# Search for transitions from $\Upsilon(4 S)$ and $\Upsilon(5 S)$ to $\eta_{b}(1 S)$ and $\eta_{b}(2 S)$ with emission of an $\omega$ meson 

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#### Abstract

Using data collected in the Belle experiment at the KEKB asymmetric-energy $e^{+} e^{-}$collider we search for transitions $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega, \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$. No significant signals are observed and we set $90 \%$ confidence level upper limits on the corresponding visible cross sections: 0.2 pb , 0.4 pb and 1.9 pb , respectively.


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## I. INTRODUCTION

Recently Belle observed the $\Upsilon(4 S) \rightarrow h_{b}(1 P) \eta$ transition and measured its branching fraction to be $\mathcal{B}[\Upsilon(4 S) \rightarrow$ $\left.h_{b}(1 P) \eta\right]=(2.18 \pm 0.21) \times 10^{-3}[1]$. This value is unexpectedly large in comparison with branching fractions of the $\Upsilon(4 S) \rightarrow \Upsilon(1 S, 2 S) \pi^{+} \pi^{-}$decays [2-4], and represents a strong violation of the heavy quark spin symmetry (HQSS) [5]. A possible mechanism for the HQSS breaking is an admixture of $B \bar{B}$ pairs in the $\Upsilon(4 S)$ state [6]. Indeed, the $B \bar{B}$ pair is not an eigenstate of the $b$ quark spin and contains $b \bar{b}$ quarks in both spin-triplet and spin-singlet states. The $\Upsilon(4 S) \rightarrow h_{b}(1 P) \eta$ transition proceeds via the spin-singlet component. The transition $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ might also proceed via the spin-singlet component and thus could be enhanced [6]. Here we perform a search for the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ as well as $\Upsilon(10860) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(10860) \rightarrow \eta_{b}(2 S) \omega$ transitions. For brevity, the $\Upsilon(10860)$ state is denoted as $\Upsilon(5 S)$ according to its quark model assignment.

We use the full data samples of $711 \mathrm{fb}^{-1}$ and $121 \mathrm{fb}^{-1}$ collected at the $\Upsilon(4 S)$ and $\Upsilon(5 S)$ resonances by the Belle detector [7] at the KEKB asymmetric-energy $e^{+} e^{-}$collider [8]. The average center-of-mass (c.m.) energy of the $\Upsilon(10860)$ sample is $\sqrt{s}=10.867 \mathrm{GeV}$. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50-layer central drift chamber, an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter comprised of $\mathrm{CsI}(\mathrm{Tl})$ crystals
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(ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K_{L}^{0}$ mesons and to identify muons.

For the Monte Carlo (MC) simulation, we use EvtGen [9] VectorISR model, which correctly describes the angular distribution of ISR photons but uses a flat distribution in photon-energy radiator function. We apply corrections on the ISR photon energy according to Ref. [10]. We use the GEANT3 [11] package to simulate the detector response.

## II. EVENT SELECTION

Since the $\eta_{b}(1 S, 2 S)$ mesons do not have decay channels that are convenient to reconstruct, we reconstruct only $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and use the recoil mass $M_{\text {recoil }}(\omega)=$ $\sqrt{\left(\sqrt{s}-E_{\omega}^{*}\right)^{2}-\left(p_{\omega}^{*}\right)^{2}}$ to identify the signal, where $E_{\omega}^{*}$ and $p_{\omega}^{*}$ are the energy and momentum of the $\omega$ meson in the c.m. frame.

We use a generic hadronic event selection with requirements on the position of the primary vertex, track multiplicity, and the total energy and momentum of the event [12]. For charged pions we require the distance of closest approach to the interaction point to be within 2 cm along the beam direction and within 0.2 cm in the plane transverse to the beam direction. We apply loose particle identification requirements to separate charged pions from kaons, protons and electrons. The energy of a photon in the laboratory frame is required to be greater than 50 MeV for the barrel part of the ECL and greater than 100 MeV for the endcap part of the ECL. To further suppress the background from low-energy photons, we require the momentum of the $\pi^{0}$ in the c.m. frame to be above $240 \mathrm{MeV} / c, 270 \mathrm{MeV} / c$ and $140 \mathrm{MeV} / c$ for the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega, \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ transitions, respectively. The masses of the $\pi^{0}$ and $\omega$ candidates should satisfy $\left|M(\gamma \gamma)-m_{\pi^{0}}\right|<8 \mathrm{MeV} / c^{2}$ and
$\left|M\left(\pi^{+} \pi^{-} \pi^{0}\right)-m_{\omega}\right|<12 \mathrm{MeV} / c^{2} \quad$ [13]. The resolutions in $M(\gamma \gamma)$ and $M\left(\pi^{+} \pi^{-} \pi^{0}\right)$ are $5.5 \mathrm{MeV} / c^{2}$ and $8 \mathrm{MeV} / c^{2}$, respectively.

The $\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}$ events predominantly populate the central part of the Dalitz plot (DP), while background events populate the region near the boundaries. Therefore we require the normalized distance from the DP center, $r$, to be lower than 0.84 . The variable $r$ takes values from $r=0$ at the DP center to $r=1$ at its boundary [14].

To suppress background from continuum events $e^{+} e^{-} \rightarrow$ $q \bar{q}(q=u, d, s, c)$ that have a jetlike shape, we use the angle $\theta_{\text {thrust }}$ between the thrust axis of $\pi^{+} \pi^{-} \pi^{0}$ and the thrust axis of the rest of the event in the $\eta_{b}$-meson candidate rest frame. The thrust axis is defined as the unit vector $\vec{n}_{T}$, which maximizes the thrust value: $T=\sum_{i}\left|\vec{p}_{i} \cdot \vec{n}_{T}\right| /$ $\sum_{i}\left|\vec{p}_{i}\right|$. From the MC simulation we find that the distributions in $\cos \left(\theta_{\text {thrust }}\right)$ for the signal transitions are uniform; this is because there is no favored direction in the $\eta_{b}$ rest frame as its spin is zero. The selection criteria are $\left|\cos \left(\theta_{\text {thrust }}\right)\right|<0.90$ and $\left|\cos \left(\theta_{\text {thrust }}\right)\right|<0.70$ for the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ transitions, respectively, while for the $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ transition no criteria on $\cos \left(\theta_{\text {thrust }}\right)$ are applied.

The selection requirements described above are optimized using a figure of merit $N_{\text {sig }} / \sqrt{N_{\text {bkg }}}$, where the signal yield is estimated using MC simulation, and the background yield is estimated using the signal region in data, assuming the signal fraction is very small. For optimization of various requirements we use an iterative procedure. The $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ is required to be in the interval $(9.20,9.60) \mathrm{GeV} / c^{2}$ for the $\eta_{b}(1 S)$ candidates and $(9.90,10.05) \mathrm{GeV} / c^{2}$ for the $\eta_{b}(2 S)$ candidates. These intervals are used in the fits described below.

The $M\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distribution for the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ candidates without the $\omega$ mass requirement is shown in Fig. 1. One can see clear signals of the $\eta$ and $\omega$ mesons. The purity of the $\omega$-meson signal is estimated to be $13 \%$. In


FIG. 1. The $M\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distribution for the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ candidates. Red lines indicate the $\omega$ mass requirement.
the $\Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ transitions the purities are $24 \%$ and $6 \%$, respectively.

We find on average $1.33,1.12$ and 1.63 candidates per event for the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega, \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ transitions, respectively. The multiple candidates are not correlated in $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$, therefore we do not perform a best candidate selection. The total selection efficiencies are $5.5 \%, 5.6 \%$ and $6.6 \%$ for the three transitions, respectively.

## III. FIT TO DATA

We perform a binned $\chi^{2}$ fit to the $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distributions in the $\eta_{b}(1 S)$ and $\eta_{b}(2 S)$ mass regions. The fit function is a sum of signal and background components. The signal component is described with a two-sided Crystal Ball (CB) function, which consists of a Gaussian core portion and power-law tails; the function and its first derivative are both continuous [15]. The parameters of the CB function are fixed using MC simulation; the $\sigma$-parameters of the core Gaussian are $12.7 \mathrm{MeV} / c^{2}, 14.6 \mathrm{MeV} / c^{2}$ and $9.2 \mathrm{MeV} / c^{2}$ for $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega, \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$, respectively. The integral of the signal function over the fit range is taken as a signal yield. The background component is described with a Chebyshev polynomial; its order is chosen as the one that gives the maximum $p$-value for the fit. The polynomial orders are 8,5 and 6 for the three transitions, respectively. The


FIG. 2. The $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distribution for the $\Upsilon(4 S) \rightarrow$ $\eta_{b}(1 S) \omega$ candidates. Top: data points with the fit function overlaid; note the suppressed zero on the vertical scale. Bottom: residuals between the data and the fit function. The solid line shows the fit function for the best fit; the dashed line shows the same function with the yield fixed to the upper limit.


FIG. 3. The $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distribution for the $\Upsilon(5 S) \rightarrow$ $\eta_{b}(1 S) \omega$ candidates; note the suppressed zero on the vertical scale. Symbols are the same as those in Fig. 2.
$M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distributions and fit results for $\Upsilon(4 S) \rightarrow$ $\eta_{b}(1 S) \omega, \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ are shown in Figs. 2-4, respectively. We use 1 MeV bins for fitting and 10 MeV bins for visualization to improve clarity. No significant signals are observed. The obtained signal yields for each transition are presented in Table I.


FIG. 4. The $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distribution for the $\Upsilon(5 S) \rightarrow$ $\eta_{b}(2 S) \omega$ candidates; note the suppressed zero on the vertical scale. Symbols are the same as those in Fig. 2.

TABLE I. Signal yields for various transitions obtained from the fits in units of $10^{3}$.

| $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ | $\Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ | $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ |
| :--- | :---: | :---: |
| $-5.0 \pm 6.6_{-2.3}^{+1.5}$ | $0.8 \pm 1.0_{-0.1}^{+0.3}$ | $2.4 \pm 5.8_{-10.8}^{+3.2}$ |

TABLE II. Systematic uncertainties in the yields of various transitions in units of $10^{2}$.

|  | $\Upsilon(4 S) \rightarrow$ | $\Upsilon(5 S) \rightarrow$ |  |
| :--- | :---: | :---: | :---: |
|  | $\eta_{b}(1 S) \omega$ | $\eta_{b}(1 S) \omega$ | $\eta_{b}(2 S) \omega$ |
| $\eta_{b}$ mass | ${ }_{-3}^{+6}$ | $\pm 0$ | ${ }_{-34}^{+27}$ |
| $\eta_{b}$ width | $-_{-7}^{+6}$ | $\pm 1$ | ${ }_{-4}^{+5}$ |
| ISR tail | ${ }_{-12}^{+0}$ | ${ }^{+2}$ | ${ }_{-0}^{+2}$ |
| Background | ${ }_{-17}^{+11}$ | ${ }_{-1}^{+3}$ | ${ }_{-0}^{+17}$ |
| Total | ${ }_{-23}^{+15}$ | ${ }_{-1}^{+3}$ | ${ }_{-102}^{+32}$ |

To set upper limits on the branching fractions, we study the systematic uncertainties in the yields. We vary the $\eta_{b}(1 S)$ and $\eta_{b}(2 S)$ masses and widths within one standard deviation [13]. The $\eta_{b}(2 S)$ width is estimated using a model-independent relation [16]:

$$
\begin{align*}
\Gamma\left[\eta_{b}(2 S)\right] & =\Gamma\left[\eta_{b}(1 S)\right] \cdot \frac{\Gamma\left[\Upsilon(2 S) \rightarrow e^{+} e^{-}\right]}{\Gamma\left[\Upsilon(1 S) \rightarrow e^{+} e^{-}\right]} \\
& =4.6_{-1.8}^{+2.3} \mathrm{MeV} \tag{1}
\end{align*}
$$

The ISR tails are sensitive to the energy dependence of the $e^{+} e^{-} \rightarrow \eta_{b}(n S) \omega$ cross sections. Instead of a resonant production via $\Upsilon(4 S)$ or $\Upsilon(5 S)$, we consider also the cross sections that are energy independent. We generate MC samples for each modification and use them to determine signal shapes. To estimate the uncertainties due to background parametrization we vary the fit range and increase or decrease the polynomial order by one. Maximal deviations in each case are considered to be a systematic uncertainty. The summary of the uncertainties in the yields is presented in Table II. The total systematic uncertainty in the yield is found by adding various contributions in quadrature.

## IV. UPPER LIMITS ON VISIBLE CROSS SECTIONS AND BRANCHING FRACTIONS

The visible cross sections are calculated as:
$\sigma_{\text {vis }}\left[e^{+} e^{-} \rightarrow \eta_{b}(m S) \omega\right]=\frac{N}{\epsilon \cdot \mathcal{B}\left[\omega \rightarrow \pi^{+} \pi^{-} \pi^{0}\right] \cdot L_{\mathrm{Y}(n S)}}$,
where $n=4,5$ and $m=1,2 ; N$ is the signal yield, $\epsilon$ is the selection efficiency, $L_{r(n S)}$ is the integrated luminosity, $L_{\Upsilon(4 S)}=711 \mathrm{fb}^{-1}$ and $L_{\Upsilon(5 S)}=121.4 \mathrm{fb}^{-1}$.

TABLE III. Visible cross sections and upper limits at the $90 \%$ confidence level in pb .

|  | Visible cross section | Upper limit |
| :--- | :---: | :---: |
| $e^{+} e^{-} \rightarrow \eta_{b}(1 S) \omega$ at $\Upsilon(4 S)$ | $-0.14_{-0.20}^{+0.19}$ | 0.2 |
| $e^{+} e^{-} \rightarrow \eta_{b}(1 S) \omega$ at $\Upsilon(5 S)$ | $0.13_{-0.18}^{+0.18}$ | 0.4 |
| $e^{+} e^{-} \rightarrow \eta_{b}(2 S) \omega$ at $\Upsilon(5 S)$ | $0.3_{-1.7}^{+0.9}$ | 1.9 |

We take into account the uncertainty in the efficiency due to possible discrepancies between data and MC simulation ( $1 \%$ per track and $2.2 \%$ for $\pi^{0}$ ), the uncertainty in $\mathcal{B}[\omega \rightarrow$ $\pi^{+} \pi^{-} \pi^{0}$ ] [13] and the uncertainty in the $\Upsilon(4 S)$ and $\Upsilon(5 S)$ integrated luminosity. The total multiplicative uncertainty in the visible cross section is $4.5 \%$.

To combine the uncertainty in the yield, $\delta_{N}$, which is obtained by adding corresponding statistical and systematic uncertainties in quadrature, and the multiplicative uncertainty $\delta$, we use formula:

$$
\begin{equation*}
\left(N \pm \delta_{N}\right) \cdot(1 \pm \delta)=N \pm\left(\delta_{N} \oplus N \delta \oplus \delta_{N} \delta\right) \tag{3}
\end{equation*}
$$

where the symbol $\oplus$ denotes addition in quadrature.
Estimated visible cross sections and upper limits set using the Feldman-Cousins method [17] at the $90 \%$ confidence level are presented in Table III.

We also estimate branching fractions of the $\Upsilon(4 S) \rightarrow$ $\eta_{b}(1 S) \omega, \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ and $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ transitions using the number of $\Upsilon(4 S)$ or $\Upsilon(5 S)$ instead of the luminosity in the denominator of Eq. (2). The number of $\Upsilon(4 S)$ is $(771.6 \pm 10.6) \times 10^{6}$, while the number of $\Upsilon(5 S)$ is estimated as $L_{\Upsilon(5 S)} \cdot \sigma_{b \bar{b}}$, where $\sigma_{b \bar{b}}=(0.340 \pm$ $0.016) \mathrm{nb}$ [18]. The total multiplicative uncertainty in the branching fraction is $4.5 \%$ and $6.5 \%$ for transitions from $\Upsilon(4 S)$ and $\Upsilon(5 S)$, respectively. Estimated branching fractions and upper limits at the $90 \%$ confidence level are presented in Table IV.

We also set the upper limit on the ratio:

$$
\begin{equation*}
\frac{\mathcal{B}\left[\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega\right]}{\mathcal{B}\left[\Upsilon(4 S) \rightarrow h_{b}(1 P) \eta\right]}<8.4 \times 10^{-2} \tag{4}
\end{equation*}
$$

at the $90 \%$ confidence level.

## V. CROSS-CHECK WITH $\Upsilon(5 S) \rightarrow \chi_{b J}(1 P) \omega$

As a cross-check, we perform a search for the $\Upsilon(5 S) \rightarrow$ $\chi_{b J}(1 P) \omega$ transitions that were observed previously using

TABLE IV. Branching fractions and upper limits at the $90 \%$ confidence level.

|  | Branching fraction | Upper limit |
| :---: | :---: | :---: |
| $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ | $\left(-1.3_{-1.9}^{+1.8}\right) \times 10^{-4}$ | $1.8 \times 10^{-4}$ |
| $\Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega$ | $\left(3.7_{-5.1}^{+5.4}\right) \times 10^{-4}$ | $1.3 \times 10^{-3}$ |
| $\Upsilon(5 S) \rightarrow \eta_{b}(2 S) \omega$ | $\left(1.0_{-5.1}^{+2.7}\right) \times 10^{-3}$ | $5.6 \times 10^{-3}$ |



FIG. 5. The $M_{\text {recoil }}\left(\pi^{+} \pi^{-} \pi^{0}\right)$ distribution for the $\Upsilon(5 S) \rightarrow$ $\chi_{b J}(1 P) \omega$ candidates; note the suppressed zero on the vertical scale. Symbols are the same as those in Fig. 2.
exclusive reconstruction [19]. The analysis procedure is the same as for the $\eta_{b}(n S) \omega$ transitions. To fit the signal region we use the sum of three CB functions corresponding to $\chi_{b 0}(1 P), \chi_{b 1}(1 P)$ and $\chi_{b 2}(1 P)$ signals. Since we do not have enough resolution to measure $\chi_{b 1}(1 P)$ and $\chi_{b 2}(1 P)$ yields individually, we fix the ratio between them according to the known values [19]. To fit the background contribution we use a ninth-order Chebyshev polynomial. The fit result is shown in Fig. 5.

There are no significant signals. The obtained upper limits on branching fractions at the $90 \%$ confidence level are

$$
\begin{align*}
& \mathcal{B}\left[\Upsilon(5 S) \rightarrow \chi_{b 0}(1 P) \omega\right]<3.0 \times 10^{-3}, \\
& \mathcal{B}\left[\Upsilon(5 S) \rightarrow \chi_{b 1}(1 P) \omega\right]<3.2 \times 10^{-3} . \tag{5}
\end{align*}
$$

Since the ratio between $\chi_{b 1}(1 P)$ and $\chi_{b 2}(1 P)$ is fixed, we do not give an upper limit for $\chi_{b 2}(1 P)$. The obtained upper limits are consistent with the exclusive measurement [19]:

$$
\begin{align*}
& \mathcal{B}\left[\Upsilon(5 S) \rightarrow \chi_{b 0}(1 P) \omega\right]<3.9 \times 10^{-3} \\
& \mathcal{B}\left[\Upsilon(5 S) \rightarrow \chi_{b 1}(1 P) \omega\right]=(1.57 \pm 0.30) \times 10^{-3} \tag{6}
\end{align*}
$$

## VI. CONCLUSION

In summary, we perform a search for the transitions $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega, \quad \Upsilon(5 S) \rightarrow \eta_{b}(1 S) \omega \quad$ and $\quad \Upsilon(5 S) \rightarrow$ $\eta_{b}(2 S) \omega$. No significant signals are observed and we set upper limits on visible cross sections and branching fractions presented in Tables III and IV. The upper limit for $\Upsilon(4 S) \longrightarrow \eta_{b}(1 S) \omega$ is order of magnitude lower than
the value for the similar transition to a spin-singlet state, $\Upsilon(4 S) \rightarrow h_{b}(1 P) \eta$ [1]. We set the upper limit on the ratio:

$$
\begin{equation*}
\frac{\mathcal{B}\left[\Upsilon(4 S) \longrightarrow \eta_{b}(1 S) \omega\right]}{\mathcal{B}\left[\Upsilon(4 S) \rightarrow h_{b}(1 P) \eta\right]}<8.4 \times 10^{-2} \tag{7}
\end{equation*}
$$

at the $90 \%$ confidence level. As both transitions are expected to proceed via the $B \bar{B}$ admixture in the $\Upsilon(4 S)$ state [6], our result will help to better understand this mechanism. The suppression of the $\Upsilon(4 S) \rightarrow \eta_{b}(1 S) \omega$ transition relative to the $\Upsilon(4 S) \rightarrow h_{b}(1 P) \eta$ one could be due to different overlaps between the initial state and the bottomonium in the final state or the details of the $\eta$ and $\omega$ meson production.

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