \qquad \text{Eq. 4}$$

This equation is calculated for an average stress equal to the ultimate stress of the flange element. It assumes local buckling of the web is not critical, i.e. $B > D/3$. For $B < D/3$ or $\lambda_{py} < 1.0$, *e* should be taken as zero.

4. Finite element models and results

4.1. Finite element model description

For the assessment of the proposed expression finite element models have been developed using ABAQUS [3]. Plain channel sections (C-sections) were modelled using S4R elements and elastic-perfectly-plastic material properties defined through the Young's modulus (E=210 GPa) and the yield stress f_y have been assigned to the specimens. Table 1 summarizes the web height D, flange width B, section thickness t and yield stress f_y values considered in the parametric study. These sections represent most of the plain channel sections commercially available in terms of D/B-ratios and cross-section slendernesses. The study also considered relatively low and high yield stress f_y values, covering the range of f_y commonly found in cold-formed steel sections. The length of each sample was defined as the corresponding local buckling half-wavelength L_{cs}, which was estimated using CUFSM [4].

Table 1. Summary of parameter values considered in the study.

D [mm]	B [mm]	t [mm]	f_y [MPa]
100	25	1.53, 1.02, 0.76	300, 500
100	30	2.03, 1.36, 1.02	300, 500
100	40	2.45, 1.63, 1.23	300, 500
100	50	3.05, 2.03, 1.53	300, 500

Simply-supported end conditions were defined following the recommendations in [5] to ensure uniform longitudinal stress distribution throughout the length of the member, as assumed in the finite strip formulation. The degree of freedom corresponding to the longitudinal displacement of one of the end sections was kinematically coupled to a reference point located at the mid-height of the web, to which the longitudinal displacement was restrained. This allowed the direct extraction of the load P and moment M reactions for the calculation of the position of the centroid at each loading step, being equal to the load eccentricity M/P.

Geometrically and materially nonlinear FE analyses were carried out using the modified Riks method. These analyses included initial imperfections introduced in the form of elastic buckling mode shapes obtained from prior buckling analyses and with amplitudes equal to B/200.

4.2. FE results

The failure stress of each specimen was defined as the maximum average stress σ_u, and the position of the centroid of the effective cross-sections at failure was computed from the corresponding load P_u (= σ_u A) and moment M_u reactions, $\xi_{e,FE}$=M_u/P_u. The shift in the centroidal axis e_{FE} due to local buckling was then calculated as the difference between the predicted position of the centroid at failure and the position of the centroid while the section was fully effective (i.e. at the first loading step), $e_{FE} = \xi - \xi_e$. Figure 4 shows a typical stress distribution at failure for the specimen C100-40-1.23 for f_y=300 MPa, while Figure 5 presents the average stress *vs* average strain curve for the same specimen.

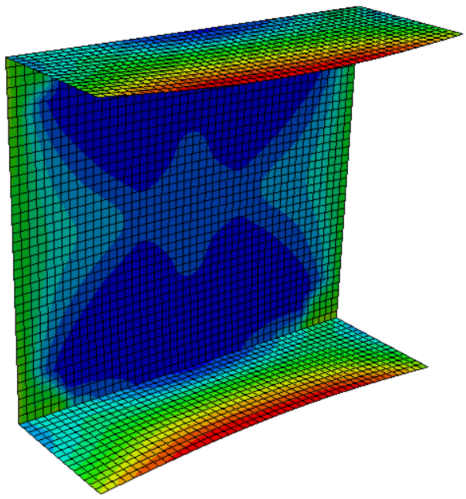

Figure 4. Typical stress distribution at failure for specimen C100-40-1.23 for f_y=300MPa.

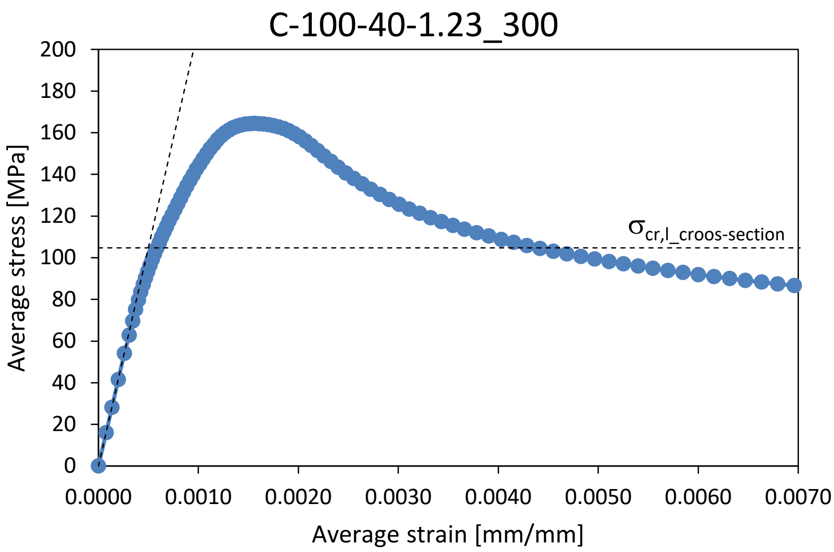

Figure 5. Average stress-average strain curve predicted for specimen C100-40-1.23 for f_y=300MPa.

5. Assessment of eccentricity equation

The equation proposed to estimate the shift of the effective centroid of plain channel cross-sections is assessed herein against the numerical results obtained from the FE simulations.

The shift of the effective centroid obtained from the FE simulations at the ultimate stress (maximum average stress) e_{FE} is compared with the shifts predicted by the proposal developed in this study e_{prop}, which is given by Eq. 17. The results are compared in Table 2 for the 24 C-sections investigated, which consider a range of D/B ratios, flange slenderness and yield stress values. In addition, the shifts of the effective centroids of the same sections predicted by the Effective Width Method e_{EWM} using the equations provided in ASCE-8 [6] are also reported in Table 2 for comparison purposes.

Table 2. Assessment of the proposal to estimate the shift of the effective centroid in C-sections.

f_y [MPa]	D [mm]	B [mm]	t [mm]	λ_{py}	e_{FE} [mm]	e_{prop} [mm]	e_{EWM} [mm]
300	100	25	1.53	1.00	-0.6	0.0	-0.6
300	100	25	1.02	1.50	-0.2	0.0	0.6
300	100	25	0.76	2.00	0.3	0.0	1.3
300	100	33	2.03	1.00	-0.1	0.0	-0.1
300	100	33	1.36	1.50	0.6	1.7	1.7
300	100	33	1.02	2.00	1.5	2.6	2.8
300	100	40	2.45	1.00	0.2	0.0	0.4
300	100	40	1.63	1.50	1.4	2.1	2.7
300	100	40	1.23	1.99	3.3	3.1	4.1
300	100	50	3.05	1.00	0.3	0.0	0.9
300	100	50	2.03	1.50	2.5	2.6	4.4
300	100	50	1.53	2.00	4.3	3.9	6.3
500	100	25	1.53	1.29	-0.6	0.0	0.0
500	100	25	1.02	1.94	0.1	0.0	1.1
500	100	25	0.76	2.58	0.6	0.0	1.8
500	100	33	2.03	1.29	0.1	1.2	0.8
500	100	33	1.36	1.94	1.3	2.5	2.5
500	100	33	1.02	2.58	1.9	3.2	3.4
500	100	40	2.45	1.29	0.6	1.4	1.6
500	100	40	1.63	1.93	2.4	3.0	3.7
500	100	40	1.23	2.57	4.2	3.8	4.9
500	100	50	3.05	1.29	1.2	1.8	2.8
500	100	50	2.03	1.94	4.1	3.8	5.8
500	100	50	1.53	2.58	5.4	4.8	7.3

Note: Positive *e* indicates that the centroid shifts towards the web.

The results shown in Table 2 suggest that Eq. 17 provides a reasonably accurate prediction of the shift of the effective centroid of plain channel cross-sections and can be used for the design of plain C-section columns with slender flanges. It produces considerably more accurate results than the Effective Width Method, especially for plain channels with high-B/D ratios. The derived equation is especially useful for determining the eccentricity when the column is designed using the Direct Strength Method.

6. References

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