WORKING PAPER
ITLS-WP-21-13
A Framework to Model and Compare
Rest Break Policies in Logistics Industry
By
Saman Eskandarzadeh and Behnam
Fahimnia
Institute of Transport and Logistics Studies (ITLS),
The University of Sydney, Australia
July 2021
ISSN 1832-570x

## INSTITUTE of TRANSPORT and LOGISTICS STUDIES <br> The Australian Key Centre in <br> Transport and Logistics Management <br> The University of Sydney

## TITLE:

## A Framework to Model and Compare Rest Break Policies in Logistics Industry

## ABSTRACT:

## AUTHORS:

## CONTACT:

INSTITUTE OF TRANSPORT AND LOGISTICS STUDIES (H04)

The Australian Key Centre in Transport and Logistics Management

The University of Sydney NSW 2006 Australia
Telephone: +6129114 1813
E-mail: business.itlsinfo@sydney.edu.au
Internet: http://sydney.edu.au/business/itls

DATE:

## 1 Introduction and Background

Carriers and postal companies have been under increasing pressure to reduce prices and increase their service levels, often measured in form of delivery times ( $\mathrm{PwC}(2016)$, Briest et al. (2019)). They also feel obligated to their customers and the next generations to reduce their carbon footprint 1 by adopting greener strategies (Fahimnia et al. (2015)). One approach to tackle this pressure is to become more productive and cost-efficient through efficient planning and optimisation of pickup and delivery operations (Briest et al. (2019)). The primary service of carriers is to collect customer products (mails or parcels) and deliver them to given destinations/customers. If the volume of the products in one shipment is much less than a truckload, direct shipping may be too cost inefficient; in which case, the products destined to the same region are consolidated to better utilise the economies of scale.

Large carriers have multiple consolidation centres allowing them to coordinate the flow of the freight between these centres and between the centres and the customers in order to reduce costs and maintain/improve service levels. An effective robust scheduling is essential to plan the resourcing and timing of the pickup and delivery jobs. This is a challenging scheduling problem, the classic form of which is known as Pickup and Delivery Problem with Time Windows (PDPTW).

A restricting factor in a PDPTW problem is the planning of the rest breaks. Most countries have certain rest break regulations for truck drivers. Australia (NHVR (2020)), Canada (Justice Laws (2020)), European Union (EU) countries (Europa 1 (2020); Europa 2 (2020)), and US (Federal Register (2020)) have applied strict rest break regulations to reduce the risk of accidents due to prolonged hours of work with insufficient rest. The rest break regulations in Australia are quite unique with less structural resemblance to that of the rest of the world. There are multiple regulatory frameworks for rest breaks, but the most commonly used framework is based on the standard hours regulations for solo truck drivers. We refer to this as SH framework/rules $\mathbb{T}^{2}$. Our focus in this paper is on scheduling short Hours of Service (HOS) (i.e., less than 13 hours service per day) which is the common practice in major cities and many regional areas. According to the SH framework, for short HOS, each truck driver cannot work continuously more than $5: 15,7: 30$, and 10 hours without taking break(s) of at least 15,30 and 60 minutes, respectively (in 15 -minute resting blocks).

Truck drivers in cities and regional areas complete multiple pickup and delivery jobs in every shift. Finding an optimal schedule with respect to time windows for pickup and delivery while also considering the SH rules is a formidable challenge. The existing methodologies either lack the flexibility to easily accommodate carrier specific preferences or fail to utilise the full flexibility provided by SH rules; hence, not quite practical - especially for short HOS.

To illustrate the significance of break rules in terms of their cost, we provide an example.
Example 1. There are three customers to be visited by a truck driver for daily product collection. The collected products are delivered to the depot. Service time at each stop is 20 minutes. Customers 1, 2, and 3 have to be visited within time windows [100, 260], [ 0,1440$]$, and[100, 420], respectively; where $[a, b]$ denotes the time window with earliest start time $a$ and latest start time $b$ for the visit. Times $a$ and $b$ are measured in minutes passed from 6:00 AM. The distance matrix (in minutes) between customer locations and the depot (with index 0 ) is as follows:

$$
D=\left[\begin{array}{cccc}
0 & 96 & 112 & 136 \\
96 & 0 & 120 & 144 \\
112 & 120 & 0 & 232 \\
136 & 144 & 232 & 0
\end{array}\right]
$$

Without breaks, the optimal sequence of visits is 0-2-1-3-0 with a total duration of 592 minutes. When breaks are taken into consideration, this sequence is not feasible anymore. The reason is that it takes 416 minutes for a truck to get to Customer 3 and since, according to SH rules, a break of 15 minutes is required before servicing Customer 3, the truck can only start servicing Customer 3 at 431 minutes at earliest, which is outside the acceptable time window for Customer 3. Under the SH framework, an optimal tour is $0-1-3-2-0$ with a duration of 724 minutes with two 30 -minute breaks at Customer 2 and

[^0]Customer 3. So, if the break times are excluded, the work time is increased from 592 minutes to 663 minutes from the first optimal sequence to the second optimal sequence. This is a significant increase of $12 \%$ in total work time.

Our research was motivated by an optimisation/scheduling problem facing a major postal carrier in Australia. Typical daily operations of the proposed company are as follows. Parcels and mail are collected from collection facilities and bulk customers. They are then shipped to middle sortation facilities and subsequently to delivery centres. From delivery centres, they are carried to destination points. The shipping between bulk customers and facilities is mainly handled by trucks. The aim is to maximise the truck utilisation. We study this problem for a single vehicle and single driver operating under the Australian HOS regulations. We are given a set of pickup and delivery jobs where each job requires some parcels to be picked up from one location and delivered to another location. The pickup as well as the delivery should be done within pre-specified time windows. Consistent with practice, demand is measured in cubic meters. For heavy items, more space is allocated in the truck to keep the load balance of the truck. Therefore, weight can be translated into cubic meters. We aim to schedule all jobs in a single tour while respecting the SH break rules.

The SH rules do not require the drivers to return to the depot for taking breaks. However, some logistic providers, including the postal company that motivated this study, schedule the tours in a way that the drivers end up taking their breaks in the depot which is fully equipped for rest taking and also allows the drivers to socialise. In our analysis, we also provide a cost comparison of the two policies.

The problem will be formulated using Mixed-Integer Programming (MIP) which is known to be a suitable methodology for tactical and operational decision-making problems in logistics and supply chain management. This is evidenced by numerous commercial solvers and academic papers published on both theory and applications (Dong et al. (2020); Zhen et al. (2020); Li et al. (2020); Schiffer et al. (2019)). We develop an exact MIP model to solve a single-vehicle PDPTW problem under SH rules. We model the problem under SH rules with (scenario 1) and without (scenario 1) restriction for break location. A "unified" methodology will then be developed to compare the two scenarios in terms of the tour length. We refer to this as a unified approach/methodology since both scenarios are modelled in the same fashion. Finally, we use the models and the methodology on a real dataset provided by a major postal carrier.

Our contribution to the literature of TDSP is threefold. (1) We pioneer the development of an MIP model that allows for a flexible break location (i.e., a break may take place between tasks at any location, not just at the depot). The model is tractable by existing commercial optimisation packages. MIP models are much easier to use or extend compared to other exact approaches such as dynamic programming which require customisation and time-consuming implementation. For dynamic programming, no generic solver exists in the market; while there are numerous commercial and open-source packages to solve MIP models. (2) We develop a unified methodology for modelling various restrictions on break locations. (3) We use real data to compare rest break practices using the proposed models and methodology.

The rest of the paper is organised as follows. Section 2 presents a review of the literature on rest break optimisation under hours-of-service constraints and related policies. A formal description of the problem under investigation is presented in Section 3. Section 4 presents a set of necessary and sufficient conditions for tour feasibility under SH rules. These conditions will then enable us to develop an exact MIP model that can be customised to formulate and evaluate different policies. Section 5 compares the impact of two rest break scenarios on a real dataset obtained from a major postal carrier in Australia. We also estimate the price of each scenario compared to a situation with no rest break. Finally, Section 6 presents a summary of the key findings as well as directions for future work in this domain.

## 2 Literature review

There are two streams of research relevant to this research. The first stream considers HOS regulations in scheduling of truck deliveries for given time windows. The classic problem in this stream is the so-called Truck Driver Scheduling Problem (TDSP). In a TDSP problem, there is a single vehicle, and the sequence of delivery tasks is given. Therefore, the sequencing of the tasks, which affects the total driving times, is not a decision variable. In this stream of research, there are also problems in which TDSP is integrated with Vehicle Routing Problem (VRP). The focus is more on developing solution methods or ideas to help improve operational scheduling/planning. The second stream of research, on the other hand, aims to study TDSP or its variants from tactical planning and/or policymaking perspectives.

Our review of the first stream of literature starts by the work of Xu et al. (2003), the first study to integrate TDSP with Pickup and Delivery Problem under HOS regulations in USA. A column generation based heuristic algorithm is presented as a solution method. Archetti and Savelsbergh (2009) study TDSP under USA's HOS regulations and propose a polynomial time algorithm that either finds a feasible solution for the problem with minimum total rest time or establishes infeasibility. In their setting, time windows are only defined for pickup tasks.

Goel (2009) and Ceselli et al. (2009) were the first to study TDSP under EU HOS regulations. Both papers solve the TDSP integrated with routing decisions using heuristic algorithms. The problem setting in Ceselli et al. (2009) was used in a decision support system of a transport company in Italy. Similar to our setting, they consider a short less-than-one-day time horizon. Goel (2010) develops the first exact algorithm for TDSP under EU regulations. Kok et al. (2010) and Prescott-Gagnon et al. (2010) develop heuristic algorithms for integrated TDSP with routing under EU HOS regulations.

Goel (2012b) is the closest paper to our work and the first study that suggests an MIP model for the TDSP under Australian HOS regulations. Our work differs from the work of Goel in four dimensions. (1) We relax the assumption that the sequence of tasks is known; therefore, we also incorporate routing decisions into our model. (2) We schedule tasks for a single tour for short HOS, less than 13 hours. In our application in Sydney Metropolitan area, the maximum shift time for drivers is 12 hours which seems to be a standard characteristic of city transportation jobs around the globe. In Goel's model the focus is on a longer time horizon over one week which usually applies to intercity transport. (3) Our primary aim is to develop a framework for evaluation of HOS regulations in practice, not a methodology for generating schedules. Nevertheless, we show that the MIP models presented in this paper perform very well on real data and, in most instances, generate optimal tours in less than one minute. (4) We also relax some of the other assumptions that may not replicate the reality. Specifically, Goel (2012b) assumes that rest breaks can only be taken immediately after arrival at a location and before starting to work at that location. It also assumes all time values are a multiple of 15 minutes. These two critical assumptions make the study and the resulting models too restrictive when compared to real scenarios. In our paper, we relax these two restrictions.

In another study, Goel et al. (2012) investigate TDSP under Australian HOS regulations using a dynamic programming approach. An exact dynamic programming algorithm and a set of heuristics are presented to find a feasible solution. If no feasible solution exists, the algorithm reports infeasibility. Although they do not restrict the resting locations, they model all time values in multiples of 15 minutes. In the same year, Goel (2012a) introduces a generic MIP and dynamic programming approach for solving TDSP under EU and U.S. regulations. The MIP model imposes restriction on rest break locations, but the dynamic programming relaxes this assumption. Goel and Rousseau (2012) introduce an approach for solving TDSP under Canadian regulations. It presents an exact algorithm for either finding a feasible solution or proving infeasibility.

The second stream of research is not as mature and established. Goel and Vidal (2014) propose a metaheuristic to compare HOS regulations in EU, Canada, Australia and the U.S. in terms of accident risk and operating costs, considering total distance and fleet sizes. They compare these regulations on instances with 100 customers for a planning horizon of 144 hours. The average time window in their study is 7 hours which is quite large compared to urban services where the average time window is less than an hour. For express post, the time windows are even shorter. They use a modified version of the heuristic algorithm introduced by Goel et al. (2012) for assessing compliance with the Australian regulations. The original algorithm assumes all time values in multiple of 15 minutes which restricts the application of this approach. This assumption is rather relaxed in the modified algorithm. However, the start and end times of all off-duty periods are still multiples of 15 minutes in the solutions generated by the modified algorithm. This leads to allocation of redundant times to a tour which is particularly not desirable for short HOS in which 15 minutes can be allocated to a stand-alone piece of work. Our model is free of this restriction.

Goel (2014) assesses the impact of new HOS regulations in the U.S. (changed in 2013) on the operating costs of transport companies using a simulation-based methodology, initially proposed by Goel and Vidal (2014). The study uses monetized accident risk, and time-based and distance-based costs as performance metrics. In another study, Koç et al. (2017) investigates TDSP with a rich objective function that takes into account the cost associated with engine idling. The authors develop an MIP model considering the U.S. HOS regulations to assess the impact of engine idling and its policy implications.

Indeed, the second stream of research focuses primarily on the operating and safety measures in

Table 1: Summary of Studies on Truck Driver Scheduling Problem

| Article | Year | Problem | HOS Regulations |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | AUS | CAN | EU | US |
| Xu et al. (2003) | 2003 | TDSP+Routing |  |  |  | $\checkmark$ |
| Archetti and Savelsbergh (2009) | 2009 | TDSP |  |  |  | $\checkmark$ |
| Goel (2009) | 2009 | TDSP+Routing |  |  | $\checkmark$ |  |
| Ceselli et al. 2009 | 2009 | TDSP+Routing |  |  | $\checkmark$ |  |
| Goel (2010) | 2010 | TDSP |  |  | $\checkmark$ |  |
| Kok et al. (2010), <br> Prescott-Gagnon et al. (2010) | 2010 | TDSP+Routing |  |  | $\checkmark$ |  |
| Goel (2012b), Goel et al. (2012) | 2012 | TDSP | $\checkmark$ |  |  |  |
| Goel (2012a) | 2012 | TDSP |  |  | $\checkmark$ | $\checkmark$ |
| Goel and Rousseau (2012) | 2012 | TDSP |  | $\checkmark$ |  |  |
| Rancourt et al. (2013) | 2013 | TDSP+Routing |  |  |  | $\checkmark$ |
| Goel (2014) | 2014 | TDSP+Routing |  |  |  | $\checkmark$ |
| Goel and Vidal (2014) | 2014 | TDSP+Routing | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Koç et al. (2016) | 2016 | TDSP |  |  |  | $\checkmark$ |
| Goel and Irnich (2017) | 2017 | TDSP+Routing |  |  | $\checkmark$ | $\checkmark$ |
| Koç et al. (2017) | 2017 | TDSP+Routing |  |  |  | $\checkmark$ |
| Tilk and Goel (2020) | 2020 | TDSP+Routing |  |  | $\checkmark$ | $\checkmark$ |

analysing HOS regulations. Research in this area overlooks the implementation challenges and the direction/intensity of the impacts of such regulations. It is our intention in this paper to explore this topic from the perspective of rest break location. In addition, previous studies have often developed hard-to-use, relatively inflexible, and highly specialised methodologies to tackle the related problems. It is not possible to conveniently accommodate various criteria or preferences in such complex models. We will address this issue by developing a simple and highly flexible MIP-based methodology which particularly useful in real world situations where rapid decision making is essential. Table 1 provides a comprehensive summary of studies on TDSP and its variants.

## 3 Problem Description

We are given a set of tasks $i \in\{0, \cdots, n+1\}$ with earliest start time $t_{i}^{e}$, latest start time $t_{i}^{l}$, location $l_{i}$, pickup quantity $q_{i}$, and service time $s_{i}$. At each location $s_{l}$, there is a fixed preparation time prior the commencement of the service, denoted by $s_{l}^{1}$, and after the completion of the service, denoted by $s_{l}^{2}$. Preparation times could be related to parking and unparking times or loading and unloading times at location $l$. Since multiple tasks could be scheduled at one location, the preparation times cannot be incorporated into the task service time; hence, we define them separately. For each pickup task $i$ there is an associated delivery task $j$. We denote its pickup quantity (i.e., $q_{j}$ ) by $-q_{i}$. Let P and D denote the pickup and delivery tasks respectively. We are also given set $O$ composed of jobs $(i, j)$ where task $i \in P$ is a pickup task and task $j \in D$ is its associated delivery task. Given the definition for each job $(i, j)$, we have $q_{i}=-q_{j}$. We denote the maximum tour time by $t_{\max }$. The quantity carried by the truck at any time should be less than its capacity $c$. The time distance between locations of task $i$ and task $j$ is denoted by $d_{i j}$. Each contiguous blocks of breaks is $b$ minutes long where $b$ is a multiple of 15 . The problem is to find a minimum length tour such that all pickup and delivery tasks are serviced within their time windows. Without break regulations, this is a classic vehicle routing problem with pickup and delivery time windows. A tour starts with first task at depot indexed 0 and finishes with the last task at depot indexed $n+1$.

### 3.1 Break time

There are three established regulatory frameworks in Australia including standard hours (SH), Basic Fatigue Management (BFM), and Advanced Fatigue Management (AFM). These frameworks are structurally identical. In this paper, we merely focus on the SH regulations which is the most broadly adopted
framework in industry. The BFM and AFM regulations allow longer hours of work but are only applicable to accredited transport operators. In any case, within cities and regional areas the working hours are often limited to maximum 12 hours. Considering the short trips and short HOS, we do not need to incorporate restrictions on night shifts as well as the rules requiring drivers to take long rest breaks out of the vehicle or inside the vehicle in certain conditions ${ }^{4}$. Such rules and restrictions are usually taken into account by planners/schedulers for rostering and task assignment purposes. We consider this outside the scope of our analysis.

According to SH rules, the minimum break is 15 minutes with no restrictions with regards to the location of the break. There are however Australian carriers who prefer the breaks to be taken at the depot, if possible. We are interested to know how this alternative practice compares financially to the standard SH rules. Since the length of a tour is the primary cost driver for that tour, we compare these alternative policies using the length of the tours as the primary measure. We specifically compare two policies: policy Standard Hours (SH) and policy Standard Hours at Depot (SHD). In the SH policy, all breaks are scheduled according to the SH rules. In the SHD policy, the breaks are still scheduled according to the SH rules taking into account an extra restriction that all breaks need to be scheduled at the depot. This restriction of the SHD policy may incur a significant cost as illustrated in the following examples.

Example 2. We have 3 customers with some parcels for collection. All parcels need to be transported to the depot. Service duration is 20 minutes at each location irrespective of the number of parcels loaded or unloaded. We set the index of the depot to 0 . The distance matrix in minutes is equal to:

$$
D=\left[\begin{array}{cccc}
0 & 90 & 90 & 120 \\
90 & 0 & 10 & 60 \\
90 & 10 & 0 & 50 \\
120 & 60 & 50 & 0
\end{array}\right]
$$

We assume that the truck allocated to service the customers has enough capacity to complete all tasks without the need to go back to the depot in the middle of the tour. For SH policy, an optimal tour is $0-1-2-3-0$ with a duration of 365 minutes and a 15 -minute break at customer 3 location. However, an optimal tour for SHD policy would be $0-3-2-0-1-0$ with a duration of 570 minutes and a break of 30 minutes at the depot given that the truck must visit the depot to take a break in the middle of the tour. Not only SHD policy caused a significant increase in tour duration, but it also affected the order of the visits. The SHD policy made the tour long enough to need a 30-minute break, instead of 15 minutes as required by the SH rules. We note that since the break location is not restricted in the SH policy, the only way that SH policy could change the optimal sequence of visits to customers is when strict time windows are applied. Just consider a sequence. In order to make it compliant with SH policy, we just need to schedule breaks at appropriate times. Adding breaks, does not require us to change the sequence, nor does it increase the total tour length excluding the total break time.

Example 3. Consider the data in Example 2 with the following distance matrix:

$$
D=\left[\begin{array}{cccc}
0 & 96 & 112 & 136 \\
96 & 0 & 120 & 144 \\
112 & 120 & 0 & 232 \\
136 & 144 & 232 & 0
\end{array}\right]
$$

Customer 3 has to be visited within time window [100, 420] and Customer 1 has to visited within time window $[100,260]$. If we ignore both policies, optimal tour is $0-2-1-3-0$ with a duration of 592 minutes. Under SH policy, this tour is not feasible as it would take 416 minutes for the truck to get to Customer 3 without break and since a break of 15 minutes is required before getting to Customer 3 , the truck could not start servicing Customer 3 before 431 min - which is obviously outside the acceptable timeframe. Under SH policy, an optimal tour is $0-1-3-2-0$ with a duration of 724 minutes, a 30-minute break at Customer 3, and a 30-minute break at Customer 2.

[^1]SHD policy has two main advantages over SH policy. First, it is more convenient for the drivers since they can socialise with the other drivers and use the existing amenities at the depot. Second, it is easier to plan for compliance with the HOS regulations using SHD policy as the plan is tightly restricted. However, the associated logistics cost could be excessively high; hence the need for a thorough cost/benefit analysis.

In all these policies, we assume that the break times cannot happen during service times. This is consistent with postal service practice, since a service at a pickup and delivery location cannot be interrupted by a break. There are however certain activities that could be interrupted by a break (e.g., driving times). Regardless of the rest break policy, we assume in all our models and experiments that service times at the pickup and delivery locations cannot be interrupted.

## 4 The Models

In this section, we present an MIP model for each policy. Before that, we prove an important theorem which gives us both necessary and sufficient conditions for compliance of a given tour. We can think of a tour as a sequence of two types of periods. We call them flexible and inflexible periods. A flexible period has two features. First, it can be all work time or all break time. Second, a break can start at any time during the period. In contrast, an inflexible period is a working time with no breaks.

Let $[n]=\{1, \cdots, n\}$. For inflexible period $i$, we denote its start time and end time by $s_{i}$ and $f_{i}$, respectively. We denote a tour with $n$ inflexible periods $i$ by $\left\{\left(s_{i}, f_{i}\right)\right\}_{[n]}$ where $f_{i} \leqslant s_{i+1}$ for all $i \in[n-1]$. All the periods between consecutive inflexible periods are flexible periods. We assume that the period ending at $s_{1}$, and the period starting after $f_{n}$ are rest breaks. An SH-type break rule can be defined by a positive and real parameter $a$ and two positive integers $b$ and $\delta$. We refer to an SH-type rule by triple $(a, b, \delta)$. In any period of length $a$ minutes, there should be at least $b$ break blocks of length $\delta$ minutes each. For example, for the first SH break rule $a=330, b=1$, and $\delta=15$. We first prove a general theorem for a single SH-type rule which sets necessary conditions for feasibility of a given tour $\left\{\left(s_{i}, f_{i}\right)\right\}_{[n]}$. Consider tour $\left\{\left(s_{i}, f_{i}\right)\right\}_{[n]}$, and SH-type rule $(a, b, \delta)$ with $b_{i}$ breaks in flexible period $\left[f_{i}, s_{i+1}\right]$ for all $i \in[n-1]$.
Lemma 1. If there exists an SH-feasible schedule of breaks with $b_{i}$ breaks in each flexible period $i$ then for all $i, j \in[n]$ where $i \leqslant j$, and for all $r \in\{0, \cdots, b-1\}$, we have $\sum_{k=i}^{j-1} b_{k} \geqslant b-r$ if

$$
\begin{equation*}
f_{j}-s_{i}>a-(2+r) \delta \tag{1}
\end{equation*}
$$

If the times and parameters are all integers then condition 1 boils down to

$$
\begin{equation*}
f_{j}-s_{i} \geqslant a-(2+r) \delta+1 \tag{2}
\end{equation*}
$$

Now, consider three SH rules $(330,1,15),(480,2,15)$, and $(660,4,15)$. We assume that the length of the tour, i.e., $f_{n}-s_{1}$, does not exceed 13 hours or equivalently 780 minutes. We further assume all times are integer values.

Theorem 1. There exists an SH-feasible schedule of breaks if and only if for all $i, j \in[n], i \leqslant j$, we have:

1. if $f_{j}-s_{i} \geqslant 301$, then $\sum_{l=i}^{j-1} b_{i} \geqslant 1$,
2. if $f_{j}-s_{i} \geqslant 451$, then $\sum_{l=i}^{j-1} b_{i} \geqslant 2$,
3. if $f_{j}-s_{i} \geqslant 616$, then $\sum_{l=i}^{j-1} b_{i} \geqslant 3$,
4. if $f_{j}-s_{i} \geqslant 631$, then $\sum_{l=i}^{j-1} b_{i} \geqslant 4$.

Furthermore, this feasible schedule can be obtained by just scheduling $b_{i}$ break block(s) within flexible period $\left[f_{i}, s_{i+1}\right]$ comprised of $b_{i} \delta$ break time and $s_{i+1}-b_{i} \delta-f_{i}$ combined preemptive work time and idle time for every $i \in[n-1]$.

Given the objective of tour length minimisation, the immediate consequence of the above theorem is that it allows us to explicitly schedule jobs without the need to schedule breaks. Therefore, in the first stage, we find an optimal schedule of jobs. Then, in the second stage, we can find an explicit schedule of breaks without affecting the optimality or the scheduled start time of the jobs.

Under both policies, all tours should start and end at the depot. We denote the start location and the end location of a tour by 0 and $n+1$, respectively. Since there is only one depot, the start and end locations are identical. Let us define some notations (refer to Table 2 for primary notations and all notations not defined within the text). We denote the set of all possible pairs of consecutive tasks by $H$ which is defined as $\{(i, j): i \in P \cup D \cup\{0\}, j \in P \cup D \cup\{n+1\}\}$. In real applications, some of these pairs might not be acceptable due to operational, regulatory, or contractual requirements. Let $A \subseteq H$ be the set of acceptable pairs. The objective function is to minimise the length of the tour, that is,

$$
\begin{equation*}
\min x_{n+1}^{s}-x_{0}^{s} . \tag{3}
\end{equation*}
$$

Table 2: Notations

| Symbol | Definition |
| :---: | :--- |
| $[n]$ | $\{1, \cdots, n\}, n$ is a positive integer |
| $O$ | set of all pickup or delivery orders or jobs $(i, j)$ where $i$ is a |
| $P$ | collection task and $j$ is a delivery task |
| $D$ | set of collection tasks delivery tasks |
| $H$ | set of all possible pairs of consecutive tasks, or equivalently $\{(i, j):$ |
|  | $i \in P \cup D \cup\{0\}, j \in P \cup D \cup\{n+1\}\}$ |
| $A$ | a given subset of $H$ |
| $\mathcal{B}$ | $\left\{15 k: k \in\left[k_{m a x}\right]\right\}$ |
| $k_{m a x}$ | the maximum number of break blocks required between any two |
|  | consecutive tasks under SH rules |
| $t_{m a x}$ | maximum tour time, a number less than 780 minute or 13 hours |
| $l_{i}$ | location of task $i$ |
| $t_{i}^{e}, t_{i}^{l}$ | earliest start time and latest start time of task $i$ |
| $s_{i}$ | service time specific to task $i$ |
| $s_{l}^{1}$ | preparation time specific to location $l$ before service starts |
| $s_{l}^{2}$ | preparation time specific to location $l$ after service ends |
| $q_{i}$ | quantity of task $i$ for collection (if task $i$ is delivery, $q_{i}$ is negative) |
| $d_{i j}$ | distance in time between location of task $i$ and $j$ |
| $c$ | capacity of a truck |
| $x_{i j}$ | value 1 indicates task $j$ is immediately after task $i$ |
| $x_{i}^{s}$ | service start time for task $i$ |
| $x_{i}^{B}$ | accumulated break time by the start time of task $i$ |
| $x_{i}^{c}$ | vehicle's available capacity at the start time of task $i$ |
| $y_{i j}^{b}$ | value 1 indicates that there should be $b$ minutes break between |
| $x_{i j}^{b}$ | finish time of task $i$ and start time of task $j$ |
|  | value 1 indicates there is $b$ minutes break between finish time of $i$ and start time of next task $j$ |
|  |  |

All the given tasks must be completed in a single tour. To make sure we have a sequence of tasks in the solution, each task $i \in P \cup D$ can be succeeded and preceded by only one task. Let us name the set of constraints for modelling of this requirement as "service constraints".

## Service constraints:

$$
\begin{array}{ll}
\sum_{(i, j) \in A} x_{i j}=1 & \text { for all } i \in P \cup D \cup\{0\} \\
\sum_{(j, i) \in A} x_{j i}=1 & \text { for all } i \in P \cup D \cup\{n+1\} \tag{5}
\end{array}
$$

The capacity of a truck is $c$ cubic meters. After each collection $i$, the available capacity drops by $q_{i}$; and after each delivery $j$, the available capacity is increased by $-q_{j}$.

## Capacity constraints:

$$
\begin{align*}
x_{j}^{c} & \leqslant x_{i}^{c}-q_{i}+M\left(1-x_{i j}\right)  \tag{6}\\
0 \leqslant x_{i}^{c} & \leqslant c  \tag{7}\\
x_{0}^{c} & =c . \tag{8}
\end{align*}
$$

$$
\text { for all }(i, j) \in A \text {, }
$$

$$
\text { for all } i \in P \cup D \cup\{n+1\} \text {, }
$$

$M$ is a number bigger than $c$. Constraint 6 ensures that the available capacity of the truck at the start of servicing task $j$ is not more than the available capacity at the end of the immediate preceding task $i$, that is, $x_{i}^{c}-q_{i}$.

## Time constraints:

$$
\begin{array}{cc}
t_{i}^{e} \leqslant x_{i}^{s} \leqslant t_{j}^{l} & \text { for all } i \in P \cup D \cup\{0, n+1\}, \\
x_{n+1}^{s}+s_{n+1}+s_{l_{n+1}}^{1}-\left(x_{0}^{s}+s_{l_{0}}^{2}\right) \leqslant t_{\text {max }} . & \tag{10}
\end{array}
$$

Constraints 9 ensures time window for each collection or delivery is respected. Constraint 10 ensures the length of the tour does not exceed the maximum tour length $t_{\text {max }}$.

The following sub-sections present the constraints specific to each policy. The break constraints are the direct consequence of Theorem 1.

### 4.1 SH policy

If we look at each pair of tasks $(i, j)$ in $A \backslash O$, there are four possibilities in a feasible solution.

1. Task $j$ is the next task after $i$,
2. Task $i$ is the next task after $j$,
3. Task $j$ is not the next task after $i$ but succeed task $i$,
4. Task $i$ is not the next task after $j$ but succeed task $j$.

For jobs $(i, j) \in O$, the order is predetermined. The order of all other tasks is decided by the model. We name the constraints for the modelling of this requirement as "precedence constraints". Let set $\mathcal{B}=\left\{15 k: k \in\left[k_{\max }\right]\right\}$, where $k_{\max }$ is the maximum number of break blocks required between tasks $i$ and $j$ in any feasible solution with respect to SH rules. Following Theorem 1 and knowing that tasks $i$ and $j$ are consecutive tasks, it is obvious that the maximum number of breaks is 4 .

Let $s_{i j}$ denote the total service time and preparation time between start of task $i$ and start of next task $j$. Total service time at each location is the total service time of consecutive tasks done at the location plus the total preparation time. Preparation time before the first task at a location can include parking and any sort of preparation that is needed for collection/delivery. Preparation time after the last task at a location can include any activity that is needed prior to departure including packing up and unparking.

$$
s_{i j}= \begin{cases}s_{i} & \text { if } l_{i}=l_{j} \\ s_{i}+s_{l_{i}}^{2}+s_{l_{j}}^{1} & \text { otherwise }\end{cases}
$$

## Precedence constraints:

$$
\begin{array}{ll}
x_{i}^{s}+s_{i j}+d_{i j}+\sum_{b \in \mathcal{B}} b x_{i j}^{b} \leqslant x_{j}^{s}+M\left(1-x_{i j}\right) & \text { for all }(i, j) \in A \backslash O \\
x_{i}^{s}+s_{i j}+d_{i j}+\sum_{b \in \mathcal{B}} b x_{i j}^{b} \leqslant x_{j}^{s} & \text { for all }(i, j), \in O \tag{12}
\end{array}
$$

$M$ denotes a sufficiently big number. For precedence constraint, $M \geqslant T$ is sufficient. Consider pair of tasks $(i, j) \in A \backslash O$. Constraint 11 enforces possibilities 1 or 2 by the value of binary variable $x_{i j}$. If $i$ and $j$ are not consecutive, then Constraint 11 becomes inactive for $(i, j)$. However, by service constraints there is a set of tasks that come between $i$ and $j$. Constraint 11 indirectly enforces possibilities 4 and 5 in this case for $(i, j)$ by directly enforcing possibilities 1 and 2 on all pairs of consecutive tasks from $i$ to
$j$. For $(i, j) \in O$, Constraint 12 imposes that task $j$ should succeed task $i$ because it is a delivery task. For job $(i, j)$ only possibilities 1 and 3 can happen in a feasible solution. The other two possibilities are ruled out by Constraint 12. These two possibilities are enforced by combination of service constraint and precedence constraints.

We assume drivers cannot take a break once the preparation time starts in a location until the after-task preparation ends. This assumption is in line with our motivating postal service application (and with general logistics practice for short service times) and is consistent with the previous literature in this domain (e.g., Goel et al. (2012); Goel (2012b). This is an inflexible period in terms of Theorem 1 . However, drivers are free to take a break at any other times.

## Break constraints:

By leveraging the theorem, we can model the SH rules as follows:

$$
\begin{array}{cl}
x_{j}^{B} \leqslant x_{i}^{B}+\sum_{b \in \mathcal{B}} b x_{i j}^{b}+M\left(1-x_{i j}\right), & \text { for all }(i, j) \in A, \text { if } l_{i} \neq l_{j} \\
x_{j}^{B} \leqslant x_{i}^{B}+M\left(1-x_{i j}\right) & \text { for all }(i, j) \in A, \text { if } l_{i}=l_{j}, \\
\left(x_{j}^{s}+s_{j}+s_{l_{j}}^{2}\right)-\left(x_{i}^{s}-s_{l_{i}}^{1}\right) \leqslant & \\
301+150 y_{i j}^{15}+315 y_{i j}^{30}+330 y_{i j}^{45}+\left(t_{\text {max }}-301\right) y_{i j}^{60}, & \text { for all }(i, j) \in H \\
x_{j}^{B}-x_{i}^{B} \geqslant & \text { for all }(i, j) \in H \\
15 y_{i j}^{15}+30 y_{i j}^{30}+45 y_{i j}^{45}+60 y_{i j}^{60}-M\left(1-\sum_{b} y_{i j}^{b}\right), & \text { for all }(i, j) \in H \\
\sum_{b} y_{i j}^{b} \leqslant 1 & \text { for all }(i, j) \in A, b \in \mathcal{B}, \\
x_{i j}^{b} \leqslant x_{i j} & \text { for all }(i, j) \in A . \\
\sum_{b \in \mathcal{B}} x_{i j}^{b} \leqslant 1 & \tag{19}
\end{array}
$$

Constraint 13 ensures that the total break time by the start of task $j$ does not exceed the total break time by the start of the previous task $i$ plus the total break taken between task $i$ and $j$; that is, $\sum_{b \in \mathcal{B}} b x_{i j}^{b}+M\left(1-x_{i j}\right)$. If task $i$ and $j$ are at the same location, the model does not allow a break between $i$ and $j$. This is enforced by Constraint 14. Constraints 15 and 17 are the direct result of Theorem 11 and indicate the length of all intervals starting and ending with inflexible periods by variables $y_{i j}^{b}$. For every $(i, j) \in H$, the length of interval $\left[x_{i}^{s}-s_{l_{i}}^{1}, x_{j}^{s}+s_{j}+s_{l_{j}}^{2}\right]$ is either less than 302 , or in $[302,452]$, or in $[453,617]$, or in $[618,632]$, or in $\left[633, t_{\max }\right]$, which by Theorem 1 requires $0,15,30,45,60$ minutes break respectively. The minimum break time in intervals [ $f_{i}, s_{i+1}$ ] is enforced by Constraint 16 . Constraint 18 ensures that $x_{i j}^{b}$ takes value zero when task $j$ does not succeed task $i$. Constraint 19 imposes that at most only one break variable for each pair of consecutive tasks can take a non-zero value. For example, if we have a 45-minute break, there are two possibilities without Constraint $19 x_{i j}^{15}=1, x_{i j}^{30}=1$ or $x_{i j}^{45}=1$. But the first possibility is not consistent with the definition of variables $x_{i j}^{b}$.

It may be of interest to some readers to know how we can deal with this problem when breaks can be scheduled at any time and at any location. Under SH policy, we have the flexibility of any location but not the flexibility of any time. For simplicity, assume that the preparation times are zero. Under any time and any location scenario, for each task the time window should be defined for the whole service time, and the service time should be contained in the time window. In addition to the task start time variable, we need to define a task end time variable. We also need to define additional variables for breaks happening within tasks and between tasks at the same location.
Remark 1. Goel (2012b) assumed that breaks can only take place at customer locations before the service starts. This restriction can be modelled by a slight modification to the model for SH policy. Constraint 15 will change to:

$$
\begin{align*}
& \left(x_{j}^{s}+s_{j}+s_{l_{j}}^{2}+\sum_{(j, k) \in H} d_{j k} x_{j k}\right)-\left(x_{i}^{s}-s_{l_{i}}^{1}\right) \leqslant \\
& \quad 301+150 y_{i j}^{15}+315 y_{i j}^{30}+330 y_{i j}^{45}+\left(t_{\max }-301\right) y_{i j}^{60}, \quad \text { for all }(i, j) \in H \tag{20}
\end{align*}
$$

This constraint for each $(i, j)$ corresponds to the interval

$$
\left.\left[x_{i}^{s}-s_{l_{i}}^{1}, x_{j}^{s}+s_{j}+s_{l_{j}}^{2}+\sum_{(j, k) \in H} d_{j k} x_{j k}\right)\right]
$$

which contains the inflexible interval $\left[x_{i}^{s}-s_{l_{i}}^{1}, x_{i}^{s}+s_{i}+s_{l_{i}}^{2}\right]$ at the start and the inflexible interval

$$
\left.\left[x_{j}^{s}-s_{l_{j}}^{1}, x_{j}^{s}+s_{j}+s_{l_{j}}^{2}+\sum_{(j, k) \in H} d_{j k} x_{j k}\right)\right]
$$

at the end.The latter interval contains service period at location $j$ and the subsequent travel period. Since no break can take place after the service $j$ or while travelling, Constraint 20 immediately follows from Theorem 1

### 4.2 SHD policy

Under SHD policy, breaks can only occur at a depot. In a situation with only one depot (which is what we assume in our study), if there needs to be a break between two consecutive tasks, the driver should go to the depot, take the break, and continue the tour. The model for the SH policy needs two major changes in the precedence constraints and the break constraints.

## Precedence constraints:

$$
\begin{array}{ll}
x_{i}^{s}+s_{i j}+d_{i j}\left(1-\sum_{b} x_{i j}^{b}\right)+\left(d_{i 0}+d_{0 j}\right) \sum_{b} x_{i j}^{b}+\sum_{b \in \mathcal{B}} b x_{i j}^{b} \leqslant x_{j}^{s}+M\left(1-x_{i j}\right) & \text { for all }(i, j) \in A \backslash O, \\
x_{i}^{s}+s_{i j}+d_{i j}\left(1-\sum_{b} x_{i j}^{b}\right)+\left(d_{i 0}+d_{0 j}\right) \sum_{b} x_{i j}^{b}+\sum_{b \in \mathcal{B}} b x_{i j}^{b} \leqslant x_{j}^{s} & \text { for all }(i, j), \in O, \tag{22}
\end{array}
$$

If we have a break between task $i$ and task $j$, the driver needs to drive from location $l_{i}$ to depot, take break, and then drive to location $l_{j}$. Total driving time in this case between task $i$ and $j$ is $\left(d_{i 0}+d_{0 j}\right)$.

## Break constraints:

In this setting, inflexible intervals either commence at the start of a preparation or at the departure from the depot after taking a break. Moreover, they end either at the end of an after-task preparation or on arrival at the depot before the break.

$$
\begin{array}{cl}
x_{j}^{B} \leqslant x_{i}^{B}+\sum_{b \in \mathcal{B}} b x_{i j}^{b}+M\left(1-x_{i j}\right), & \forall(i, j) \in A, \text { if } l_{i} \neq l_{j} \\
\left(x_{j}^{s}+d_{j 0}\left(\sum_{\substack{\left.\left.k:(j, k) \in A, b \in \mathcal{B} \\
301+150 y_{i j}^{15}+315 y_{i j}^{30}+330 y_{i j}^{45}+\left(t_{\max }^{B}-301\right) y_{i j}^{60}, x_{j k}^{B}\right)+s_{j}^{B}+s_{l_{j}}^{2}\right)-\left(x_{i}^{s}-d_{0 i}\left(\sum_{\begin{subarray}{c}{i(k, i) \in A, b} }} x_{k i}^{b}\right)-s_{l_{i}}^{1}\right) \leqslant}\end{subarray}} \quad \forall(i, j) \in A, \text { if } l_{i}=l_{j},\right.\right. \\
\geqslant 15 y_{i j}^{15}+30 y_{i j}^{30}+45 y_{i j}^{45}+60 y_{i j}^{60}-M\left(1-\sum_{b} y_{i j}^{b}\right), & \forall(i, j) \in H \\
\sum_{b} y_{i j}^{b} \leqslant 1 & \forall(i, j) \in H \\
\sum_{b \in \mathcal{B}} x_{i j}^{b} \leqslant 1 & \text { for all }(i, j) \in A, b \in x_{i j}^{b}
\end{array}
$$

Note that the inflexible intervals might have a different structure compared to those of the SH policy.

The inflexible interval for $(i, j)$ has this structure:

$$
\left[x_{i}^{s}-d_{0 i}\left(\sum_{\substack{i:(k, i) \in A, b \in \mathcal{B}}} x_{k i}^{b}\right)-s_{l_{i}}^{1}, x_{j}^{s}+d_{j 0}\left(\sum_{\substack{k:(j, k) \in A, b \in \mathcal{B}}} x_{j k}^{b}\right)+s_{j}+s_{l_{j}}^{2}\right]
$$

If there is no break between task $i$ and its previous task, and there is no break between task $j$ and its next task, then the interval has the same structure as the corresponding interval in SH policy, that is,

$$
\left[x_{i}^{s}-s_{l_{i}}^{1}, x_{j}^{s}+s_{j}+s_{l_{j}}^{2}\right]
$$

If there is a break between task $i$ and its previous task, since that break should be taken at the depot, the driver cannot take another break after departure from the depot and before task $i$. Analogously, if there is no break between task $j$ and its next task, the driver cannot take another break after departure from task $j$ and before arrival at the depot.

## 5 Computational Experiments and Discussion

In our computational studies, we used Gurobi 8.1 Optimiser on a 64 -bit Windows 10 Machine with 16 GB of RAM and Intel 4.8 GHz i7-8665U processor. To compare policies, we generate 176 benchmark classes each comprised of 9 instances. Each class corresponds to an actual tour that is run daily by a major postal company in Australia (data is related to the tours in Sydney Metropolitan and its regional areas). In each class, the instances differ only in their time windows. For each tour, we have the tasks to be completed and the capacity of the truck assigned to it. For each task, we have its duration, the scheduled start time, and the location. For each pickup task, we know its associated delivery task.

We observe in our dataset that the deliveries are not tightly scheduled by the company. This is consistent with the current practice and industry norms for non-express postal items. Since most items in our database are classified as non-express, we consider no specific delivery time for daily delivery tasks. Practically, the delivery times are usually bounded by the maximum length of the tours which is 12 hours. Therefore, we only need to generate time windows for pickup tasks.

In each class, we have 9 instances corresponding to $O=\{5,20,35,50,65,95,125,155,185\}$. Let $t_{i}^{s}$ denote the scheduled start time of pickup task $i$ in instance $o \in O$. The time window for task $i$, i.e. $\left[t_{i}^{e}, t_{i}^{l}\right]$, is equal to $\left[t_{i}^{s}-o, t_{i}^{s}+s_{i}+o\right]$. All preparation times are set to zero. Service duration ranges from 5 minutes to 55 minutes with an average of approximately 12 minutes. Distance matrix (in minutes) is pulled from google maps database. We use peak travel times on Tuesday, 24 September 2019, at 4:30PM as the tours were run on that date.

Locations of all tasks are depicted in Figure 1. It contains 240 locations spread over Sydney Metropolitan which is a relatively compact area. The top two pickup locations comprise $37 \%$ of all visits, and the top two delivery locations comprise $47 \%$ of all visits. These locations host the two major sortation facilities for parcels and mail.

Most of the tours require trucks with capacities of 37 cubic meters ( 62 percent of the tours) and 58 cubic meters ( 19 percent of the tours). The capacity requirement of a tour is an input to the model which can be determined based on the road access restrictions and customer requirements.

In Table 3, we presented a typical tour as planned by the postal carrier. A 30-minute break is scheduled at depot. The tour has 12 tasks with the first task starting at 12:10 at the depot and the last task ending at 21:05 at the depot. This tour has to be executed from Monday to Friday every day.

Tables 45 and Tables 67 compare the optimal tour times of the SH policy (Tables 45) and the SHD policy (67) with the no break scheme. The columns, from left to right, show instance classes O, mean difference, standard deviation, number of instances with an optimal solution within one-minute time limit, minimum difference, first quantile, second quantile, third quantile, and maximum difference. For each class $o \in O$, we only consider tours that have optimal solutions under all three policies.

The second column shows the mean duration difference of optimal tours when compared to the no break scheme. Let us refer to class $o \in O$, offset-o class. For offset- 5 in Table 4 the mean difference is 3.2 which means that the average duration of optimal tours under SH policy is 3.2 minutes longer than tour

Figure 1: Map of locations of stops in Sydney

durations under no break scheme. As the offset gets bigger, the mean increases until offset 125 where the pattern is reversed.

There are two factors at play here. The first factor is the time window and the second is the break policy. Bigger time windows allow for more flexible sequencing of tasks in each tour. Of course, requiring a break limits this positive effect of wider time windows on the tour length. As time windows get larger, the mean difference, unsurprisingly, converges to the minimum required break. An evidence of this would be the quantile statistics with the majority being at 0,15 , and 30 minutes.

Under SHD policy (Tables 67), the mean difference compared to the no break scheme is much larger than what is reported in Table 4 . This is pretty much expected since the drivers under the SHD policy have to travel the extra miles to take rest at the depot. Perhaps the more interesting difference compared to the SH policy is the standard deviation being almost twice as much for the SHD policy when compared to the SH policy. The maximum difference also is much higher for the SHD policy compared to the SH policy.

To facilitate a better comparison between the two schemes, we can provide a direct comparison between the SHD policy and the SH policy. The results are presented in Tables 89. Quantiles suggest that in a majority of instances, both policies have rather identical performance in terms of tour duration. This can be explained by the high frequency of visits to two of strategically-located sortation facilities. These facilities are very close to the depot which makes it very convenient as they are visited frequently in most tours. This way, taking a break at the depot becomes much less costly as the drivers do not require travelling significant extra miles just for the purpose of a break. This justifies a small different of only 1-1.5 percent between the average tour length of the SHD and that of the SH policies. The gains may seem slim in the short term, but the overall benefits of the SH policy become more pronounced in the long term.

The distribution of benefits across all tours is not uniform and there are extreme situations in which the extra miles are significant (see the last column of Tables 89 ). In those cases, the break needs to be scheduled at different locations than only at the depot. There are, therefore, tours that may benefit from scheduling the rest breaks in locations other than the depot. The benefits in these situations can be as large $30 \%$ reduction in tour length which is quite significant.

Under SHD policy, tours have to travel extra miles on average compared to SH policy. Extra miles

Table 3: A typical tour time and duration (locations anonymized)
Transport Tour: 341
Monday to Friday 12:10-21:05

| Arrive | Location | Instructions | Depart |
| :--- | :--- | :--- | :--- |
| $12: 10$ | Depot | Prepare a small truck | $12: 25$ |
| $12: 50$ | Customer 1 | Collect All Available for Hub 1 | $13: 05$ |
| $14: 10$ | Hub 1 | Deliver All Available ex - Customer 1 | $14: 20$ |
| $15: 05$ | Facility 1 | Collect All Available for Hub 2 | $15: 25$ |
| $15: 50$ | Hub 2 | Deliver All Available ex - Facility 1 | $16: 00$ |
| $16: 05$ | Depot | Rest Break | $16: 35$ |
| $17: 40$ | Customer 2 | Collect All Available for Customer 3 | $18: 35$ |
| $18: 25$ | Customer 3 | Deliver All Available ex - Customer 2 |  |
|  |  | Collect All Available for Hub 2 | $18: 35$ |
| $19: 20$ | Facility 2 | Collect All Available for Hub 1, Hub 2 | $19: 40$ |
| $20: 25$ | Hub 2 | Deliver All Available ex - Facility 2, Customer 3 | $20: 35$ |
| $20: 45$ | Hub 1 | Deliver All Available ex - Facility 2 | $20: 55$ |
| $21: 00$ | Depot | Return and Refuel Vehicle | $21: 05$ |

are not generally desirable as they prompt further uncertainties in travel times. This is a bigger issue in larger cities like Sydney with very congested roads. For parcel collection services, this is even more pronounced when s large share of tasks is scheduled during the peak traffic hours (i.e., 7:00-10:00 am and 4:00-7:00 pm) (Liao et al. (2020)). In our dataset, around 21 percent of all tasks are scheduled in the peak hours. Any reduction in the frequency and length of the tours, especially in the most congested areas, could reduce uncertainties in travel times for planning purposes, and contribute to improved city

|  | Table 4: SH Policy vs. No Break (minute) |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| offset | mean | std | count | min | FQ | SQ | TQ | max |  |
| 5 | 3.2 | 7.5 | 140 | 0 | 0 | 0 | 0 | 30 |  |
| 20 | 7.2 | 10.8 | 148 | 0 | 0 | 0 | 12 | 30 |  |
| 35 | 11.6 | 12.0 | 137 | 0 | 0 | 10 | 19 | 30 |  |
| 50 | 14.5 | 11.7 | 127 | 0 | 1 | 15 | 30 | 30 |  |
| 65 | 17.2 | 11.3 | 121 | 0 | 14 | 15 | 30 | 30 |  |
| 95 | 18.3 | 10.3 | 112 | 0 | 15 | 15 | 30 | 30 |  |
| 125 | 18.6 | 10.2 | 101 | 0 | 15 | 15 | 30 | 30 |  |
| 155 | 18.1 | 9.9 | 88 | 0 | 15 | 15 | 30 | 32 |  |
| 185 | 17.7 | 10.0 | 80 | 0 | 15 | 15 | 30 | 30 |  |

Table 5: SH Policy vs. no break (percentage)

| offset | mean | std | count | $\min$ | FQ | SQ | TQ | $\max$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 0.6 | 1.5 | 140 | 0 | 0 | 0 | 0 | 7 |
| 20 | 1.5 | 2.2 | 148 | 0 | 0 | 0 | 3 | 7 |
| 35 | 2.5 | 2.5 | 137 | 0 | 0 | 2 | 4 | 7 |
| 50 | 3.2 | 2.4 | 127 | 0 | 0 | 4 | 6 | 7 |
| 65 | 3.8 | 2.3 | 121 | 0 | 3 | 4 | 6 | 7 |
| 95 | 4.2 | 2.0 | 112 | 0 | 4 | 4 | 6 | 7 |
| 125 | 4.2 | 2.0 | 101 | 0 | 4 | 4 | 6 | 7 |
| 155 | 4.2 | 1.9 | 88 | 0 | 4 | 4 | 6 | 7 |
| 185 | 4.1 | 2.0 | 80 | 0 | 4 | 4 | 6 | 7 |

congestion as a whole.

Table 6: SHD Policy vs. No Break (minute)

| offset | mean | std | count | $\min$ | FQ | SQ | TQ | max |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 9.0 | 24.8 | 140 | 0 | 0 | 0 | 7 | 172 |
| 20 | 11.7 | 19.8 | 148 | 0 | 0 | 0 | 16 | 137 |
| 35 | 16.0 | 19.2 | 137 | 0 | 0 | 13 | 30 | 136 |
| 50 | 19.0 | 18.7 | 127 | 0 | 6 | 15 | 30 | 136 |
| 65 | 22.6 | 19.2 | 121 | 0 | 15 | 15 | 30 | 136 |
| 95 | 24.4 | 21.0 | 112 | 0 | 15 | 15 | 30 | 136 |
| 125 | 23.0 | 18.9 | 101 | 0 | 15 | 15 | 30 | 133 |
| 155 | 23.4 | 19.8 | 88 | 0 | 15 | 15 | 30 | 133 |
| 185 | 23.0 | 20.2 | 80 | 0 | 15 | 15 | 30 | 133 |

Table 7: SHD Policy vs. no break (percentage)

| offset | mean | std | count | $\min$ | FQ | SQ | TQ | $\max$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 1.8 | 4.9 | 140 | 0 | 0 | 0 | 1 | 35 |
| 20 | 2.4 | 4.1 | 148 | 0 | 0 | 0 | 4 | 30 |
| 35 | 3.5 | 4.1 | 137 | 0 | 0 | 3 | 6 | 30 |
| 50 | 4.2 | 4.0 | 127 | 0 | 2 | 4 | 6 | 30 |
| 65 | 5.1 | 4.2 | 121 | 0 | 4 | 5 | 6 | 30 |
| 95 | 5.6 | 4.8 | 112 | 0 | 4 | 5 | 6 | 30 |
| 125 | 5.3 | 4.3 | 101 | 0 | 4 | 5 | 6 | 30 |
| 155 | 5.5 | 4.7 | 88 | 0 | 4 | 5 | 6 | 30 |
| 185 | 5.4 | 4.7 | 80 | 0 | 4 | 5 | 6 | 30 |

Table 8: SHD Policy vs. SH Policy (minute)

| offset | mean | std | count | min | FQ | SQ | TQ | $\max$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 5.9 | 22.1 | 140 | 0 | 0 | 0 | 0 | 148 |
| 20 | 4.5 | 14.6 | 148 | 0 | 0 | 0 | 0 | 111 |
| 35 | 4.4 | 13.0 | 137 | 0 | 0 | 0 | 1 | 107 |
| 50 | 4.5 | 13.6 | 127 | 0 | 0 | 0 | 1 | 106 |
| 65 | 5.4 | 15.9 | 121 | 0 | 0 | 0 | 1 | 106 |
| 95 | 6.2 | 18.3 | 112 | 0 | 0 | 0 | 3 | 112 |
| 125 | 4.4 | 14.6 | 101 | 0 | 0 | 0 | 0 | 103 |
| 155 | 5.2 | 16.3 | 88 | 0 | 0 | 0 | 0 | 103 |
| 185 | 5.2 | 16.2 | 80 | 0 | 0 | 0 | 0 | 103 |

Table 9: SHD Policy vs. SH Policy (percentage)

| offset | mean | std | count | min | FQ | SQ | TQ | max |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 1.1 | 4.4 | 140 | 0 | 0 | 0 | 0 | 30 |
| 20 | 1.0 | 3.1 | 148 | 0 | 0 | 0 | 0 | 23 |
| 35 | 1.0 | 2.9 | 137 | 0 | 0 | 0 | 0 | 24 |
| 50 | 1.0 | 3.1 | 127 | 0 | 0 | 0 | 0 | 24 |
| 65 | 1.3 | 3.7 | 121 | 0 | 0 | 0 | 0 | 24 |
| 95 | 1.5 | 4.3 | 112 | 0 | 0 | 0 | 1 | 26 |
| 125 | 1.0 | 3.6 | 101 | 0 | 0 | 0 | 0 | 26 |
| 155 | 1.3 | 4.1 | 88 | 0 | 0 | 0 | 0 | 26 |
| 185 | 1.3 | 4.0 | 80 | 0 | 0 | 0 | 0 | 26 |

In our analysis, we compared two representative policies. One policy with no restriction on rest break location and the other with a restriction to only take a break at the depot. A policy with multiple designated rest break locations cannot be more expensive than SHD policy (in terms of tour duration), and cannot be less expensive than SH policy either. We thus studied the two extreme situations to provide a good perspective for further analysis of this type. The developed model for SHD policy (as mentioned in Remark 1), can be utilised for analysis of any other rest break scenario.

## 6 Conclusion

This paper presented a methodology to compare two representative policies of rest break scheduling for truck drivers. In the SH policy, all breaks are scheduled according to the standard hours regulations. In the SHD policy, the breaks are still scheduled according to the standard hours regulations with an extra constraint that enforces the breaks to be only taken at the depot. Logically, the SHD policy should be more expensive in terms of tour length since it is more restrictive in terms of the rest break location. However, we observe in a real case study from postal services in Australia that the magnitude of benefits that can be obtained from the SH policy is not as large as one would expect. Our computational analysis suggests that the SHD policy, on average, is between 1 to 1.5 percent more expensive than the SH policy. The reason is the fact that most of the tours in the concerned service area (Sydney metropolitan) occur in the vicinity of the depot (where the rest breaks occur in the SHD policy), hence, the extra miles that need to be travelled to return to the depot to take a break may not be as significant. This finding underlines the role that facility location (depot location in this case) plays in policy impact analysis. This level of analysis is crucial when making supply chain network design decisions as the outcomes not only impacts the internal scheduling decisions, but they also contribute to our pick-time traffic congestion in a broader perspective.

We proposed novel MIP models for analysing rest break policies. The models are developed taking into consideration necessary and sufficient conditions pertaining to the application of SH rules for short HOS. These conditions relieve us from explicitly scheduling breaks to find an optimal tour. They also enable us to model rest break requirements without losing the tractability using the existing MIP solvers. However, proving these conditions imply that we are not seeking to find an exact solution (schedule of breaks). This makes sense for the purpose of our analysis because having an exact schedule of breaks is not a requirement for finding an optimal tour. An interesting direction for future research could be to develop algorithms to find an exact schedule of breaks in an optimal tour (obtained from our methodology) under SH policy with unrestricted rest break location. Given that our approach gives the duration of rest break between tasks, finding an exact schedule of breaks should be plausible. Another direction for future research would be to extend the developed models to accommodate longer HOS that is essential for intercity planning.

## References

Archetti, C. and M. Savelsbergh (2009, November). The Trip Scheduling Problem. Transportation Science 43(4), 417-431.

Briest, P., J. Dragendorf, T. Ecker, D. Mohr, and F. Neuhaus (2019). The Endgame for Postal Networks: How to Win in the Age of e-Commerce. McKinsey Company.

Ceselli, A., G. Righini, and M. Salani (2009, February). A Column Generation Algorithm for a Rich Vehicle-Routing Problem. Transportation Science 43(1), 56-69.

Dong, B., M. Christiansen, K. Fagerholt, and S. Chandra (2020, November). Design of a sustainable maritime multi-modal distribution network - Case study from automotive logistics. Transportation Research Part E: Logistics and Transportation Review 143, 102086. 00003.

Europa 1 (accessed on July 30,2020). Human resources:Transport sector workers:Road transportation workers. https://europa.eu/youreurope/business/human-resources/transport-sector-workers/ road-transportation-workers/index_en.htm

Europa 2 (accessed on July 30,2020). Regulation (EC) No 561/2006 of the European Parliament and of the Council of 15 March 2006. https://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX: 32006R0561:EN:HTML\#d1e817-1-1.

Fahimnia, B., M. Bell GH, D. Hensher A, and J. Sarkis (2015). Green Logistics \& Transportation: A Sustainable Supply Chain Perspective. Springer.

Federal Register (accessed on July 30,2020). Hours of Service of Drivers. https://www.federalregister gov/documents/2020/06/01/2020-11469/hours-of-service-of-drivers.

Goel, A. (2009, February). Vehicle Scheduling and Routing with Drivers' Working Hours. Transportation Science 43 (1), 17-26.

Goel, A. (2010, November). Truck Driver Scheduling in the European Union. Transportation Science 44 (4), 429-441.

Goel, A. (2012a, December). The minimum duration truck driver scheduling problem. EURO Journal on Transportation and Logistics 1(4), 285-306.

Goel, A. (2012b, August). A mixed integer programming formulation and effective cuts for minimising schedule durations of Australian truck drivers. Journal of Scheduling.

Goel, A. (2014, May). Hours of service regulations in the United States and the 2013 rule change. Transport Policy 33, 48-55.

Goel, A., C. Archetti, and M. Savelsbergh (2012, May). Truck driver scheduling in Australia. Computers © Operations Research 39(5), 1122-1132.

Goel, A. and S. Irnich (2017, May). An Exact Method for Vehicle Routing and Truck Driver Scheduling Problems. Transportation Science 51(2), 737-754.

Goel, A. and L.-M. Rousseau (2012, December). Truck driver scheduling in Canada. Journal of Scheduling 15(6), 783-799.

Goel, A. and T. Vidal (2014, August). Hours of Service Regulations in Road Freight Transport: An Optimization-Based International Assessment. Transportation Science 48(3), 391-412.

Justice Laws (accessed on July 30,2020). Commercial Vehicle Drivers Hours of Service Regulations. https://laws-lois.justice.gc.ca/eng/regulations/SOR-2005-313/.

Koç, Ç., T. Bektaş, O. Jabali, and G. Laporte (2016, November). A comparison of three idling options in long-haul truck scheduling. Transportation Research Part B: Methodological 93, 631-647.

Koç, Ç., O. Jabali, and G. Laporte (2017, April). Long-haul vehicle routing and scheduling with idling options. Journal of the Operational Research Society.

Kok, A. L., C. M. Meyer, H. Kopfer, and J. M. J. Schutten (2010, September). A Dynamic Programming Heuristic for the Vehicle Routing Problem with Time Windows and European Community Social Legislation. Transportation Science 44 (4), 442-454.

Li, X., Y. Ding, K. Pan, D. Jiang, and Y. Aneja (2020, August). Single-path service network design problem with resource constraints. Transportation Research Part E: Logistics and Transportation Review 140, 101945. 00003.

Liao, Y., J. Gil, R. H. M. Pereira, S. Yeh, and V. Verendel (2020, March). Disparities in travel times between car and transit: Spatiotemporal patterns in cities. Scientific Reports 10(1), 4056.

NHVR (accessed on July 30, 2020). Work and Rest Requirements. https://www.nhvr.gov.au/ safety-accreditation-compliance/fatigue-management/work-and-rest-requirements

Prescott-Gagnon, E., G. Desaulniers, M. Drexl, and L.-M. Rousseau (2010, November). European Driver Rules in Vehicle Routing with Time Windows. Transportation Science 44 (4), 455-473.

PwC (2016). Shifting Patterns: The Future of the Logistics Industry. PwC Network.
Rancourt, M.-E., J.-F. Cordeau, and G. Laporte (2013, February). Long-Haul Vehicle Routing and Scheduling with Working Hour Rules. Transportation Science 47(1), 81-107.

Schiffer, M., M. Schneider, G. Walther, and G. Laporte (2019, March). Vehicle Routing and Location Routing with Intermediate Stops: A Review. Transportation Science 53(2), 319-343.

Tilk, C. and A. Goel (2020, May). Bidirectional labeling for solving vehicle routing and truck driver scheduling problems. European Journal of Operational Research 283(1), 108-124.

Xu, H., Z.-L. Chen, S. Rajagopal, and S. Arunapuram (2003, August). Solving a Practical Pickup and Delivery Problem. Transportation Science 37(3), 347-364.

Zhen, L., C. Ma, K. Wang, L. Xiao, and W. Zhang (2020, March). Multi-depot multi-trip vehicle routing problem with time windows and release dates. Transportation Research Part E: Logistics and Transportation Review 135, 101866. 00012.


[^0]:    ${ }^{1}$ https://sustainability.ups.com/sustainability-strategy/environmental-responsibility
    ${ }^{2}$ https://www.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/work-and-rest-requirements/ standard-hours
    ${ }^{3}$ https://wWW.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/counting-time

[^1]:    ${ }^{4}$ https://www.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/work-and-rest-requirements/ standard-hours

