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Examination of the World3 Model and the Development of a Novel Model of a Multi-market, Multi-regional Economy Driven by Adaptive Heterogeneous Consumer Agents

Ashley William Heath

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Recommended Citation

Heath, Ashley William, Examination of the World3 Model and the Development of a Novel Model of a Multi-market, Multi-regional Economy Driven by Adaptive Heterogeneous Consumer Agents, Doctor of Philosophy thesis, School of Mechanical, Materials, Mechatronics and Biomedical Engineering, University of Wollongong, 2021. https://ro.uow.edu.au/theses1/1008

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Examination of the World3 Model and the Development of a Novel Model of a Multi-market, Multi-regional Economy Driven by Adaptive Heterogeneous Consumer Agents

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This thesis is presented as part of the requirements for the conferral of the degree:

Doctor of Philosophy

Supervisors:
Dr. B. Stappenbelt & Assoc. Prof. M. Ros

The University of Wollongong
Faculty of Engineering and Information Sciences

May 2, 2021

This research has been conducted with the support of an Australian Government Research Training Program Scholarship.

Declaration

I, *Ashley William Heath BMechEng(Hons1), BMath*, declare that this thesis is submitted in partial fulfilment of the requirements for the conferral of the degree *Doctor of Philosophy*, from the University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. This document has not been submitted for qualifications at any other academic institution.

Ashley William Heath ***BMechEng(Hons1), BMath***

May 2, 2021

Abstract

Ever since the human race developed consciousness we have battled against the elements to bring about prosperity and health. For millennia we closely observed the natural phenomena that seemed to influence future outcomes, gradually building and refining our conceptions of reality, our mental models. We refined the process of observation and discovery with the scientific method, and from that point on our power to control our environment grew immensely. Now our greatest foe is not only Mother Nature, but ourselves. We still act impulsively, and make decisions which seem irrational. We may guiltily watch hour after hour of Antiques Road Show, instead of spending a mere 30 minutes finishing off the final thesis chapter.

The tradition of model development is continued herein, with a focus on holistic socio-ecological models. The first part of this thesis examines the pre-existing Limits to Growth model, originally developed by Meadows et. al. in 1972. Uncertainty analysis was performed on this model to develop a better understanding of its reliability. This model is also used to better understand the trade-off relationships between common goals humans wish to achieve in the future. A genetic algorithm was used to determine the Pareto front of the seven examined goals. The final part of the thesis presents a novel model designed to allow many simulated human actors to make purchasing decisions in a self determining fashion, based on the cost of various goods. The new model simulates multi-item market-places, floating prices on goods, and spacial effects on trading and resource extraction. A preliminary version of the model is tested under eight different conditions, and the results are presented and discussed.

Acknowledgements

First, I would like to thank my supervisors, Brad[1] and Montse[2]. However, I feel they are more aptly described as mentors (and dare I say friends). Both are incredible teachers, dedicated to making the learning process interesting, engaging, and challenging. They are both beloved by their students. While most of our meetings devolved into long conversations about the struggles of being a university lecturer, I wouldn't want it any other way.

I would also like to thank my father[3] who was my original mentor and teacher; with his many (occasionally bordering on endless) lessons about analysing farming problems and thinking critically about solutions.

My mother[4] for always being loving, kind, and concerned about me. She has always been a great source of insightful points of view about current news topics and politics.

I should mention my ever-amusing brother[5], who would always ask how *his* thesis was going (for he was a tax paying citizen and my PhD scholarship was government funded).

My grandparents, Poppy Ross[6], Granny Rene[7], and Grandma Marion[8], who have always been encouraging and have taken a great interest in my work and how I was going with it. They are all beacons of hard work and generosity, whose work to support our families has been invaluable.

I would also like to thank my close friends Todd[9] who, (apart from being a great friend), is always ready and able to help me out, and Fran[10] who provided me with hundreds of cups of tea, and non-stop conversation.

I would also like to thank Bob[11] who is always a positive inspiration. He was a huge asset to the University during his time there and very much beloved by the students. Bob, I hope I can emulate your bottomless pit of dedication to giving back to the community and making learning fun.

Thank you also to the teachers who are no longer with us: Gramps[12] for teaching me

---

[1] Brad Stappenbelt
[2] Montserrat Ros
[3] Craig Heath
[4] Caroline Heath
[5] Angus Heath
[6] Ross Wells
[7] Irene Wells
[8] Marion Heath
[9] Todd Sillis
[10] Frances Burns
[11] Bob Wheway
[12] Glen Heath Snr.

the art of fire lighting (and lire fighting); Uncle John[13] for teaching me how to make a central pivot irrigator out of Lego; and Mark Wettenhall for teaching us all that you can plug a leaky tractor radiator with cracked pepper.

While I cannot thank everyone individually, I would like to thank some broad groups of people: the neighbours (the Gregorys, the Fords, the other Gregorys, and the Sleighs); my high school friends and their parents; my lovely in-laws (seemingly endless in number and energy); my uni friends, and teachers; and the other friends who've expressed support over these years.

And finally, the most important person in my life, my darling wife, Ellen Dando. She has been nothing but encouraging and supportive throughout this time. I hope I've made you proud, and continue to do so until I'm old and withered. I promise I'll never do a PhD ever again.

---

[13] John Wells

Quotes

“All models are wrong, but some are useful.”

George Box - Statistics for Experimenters

“The Fundamental cause of the trouble is that in the modern world the stupid are cocksure while the intelligent are full of doubt.”

Bertrand Russell - Mortals and Others: American Essays [“The Triumph of Stupidit”]

“In short, one may say anything about the history of the world, anything that might enter the most disordered imagination. The only thing one can’t say is that it’s rational. The very word sticks in one’s throat. And, indeed, this is the odd thing that is continually happening: there are continually turning up in life moral and rational persons, sages and lovers of humanity who make it their object to live all their lives as morally and rationally as possible, to be, so to speak, a light to their neighbours simply in order to show them that it is possible to live morally and rationally in this world. And yet we all know that those very people sooner or later have been false to themselves, playing some queer trick, often a most unseemly one. Now I ask you: what can be expected of man since he is a being endowed with strange qualities? Shower upon him every earthly blessing, drown him in a sea of happiness, so that nothing but bubbles of bliss can be seen on the surface; give him economic prosperity, such that he should have nothing else to do but sleep, eat cakes and busy himself with the continuation of his species, and even then out of sheer ingratitude, sheer spite, man would play you some nasty trick. He would even risk his cakes and would deliberately desire the most fatal rubbish, the most uneconomical absurdity, simply to introduce into all this positive good sense his fatal fantastic element. It is just his fantastic dreams, his vulgar folly that he will desire to retain, simply in order to prove to himself, as though that were so necessary, that men still are men and not the keys of a piano, which the laws of nature threaten to control so completely that soon one will be able to desire nothing but by the calendar. And that is not all: even if man really were nothing but a piano-key, even if this were proved to him by natural science and mathematics, even then he would not become reasonable, but would purposely do something perverse out of simple ingratitude, simply to gain his point. And if he does not find means he will contrive destruction and chaos, will contrive sufferings of all sorts, only to gain his point! He will launch a curse upon the world, and as only man can curse (it is his privilege, the primary distinction between him and other animals), may be by his curse alone he will attain his object, that is, convince himself that he is a man and not a piano-key! If you say that all this, too, can be calculated and tabulated, chaos and darkness and curses, so that the mere possibility of calculating it all beforehand would stop it all, and reason would reassert itself, then man would purposely go mad in order to be rid of reason and gain his point! I believe in it, I answer for it, for the whole work of man really seems to consist in nothing but proving to himself every minute that he is a man and not a piano-key! It may be at the cost of his skin, it may be by cannibalism! And this being so, can one help being tempted to rejoice that it has not yet come off, and that desire still depends on something we don't know?

You will scream at me (that is, if you condescend to do so) that no one is touching my free will, that all they are concerned with is that my will should of itself, of its own free will, coincide with my own normal interests, with the laws of nature and arithmetic.

Good heavens, gentlemen, what sort of free will is left when we come to tabulation and arithmetic, when it will all be a case of twice two make four? Twice two makes four without my will. As if free will meant that!"

Fyodor Dostoyevsky - Notes from Underground [ch. 8]

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Preface

The purpose of this thesis is to further investigate global sustainability issues from a holistic perspective on a macro, rather than micro, scale. Although there is significant and valuable work being done within specific disciplines, it is crucial to also examine the relationships between these areas of study and how each impacts the other. For example, in order to engage in useful projections of the future impact of carbon emissions, a study of the ecological impact of climate change should also include an understanding of how economic factors drive carbon emissions.

The interplay between these issues creates complex and multifaceted problems when studying and making recommendations about issues of sustainability. A common method for approaching and understanding these complex issues is socio-ecological modelling. This thesis seeks to extract further information from existing models, and to develop a novel model to help discover new insights into socio-ecological systems.

This objective has been achieved in three broad ways:

1. increasing the understanding of uncertainty surrounding the Limits to Growth Model;
2. increasing the understanding of the trade-off relationships between sustainability objectives based on the Limits to Growth model; and
3. the development of a novel framework to allow for the investigation of non-homogeneous agents actions in a socio-ecological model.

Chapter 1 begins with a broad overview of the issues of sustainability. This is then followed by two sections, one on the creation process of models, the other on the role models are playing in policy production. The final section of the chapter provides an overview of the Limits to Growth model, as this model forms the foundation of the research conducted in chapters 2 and 4.

Chapter 2 is an examination of the uncertainty surrounding the Limits to Growth model. The chapter begins with an introduction to the importance of uncertainty analysis and its previous application to the Limits to Growth model. It then moves into a description of the goals of the research, methodology and results. The chapter closes with a discussion on the uncertainty of the Limits to Growth model. The Limits to Growth Model is used to analyse trade-offs between real world objectives in Chapter 4. The research conducted in this chapter is important for solidifying the stability of the World3 model, as the research conducted in Chapter 4 is moot if it is founded on an unreliable model.

Chapter 3 describes the graphical user interface that was developed to analyse multi-dimensional Pareto fronts. This software was used to display and analyse the Pareto front data generated in Chapter 4.

Chapter 4 is an examination of the trade-off relationships between desired objectives of human kind going into a future with constrains on its resources. The World3 model used to derive the relationships. The chapter opens with a overview of multi-objective decision making and the tools used to discover solutions. This flows into a section regarding past optimisation analysis of the Limits to Growth model. The methodology used to conduct the research is presented followed by the results. The trade-off relationships of the objectives are noted and the implications discussed.

Chapter 5 presents a novel model for investigating human behaviour within socio-ecological systems. The model was developed with the hopes of addressing some of the shortcomings of models (such as the World3 model) that aggregate populations, resources, or goods, into a single entity. The model is also designed to address spacial effects on populations. The chapter begins by examining the broad spectrum of models which have been used to investigate sustainability issues. This examination looks at the various types of models that exist, their purpose for existing (be it analytical or some other reason), the theories that underpin some models, and the issues that have been examined using models. The proposed novel model is then presented, followed by the results of eight trial runs. The trials were conducted to examine the behaviour of the model and demonstrate its ability to investigate a wide variety of issues. The chapter closes with a discussion on validating the model.

Chapter 6 gives a summary of the thesis, outlining its significance to the field of socio-ecological modelling. This chapter ends with a discussion on future directions the work presented could take.

Chapter 1

Background Of Thesis

1.1 Sustainability

As the world's population has grown over time, so too has our ability to address humankind's basic requirements. In developed countries, those basic requirements – reliable clean water, clothing and shelter, food, treatment for injury and disease – have been far surpassed through tremendous leaps in technology over the 19th and 20th Centuries. The challenge before the world now lies in closing the standard of living gulf between developing and developed countries, and managing the enormous impact on the world's natural resources.

The modern notion of managing that impact in a sustainable way arose from data suggesting that the earth's carrying capacity was being outpaced by human population and resource consumption. Since the mid-20th Century, studies across scientific and other academic fields have confirmed that current trends cannot be maintained without significant and irreversible damage to the earth's biosphere. Today sustainability is widely regarded by the public as a key area of concern.

1.1.1 How is Sustainability Defined

There is a wide variety of definitions for the concept of sustainability. A widely used and very popular definition of sustainability is:

Sustainability - "meet[ing] the needs of the present without compromising the ability of future generations to meet their own needs" [175].

This definition of sustainability appeared in the United Nations (UN) 1987 report "Our Common Future". Since the Brundtland Report, as it became more widely known, the definition gained widespread use in academic literature and the public arena [80, 120]. The essence of this definition is that the activities of today's generations do not burden the generations to come, i.e. a kind of intergenerational equality is achieved.

The concept of sustainability has been long debated and refined. Today there is a general consensus that the term refers to the interlinking of three core dimensions of human activity. These dimensions are environmental, social, and economic [1, 96]. Each dimension is dependent on the other and for the system to survive indefinitely into the future, all three must remain intact [80, 160]. Systems of this type are frequently referred to as socio-ecological[1] systems.

This idea was illustrated in the International Union for Conservation of Nature (IUCN) Council's report 'The Future of Sustainability', as shown in Figure 1.1 [1]. This report neatly depicts the notion that for sustainability to be upheld the three dimensions, or "pillars", must all be present, i.e. economic health (often defined as economic growth), environmental protection, and social progress all must be maintained to ensure sustainability.

Some of the earliest discussions on the topic of sustainability date back to 1969 when the International Union for Conservation of Nature integrated it into their mandate. The UN raised the issue again in 1972 at the Human Environment Conference [1]. In 1972 the book 'The Limits to Growth' was published which generated much debate on the issue of continual growth [1]. Since the book, a string of conventions have repeatedly raised the issue, the most notable of which being the 'World Conservation Strategy' 1980 [1], 'The Brundland Report' 1987 [1, 80], Rio 'Earth Summit' 1992 [120, 1, 80], 'Kyoto Protocol' 1997 [4], 'Rio+20' 2012 [81], and the Paris 'Climate Change Conference' 2015 [153]. These, along with many other smaller events, have slowly developed the ideas of sustainability [177].

Over many years the definition of sustainability has been built upon, and edited [1] by academics who have created their own definitions. A paper by Hjorth and Bagheri [80] mentioned several definitions of sustainability developed by various academics. These examples defined sustainability as:

- "preserving the production capacity for a long future" - Solow [80],
- "a vector of desired social goals which the society tries to maximise by working on its components. The components of the vector are: increase in real per capita income, improvement in hygiene and nutrition, educational successes, access to resources, equitable distribution of wealth, and increase in liberty. Sustainable development is a condition in which the vector of development does not decrease" - Pearce et al. [80],

---

[1] We include economic systems under the socio term as economics is a societal construct.

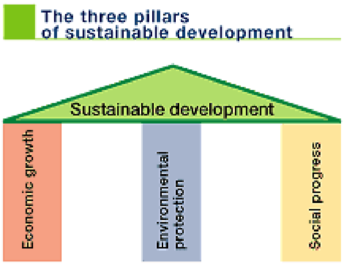

Figure 1.1: Sustainability represented as "The three pillars of sustainable development", economic growth, environmental protection, and social progress. Figure sourced from [1].

- “preserving or improving the integrity of the life supporting systems on the earth” - Fuwa [80],
- “preserving a condition” - Klauer [80],
- “adapting to the frequencies of oscillation of natural capital that perform best” - Odum [80],
- “economic development (that) keeps ... demands clearly within biological and physical limits” - Woodwell [188].

Varied definitions of sustainability also abound outside of academic circles, as it becomes a buzzword in political, economic, and social circles. Each definition is unique to its author’s priorities and knowledge of the various aspects of sustainability. The management of competing priorities makes political action on climate change and sustainability a particularly difficult task.

The scientific consensus that current overconsumption of resources and methods of waste management cannot be sustained long term is slowly becoming publicly accepted knowledge [1, 4]. Leaders at all levels of politics, whether local, national, or international, are finding that their constituents expect sustainability to be accounted for in policy decisions [4, 175, 66, 1, 120]. That trend is also reflected across the private economy, where businesses and consumers are also becoming aware of their role in facilitating change [152]. According to Todorov and Marinova [160] the gravity of the issue is enhanced by three factors. These are:

1. the scale of globalisation and its effects;
2. the complexity of the man-made world and its governing laws;
3. and that the policy making tools developed thus far have only been capable of dealing with the short to medium term.

Presently most of the worlds resources are consumed by a minor fraction of the worlds population. Turner has calculated that for a sustainable and equitable lifestyle to be reached for all humans on the planet, current consumption rates in the affluent countries would have to decrease by 1/6 [129]. This represents a dramatic shift in living patterns for almost all affluent peoples. The enormity of this shift is a major challenge for political leaders to coordinate, and for wealthy individuals to adapt to.

1.1.2 Issues and Past Progress

Striving for “sustainability” is high on the agenda for many leaders and policy makers around the world, as evident from the United Nations Conferences on Sustainable Development in Rio (2012) [81] and Paris (2015) [153]. However, while on the agenda since the Brundtland Report, the effects of change have since been slow moving and the subject of significant controversy. The IUCN’s 2006 report ‘The Future of Sustainability’ summarised the findings of the Millennium Ecosystem Assessment, as shown in Figure A.1 in

Appendix A [1]. The findings were that many key sustainability issues were not making progress, or even going backwards. The UN's 2014 report 'Prototype' again found similar trends. The summary of the reports findings are shown in Figure A.2 in appendix A [174]. It can be noted from the 2014 report that all categories had mixed results (with the exception of 'nature' which had only negative results). For example, the economy had doubled in the past two decades, however income inequality had increased in many countries. The latest reports from the UN still indicate that there are many issues left to overcome [170].

Some of the issues facing humanity include, climate change, loss of biodiversity, fresh water availability, ocean acidification, soil saturation of nitrogen/phosphorus, ozone depletion, arable land degradation, chemical pollution, population growth, poverty, resource distribution, urbanisation, illiteracy, access to health services, wealth inequality, and human rights injustices [160, 36].

To date, no country has met all sustainability targets [81], and many have a long way to go as shown in Figure 1.2 [81]. This figure gives the impression that some countries are meeting ecological footprint targets, however China and India are rapidly increasing their ecological footprint as they rapidly become wealthier and will potentially move above the ecological threshold.

Another issue confronting anyone developing policies relating to sustainability is how to decide what value to place on different outcomes [131]. Even once a decision is made about which factors or outcomes should be incorporated into a policy, subjective questions of value are difficult to adequately model across populations. That task is made harder again for any model attempting to project the long-term impact of a policy (intergenerational) [131, 20]; accounting for the costs and benefits over time means taking into consideration a complex network of competing factors.

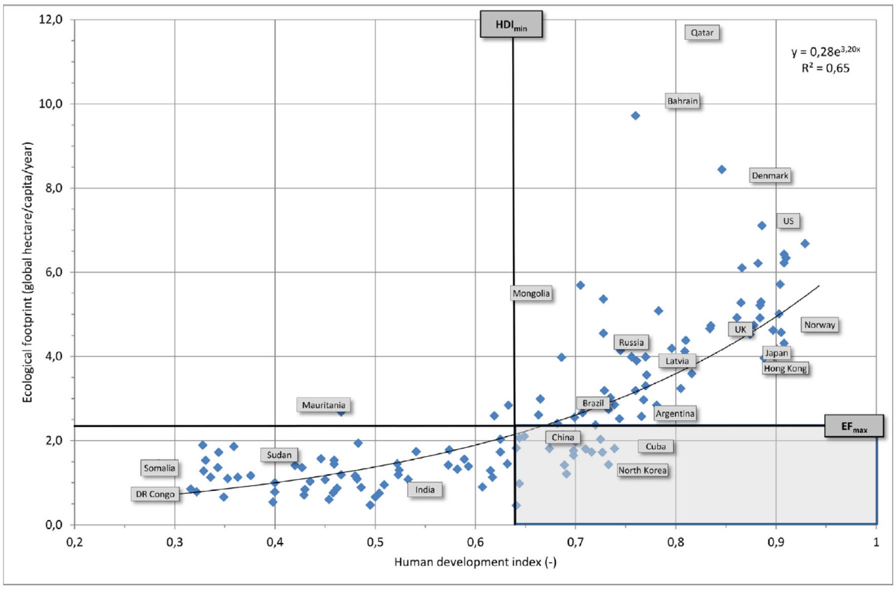

Figure 1.2: Ecological footprint index with respect to human development index. The shaded bottom right corner indicates countries meeting both criteria for sustainability. Figure sourced from [81].

1.1.3 Commentary

There is often a misunderstanding around the concept of sustainable development. Hjorth and Bagheri [80] believe that many scientists perceive it as a project that has a clear finish point and that due to reductionist thinking of the science community it could become a "meaningless buzzword". Meadows believes that the term is "widely abused" by people who do not fully comprehend the idea [80, 81].

The concise definition of sustainability cited above from the Brundtland Report became a foundation for a variety of interpretations and definitions by governments, environmentalists, economists and private businesses [1]. Those interpretations often overlooked two caveats also present in the Report, which clarified key concepts in the original definition:

1. "the concept of 'needs', in particular the essential needs of the world's poor, to which overriding priority should be given," [175, 2.I.1]
2. "the idea of limitations imposed by the state of technology and social organization on the environment's ability to meet present and future needs." [175, 2.I.1]

In relation to the first caveat, the idea of 'needs' is often not viewed as the 'basic needs' of the poor. This allows for interpretations to come about that permit highly developed countries to continue pushing their own economic growth.

The 'Three Pillar' model of sustainability has been criticised for "imply(ing) that trade-offs can always be made" between the three socio-ecological dimensions. The term 'strong sustainability' was coined to denote when no trade-offs are made, and 'weak sustainability' when trade-offs are made. It is thought that the idea of weak sustainability is a reason why environmental quality is still declining and economic growth remains key [1].

It has been believed by many that technology will continually find new ways to overcome resource scarcity and environmental damage. However, this confidence is slowly beginning to disappear [80]. The idea was first challenged in the book 'The Limits to Growth' which demonstrated that unbounded technological increase could only delay the degradation of human society, if current consumption and growth trends continued [120].

The notion that humans will be able to decouple economic production from environmental impact [74] persists today among academics. Other academics however, question the notion that technology will solve our future problems [162, 164, 163, 104, 32]. Some commentators have noted that due to our past exposure to great technical advances, we may have over inflated expectations of what can be achieved by technology [35]. It is often feared that renewable energies are going to be too insufficient or too costly to be able to meet the ever rising global energy demand.

The mood of these authors is neatly captured in the following quote; "Unfortunately, our collective dream of a technological salvation beyond peak oil tends to rest on such frail foundations. With all due respects to light bulbs, after fifty years of rhetoric about solar power it would be fortifying to finally see locomotives, tractors, or bulldozers propelled by the sun" [82].

A tough challenge for people pushing sustainable initiatives is that in democratic countries the will of the government is in principle derived from the will of the people. Therefore sustainability initiatives produced by governments can only be enacted if the population desires it. If sustainability goals are not met, a potential point of hope is that much of the developed world's consumption of goods is going towards luxury items, and that the basics of human welfare will hopefully still be met comfortably if we are willing to reduce our consumption of these luxuries.

1.2 Modelling

Human beings are problem solving creatures. Whenever we identify a problem, we toil away at the issue until we have generated a solution. Before the scientific revolution we looked to sacred texts for guidance, now we try and solve problems through the study of the natural world [71]. To solve the problem, it is broken down into many smaller parts (i.e. reductionism) and then each individual part is studied independently to understand the working of the whole system [80].

An important aspect the study of the natural world is the development and testing of models and theories. We can define models as a formalised representation of our conception of the workings of the natural world [88]. Once a model has been established, it can be used to generate hypotheses, which are subsequently tested in order to try and invalidate the model. A model is kept until a problematic discrepancy between model and reality is observed, at which point the model is either edited or completely re-invented [99].

Models are a tool to help humans analyse and understand the world around them. In terms of sustainability, they can increase our understanding of the workings behind the complex problems facing mankind [160], and produce possible avenues for intervention. This manipulation of models removes the huge barrier of conducting real world experimentation. Using a model removes much of the time and monetary costs of real world experimentation as well as ethical barriers of human examination [109]. However, a draw back associated with models is their inability to perfectly replicate the real world.

All models are different, however they often share common traits. Two key characteristics of models are: they give a simplified version of reality with only critical aspects included, and they can be tested and validated [160].

Many researchers have devoted much time and effort to developing models that capture the dynamic nature of an entity or entities over time. The entities modelled could be (but not limited to) economic, environmental, resource, human, or a combination of these. These models are designed to further the knowledge of how the entities behave in the real world and how they can be better controlled or understood.

1.2.1 Model Development Steps

There are many steps in the process of creating a model. In this case we are using the term step to signify a part of the general process in model creation. The first step is the formalisation of the theory believed to explain some sort of real world phenomena. This will usually involve a written description of the model and can involve the production of mathematical equations to describe the interactions of objects. In this form the model is still relatively conceptual, however it has materialised to the point of being able to be communicated to other people. At this point it can also be used to generate hypothesis about the behaviour of real world.

Another step in the creation process is the evaluation of model parameters. This involves the collection of real world data to calculate the values assigned to the model's parameters. This can sometimes be a difficult task as often data from the real world does not perfectly map onto the parameters. For example, it would be difficult to find data to evaluate a parameter that signified the rate of food consumption to house hold income.

A further step in the model creation process is the comparison of the models output to real world data. This is the validation step and it is very important. If the model does not accurately represent the real world, then it's usefulness in informing future decisions is low. If the model is found to be unable to account for real world behaviour, the model creator will either go back to the first conceptual step and modify the original theory, or potentially start a new theory from scratch. If the model predicts real world behaviour for some cases, it might be kept and some caveat will be attached to the model to signify under what circumstances the model is acceptable for use [99].

Once trust has been established in the model, the final step in the model's life cycle is its application to predict or examine hypothetical scenarios. This stage can be thought of as the probing stage, where users of the model can query it and mine information or knowledge out of the model in the form of predicted outcomes.

Validity

Validating a model is of great importance [17] as it allows the user to have confidence in the output of the model. All model output will contain error, as no model can perfectly represent the real world. This is particularly prominent in complex socio-ecological models [96] due to the simplified nature of these models compared to the real world. Knowing the magnitude of this error is of great importance for understanding the level of trust one can put in the output of the model [17]. It is important for policy makers to understand the potential for error in their decisions which may have been informed by a model [57, 17, 176].

Error can be introduced into a model in three different ways. These are model structure, parametrisation of the model, and the assumed initial conditions or input variables [96, 176, 20]. The first is through errors in its formulation. A model may knowingly or unknowingly exclude or include a feature, or poorly represent a feature of the system being studied. This is a tough issue to analyse using current standard statistical approaches for examining model error [20]. Another way is through errors in the calibration of parameters. The final way error can enter into the output of a model is through error in the assumed initial conditions or input variables. These three error types are depicted in Figure 1.3. In some cases error may be tolerable if the purpose of the model is not for forecasting, but rather demonstration or educational purposes [96].

The most basic and important method of investigating model validity is by qualitatively comparing the models output to real world data [17]. Sometimes this data may be unavailable, or will not be know until future events have occurred [96], in which case

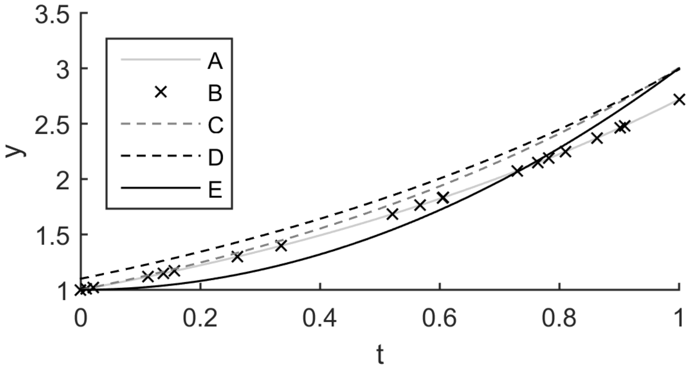

Figure 1.3: Different ways error can enter a model. (A) A real world phenomenon which follows the function $y = ae^{bt}$. (B) Real world data measured through experimentation. (C) Example of parametrisation error such that $y = ae^{\lambda bt}$. (D) Example of initialisation error such that $y = \lambda ae^{bt}$. (E) Example of structural error such that $y = c + dt^2$. In this example $a = 1$, $b = 1$, $c = 1$, $d = 2$, and the error fraction $\lambda = 1.1$.

verification can be effectively impossible. While this form of validation will give a starting foundation for confidence in the model structure and its calibration, there are still many other questions left unanswered by this method. For example, the sensitivity of the output to any one particular parameter is left unknown by this method. When it comes to calibration, it should be considered weather the model has been trained to fit a single set of data, and if it can match another set of real world data accurately [17]. This aspect of model validation comes under the headings of cross-validation and bootstrapping.

Two common analytical methods used to examine parametrisation and initial condition error are uncertainty analysis and sensitivity analysis [17]. Both are very similar in nature, however, uncertainty analysis is focused on the output error of a solution, while sensitivity analysis is concerned about the sensitivity of the output to error from each input or parameter. Uncertainty analysis is of particular concern for decision makers using a model as they are generally only interested in how much error might be in their decision. Sensitivity analysis is important for model creators and developers as they are interested in producing more accurate predictions and refining the model where possible.

Sensitivity analysis is performed for a large variety of reasons. Sensitivity analysis can be used to determine the sensitivity of the model to parameter changes, importance of input variables, the accuracy of the model, and redundant parameters [145, 11, 135], all of which are important aspects of the model creation process. Careful sensitivity analysis reveals the error attributed to each input, thus showing which inputs should receive attention in refinement [134]. In some cases, sensitivity analysis coupled with a small extension of the model can produce a risk analysis [177] if the cost of all outcomes is known.

There are many methods for analysing the sensitivity of a model. These can be grouped into three categories [145]: factor screening - each input variable is varied by an “extreme” amount and based on the outputs, a ranking of importance of input variables is established; local sensitivity analysis - each input variable is varied (one at a time) by a set fraction of its nominal value, which is then compared to the fractional change of the output; and global sensitivity analysis - a large set of input variable values (usually dictated by a probability density function) are put through the model to establish the range of uncertainty for the outputs.

The major difference between local and global sensitivity analysis is the extent to which the input parameters are varied. In local sensitivity analysis, each parameter is changed and its effects measured while holding all others to their nominal value. In global sensitivity analysis all input parameters are changed at once and the effects on the output are recorded. This process is repeated until a clear probability distribution of the output(s) is generated [145].

Because global sensitivity analysis is the most time consuming [134], most investigations are limited to factor screening[2] or local sensitivity analysis. Local sensitivity analysis produces a basic understanding of a model's sensitivity to input parameters. For non-linear models, it is particularly important to perform global sensitivity analysis to obtain the most accurate understanding of the model's error [176].

Uncertainty analysis is very similar to sensitivity analysis, the only real difference being the level of concern about the source of the error. The main purpose of uncertainty analysis is the assessment of how much error is expected to be in a solution produced by a model. The most common technique for achieving this is the Monte Carlo method. This method involves running the model many times, each time picking the inputs and parameters based upon probability density functions of the inputs and parameters [176]. The output of each model run is collated and then converted into probability density functions [176]. The major hurdle in performing a Monte Carlo analysis is the identification of the probability density functions of the inputs and parameters. If these are easy to calculate then the analysis is relatively straight forward [177].

Uncertainty analysis can also involve the altering of the model's structure to investigate how the structure affects the error in the model's output [96]. This approach, however, appears to be rarely performed as it requires large amounts of time and effort to conduct. Thus the model structure must be judged by the creator as being the best it can be given the constraints of the model creator [17].

The sampling of the parameter/input space can be more complex than pure random sampling. If simulation runs are computationally heavy and time demanding then quasi-random sampling methods may be required to help ensure good coverage of the sample space. Some quasi-random sampling methods include; fractional factorial sampling, Latin hypercube sampling, and multi-variate stratified sampling. Another consideration may be influential parameters, i.e. the value of one input parameter may be influential on another parameter, thus the two parameters cannot be sampled as if they are fully independent [144].

Probing

The ultimate aim of most models is to use them to discover insights into the issue it was designed to investigate. The insights might be new understanding of how the system behaves, or it might be to identify the best arrangement of parameters to achieve a set of goals. There are two primary ways in which this can be accomplished. The first

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[2] Factor screening is very similar in nature to scenario testing.

way is by manually changing parameters and input conditions and observing the changes in the results. This approach is the most common when the aim is to learn how the system behaves. The other method is to use computer algorithms to test combinations of parameters and inputs to find some optimal arrangements to meet predefined goals. This practice is called optimisation analysis.

Optimisation analysis can be time consuming, as it requires another level of analytical work to be put into the modelling process. It should be noted that setting up optimisation analysis requires a modeller to define functions which describe objectives. This is a valuable practice as it can refocus the modeller's attention back onto the objectives of their research (i.e. the issues the model was likely to be created to investigate). These objectives can become obscured or lost during the model creating process, and thus defining objectives can help ground a research project.

As we become more computer literate, the amount of real world problems being converted into computational optimisation problems has increased. A few examples include: the selection of regions for forest habitat creation (to minimising costs and distance between connected reserves) [10], catchment rehabilitation plans [78], optimal power generation blends (to minimise energy generation costs and environmental impact) [15], and pandemic modelling [current life]. Probably the most ubiquitous (and often unnoticed) optimisation problem that has been computerised is that of path finding between two locations in a city. This is performed millions of times each day by people using GPS directions through either their mobile phone or car navigation system.

Optimisation can aim to optimise a single objective or multiple objectives. The algorithm will often (especially if the number of potential inputs is large) attempt to find quasi-optimal solutions. The solutions are often quasi-optimal because the true global optimal solution may be too computationally expensive to compute. The field of optimisation algorithms is large and diverse. Various techniques have been developed such as: particle swarm, differential evolution, gradient (hill climbing), ant colony, simulated annealing, genetic algorithm, and bio-geographical-based [151]. Optimisation algorithms are very popular for solving complex optimisation problems as they are often more suitable to use than analytical or brute force approaches [40]. While not guaranteed (unlike analytical and brute force approaches), multi-objective optimisation algorithms are usually effective at discovering converged (close to true) and diversified (mostly discovered) Pareto fronts [75, 40]. A more comprehensive covering of genetic algorithm optimisation analysis is given in Chapter 4.

In some circumstances, the structure of the model is changed to examine how the model reacts under a different set of assumptions. This is vary rarely done, as changing the structure of a model can be a cumbersome process. This is why most examination involves simply the varying of parameters and inputs.

1.3 Policy Making Based on Models

To evaluate is to make a judgement. Everyday, humans have to evaluate aspects of their lives and make decisions about what to do next. Some decisions are relatively trivial and thus require minimal evaluation, such as deciding what to eat for dinner. Others however, have far greater consequences and thus require highly detailed evaluations. The decision of moving to a new town to find employment brings with it many considerations. How will the cost of living differ? Will there be work available in the long term? Will it limit contact with family? These questions, and more, have to be evaluated before an educated decision can be made on the most appropriate course of action.

Today, governments and organisations are increasingly evaluating their policies in the context of sustainability. Evaluating sustainability requires special assessment which takes into account economic, social, and environmental dimensions [150, 7, 4, 175, 66, 152, 1, 120, 160, 20]. Sustainability issues "[represent] a global, collective decision-making problem unprecedented in scale and complexity" [61]. For policies to be effective in all three dimensions, they must be well informed. Models are excellent tools for helping governments and organisations analyse the complex problems facing mankind [160, 176, 20]. Models can help guide predictions of future trends and investigate avenues for intervention. This in turn develops better informed policies [160, 88, 176, 96, 98].

It is of great importance to have accurate knowledge of how to best solve a problem. Sometimes we may implement plans which we believe are the correct course of action, however this may be a false belief. An archaic and extreme example of this would be the sacrificing of people to bring about a good harvest. This point is made by Wynes and Nicholas in [189], and is demonstrated through Figure 1.4. The impact various courses of action have on the reducing CO_2 emissions is presented in (A). How prolific these issues are in high school science textbooks is presented in (B). It can be noted that the efficacy of the emissions reduction activity does not correlate with the prevalence to which the activity is discussed (high impact solutions are often overlooked), hinting that students may develop false ideas about the most effective reduction strategies or become overly focussed on low impact solutions.

The value of modelling depends on a simple question, "what is the best alternative to understanding the problem or making a decision?". If the alternative is expert opinion, modelling may look attractive if the issue is complex, as it is very difficult for one person to account for many factors and mentally compute future scenarios [98].

Van den Bergh and Nijkamp have noted that "there is a strange relationship between time and uncertainty. Over both very short and very long time periods the final outcome is, by and large, more predictable than for periods of intermediate length" [177]. They also note that "long-term uncertainty is usually perceived with reservation. Faith and hope for intermediate solutions and favourable turns will support this stance" [177]. Given the gravity of the current sustainability issues facing the world, it is important that the likelihood of long-term prospects is well understood. If models show a high level of uncertainty in long term predictions, then having faith and hope could possibly be justified. However if the long term solutions are relatively certain then serious consideration needs to be made as to whether actions need to be taken or not.

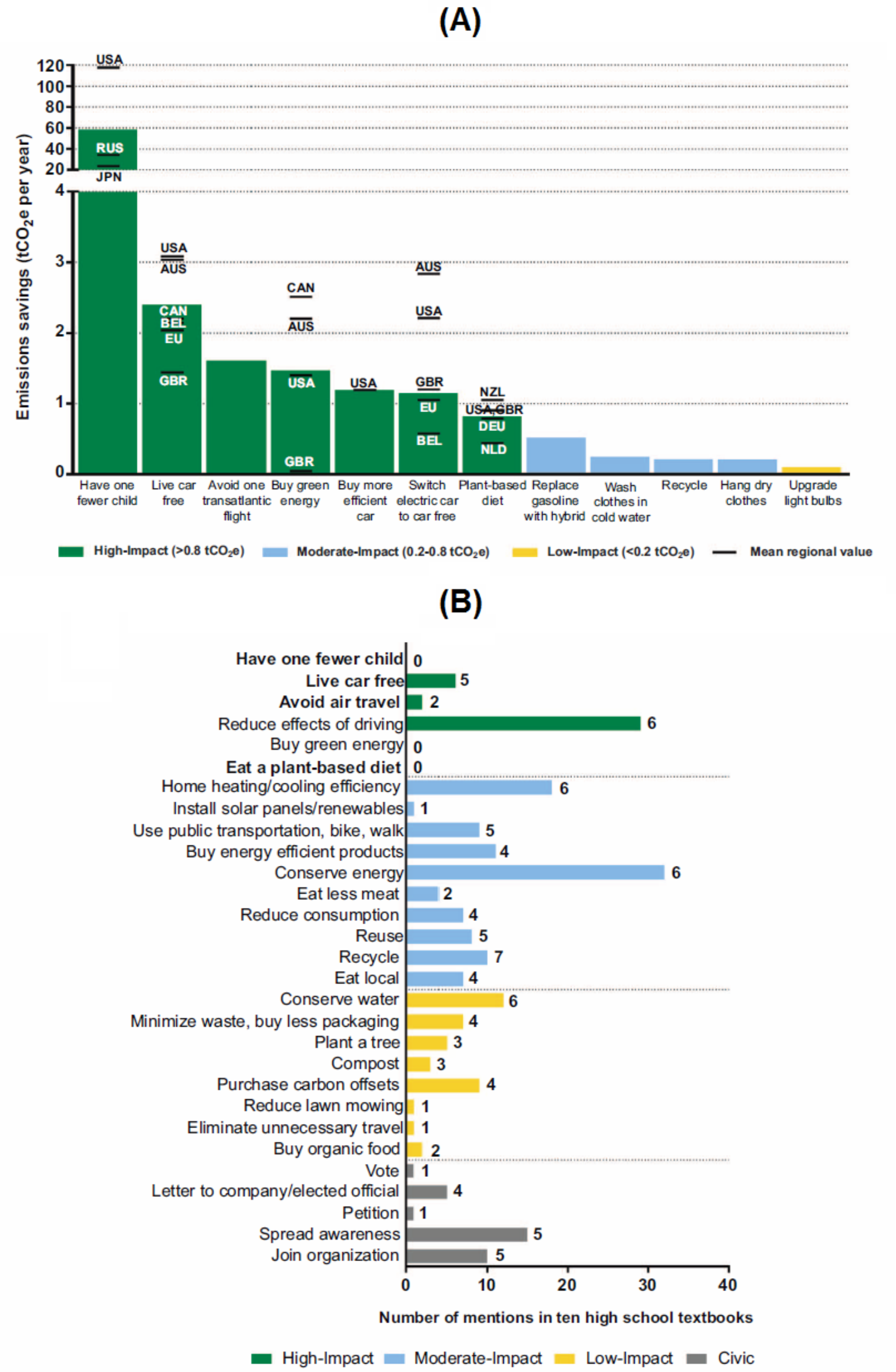

Figure 1.4: (A) Reductions in yearly CO_2 emissions for various activities. (B) Frequency at which the activity is discussed, and number of unique textbooks which discuss the activity (based on a sample of ten Canadian high school science textbooks). Figure sourced from [189].

1.3.1 Importance of Holistic Approaches for Policy

Many models in the sustainability domain focus on a very niche problem or area of concern, e.g. optimal mangrove and fish management [63], product uptake of carbon labelled products [191], efficient trading heuristics within a local economy [194], or factors effecting farmers land management decisions [158]. Others such as the Limits to Growth model [111] take a far more holistic perspective. While specific studies are very important in developing a very clear understanding of a niche problem, in the context of sustainability, they are unable to account for the wider context of which the problem is a part. A life cycle analysis can indicate better products for development, however the analysis cannot place the product in the broader context of the entire world economy or its potential over-all impact on the environment if all consumers purchased it. Life cycle analysis also assumes no large structural changes to the economy; it's relevance is for marginal change. Because of these limitations it is important to undergo efforts to create models which aim at taking a holistic perspective of our society and the wider environment [80].

Meadows et al. in their book "The Limits to Growth" [111] commented on the issues most people thought about. This was depicted in Figure 1.5. It illustrates how as time span or scale increase, the number of people concerned with the issue decreases. They posit that most people in the world are focused on small issues that affect themselves or maybe their local community within the next week to few years. This is a reasonable proposition as large scale and long time horizon issues can feel largely outside of the control of most individuals. This graphic encapsulates how large complex issues can go addressed for long periods of time, often sitting in the 'too hard basket'.

The issues defined as "sustainability issues" are often very broad and complex. Thus the widest scope should be used to understand the issues facing our world. Much knowledge is very niche and occupies a small space of all worldly knowledge, and so collaboration is important to understand sustainability issues [48]. As Herman Daly and Joshua Farley noted, "real problems in complex systems do not respect academic boundaries" [169].

Sustainability is a highly complex subject to study when viewed holistically. It can be studied through multiple academic disciplines and thus from very varied perspectives. Simpler models such as Ehrlich and Holdren's IPAT model fail to capture the full complexity and thus have to make way for more detailed analysis on longer time scales e.g. system dynamics modelling [129]. System dynamics models (see section 5.1.2 for a description of system dynamics modelling) are thus well suited for the study of sustainability as they handle complexity with relative ease and can be applied to numerous disciplines [20, 66, 80].

The combining of different academic disciplines into a single model can be defined as "cognitive integration" [20]. Numerous authors have called for greater cognitive integration in sustainability modelling [80, 160, 177, 129] thus increasing the appeal of system dynamics as a preferable method of modelling. Because the system dynamics modelling process is relatively easy to understand a model can be collaborated on by multiple parties (even individuals outside academia). Another benefit is that the assumptions that produce the base equations are formulated independent of the model's behaviour. It is therefore

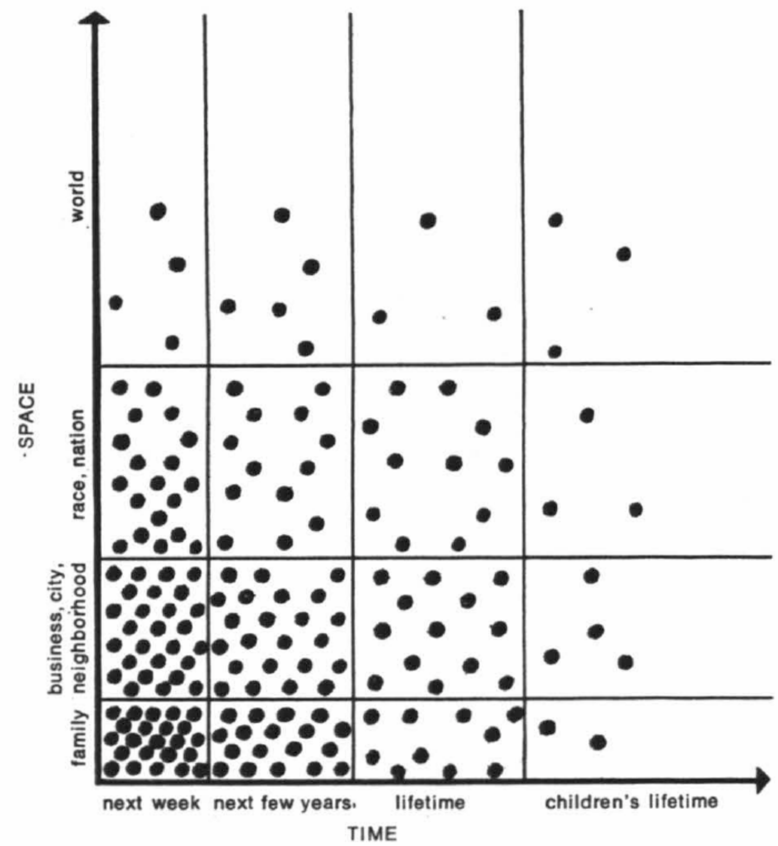

Figure 1.5: Illustration of the number of people concerned with various issues with differing magnitudes of time and scale. Figure sourced from [111].

easier to get agreement between parties on the merit of the model [7].

Cognitive integration is often a tricky task, as our beliefs and biases shape our views on the world, and when different people collaborate often their views may not mesh. The opinions on what should be done to help humanity and what will happen in the future are very wide ranging. Proponents of free markets and enterprise will argue that a free market can solve our problems through encouraging entrepreneurial developments and product substitution. Environmentalists will claim we need to place restrictions on corporations to limit their waste generation, and invest in renewable energy or processes. Climate sceptics will say their is nothing wrong and claim the climate has always been changing. Technologists will claim that scientists will discover new technologies and means to resolve the issues we come up against. It is very hard to tell to what degree each view point is correct. Knock-on effects are very difficult to foresee. An example of knock-on effects was The Dust Bowl created in the US in the 1930's. Rapid growth in the agricultural sector triggered by the First World War [56] followed by drought meant that many new and heavily indebted farms had to foreclose, leaving huge areas of land bare. This created the perfect conditions for wide spread dust storms and is regarded as a disaster caused by knock-on effects of human decision making coupled with unforeseen ecological consequences.

No single person has a complete understanding of the world [154] and thus we cannot hope to forecast into the future without the aid of computational models [5]. Even then models can only ever be a crude approximation of the world. However, they can be far superior than even the most intellectual person speculating[3], as people are incapable of calculating in their mind the evolution of a system with many feedback loops and layers [92]. This idea is captured in the quote, "the march of history is really more like a drunkard's rambling walk. No matter what the wisest and most powerful leaders carefully plan, history has a habit of going it's own way" [56]. While physical and environmental systems are far more predictable than human history (revolts, wars, etc.), the quote is still generally applicable as we cannot know exactly what technological breakthroughs are going to be encountered in the future.

Currently many socio-ecological models do not consider the decision making processes undertaken by people, and that people are not always logical. To take these processes into account requires a great deal of collaboration between economists and ecologists with social scientists [5]. Because the course of history is shaped by our decisions, it is important to examine the choices people will make of their own free will, given various conditions of their surroundings. With the integration of many disciplines in the creation of socio-ecological models, there needs to be a corollary integration of disciplines in the interpretation and examination of results [20].

Van den Bergh and Nijkamp neatly summarised in [177] the key features a model must include if it is to come close to accurately depicting our socio-ecological world. These features were laid out in 9 key points and were described as minimum requirements. They have been summarised here as follows:

1. A full description of the economic structure as well as the environment.
2. Economic activities must be linked to extraction of renewable and non-renewable resources as well as the releasing of wastes to all spheres of the environment.
3. Feedback to the economy from perceptions of ecological health, i.e. governments enacting regulations based upon ecological health indicators.
4. Wherever possible the model should take into account non-material measures for the environment (e.g. soil quality) and human development (e.g. human development index).
5. Inclusion of multiple generations.
6. A long time frame.
7. Non-linearities must be able to be represented, for example irreversibilities, tipping points and time delays.
8. Physical realities about the production of items must not violated. Substitution of materials in product production must be kept within reasonable bounds.
9. Consideration of human welfare should be explicitly taken into account.

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[3] As well as other benefits, such as providing a means for rigorous and quantitative exposure of assumptions and relationships.

We can see that it is quite an extensive list and creates a demanding job for anyone attempting to generate a model that adequately cover all of the aspects.

1.4 The Limits to Growth Model

Since the beginning of the modern environmental movement, computer modelling has played a role in examining socio-ecological problems [54]. The advent of the computer gave humankind the ability to test models which require many calculations over a large number of iterations. One of the first and most influential models was Forrester's "Limits to Growth" model (often referred to as World3) [111]. Since the publication of the Limits to Growth book in 1972, many more models have been developed that advance the work of Meadows and his MIT team.

Forrester's World Dynamics Model which formed the basis of Meadows' book "The Limits to Growth" has received much attention since the books publication in 1972. Some consider it the "most important work of Neo-Malthusian[4] literature" [147]. The model combines "concepts from demography, economics, agriculture and technology" [11]. The model's primary objective is to analysis the dynamic relationships between global human population and it's resources; where the definition of resources is extended to man-made capital [112]. The model can be broken up into five main sections: population, capital, agriculture, resources, and pollution. Each sector is connected via feedback loops, making the model dynamic over time. The interrelationships between these sectors can be seen in Figure 1.6 [112, p. 29].

The Limits to Growth model is a type of model falling under the category of system dynamics modelling. System dynamics is a modelling technique that allows for a structured representation of complex systems [96]. This method allows for the study of real world systems through the representation of stocks/amount (system variables), that have in and out flows (e.g., births and deaths of a population) that are dependent on other variables and external parameters [37, 20, 96]. System dynamics modelling allows for analysis of highly complex systems without complicated theory or mathematical equations [80]. Very simple and linear relationships, when combined, can create highly complicated behaviour. System dynamics models can be easily expanded to give a holistic representation of a system [80]. Through the development of a holistic representation, a better understanding can be gained of the represented system. The simplistic nature of system dynamics models allows them to be communicated to a wide audience including academics outside the field being studied, political leaders, and the general public.

The purpose of the Limits to Growth model was to provide "imprecise projections" of the dynamic behaviour of the world system. This is due to the unpredictable nature of social systems and also the lack of data available to the researchers. However, while imprecise, the knowledge gained about the behaviour of the system is valuable for policy makers. Decisions regarding "population control, energy consumption, and investment in new technologies" can better informed through the studying of the model [112].

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[4] Malthusian philosophy is based around the theory of inevitable scarcity due to population growth during bountiful periods of time.

The parameters in the model were derived through the collection and analysis of a large amount of published data. The details of the data, analysis process, and justifications used in the parameter selection can all be found in the book "Dynamics of Growth in a Finite World" [112]. While great efforts were taken to get the most accurate data, sometimes this

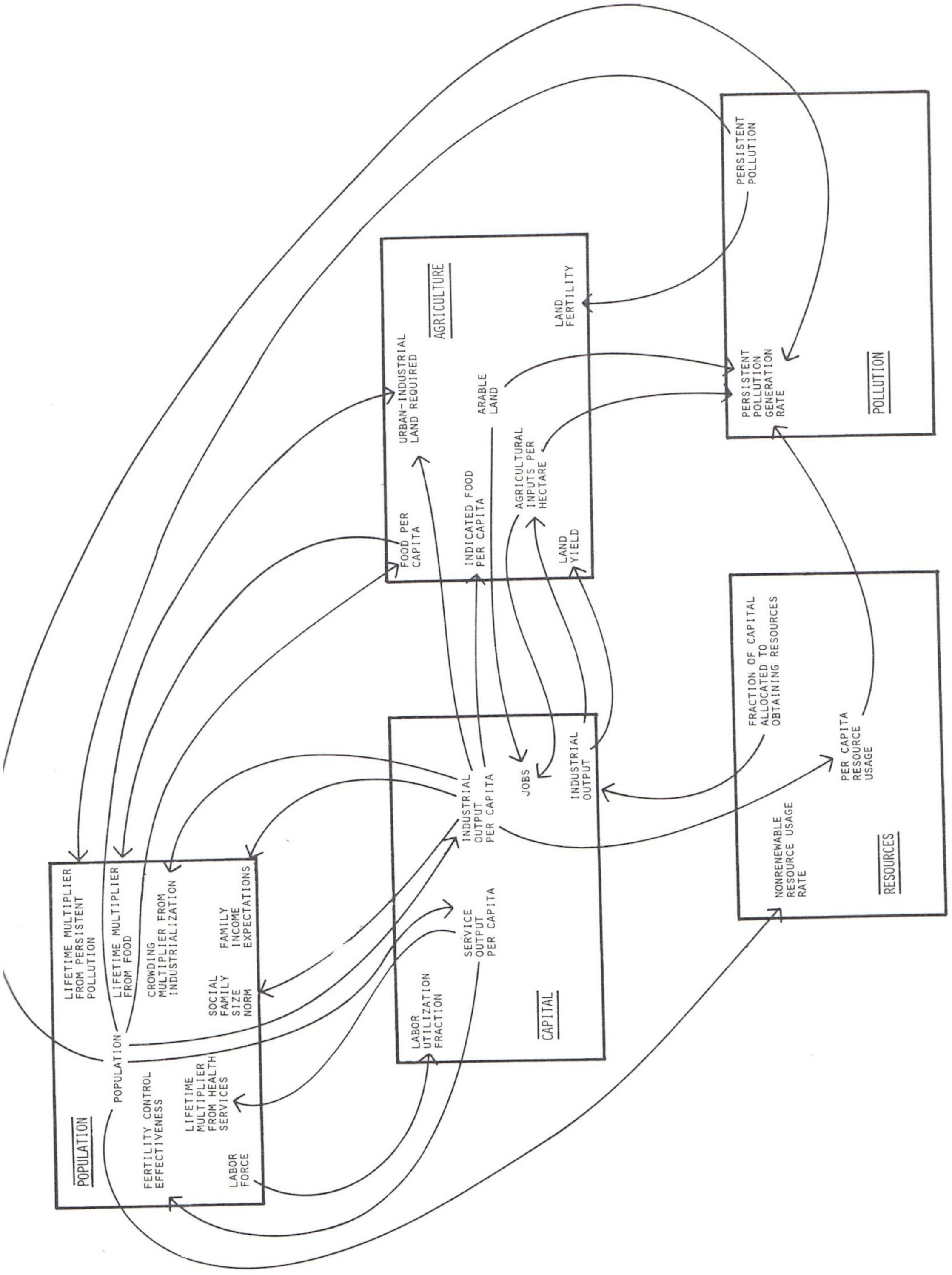

Figure 1.6: Diagram showing the interrelationships between the 5 key sectors. Figure sourced from [112].

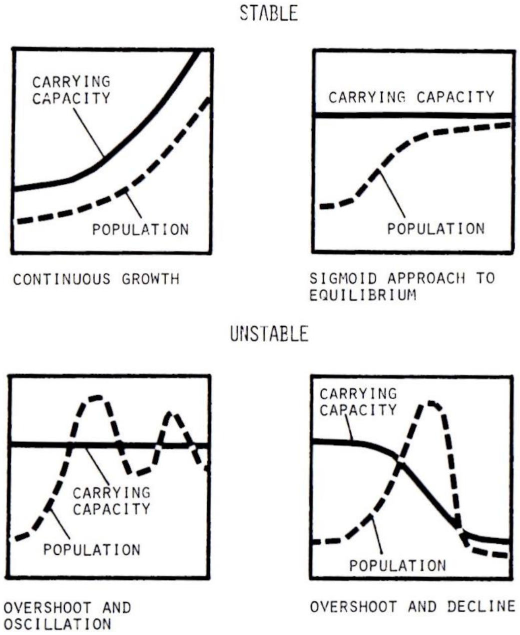

Figure 1.7: Diagram showing four possible modes of population growth into the future. Figure sourced from [112].

data was scarce, of low quality, or hard to find due to the peculiar nature of some model parameters (e.g., desired complete family size, initial non-renewable resources, persistent pollution decay rate) [44, 112, 133].

Meadows and his team produced the illustration shown in Figure 1.7 [112, p. 8] highlighting some possible trajectories for the worlds population and the Earth's long-term carrying capacity. These were classed into two groups: stable and unstable. If the population does not exceed the carrying capacity then the growth of the population is classed as stable, otherwise it is unstable.

The results of the model showed that current trends (standard run) would lead to sharp deterioration of the global communities welfare. Resources would become scarce and the carrying capacity would decline, in turn bringing about a sharp reduction in the human population as depicted in the bottom right scenario of Figure 1.7 [111]. This scenario was dubbed the "standard run" or "business as usual".

To test the behaviour of the model, parameters were changed, e.g. to simulate different social behaviours, resource conditions, environmental policies or technological advances. A set of ten scenarios were presented in the book "The Limits to Growth". While each set of changes in parameters brought about a different result, the general shape of trajectories and outcomes were similar to the "standard run", unless drastic changes were made in a large set of parameters covering technological and social factors. An example of a drastic change was to limit family size to two children and capital investment be equal to capital depreciation [111].

The output of the "standard run" of the original World3 model is shown in Figure 1.8. The simulation shows that population, food per capita, and industrial output per capita would all rise as crude birth and death rate both fell. As industrial capital grows, the

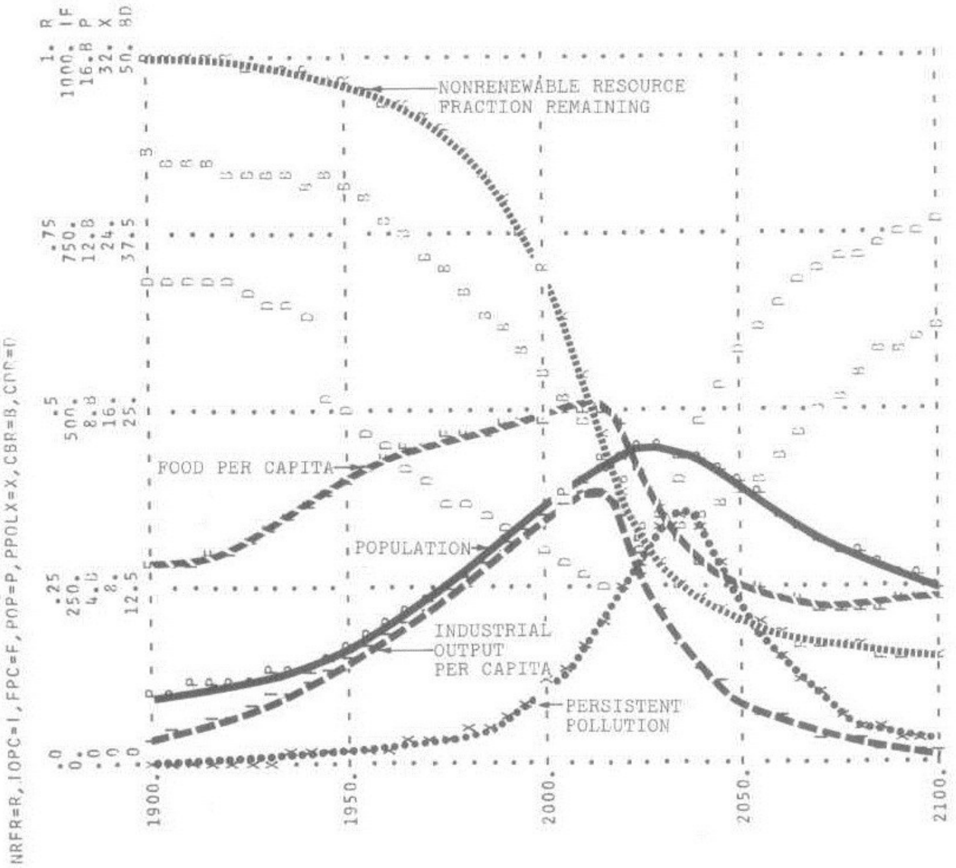

Figure 1.8: Output from the "standard run" of the World3 model. Figure sourced from [112].

rate of consumption of non-renewable resources increases and persistent pollution grows. Eventually these trends reverse (around the year 2010), as the amount of industrial capital rapidly declines due to constraints on industrial capital investment. This is due to an increasing difficulty in extracting non-renewable resources (as only lower grade materials are accessible), and the ever increasing needs of industrial output in the service and agriculture sector. Beyond this point, the previous trends reverse. Food per capita and population both begin to decline as the crude birth and death rate begin to rise. Persistent pollution eventually begins to fall approximately 30 years after the peak in industrial output.

The World3 model underwent severe scrutiny when it was first published. Bardi describes the reaction to the study as "harsh" [13], with many critics dismissing it as a flawed study. The debate surrounding the model quickly turned from scientific to political and emotional [92], and the model had a difficult time gaining respect [147]. Often the sensitivity of the model would be called into question as small changes to parameters could produce large changes in the output [44].

Forester was one of the first to use system dynamics models in terms of sustainable development. This work was published in the book 'The Limits to Growth' [120]. Unfortunately the work published in the book was taken as a prediction of what would happen, rather then a description of what could possibly happen [80]. At the time, this misunderstanding fuelled much backlash from mainstream economists and thus system dynamics fell out of favour with economists and political science [20].

Criticism of the model has died down as more data becomes available to validate the model. The work of Graham Turner showed that data collected from 1970 to 2010 matched the standard run scenario of the model well, with a further note that “there do not appear to be other economy environment models that have demonstrated such comprehensive and long-term data agreement” [167].

Interest in the Limits to Growth model has been long lasting. It has been noted that there has been a renewal in interest in the Limits to Growth model [167], even after 40 year after its initial publication. Examples of this renewed interest include [69, 169, 167, 133, 13, 97].

Chapter 2

Uncertainty Analysis of the World3 Model

Abstract

Uncertainty analysis is an important step in determining the reliability of a model. Models which are used to determine policies or guide decisions must be reliable to ensure sound choices are made. The “Limits to Growth” model by Donella Meadows and colleagues was one of the first computer models to investigate global issues of population growth and resource constraints. The model received much attention and criticism, sometimes being accused of being too sensitive to variations in input parameters. This paper studies the model’s sensitivity to input error through an uncertainty analysis, and examines if this sort of analysis could have affected the debate surrounding the model’s reliability and usefulness. Results showed that given the data used to calibrate the model, the output was susceptible to large variations, with the population variable returning a normalised standard deviation of 0.43. However, despite input error, the trends of the variables remain predictable.

2.1 Introduction

2.1.1 Uncertainty Analysis

Given that socio-ecological models are aimed at informing policy makers, it is important to ensure the sensitivity of each model is well understood [98, 96, 176]. Most socio-ecological models are non-linear, involving many feedback loops, thus small perturbations in parameters can induce large changes in the outputs [145, p. 68]. This property exacerbates the need for a global sensitivity analysis when working with socio-ecological models.

2.1.2 Previous Uncertainty Analysis and the World3 Model

There have been several sensitivity studies performed on the World3 model. The team that developed the World3 model, lead by D. L. Meadows, intuitively tested it by manually varying parameters and inspecting the changes in output. Many other researchers conducted their own tests in the same fashion as Meadows' team [31]. The work of Austin and Cottler [11], Vermeulen [182] and De Jongh [44] are clear examples of methodical and rigorous analysis. However, all of the analyses to date[1] have been local sensitivity analysis studies which have only provided a basic understanding of the models behaviour and characteristics. The reason for limiting the sensitivity analysis to simple local sensitivity analysis is due to the "enormity and laboriousness" of global sensitivity analysis [31, 76].

A study performed by Vermeulen and De Jongh [182] found that some of the most sensitive variables were Life Expectancy Normal (LEN), Reproduction Lifetime (RL), Desired Completed Family Size Normal (DCFSN), Industrial Capital-Output Ratio (ICOR), Average Lifetime of Industrial Capital (ALIC), Fraction of Industrial Output Allocated to Consumption (FIOAC), Non-renewable Resource Using Fraction (NRUF), Land Yield Factor (LYF), Land Fraction Harvested (LFH), Inherent Land Fertility (ILF), Persistent Pollution Transmutation Delay (PPTD). This was achieved through local sensitivity analysis, i.e. altering each parameter by a small percentage and comparing the percentage change of the main variables.

These variables were then changed in combination and by 10% to examine plausible responses of the model. Figure 2.1 shows some of the results obtained by [182]. From these results, Vermeulen and De Jongh claim that "[b]y changing three parameters by 10% each in 1975 the world population collapse predicted by the model is averted" [182].

Two years later, a continuation of the study was published but had rephrased their statement to say "(b)y changing three economic parameters by 10% each in 1975 it is possible to postpone the population collapse to beyond the simulation interval, i.e. to beyond the year 2100" [44]. It is quite evident from Figure 2.1 that the collapse was not conclusively averted as claimed by the Vermeulen and De Jongh in their original statement. It also seems that the combination of parameter changes effectively constitutes an alternative scenario, similar to others published in the Limits to Growth work. This

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[1] [41] tests sensitivity analysis features of OpenModelica software, and uses the World3 as it's dummy model. This paper was published in December of 2017 and was only discovered recently (post publication of our paper).

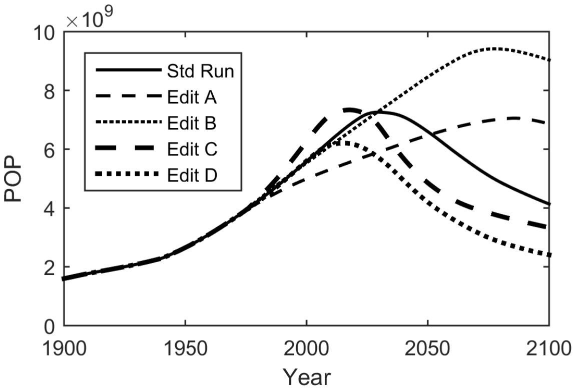

Figure 2.1: Sensitivity analysis performed by [182]. Edit A: ICOR and FIOAC increased by 10%, and ALIC decreased by 10%. Edit B: Edit A plus RLT decreased by 10%, and DCFSN increased by 10%. Edit C: ICOR and FIOAC decreased by 10%, and ALIC increased by 10%. Edit D: Edit C plus RLT increased by 10%, and DCFSN decreased by 10%.

also points to the importance of 'direction' of change in parameters i.e., weather positive or negative when made in combinations.

Austin and Cottler argue that if a model (or system) is sensitive to small variations in input parameters then the output is of "little practical interest" [11]. However, if the real world system, from which the model is based, is sensitive to small changes then, so too should the model. If it was not sensitive then the model would not accurately represent the real system. Thus it can be said the sensitivity to perturbations in inputs is not indicative of a model's validity.

As Turner points out, despite the sensitivity of the model the outcomes of overshoot and collapse are not averted as claimed by many critics [166]. He also raised the question that if the model was as sensitive as claimed, how could the data from 1972 to 2012 [167] resemble any alignment with the World3 standard run [166]. This question points out that a more holistic uncertainty analysis should be performed to better understand the models apparent alignment with real data.

While the results of previous studies gave some insight into the accuracy and behaviour of the model, the question of the model's true uncertainty was largely unknown. An uncertainty analysis was required to fully understand the model's susceptibility to input error.

A literature review, along with comments by Turner [166] indicate that to date, an uncertainty analysis has not been performed on the World3 model. The purpose of this study is to undertake an uncertainty analysis of the World 3 model to better ascertain its sensitivity to input error, and investigate the validity of past concerns about the models sensitivity.

2.1.3 Monte Carlo Analysis

One of the most established and well used methods for uncertainty analysis is the Monte Carlo method. A model is run multiple times using randomly selected input data. The output of each run is combined to form either a probability distribution function (PDF) or statistical data (e.g. mean and standard deviation). With further analysis the uncertainty of an output can be apportioned to each input variable [145].

The Monte Carlo process can be broken into 5 steps:

1. development of PDF for input variables;
2. generation of sets of input values;
3. running of the model using input sets;
4. analysis of the uncertainty of the output PDF;
5. sensitivity analysis of the models output with respect to input variables [145].

There are several variations of the Monte Carlo method. The variation in these methods is derived from their sampling process. Sampling can be random, stratified, or quasi-random. An example of stratified sampling is Latin hypercube sampling. An example of a quasi-random sample is the Sobol sequence [145]. While stratified and quasi-random sampling generally outperform random sampling, they do have drawbacks in some cases.

Often the major challenge with Monte Carlo analysis is the development of PDFs for the input variables. If the probability density functions are known, then the analysis is simple to implement [177]. Although, often the confidence of input data is largely unknown or widely varied and thus the error associated with each input is also unknown. However,"a crude characterisation may be adequate, especially if the analysis is primarily exploratory. ... [P]lausible arguments might be used to establish the ranges (of input variables)" [145]. It can also be noted that the ranges of the input variables often has a larger effect than the distribution type of the PDF [145].

An interesting addition to the Monte Carlo analysis is the comparison of solutions to historical data. If a solution trajectory deviated significantly from the historic data then it can be discarded as improbable and removed from the analysis [76].

2.2 Aims

The aim of this research is to evaluate the uncertainty of the World3 model due to input error. The results allow us to examine what could have been learnt by Meadows and his team in 1970 if the analysis had been conducted at that time. A major criticism of the model is its sensitivity to input changes, and so an uncertainty analysis is useful to inform modellers and users about the inherent characteristics of the model.

2.3 Justification

Some experts in the field believe that this study is worthy of pursuit. Turner wrote that "it would be valuable to have a complete sensitivity analysis performed on World3 outputs, if this complements broader understanding of the model behaviour and real-world developments. Since an exhaustive sensitivity analysis hasn't been undertaken and the implications of the [standard] scenario are considerable, it is appropriate that both the

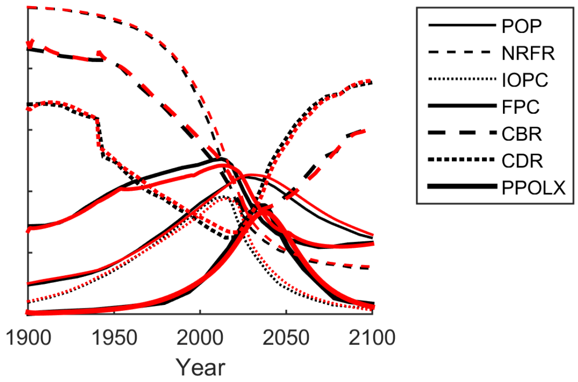

Figure 2.2: Validation of reproduced model operating correctly. Reproduced results (black), published results from Dynamics of Growth in a Finite World [112] (red). Error is largely due to low resolution of the original published graphs. 'Y-axis' values can be found in later figures.

close of my paper and Castro's critique advocate a "risk-based approach" [166]. Castro has explicitly stated that the sensitivity analysis of the World3 model is "incomplete" and that due to the uncertainties in the input value of the World3 model, sensitivity analysis is a "crucial property" [31]. As the World3 model is non-linear, it will undergo substantial changes in output values given small perturbations in parameters [31]. Shifts in the nominal values can therefore produce utterly different sensitivity analysis results [145]. This exacerbates the need for a global sensitivity analysis.

Given the comments of Turner and Castro, along with Turner's extensive modelling research [168], as well as this literature review, it appears that to date no global sensitivity analysis of the World3 model (or any other similar models) has been conducted. All previous studies have been local sensitivity analysis, leaving a gap in our understanding of the model's behaviour and allowing vast room for debate. While the results of previous local sensitivity analysis are interesting, they still leave many questions about the model's behaviour unanswered; for example, how much variation could be reasonably expected? Are the cases shown likely to happen? Are worse cases possible/probable? This is due to the inability of local sensitivity analysis to assess different variations of input parameters methodically [145].

2.4 Method

The World3 model was taken from the book "Dynamics of Growth in a Finite World" [112, p. 549]. The code was rewritten in the C++ language. To ensure the model was replicated correctly the Standard Run was executed and then plotted against the published results for the Standard Run [112, p. 501] as shown in Figure 2.2. The error between the results can be attributed to the low quality of the published plots and extracting this data to a digital form. From this plot it was concluded that the model had been reproduced to an acceptable standard for the purpose of this thesis.

A Monte-Carlo analysis was used to conduct the uncertainty analysis. The input Probability Density Functions (PDFs) were sampled and passed to the model one million times. Independence is assumed between all parameters. The PDFs for the World3 model parameters were derived through analysis of the data presented in [112], as this data was used to calibrate the model. Where possible, the standard deviation was calculated using the same data. The distributions were assumed to be normal or log normal for simplicity. The calculation of the standard error of the mean (SEM) followed the basic methods of calculation. However, some of the data needed to calibrate the World3 model was sparse or not available, making it difficult to calculate the standard deviation. For these cases a reasonable standard deviation was assigned to the parameter.[2] For parameters based on intuition or a single source of data, a SD of 15-20% of the parameter's nominal value would be allocated. Parameters based on two or three sources received 10-15%, and those based on three or more sources received 5-10%.

An important aspect of the World3 model is its inclusion of some parameters as table functions, i.e. a parameter depends on an input variable according to a predefined static transfer function. Tables are defined by a set of points T on a graph. Linear interpolation is used to determine intermediate points. This presents a major challenge when conducting a global SA, as the tables need to be easily changed with each simulation run.

For tabular variables the standard deviation SD_{data} was calculated according to equation 2.1, given a set of data points $D = \{(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)\}$.

$$SD_{data}(x) = \sqrt{\sum_{i=1}^{n} (y_i - T(x_i))^2 W_i(x)} \tag{2.1}$$

$$W_i(x) = \frac{\exp(ln(\lambda)|\frac{x_i - x}{x_d}|)}{\sum_{j=1}^{n} \exp(ln(\lambda)|\frac{x_j - x}{x_d}|)} \tag{2.2}$$

Here n equals the number of data points, λ is the shape factor for which we set to $\frac{1}{10}$[3], and $x_d = max(x) - min(x)$. W_i represents a weighting factor for each data point. The further away a data point is from x the smaller it's effect on the standard deviation at x. T is a table function.

Figure 2.3 shows the calculated standard deviation for the table input ISOPC (indicated service output per capita).

To assign a PDF to a table variable, a set of standard deviations (SD) was created. The ith element of SD corresponds to the ith element of T. Figure 2.4 shows a hypothetical table variable $T = \{(2, 0), (4, 100), (5, 200), (5.5, 300)\}$ with standard deviation of $SD = \{0.2, 0.3, 0.5, 0.6\}$. The four bell shaped curves show the probability density functions assigned to each element of T. In this example we can see that y increases with x, and the standard deviation assigned to this table variable also increases with x.

To sample the table variables PDF and generate a new table $\bar{T}$, a number P is chosen at random from the set $(0, 1)$. If the ith element of T is $T_i = (y_i, x_i)$ then the ith element

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[2] The assigned standard deviation of these parameters is essentially arbitrary. The values were chosen to ensure variation in parameters without changing them by orders of magnitude. To illustrate the quandary, imagine trying to assign a standard deviation to a set of data containing only one data point.

[3] A value of $\frac{1}{10}$ signifies that a variable with a distance x_d from x will contribute $\frac{1}{10}$ of it's value compared to a variable at x. This relationship is exponential with respect to x. The weighting function is an attempt to better describe the SD of the table function, and is not meant to be a rigorous formulation. In the future we want to apply a suitable confidence measure for these GAM models.

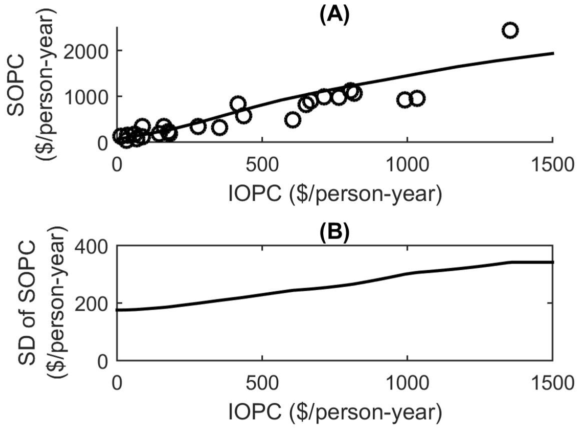

Figure 2.3: Example of the calculated standard deviation for a table variable. Plot (A) shows the location of data points (circles) along with the trend line used in the World3 model. Plot (B) shows how the standard deviation of the data changes with respect to x, calculated from equation 2.1.

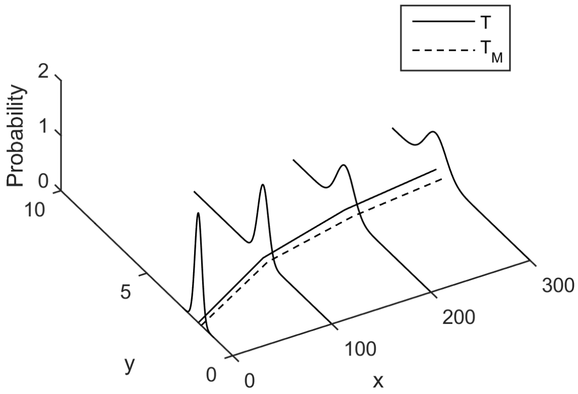

Figure 2.4: Assigning a probability density function to a table function. T (solid) indicates the original table function, $\bar{T}$ (dashed) indicates the altered table function. Bell shaped lines indicate the probability of $\bar{T}$'s position.

of $\bar{T}$ is defined as,

$$\bar{T_i} = (C^{-1}(P|_{y_i,SD_i}), x_i) \tag{2.3}$$

where C is the cumulative distribution function,

$$P = C(x|_{\mu,\sigma}) \tag{2.4}$$

with μ and σ denoting mean and standard deviation. C^{-1} is the inverse of C.

An example $\bar{T}$ was generated with P equal to 0.2. The new table has been plotted in Figure 2.4 as a dashed line. We can see that this new table is similar to the original table T but has been shifted slightly. As x increases $\bar{T}$ deviates from T due to the increase in SD. For every simulation, a new $\bar{T}$ is generated.

Because the number of data samples affects the standard deviation of the mean of the sample, the standard deviation of each World3 input SD_{input} was calculated according to equation 2.5,

$$SD_{input} = \frac{SD_{data}}{\sqrt{N}} \tag{2.5}$$

where N represents the number of data samples taken.

Table B.1 of Appendix B summarises the standard deviations assigned to each variable, as a percentage of its nominal value. For World3 table variables the average standard deviation is reported. The data for each variable can be found at the indicated page number in the book "Dynamics of Growth in a Finite World" [112].

For copies of the software please contact the author at aheath@uow.edu.au.

2.5 Results

In this section the data from the study will be presented in a variety of methods, that is, trajectory lines, probability density maps, probability density functions, percentiles and averages, and standard deviation. Each method allows for a different understanding of the results.

The main variables examined from the World3 model were human population (POP), fraction remaining of non-renewable resources (NRFR), industrial output per capita (IOPC), food per capita (FPC), crude birth rate (CBR), crude death rate (CDR), and persistent pollution normalised with 1970 levels (PPOLX). The units for these variables respectively are; people, unit-less, US dollars per person-year, vegetable equivalent kilograms per person, births per 1000 people-year, deaths per 1000 people-year, and unit-less. These variables will be the main focus of discussion.

2.5.1 Trajectories

Firstly, to understand the variety of outcomes, the trajectories of 100 randomly sampled simulations were plotted. This allows for a clear representation of how each trajectory behaves. This is limited to a relatively small number of examples (100 of the 1 million simulations) as the plot becomes overly crowded with the addition of more simulation trajectories.

Figure 2.5 shows the trajectory of the population in the World3 model for 100 runs. It can be seen that there is a wide variation of trajectories, however most follow a similar path to that of the "standard run" shown by the grey line. Some trajectories manage to reach a maximum in excess of 10 billion people. This is approximately 3 billion greater then the maximum of the "standard run". Some trajectories have very high values in the year 2100 compared to the "standard run".

One trajectory stays low for all years, showing how the model can react dramatically for some permutations of the variables. Two trajectories can be seen very slowly increasing for the whole simulation. This would indicate a favourable and stable future for humanity for the period simulated, that is, no sudden fall in population in the 21st century.

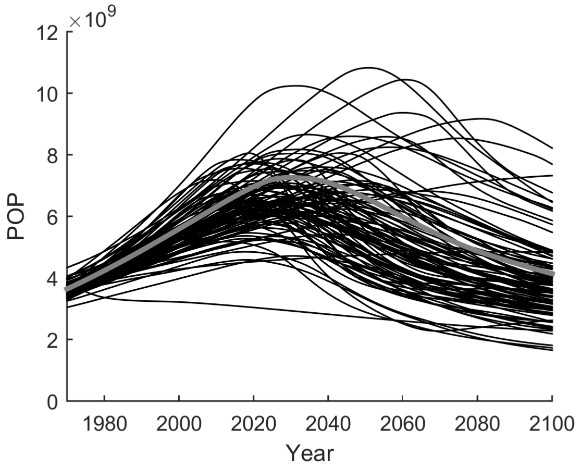

Figure 2.5: Population trajectories for 100 simulation runs of the Monte Carlo analysis.

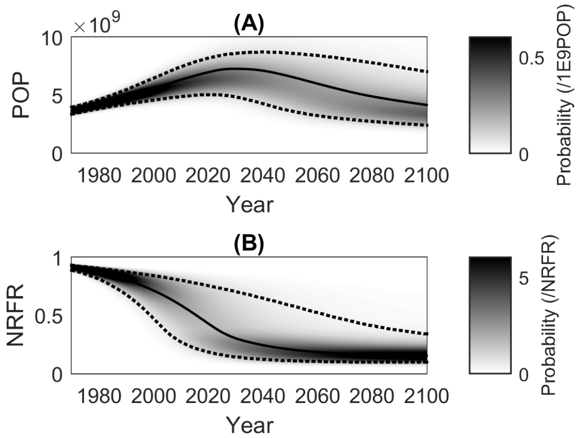

Figure 2.6: (A) - Probability density of population against time. Units of /(1e9POP) (B) - Probability density of remaining fraction of non-renewable resources against time. Units of /NRFR. Darker regions indicate a higher probability of a trajectory passing through that point. White indicates zero probability. Thin black line indicates the "standard run".

2.5.2 Probability Density Maps

To overcome the crowding of the trajectory plot, probability density maps were produced. These maps show the concentration of trajectory lines, thus darker areas of the plot represent a higher probability of a trajectory passing through that point.

Figure 2.6 (A) shows the probability density of the population variable. In Figure 2.5, areas with a high density of lines correlate to darker areas in Figure 2.6 (A). The probability is spread thinly after the year 2010, denoted by the lightening of the grey. The probability remains thinly spread for the first half of the 21st century. We can note a slight converging after about 2070, denoted by a darkening, as most trajectories fall to lower levels of around 3.5 billion.

Figure 2.6 (B) shows the probability density of the fraction remaining of non-renewable resources. It is evident that the probability is spread thinly between the years 2000 and 2030. There is a clear increase in probability density for the final decades of the simulation for low values of remaining resources. The darker regions follow the trajectory of the "standard run" denoted by the thin black line.

2.5.3 Probability Density Functions

To better understand the evolution of the probability density it was plotted for individual times, that is, the initial year (1970) and each subsequent 20 years. Figure 2.7 (A) shows the progression of the population's PDF over time. We can see a very concentrated PDF in 1970. This is representative of the limited uncertainty of world population data which was used to initialise the model. The PDFs widen with time, accompanied by a fall in peak probability. The year 2050 has the lowest peak and greatest distribution of probability indicating the least amount of certainty for the population.

The certainty of the model begins to increase after 2050 as evidenced by the peak probability increasing. The PDFs maintain a log normal shape for all times. We can see that the peak of each PDF loosely follows the "standard run" indicated by the vertical dashed line.

Figure 2.7 (B) shows the PDF over time for the fraction remaining of non-renewable resources variable. Again, there is a concentrated PDF for the year 1970. As the model progresses through time the certainty quickly diminishes with the lowest peak in 2010. The peak begins to increase after 2010 as the fraction remaining of non-renewable resources variable converges to a low of around 0.2. Again the peak of the PDF follows the "standard run".

2.5.4 Percentiles

A useful approach to analyse the results is with the use of percentiles. The percentiles chosen were the 5th, 25th, 50th (median), 75th and 95th. By examining the 5th and 95th percentiles the range of 90% of all trajectories can be easily determined. Likewise, the 25th and 75th percentiles show the range of 50% of all trajectories.

Figure 2.8 shows the percentiles, mean and "standard run" for six of the main variables of the World3 model. In plot (A) we can observe that the percentile lines begin very close together, showing a high certainty of the population value. The percentile lines then "fan out" as time progresses, indicating the increasing uncertainty. The lines reach a wide spread by 2030. At this point, 50% of all trajectories are contained within a range of 1.5 billion, 90% within 3.76 billion. Beyond 2030 the percentile lines remain approximately the same width apart. The percentile lines retain a similar trajectory to that of the "standard run". For the other variables we see that the percentile lines also follow similar paths to that of the "standard run".

Fraction remaining of non-renewable resources and industrial output per capita both undergo an increase in uncertainty in the early decades, followed by a convergence in later years. The increase in certainty comes from the fall of non-renewable resources and collapse in industrial output towards their limit of zero. The variables food per capita, crude birth rate, and crude death rate have a more consistent level of uncertainty as indicated by the consistent spacing between percentile lines.

It is interesting to note the average and median of the population remains well below the "standard run". This suggests that population is more likely to stay at a level lower than that of the "standard run".

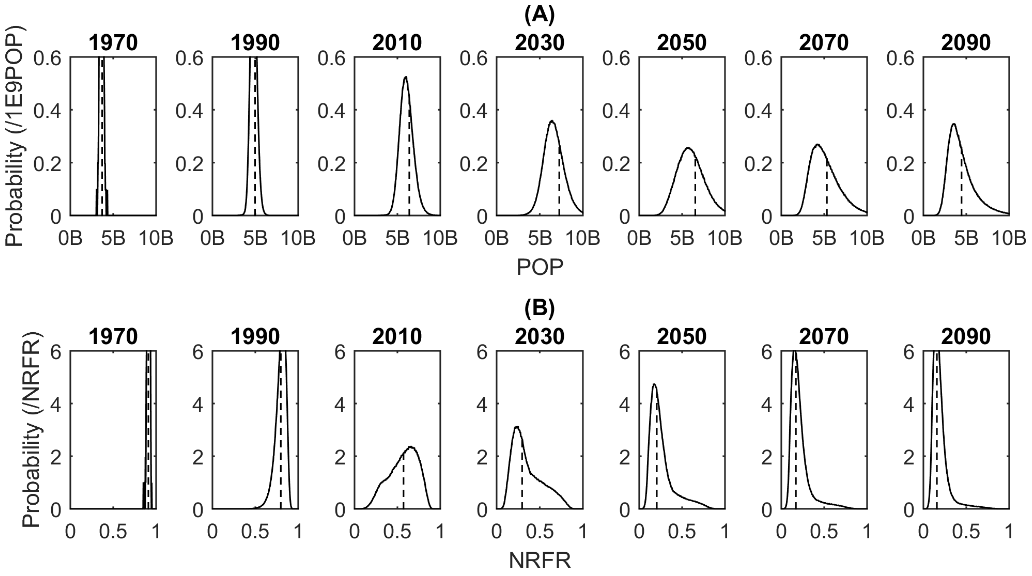

Figure 2.7: (A) Probability density functions of the population (POP) at various times during the simulation. (B) Probability density functions of the fraction remaining of non-renewable resources (NRFR) at various times during the simulation. Dashed line indicates the position of the variables during the standard run.

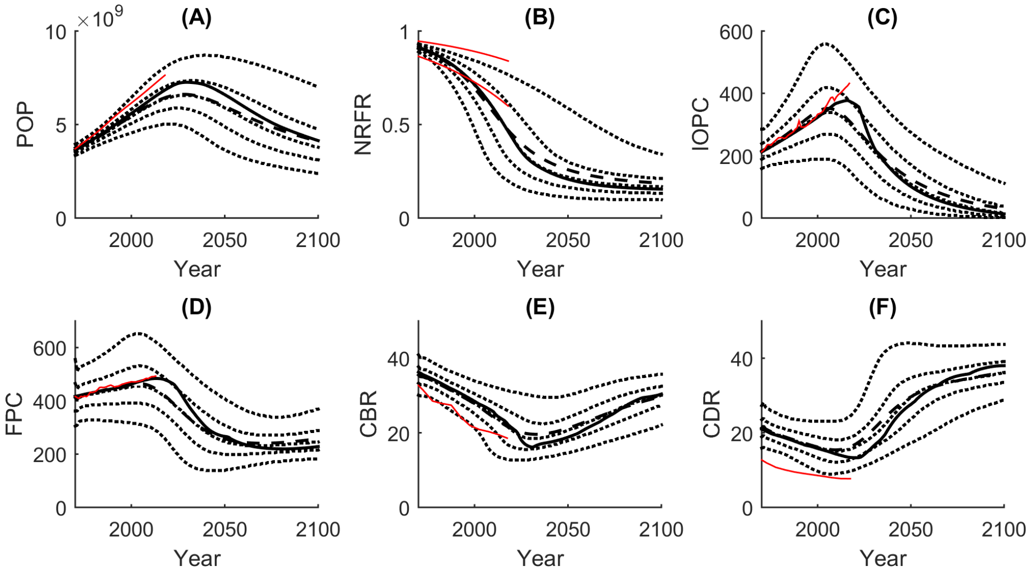

Figure 2.8: Graphs of percentiles, mean and standard run for (A) population (POP)(people), (B) fraction remaining of non-renewable resources (NRFR)(unit less), (C) industrial output per capita (IOPC)(1970 $US), (D) food per capita (FPC)(vegetable kg), (E) crude birth rate (CBR) (births per 1000 people), and (F) crude death rate (CDR) (deaths per 1000 people). Percentiles (5, 25, 50, 75, 95) (dotted lines), mean (dashed line), standard run (solid line), and real world data (red line). There are two red lines in plot (B), as an upper and lower estimate for total fuel reserves.

2.5.5 Standard Deviation

To fully understand how the uncertainty of the output varies with time, the standard deviation of the PDFs at each time interval was calculated. Figure 2.9 shows the SD of the population (A), fraction remaining of non-renewable resources (B), and industrial output per capita (C). We can see that the population's SD gradually increases to a maximum of approximately 1.5 billion. This is a large value considering the maximum population of the standard run is approximately 7 billion. It begins to decrease after 2070 as most trajectories fall to lower values.

The SD of the fraction remaining of non-renewable resource quickly climbs to a maximum of 0.18 by 2020. This is a large SD given the variable's bounds of 0 to 1. After 2020 the SD begins to fall as trajectories converge as seen in Figure 2.8 (B). Industrial output per capita likewise has a very sharp increase in SD. It peaks at around 110 dollars per person in the year 2000, a significant value compared to the 300 dollars per person of the "standard run" at the same point in time.

Figure 2.10 shows the standard deviation of each variable normalised by its 1970 "standard run" value. The normalised SDs of the variables food per capita, fraction remaining of non-renewable resources, and crude birth rate remain reasonably steady for the years simulated, reaching maxima of 0.25, 0.20, and 0.16 respectively. Crude death rate is steady at approximately 0.2, for the first 5 decades and then experiences a sharp spike reaching 0.41 in the year 2040. This period corresponds to the beginning of the population decline in the model.

We can see from this that the severity of the decline varies strongly between simulation runs. The normalised SD of industrial output per capita quickly rises to a high of 0.54 around the turn of the millennium. It then proceeds to decline to a comparable level to that of food per capita, fraction remaining of non-renewable resources, and crude birth rate. The normalised SD of population continually rises to a maximum 0.43 and then declines slowly. Normalised persistent pollution normalised SD reaches a high of around 3.6 between the years 2030 to 2060. This is the most uncertain variable as the modelling of pollution is very difficult, and due to the large increase in pollution from 1970 levels.

2.5.6 Real World Comparison

Figure 2.11 zooms into the comparison of the standard run with real world data from 1970 to 2017 as indicated in Figure 2.8. The data for population (POP), deaths (CDR), and births (CBR) were obtained from the United Nations' (UN) Department of Economic and Social Affairs' (DESA) Population Division (UN DESA Population Division, 2017). Industrial output per capita (IOPC) data were approximated using global GDP data and the population data. GDP data were taken from the UN DESA Statistics Division (2018). This was scaled to align with the 1970 standard run value, allowing for easy comparison of trends. Fraction remaining of non-renewable resources data were taken from the British Petroleum Energy Outlook [22]. An exponential function was fitted to the data to estimate missing data (i.e. pre-1970 records). An upper (150 zettajoule) and lower (60 zettajoule) estimate for total fuel reserves was used to mimic Turner's [167] estimates. Food per

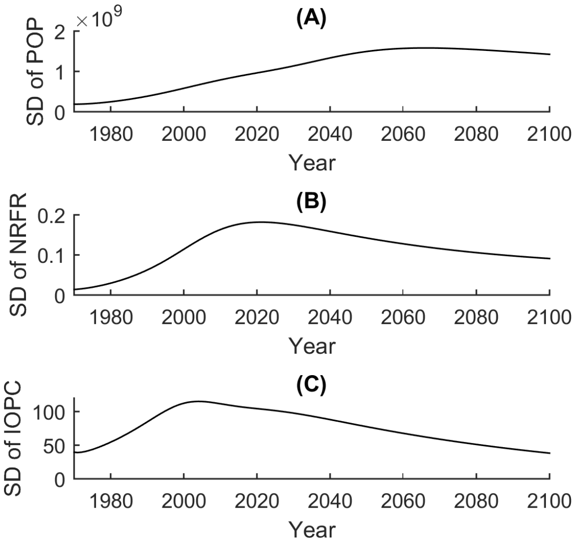

Figure 2.9: Standard deviation of population (A), fraction remaining of non-renewable resources (B), and industrial output per capita (C) over time.

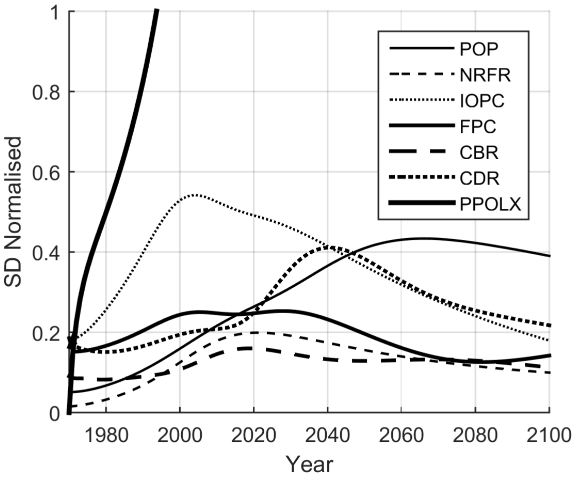

Figure 2.10: Standard deviation normalised with 1970 "standard run" value. Variables are population (POP), fraction remaining of non-renewable resources (NRFR), industrial output per capita (IOPC), food per capita (FPC), crude birth rate (CBR), crude death rate (CDR), and normalised persistent pollution (PPOLX).

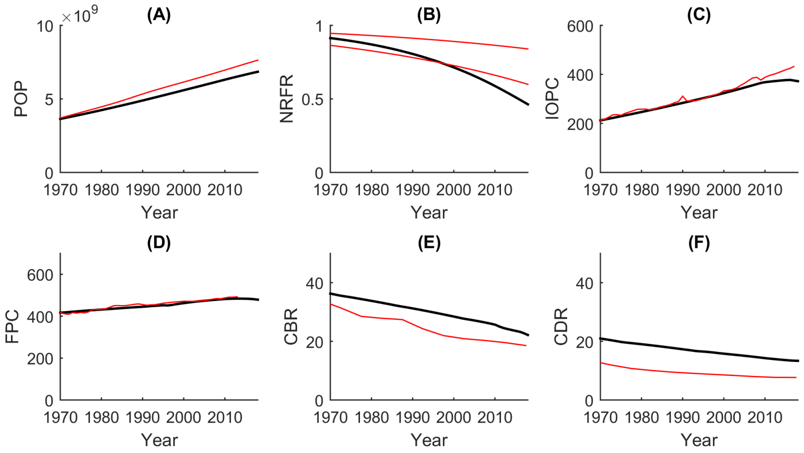

Figure 2.11: Comparison of the standard run of the World3 model (black) against real world data (red). (A) population (POP) (people), (B) fraction remaining of non-renewable resources (NRFR) (unit less), (C) industrial output per capita (IOPC) (1970 $US), (D) food per capita (FPC)(vegetable kg), (E) crude birth rate (CBR) (births per 1000 people), and (F) crude death rate (CDR)(deaths per 1000 people).

capita data were taken from calories/capita/day data from the Food and Agricultural Organisation [58]. This too was scaled to align with the 1970 standard run value.

The real world data for industrial output per capita and food per capital align very well. However, industrial output per capita is now (in the last few years) beginning to exceed the standard run values. Real world birth rates and even more so death rates have been lower than predicted, and significantly lower for death rates. These two deviations between predicted and realised trajectories, however, partially neutralise each other, resulting in the real world population now being slightly higher (≈11%) than predicted. The fraction of non-renewable resources remaining in the real world is well above that predicted by the standard run. In the World3 model, the dwindling resource pool is what prompts the down turn of the industrial output. As the real world data have not reached such a level by now, the World3 model would suggest that the decline in industrial output is yet to come, as evident in the real world data of GDP per capita, or IOPC.

It should be noted that data agreement is not conclusive of model validity. Many different models could produce similar alignment. An ordinary exponential function (simply calibrated to fit real world data) would also align nicely with population and industrial output per capita development to date. However, it would offer little insight into the actual mechanics to the phenomenon behind the data. The only way to begin to validate the standard run simulation is to actually observe a collapse (by which point it is too late to avoid it) in the real world. This is a very hard test to satisfy as the data and variables used to calibrate the model have most likely changed as our society has progressed, e.g. concerning efficiencies in material usage or the introduction of unconventional fossil fuels. Thus the collapse would most likely be different to that shown in the standard run.

Meadows and his team understood this issue very well. The World3 model was also used to examine other transitions into the future. The model has mechanisms that allow for modifications in parameters to be made at a particular point in time. This was designed to enable the simulation of events like government policy changes or technological advancements. The standard run can only represent a society trapped in 1970's behaviour and technology.

An issue regarding the model is the exclusion of an explicit renewable resource sector. Because of this the model is nearly guaranteed to produce a "collapse". While the addition of a renewable energy or resource pool would allow for a greater chance of a sustainable future (one in which variables plateau), it would potentially still result in a "collapse". For example, the energy study of Dale et al. (2012), which investigated the futures of non-renewable and renewable energy production, showed a sharp downturn in total energy production after the year 2060, despite strong growth in the renewable energy sector.

Another issue is that energy and material resources are lumped together in a single variable. This limits the complexity of the possible scenarios. It is clear that many intricacies of the world have been consolidated into single variables in the World3 model. This causes many issues regarding model usability and structural accuracy and could cause larger errors than parameter uncertainty.

Other investigations have been made into the relationship of the World3 scenarios and real world data. Examples include [167] and [133] .

2.6 Discussion

After performing the uncertainty analysis, our knowledge of the World3 model has greatly improved. While past local sensitivity analysis studies allowed sensitive parameters to be identified and then hypothetical scenarios tested, this uncertainty analysis identifies more accurately the model's sensitivity to input errors.

Key variables of the model diverge for the first half of the simulation. This is unsurprising given the complexity of the World3 model. This divergence however reverses in the second half of the simulation. This finding is in keeping with many of the notes made by the original authors on the behaviour of the model (i.e. simulations tend to end in the same manner).

It is evident that the model as such is sensitive to the input error assumed in this study, however the trends, i.e. the shapes of the trajectories, are less so. Thus, while the exact numbers produced by the model maybe of little practical use, the behaviours exhibited by the model would still be of interest.

This study does highlight the need for concern about the future welfare of human kind, as the World3 model has thus far shown considerable agreement with past data. This statement will remain true until the World3 model is proven to be incorrect.

2.7 Limitations

One of the main limitations of this study is the assessment of standard deviation of inputs with a small number of data points. If the magnitude of the standard deviation is changed, then so to will the spread of results in the final analysis. This must be keep in mind when evaluating this body of work. The reader should consider if the standard deviations selected are realistic, and what the consequences of unrealistic assumptions might be.

Another issue is the assumption of normal and log-normal distributions. These distributions were chosen for convenience, rather than through a rigorous examination of the data sources. The independence between input variables, and the effect of the weighting factors (used in the establishment of standard deviation for table functions) on the assumed PDFs.

The results presented in this chapter demonstrate the error in the "standard run" scenario. The results are only relevant for a situation in which the parameters stay fixed to their initial values, that is, as they were in the first half of the 20th century. An addition to this study would be to allow "stabilising policies" to be implemented at a certain year in the simulation. The year in which the policies are implemented would be given a PDF to illustrate the uncertainty of when society would make a significant change to its behaviour. This addition could improve our understanding of uncertainties in the World3 model.

This study only investigates uncertainty caused by model input errors. The question of the accuracy of the model itself is still open for debate. While the model captures many economic and ecological relationships in our world, it still omits or aggregates most aspects.

The removal of trajectories which drastically diverge from historic data, e.g. the lowest trajectory in Figure 2.5, would improve the accuracy of the results. However, as they contribute a low proportion of all simulation runs their effect on the results are negligible, thus it was deemed unnecessary to remove them for this study.

Another limitation of this study is that only one set of standard deviation values was examined. The results of the study may give an overestimation of variance if the standard deviation values are likewise overestimated. To fully understand the expected variance of the model, further tests should be conducted which take other standard deviation values into consideration. Given that the standard deviation values (that had to be approximated) were selected to be overestimates, it is most likely the case that the data presented in this study has a higher variance then in reality. If this is the case, then it can be noted that the trends seen in the output would be more consistent then stated previously.

2.8 Conclusion

The results show that the unpredictability of the model is noteworthy. The variables population (POP), industrial output per capita (IOPC), fraction remaining of non-renewable resources (NRFR), and normalised persistent pollution (PPOLX) produced normalised standard deviations of 0.43, 0.54, 0.20, and 3.6 respectively. The general trends of the variables are predictable, with more then 95% of simulation runs producing nearly identically shaped trend lines. This is evident by the similar shape of each percentile line, and that these lines do not drastically diverge as time progresses.

It is evident that trajectories can be found that indicate a favourable future for humanity, i.e. no sudden downturn in population or access to resources, without policy intervention for the time period modelled. However these trajectories reside in areas of low probability, and thus should not be considered as likely outcomes.

For the authors of the “The Limits to Growth”, a study such as this might have proved useful in defining the models behaviour to its critics, and demonstrating it’s main purpose i.e., allowing the study of possible trends, not producing precise predictions.

Chapter 3

Creation Process of GUI for High Dimensional Pareto Front Analysis

Abstract

To analyse the data (a Pareto front[1]) generated in Chapter 4 a graphical user interface (GUI) was developed to display and navigate the data. This chapter summarises the importance of data visualisation in decision making problems and gives a description of the developed GUI. This is not a major component of the overall thesis, but is important for clarifying some of the figures presented in Chapter 4 as there were generated using the GUI.

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[1] A Pareto front is a set of data points which represent the best possible solutions in a multi-objective problem. Each point has its own strengths and weaknesses in terms of the problem objectives.

Problem Statement

To frame this chapter we will take a hypothetical situation and consider how it could be approached. This will help give context to the following sections.

- Imagine a large group of people who are faced with a problem. This problem has many objectives which need to be maximised / minimised. The group has developed a model[2] and have used it to discover all of the best (Pareto optimal) options available to them. The group now has to select one option to pursue from the wide selection of options generated by the model.

3.0.1 Group Decision Making

Group decision making is the process by which a group of people select a course of action to pursue [138, 101]. Due to increases in information technology, it is now possible to involve many more people in the decision making process, thus creating a new class of group decision making, namely large scale group decision making [101]. The theory behind large scale group decision making is that by involving greater numbers of stakeholders in the decision making process, the final decision will be better accepted [119, 101], producing greater follow through and support for the decided actions post decision. An example of this shift in group decision making is the attempt of a research team in Hamburg, Germany to involve more people in an urban planning project through gamification of the planning problem [137]. There are still many issues to be addressed in large scale decision making problems, e.g. added human resources costs and time [101].

With traditional group decision making processes the group attempts to reach unanimity [79] in their opinion through a series of negotiations and discussions [138, 79] which will often be guided by a mediator [79]. However, in large groups the opinions are likely to be more diverse and thus unanimity is harder to achieve. Because of this other criteria may have to be used to determine when consensus has been achieved. This might be unanimity minus n, i.e. everyone except for n people agree on the decision. Other methods include ensuring a majority agree, or some fraction of the group agree [79].

A non-traditional method is emerging in which “soft consensus” is used to decide if an option is supported enough to be accepted [79]. Soft consensus examines the “degree” [79] to which each individual supports an option, rather than a binary aye or nay degree of support.

The problem of selecting the best solution (from our hypothetical set) can be addressed in two ways:

1. The solution set can be reduced to a small set of options (< 10) by a subset of the group, i.e. experts of elected representatives. The group at large can then select an option from the small set of options; or

2. Each group member examines the large set of options for a solution they most prefer. This information is then used in the decision making process.

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[2] We will assume the people have agreed that the model is reliable.

Method one has the advantage that the reduced data set (potential options) is easier to examine (for the group at large) as the number of data points is small. If the number of points is small then the data could conceivably be placed on a single table with a row for each solution, and columns for each objective.

If method two is selected then the data has to be presented in an easy to interpret way. High dimensional data (note that this hypothetical problem has many objectives and thus many dimensions) is often “difficult for experts to navigate let alone non-experts” [110], and so making the data easy to interpret is not a simple task.

Once each group member has selected an option, soft consensus could be used to decide on an ultimate option, i.e. the one that aligns most closely to every group members most desired option. The other approach is to use the selection process (the exploring of the data) as a mechanism for informing the individual [110]. This then gives them knowledge which they can use in the discussion and negotiation stages of a traditional group decision making process.

3.1 Information Encoding

There are many different ways researchers can present data. This ‘presenting’ of data can be thought of as information encoding. Humans use their senses to decode the information into mental representations/thoughts. In Figure 3.1 [84] three dimensions of data have been encoded onto a 3D plot. In this case, position is the encoding format. In Figure 3.2 [84] four more dimensions have been encoded into the graphic by replacing the dots with a special marker. The markers have a direction (in which they point), size, colour and transparency. Other possible types of encoding include, length, pattern (spotted, stripped), shape (number of corners), or focus.

While the primary method of data absorption is sight (the previous encoding examples were all sight based), there are many other ways the human body can receive information. For example we are able to acquire information through our sense of:

- hearing - e.g.: pitch, volume, direction;
- smell - e.g.: floral, rotten;
- taste - e.g.: sweet, sour, salty;
- touch - e.g.: pressure, pain, temperature, resistance;
- spacial awareness - e.g.: location, time, direction, speed; and
- emotional state - e.g.: hungry, sleepy, nauseous, muscle fatigue, intoxication.

While these senses are hard to incorporate in traditional data representation forms (i.e. print media), future technologies might be able to exploit these senses for data exploration.

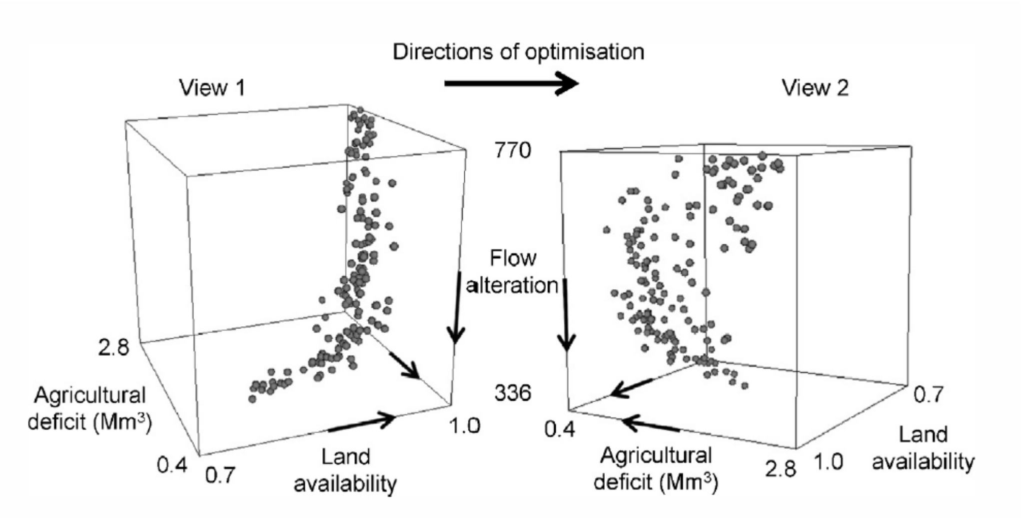

Figure 3.1: A typical three dimensional plot with information encoded by position. Figure sourced from [84].

Same Data, Different Encoding

To explore how the same piece of data can be encoded in different formats, let's take an example of a doctor checking a patient's heart. The doctor could take an electrocardiograph and examine the printed results by eye, or they could use a stethoscope to listen with their ear, to hear the beat of the heart. Both tests would then give the doctor knowledge of the heart which they can use to give a diagnosis.

From this example it is easy to understand that the same piece of data can be encoded in many different ways. Some data, e.g. households income per year, can be represented in table (numbers), bar chart (length), or plot (position) format. The speech of a politician could be recorded in print (text) or in an audio file (sound). Theories can be explained with descriptions and example diagrams (text and imagery) or mathematically (equations). This concept is important to remember when trying to create a method for transferring information from the real world into human consciousness.

3.2 Information Absorption

The act of making a decision has two stages. In the first stage a person absorbs information about the world around them, and then generates a mental representation. This process is not error free (especially in the case of large complex systems). This can be due to: errors of the sensors, errors in the information, and our inability to hold a perfect representation of complex systems in our minds. In the second stage, a person then acts based upon assessments of their mental representation. This concept is depicted in Figure 3.3.

It is important to be able to easily understand and interpret information. By having easy to understand information the time it takes to develop a mental model and then pick a course of action is reduced [49]. The process of absorbing information falls into the field of visual analytics. This field examines how computer programs can aid in the process of knowledge development when working with large data sets [165].

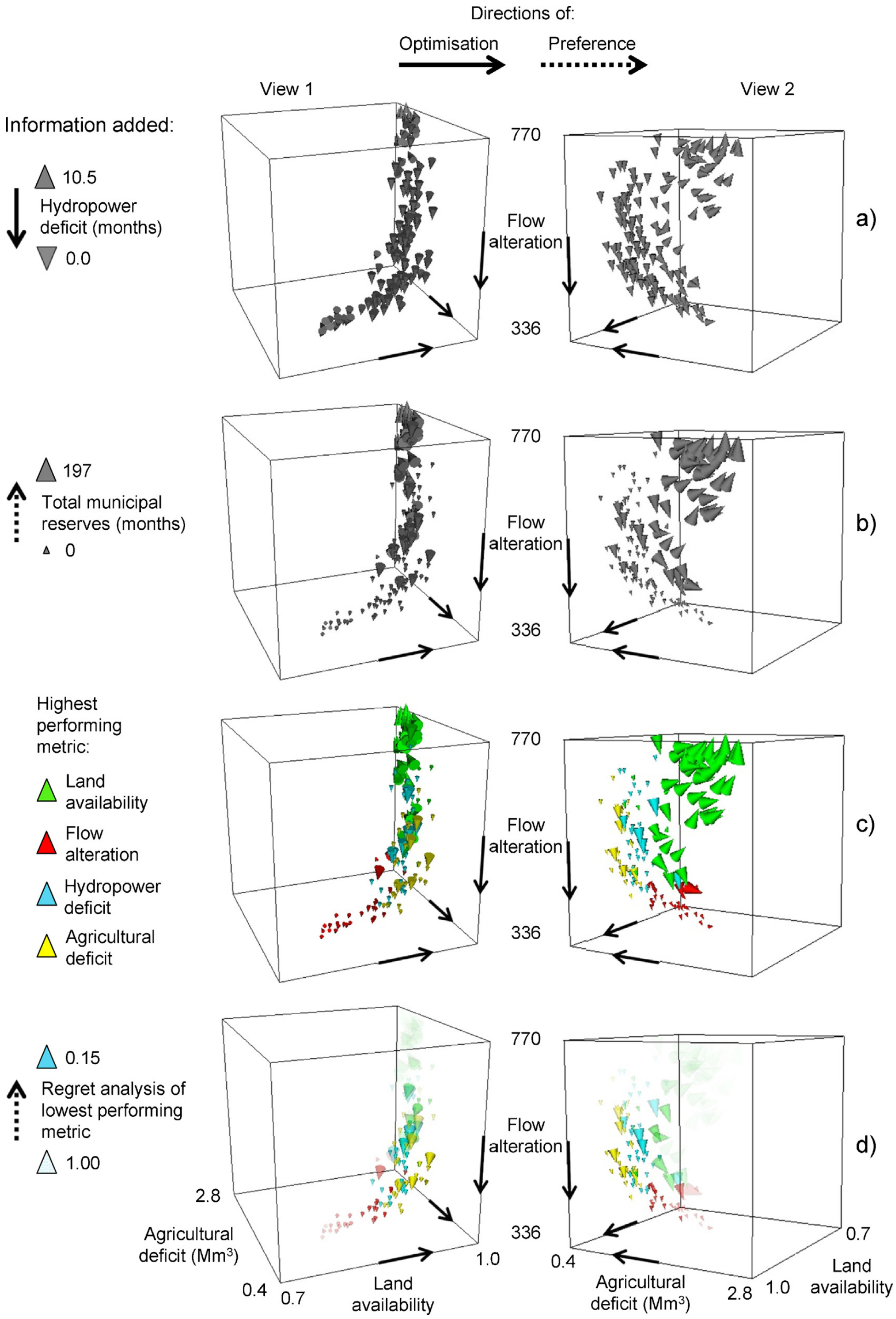

Figure 3.2: Examples of multiple information encoding. Information is encoded in; position (in three axis) plus (1) arrow direction, (2) arrow size, (3) arrow colour, and (4) transparency. Figure sourced from [84].

Figure 3.3: The causal chain of events in a decision making process. Information about the world is absorbed through sensory inputs. This produces a mental representation which is then examined. The resulting action as based off the mental understanding of the real world. In this illustration the mental image is blurred to represent the error associated with interpreting the real world.

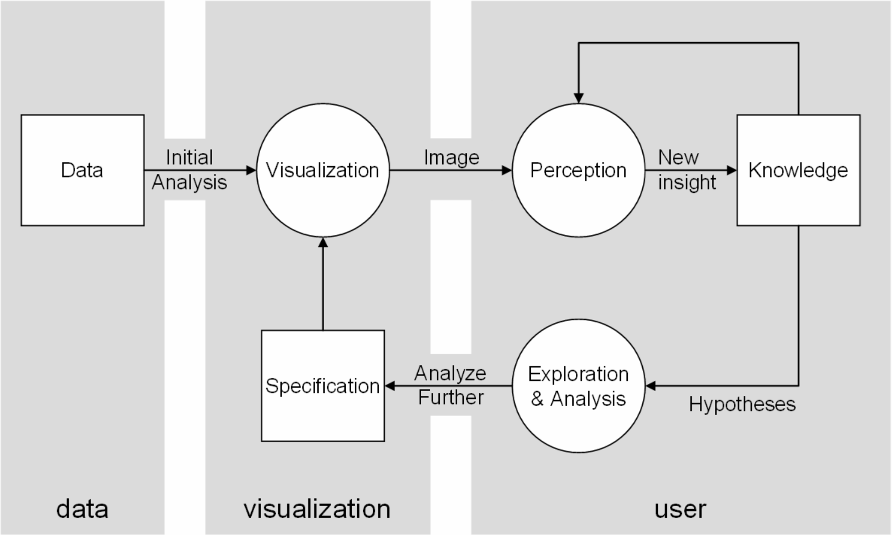

Figure 3.4: The concept of visual analysis. Figure sourced from [95].

The visual analytic process is outlined in [95] (adapted from [180]) and shown in Figure 3.4. It is a cyclical process by which the user is informed by some initial visualisation of the data. Once this knowledge has been acquired the user updates the specifications of the program to produce new visuals which are then interpreted to produce further knowledge. Based upon these descriptions of visual analytics, the GUI developed in this section would be classed as a visual analytics tool.

Information Impact

Increasing participation in decision making process requires information to be conveyed in an easy to understand way [105], otherwise some participants may be alienated from the process if they are unable to understand the information.

Morseletto [105] asserts that there are three key features to making information impactful. These features are: understandable; meaningful; and engaging. Information that meets these three criteria will be absorbed more readily. The concept of ‘understandable’ covers the ease of which the information is absorbed. ‘Meaningful’ denotes the relatability of the information to the audience, e.g. a kilogram figure of CO_2 may be meaningless to most people, however if it is converted to a metric like eco-time[3] [149] it may have more meaningfulness. ‘Engaging’ encompasses how “eye catching” and memorable the information is.

Images can be very useful in increasing the ease at which information is absorbed [124]. Three examples of this are given in figures 3.5, 3.6, and 3.7 [173, 105]. These figures have combined traditional information encoding methods (numbers, size, and line plotting respectively) with images to make the data easier to interpret. In Figure 3.5 the information has been represented in number / text form and has been paired with a graphic to denote the thing being examined. For Figure 3.6 the information about the size of the fish is indicated by the size of the image of the fish. The fish image immediately gives context to the subject matter of the graphic. In Figure 3.7 the end point of each line plot has a small image to denote the thing being plotted. This again cuts down mental load when viewing the plots as no time is spent analysing legends/keys or text to understand what each line denotes.

Another option is to have all of the information displayed in image form [148]. In Figure 3.8 all of the information is in pure image form. The image represents possible social arrangements. The raw information about population density, agricultural productivity, environmental health, wealth, consumption habits and waste generation has been depicted in the images.

With the advent of virtual reality, and augmented reality technologies, we can begin exploring other methods of information acquisition. These technologies may help us exploit our other senses to increase our information absorption abilities.

---

[3] Eco-time is the measure of CO_2 given in terms of time (seconds, minutes, hours), such that a person should not “spend” eco-time faster than time moves, i.e. they should spend less than one day of eco-time per day.

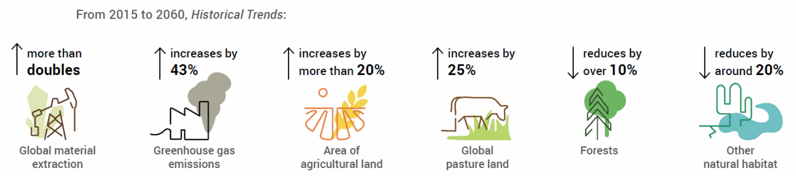

Figure 3.5: Graphic which blends numbers and images. Figure sourced from [173].

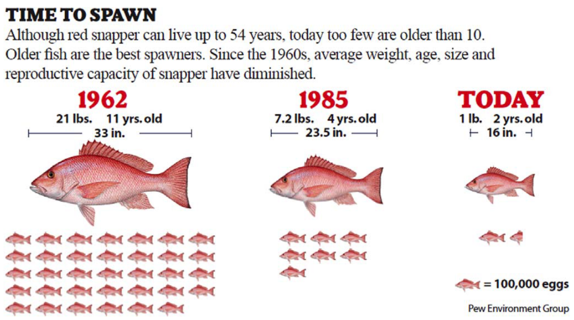

Figure 3.6: Graphic which blends size and images. Figure sourced from [105].

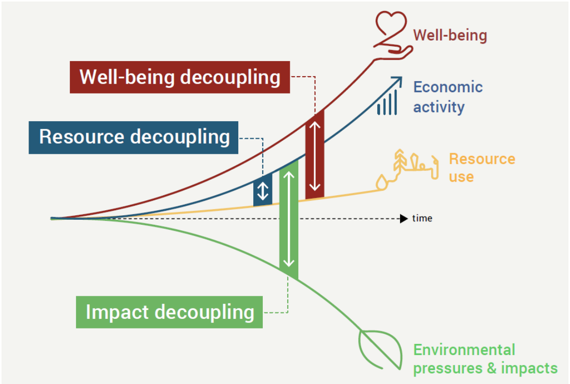

Figure 3.7: Graphic which blends line plots and images. Figure sourced from [173].

Figure 3.8: Graphic in which all information is encoded in image form. Figure sourced from [148].

3.3 Visualisation Techniques and Dimension Reduction

There are many techniques that have been developed to represent highly dimensional data. Walker et al. presented several of these in [183]. The types presented were: 2/3D scatter plots, parallel coordinate (textile) plots, pairwise plotting, heatmaps, principal component analysis plots, self-organising maps, generative topographical maps, and neuroscale plots. Figure 3.9 shows each of these plot types.

Scatter plots are a very common occurrence in scientific literature. Most often the plot will be two dimensional as no information is lost, unlike when a three dimensional plot is printed on a two dimensional piece of paper. In [55] a three dimensional plot is used along with two dimensional plots (pairwise) to examine the data of a three objective optimisation problem (the problem construction work sequencing). This paper has been selected simply to demonstrate this standard method of data representation.

Parallel coordinate / textile plots are graphics in which a data point is represented by a

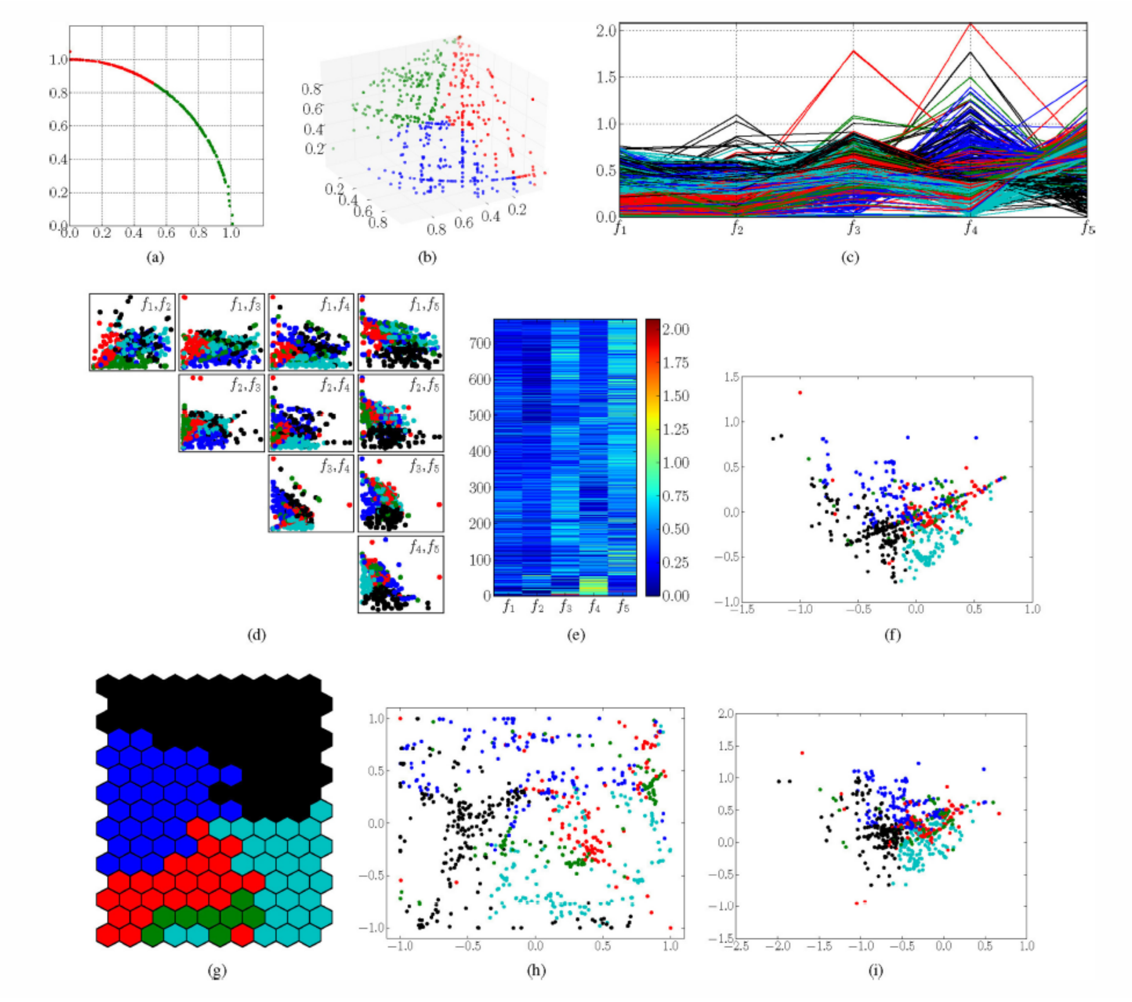

Figure 3.9: Examples of data representation for optimisation problems. (a) 2D plot, (b) 3D plot, (c) 5D parallel coordinate (textile) plot (1 line = 1 solution), (d) pairwise plots, (e) 5D heatmap (y axis denotes solution number, colour denotes dimension value), (f) principal component analysis plot, (g) self-organising map, (h) generative topographical map, (i) neuroscale plot. Figure sourced from [183].

line which zigzags horizontally across the plot, passing over vertical dimension lines. The point at which each line crosses a dimension line denotes its value in that dimension. When examining the data on a textile plot, patterns can be deduced. Two kinds of patterns are "knots and parallel wefts" [100]. Knots indicate dimensions which are in conflict (i.e. increases in one dimension produce decreases in the other), while wefts indicate harmony (i.e. where an increase in one dimension produces an increase in the other) [42]. Figure 3.10 [100] shows measurement data of human body parts presented in textile plot format. Each grey line represents an individual who was measured. To assist in analysing the plot, dimensions which are the most similar are placed together. The hierarchical clustering process is depicted by the dendrogram above the plot. This is analogous to the branches of the evolution tree. In this sense pairs further along the branches of the tree are closer in similarity.

Examples of parallel / textile plotting can be found in [107, 42]. In [107] the data (calibration options for an engine) is grouped into clusters and colour-coded accordingly (shown in Figure 3.11). The clustering was achieved via k-mean clustering. In the study principle component analysis was used on each cluster to try to reduce the complexity of each cluster. In [42] hierarchical clustering is used to finding harmony and conflict between the objectives. This is done to aid in visualisation and analysis of the data.

Pairwise plotting uses many simple 2D scatter plots to represent the data. With this method, a plot is produced for every pair of dimensions. This is a simple technique, however the number of plots needed to display each dimension pair grows rapidly as dimensions increase. Another detracting factor is that information can only be shown in two dimensions at a time, meaning that all other information (about the other dimensions not shown in the sub-plot) is lost.

Heatmaps are simple to understand as they are similar to table data, with solutions organised by the row, objective dimensions by column, and colour denoting the values in the table. Heat maps can appear noisy if left unsorted. To help with the interpretation of heatmaps hierarchical clustering is used to arrange the solutions so that similar solutions (and solution clusters) are close together. A dendrogram may or may not be included in the figure.

Principle component analysis is frequently used to reduce the dimensions of a high dimensional data set. Finding the first principal requires finding the vector which most closely aligns with the data, i.e the average squared distance from each data point to the principal vector is minimised. Once the first principal component is found, the second principal component can be found. The second vector must be perpendicular to the first. Figure 3.12 [187] shows an example of the principal components of a data set.

The process of finding principal components can be repeated continually, however a point will come where the number of principal components is equal to the number of dimensions of the original data set. The amount of information retained by the analysis depends on the number of principal components calculated. The more principal components used, the higher the information retention. The first component always retains the greatest amount of information, followed by the second, etc. [49]. If the number of principal components is equal to the number of data dimensions, then the information contained in the data is fully retained.

Principle component analysis is a major component in “GAIA plane” representation [49]. In this method the solutions and objective axes are projected onto the first and second principle components. If weightings are given to the objectives/criteria (i.e. varying importance given to each objective) then a “decision stick” can also be mapped onto the GAIA plane to indicate the direction in which the most desirable solutions will occur. An indicator may also be given to describe the degree of information loss in the projection process [49]. A GAIA plane example is shown in Figure 3.13 [49].

Dimensional reduction is a feature of self-organising maps, generative topographical maps, and neuroscale plots. These approaches are designed to transfer data points from a highly dimensional data space onto a 2D surface, while retaining information about the closeness of data points, i.e. close points in the data space should remain close in the 2D space [183]. For example, the neuroscale algorithm trains a neural network to map points from the data space onto a 2D space (called the feature space). The goal of the network is to minimise the difference in the distance between points in the data space and the distance between points in the feature space. This is illustrated in Figure 3.14.

Level diagrams [19, 140] are another method that have been proposed for representing data. Following this method, a solution is scored by calculating its distance from an ideal point. This score is then plotted against the control variables to allow the user to quickly select optimal control settings.

While GAIA planes and level diagrams do seem to make data visualisation easier, they both suffer from the fact that a value judgement must be made at the beginning of the process as to what constitutes an ideal solution. This is similar to applying a weighting function to an optimisation problem to reduce it into a single objective problem. This in a way runs counter to the reasons to conduct a multi-objective optimisation analysis. It also introduced the issue of normalising each dimension so that scales of magnitude do not bias the results.

Another data analysis method is correlation analysis. In this process the correlation between objectives is calculated and thus an understanding of which objectives are in conflict and which are in harmony can be determined. An example of this is given in Figure 3.15 [49].

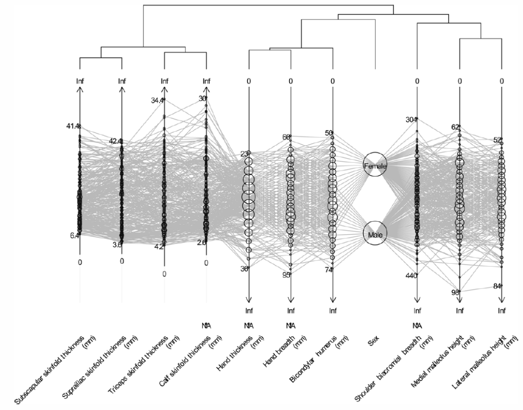

Figure 3.10: Human body part measurements plotted on a textile plot. A dendrogram is positioned above the plot to record the hierarchy clustering. Figure sourced from [100].

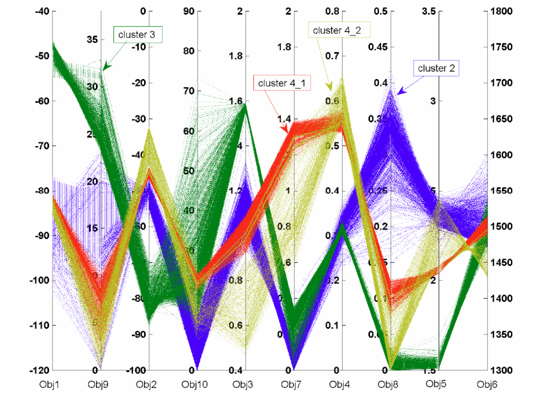

Figure 3.11: Engine performance metrics plotted on a parallel coordinate plot. Colour denotes clusters of similar solutions. Figure sourced from [107].

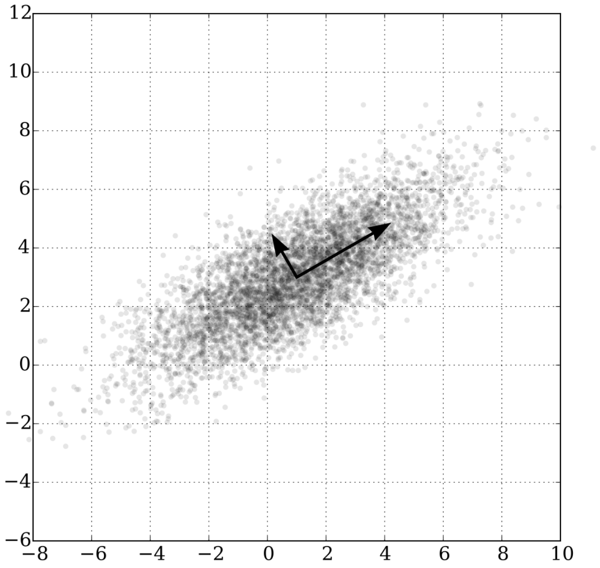

Figure 3.12: Example showing the first (large vector) and second (small vector) principal components of a set of data points. Figure sourced from [187].

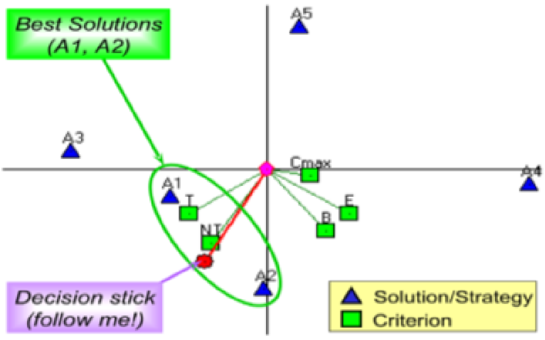

Figure 3.13: The GAIA plane has solutions plotted on the first and second principal components. Figure sourced from [49].

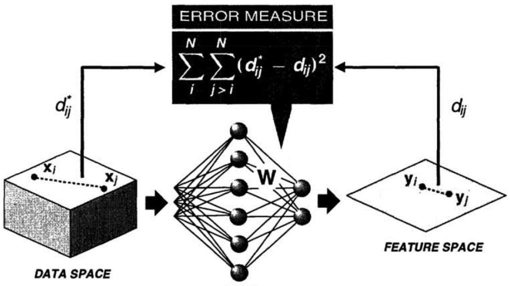

Figure 3.14: NeuroScale error measurement. A neural network (W) transfers points (x) in the data space to points (y) in the feature space for viewing. The aim of the neural network is to minimise the discrepancy of the distance between points in the data space and the feature space. Figure sourced from [106].

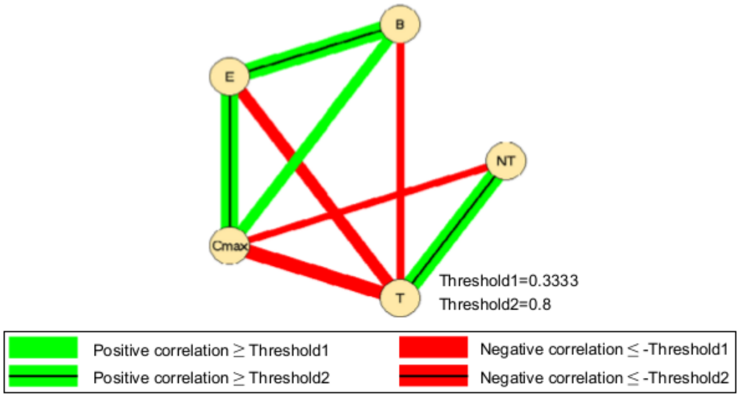

Figure 3.15: Correlation between five objectives. Green indicates a positive correlation, red a negative correlation. Figure sourced from [49].

3.4 Purpose of GUI

A critical question of this chapter is “how can someone examine a highly dimensional Pareto front, such that the information is easily absorbed to allow for a decision to be made within a reasonable time frame?”

To try to solve this problem a GUI was created that facilitates the exploration of highly dimensioned Pareto fronts. While many data visualisation methods aim to reduce the dimensionality of the data (so it can be displayed statically), the developed GUI focuses on improving the rate to which a decision maker can absorb the Pareto front data and come to a decision. This GUI leaves the data in a relatively raw form, and allows a decision maker to experiment with the data (in real time) to see how it behaves, rather than being “told” how it behaves.

The GUI was made for the purpose of exploring the data of this thesis and for future experimentations regrading knowledge acquisition with student subjects. This meant the GUI had to be accessible to most people and therefore easy to understand.

3.5 GUI Description

Figure 3.16 shows this interface[4]. The GUI allows the user to set up minimum criteria for each objective. If a solution is to be considered valid, and thus allowed to be printed on a graph, it must pass the criteria for each objective. The objectives name (A) and its minimum criteria (B) are displayed on the GUI. The number of solutions meeting all criteria (G) is shown in the top left corner. Suggested new criteria (b1-6) are displayed along with the change in the number of options, if the suggestion was selected. The criteria can be changed by clicking these buttons, or they can be changed by manually typing in a number. Criteria can be saved and reset (C) in the bottom left corner. The criteria set saved in the bottom location (D) is used as a reference set. Any solutions meeting this criteria set are displayed as grey circles on graphs. Pairwise graphs can be printed (to the screen) (E) and saved (F) along with a text file containing the model input values for each viable solution. The “release” button and jump percentage are located at (x). The release buttons allow the user to quickly release (relax) the criteria placed on the objective, i.e. the criteria value is as non-restrictive as possible. The jump percentage changes the difference between the values of the suggested criteria (based upon a predefined normalising value), i.e. the smaller the percentage, the smaller the jumps in values will be.

The suggested new criteria buttons allows the user to quickly and easily change criteria to find a set of solutions they find optimal. The change in options number can guide the user to where the most gains or losses can be made by changing the criteria.

The print button was added because plotting the pairwise graphs takes a couple of seconds to execute. If the graphs are automatically updated every time the criteria are altered, then the process of altering the criteria is slowed dramatically, as producing the detailed pairwise plots it computationally heavy.

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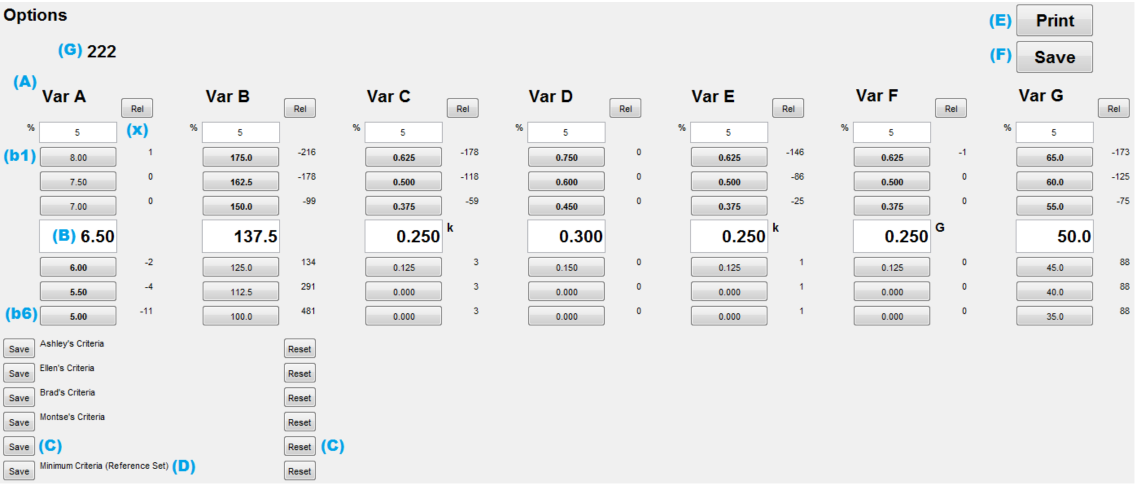
[4] For copies of the software please contact the author at aheath@uow.edu.au.



Var A		Var B		Var C		Var D		Var E		Var F		Var G	
% 5		% 5		% 5		% 5		% 5		% 5		% 5	
8.00	1	175.0	-216	0.625	-178	0.750	0	0.625	-146	0.625	-1	65.0	-173
7.50	0	162.5	-178	0.500	-118	0.600	0	0.500	-86	0.500	0	60.0	-125
7.00	0	150.0	-99	0.375	-59	0.450	0	0.375	-25	0.375	0	55.0	-75
6.50		137.5		0.250	k	0.300		0.250	k	0.250	G	50.0	
6.00	-2	125.0	134	0.125	3	0.150	0	0.125	1	0.125	0	45.0	88
5.50	-4	112.5	291	0.000	3	0.000	0	0.000	1	0.000	0	40.0	88
5.00	-11	100.0	481	0.000	3	0.000	0	0.000	1	0.000	0	35.0	88

Figure 3.16: The main control panel of the Pareto front exploring graphical user interface.

The procedure for reducing the options to a few solutions begins by releasing each objective criteria. Then the most lacking criteria[5] are increased by a small amount. The decision maker can now produce an overview plot to examine how these changes have altered the available options. They can again make small changes to the most lacking criteria and reproduce the overview graph. This process is repeated until a single solution remains. Decision makers with different goals will arrive at different final solutions.

Figure 3.17 shows the 'simple' plot. This plot is designed to give the user a crude sense of how the objectives behave in relation to each other. The plot is always visible so the user can receive real time feedback. This plot is used to show the state of each objective. The blue bars represent the level guaranteed for each objective given the criteria currently set by the user. The black lines above each of the bar represent the best possible level available to the decision maker for each objective if no gains are to be expected for any other criteria. The red lines mark the best and worst levels of the reference set.

By examining the movement of the black bars in relationship to the top red bars, the user can sense how quickly other objectives deteriorate as they improve a specific objective. This gives the user a crude sense of how the objectives are related to each other through experimentation.

Figure 3.18 is an example of a printed overview graph. Each sub plot shows the relationship between two objectives. The objective plotted on the y axis is identical across rows, the x axis along columns. Grey circles represent solutions which meet the reference criteria. Green dots represent solutions which passed the current criteria. Red lines show the current criteria values. The axes of each graph are oriented (based on if it is a maximisation or minimisation objective) such that the upper right corner is the most desirable position. By plotting the reference set in grey, the user can easily see how the changes they make to the criteria affect the remaining possibilities. Enlarged plots of each sub-plot can be viewed on a computer monitor in real time by clicking an enlarge button at the top right hand corner of each sub-plot. This is shown in Figure 3.19.

The enlarged plot can be split into four sections based on the red lines. The bottom

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[5] According to the decision maker.

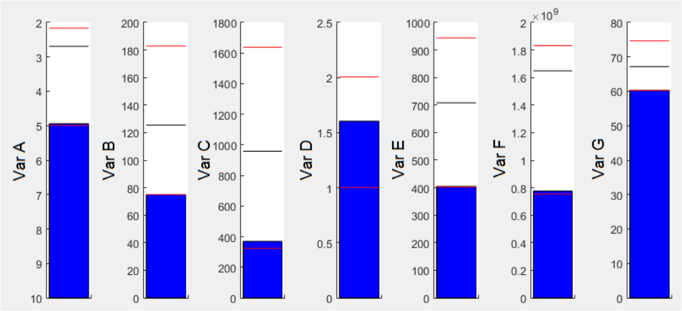

Figure 3.17: Simple plot - a plot of the state of each objective. The blue bars represents the level guaranteed for each objective by the criteria currently set by the user. The black line above each bar represents the best possible level available to the decision maker for each objective. The red lines mark the best and worst options of the reference set.

left section contains points which have failed to meet the criteria set for both objectives[6]. The top right section contains solutions that pass both criteria. The green dots represent solutions that have passed all criteria (not just the two axis of this particular plot). Grey dots in this section represent solutions which have passed the criteria of the two objectives that form the axes of the graph, but not all of the current criteria. Points in the top left and bottom right sections have passed one of the axes criteria, but not the other.

Once several decision makers have finished picking their most desirable solution, a summary chart could be produced and taken to a larger audience for discussion, debate, and final decision. An example is shown in Figure 3.20. Each decision maker is represented by a different colour. This form of representation is simple and well establish with the wider public. Further processing could produce even more appealing graphics, e.g. pictorial notation.

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[6] Both of these objectives are to be maximised.

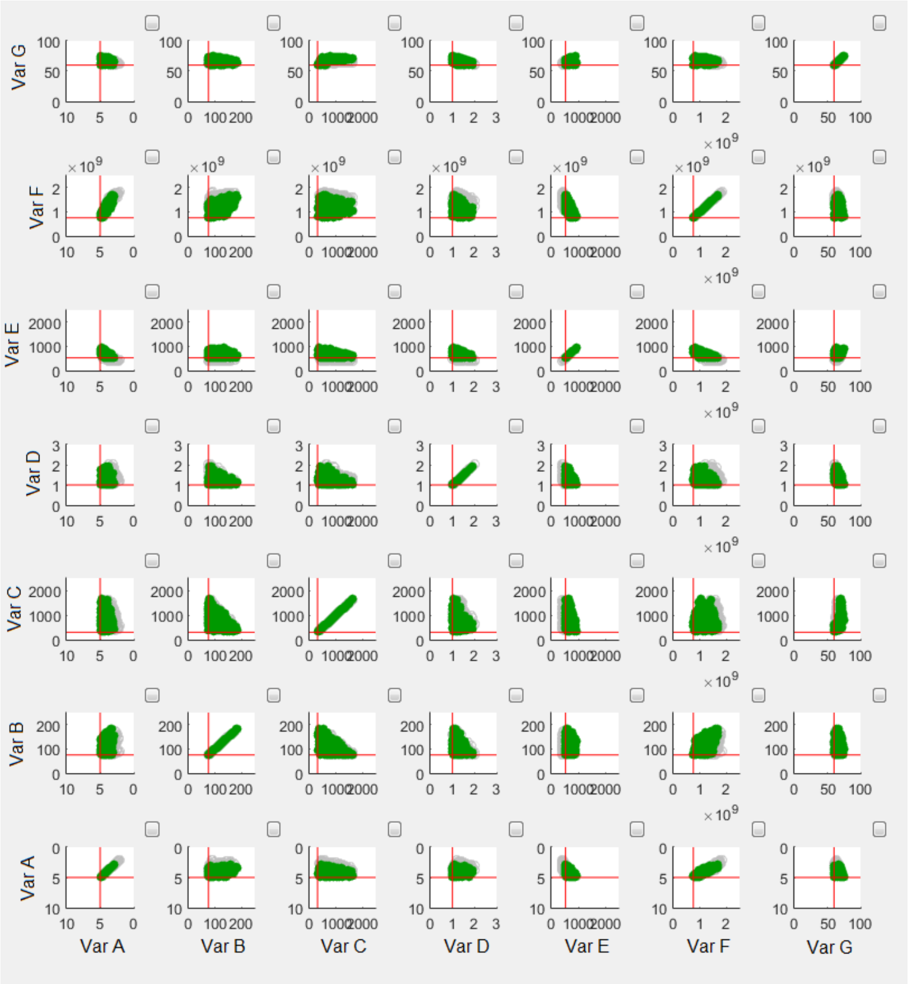

Figure 3.18: Overview plot - every combination of two objectives plotted against each other. Allows for easy identification of relationships between variables. This is produced when the print button is pressed. Red lines indicate the current criteria.

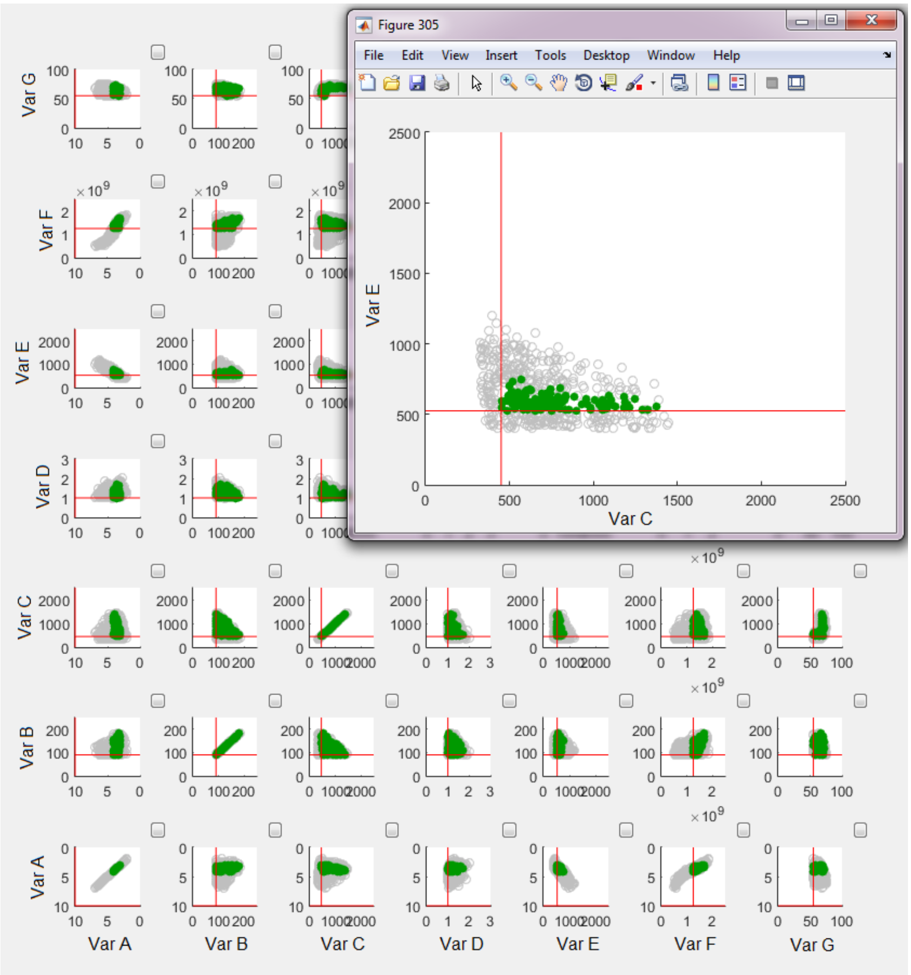

Figure 3.19: An enlarged version of one of the sub-plots in Figure 3.18.

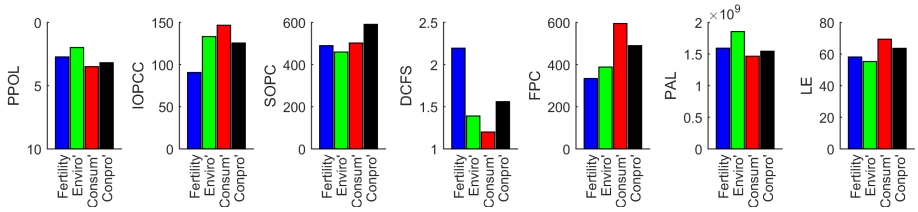

Figure 3.20: Simple bar charts of each objective comparing each final solution.

3.6 Overall Effect of Using the GUI

This GUI does not explicitly show the user the location of the Pareto front, but rather allows the user to discover it by navigating the space with the GUI, i.e. changing criteria and examining the number of solutions meeting the criteria. This concept is shown in Figure 3.21.

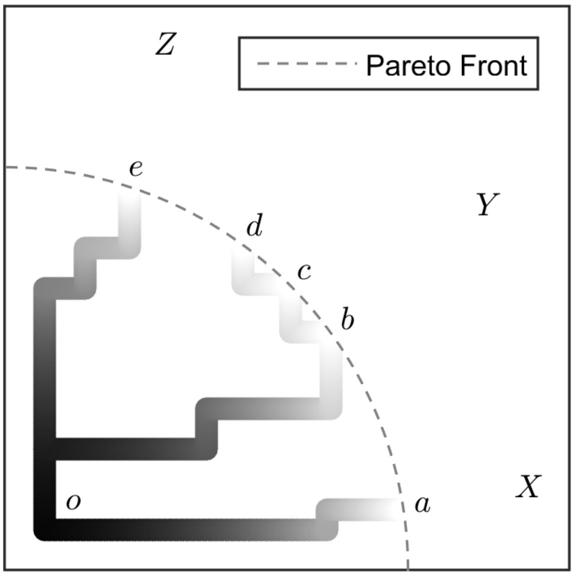

Figure 3.21: An example of someone discovering a Pareto front using the graphical user interface. The Pareto front is not explicitly shown in the GUI and so is denoted with a dashed line. The user begins by setting the criteria such that they are at point o. The user moves through the space trying to reach an ideal point (of their own imagination) X, Y, or Z by changing the criteria. As they move through the space towards the Pareto front the number of solutions meeting the criteria diminishes. This diminishing is denoted by the lighter colour nearer the Pareto front. The user knows they have reached the Pareto front once their criteria set excludes all of the solutions, as is the case at points a, b, c, d, or e.

Chapter 4

Pareto Front Discovery of the World3 Model

Abstract

Probing a model is an important step in generating new knowledge about an issue[1]. Two mechanisms for extracting knowledge are manual experimentation and optimisation analysis. Optimisation analysis is the process of discovering the best potential options a decision maker can choose from, given a set of constraints. We have applied an evolutionary optimisation algorithm (NSGA-II) to the Meadows et. al. World3 model [111]. This was done to examine the trade-offs between seven real world optimisation objectives. The objectives examined are; minimise persistent pollution, maximise desired complete family size, maximise industrial output per capita consumed, maximise service output per capita, maximise food per capita, maximise potentially arable land (i.e. conserved land which has not been converted to agricultural land), and maximise life expectancy. The most obstructive objective (i.e. diminishes options of other objectives) appears to be maximising desired complete family size, followed by maximising food per capita. The least obstructive objective appears to be minimising persistent pollution.

---

[1] Assuming the model accurately depicts the issue / real world phenomenon.

4.1 Introduction

Optimisation is a common part of every day human life [151, p. 11]. It is the selection of variables to produce the greatest desired result, i.e. the optimal solution. There are several ways in which we can search for the optimal solution. One method is to solve the problem analytically, e.g. using the Hamiltonian function to derive the optimal solution [62, 30]. This gives the exact solution, however it is not always possible. Alternatively the problem can be solved numerically. An answer can be found either through scenario testing, or executing an optimisation algorithm. Scenario testing is the process by which the model user manually changes inputs and studies the outcomes. The use of an optimisation algorithm removes the task of changing the inputs from the user, and returns the best solutions to the user.

Some problems only have a single objective to optimise; of course most real world problems have more then one objective [176]. As objectives are often conflicting, it is thus hard to determine the best solution. The renewable energy sector has received some attention in regards to optimisation [85, 9] and thus is a good example of the importance of optimisation. There are many variables that can be optimised in a renewable energy system: costs, energy output, reliability, profits, life span, and many others [85]. With so many variables, it can become complicated to find a good solution without the help of optimisation algorithms.

To illustrate some aspects of optimisation, the age old problem of path finding will be used. The task of moving from point A to point B in a large building is a simple example of path finding. Often the desired goal is to get from A to B in the quickest time possible. There are often numerous paths (solutions) for getting from A to B; some will be almost the quickest path (quasi-optimal) however only one path will be the fastest (optimal solution). For simple every day problems, humans will often derive quasi-optimal solutions intuitively.

For larger more complex problems intuition may not suffice. In these cases other methods for optimisation need to be applied to find a solution. Heuristic techniques (colloquially known as "rules of thumb") are methods used to find quasi-optimal solutions to optimisation problems. Finding the fastest path from point A to point B across a large city is a problem with thousands of solutions and no clear optimal solution. In this case the use of a heuristic can facilitate in deriving a suitable solution. In this example a heuristic (rule) could be to follow the most direct roads and predominately taking major roads. However if traffic conditions (e.g. traffic jams or road works) need to be accounted for then the problem becomes even more complicated. Highly complex problems generally require computers to generate optimal solutions.

4.1.1 Multi-objective Analysis

A single objective problem is one in which there is only one output to optimise, e.g. finding a path that requires the least time to get from point A to B. In this example the objective is to minimise travel time. A multi-objective problem is one in which there is more the one objective to optimise. Almost all real world optimisation problems are multi-objective [151, p. 16]. Most multi-objective problems have conflicting objectives, i.e. a gain in one objective means a decline in another objective [184]. Economic, social, and environmental objects are often clear examples of conflicting view points.

Let us consider an optimisation problem with n objectives f which form a vector $y = (f_1, f_2, ..., f_n)$. Each objective function f is to maximised, i.e. we wish to find $\max(f_i) \forall i | i \in 1, 2, ..., n$. It should be noted that any minimisation problem can be converted into a maximisation problem and vis versa, as $\min(f_i) = \max(-f_i)$. The model/problem has m inputs/controls s which form a vector $x = (s_1, s_2, ..., s_m)$. In terms of modelling x is the input space (domain), while y is the output space (range).

One approach to simplify multi-objective problems is to aggregate the objectives using a weighting system, thus bringing the problem back to a single objective problem [151]. This can be done by assigning a weighting scheme to each objective and summing the combined "scores" of each objective, there by giving each solution a single score. We can call this value g and then attempt to maximise g. A simple example of aggregation is $g = w_1 f_1 + w_2 f_2 + w_3 f_3 + ... + w_n f_n$, where w_i represents the weightings assigned to each function. The function g could also be evaluated via some other function h, such that $g = h(f_1, f_2, ..., f_n)$. This function h could be the Euclidian distance from the solutions position to a fixed position in the output space.

While aggregating simplifies the problem, it does restrict the designers freedom in exploring many possible solutions and can beg the question of how the weightings were chosen along with extra communication difficulties. Decision makers have been increasingly employing multi objective approaches which avoid the myopia of an aggregated objective [184].

Another option is to find the Edgeworth-Pareto (Pareto) optimality [40]. A solution that is Pareto optimal is one by which no other solution dominates it. For a solution to dominate another solution it must be better than or equal to the other solution for all objectives, and at least better for one objective. The solutions which satisfy the Pareto optimality condition are called the Pareto set [151, p. 18]. It is up to the designer to choose a solution from the set to be the optimal solution. This is done by considering the trade-offs between all objectives and thus another designer may choose a different solution as personal views dictate trade-off judgements.

The Pareto set is called a non-dominant set because no solution in the set is better then another for all objectives, i.e. no solution dominates another solution (within the Pareto set). A succinct way to describe this is that "For each one of the Pareto arrangements, it is impossible to improve one objective without worsening another" [15]. In this way each solution in a set of Pareto optimal solutions has it's own strengths and weaknesses in regards to the objectives [40]. Formally, if solution x_b is dominated by solution x_a then for all i, $f_i(x_a) \geq f_i(x_b)$, and[2] there exists an i such that $f_i(x_a) > f_i(x_b)$. Because b is dominated by a it is excluded from the Pareto set.

Once the Pareto set has been established, a Pareto front can be determined [184]. The

---

[2] The previous condition does not exclude the possibility that all for all i $f_i(x_a) = f_i(x_b)$

Pareto front is the output of the objective functions when the Pareto set is applied to them. Plotting the Pareto front allows a designer to easily visualise (for < 4 objectives) the set of options [184]. It is then up to the decision maker to choose a solution from the Pareto set to be the optimal solution. This is done by analysing the Pareto front and considering the trade-offs between all objectives. Thus another decision maker may choose a different solution as personal views dictate trade-off judgements [151, p. 18].

In the literature there is a tendency to create a distinction between the number of objective functions a problem has. Problems with 2 to 5 functions are defined as multi-objective, while problems with more then 5 are defined as many-objective [75]. As the number of objective dimensions increase a performance drop is noticed with standard algorithms. Because of this improved algorithms have been developed to combat this issue [75].

Not only does the performance of standard algorithms drop as dimensions increase, but computational demands increase and interpretation of data also becomes increasingly difficult [75]. For example a three dimensioned problem could have it's objectives plotted on a single 3 axis graph. Another dimension could be added to this graph through the use of colour, however beyond this visualisation becomes complex.

4.1.2 Genetic Algorithm Optimisation

Genetic Algorithms mimic the process of Darwin's natural selection and are one of the most commonly used optimisation methods [40, 151, p. 35]. They were originally developed to study the evolutionary process of species, however have now been developed into a large class of optimisation algorithms. They have proven to be simple to implement, while also being effective and reliable [40]. They can be applied to a broad set of problems, including multi-objective problems for which a Pareto front is of interest [40]. They also appeal to researchers because no special knowledge is required to implement them [40].

Genetic optimisation algorithms are very popular for multi-objective optimisation problems. A genetic optimisation algorithm is implemented by taking a set of possible input parameters (solutions) and inputting them into the model. The output of each solution is evaluated to determine its suitability as an optimal solution. This optimality is often refereed to as fitness. New solutions are created through combination and mutation of the fittest solutions [151]. The new solutions are then input into the model and the cycle is repeated. This process is repeated many times until a set of quasi-optimal solutions is discovered.

Terminology

In genetic algorithms the "population" is analogous to a group of animals from one type of species. As the genetic algorithm updates the population with each iteration, the process of evolution is simulated. After many iterations the characteristics of the population will change in a positive manner.

The process of evolution requires a "pressure" to be placed on the population. For animals this pressure comes from the hazards of the environment they live in. "Fitness" refers to how suited an animal is to its environmental pressure. In genetic algorithms, fitness is simulated by comparing how well individuals of the population compare to each other in terms of the problem's objectives [40].

In nature, the most fit individuals are usually the ones to pass on their genes to the next generation. To simulate this a genetic algorithm will probabilistically select individuals form the population to go into a "mating pool" based on their fitness [40]. This is done such that individuals with greater fitness are more likely to be selected and thus more likely to pass on their properties.

One simple way to determine an individual's probability of being selected into the mating pool is to create a fraction based on the fitness of the population. This can be expressed as $P_q = \frac{F_q}{\sum_{q=1}^{N} F_q}$ where P_q is the probability of individual q with fitness F being selected for the mating pool out of all N individuals in the population [40].

The process of selecting individuals is of great importance to the development of the population. If the population experiences too much pressure it could converge quickly on what might be a local optimal solution (but not globally optimal) and then become trapped. If the pressure is weak the population will be able to better explore the search domain but will take a longer time to find an appropriate solution [40].

To generate a new "offspring" solution two individual "parents" are selected from the mating pool. Their solution properties (i.e. the values of their input parameters) are then mixed together to form a new solution [40]. While this process allows the population to explore the domain it does restrict the number of possible variations to a maximum of N^n combinations. To help jump the populations to new regions of the domain, mutations are allowed to occur. In this process a new solution's variables will change with a low probability to a random value [40].

Holland, a pioneer of genetic algorithms, developed a method for accomplishing the process of crossover (mixing) and mutation on a bit string input vector, as shown in figures 4.1 and 4.2 respectively [40]. The crossover cutting point is randomly selected for each pairing of parents. This technique of mutating inputs is not the only technique that can be used, however it neatly demonstrates the main concepts of crossover and mutation.

"Elitism" is a process where the most fit individuals always survive to the next iteration of the algorithm. Some earlier genetic algorithms would replace the entire population with offspring. However it can be shown that it is favourable to keep the fittest portion of the old population, i.e. elitism. Keeping the best individuals guarantees the top of the next generation will be equally fit if not more fit [40].

Figure 4.3 [151] illustrates the performance of genetic algorithm with and without elitism. It is clearly shown the performance of the elitism population always improves, while the non-elitism population can perform worse from one iteration to the next.

In nature, animal species differ significantly and exhibit a large variety of behaviour. Diet is one example of how species can wildly differ. The process of differing ones attributes to gain an advantage is called "niching", i.e. niching causes species to change their behaviour to take advantage of under-utilised resources. For example: a set of species could all be competing for a food source A, thus a species could gain an advantage by adapting

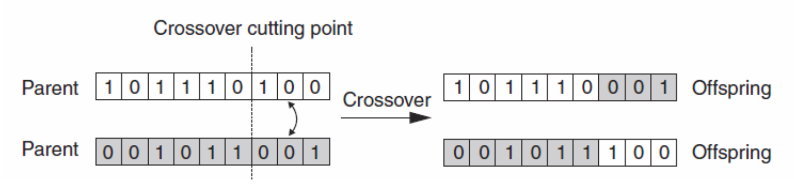

Figure 4.1: The process of mixing parents to produce two offspring. Figure sourced from [40].

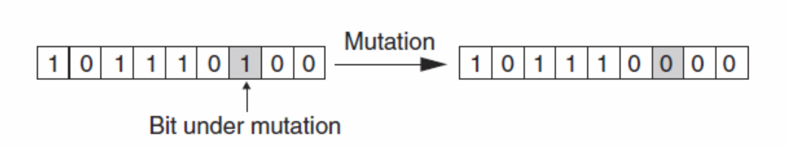

Figure 4.2: The process of mutating an input variable. Figure sourced from [40].

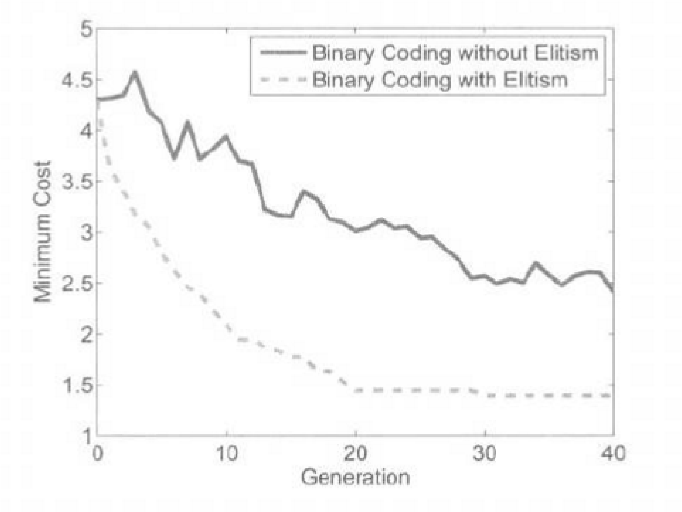

Figure 4.3: The improved performance of a genetic algorithm due to elitism. Figure sourced from [151].

to eat the plentiful food source B.

The application of niching in genetic algorithms forces individuals to spread themselves more sparsely in the search domain [151, p. 194]. For a multi-objective problem, niching could be achieved by forcing individuals in a Pareto set to share their fitness if they are close together [40]. This has the effect of favouring individuals in less populated regions of output space and thus encourages diversity in the population.

"Archiving" is a term which is sometimes used in the evolutionary algorithm literature. It refers to the process of storing all of the best known solutions found by the algorithm. The importance of this can be realised when we consider a population in which more than half of the population are Pareto optimal solutions. If the algorithm has a fixed population size and each generation half of the population is replace by offspring, then we have the condition where some Pareto optimal solutions will be lost from memory. The purpose of implementing an archive is to ensure that these Pareto optimal individuals are not lost from memory.

4.1.3 Types of Genetic Algorithms

Over the past two decades many multi-objective evolutionary algorithms have been developed [184, 192] to improve the process of finding Pareto fronts and sets. Some of these include the Borg multi-objective evolutionary algorithm (MOEA), the Non-dominated Sorting Genetic Algorith (NSGA), the Generalised Differential Evolution 3 (GDE3), the Strength Pareto Evolutionary Algorithm (SPEA), the Pareto Archived Evolution Strategy (PAES), the Pareto Envelope-based Selection Algorithm (PESA), and the Grid based Evolutionary Algorithms (GrEA). Most of these have several different variants, such as MOEA/D, ϵ-MOEA, NSGA-II, FD-NSGA-II, MO-NSGA-II, SPEA2, and PESA2 [184, 40, 75]. Newer versions are often designed to address issues of selection pressure in many objective problems. The newer techniques all follow the basic structure of evolutionary optimisation algorithms, but include slight adjustments to help improve the diversity and niching of the population when searching for the Pareto front in high dimensions (many objective problems) [75].

Basic Genetic Algorithm Structure

As genetic algorithms are executed on computers, it is necessary to know the programming behind them. The pseudo code used to implement a genetic algorithm is outlined below in Figure 4.4 [40]. This outline forms the basis for all evolutionary algorithms, with minor changes to the calculation of fitness or offspring generation separating most algorithms.

NSGA II

The NSGA-II is the most widely used genetic algorithm in the fields of science and engineering [40]. This is due to "its elitism, efficient non-dominated sorting, and parameter free diversity maintenance" [40]. The algorithm is very easy to set up and understand, and thus makes it accessible to anyone with basic programming skills.

```
GA{
    Initialize population;
    Evaluate population, i.e. run the inputs of each solution through
        the model and record the output;
    while( termination condition not met ){
        Calculate the fitness of each member of the population
            and create a mating pool from the fittest members;
        Remove the least fit members from the population;
        Create offspring from the mating pool and add them
            to the population;
        Evaluate the offspring;
    };
};
```

Figure 4.4: Pseudo code of a genetic algorithm

Two major components of the algorithm are shown in figures 4.5 and 4.6. In Figure 4.5 we can see how the algorithm selects a mating pool. First of all the population is sorted into non-dominance ranked groups. All Pareto optimal solutions found are placed into the group F_1. This group is then removed from the population and the procedure is repeated to find the new non-dominated group F_2. This is repeated until there are no individuals left in the population [45].

Once the population is divided into non-dominated groups, the crowding distance is calculated for each solution of that group (crowding distance is based on the position of other group members). This is done by summing the distances (in one dimension) to the adjacent solutions for each objective dimension. The crowding distance is the average side length of the cuboid as illustrated in 4.6. For points on the outer edge, a crowding distance of infinity is given. The crowding distance is then used to sort the members of each group, with a large crowding distance being more desirable [45].

Once the population has been fully sorted the bottom fraction of population is removed and replaced by new offspring spawned from the remaining population (the top fraction), i.e. the mating pool.

4.1.4 Multi-objective Analysis of the World3 Model

Despite the scrutiny applied to the "Limits to Growth" work, it has only had minimal attention in terms of optimisation. There are many cases of parameters being changed "manually" to find solutions that avoid population collapse (e.g. [111, 182, 11]). In the process of this literature review, only one article by Hearne [76] was found that presented a methodical analysis. This analysis however was limited in scope and contained only a vague outline of key results.

The study by Hearne converted the "Limits to Growth" model into a single objective optimisation problem, which can be succinctly described as $min(\int^t \exp(\frac{dP_N}{dt}))$, where P_N is the normalised population and t is time. The domain of the input parameters $\mathbf{x}$ was restricted to deviations of no more then 20% from their nominal values $\mathbf{x}^*$, i.e. $0.8\mathbf{x}^* \leq$

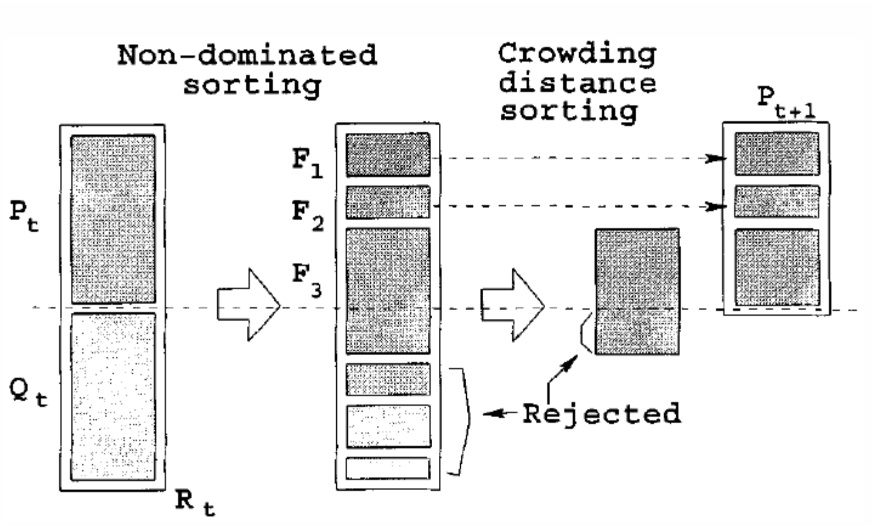

Figure 4.5: The process of selecting a mating pool in NSGA-II. Figure sourced from [45].

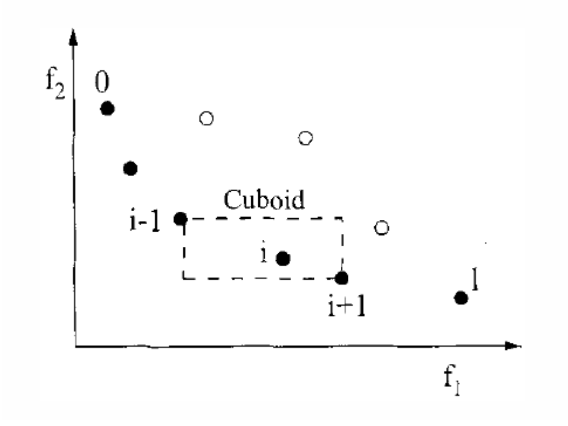

Figure 4.6: Method for calculating crowding in NSGA-II. Figure sourced from [45].

$\mathbf{x} \leq 1.2\mathbf{x}^*$.

The optimisation was performed using the FORTRAN - NAG (Numerical Algorithms Group) Library routine EO4JAF. The routine employs the quasi-Newton method for finding the minimum of a function, which falls into the gradient based class of optimisation algorithms. The simulation found that "at the optimum, ten parameters were on their bounds (20% change from their nominal values) while the remaining four parameters differed from their nominal values in a range from 12% to 4% ... [The optimal] population trajectory does not decline until about 15 years from the turn of the century and the decrease over this period is less than 1%" [76]. The summary quote provided encapsulates the entirety of the study's findings. No figures showing the new trajectory are provided nor are the exact values of the parameters, making it difficult to draw useful knowledge from this paper.

4.2 Aim

The aim of the study was to determine the Pareto front of competing objectives in the Limits to Growth model. By determining the Pareto front, a thorough understanding of how different objectives compete with each other can be obtained.

4.3 Justification

There had been very few attempts to apply optimisation algorithms to the World3 model (or other similar models). Almost all attempts so far have been done by observing sensitive parameters and then "manually" altering these to produce an improved solution (compared to the nominal solution). Additionally, most papers have only been concerned with optimising the trajectory of the population. While this does provide some insights, it does leave many questions unanswered about the optimal parameters. What if there were other objectives to be achieve, e.g.: minimise pollution, maximise services per capita, maximise food per capita, minimise crowding, minimise resources depletion, or maximise jobs? What would the Pareto front look like for a multi-objective problem? What outcomes are mutually exclusive? To answer these questions, it is proposed that a multi-objective optimisation analysis be performed on the World3 model.

By performing a multi-objective optimisation analysis of World3, a richer understanding of the model can be achieved, and greater insights into the working of the real world. Insights gained through studies such as this can give future decision makers a broader knowledge of the trade-offs between objectives, and hence lead to better informed policies. Leaders are increasingly making decisions and crafting policies based on sustainability goals. Knowledge gained from optimisation analysis of socio-ecological models can give leaders a clearer understanding of potential outcomes of policies in relation to sustainability. Without thorough analysis there is less certainty of whether a policy will bring about the outcomes it is intended to produce.

4.4 Method

As described in Section 2.4 The Limits to Growth model was reproduced in the C++ language. To ensure the models was performing correctly, results from the model were compared to the "standard run" results published in "Dynamics of Growth in a Finite World" [112]. Once it was established that the model was running correctly[3], some minor changes were made to the model to better simulate a society conscious of meeting social objectives. In the original model, consumption rates of consumables, services, and food were all functions of industrial output per capita. Giving the "people" in the model the ability to choose their consumption levels (so they can explore different options) meant that some modifications to the code were made. The largest of these changes was the calculation of the industrial capital investment rate (ICIR). The modified equations can be found in Appendix C.

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[3] See chapter 2 for validation.

To establish potential objectives, issues appearing in several UN reports were examined [175, 1, 174, 171, 170]. These texts were chosen due to their broad nature and inclination to examine global issues.

The objectives investigated in the study were: minimise persistent pollution (PPOL), maximise industrial output per capita consumed in 2100 (IOPCC), maximise service output per capita in 2100 (SOPC), maximise desired complete family size after 1975 (DCFS), maximise food per capita in 2100 (FPC), life expectancy in 2100 (LE), and maximise potentially arable land in 2100 (PAL). The objective of minimising persistent pollution represents humanity's desire to keep pollution and wastes in the environment to a minimum to conserve biodiversity and soil productivity, as well as keep toxic materials out of food chains. Maximising industrial output per capita consumed, and service output per capita represents humankind's desire to consume items such as cars, washing machines, houses, and other service based commodities such as health and education. To maximise desired complete family size represents our desire not to be restricted with respects to family planning. Maximising food per capita represents our desire to consume not just grains and vegetables, but also animal proteins that can in some cases require far more grain calories to produce than they provide. Maximising life expectancy represents our desire to live as long as possible and as healthily as possible. Maximising potentially arable land represent our desire to keep some habitat on earth untouched to allow other species to continue roaming around in the wild, e.g. elephants in Africa or bison in North America.

Variables that could be changed were: desired industrial output consumed per capita (IOPCCD), desired service output per capita (SOPCD), health services allocations per capita multiplier (HSAPCM), indicated food per capita (IFPC), desired complete family size (DCFS), fraction of inputs allocated to land maintenance multiplier (FALMM), and fraction of inputs allocated to land development multiplier (FIALDM). It should be noted that many more input variables could have been altered, however these left unaltered to simplify the analysis.

Figure 4.7 shows the output of two simulation runs. The inputs have been selected to produce a stable and an unstable outcome until the final year of the simulation. The simulation period of 1900 to 2100 has been kept from the original model. The output of the stable solution shows that the consumption of industrial output, service output, and food remains steady until the end of the simulation. The unstable solution however shows a dramatic decline of these variables after the year 2060 due to dwindling non-renewable resource levels. This is due to diminishing returns on investment as higher quality stocks are removed first, leaving expensive to extract material for later consumption.

The algorithm used to search for the Pareto Front was the NSGA-II algorithm [45]. NSGA-II was chosen for its simplicity and ease of use. The NSGA-II algorithm operates by sorting a set of solutions by non-domination rank. Each rank is then sorted by crowding distance, i.e. "the perimeter of the cuboid formed by using it's nearest neighbours" [45]. Solutions were normalised to the maximum and minimum of each dimension, so that output was between 0 and 1. This was important to avoid output dimensions with large

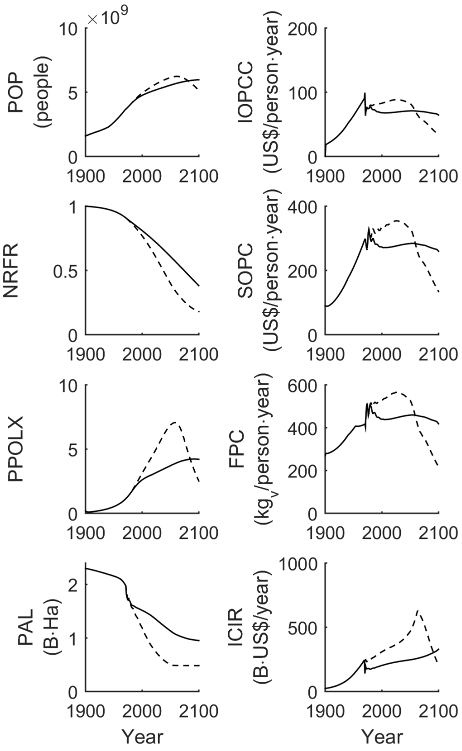

Figure 4.7: Stable solution (solid). Unstable solution (dashed). In the unstable solution the desired consumption of consumables, services, and food were increase by approximately 1/3 from the levels desired in the stable solution.

magnitudes[4] dominating performance indicators. The bottom half of the solutions (once ordered by rank and crowding distance) are discarded, and subsequently replaced using crossover and mutation from the remaining solutions [45].

An ϵ-box dominance [68] archive was maintained during each run of the NSGA-II. Each generation of solutions was compared with this archive of best solutions. If a new solution was found that ϵ-box dominated an archive solution, this solution would replace the archive solution. If no archive solution dominated the new solution it would be added to the archive. The ϵ-box archive minimises number of solutions that have to be kept in memory, at the cost of some detail in the Pareto front.

The algorithm was run with a population of 500 solutions for 250 generations. The ϵ-box archive was saved at the end of the process. After the first 50 generations the objective space was constrained to concentrate the search to solutions of interest (the Minimum Set discussed later). The objective space had to initially be unconstrained to help the algorithm get started. The algorithm was run 20 times to minimise the chance of convergence to a local optimum and help reduce the effects of random chance results (i.e. repeating the experiment to develop confidence in the results). The 20 saved archives A_{1-20} were then combined to determine the best Pareto set A.

The data points shown in later sections were generated by the 20 runs of the NASGA-II algorithm. Each point represents a solution found by the algorithm that is a dominant solution, i.e part of the Pareto set. The data points were passed to the GUI (described in

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[4] An objective which ranges in values from 0 to 1000 could drown out the effects of an objective ranging 0 to 5.

Chapter 3) for visualisation.

Figure 4.8 shows the ϵ-progress of the NSGA-II algorithm with respect to each generation. Each time a better solution is found, the ϵ-progress counter is incremented, thus keeping a record of the progress the algorithm is making. Each line represents a different run of the algorithm. The ϵ-progress counter slows to < 0.1 progressions per generation by the 200th generation (after being constrained). This shows that the rate at which new Pareto optimal solutions are being discovered slows dramatically by the end of the run. This means that continuing the run for more generations is a waste of computing time.

To measure the performance of the NSGA-II for each of its runs, three measures were taken, namely average minimum Euclidean distance (often referred to as generational distance), ϵ-indicator and hypervolume[5]. These indicators were calculated to help determine the reliability of the algorithm's results. The indicators help evaluate the convergence and diversity of a set.

In this study, generational distance [181] is the average Euclidean distance from each element of an archive set A_i to the best Pareto set A. Figure 4.9 shows the average distance of each optimisation run along with the standard deviation. The average distance for all runs was < 0.06 with a standard deviation ≈ 0.02. This indicates that each run of the algorithm produced a set A_i of solutions which were on average less than 0.06 normalised units away from the best Pareto set A.

An epsilon indicator [193] was used to establish the size of gaps (sections of a Pareto front with no solutions) within each of the archive sets. This indicator denotes the degree to which the solutions of a run A_i need to be inflated for the set to dominate the best Pareto set A. Figure 4.10 shows the ϵ-indicator of each run. The ϵ-indicator $\lessapprox 1.5$ indicates that most A_i sets would have to be inflated by 50% to make them dominate the best Pareto set A. This indicates that there are substantial holes (or possibly only one hole) in each set A_i. This indicates that it was necessary to repeatedly run the algorithm to get a clearer Pareto front with fewer holes.

Hypervolume of each run was also calculated to measure the convergence and diversity of each run. The hypervolume was normalised using the best Pareto sets hypervolume. Figure 4.11 shows the hypervolume fraction H in grey, and a scale factor[6] $H^{\frac{1}{d}}$ in black. Hypervolume fraction is ≈ 0.75 and shrinkage factor of ≈ 0.95 for all runs. This means that each run of the algorithm discovers 75% of the best Pareto set's hypervolume. If the volume was a 7 dimensional square box, the length of a side would be 95% of the best Pareto set's box.

Given the indicators shown in figures 4.8 to 4.11, the likely hood that a decent approximation of the true Pareto front has been found is high. This is largely confirmed by the hypervolume indicator scale factor, which suggests that each run gets approximately 95% of the way to reaching the best Pareto set. As 20 runs were conducted, it is reasonable to

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[5] The same indicators as those used by Ward et al. [184]

[6] This represents the length of each side of a hypercube with D dimensions and hypervolume H. This study investigated 7 objectives, thus $d = 7$. This is important as increases in dimensions affect the relationship of side length and hypervolume. If a 2D box has its sides doubled in length its volume increased 4 fold. If a 3D box has its sides doubled in length its volume increased 8 fold. 4D box, 16 fold, etc.

assume a diverse and converged front has been found.

For copies of the software please contact the author at aheath@uow.edu.au.

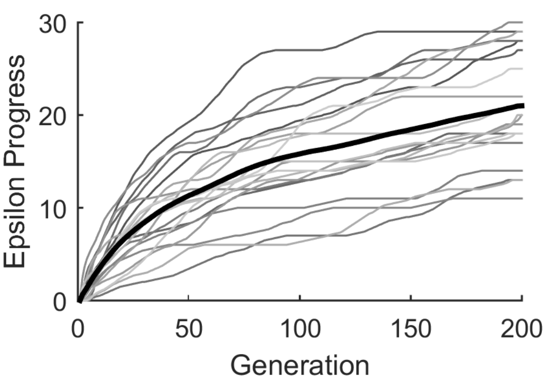

Figure 4.8: ϵ-progress counter with respect to each generation. Fine grey lines show a smoothed result of a single run. Thick black line is the average of all 20 runs.

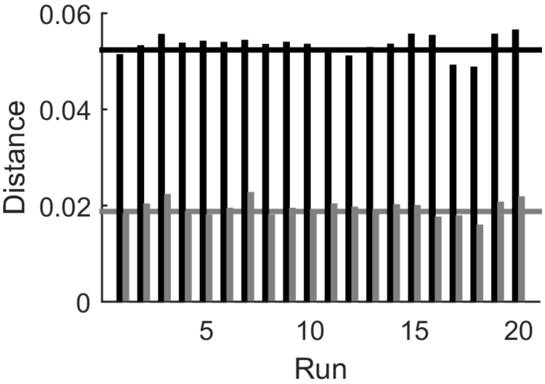

Figure 4.9: Average minimum Euclidean distance (black), and standard deviation (grey) of each run with respect to the reference set. The black horizontal line indicate the average distance of each point in the best approximate set to its closest neighbour. The grey horizontal line indicates the standard deviation.

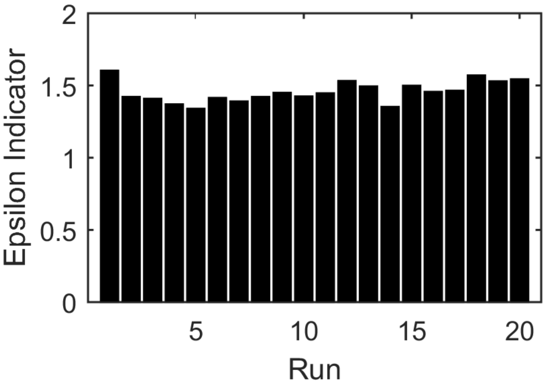

Figure 4.10: ϵ-indicator of each run.

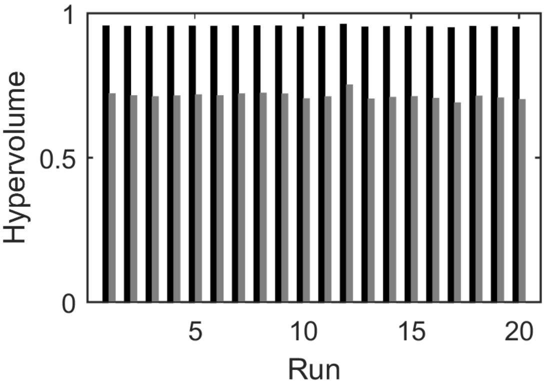

Figure 4.11: Hypervolume (grey), and scale factor (black) for each run.

4.5 Results

Before exploring the results, the author wishes to state that this study has been conducted in a retrospective fashion, i.e. what are some possible conclusions the Limits to Growth team could have reached if a study of this type was conducted around the time of their first publication. Care should be taken when inferring potential policies as the accuracy of any model can never be certain. Dollar values used are consistent with the values used in the model, i.e. 1970 US dollar values, hence they appear small due to inflation and are global averages. An obvious question to raise when critiquing this work is, "why was the model left in its original form?" Or "why were the variables not recalibrated to represent the situation of today?" The main point of the study is to find the relationship between the objectives, not to find the exact options available to human kind. If we were to try to find exact answers, the next question would be, is the World3 model a perfect representation of the real world? Given its broad nature, it cannot perfectly capture the exact workings of the world (nor can any other model). Taking the results from [133] and re-calibrating the model would be a very laborious task, with minimal benefit to the utility of the study. The relationships between variables are still the same as they used to be. For example, an increase in the production of industrial capital still today produces an increase in wastes and pollution. The exact degree of affect may have changed but the general relationship has not [179].

4.5.1 Minimal Set

To begin the analysis of the data a base criteria set was created. This set was used to show the full range of possible solutions given a very loose set of constraints. This set was termed the minimal set to denote its minimal constraint on the objectives.

The minimal criteria was selected based on the values from the model in the year 1970. The values were chosen to narrow down the options to those that are potentially realistic[7]. This was done to remove solutions that would broadly be considered intolerable, e.g. allowing food per capita to fall to almost zero. To give perspective, in the year 1970 of the simulation the values of the variables were as such: persistent pollution (PPOL) = 1, industrial output per capita consumed (IOPCC) $\approx 85 US\$/person \cdot year$, service output per capita (SOPC) $\approx 300 US\$/person \cdot year$, food per capita (FPC) $\approx 400 kg_v/person \cdot year$, life expectancy (LE) $\approx 45 years$, and desired complete family size (DCFS) $> 2 children/family$. At the beginning of the simulation potential arable land (PAL) $= 2.3 G \cdot Ha$. Using this information, the criteria for persistent pollution was set to 10, i.e. not allowing pollution levels to elevate to over 10 times the 1970 level. Industrial output per capita was set at $90 US\$/person \cdot year$, service output per capita to $325 US\$/person \cdot year$, and food output per capita to $400 kg_v/person \cdot year$, corresponding to very slight improvements on the values of the time. Desired complete family size was set to $1 children/family$, implying one child per family. Potentially arable land was arbitrarily set to $0.5 G \cdot Ha$, around a quarter of it's initial value. Life expectancy was slightly improved to $55 years$. With the minimum

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[7] Who can say what is a realistic expectation or not.

	Minimal Set	Alteration
PPOL	10	-6
IOPCC ($US\$/person \cdot year$)	90	+25
SOPC ($US\$/person \cdot year$)	325	+375
DCFS ($children/family$)	1.0	+0.3
FPC ($kg_v/person \cdot year$)	400	+250
PAL ($G \cdot Ha$)	0.5	+0.7
LE ($years$)	55	+10

Table 4.1: Table showing the values used for the minimal set and the amount each variable was shifted to explore its effect on the Pareto Front.

criteria as such, 543 solutions remained of the initial 2642 Pareto optimal solutions. The exact values chosen for the minimum criteria are not of great importance. What is important is that there are still many solutions remaining so that the relationships between objectives can be explored, i.e. if only 3 solutions remained after the minimum criteria was applied, it would then be hard to derive relationships based on 3 data points.

Table 4.1 lists the values of each objective used for the minimal set. Also included in the table are the amount each objective criteria will be altered to study its relationship to other objectives. This is explored in later sections.

4.5.2 Pairwise Relationships

Description of types of relationships

To begin the analysis of the Pareto front, scatter plots of each pair of objectives were created[8]. This was done to determine the nature of the trade-offs of the minimum set. Depending on the relationship objectives have with each other, various patterns can be observed. Figure 4.12 shows seven different categories of potential relationships that objectives can have with each other.

The plots shown in Figure 4.12 are pairwise scatter plots of two objectives, f_1 and f_2, of a Pareto front with three or more objectives. The f_1 and f_2 objectives are to be maximised. There are four broad ways in which the relationships between two objectives can appear.

The first variant is the linear trade-off relationship. In this mode the trade-off ratio of the objectives (i.e. how much one objective must be sacrificed for a gain in the other) is constant. This produces a linear line on the outer-most edge[9]. This is shown in plots C and Y of Figure 4.12.

The next is the small (weak) trade-off. In this case, a small reduction in best outcomes of one objectives gives a large improvement in the other. This is a more desirable relationship for decision makers compared to a linear trade-off, as less of each objective needs to be sacrificed to reach a highly desirable point (a point close to the top right hand corner). This relationship is shown in B and X of Figure 4.12.

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[8] Plots have been presented such that most desirable outcome is located in the top right corner. This means that for objectives that are to be minimised, the axis is reversed.

[9] When we speak of trade-offs in this section we are referring to the Pareto front of the two objectives f_1 and f_2, not of the Pareto front of all objectives.

The next is the large (strong) trade-off. This relationship is characterised by the fact that a large reduction in the best options of one objective, gives a small improvement of the other. This is an undesirable property for decision makers. It is shown in D and Z of Figure 4.12.

The final trade-off variant is the zero trade-off. Here, little to no sacrifice needs to be made of the best option of one objective to achieve large gains in the other. This is the ultimate outcome for a decision maker as no compromises need to be made. This is demonstrated in A, L, and W of Figure 4.12.

Another feature is the spread of points that lay behind the f_1-f_2 Pareto front. In plots W, X, Y, Z, and L of Figure 4.12 we can see significant separation of points from the Pareto front. This signifies that the f_1-f_2 Pareto front is dependent on the other objectives. For the points behind the Pareto front to exist, they must have an advantage in another objective compared to those on the outer front.

Another feature is how correlated the two objectives are to each other. For highly correlated objectives, if the value of one objective is fixed, then the range of the other variable is small. This is shown in figures A, B, C, D, and L.

Table 4.3 defines a list of short-hand notation that describes relationships exhibited between objectives.

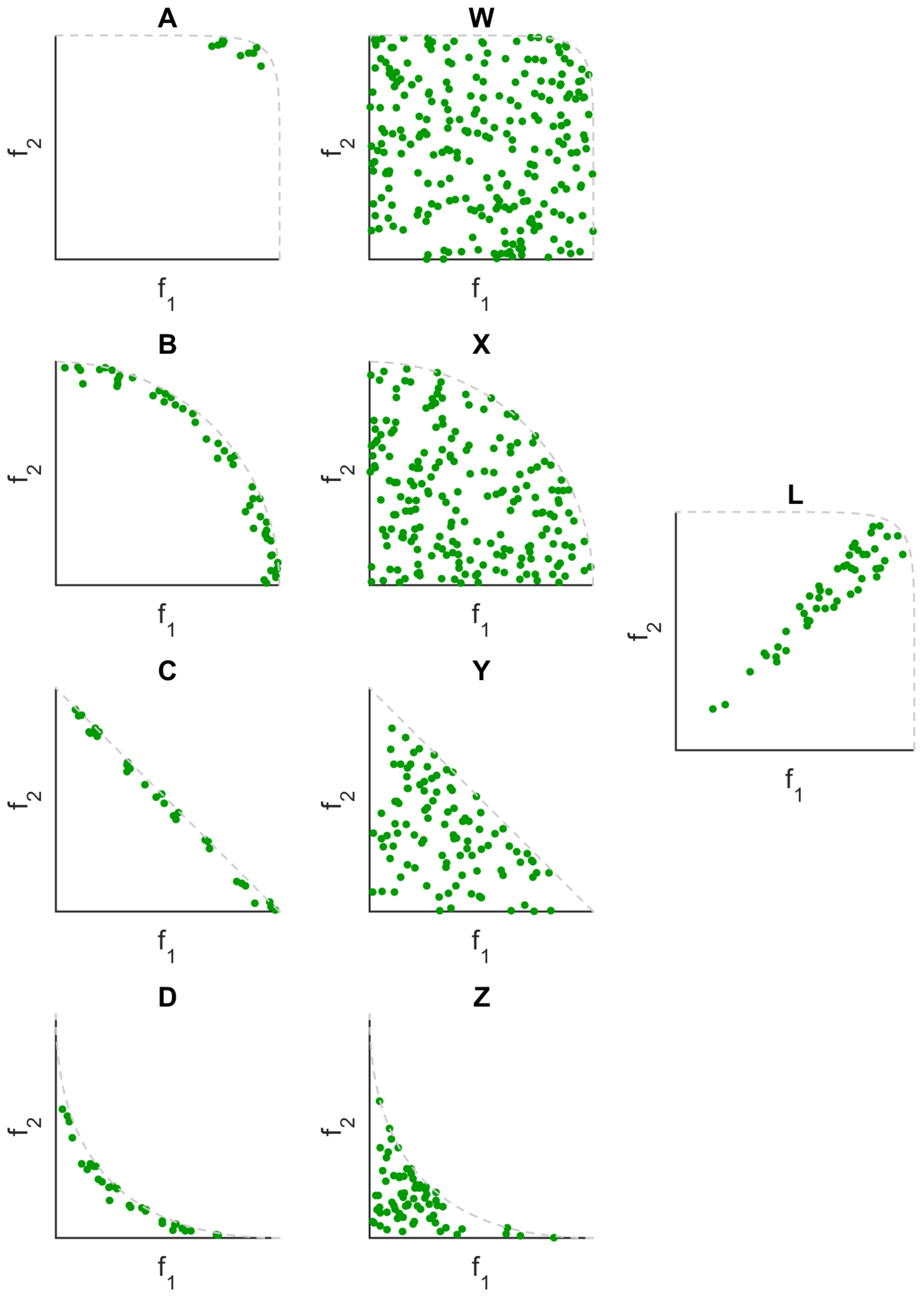

Figure 4.12: Diagrams of possible relationships between two objectives in Pareto front with more than two (3+) objectives. (A) Zero Trade-off. (B) Small Trade-off, Strong Correlation. (C) Linear Trade-off, Strong Correlation. (D) Large Trade-off, Strong Correlation. (L) Zero Trade-off, Strong Correlation, Strongly Affected by Others. (W) Zero Trade-off, Strongly Affected by Others. (X) Small Trade-off, Strongly Affected by Others. (Y) Linear Trade-off, Strongly Affected by Others. (Z) Large Trade-off, Strongly Affected by Others.

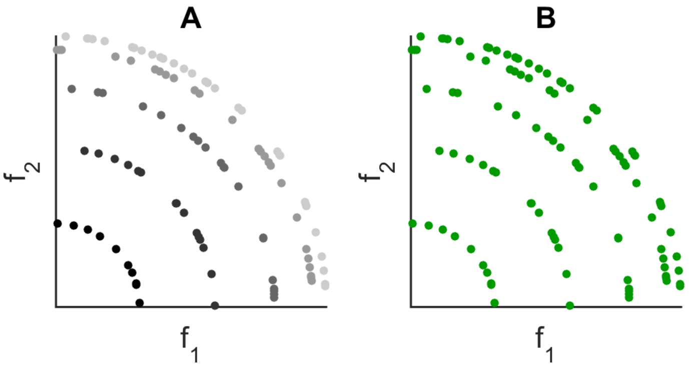

Figure 4.13: A simple three objective Pareto front reduced to show only two objectives. The Pareto front follows the formula $f_1^2 + f_2^2 + f_3^2 = 1$, i.e. the surface of a sphere. In plot (A) the value of the third objective f_3 is denoted by the shade of grey (light being a low value, dark a high value). In plot (B) the information about the third objective has been removed due to the singular colour. Using a singular colour is necessary when plotting a Pareto front with more than three objectives (4+).

4.5.3 Hidden relationships

To get a clearer understanding of the pairwise plots, it is important to understand how other objectives might be behaving. In Figure 4.13 a simple three dimensional Pareto front is shown. This front follows the formula $f_1^2+f_2^2+f_3^2 = 1$. In plot (A) the value of the third objective is denoted by the shade of grey, with darker shades indicating a larger value. In plot (B) this information has been removed. A solution that appears to be dominated in plot (B) (i.e. there are solutions that are better in terms of the two objectives that have been plotted) must be superior in the third objective. This is an important feature to understand when examining pairwise plots.

While it may be noted that a relationship is linear, this may not actually be the case if another variable is held constant. Figure 4.14 shows two different shaped Pareto fronts with three objectives. Both appear to have a linear relationship (when looking at the outer most edge). The shade of grey in plots (A) and (C) again indicates the value of the third objective. In plots (A) and (B) if the f_3 objective is held constant the relationship between the objectives f_1 and f_2 is linear. In plots (C) and (D) if the f_3 objective is held constant the relationship between the objectives f_1 and f_2 is non-linear (curved). This is an example of how the outer edge may appear linear, and hence it would be called a linear relationship, but in reality the relationship between the two objectives is not actually linear. This is an important feature to understand when examining pairwise plots.

A similar variant of this property is shown in Figure 4.15. In plot (B) the relationship would be classified as a small trade-off (based on the outer most edge). However in plot (A) it can be noted that this relationship changes depending on the criteria applied to f_3. As the criteria for f_3 is increased the relationship between f_1 and f_2 goes from small, to linear, to large. This highlights the need to be mindful of generalised claims about the relationship between objectives as these relationships can change depending on the criteria applied to other objective.

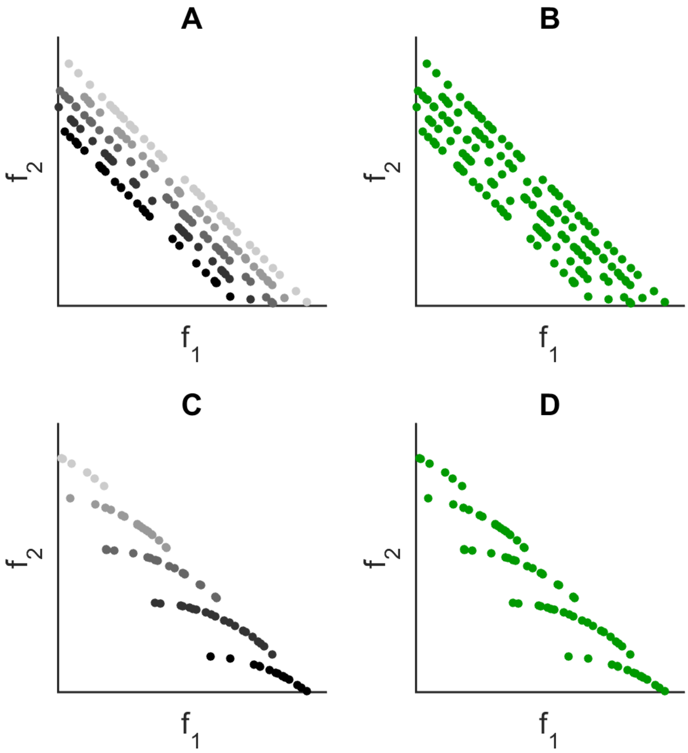

Figure 4.14: An example of a linear relationship deception. Plots (A) and (B) are examples of a true linear relationship between f_1 and f_2. Plots (C) shows (D) show an example of a Pareto front which appears linear in plot (D), however is clearly non-linear in plot (C).

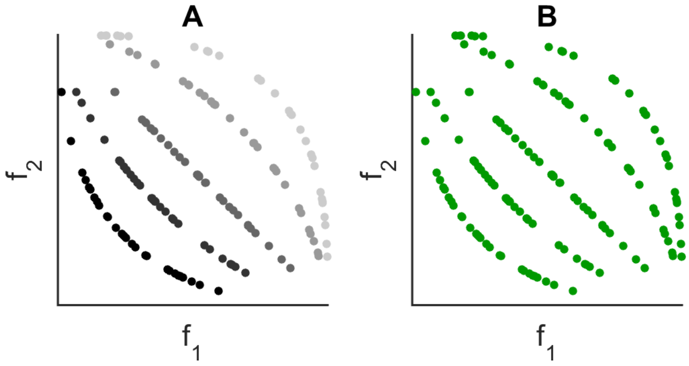

Figure 4.15: The influence another objective can have on the Pareto front shape. Changing the minimum criteria of f_3 alters the outer most edge.

Pairwise example from the study

Figure 4.16 shows the relationship between consumed industrial output per capita and desired complete family size. There is a clear linear trade-off between the two objectives. Being linear, the trade-off is linear for any point along its outer front. The large spread of solutions behind the Pareto-Front of these two objectives indicates a dependency on other objectives.

Figure 4.17 shows the relationship between life expectancy and persistent pollution. In this example we can see that there is a very small trade-off between the two objectives, i.e. there are solutions that can be reached through minor reductions of the objectives maximal values. This is a good outcome for decision makers as there is little compromising.

Figure 4.18 shows the relationship between potential arable land and food per capita. The trade-off between these objectives is linear and strongly correlated. The strong correlation means that by knowing the value of one of the objectives, the value of the other objective is known with a fair degree of accuracy.

Figure 4.19 shows the relationship between potential arable land and persistent pollution. There is no trade-off between the two objectives. A constriction in the criteria of one objective does not remove the best option for the other objective. There is a strong correlation between the two objectives and this must be linked to improvements of other objectives, otherwise the points closer to the bottom left corner could not exist.

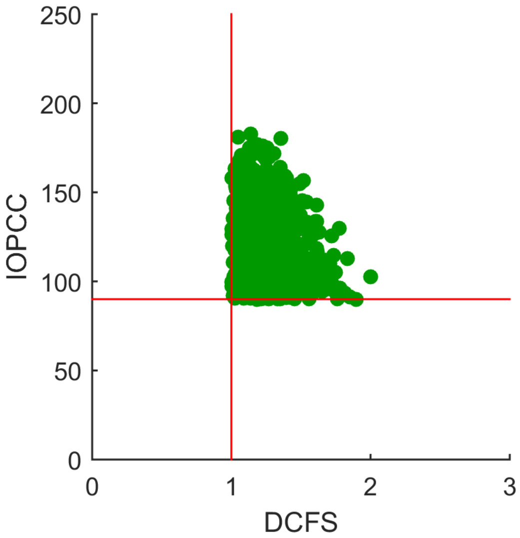

Figure 4.16: Industrial output per capita consumed vs. desired complete family size. A linear trade-off relationship can be seen between the two objectives.

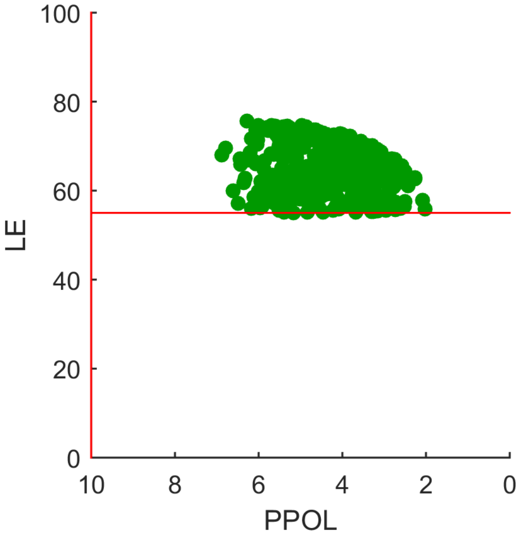

Figure 4.17: Life expectancy vs. persistent pollution. A small trade-off relationship can be seen between the two objectives.

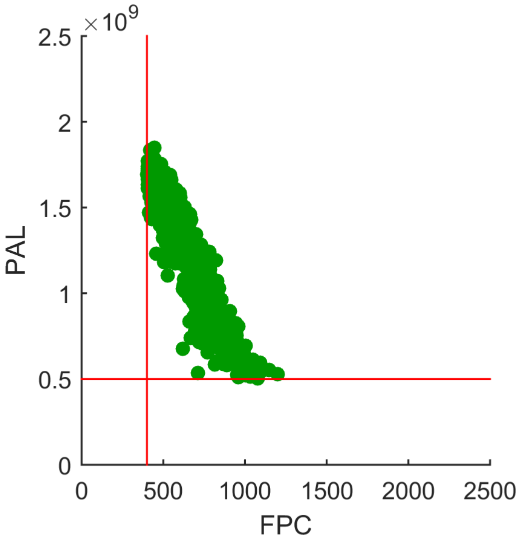

Figure 4.18: Potentially arable land vs. food per capita. A linear trade-off relationship can be seen between the two objectives with a strong correlation.

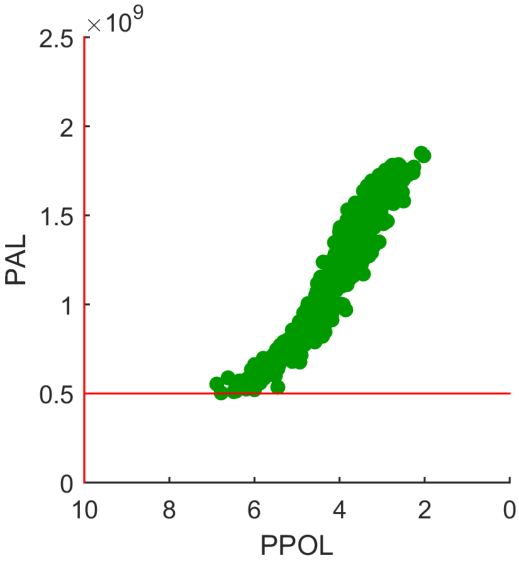

Figure 4.19: Potentially arable land vs. persistent pollution. Zero trade-off relationship can be seen between the two objectives with a strong correlation.

Summary of minimum set relationships

Table 4.2 summarises the relationship between each pair of objectives when examining the minimum set. To help categorise each pair, four statistics were measured[10], T, C, V, and R[11]. T gives a measure of the trade-off between variables (i.e. none, small, linear, or large). This is done by comparing the size of the smallest box able to contain the data points, with the size of dominated space of those points. C represents the correlation between the two objectives, V is the area created by the domains of the objectives and can represent the reliability/sample size. R is the trade-off ratio between the objectives, i.e. the ratio between the maximums of each objective.

To begin we will examine the relationships each objective has to the other objectives. This will largely involve examining the degree to which one objective adversely restricts another (large trade-offs being the most adversely restrictive). If one objective restricts most of the other objectives, it can be said that it is an intrusive[12] objective, i.e. small improvements of this objective quickly remove the most desirable solutions of other objectives.

Persistent pollution has only a small trade-off with all other objectives, apart from the linear trade-off with food per capita. There is a strong correlation between persistent pollution and food per capita, and potentially arable land. The zero trade-off and strong correlation between potentially arable land and persistent pollution mean that excluding poorer results in one of these objectives will naturally exclude poorer results in the other. This is a great relationship to have between two objective functions.

After examining the table, persistent pollution appears to be the least intrusive objective as it competes with only one other objective. Potentially arable land is close to persistent pollution in terms of intrusiveness. It has a linear trade-off with food per capita and a small trade-off with life expectancy.

Life expectancy is not very intrusive. It has a linear trade-off with desired complete family size. All other objectives have a small trade-off with life expectancy, except for food per capita which has a zero trade-off relationship.

Desired complete family size is the most intrusive objective. It has a linear trade-off with consumed industrial output per capita, service output per capita, food per capita, and life expectancy. This is problematic for a decision maker, as an increase in the constraint of this objective will rapidly remove the best options of the other four objectives. It does however not appear to affect persistent pollution or potentially arable land.

Another very intrusive objective is food per capita. It has a linear trade off with persistent pollution, service output per capita, desired complete family size, and potentially

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[10] The final decision of which category the relationship fell into was ultimately made by the author.

[11] $T = 1 - H(A)/V$, $C = \mathrm{E}(\mathrm{P}(A), A)$, $R = max(A_y)/max(A_x)$, and $V = max(A_x) \cdot max(A_y)$. A is the set of solutions meeting the current criteria reduced to just the x and y objectives. To deduce A_x we take the x-objective values minus the x-criteria divided by the x-normalisation value. H is the hypervolume, P the Pareto front points, $\mathrm{E}(A, B)$ the average minimum Euclidean distance from each element of a set B to a reference set A, and max is the maximum.

[12] We do not have a statistical measurement for this property, and have not identified one during the literature review. A possible avenue for measuring this is by studying the correlation or curl between objectives.

	PP'	IO'	SO'	DC'	FP'	PA'	LE
PPOL	NA	X	X	X	\	/	!)
IOPCC	X	NA	!\	!\	!)	X	!)
SOPC	X	!\	NA	!\	!\	X	!)
DCFS	X	!\	!\	NA	!\	X	!\
FPC	\	!)	!\	!\	NA	\	X
PAL	/	X	X	X	\	NA	!)
LE	!)	!)	!)	!\	X	!)	NA

Table 4.2: Shorthand notation describing the relationship between objectives. See Table 4.3 for notation definition.

Symbol	Plot	Description
*	A	Zero Trade-off
)	B	Small Trade-off, Strong Correlation
\	C	Linear Trade-off, Strong Correlation
(	D	Large Trade-off, Strong Correlation
/	L	Zero Trade-off, Strong Correlation, Strongly Affected by Others
X	W	Zero Trade-off, Strongly Affected by Others
!)	X	Small Trade-off, Strongly Affected by Others
!\	Y	Linear Trade-off, Strongly Affected by Others
!(	Z	Large Trade-off, Strongly Affected by Others

Table 4.3: Short-hand relationship notation. Plots are shown in Figure 4.12.

arable land. The only objective not impinged on by food per capita, is life expectancy. Industrial output per capita consumed has only a small trade-off.

Service output per capita and industrial output per capita consumed both have mild restrictions on other objectives. They have a linear relationship with each other and with desired complete family size. All other objectives are small or zero trade-offs.

The small trade-off noted between life expectancy and persistent pollution is unexpected, for in the model persistent pollution reduces life expectancy. The cause of this is probably due to the fact that the positive effects of increasing food per capita (which increases persistent pollution) on life expectancy outweigh the negative consequences of increased pollution. It was assumed before the analysis that life expectancy would have zero trade off with pollution, as it was expected that reducing pollution would naturally increase life expectancy.

Explanations for pairwise relationships

Explanations for each of the observed pairwise relationships are given in Appendix E.

4.5.4 Alterations

To analyse the impact each objective has on the Pareto Front, the criteria for a single objective would be altered by the amount shown in table 4.1. The alteration amount chosen would reduce the number of options by approximately half. New plots were produced to show the resulting reduction in options. Solutions that no longer meet the criteria set are depicted with grey circles. As before, solutions meeting the criteria were marked with green dots. Figures 4.20 to 4.34 show some noteworthy graphs that give us insight into how the objectives react when the criteria of one objective (noted in the title of the figure) is restricted.

Figure 4.23 shows the effect of increasing the criteria for food per capita from $400kg_v/person\cdot year$ to $650kg_v/person\cdot year$. It shows that this has a strong effect on the industrial output per capita consumed and service output per capita. The outer Pareto front of industrial output per capita and service output per capita has been pushed inwards removing the best combinations of these two objectives. It indicates the strong relationship these three objectives have to each other[13]. This strong relationship is also demonstrated in Figure 4.30, however in this case the criteria for service output per capita is increased. This is however of little surprise as the output of industrial capital must be divided between between these three sectors[14].

We can also see in Figure 4.24 the very strong effect food per capita has on persistent pollution and potentially arable land. Its increase quickly removes best options for both objectives. This shows there is a very clear relationship between food per capita, persistent pollution and potentially arable land. A decision maker should keep this in mind when increasing the criteria for food per capita.

The effects of increasing the criteria for desired complete family size from $1.0 children/family$ to $1.3 children/family$ are shown in figures 4.20, 4.21, and 4.22. It can be observed that this increase has a notable effect on the food per capita, service output per capita and industrial output per capita consumed. Interestingly the desired complete family size does not appear to hinder persistent pollution or potentially arable land. This was a surprise as it was expected that persistent pollution and potentially arable land would, like the other objectives, also be heavily influenced by desired complete family size.

Increasing life expectancy from $55 years$ to $65 years$ has minimal impact on other objectives, apart from the desired complete family size and industrial output per capita consumed, as shown in Figure 4.26. In this figure we can see the removal of the best combinations of these two objectives. Due to higher life expectancy the global population will increase, therefore, either less resources are consumed per person, or the birth rate must be reduced to offset the population increase.

We can see some interesting effects occurring when service output per capita is increased from $325US\$/person\cdot year$ to $700US\$/person\cdot year$. We know that this will heavily effect food per capita and industrial output per capita consumed due to their strong relationship. This is clearly marked in Figure 4.30.

A peculiar feature regarding service output per capita's relationship to life expectancy, food per capita and potentially arable land is presented in figures 4.32, and 4.33. This is

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[13] That is an increase in one objective has adverse effects on the others

[14] One may wonder, if we already know the relationships between some variables, what is the worth of analysis such as this? The problem with analysing the formula/model structure is that sometimes unforeseen relationships can arise, which would require a lot of very close and careful study of formulas to reveal. This method (i.e. performing an optimisation analysis and examining the output) however is much quicker, easier, and requires no special mathematical knowledge to perform.

attributed to the characteristics of life expectancy and service output per capita shown in Figure 4.34. Strangely, as service output per capita is increased to above approximately $700 US\$/person \cdot year$ the lower values of life expectancy disappear.

Figure 4.32 shows a well correlated zero trade-off relationship between food per capita and life expectancy. This is a good relationship to have as no compromises need to be made between these two objectives.

It can be noted from Figure 4.29 that an alteration that removes the best food per capita solutions (such as decreasing persistent pollution) produces reduction in the best life expectancy solutions. Unfortunately, a restriction in the criteria of almost all other objectives creates a reduction in the best food per capita solutions, thus also reducing life expectancy.

Figure 4.27 shows that an increase in the potentially arable land criteria removes the best options of food per capita, however it does remove the worst solutions in terms of persistent pollution. This effect is similarly seen when the persistent pollution criteria is decreased. In this case the best food per capita are again removed however the worst potentially arable land solutions are also removed.

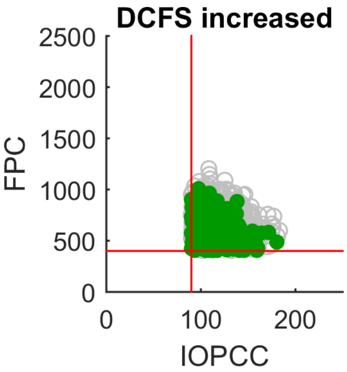

Figure 4.20: The effect of increasing desired complete family size on food per capita and industrial output per capita.

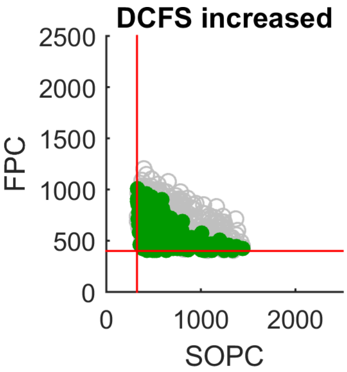

Figure 4.21: The effect of increasing desired complete family size on food per capita and service output per capita.

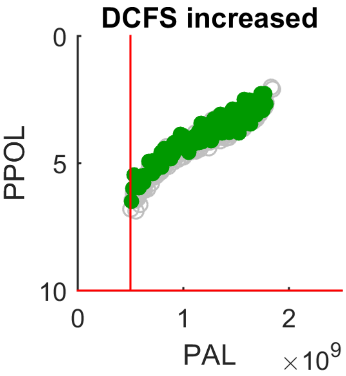

Figure 4.22: The effect of increasing desired complete family size on persistent pollution and potentially arable land.

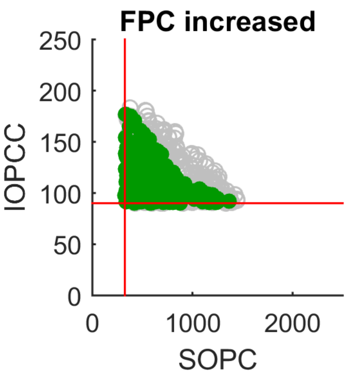

Figure 4.23: The effect of increasing food per capita on industrial output per capita consumed and service output per capita.

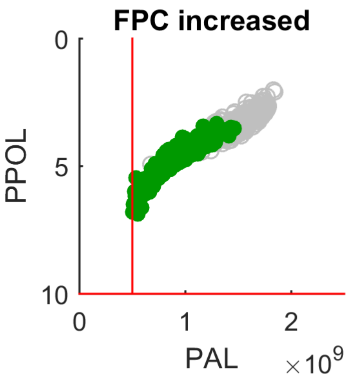

Figure 4.24: The effect of increasing food per capita on persistent pollution and potentially arable land.

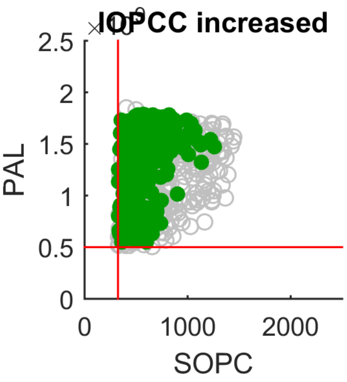

Figure 4.25: The effect of increasing industrial output per capita consumed on potentially arable land and service output per capita.

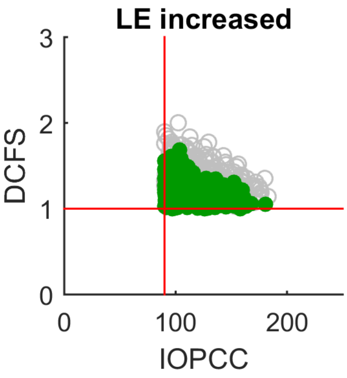

Figure 4.26: The effect of increasing life expectancy on desired complete family size and industrial output per capita.

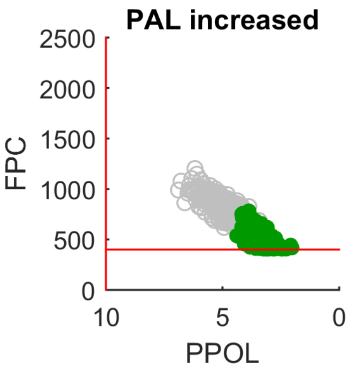

Figure 4.27: The effect of increasing life expectancy on food per capita and persistent pollution.

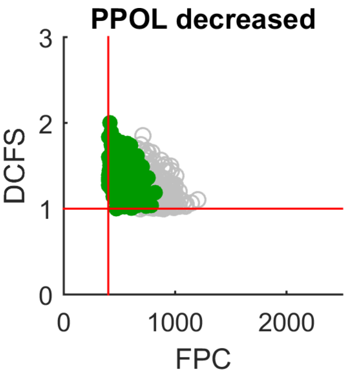

Figure 4.28: The effect of decreasing persistent pollution on desired complete family size and food per capita.

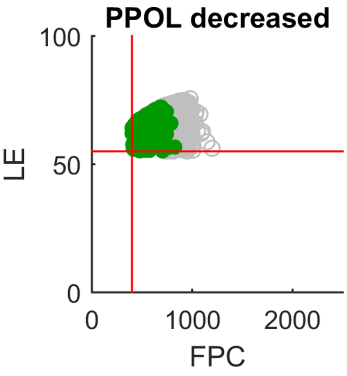

Figure 4.29: The effect of decreasing persistent pollution on life expectancy and food per capita.

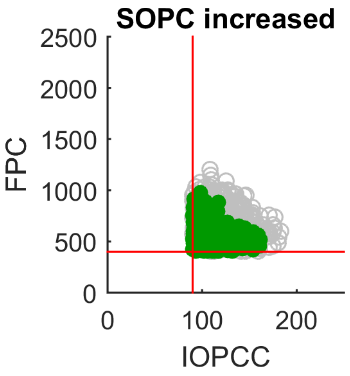

Figure 4.30: The effect of increasing service output per capita on food per capita and industrial output per capita consumed.

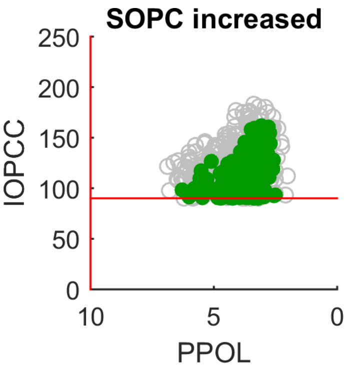

Figure 4.31: The effect of increasing service output per capita on industrial output per capita consumed and persistent pollution.

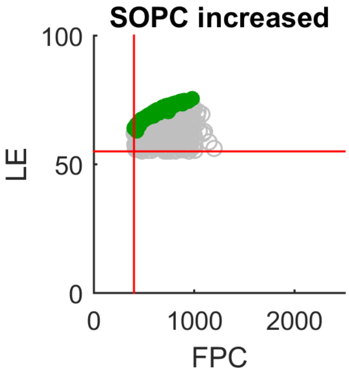

Figure 4.32: The effect of increasing service output per capita on life expectancy and food per capita.

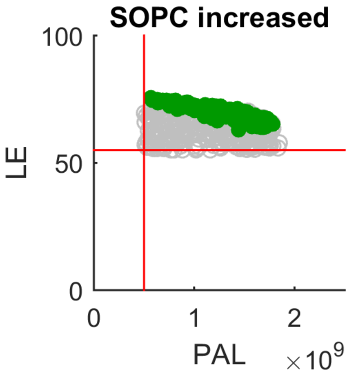

Figure 4.33: The effect of increasing service output per capita on life expectancy and potentially arable land.

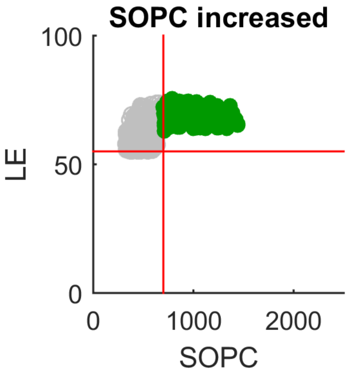

Figure 4.34: The effect of increasing service output per capita on life expectancy and service output per capita.

4.6 Discussion

To begin, it must be noted that it is not the intention to prescribe value judgements on the objects being explored here. We will be attempting to discuss potential options decision makers have at their disposal without judgement as to their ethics.

A possibly contentious objective in this analysis is the desired complete family size, as family planning is a very personal and emotionally charged issue for some people. Most people have a strong aversion to the idea of an outside party having control over another persons ability to have a family. Unfortunately, family size provides a large source of impingement on most other objectives. There have been many unforeseen negative consequences of China's one child policy [33, 28], one of which being some female children being hidden from the government at birth, and thus not having any birth certificate or other identifying documentation. It is also very hard to imagine how a policy affecting this would even be implemented in a democratic society. So far birth rates have been naturally falling as higher standards of living are allowing for better education and access to birth control for women.

The fact that food per capita is linked with potentially arable land (i.e. virgin land) is not overly surprising as most people understand that farming requires land. However, the fact that consumption levels of industrial output and services output per capita appears to have minimal impact on potentially arable land is more interesting as many people are concerned with urban sprawl (which is linked to industrial development). This model suggests that these issues are of minimal importance in the protection of untouched land. One way to improve potentially arable land and persistent pollution could involve encouraging people to eat a more plant based diet, which would naturally reduce the food per capita variable. This is because livestock grown in feed-lots require large amounts of grain to produce the meat, and for every calorie of grain put into a feed-lot a small fraction of that calorie is returned in terms of meat [112].

One way of invoking change is to push policies that have a direct affect on the objectives. The other is to produce an indirect effect through indirect consequence. An example of a direct policy would be taxing pollution generation to encourage reduced pollution production. Another more indirect way to produce the same effect could be to reduce the cost of public transportation which would also have a reduction in pollution (due to a presumed increase in public transport use).

Pushing for an increase of peoples' consumption of services will help solve some other issues such as pollution and life expectancy. By increasing the portion of the industrial output directed to services, the amount of capital funnelled to agriculture and industry is reduced (as illustrated by the relationship between service output per capita desired and food per capita in Figure 4.35), thus decreasing the waste products and pollution produced by these activities (as illustrated by the relationship between service output per capita desired and persistent pollution in Figure 4.36).

Many unforeseen interactions (or lack there of) appeared between the objectives. There was an unforeseen interaction between life expectancy and persistent pollution. It was originally expected that there would be a zero trade-off relationship between these two

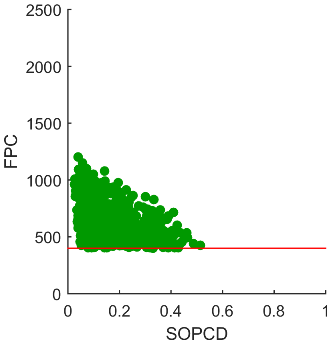

Figure 4.35: The relationship between service output per capita desired (normalised by $1000US\$/person \cdot year$) and food per person ($kg_v/person \cdot year$). As service output per capita increases food per capita decreases.

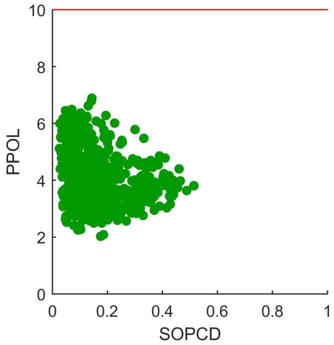

Figure 4.36: The relationship between service output per capita desired (normalised by $1000US\$/person \cdot year$) and persistent pollution. As service output per capita increases pollution decreases.

objectives, i.e. as persistent pollution decreased life expectancy would increase, as this is predicted by examining the Lifetime Multiplier from Persistent Pollution Table (LMPT). This however was not the case, and to the contrary increasing pollution increased life expectancy, presumably due to a greater abundance of food facilitated by the increased pesticides etc.. It was also expected that there would be a linear trade-off between desired complete family size and the objectives persistent pollution and potentially arable land, however this was not found to be the case. The non-trade off of persistent pollution with the objectives industrial output per capita consumed, service output per capita, desired complete family size and potentially arable land was also unexpected, as these were assumed to be factors which would increase persistent pollution. The strong correlation between potentially arable land and persistent pollution was also a surprising revelation.

This study highlights the complexity of the trade-offs humans face, both now and into the future. Without examining our values, we may inadvertently end-up cornering ourselves into a space with only a few options. The trade-offs should be examined, followed by carefully crafted policies to try and steer us towards the future we desire.

4.7 Conclusion

This study has examined the relationships embedded in World3 between some common global objectives, these being minimising pollution (persistent pollution), maximising material wealth (industrial output per capita), maximising services (service output per capita), minimising restrictions on family size (desired complete family size), maximising food consumption (food per capita), maximising wilderness reserves (potentially arable land), and maximising life expectancy (life expectancy).

The persistent pollution and and life expectancy objectives were found to have minimal intrusiveness on most other objectives. Desired complete family size was found to be the most restrictive objective. This objective displayed a linear trade-off with industrial and service output per capita, food per capita, and life expectancy. It had minimal trade-offs with persistent pollution and potentially arable land.

The remaining objectives (industrial and service output per capita, food per capita, and potentially arable land) showed moderate intrusiveness. Understanding the relationship each objective has to the others, allows for more informed policy decisions, and a better understanding of the limits of possible outcomes.

To the authors knowledge, assessment of objective trade-offs (in terms of macro sustainability issues) using a multi-dimensional Pareto front has never before been undertaken.

Chapter 5

Novel Agent Based Socio-ecological Model

Abstract

While models such as World3 are very helpful for understanding our world, they often suffer from extensive aggregation. For example, the population in World3 has no links to regions (different continents or countries), nor are there any variations in wealth among the population. Human decision making is often seen as key to understanding how future scenarios will play out, and because of this it is important that the models we develop try to accommodate this aspect. In this chapter we present a novel model designed to better examine complex socio-ecological issues. A major component of this is the inclusion of many autonomous human agents, capable of making consumption choices based on their unique circumstances. Other important components are the inclusion of multi-item marketplaces (so the people have choices), floating prices on goods (i.e. prices are not predefined), and spacial effects (e.g. transportation and natural resource availability). The model is described and the results of eight preliminary tests are discussed. The model appears to produce reasonable behaviour and proves itself to be capable of examining a wide variety of scenarios.[1]

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[1] Please note that this body of work is not meant to directly link back to previous chapters.

5.1 Introduction

Before any researcher can contribute to a field of study, they must have a solid understanding of the state of the field. The current problems being tackled by the researchers and the methods employed to study them should be known. In the case of socio-ecological modelling, the two main questions which must first be answered are: what issues are being modelled (both now and in the past), and how have these issues been modelled?

There is an enormous amount of literature surrounding the modelling of sustainability issues. This is because of two factors: 1) the plethora of issues that can be tagged as sustainability issues[2], and 2) the numerous ways a set of sustainability issues can be conceptualised and modelled.

The following is an attempt to document and describe the relevant aspects and theories pertaining to the practice of sustainability modelling. The major sections are:

- Purpose of Modelling- investigation of the various reasons models are created.
- Modelling Techniques- outline of the various techniques which are employed to model an issue.
- Theories, Algorithms, and Concepts- discussion of some of the theories (economic, cognitive, natural science) that underpin, and some of the algorithms that are embedded in some models.
- Issues Explored and Components of Models- details of the different issues that have been modelled in the past.

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[2] Because sustainability can refer to the viability of our socio-ecological systems, almost any social or natural system falls into this category.

5.1.1 Purpose of Modelling

Here we will explore some of the reasons models are created.

Toy Models / Model Behaviour Examination

It is easy to assume that every model created to explore a sustainability issue is intended to derive an exact answer. This is not the case. A toy model is one in which the purpose for its creation is to investigate the behaviour of a system. These models are used to answer questions such as, if I change parameter x, what is the change in outcome for variable y. In these cases the magnitude of values is of minor importance, compared to models that have been designed for analytical purposes. Any model which is going to be used for analytical purposes but is yet to be calibrated, can be considered a toy model. Once the model's creator has calibrated the model's parameters with real world data, it transitions from a toy model to an analytical model.

Often if the model produces behaviour which is deemed commensurate with expected or observed real world phenomenon, then the model creator will deem the model a useful tool for exploring further behaviours. This can be justified if the models purpose is to conduct preliminary investigation into possible behaviour [126].

Sometimes the model may be set in an entirely fictitious world with all parameters selected based on the developers intuition [88, 184]. In other cases it may be an extremely simplified scenario designed to examine course behaviour of a system [125].

The advantage of working with a toy model is that it reduces the amount of time needed in the creation process. This is primarily due to the developer not having to invest time towards tracking down real world data to calibrate the model. Time spent figuring out how to calibrate the variables is also saved. This process can take time as most real world data does not always neatly map onto all model parameters. This advantage is also a great danger, as checking the agreement with real world data is of importance in ensuring model accuracy and reliability. It also means that the output does not directly translate into real world equivalents.

A toy model can be thought of as a prototype for a fully developed model. The basics of the model and initial results can be published and then critiqued by others. If it is deemed to have potential it can be further refined and potentially turned into an analytical model [52].

Analytical Examination

An analytical model can be thought of as a fully developed and calibrated toy model. An analytical model is one in which the model is attempting to directly describe the real world. This means that the parameters and variables used in the model must be calibrated to ensure that the output produced is meaningful.

Calibration can be achieved in two ways. One way is to assign values to parameters based on data specific to the parameter. This was the case in the World3 model [112]. The other is to compute the value of the parameter based upon curve fitting of a secondary variable. This type of calibration can be seen in [133]. In this study some of the World3 parameters were recalibrated so that the model's output matched as closely as possible to historical data from the years 1995 to 2012. The variables matched were; population, birth rate, life expectancy, industrial output, service output, and arable land. Data for these variables was sourced from the UN population division, the world bank national accounts, and the UN food and agriculture organisation.

Examples of models, in which efforts have been taken to ensure the model correlates with real world data, can be found in [38, 190, 136, 159, 168, 39, 65, 66]. The purpose of these models is to help guide real world policy.

While comparison analysis of model output with real world data does help validate the model, there still remains the subjective matter of what level of agreement (between the model and real world data) constitutes a valid representation of the real world. In [65] a relative error of less than 10% was deemed as verification that the model was correct, while in [66] 5% was used as the measure for validity. The allowable amount of error appears to be dependent on the judgement of the model creator/user.

Conceptualising, Learning, Education

Sometimes a model is created in the course of an educational experience. In some cases, a collection of stakeholders will come together and work out a conceptual model of a relevant issue. This process has been termed participatory modelling[3] [52]. In the process of constructing the model the participants gain a better understanding of the issue they are facing, and can learn new approaches to tackling the problem [7]. During the process of model development participants may learn new points of view about the issue at hand, and develop stronger ties with other stakeholders. The likelihood that participants of the model building exercise will commit to the drawn conclusions increases due to the fact that the stakeholders should all have the same conceptual model at the end of the process [7].

Because system dynamics modelling is relatively easy to understand, it is often the modelling type of choice for participatory modelling [52]. A model can be constructed and collaborated on by multiple parties, including individuals outside of academia. Another benefit is that the assumptions that produce the base equations are formulated independent of the models behaviour. Thus it is easier to get agreement between parties on the merit of the model [7]. The equations used in the model can be generated through subjective means, i.e. "anecdotal information", or more rigorous empirical means, i.e. "empirical data, survey data, [or] literature" [52]. The downsides of participatory modelling are the extensive time costs (of the stakeholders) and the need for a systems dynamics modelling expert to help guide the creation of the model [52].

The process by which a mental model is turned into a system dynamics model was well documented by Colin Eden [50]. The process of creating a mental model of the world begins when we are born. We usually call this process "learning", and often do not imagine

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[3] Other names given to this process have been; mediated modelling, system dynamics learning, and group model building [52].

it as part of a model building process. Figure 5.1 [50] shows the process by which a person views the world and develops an understanding of a problem. First, events in the real world are picked up through our sensory system, which “filters in” information, i.e. our conscious mind receives only a small fraction of the total information being delivered to the brain (think mild traffic noises outside of an office that go unnoticed). We then take this information and make sense of what it means. Our interpretation of this information is based upon our belief system, belief being our understanding of causality. Once we have an understanding of the situation we can apply our value system to the situation to create an assessment of the problem and decide what should be done to address it [50].

The next step in developing a formalised model is the creation of an influence diagram. This is shown in Figure 5.2 [50]. Influence diagrams allow a person to sketch out the elements in a model (in this diagram they are denoted with a letter, however they could be descriptions or short sentences) and their influence on each other (denoted by arrows). These diagrams allow the creator(s) to get an overall sense of the system they are modelling and easily see where feedbacks exist in the system by identifying loops in the diagram. As the model develops more detail can be added as shown on the right hand side of the diagram.

The final step involves codifying the influence diagrams into mathematical expressions. This step requires the help of someone knowledgeable in developing system dynamics models. The overall process is shown in Figure 5.3 [50]. We can see that the conceptual model informs the creation of the influence diagram, which leads to the system dynamics model. In the process of creating and testing the system dynamics model and influence diagram, new insights are fed back into previous steps.

Sometimes a simple model is created and turned into a game (often computer based) to help educate people or students about a complex topic [123, 94]. Serious games aim to be immersive, educational, and engaging [137, 108, 94]. Some examples of serious games have had the user pretend to; run a local government [148], be a business owner [12], be a water resource manager [46], or be an urban planner [137]. With a serious game, the act of playing the game educates the student about the issue. This can be done in conjunction with the classical learning of equations and theories.

While serious games are an interesting new development (in terms of computer mediated games), more research still needs to be conducted in order to validated their usefulness as an educational tool [94, 108]. Gamification is also not a universal principle that can be applied to all models [108].

In other scenarios the model is described and produced for the task of disseminating the ideas or theory thought to be driving a particular issue. In these cases the model may never actually be run through a computer as it is simply created to formally describe the relationships between objects in the real world for educational purposes. An example of this is presented in [80], where a model is formulated but not turned into a computer simulation.

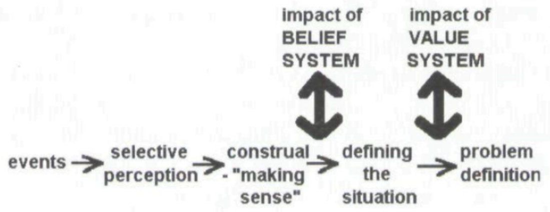

Figure 5.1: A theory of perception and problem formulation. Figure sourced from [50].

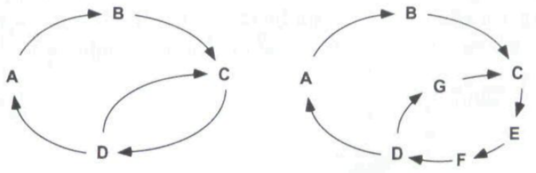

Figure 5.2: A generic influence diagram. Left - a simple mental model. Right - the same model but with greater detail. Figure sourced from [50].

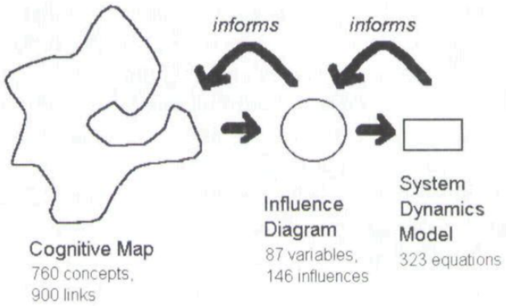

Figure 5.3: The general process of converting a mental model into a system dynamics model. Figure sourced from [50].

The issue of complexity

There is a wide variety of issues which researchers attempt to model. For some researchers, different aspects (of the world) are of greater interest than others, and hence these aspects receive a detailed description in the model. Other factors which are of less interest are described in a more simplistic way [52]. Some aspects may even be ignored or omitted for simplicity or due to a lack of relevance. Concepts such as Occam's razor are often applied to ensure a model remains as simple as possible while still performing it's required function.

The tension of including enough detail and keeping a model simple is a large part of designing a model [109]. This can be thought of as a balance between completeness and cumbersomeness. It is often guided by intuition and experience as every issue that is modelled is unique [168], and so there are no clear rules for defining what should be included or excluded. Unfortunately socio-ecological issues are highly complex, and much detail has to be kept in order for the model to be representative of the real world [109]. This creates a tough situation for modellers who have to decide on an appropriate level of detail for their model.

While including fine details can broaden the knowledge of how a system works, it can also have draw backs. These drawbacks can be due to an increase in complexity of: the model's set-up, experimental study[4], time required to run a simulation[5], interpretation of results, and the communication of the model and its results to others. Thus a balance needs to be struck between complexity and simplification, to allow a model to be both insightful, useful, and practical.

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[4] The ease at which inputs and parameters can be changed and managed.

[5] Sometimes simulation time is a non-issue.

5.1.2 Modelling Techniques

There are many types of models researchers can select from to analyse socio-ecological systems [96]. Two commonly used types are system dynamics and agent-based models. However, there are many other model types. The type chosen by a researcher depends on the specifics of the issue they are studying and the objectives of the research [17]. It should be noted that the model a researcher develops may not neatly fall into a predefined category. In many cases the model will exhibit aspects of multiple types.

Conceptual

This type of model exists only in the mind of an individual. It is a theory of how something works. It can be communicated to others either through speech or text. Philosophical models of how the world works would fall under this category. There is often no underlying mathematical formulations to these models. These are of a philosophical flavour, as is the case in a paper by Hjorth and Bagheri [80]. In this paper they give a describe of a[6] conceptualisation of the world, and also provide an influence diagram with stocks and flows.

Pictorial

Of all the methods for examining sustainability, pictorial models are the simplest and easiest to interpret. Examples include Venn diagrams, pictures/drawings, and flow charts [160]. This makes them suitable for reaching the general public but unsuitable for in depth assessment [160]. An example of a pictorial model is shown in Figure 5.4 [186]. Pictorial models are similar to conceptual models, in that they may not necessarily have any mathematical underpinning. The main purpose of the picture is to communicate the ideas of a theory about the workings of the world.

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[6] Presumably their own.

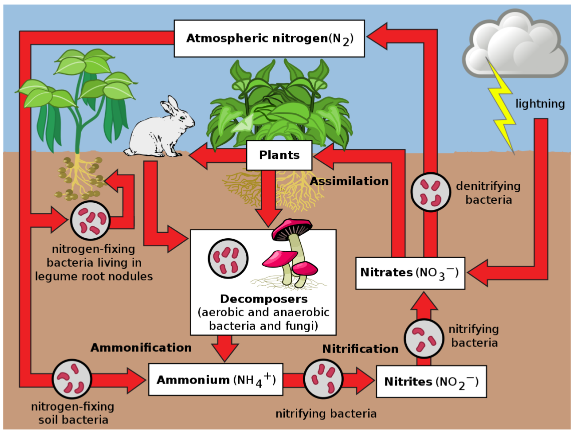

Figure 5.4: The nitrogen cycle. Figure sourced from [186].

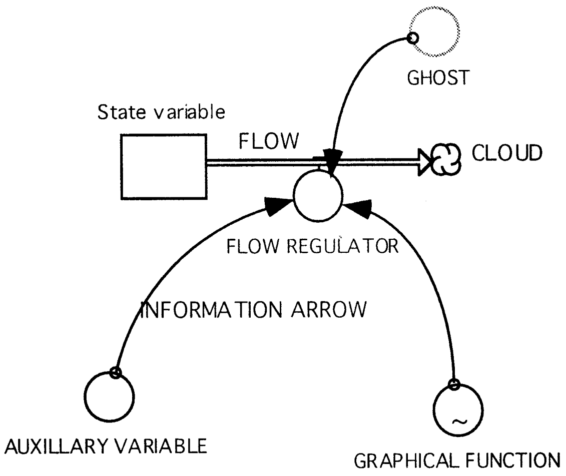

Figure 5.5: Stocks, Flows, and Parameters concept primarily used in System Dynamics. Figure sourced from [37].

Physical Models

Physical models are models which physically exist. Scale models (e.g. vehicle aerodynamic testing in a wind tunnel) are a clear example of this. While physical models can be highly accurate, they can also be costly, resource intensive, time consuming, or unsuited to the phenomenon being modelled.

Input and Output Models

Input/Output models are widely used in economics to determine the relationships between economic sectors [4, 3]. The core of the input/output model is a matrix which records the amount of products required to produce other products. These ratios are fixed and so these models are static [120]. The amount of product produced by the system can be specified (i.e. consumer demand). The required production rates of each economic sector can then be determined. Varying the economic output of the system (demands), will cause shifts in the production rates of each sector, and hence consumption habit effects can be observed.

System Dynamics

System Dynamics is the study of real world systems through the representation of stocks (system variables), which change with time due to inflows and outflows. The inflows and outflows vary over time depending on stock levels and external parameters [37, 20]. A graphical representation of this can be seen in Figure 5.5 [37] which depicts a single stock with one outflow.

System dynamics modelling allows for analysis of highly complex systems without complicated theory or mathematical equations [80]. Very simple and linear relationships, when combined, can create highly complicated behaviour. System dynamics models can be easily expanded to give a holistic representation of a system [80]. Through the development of a holistic representation, a deeper knowledge can be formed of the represented system.

System dynamics analysis is based around the study of non-linear systems and control theory [7]. The behaviour/output of a system dynamics model derives predominately from the systems structure i.e. the links between variables [7, 80]. Thus an analyst of systems is interested in the structure of a system and the resulting behaviour [76, 16].

Feedback is a major component of many system dynamics models. A simple example is population growth, i.e. people have babies, babies increase the population, the larger the population the greater the number of babies born each year. This is an example of a positive feedback loop, i.e. where growth of a stock produces an increase in that stocks growth rate. Negative feedback occurs when growth of a stock reduces it growth rate. Positive and negative feedback loops can also be referred to as 'amplifying' and 'damping' loops respectively [39]. These loops can also be thought of as reinforcing (drive the system to an extreme) or balancing (drive the system to equilibrium) [80].

The field of system dynamics has allowed for a simple graphical representation of complicated systems and thus communication of a system to wider audiences [80]. This wider audience includes academics outside the field being studied, political leaders, and even to the general public. Figure 5.6 [80] shows an example of an influence diagram. These are easy to understand and can be effectively communicated. Many software packages have been created to aid in the construction process of system dynamics models. Some of the more notable packages are Stella, Powersim, Geonamica, Simile, Vensim [52], and OpenModelica [41]

Agent Based Models

A multiple agent model (often referred to as agent based modelling (ABM)) simulates many entities interacting with each other and their surroundings/environment [20, 96]. The entities follow simple rules that dictate how they behave, and will adjust their actions or strategies based on their awareness of the environment [5]. By having many entities interacting the global behaviour of the entities can be simulated and studied. This global behaviour is referred to as the emergent behaviour [96, 5]. The emergence of macro-level behaviour from micro-level behaviour is the key focus of agent based models [96].

A model of a colony of ants searching for food is a simple agent-based model to imagine. The agents (the ants) follow simple rules that dictate their behaviour. By having many agents interacting with each other the global behaviour of the agents can be simulated and studied. In the ant colony model, the global behaviour of interest is the path most travelled by the ants as this is usually a quasi-optimal path.

In other modelling methods, agents are generally summed (aggregated) into one large homogeneous agent to simplify the model [96, 179]. This however removes many fine details. Agent based modelling allows for these details (heterogeneous actors and actor interactions), and hence can be used for more complex systems [88, 12].

The main advantage of agent based modelling is it's ability to represent collective behaviour given different assumptions about micro level behaviour (i.e. the rule agents follow when reacting to their environment [96]) [16]. It can take into account aspects such as evolutionary learning (i.e. agents adapting their behaviour based on analysis of past failings

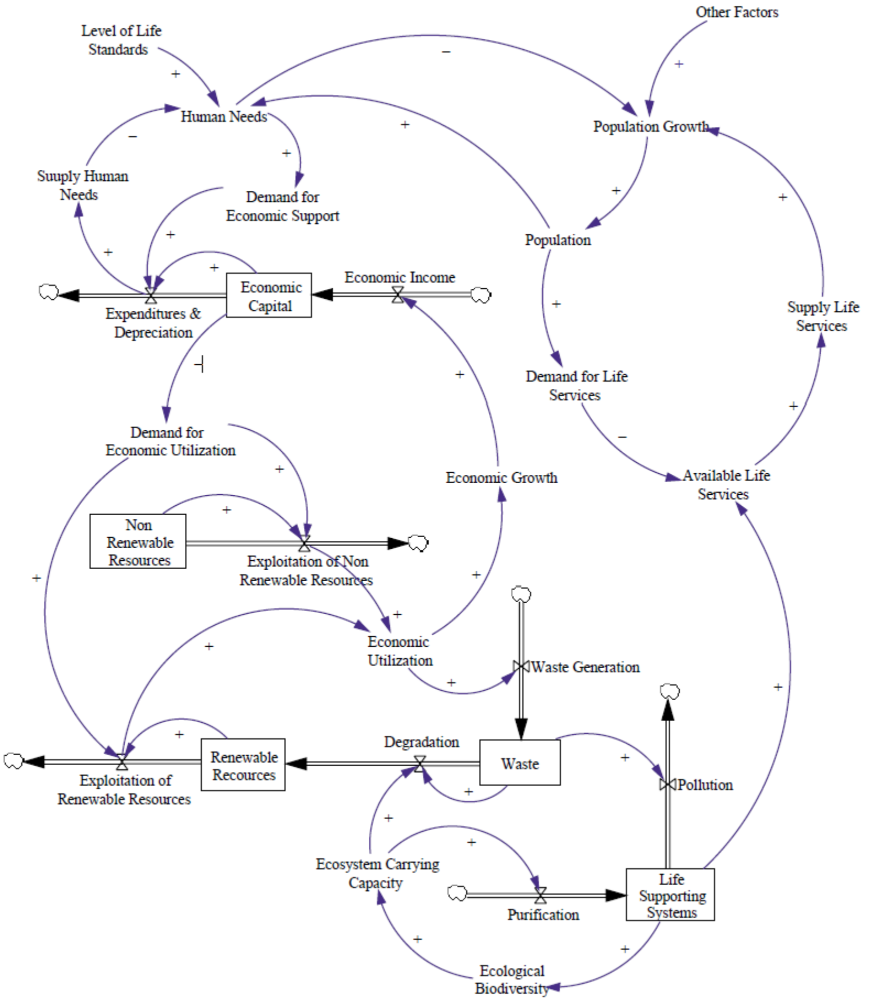

Figure 5.6: A generic example of a system dynamics model. This model gives an overview of the connections between the economy, the ecosystem, and the human population. Figure sourced from [80].

and successes), spacial effects, and non-equilibrium dynamics [57, 96]. These properties mean that agent based modelling is able to analyse "complex adaptive systems such as economies" and governmental "public policy impacts" [179, 16].

The assumptions made about the behaviour of agents (human agents) can vary greatly from one model to the next. Some models may base the behaviour of the agent on behavioural theories (e.g. behavioural theories from psychology [57]), while others may ignore these theories and attempt to replicate (at the macro level) real world empirical data [57].

Decision making can be broken into two major categories, namely rational and heuristic. Rational behaviour models assume that agents will choose the action which maximises their utility based on full access to knowledge. In reality, humans are more likely to follow heuristics rather than calculate the utility of all possible options. This is due to limitations that are placed on real world humans, e.g. time constraints, bounded rationality, information error, and social norms [88, 51, 6]. It is important for policy makers to understand "how [people] actually make decisions", as opposed to "how people should" [51], as the differences in behaviour can have significant effects.

While multi agent models are a great tool for modelling complex systems, the laws used to control the agents are often hard to validate with empirical data [61]; thus often the model is validated by comparing the macro "emergent" behaviour of the group to empirical information [88]. The difficulty in validation forms a major weakness of the method [6], as any model should have its predictive power validated [57]. While the underlying laws that govern the agents behaviour might be easy to communicate to others with minimal technical knowledge, however often the results (emergent behaviour) are harder to express [96].

There are many methodological issues that must be addressed before agent based modelling can be used in policy development [20]. A major issue is the lack of standardisation between reporting and transparency of model structure, making it hard to compare and critique various models. Attempts have been made to create a standard set of questions that guide the reporting of an agent based model's structure. This standardisation process is examined in [118].

Agent based modelling is a relatively new technique. It has been pushed forward by the social science, game theory, and psychology fields, which are continually striving to make more realistic representations of social phenomena [8, 6]. These fields have adopted the modelling practice as it is well suited to the study of cognitive processes and their effects on the macro social outcomes [96]. The use of agent based models has grown due to the ever increasing ease at which computer programs can be written and managed, as well as the development of software packages [20]. Agent based modelling is beginning to be used more often in economic models [29]. In these models all transactions are tracked at an individual level. This newer approach is termed the flows of funds method. It is an approach that better represents the economy and financial system and can be valuable in identifying fragility in these systems [29].

Mathematical Formulations

A mathematical model is one in which the model is designed to be solved and examined analytically, as opposed to numerically. This means that the model focuses on a small number of equations and variables as opposed to a large set (e.g. World3 model) that require a computer to process [73]. The analysis of a mathematical model will often involve examining transition diagrams to establish the location of critical points, e.g. saddle points and stable nodes. A transition diagram is shown in Figure 5.7 [23]. Mathematical models may be used to calculate optimal outcomes. In these cases the Hamiltonian function is applied to find the optimal conditions, e.g. in [30]. Some models could be classed as mathematical representations can be found in [125, 130, 34, 23].

Para-modelling Techniques

While the content of the following sections cannot be described as modelling techniques (in the same way mathematical, system dynamics, and agent based models are), it is of

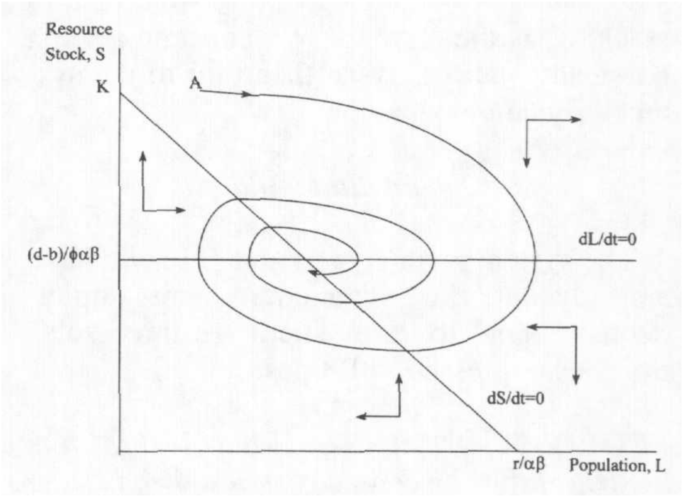

Figure 5.7: Transition diagram. Figure sourced from [23].

prominence within the sustainability literature and thus has been given a place in this literature review.

Indicators

While indicators are not models, they are a prolific component of research work based around sustainability. An indicator is a measure of the current state of an item, which is of concern [86]. Indicators can be easily applied across a wide variety of fields. The simple output of indicators makes them easy to interpret and analyse [152] and thus are often used by policy makers and for the informing of the general public [152].

Much research has been directed towards generating and improving indicators [160]. This is partially due to the role indicators play in measuring sustainability [120]. By defining a reference value, to which the indicator can be compared, the 'health' of an issue can be determined. Without a reference value an indicator is merely a number [152]. This is a simple method for determining if something is on a sustainable course or not [152].

As Meadows notes, "indicators arise from values, and they create values" [152], i.e. matters of interest are measured and what is measured becomes a matter of interest. This statement proposes that the creation of an indicator can prompt concern for the object of measurement.

Most indicators (and all composite indicators) perform aggregation to give a final "score". Composite indicators are derived by aggregating multiple indices into one, in an attempt to produce a more holistic indicator. While composite indicators do summarise more information into a single value, they open up many avenues for subjectivity. Subjectivity can be introduced via the: selection and quality of data, selection of indicators, normalising schemes, weightings, and aggregation methods [152].

A paper produced by Singh et [152] lists a large collection (approximately 41) of indicators that have been developed over the years. Some well known indicators are "Ecological Footprint, Human Development Index [172], and Genuine Progress Indicator" [168].

While indicators are simple to use and interpret, they can only give retrospective assessments, hence they cannot be used for making predictions on a long term scale [120]. Indicators offer little "explanation of the phenomena" they represent and have been criticised for "distract[ing] from the real phenomenon". Policy makers can become fixated on a target number rather then understanding and fixing the underlying phenomenon causing the issue [160]. Another drawback of indicators is that they need to be kept updated to remain accurate [83], and establishing data banks can take a large amount of time and resources [83].

General Equilibrium Models

General equilibrium models are economic models which assumes that all markets have reached an equilibrium state. This draws from the theory that in the long run an economy will reach a steady state if no technological changes or other disruptions occur. This means that supply and demand are balanced [120] and that prices will have come to rest at their appropriate level according to a supply demand curve. A basic supply and demand curve is shown in Figure 5.8 [26]. These assumptions are often applied out of necessity to make solving the problem possible with a certain technique [169].

The simplifying assumption of general equilibrium removes many aspects of the real world from these models. Some of these aspects are; feedback loops, agent interactions, agents short term behaviours, trends in fashion, technological change, financial speculation, or transaction costs [115, 120]. Because of these disadvantages, some economic modellers have begun encouraging a shift away from the general equilibrium models to other out-of-equilibrium models [169], such as system dynamics and agent based models.

Impact Analysis

Product-related assessment is focused on analysing the amount of energy, materials, or money consumed in the life cycle of a product [128]. This is an important aspect in the

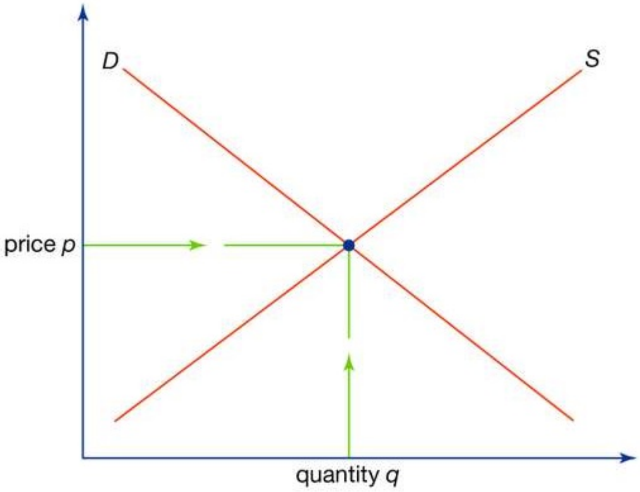

Figure 5.8: A simple supply and demand curve. S - supply, D - demand. As price increases the quantity of product demanded decreases and the quantity of the product supplied increases. Figure sourced from [26].

design of products, as improvements in these factors can help reduce the impact of humans on the environment. The downside to these techniques is that they examine a very small and niche portion of the global system, i.e. the product being examined is removed from the global context. This analysis can only tell us information about the product, not the global system as a whole [128].

Integrated Assessment Models

Integrated assessment models include representations of both environmental elements (natural world) and social elements (man made world). This means that integrated assessment is not a modelling technique, but rather a category of model which depicts both the social and natural elements of the real world and the relations these have with each other [61]. The level of detail in an integrated assessment model can vary dramatically, with some using very simple representation, while others combine specific knowledge from many fields, e.g. economics, chemistry, biology, and physics [103]. The World3 model [111] is relatively simple, while the GUMBO model [21] contains far more detail. Integrated assessment may involve combining two or more pre-existing models into a meta-model, with minor adjustments made to link the variables of each model together, i.e. the output of one sub-model is used as input for the other sub-models, and vis versa.

Multi-Criteria Assessment

While not an actual model, it is important to mention multi-criteria assessment as it is widely used by policy makers as a tool for deciding courses of action given a list of criteria [7]. A decision maker will have a number of options and a list of criteria. Each option is rated with respect to each criteria. The ratings are aggregated, often with weightings, to give a final score for each of the options. The option with the highest score should theoretically correspond to the most suitable of the options.

5.1.3 Theories, Algorithms, and Concepts

The following will cover some theories, algorithms, and concepts that appear in the literature surrounding socio-ecological modelling.

Endogenous and exogenous factors

The distinction between endogenous and exogenous variables and parameters in a model is very important. An exogenous variable is a value that is controlled from outside the modelled system. An example of this would be a production efficiency coefficient that is either held constant or varies with time according to a set function [121]. An endogenous variable is controlled by the system. To use the example from before, if the efficiency is dependent on investment from within the model, then the change in efficiency would be endogenous. In the model presented in [61] "prices, wages, energy use, and technological change are determined endogenously".

Production functions

The way production of goods and services is handled in models can vary significantly. One method is the Douglas-Cobb production function, which assumes that factors of production can be substituted with diminishing returns [188]. For example, if the production rate of product (p) depends on a flow of resources (r) and human labour (l), then production is calculated by $p = r^a l^b$, where a and b sum to 1. The advantage of the Cobb-Douglas function is that it does not require conditionals to determine if a flow is being under-supplied. While this function is simple to implement and analyse [169] it can bring about assumptions that can be unrealistic. Models which have employed this function can be found in [188], [66], [126], and [169].

Another production function is the Leontief function. This function assumes that no substitutions can be made between inputs. To use the example before, the production rate would follow $p = min(r, l)$. This type of production function can be found in [121], and [38].

Figure 5.9 [116] shows a comparison of a linear, Leontief, and Cobb-Douglas substitution of production function. Another expression is the constant elasticity of substitution function, which takes the form of $p = (r^\lambda + l^\lambda)^{\frac{1}{\lambda}}$, where λ is a constant [116]. This produces a curve similar to the Cobb-Douglas or the linear function depending on the value of λ.

Tipping Points, Irreversibilities, and Non-linear Functions

A tipping point is a point at which a system rapidly changes its characteristics. This is demonstrated in Figure 5.10 [93]. Defining when a system has "tipped" over a threshold is difficult, as it is not a clearly defined state [93]. An irreversible system is one in which the process path taken to one point cannot be traversed in reverse to get back to the starting state. This property is called hysteresis. This is demonstrated in Figure 5.11 [117]. These two features can appear in systems that are non-linear in nature, which is often the case for complex socio-ecological systems.

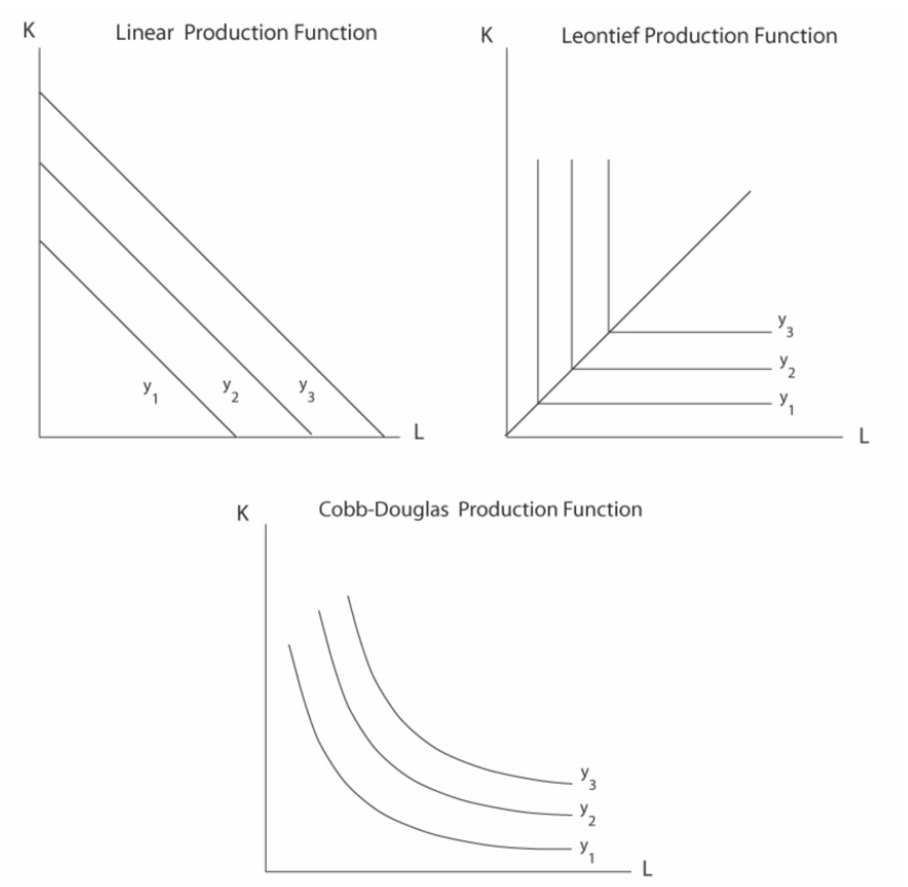

Figure 5.9: Three different forms of production functions. y - production output rate, K - capital, L - labour. Figure sourced from [116].

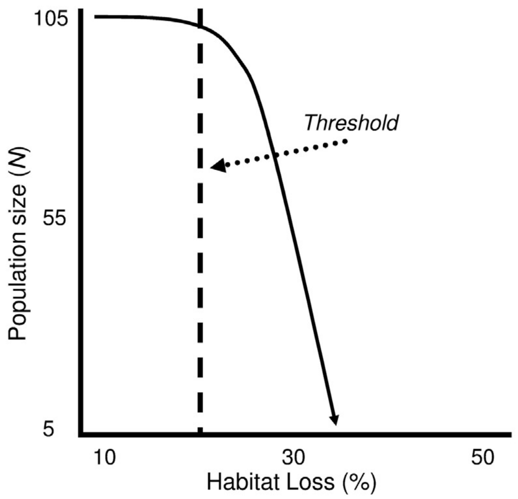

Figure 5.10: An example of a threshold. In this case the point at which population size rapidly decreases with small increases in habitat loss. Figure sourced from [93].

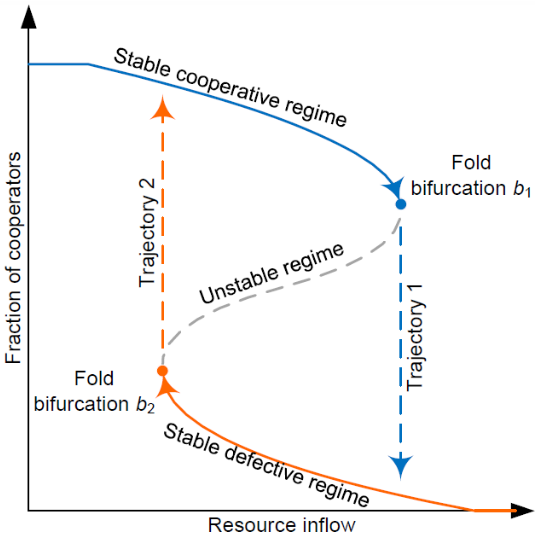

Figure 5.11: An example of a two regime system. The blue line represents the path when most people in a commons game cooperate, red when they mostly follow uncooperative behaviour. Figure sourced from [117].

Tipping points are often observed in models which examine economic stability, such as [64]. They can also appear in common resource scenarios, especially in “the lake problem” [184]. Irreversibilities and hysteresis cycles are demonstrated in the cooperation/defection resource scenario described in [117].

Beliefs, Desires, Intentions

One way of conceptualising the human mental model is through the beliefs, desires, and intentions paradigm. In this conception, actors in a model form beliefs about their environment by acquiring knowledge (either partial or complete). Agents in the model are also inbuilt with desires, which represent the outcomes the agent wishes to achieve. Intentions signify the actions the agent tries to pursue after examining their beliefs and desires [12, 159].

Bounded Rationality

Bounded rationality is the assumption that humans make choices based upon limited information. In some models an actor in the model might consider every option and then choose the best option. This is an erroneous assumption from the perspective of bounded rationality [142]. Agents with bounded rationality are given partial access to the full array of information. The fraction of this information can be varied to simulated different degrees of bounded rationality [57]. Humans are also unable compute all possible combinations of options available to them [142, 61]. This is due to time and resources limits, and so humans are often forced to use heuristics to make a decision, as opposed to perfectly calculated expected utility [131].

Repetition, Deliberation, Imitation, and Social Comparison

The repetition, deliberation, imitation, and social comparison paradigm is used to describe possible modes of deliberation in which a agent can make a decision. Repetition is the case in which an action is simply repeated from one time step to another. An agent which is content and doesn't feel compelled to change will engage in repetition. Deliberation is the choice mechanism by which the agent examines possible options and calculates the utility of each option. Imitation refers to a choice making process where the agent imitates the action choices of another actor (usually an actor which is doing well). Social comparison refers to when people compare themselves to others to evaluate their feelings of fulfilment [88].

Keynese/Frediman Economic

Keynesian economics posits that the economy is driven by demand [47]. By this theory, economic activity and job creation can be produced by increasing the demand for products. In Friedmanesque economics, consumers base their consumption habits on expected lifetime income and will prepare for unexpected shocks to income flow. Individuals with a permanent income will also base spending on the expectation that their income will remain stable. This is the basis of buffer stock saving models (precautionary saving motive) [47].

The model in [121] is both a demand driven model, and one in which the agents operate under buffer stock savings theory.

Complexity Theory

Complexity theory examines the emergence of unforeseen behaviour of a system, due to the interaction of multiple parts or agents, which when examined individually would not predict the resulting global behaviour. This field has emerged from the field of systems theory, and is now an umbrella term linking principles in a mixture of fields such as mathematics, social science, and economics [8, 6]. Complex systems often contain a large number of self-determining actors. Complexity can emerge in a variety of forms, e.g. chaotic behaviour, tipping points, self-organisation (i.e. many uncoordinated agents forming coordinated behaviour), and hysteresis (path-dependency) [6]. Many real world issues need to be examined with complex system theory in mind (e.g. climate change and economic depressions) [57]. Correctly modelling a complex system is important because simple assumption (e.g. a single homogeneous actor, or general equilibrium) may produce misleading output [57, 16, 115].

Aggregation

Aggregation is an important part of any model. To produce a model that had no aggregation would require modelling every individual element of the real world (every person, dog, tree, car, building). Thus to make any model usable there must be some aggregation of similar elements to reduce the complexity of the model. The extent of this aggregation depends largely on the model designer's discretion. Having more components in a model means more time has to be dedicated to setting up the model, more data has to be found to calibrate variables, and more effort has to be put in to analyse the output of the model.

The World3 model is a complex model with many variables and parameters, but is highly aggregated compared to the real world [92]. For example, there is a single resource supply in the model. This resource is extracted to produce all industrial capital and consumables. In the real world, the economy if fuelled by many different resources, and the depletion of one can upset parts of the economy.

Pollution can be aggregated into a single variable, as is the case in the World3 model [112], or it can be disaggregated as is the case for the models in [65], [66], and [120]. Differentiating between pollutants can be important, as different pollutants have varying roles in the economic system and produce different impacts on the environment. In the latter models pollution has been disaggregated into components such as carbon dioxide and monoxide, sulphur dioxide, nitrous oxides, soot (particulates), volatile organic components, heavy metals, solid wastes, radioactive wastes, chemical and biological oxygen demanding wastes (water), suspended solids (water), and hydrocarbons.

Another example of disaggregation can be found in the splitting of land and industries into different types. In the World3 model three types of land are depicted: undisturbed, cultivated, and urban. Production falls into three sectors: industry, agriculture, and services [112]. In the GUMBO model, energy and material flows are tracked through five different spheres (bio-, litho-, hydro-, atmo-, and enthropo-), and eleven different biomes ("open ocean, coastal ocean, forests, grasslands, wetlands, lakes/rivers, deserts, tundra, ice/rock, crop-lands, and urban) [21]. In the Australian Stock and Flows Framework, industry is not only broken down into primary and secondary industries, but primary industry is split into "agriculture, forestry, fisheries and mining" [168].

Thneed

Occasionally a "thneed" will be used to represent a unit of consumption. Thneed stands for "Total Household Normalized Energy Expenditure Division" [61]. It represents an amalgam of cars, white-goods, and other energy consuming goods. The number of thneeds a household in the model has, the greater its energy consumption. This allows the energy expenditure of individual households to be greatly simplified.

Price Setting

Setting prices is a complicated matter. In the real world factors such as input costs, advertisement costs, consumer lock-in, and staff salaries [12] will go into a business owner's pricing decision. The prices in a model might be estimated using a constant value (e.g. for an exogenous good imported into a region), a mark-up value based on input costs (e.g. [61] and [121]), exogenous functions, demand curves (e.g. [16]), or evolutionary learning [179]. In some models (e.g. housing markets) bidding algorithms may be used to find the market price. In these formulations buyers will put in an initial bid. If the buyer is unsuccessful in obtaining an item they increase their bid. For sellers, if they are unsuccessful in making a sale they decrease their asking price (e.g. [60]).

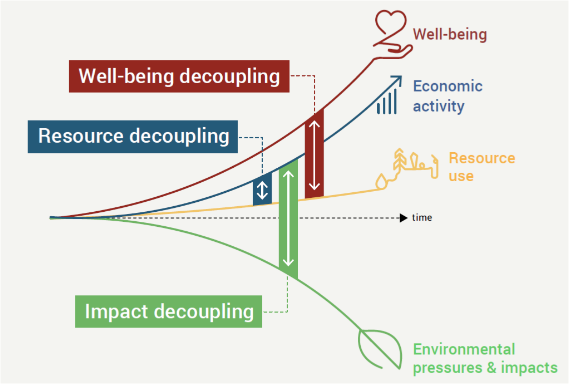

Figure 5.12: The concept of decoupling. Figure sourced from [173].

Labour Pay

Determining the price of an item is always difficult when operating within an artificial market (i.e. in a model). For a product, the price can be estimated based on a mark-up of input costs. However, this cannot be applied to labour prices as there are no monetary inputs in the production of human labour. Wages might change over time depending on labour productivity [61], or unemployment rates [121]. A skill level factor may also contribute to wage pricing [121].

Decoupling

The concept of decoupling is based on the goal of maintaining economic growth and the production of services while simultaneously reducing environmental impacts [179]. This concept is illustrated in Figure 5.12 [173].

Market Share

Often when modelling businesses competing with each other, the demand for a product is aggregated and the sales are shared between the businesses according to market share. Changes in market share are determined by the competitiveness of the businesses. Competitiveness can be a function of factors such as price, advertisement, and quality. Market share is a component in models [12], and [61].

Harvesting

Harvesting is an important aspect of any socio-ecological model because the extraction of resources is the first step in many economic processes. When modelling the harvesting of fish, the rate at which fish are removed can depend upon factors such as fish population density, the amount of time put into catching fish, and the type of technology used to catch the fish [16].

Human Needs/Driving Forces

Human needs and driving forces are critical in understanding the decisions people make. The decisions people make end up affecting the economy and environment. Maslow's hierarchy of needs is the best known theory of human action preferences [132]. In this formulation, people will fulfil their most basic needs first before trying to fulfil higher order needs. The first order needs include basic objectives such as shelter and food, while higher order needs include, education, health, and art. This is often used as a starting point for understanding human actions, however the theory does not fully match reality.

Another theory is that people have two reasoning systems. The first system is informed through emotions and 'gut' feelings. It is fast acting and automatically operates with little conscious awareness. The second system operates as a more rational agent and tries to work logically. It is slower operating and is under conscious control [114]. It is difficult to model the first system as it operates under emotional influence, emotions which are numerous (20+) and difficult to quantify [43].

People are found to often use heuristics over optimisation calculations [77]. While many models assume that humans act as "rational, self-interested economic agents termed homo economicus" [77], that assumption can prove incorrect. It has been shown that people are risk averse, not solely self-regarding, strong reciprocates and will punish defectors (selfish agents) at their own expense [77]. Another formulation of the homo economicus idea is that of homo politicus. This agent will favour morally responsible actions rather than acting in a purely material wealth-maximising fashion [53].

In [141] the decision making process is conceptualised as a five step process. The first step is recognising the existence of a need. After this, information is gathered on how the need could be fulfilled, followed by an evaluation of options and purchasing of a product. The final step is an examination of how well the product performed. Another aspect of this concept is the actual and desired state of a need, and the tolerance threshold (the difference between the actual and desired state) which must be reached before the decision making process begins.

When a person is in the evaluation stage of the decision making process, many factors may be considered. Some factors may include price, quality, status, branding, popular opinion, social pressures, or moral concerns [18]. When a course of action is being decided, factors such as the potential for social ostracisation may come into play (if the action is seen as socially unacceptable) [117]. The litany of factors that can affect a decision makes the process very difficult to model.

Some researchers have found that material wealth above a basic level fails to bring about greater levels of happiness [156]. Over long run time periods, there is also no increase of happiness with material wealth. This finding is known as the Easterlin paradox [139]. This may indicate that the happiness is dependent on the "gap between aspiration and realized income" [139], i.e. as average wealth rises, expectations rise negating the growth of wealth. These observations should also give us pause to consider other factors that affect happiness and mood, such as physical health [30], personal relationships, and other non-material assets.

Preferences

Preferences represent the difference each human has in terms of taste or goals. This can be expressed as a set of weights which an actor applies to a utility function when deciding on a course of action [142]. Sometimes it is possible that a consumer will become locked into a preference due to circumstance [146], e.g. a different product that is cheaper to use in the long run, maybe have a high initial cost which the person can never afford to pay.

Human Behaviour and the Environment

It is understood that the impacts on the environment can be traced back to the individual actions and decisions made by every person on the planet [5, 83, 88]. From this logic, it is considered crucial that models simulating socio-ecological systems contain agents equipped with the ability to learn and pursue their own self-interests [88, 54]. This must be done so that the flow-on effects of individuals decisions can be examined. This type of modelling is often used in commons scenarios, as the interest of a single actor may not align with the interests of the global community.

Debt

In some models, when businesses or individuals are unable to purchase items with their own savings, banks will lend out money. In the model presented in [61], money is lent to businesses at a maximum debt to sales ratio.

Self-defeating systems

A self defeating system is a scenario in which the winning strategy (the strategy that derives the most utility) becomes the losing strategy, and vis versa. This creates a cyclical process where the best strategy continually changing. Self defeating systems are examined in [25] and [14].

Learning behaviour

In an agent based model the agents can be equipped with the ability to learn and adapt to their situation. This requires the agent to keep a record of past information. Neural networks and machine learning algorithms can be trained to find quasi-optimal behaviour [142]. In the model in [60] agents examine past housing prices and speculate on future housing prices.

5.1.4 Issues Explored and Components of Models

The following section will highlight some issues that have been examined using models.

Population

Population is an important aspect of some socio-ecological systems. Any model that wishes to explore gross consumption levels of resources needs to take into account population, as greater numbers of people will consume greater levels of resources. The two main reasons a population will change is through births and deaths, however immigration or emigration may also need to be taken into account in some cases. For animals, births and deaths are largely a function of food supply and predator population levels. For humans, the greatest determining factors for birth rates and death rates (in the absence of famine) appear to be education (decreases crude birth rate), contraception availability (decreases crude birth rate), and health services (increases crude birth rate and decreases crude death rate), all of which correlate with material wealth [112]. Example of models in which population is an important factor can be found in [111, 188, 168];

Resource Depletion and Extraction Efficiency

An aspect of resource extraction is the efficiency at which resources can be extracted, either in terms of energy costs, material costs, or capital costs[7]. Energy efficiency is tied into the concept of energy return on investment. Energy return on investment is the ratio of energy that is produced for every unit of energy that is spent in the manufacturing process and the creation of the extraction capital.

Historically, standards of living have been very closely knit with increased energy consumption [102]. The effects of declining energy return on investment are still "uncertain, but probably adverse" [102]. Lambert et al. [102] propose a theory of social condition based upon energy return on investment. They imagine that energy expenditure is spent according to a hierarchy (see Figure 5.13), with the initial energy going into the production of energy (reinvestment) and basic essentials such as growing food. At the pinnacle of the hierarchy sits medicine and art.

Most conventional fuels (coal and oil) have a very large energy return on investment (i.e. a small energy investment for a large energy return). Non-conventional (renewable energy) sources have "substantially lower" returns [70]. If energy return on investment is high (say 50), then a small change (-1) in energy return on investment has a small effect on the energy being delivered to the wider society. However, if returns are low (say 5) then the same reduction will have a significant impact. This is described as the energy cliff and is represented in Figure 5.15 [102]. The energy cliff indicates that energy return on investment is an issue which should be monitored, as approaching "the cliff" could have significant impacts on energy production.

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[7] A distinction has been made between capital and material, as one method of extraction may need low levels of capital to set up but will have high levels of consumable material, and for another method the reverse will be true.

An energy model produced by Dale et al. [39] examined future energy production as technologies transition from conventional to renewable energies. The model output is shown in Figure 5.16 [39]. The model predicts a continued increase in energy production until the year 2060, by which point an equally rapid decline occurs.

Two models that include resource depletion and extraction and their effects on human welfare can be found in [111, 125].

Production Efficiency

The efficiency of production is an important aspect of a socio-ecological model. This is because the efficiency of material conversion alters the rate at which natural resources need to be extracted to produce a desired level of consumption. Efficiency can vary with time in a model, based upon endogenous variables or exogenous functions [127]. The change in efficiency can either be stochastic or non-stochastic [61]. The Jevons paradox (or rebound effect) is the paradox that despite increases in production efficiency, resource consumption remains the same, but consumption increases [125, 164]. Examples of models with production efficiency [103, 178].

Commons Scenarios

A commons scenario is one in which a large group of people exploit a common resource. A scenario where the resource ends up being over exploited is termed the tragedy of the commons and was first coined by Hardin [72].

The over-exploitation of a resource can be the result of many actors perusing actions which are most profitable to themselves, but not for the overall profitability of the group [88]. A well studied commons resource is fish stocks [16]. Other examples include lake problems, in which a lake is used as a reservoir for waste [184], and the management of red deer presented in [161].

Cooperation and defection can play a large part in a system's functioning [16]. The lake problem presented in [87] shows cyclical behaviour of the system as due to shifting levels of cooperation and defection [87].

The fishing commons can potentially be thought of as a self-defeating system [14]. This is due to the fact one species of fish will be the most profitable to harvest until it has been over exploited, at which point another species will become more profitable. At this point another species becomes the most profitable species to harvest and hence the catch strategy changes. This process however can only continue if the total harvest level is below the replenishing rate of the total fish stocks.

Uncertainty Effects

Uncertainty is a common element in many real world problems. Things such as the weather, market prices, government policy, and technology changes are often very unpredictable. "It is also unclear whether climate policies may influence access to food, water and energy, and - if so - how" [115].

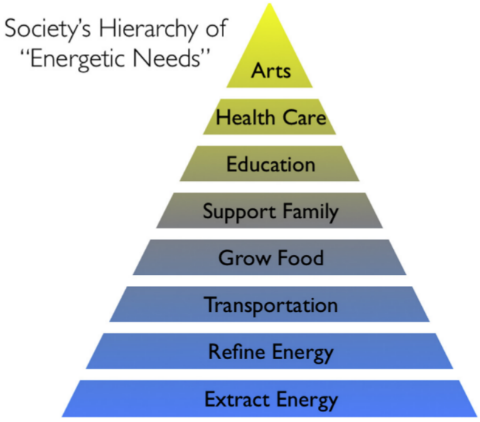

Figure 5.13: Hierarchy of energy needs. Figure sourced from [102].

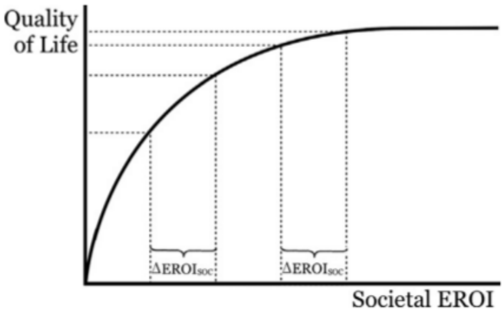

Figure 5.14: Quality of life verse energy return on investment. Figure sourced from [102].

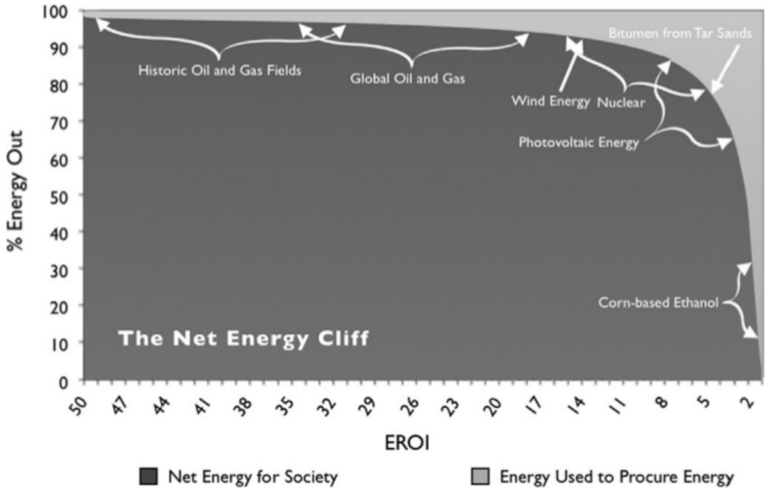

Figure 5.15: The net energy cliff. As energy return on investment (EROI) decreases to lower levels, the energy delivered by the system (per energy unit invested) dramatically decreases. Figure sourced from [102].

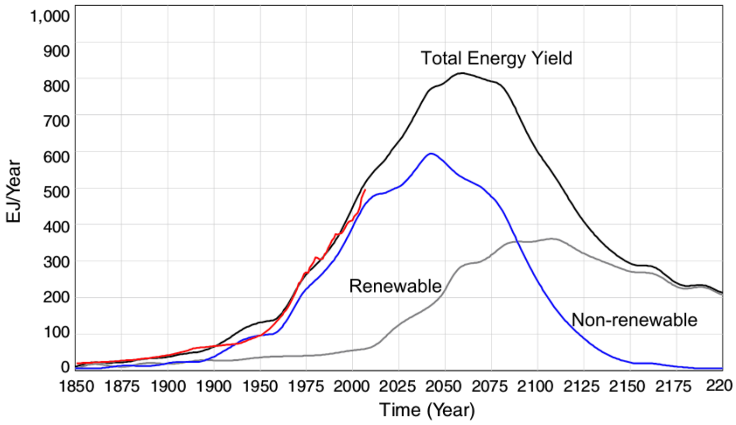

Figure 5.16: Potential energy production outlook. Figure sourced from [39].

In [184] a lake is modelled, and phosphorous enters into the lake via two methods. The first method is via pollution generation due to economic activity, the second is through stochastic natural releases of phosphorous. The citizens living next to the lake wish to maximise their economic activity without crossing a critical tipping point. This natural release adds a uncertain element into the model.

Uncertainty can also exist in the minds of the actors of the model [89]. The actors might be uncertain as to how well their actions will pay-off later on. This uncertainty can effect their choices and hence have an impact of the simulation results [89].

Government Policy

Some models are designed to investigate how government policy effects a socio-ecological systems. The model presented in [61] shows the effects of taxation expenditure policy on development of green technology. [136] studies the effects of tariff policies on investment in renewable energy production. The models in [27] and [150] examine the affects of agricultural and land use policies. In one model [122] agents could vote in and out different types of government which would effect the policies.

Financial Stability, Lending Practices, and Minsky Moments

One important aspect often studies by economists is financial stability and lending practices. The financial sector can often cause large shocks to economic systems which have major consequences on the lives of ordinary people.

The model described in [47] investigates the effect inequality of income distribution has on economic stability. The study concluded that inequality adversely affected economic and financial stability due to “higher credit demands, higher unemployment rates, economic volatility, and financial fragility”.

Another model [60], investigated the effect leniency in loan lending has on housing prices. Some parameters in the model include maximum debt-to-income, and minimum down payment. A two different runs of the model are shown in 5.17. The left plot shows in which banks have low leniency when lending money, while the other plot shows high leniency. The price of housing in shown by the black line, while the red shows foreclosures. It is clear that the system is far more stable when low leniency is practised by the banks.

Minsky moments are the turning point of the value of an asset. They are named after the economist Hyman Minsky [47]. Economic booms and busts are continually occurring, the basic premiss being that as an economy comes out of a slump, banks begin lending money to businesses. As the economy begins moving again confidence begins to grow and banks gain more confidence in lending, creating more economic activity. When investors see high economic activity, they begin heavily investing, and can over-estimate the value of an asset. As soon as the economy stabilises and growth slows, investors can lose confidence in some assets and their value rapidly decreases causing a bust. This type of behaviour is common of housing bubbles [60]

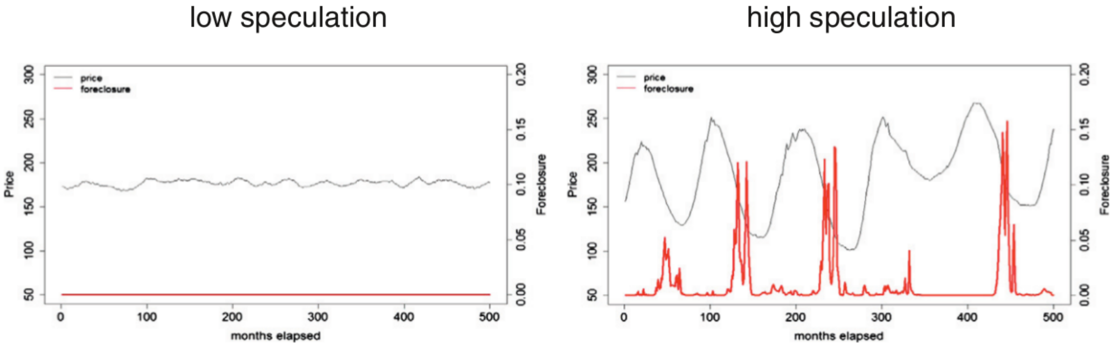

Figure 5.17: Housing price volatility under two lending practices. Gray line - prices. Red line - foreclosures Figure sourced from [60].

Inequalities

Economic inequality is a concern for many people, and so some models have been developed to try and examine the driving forces behind it.

In the model presented in [121], there are two classes of individuals. A working class individual earns their money solely by selling labour for a wage, while capitalist agents earn money through government bonds and dividends. In [47] the model depicts inequality through varying the productivity of the individual in the model. A shape factor allows the inequality to be varied for different scenarios.

Climate Change and Natural Shocks to Economies

Climate change is predicted to have many adverse effects. Some of these are: damages to infrastructure and crops due to extreme weather, relocation of low lying coastal areas due to sea-level rise, increased risk of infectious disease (more favourable conditions for infections), increased natural disasters (e.g. floods), and more strenuous outdoor working conditions [38].

Some models attempt to capture these effects by incorporating a climate model into an economic model. The climate aspect of the model is then able to deliver shocks to the economic sector (labour force productivity, and firm capital/inventory) in proportion to the level of climate change experienced. This is demonstrated in [103].

Another example [62] looked the strategies most effective for combating flooding (based off different economies) at the damages on capital due to flooding, influence by climate change. Figure 5.18 [62] depicts the model.

Product Adoption

Product adoption is important in understanding market and economic shifts. This is relevant to sustainability through the adoption of more sustainable products. In models such as these the network (communication) between people, the consumer choice mechanisms, or seller-buyer interaction many be examined to determine how markets behave [179, 90].

Behaviours examined may include quiet and busy markets. Quiet markets have only a few major competitors and minor movements within them. Busy markets have many

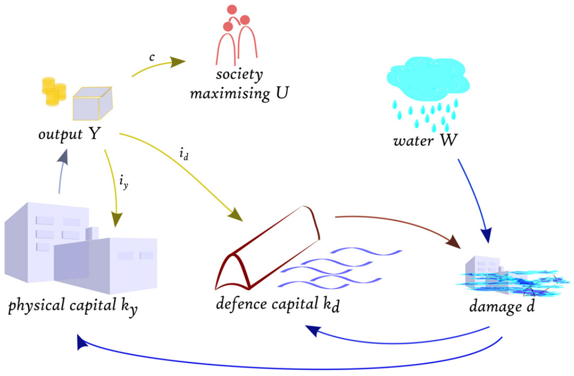

Figure 5.18: Damages Figure sourced from [62].

people entering and exiting and rapid movements in market share [91].

Agent based models are good for studying these issues as they allow for the analysis of decision making processes and how these translate into macro behaviour [91, 90]. Often the study will try and examine how people transition from being a potential consumer, to a ordinary customer, to a loyal customer, or vice versa [191].

Land Use

Some models examine how the land use of a region changes over time. For example, forest can be converted to residential or industrial land, depending on the levels of development [122]. Many land use models will simulate different agent types (government body, developers, residents) [159] to examine how their actions impact the landscape over time. Other models may however use statistical Markov chains to predict future outlooks, e.g. [67].

The complexity of land use models is continually increasing as modellers attempt to account for “climatic, political, economic and demographic” effects on land use change [126]. Models such as “CRAFTY” simulate various land holder types (crop farmers, livestock farmers, loggers) competing for land, with land productivity affecting the profitability of each agent type [126]. Models like this are able to test the effects government policy will have on land use into the future, e.g. [150].

Carbon Cycles

The carbon cycle refers to the ways in which carbon (usually carbon dioxide) moves through the biosphere, lithosphere, atmosphere, and hydrosphere. Global heat retention due to carbon dioxide depends on the quantity of CO_2 in the atmosphere. While carbon emission from fossil fuels initially enter the atmosphere, they can end up dissolved in the upper or lower ocean [38], changing the concentration in the atmosphere. Carbon dioxide can also enter via thawing of permafrost regions. An example of a model in which CO_2 migration is explicitly accounted can be found in [38].

Strategy Adoption in Multi-Agent Competitions

To understand a system, the behaviour of the people must be well represented. For this to be the case, the behaviour of the people must be open to change when better strategies are presented. A strategy that involves perfect cooperation may produce an optimal pay-off [16] (i.e. the largest total harvest); this strategy can be undermined by agent defecting from the cooperation strategy [25]. The agents in such models are usually referred to as cooperators and defectors, with a third 'agent' being an enforcer whose purpose is to punish defectors [130].

In commons harvesting games the pay-off for over harvesting (performing selfish behaviour) depends on the gains to be made (extra resources) and the losses (fines or other punishments). The losses to defectors usually come with a cost to the cooperators [117], as is the case in [24]. However, this can be a costless expense to the cooperators if the losses are in the form of social ostracising, as is the case in [117]. Some strategy models may look at the spontaneous emergence of cooperative strategies due to aligning pay-off functions (i.e. the cooperation of two agents benefits both agents). This type of behaviour was examined in [161] with the modelling of red deer harvesting and pest control.

Carrying Capacity / Biodiversity

Carrying capacity is the concept that a region has a limit to the number of people (or animals) it can sustain. The level depends upon the health or care given to the region, e.g. if a region is over stocked with grazing animals it might become bare and will have its carrying capacity diminished, or if irrigation is implemented in the area carrying capacity can be increased.

Biodiversity is scope of different species in an area. A loss of species from an area (either through extinction or habitat lose) is viewed as a reduction in biodiversity in that area. Biodiversity is thought of as a key element in retaining ecological health. Often the loss of one species will cause a loss of another, if it provided the main food source for that species.

While understanding the concepts of carrying capacity and biodiversity is easy, accounting for ecological health and resilience is very trick as it is not straight forward. The concepts do not have sufficiently specific definitions, and placing a value on it is difficult [48]. Some attempts, to evaluate the health of a system, are based on examining the total economic benefit it delivers to society (e.g. tourism, food production, or natural resources); however these estimations often have flaws [2].

Waste Generation / Pollution

The generation of waste and pollution is often an issue of concern for many people. Waste generation is often linked to the rate of economic activity, and may have an impact on the environmental spheres [178]. Understanding pollution generation is important as elements such as heavy metals or pesticides can have adverse impacts on many species.

Green Bonds

Bonds are a form of loan. They are often representative as a form of indebtedness of a government to an individual civilian. Bonds are a means for a governments to quickly raise funds, and pay-back costs later. In a model presented in [121] "green bonds" were used to offset government expenditure of green subsidies, to test if they could help with deal with the issue of advancing green technology.

Regional Effects / Spacial Effects

Many socio-ecological systems operate in a spacial context. This complicates matters when modelling as it raises questions, e.g. how are the agents in the model connected (does distance apart affect likelihood of iteration), can agents move from one region to another and if so what compels them to move, how does location affect the behaviour of the agents, etc. [57]. Implementing spacial effects is difficult in system dynamics models, but easier in agent based models.

Multi-market

There are many models which will simulate a market (a product with buyers and seller), however the model will usually only contain a single market[8], like in the case of [179].

Market Signalling

Markets can signal to other processes in an economy, e.g. a drop in price may indicate that a product is being over produced. Signals can play a role in the overall behaviour of the system. The time in which signals takes to reach the producer can determine the stability of the produced good [169].

Servicing

One potential solution to reducing material is the increased use of services, as opposed to private ownership of goods that produce services. Car sharing services (e.g. car rental) are a simple example of this. Rather then each person owning a car (which may go largely unused for people living in a city) a single car is hired. This reduces the amount of materials that are used to create a car for each and every person. This issue is modelled in [179].

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[8] General equilibrium models will have multiple markets, however these are static representations with no buyer-seller interactions.

5.2 Desired Ability of the Model

The aim of this chapter is to create a model which explicitly models individual human actors, making consumption decisions within an economy which is linked to the broader ecosystem. This would mean that economic activity in the model is dependent upon the state of wildlife stocks and mineral reserves.

The economy in this model should have many sectors, and many different items for the human actors to select from in order to meet their desires. The prices of goods should be completely endogenous. Spacial elements should be included so that trading can occur between regions, along with variations in natural resource availability and accessibility. The model should also include a government agent, capable of imposing regulations (e.g. taxes, and wealth distribution) on the people and firms. The people and firms should be able to freely adjust their actions to maximise their self-interests. The model should replicate the real world as closely as possible. In the real world there are many different goods consumers can purchase, which are sourced from a wide variety of natural resources, often imported from multiple regions.

5.3 Purpose

The purpose of the model is to be able to examine the implications of government policy for the lives of individual citizens in various regions, and their combined consumption of natural resources. It is also intended to examine the effects of shifting technological advancement and changing resource extraction efficiencies. A "toy" style model has been chosen in this iteration, as the intent is to examine the system behaviour rather than making predictions about real world situations. Agent based modelling was used so that the effects of the system can be seen on the individual level so that differences in individuals can be examined.

The hope is that questions such as: does taxing an item produce the desired effect[9], [10]? what effect does minimum wage have on the lives of people? or, how would increased transportation costs change the economy?

The new model aims to explore issues unexamined in the previous chapters. The main issue that this model can address, which is not examined in previous chapters, is inequalities between individuals in a population. Aggregation assumptions may not be appropriate when studying the complex phenomena of the world economy, especially when tipping point feedback loops are present (behaviour may appear stable when aggregated, but unstable when individuality is considered, e.g. the 2008 global financial crisis).

The results presented in this Thesis are to demonstrate the abilities of the model framework. Questions regarding what should be placed in the model framework have been left for future work. The most important aspect to be demonstrated is the ability of the model framework to capture complex knock-on effects for a wide range of factors.

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[9] For example, does taxing fish help reduce fish consumption and thus benefit wild fish stocks.

[10] There is a dark joke (that illustrates this point), that taxing cigarettes does not reduce the numbers of cigarettes smoked, but stops many poorer people from eating.

5.4 Justification

There is a great need for models to better capture the effect human decision making has on the consumption of materials and services, as this in turn affects natural resources and the environment. By better capturing the bounded rationality of human decision making, more detailed and complex behaviours can emerge from the models. Questions like ‘can certain events trigger mass changes in consumption habits?’, ‘how much do reinforced habits hinder social change?’ or, ‘can role model behaviour influence others into more sustainable ways?’ are better answered with human decision making models as they can offer more insights than standard socio-ecological models. Multi-agent modelling is a commonly used approach to incorporate the human decision making process into a model [88, 5, 91, 16].

The following quotes highlight the importance of developing models that emulate human decision making in a dynamic environment in order to simulate their corresponding impact on the wider world.

- “What is apparent is the lack of a new approach to handling what Dovers describes as “the fundamental, structural inconsistencies between natural and human systems. The causes of sustainability problems lie deep in patterns of consumption and production, settlement and governance” that any modelling, be it boosted by the unprecedented computer power, so far has left untouched” [160].

- “For further application in ecological economic models, we suggest that the consumat approach [be used] ... to simulate social processes and habitual behaviour, next to deliberate behaviour, in order to unravel the behavioural dynamics underlying consumption of common properties and to design suitable management strategies for our common good” [88].

- “In the process of truly coupling the human systems and natural systems within any [socio-ecological system], the importance of understanding how human decisions are made and then put into practice can never be exaggerated” [6].

- “Information structure is an important feedback mechanism with high-leverage. If you make information go to places it did not go before, it may well cause people to behave differently.” “Missing feedback is one of the most common causes of system malfunction. Meadows points out, we humans have a systematic tendency to avoid accountability for our own decisions and that is why so many feedback loops are missing. Thus, adding or restoring information can be a powerful intervention, usually much easier and cheaper than rebuilding physical infrastructure” [80].

- “The article encourages modellers to incorporate out-of-equilibrium aspects of an economic system as appropriate and highlights the potential for SD practitioners to contribute to economics, especially ecological economics” [169].

The cost of items can play a large role in purchasing habits. To understand these habits, it is important to arrange a model so that the simulated people can make consumption choices based upon cost.

One important issue to studying is the equality between all people, both interregionally and intergenerationally. Unfortunately this is a factor that has continued to have mixed results since the Brundtland Report in 1987 [81]. An advantage of multi-agent modelling is its ability to break down macro variables (e.g. average income) into raw statistical data (e.g. individual income). This allows for exploration of questions around the topic of inequality [88].

By defining clear guidelines for sustainable behaviours[11], decisions around consumption may become easier to make. Indicator based measures can suffer from the user having to interpret the information and then make a decision as to what to do. For a product that claims to produce 1.2kg of carbon, a consumer may wonder, "is 1.2 kg of carbon a lot, or is it negligible?" [149] which may further complicate the consumer's decisions. Also, to get a full appreciation for one's total impact, a record would have to be kept to tally all carbon emissions, a highly intensive practice unlikely to be adopted by any individual. For these reasons it is important to develop information that can be easily interpreted and applied by consumers.

As this model replicates autonomous individuals in a dynamic world, it can be easily relatable to real humans. Agents in the model can be given different personality traits to further extend their connection to real humans and to offer more insights into the models behaviour. Questions around culture can then be investigated further [88] to produce further knowledge around sustainability.

The literature review indicates that the coupling of broad socio-ecological models with multi-agent models is still in its infancy. The exercise of adding human behaviour/decisions to a socio-ecological model could extend the current knowledge of these systems. By further developing this field, it is hoped that better governance policies can be developed.

5.5 Model Description

Overview

The 'world' in this model is separated into multiple regions. Each region has its own marketplace and government. Each region has its own supplies of natural resources for which the government can sell licences in the marketplace. There are many different types of items, which can be traded at a marketplace and used by firms to produce other items. A market can purchase items from other markets (i.e. from other regions) to fulfil the demand for items. People and firms (of which there are multiple) can purchase items from the marketplace to fulfil their needs. Banks are indirectly included due to the ability of firms and people to go into negative cash levels (with zero interest loans). The basics of the model are depicted in Figure 5.19.

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[11] For example, buying high quality goods over low quality goods due to longer life cycles, or purchasing services over owning physical goods.

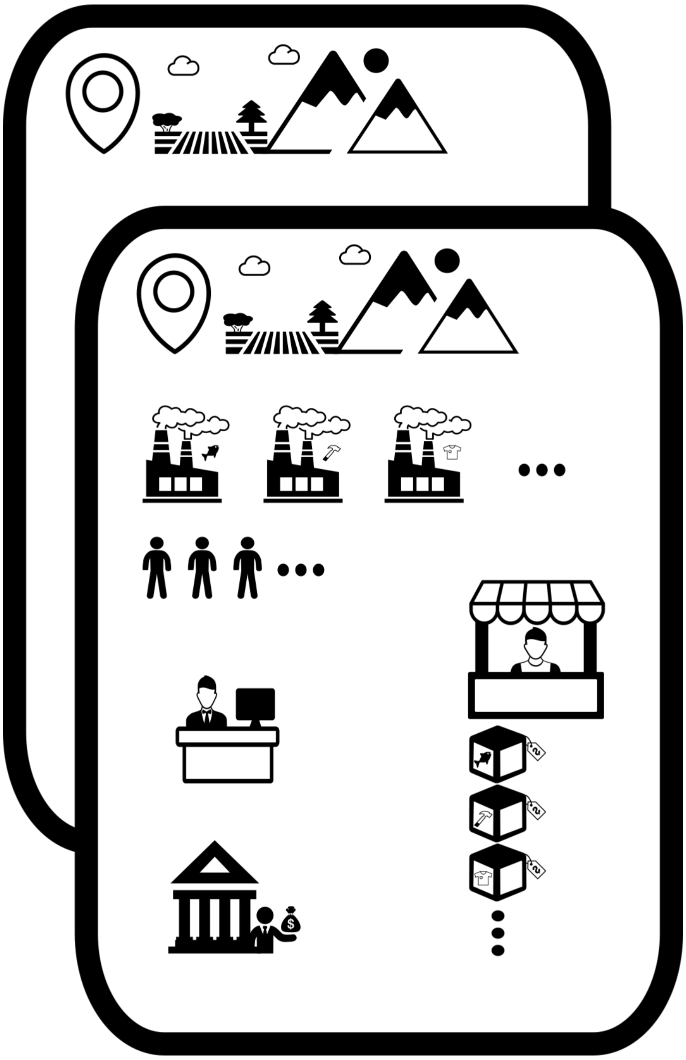

Figure 5.19: A diagram of the novel model.

Assumptions

The assumptions made in the model are as follows:

- There is a 'single' firm of each firm type in each region (i.e. one fishing, building, manufacturing, and transporting firm in each region). The single firm represents a collection of smaller firms of the same type.
- Resource extraction efficiencies are held constant. In this first iteration of the model, resources have not been modelled explicitly and are essentially assumed to be unlimited. In the future this will be changed so that the dynamics of the resource can also be examined.
- Constant efficiency of production. The rates at which resources need to be consumed to create other products is held constant.
- Production follows a Leontief production function (see section "Production functions").

- People in the model spend their money according to a hierarchy of needs (see section “Human Needs/Driving Forces”). The order of the needs is food, shelter and finally products.
- Spending practices of people have inertia (based on a mass, spring, damper dynamic).
- Prices are endogenous and based upon the supply and demand of each item (if there is an imbalance prices will shift accordingly). This price movement is in keeping with general supply and demand theory [59].
- Governments collect money from the sale of natural resources. This money is redistributed evenly amongst the citizen of the region. There is one government in each region.
- Production increases and declines based upon profitability of the firm. This is akin to investors putting capital into enterprises that give high returns, and vis versa [155].
- Firms have access to unlimited zero interest loans. When loans are given out it can be imagined that there is an investor like a bank supplying the money, however the bank is not explicitly modelled.
- Wealth is distributed according to a distribution function, rather than endogenously through ownership of a firm or shares in a firm.
- The collection of firms (which have been aggregated into one firm) partake in a tragedy of the commons scenario, where by each firm tries to acquire larger profits by increasing its production rate. This can oversupply the market and cause the price to fall as competitors undercut each-other on price, as it is assumed that all the demand would go to the cheapest supplier (even if this is by a minute margin).
- All actors have perfect information about prices, i.e. they do not have bounded knowledge.

Detailed Description

1. Setup of the model takes place. Time t is set to zero and time step Δt is defined.
2. Each person develops a ‘crude’ shopping list for the coming round of trading. The first item to be placed on the list is food. The person examines their budget and notes on their list how much food they can afford to purchase (based on the expected cost of food in their marketplace), up to a maximum of 2 food units per time unit. They then examine the remainder of their budget and note shelter, up to a maximum of 1 shelter unit per time unit. After this the remainder of the budget is spent on products[12].

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[12] These products could be thought of as items such as shirts, cars, and electronics.

The crude shopping list informs the real shopping list. Changes in the crude shopping list are smoothed using a critically damped mass-spring-damper style function (mass and spring equal to 1).

3. Each firm develops a shopping list for the coming round of trading. First, firms must decide if they will expand production or reduce production. They do this by calculating their adjusted profit ratio $p = (R/E - 1)/T$, where R is the revenue made if output items are sold at current market prices, E is the expenses of purchasing input items at expected market cost, and T is target profits per dollar spent on expenses.

   The target production rate α is modified depending on the altered profit ratio. The target production rate is used to better model the flow of investment, i.e. faster growth if profit ratios are higher. The rate of change of α depends on p, investment rate I, and divestment rate D. This is noted in equation 5.1.

$$\alpha_t = \begin{cases} 0 & p \leq -1 \\ \alpha_{t-\Delta t}(1 + pD\Delta t) & -1 < p < 0 \\ \alpha_{t-\Delta t}(1 + pI\Delta t) & 0 \leq p < 1 \\ \alpha_{t-\Delta t}(1 + I\Delta t) & 1 \leq p \end{cases} \tag{5.1}$$

   The firm then checks if there is a blockage in its production line. Blockages can be caused by either disruptions to the purchasing of input, or over stocking of output product. The actual production rate β is compared to the target production rate α. If $\beta \leq B\alpha$, where B is the blockage fraction, then $\alpha_t = \alpha_{t-\Delta t}(1 - K\Delta t)$, where K is the blockage decay rate function.

   Once these calculations have taken place, the firms calculate their desired stock levels. These levels are the amount of stock (input and output) the firm desires to have on hand. To do this, the firm calculates the rate at which it will consume inputs and produce outputs, given production ratios and its target production rate. These rates are then multiplied by the target reserves fraction to arrive at the target stock levels.

   Finally, the firm's shopping list is created by calculating the difference between the desired stock levels and actual stock levels. Each trading round firms send all of their output stock to the market.

4. Market[13] exchanges are now simulated. A diagram of a market is displayed in Figure 5.20. To begin, the marketplaces calculate the cost of purchasing each item from each marketplace (including their own). The cost is based on a per-unit basis and includes the price and cost of transporting the item. Then the market of each region takes the orders (requests for items) and their equivalent cost (based on expected cost) from each person and firm. In doing so, people or firms can go into negative

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[13] The term market will refer to a particular item in one region, while marketplace will refer to a collection of markets (many items) in one region.

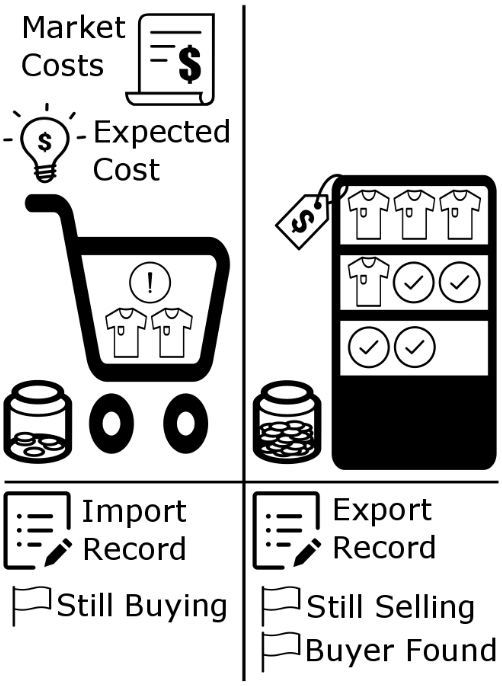

Figure 5.20: The market of an item i within a region r. The market is part way through a round of trading, i.e. there are still transactions that need to happen before the trading process is complete. There are two sides to the market. The left side represents demand, the right, supply. The cash is transferred between jars as transactions occur. The 3 icons in the shopping cart (two shirt icons and one exclamation mark) indicate a demand for 3 units of the good. Similarly the 8 icons on the shelf represent the initial supply of 8 units of the good. As two units of the good are in the shopping cart and four units have been sold from the shelf (indicated by the tick icon), we can infer that at least two units of the good have been sold to an outside market place (i.e. sold and transported to a different region).

cash holdings. The orders and cost are aggregated into a demand tally and demand cash jar. The items that the people and firms wish to sell are then added to the shelves of the market. Governments then put licences (an unlimited supply) for natural resources on the shelves.

Once the marketplaces are primed, trading begins. This process is broken down into many partial rounds of trading, where only a fraction of the demand for an item is attempted to be fulfilled. Each partial round is further broken down into a particular trade of a single item to a single marketplace. The ordering of the particular trades is randomised so that no singular item is prioritised when trading. The partial rounds are conducted to make trading a more continuous function between all items, rather than occurring in large discrete chunks.

To conduct a particular trade, a marketplace is selected at random along with an item[14]. The market cost record is examined. The market that is selling the item for the lowest cost will be approached for trading (presuming no trading restrictions are broken, the market is still stocked with the item, and there is transportation still available). The 'buyer found' flag will be activated for the approached market. The

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[14] This is repeated until all items from all marketplaces have attempted a particular trade. The process is repeated every partial trade round.

two markets will exchange the item and cash. Transportation for the item will be consumed in equal parts from both the supply marketplace and demand marketplace. Cash will be paid for the transportation out of the demand cash jar. The transaction is recorded in the import and export record. The transaction can be hindered due to a lack of supplies, cash, or transportation. These factors are appropriately accounted for in the model.

5. After trading has taken place, the actual cost of each item is calculated based on how much was purchased and the amount of cash spent. Next, items and cash are returned to people and firms. Items that were supplied to the market are returned along with the cash accumulated in the supplies cash jar. The amount originally supplied by the person or firm correlates with the size of the return.

   Items that were purchased for the people and firms are returned. Purchased goods are returned in order of altered profit ratio, with people being assigned a ratio of zero[15]. People also receive returned goods in order of wealth. It is assumed that people and firms are able to 'out bid' each other by minute fractions.

   When returning purchases, the actual cost is taken into account. If the actual cost is higher than expected, the difference is paid into the demand cash jar. People and firms that miss out on purchased goods receive money from the demand cash jar.

6. The price of each item is adjusted based on an adjustment rate C and the target to actual sales ratio σ. The target sales rate is calculated off the amount of product placed on the market shelf, divided by the target reserves fraction, and multiplied by the time step. The change in price P is given by $P_t = P_{t-\Delta t}(1 + \sigma C)$. If supplies to a market were equal to zero, then the price will move up or down depending on if the buyer found flag was activated.

7. Each firm now examines its stock levels and computes its actual production rate β. The actual production rate is based on the amount of stock available and the production ratios, with a maximum of α. Production follows a Leontief function, i.e. no substitution between inputs can be made. Once β is established the input stocks are consumed and the output stocks are created.

8. If a firm has positive cash stocks, the cash is distributed among the people of the region according to a weighting function $w = ax^c$, where c is the profit distribution factor (a higher value indicates a more uneven distribution), x is the person's identity number (ranging from 1 to the number of people in the region), and a is a scaling factor (so that the distribution weights sum to 1).

   Cash collected from the government through the sale of resource licences is redistributed to the people of the region evenly.

9. Time t is incremented by Δt and the process is repeated until the end time has been reached.

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[15] This stops odd behaviour happening, i.e. where a highly profitable firm receive a small fraction of goods because its request for goods represent a small fraction of the total demand.

Software

The model was coded in the MATLAB environment. No pre-defined software package was used to construct the model.

For copies of the software please contact the author at aheath@uow.edu.au.

5.6 Results

To begin experimentation, a ‘toy’ world was set up within the model framework previously described. The world is fictional so units have been left unspecified, as the purpose is to test model behaviour. All tests, apart from the final test, simulate a single region for simplicity purposes.

There are 9 items that are traded in the ‘toy’ world. These items are: fish, wood, iron, oil (which represent the material inputs required for the following products), food, shelter, products, freighting (transportation), and labour. The properties of these items are listed in Table 5.1. The properties that an item can have are: if it can be traded, if it can be traded regionally, the transport type required to move it around, and the number of units needed to move a single unit of the good a single unit of distance.

In this preliminary version of the model, the people follow very simple consumption rules. They examine their current cash supply and attempt to purchase food, shelter, and products in this order, until they have exhausted their cash supply. They will purchase a maximum of 2 units of food and 1 unit of shelter each time unit. Products can be purchased at an infinite rate. All people produce labour which can be sold in the market. They each produce 20 units of labour each time unit.

There are four different types of firms. The four firm types are: fishing, building, manufacturing, and transporting. Each type represents an aggregation of multiple firms producing the same type of good. Because of this, the price of the good will fall when it is over supplied to the market as the individual firms (which are hidden by the aggregate) will undercut each other to gain greater market share.

Each firm type has a production efficiency. The efficiency is related to the ratios at which inputs are converted into outputs. The ratios for each firm type are given in Table 5.2.

Eight different tests (A through to H) were conducted to examine the behaviour of the ’toy’ world. To begin, a simple base line simulation was conducted. The base line test was labelled test A. The parameter settings of test A, and the subsequent tests (B through to H) are presented in tables 5.1, 5.2 and 5.3.

Item	Tradeable	Regional Trade Allowed	Transport Type	Transport Cost (transport type units/item unit · distance)
Fish	true	true	Freighting	1
Wood	true	true	Freighting	1
Iron	true	true	Freighting	1
Oil	true	true	false	false
Food	true	true	Freighting	1
Shelter	true	false	false	false
Product	true	true	Freighting	1
Freighting	true	false	false	false
Labour	true	false	false	false

Table 5.1: Item properties used throughout all model tests. 'False' indicates that no transport is needed to move the item. This has been applied to Oil so that the inputs used in the creation of freighting transportation can always be traded, even if no transportation existed.

Input/Output	Value (A)	Altered (X)
Fishing		
Fish (input)	1.0	-
Labour (input)	1.0	0.2 (G)
Food (output)	1.0	-
Building		
Wood (input)	1.0	-
Labour (input)	1.0	0.2 (G)
Shelter (output)	1.0	-
Manufacturing		
Iron (input)	1.0	-
Labour (input)	3.0	-
Product (output)	1.0	-
Transporting		
Oil (input)	1.0	-
Labour (input)	1.0	-
Freighting (output)	1.0	-

Table 5.2: Production ratios used throughout the model tests. (X) denotes test.

Parameter	Value (A)	Altered (X)
People		
Starting Cash	50	200 (B)
Desired Consumption Rate of Food	2.00	-
Desired Consumption Rate of Shelter	1.00	-
Desired Consumption Rate of Products	∞	-
Labour Output Rate	20	-
Profit Distribution Factor	1.00	3.00 (C)
Firms		
Target Profits	0.50	1.00 (E)
Firm Investment Rate	0.10	-
Firm Divest Rate	0.05	-
Maximum Production Blockage	0.80	-
Blockage Decay Rate	0.20	-
Target Reserves of Firm Outputs	0.30	-
Target Reserves of Firm Inputs	0.30	-
Markets		
Price Change Rate	0.05	-
Distance To Market	0.10	0.30 (F)
Government		
Fish Licence Price	1.00	4.0, 0.1 (B,D)
Wood Licence Price	1.00	4.0, 0.1 (B,D)
Iron Licence Price	1.00	4.0, 0.1 (B,D)
Oil Licence Price	1.00	4.0, 0.1 (B,D)

Table 5.3: Parameter settings used throughout the model tests. (X) denotes test.

5.6.1 Base Run (A)

First, we will examine the behaviour of the markets in the model. The food market is shown in Figure 5.21 and the product market in 5.22. The top plot shows the price of the good (black), and the expected price (grey). The expected cost includes the cost of transporting the item, and so price and expected cost can vary semi-independently, as can be seen at time $t \approx 620$ in the food market.

We can see from these graphs that the price of an item is in constant flux. The price of a good falls while the good is oversupplied to the market (see the price of food from the time $t \approx 480$ to $t \approx 530$), in keeping with basic supply and demand theory [59]. Over this time the level of desired sales (grey) is higher than the actual sales (black). At time $t \approx 540$ the adjusted profits of the fishing firms dips below zero (see Figure 5.25. Investors[16] begin divesting and the production rate of food declines. This results in a dip in the desired sales rate of food (see Figure 5.21). This causes actual food sales to out-pace target food sales, causing the price of food to rise.

The consumption rates of the 1st and 5th person (i.e. the poorest and wealthiest respectively) are shown in figures 5.23 and 5.24. Both person 1 and 5 (and the other people in the model) manage to meet their consumption needs of both food and shelter the majority of the time. Person 1 however has a far lower consumption rate of products compared to that of person 5. Both person 1 and 5 have fluctuating consumption rates of products.

The production rates of each firm type are presented in figures 5.25, 5.26, 5.27, and 5.28. The production rates of the fishing and building firms are relatively constant. The production rates of the manufacturing and transporting firms are comparatively unstable

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[16] I have mentioned investors to make it easier to imagine what is happening, however the investment and divestment (i.e. growth or decline) is controlled by the target production rate variable α.

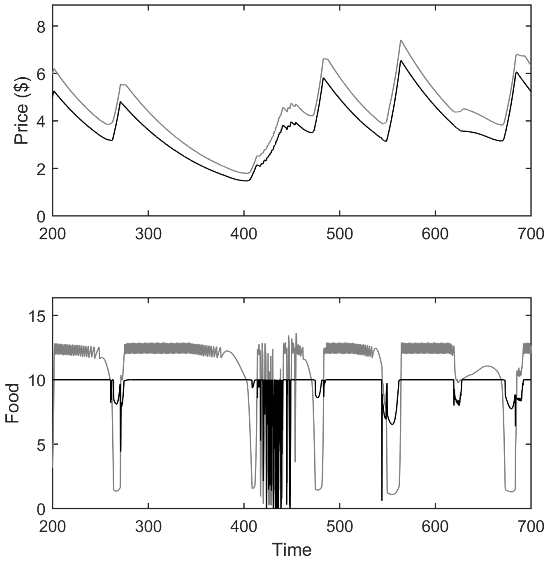

Figure 5.21: The food market in test A. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

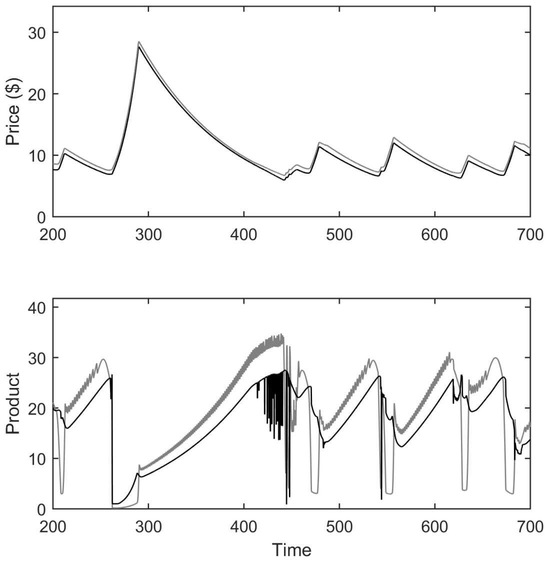

Figure 5.22: The product market in test A. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

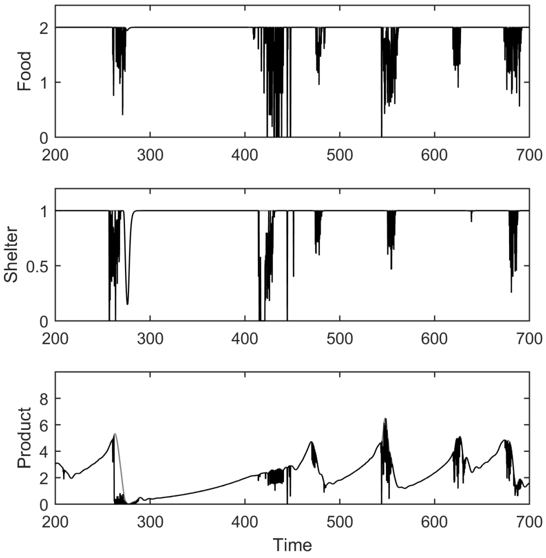

Figure 5.23: The consumption rates of person 1 in test A. Black - consumption rate; grey - desired consumption based on budget constraints.

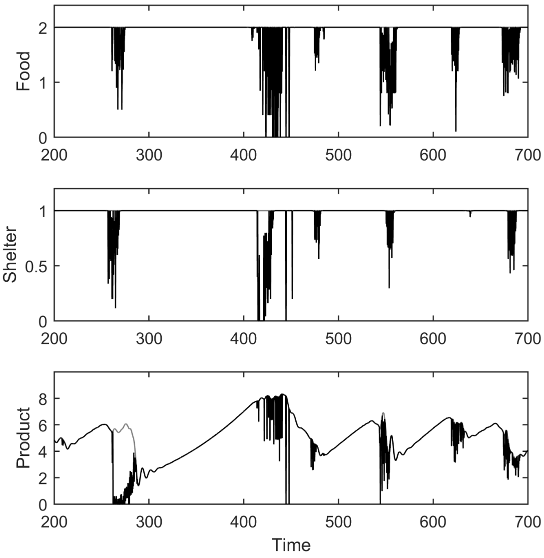

Figure 5.24: The consumption rates of person 5 in test A. Black - consumption rate; grey - desired consumption based on budget constraints.

(i.e. their rate of production is constantly changing). The drops in production rate correspond to points in time when the adjusted profit ratio (-1 indicates that expenses equal income, i.e. zero profit) falls below zero. Rises in production correspond to adjusted profit ratios being greater than zero.

The stability of the food and shelter market is due to their higher priority in the budgeting of citizens. When money becomes scarce for the citizens, the first item to be removed from their shopping lists is products. This makes the manufacturing sector unstable, which in turn destabilises the transporting sector (as reduced production of products reduces the need for freight).

In this example we can see five points at which the economy slows down (at $t \approx 280, 470, 550, 620$, and 680). At these points the profits of each firm type have dropped below the target profit margin and so production rates begin to drop. The drop in production causes the demand for labour (noted by actual sales) to diminish as seen in Figure 5.29. Just before the aforementioned times, the consumption rates of the individuals become very similar, as the profits of the firms near zero (when this happens there are no profits to be unequally distributed). This can be seen by comparing the product consumption rate of person 1 and 5.

At time $t \approx 550$ the total cash held by all people increases as shown in the bottom plot of Figure 5.30. This is because the manufacturing firms go into debt at this time as shown in the top plot of Figure 5.30. Manufacturing firms go into debt at this time because they undergo a rapid growth stage where they produce more product than they sell, which can create a situation in which expenses exceed income.

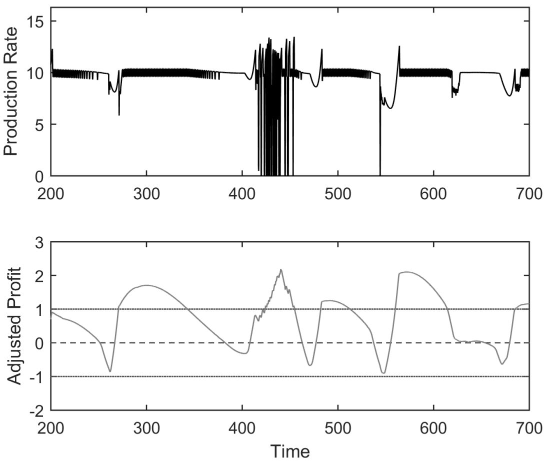

Figure 5.25: The fishing firms in test A. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

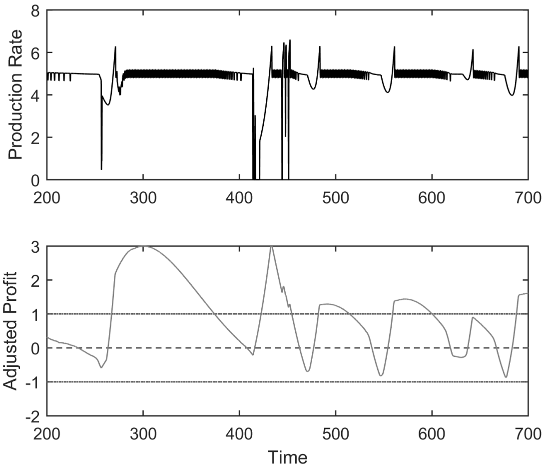

Figure 5.26: The building firms in test A. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

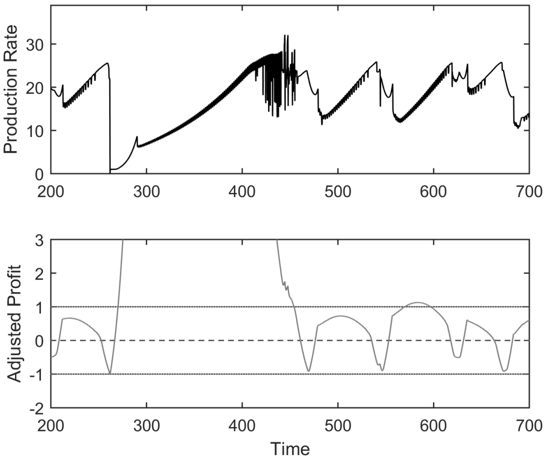

Figure 5.27: The manufacturing firms in test A. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

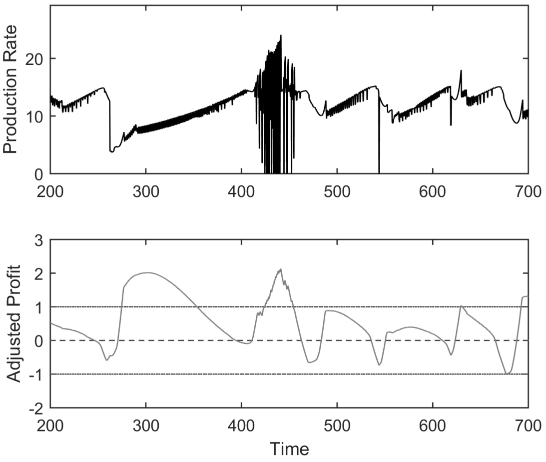

Figure 5.28: The transporting firms in test A. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

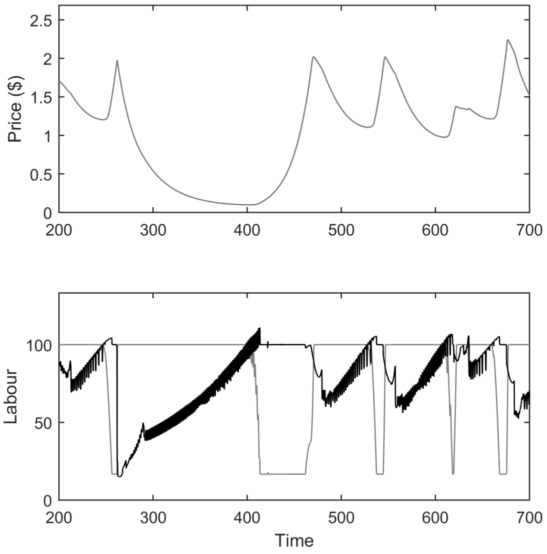

Figure 5.29: The labour market in test A. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels. Due to the price and expected cost being equal, only the grey line (expected cost) appears in the plot.

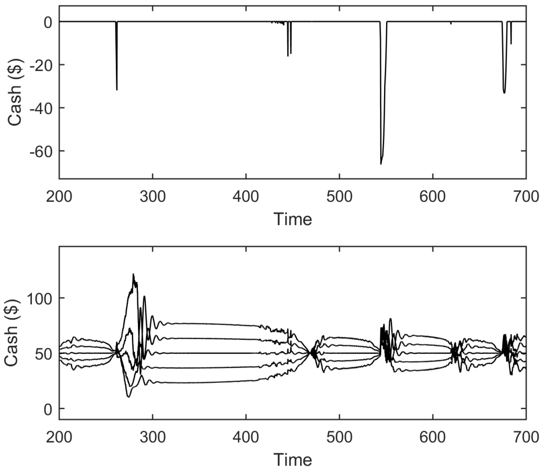

Figure 5.30: (Top) Cash reserves of the manufacturing firms (Bottom) Cash held by each person during test A.

5.6.2 Currency In Circulation (B)

The first test conducted (Test B) involved increasing the initial cash held by each person from \$50 to \$200. This had the effect of increasing the total amount of currency in circulation. The price of resource extraction licences was also increased from \$1 to \$4 to mimic the price increase seen in all other items[17]. This had no noticeable effect on the amount of goods consumed by each person. The only effect to occur was the inflation of prices by a factor of 4, i.e. 50/200. This is demonstrated by comparing the price of food and products from test A with test B in figures 5.21/5.22 and 5.31/5.32 respectively.

This demonstrates the effects of printing new money and putting it into the economy. It is now well known that the printing of money by governments causes the costs of items to rise, i.e. inflation. While this information is not new, it is important to demonstrate that the model handles this fact without any special mechanism for modelling inflation.

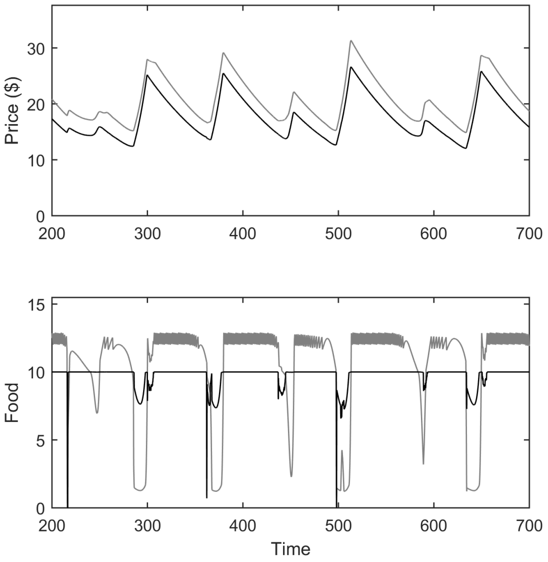

Figure 5.31: The food market in test B. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

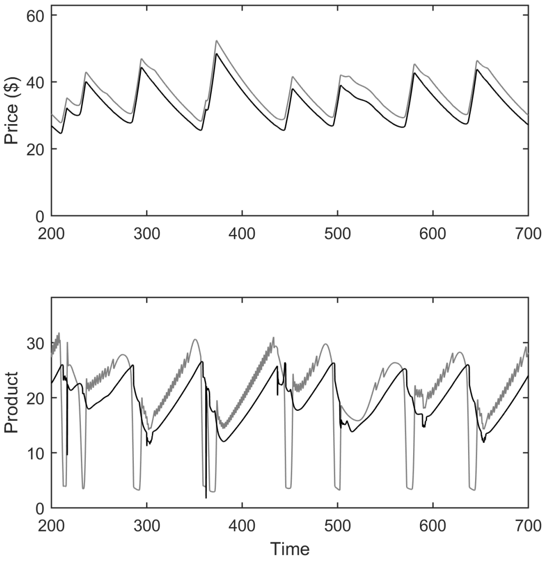

Figure 5.32: The product market in test B. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

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[17] If this is not implemented the condition of the poorer people worsens due to the same mechanism demonstrated in test D.

5.6.3 Wealth Distribution (C)

In test C the wealth distribution factor was increased from 1 to 3 to examine the effects of making wealth more unevenly distributed. The results of this test were surprising. The economy appears to be more stable when comparing the production and profit ratios of the fishing and manufacturing firms (see figures 5.33 and 5.34 respectively). A more stable economy means that production rates remain higher on average. This brought benefits to the average consumption of all people in the model, with the biggest benefit going to the poorest person (a 26% increase in product consumption). The wealthiest person received only minimal increases in their consumption of goods (a few percent). Figure 5.35 shows the consumption pattern on person 1. This is a far more robust consumption pattern compared to that of person 1 in test A (see Figure 5.23).

The effects of the stabilised economy can be seen in the levels of cash held by each person. When the profits of businesses decline, the difference in income and cash held by each person reduce because the profits being generated, and then unevenly distributed, approach zero. This appears as a converging of cash held by each person. In Figure 5.36 it is evident that the average cycle time of economic growth and shrinkage is smaller in test C (bottom plot) than in test A (top plot), indicating more frequent but less volatile rises and drops in economic output. Further testing needs to occur in order to validate this hypothesis.

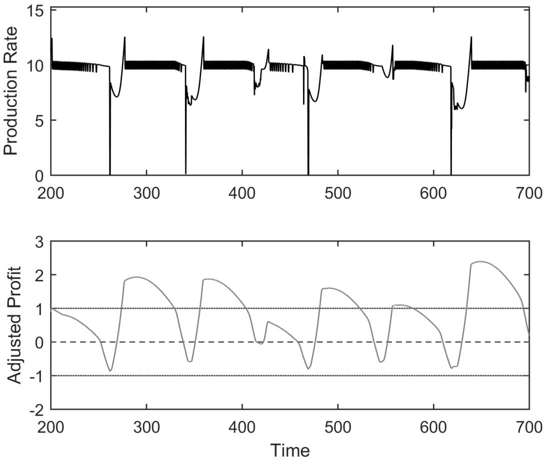

Figure 5.33: The fishing firms in test C. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

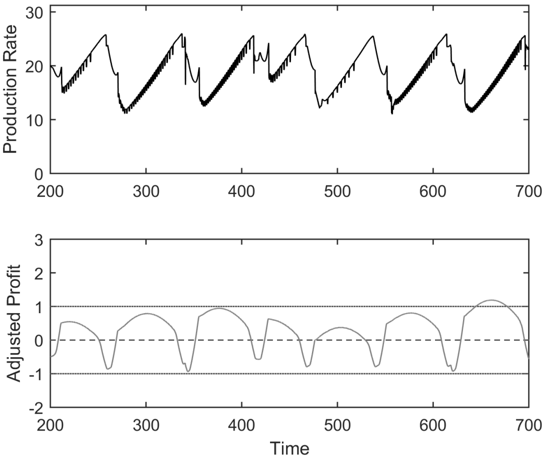

Figure 5.34: The manufacturing firms in test C. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

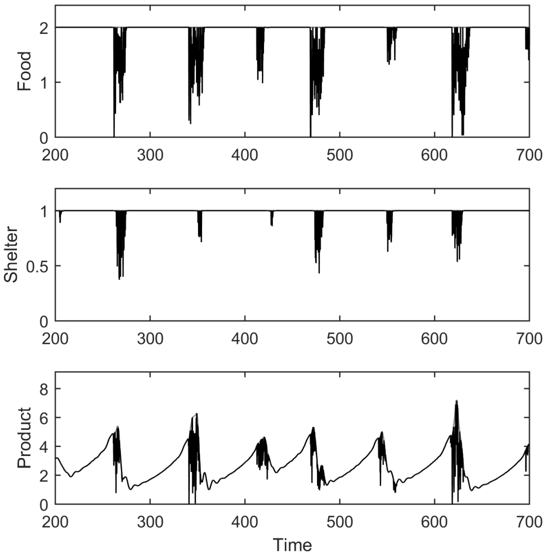

Figure 5.35: The consumption rates of person 1 in test C. Black - consumption rate; grey - desired consumption based on budget constraints.

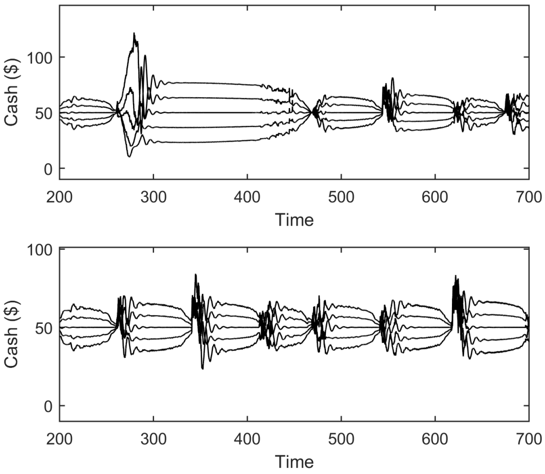

Figure 5.36: (Top) Cash held by each person during test A. (Bottom) Cash held by each person during test C.

5.6.4 Licence Cost (D)

In test D the cost of acquiring resource extraction licences was decreased from \$1.0 to \$0.1. The money collected from the sale of these licences is distributed equally between all people of a region. This alteration reduced the ability of the poorer people to consume goods, as a larger proportion of their income is dependent on these sales. Figure 5.37 shows the consumption pattern of person 1. It is clear that there is a marked reduction in their consumption of products and shelter, when compared to person 1 of test A (see Figure 5.23).

This demonstrates the knock-on effects of taxation (on natural resources) on to tangible consumption rates of individuals. The total consumption of items decreased in this scenario, indicating that the tax rate can have a macro effect on the total output of the economy.

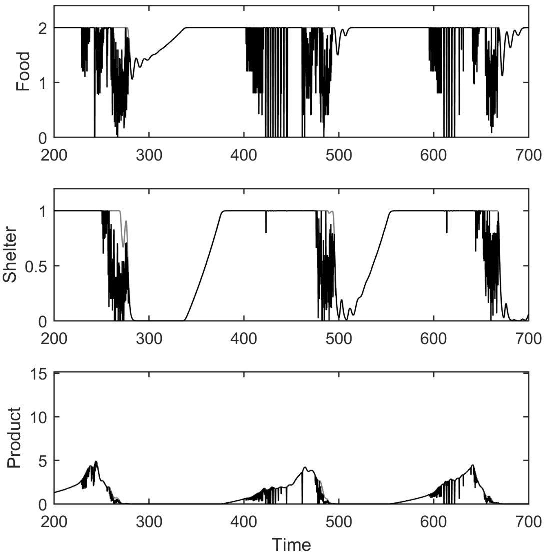

Figure 5.37: The consumption rates of person 1 in test D. Black - consumption rate; grey - desired consumption based on budget constraints.

5.6.5 Target Profit Ratio (E)

In test E the target profit ratio was increased from 0.5 to 1.0 (dollars of profit per dollar invested). This alteration increased the amount of goods consumed by all people, the only exception being the consumption of products by person 1 which fell by 2%. This result is counter-intuitive to the expected outcome, i.e. that increased profit ratios would increase disparities of consumption between person 1 and 5. A possible reason for the increased consumption of goods is the appearance of a more stable economy. Figure 5.38 show the production rate and adjusted profit ratio of the manufacturing firms. The production fluctuates in a similar way as seen in test A (see Figure 5.27), however the magnitude of the fluctuations are smaller in size, thus making the economy more stable. Further testing needs to occur in order to validate this hypothesis.

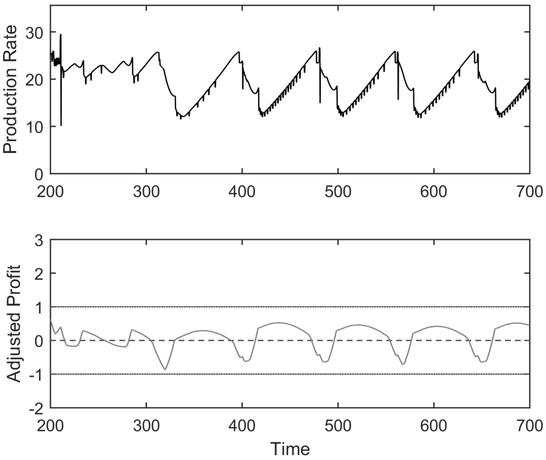

Figure 5.38: The manufacturer firms in test E. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

5.6.6 Transport Distance (F)

To examine the effect transport distance has on the consumption levels of the people, the distance from firms, people, and government to the market was increased from 0.1 to 0.3 units. We can see that when the economy is operating at full capacity, transportation is consumed at a rate of approximately 35 units per time unit (see Figure 5.40). This is compared to approximately 15 units of transport consumption of test A. The increase in transportation production negatively impacts the consumption of goods for all persons in the model.

In test A both person 1 and person 5 consume enough food and shelter to meet their maximum threshold (see figures 5.23 and 5.24). In this test, both person 1 and person 5 struggle to meet their desired consumption rates of food and shelter during the entirety of the simulation (see figures 5.43 and 5.44). The reason for this appears to be twofold. One, more resources (including labour) need to be put towards producing transport to move goods throughout the economy, and two, the economy appears to have become more unstable and slower to recover. This second point is harder to verify and would be worth further examination in future work.

It is well understood that remote communities (either due to distance or terrain) can be economically effected by their remoteness [155]. Understanding the exact degree to which these factors effect the communities is hard to model, as spacial effects need to be explicitly accounted for. Again, this test demonstrates the models ability to facilitate knock-on effects throughout the whole economy. In this case it has been demonstrated that increase transport distances may severely hinder production of goods and services.

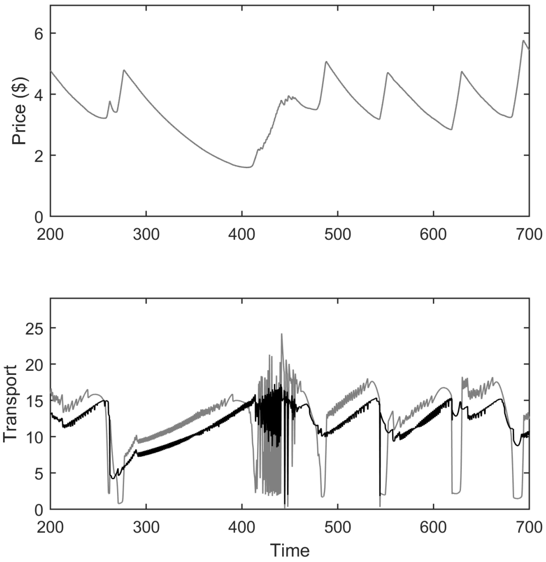

Figure 5.39: The freighting market in test A. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

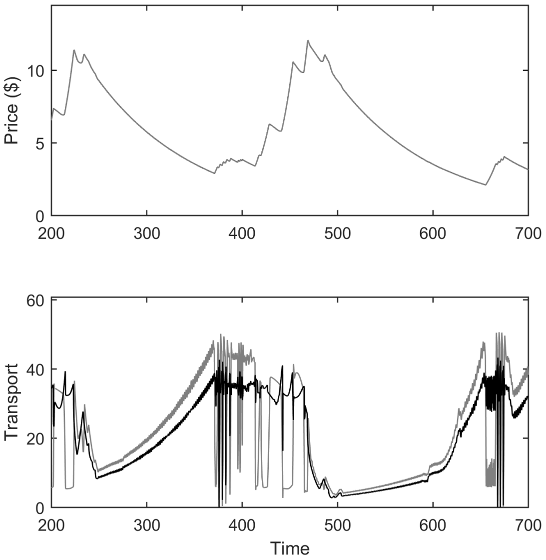

Figure 5.40: The freighting market in test F. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

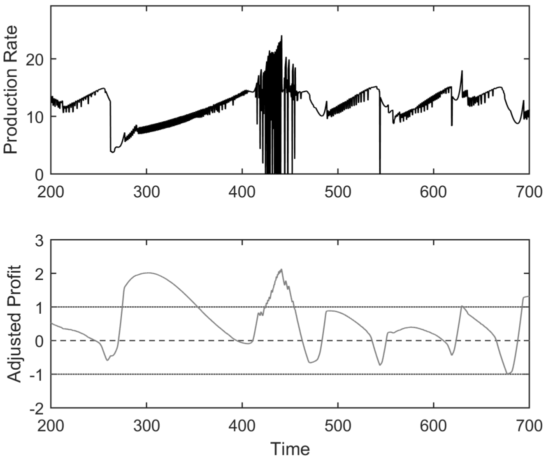

Figure 5.41: The transporting firms in test A. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

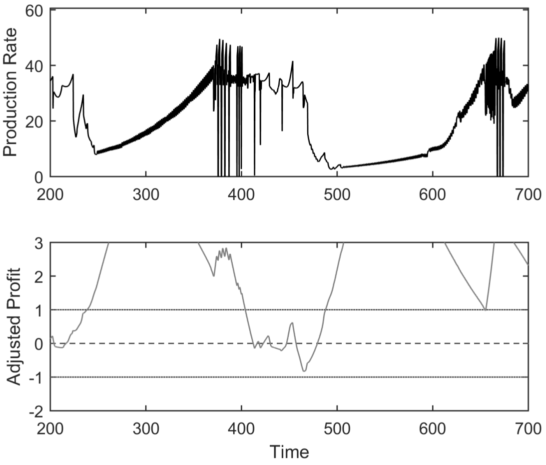

Figure 5.42: The transporting firms in test F. (Top) production rate. (Bottom) the firms' adjusted profit ratio.

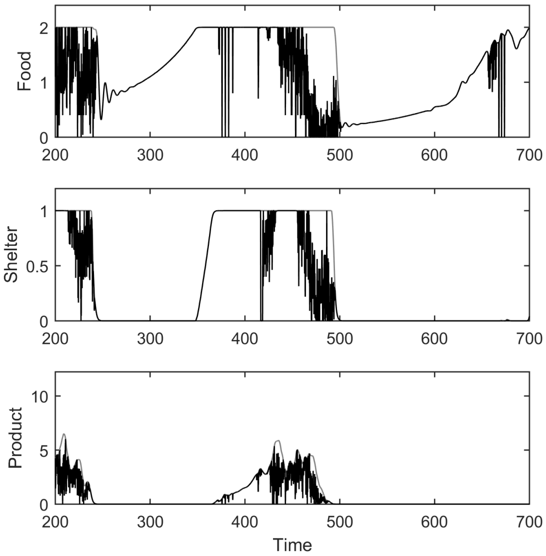

Figure 5.43: The consumption rates of person 1 in test F. Black - consumption rate; grey - desired consumption based on budget constraints.

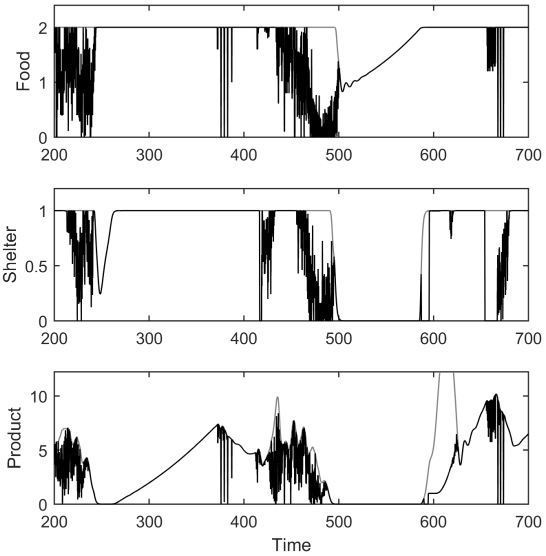

Figure 5.44: The consumption rates of person 1 in test F. Black - consumption rate; grey - desired consumption based on budget constraints.

5.6.7 Technology Shift (G)

The effect of changing the production efficiency was examined in test G. In this test the amount of labour required to produce a unit of food and shelter was changed from 1.0 to 0.2 units at time equal to 600. This naturally affected the price of food and shelter. Figure 5.45 shows a decrease in the average price of food after this point. The decrease in labour directed to the production of food and shelter should correspond to an increase in the production of products, however this is difficult to infer from examining the selling rate of products in Figure 5.46. The average consumption rate of goods increases slightly for all people, however the increase is small (a few percent). The increase was predicted to be bigger. The reason for the smaller than expected increase might be due to an increase in the instability of the economy, as more labour is dedicated to the most unstable sector (i.e. the production sector, which is the first to be discarded when people begin cutting back on their spending). This hypothesis requires further testing to validate.

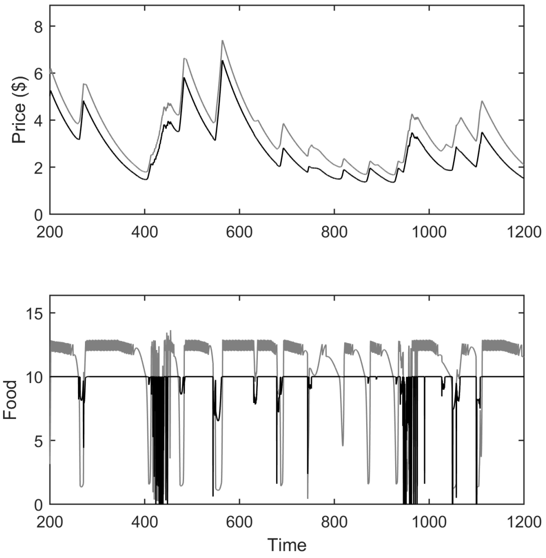

Figure 5.45: The food market in test G. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

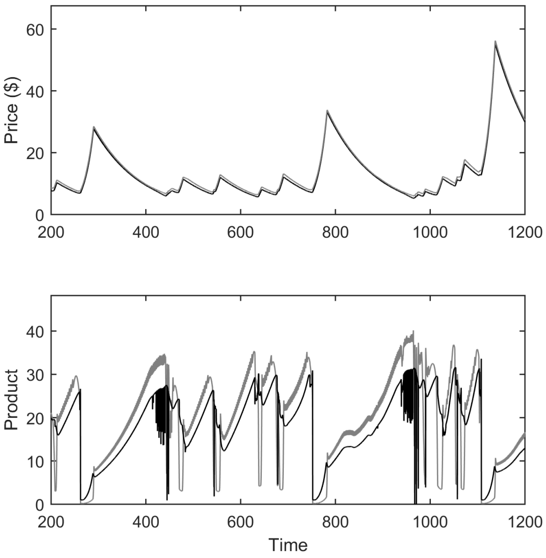

Figure 5.46: The product market in test G. (Top) black - price; grey - expected cost. (Bottom) black - actual sales; grey - target sales based on stock levels.

5.6.8 Resource Poor Region (H)

In test H another region was added into the simulation. This test was designed to examine the effect resource abundance plays on the ability of people from different regions to consume goods. The first region has access to supplies of fish, wood, iron, and oil. The second region (positioned 1 unit of distance away from region one) has access to only oil and iron (thus it must import fish and wood). This test produced a large disparity between the people of each region. The people of region one easily met their consumption needs of food and shelter, while the people of region two struggled to consume enough shelter. The combined consumption of products of the people of region two was less than that of the poorest person of region one. The test showed that the effects of having to import products can be significant and a great hindrance to the residents of the region.

This finding should give us pause when considering the economic situation of peoples living in different countries which have vastly different natural resource supplies. If economic prosperity is dependent on this, then restriction on immigration become a moral issue.

5.6.9 Alterations Summary

Table 5.4 was produced to better examine the effects each test had on the citizens of the ‘toy’ world. The table displays the average consumption rates of each citizen. Test A is displayed in raw units, while tests B through to G have been normalised against test A. Test H is displayed in raw units for each region. To produce the summary table, the time frames of the tests were increase twofold, to produce more reliable averages.

In test A, it is evident that each citizen consumes approximately the same amount of food and shelter, however differences appear in the amount of product consumed. The poorest citizen’s product consumption was around 40% of the wealthiest.

Increasing the currency in circulation, target profit ratios, and decreasing labour costs, all had a slight beneficial effect for the citizens. Increasing the inequality in wealth distribution had a slight beneficial outcome on the consumption pattern of all citizens, with the largest effect going to the poorest individual, whose average consumption of products increased by 26%. Consumption of food and shelter also rose for all individuals.

Decreasing natural resource licence costs and increasing the transportation distance both negatively affected the citizens. The poorest citizen had their consumption of products reduced by 60% in test D and 65% in test F. No major impact was observed to occur for the wealthiest person in test D, however consumption of products was reduced by 30% in test F.

The inclusion of a resource poor region produced a stark difference between the citizens of each region. The citizens of the resource rich region (r1) increased their consumption compared to test A, while the citizens of the resource poor region (r2) consumed a fraction of their counterparts.

Person	A	B_n	C_n	D_n	E_n	F_n	G_n	H_{r1}	H_{r2}
Food (2)									
1	1.87	1.04	1.03	0.89	1.03	0.71	1.03	1.95	1.89
2	1.89	1.03	1.02	0.97	1.03	0.82	1.02	1.95	1.90
3	1.88	1.03	1.02	0.97	1.03	0.88	1.02	1.95	1.90
4	1.88	1.03	1.03	0.97	1.03	0.92	1.03	1.94	1.90
5	1.88	1.03	1.03	0.97	1.03	0.94	1.03	1.95	1.90
Shelter (1)									
1	0.91	1.08	1.08	0.63	1.09	0.41	1.06	0.96	0.14
2	0.96	1.02	1.03	0.88	1.03	0.54	1.01	0.96	0.24
3	0.96	1.02	1.02	0.97	1.03	0.65	1.01	0.96	0.35
4	0.96	1.02	1.02	0.98	1.03	0.73	1.01	0.96	0.42
5	0.96	1.02	1.02	0.97	1.03	0.80	1.01	0.95	0.48
Product (∞)									
1	2.11	1.16	1.26	0.40	0.98	0.35	1.03	3.54	0.11
2	2.78	1.10	1.17	0.61	1.06	0.41	1.04	4.32	0.17
3	3.50	1.05	1.09	0.79	1.09	0.51	1.03	5.10	0.28
4	4.23	1.02	1.04	0.93	1.11	0.61	1.03	5.89	0.70
5	4.95	1.00	1.01	1.03	1.13	0.70	1.02	6.67	1.48

Table 5.4: Summary table of average consumption of each person in each test. Tests B through to G have been normalised with respect to test A. Test H has two populations in different regions. Desired consumption rates of food, shelter, and product are 2, 1, and ∞ respectively.

Test	Description
A	Basic run with a single region.
B	Increased currency in circulation.
C	Increased inequality in distribution of firm profits.
D	Decreased natural resources licence costs.
E	Increased target profits for firms.
F	Increase in transportation distance to the market.
G	Decrease in labour costs for fishing and building.
H	Addition of a region lacking in natural resource.

Table 5.5: Summary of test scenarios.

5.7 Discussion

The model developed in this chapter has shown considerable non-linear behaviour. It was expected that the model would settle to a steady state, however this was not the case. The model produced cyclical behaviour, with peaks and troughs in production rates, which naturally had consequences for consumption rates.

The model has demonstrated some potential for experimenting with a multi-regional, multi-market (many different items being traded) economy, in which the autonomous people are able to make consumption choices based on their own buying power. The model has shown itself capable of investigating a large variety of scenarios. It has been shown capable of testing issues such as wealth inequality, technological change, government taxation (in the form of resource licence costs), transportation effects, and resource availability between regions.

As the overall aim of this thesis is to better understand our world through a holistic approach, the reversion back to a 'toy' world with heavy abstraction may appear counter intuitive. The author believes that this 'toy' still fits within the holistic arch as the aim of the model is to be able to examine global issues such as regional variations, regional trade, and energy technology shifts[18].

5.7.1 Captured Theories and Concepts

The model is able to capture concepts such as EROI, technological shifts (efficiency changes in production and extraction), spacial effects and their impacts on trade and markets, wealth distribution, wages and consumer buying power.

5.8 Verification

The next, and most important step in the creation process of this model is verifying its validity[19]. This could be approached in two different ways.

First, if all assumptions made regarding the model could be shown to be perfectly valid and reliable, then the model could be (tentatively) considered representative of a real world systems. This process however is problematic as there is no concrete data used, and human decisions on what counts as a valid assumption can be prone to error.

The second option is to compare the model's output with real world data. This is also difficult as few real world examples exist which match the conditions simulated within this model. For example the production ratios in the tests were kept constant (excluding test F), however, in the real world production efficiencies are constantly changing due to technological shifts.

The model does appear to follow business cycles (booms and busts) naturally on its own accord. This is not due to outside functions, but through endogenous characteristics

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[18] While not included in great detail in this thesis, technological shifts are intended to be studied in the future.

[19] Unfortunately time constraints have limited this aspect.

of the model. This may indicate a small level of congruency between the model and the real world.

5.9 Conclusions

The aim of this chapter was to develop a novel model which is capable of capturing phenomena which previous socio-ecological models could not. An important aspect which past and current socio-ecological models[20] have lacked, is the inclusion of human agents which are free to make consumption choices, along with minimal consideration given to regional effects (on issues such as trade and production). This model has taken steps towards rectifying these underdeveloped aspects. To the authors best knowledge the model presented here is novel in formulation and conception.

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[20] As far as the author is aware.

Chapter 6

Concluding Remarks

The purpose of this thesis is to help advance humankind's understanding of the world, and how we can better handle the issues of our time. This was achieved by adding new knowledge to our collective understanding of socio-ecological models in three main ways.

First, the uncertainty surrounding the renowned (in the world of socio-ecological modelling) Limits to Growth model was quantified in greater detail. To date, only scenario testing and local sensitivity testing had been performed on the model. Both of these methods leave room for question into the reliability of the model, as complex systems often show non-linear behaviour. This issue was resolved by conducting a Monte Carlo uncertainty analysis. This involved applying probability density functions to the parameters of the Limits to Growth model and running the model one million times, to develop probability density functions of the models outputs. This provided a more complete sense of the reliability of the models projections.

Secondly, an analysis of the trade-off relationships between seven goals (objectives) of mankind was conducted. This was achieved by applying a multi-objective optimisation algorithm to the Limits to Growth model to determine the Pareto front of the objectives. The analysis found many unexpected relationships between the objective trade-offs, and gave insights into which goals are conflicting, and which are not.

Finally, a novel modelling approach was developed to examine the effects of human decision making[1] and spacial aspects on the lives of individual citizens in a socio-ecological model. The new model proved capable of capturing dynamic behaviour of a multi-market, multi-regional economy. The next step in the model's life cycle is the process of validating its output. If it can be shown to approximately represent real world systems them it could prove useful in furthering our analytic abilities.

All of the aforementioned contributions are, to the authors knowledge, unprecedented.

For copies of the software please contact the author at aheath@uow.edu.au.

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[1] In this iteration of the model it was crudely represented with consumer purchasing power and hierarchical needs for goods.

6.1 Future Works

Further work can be conducted investigating the fraction of simulations (from Chapter 2) which ended in a stable verse unstable future. This is bound to be a small fraction (based on observation), however it would make for an interesting examination.

This graphical user interface (GUI) presented in chapter 3 could be improved by adding a tool which allows the user to draw a line over a plot, which then alters the colour of solutions falling on that line. The colour could transition from red to blue along the length of the line. The user could then go and look at other plots to see where these points fall in other pairwise comparison plots.

Another way of improving the GUI could be to include a small image under the name of each objective so that it is quicker to interpret. A panel with physical dials (e.g. a sound engineers equaliser board) may also improve the experience of adjusting criteria. A physical board allows the user to capitalise on their spacial awareness to manipulate the controls, while their visual senses concentrate on the movement of the bars in the simple plot.

In the future, a study could be conducted to examine the effectiveness of learning about objective trade-offs via either manual manipulation (scenario testing), examination of underlying model equations, or use of the GUI. The ability of the students (or whom ever happens to be the guinea pig) to correctly identify the trade-offs could be measured to better understand if the GUI has any appreciable effects. The time taken for a student to make a decision on the most desirable course of action (select a solution) could also be studied, along with the variance of the most desired solution (when examining a collection of student choices).

While the optimisation[2] analysis presented in Chapter 4 has been conducted on the Limits to Growth model, this is but one of many models. It would be of great interest to apply this technique to other models to determine if the trade-off relationships hold true for other models. If the relationships hold true over several models, then a greater trust in their validity can be established.

Adding different objectives to the trade-off analysis would provide new insights to the work conducted in Chapter 4. There are thousands of objectives for which the human race could strive to achieve, and so this task is inexhaustible.

Aside from the validation work required for the novel model presented in chapter 5, there are many aspects of the model that can be advanced. For starters the simulated world could have a larger number of tradeable items. This would increase the choices available to the human agents within the model, and allow for the effects of 'taste' to be explored.

If firm types were to be broken up into individual actors (rather than being modelled as an aggregation of many similar firms) and effects of capital (economy of scale) were also included, then phenomena such as market monopolisation could be examined.

The model is also intended to be extended to explicitly model natural resource stocks, thus allowing for extraction efficiencies to be accounted for. If farming and fishing were

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[2] Optimising in the sense of finding the Pareto front of objectives.

modelled together (with fishing efficiency depending on fish stock levels), then interactions between food sources could be investigated.

Taxation (including import tariffs) and subsidisation is another component that should be properly implemented in the model, as this is an important mechanism government have at their disposal to curb and encourage certain consumption activities.

Another aspect which has been left unexamined are the effects of immigration and emigration, along with population growth. By allowing human agents to immigrate and emigrate from regions, changes in labour force size could be examined. Theories about the effectiveness of a universal basic income could be examined by implementing a redistribution scheme into the model.

Increasing the complexity of the banking agent would also be of benefit, as interest rates affect the lending of money and willingness of businesses to start up. By adding this into the model, the affect institutions such as the reserve bank have on economies could be investigated.

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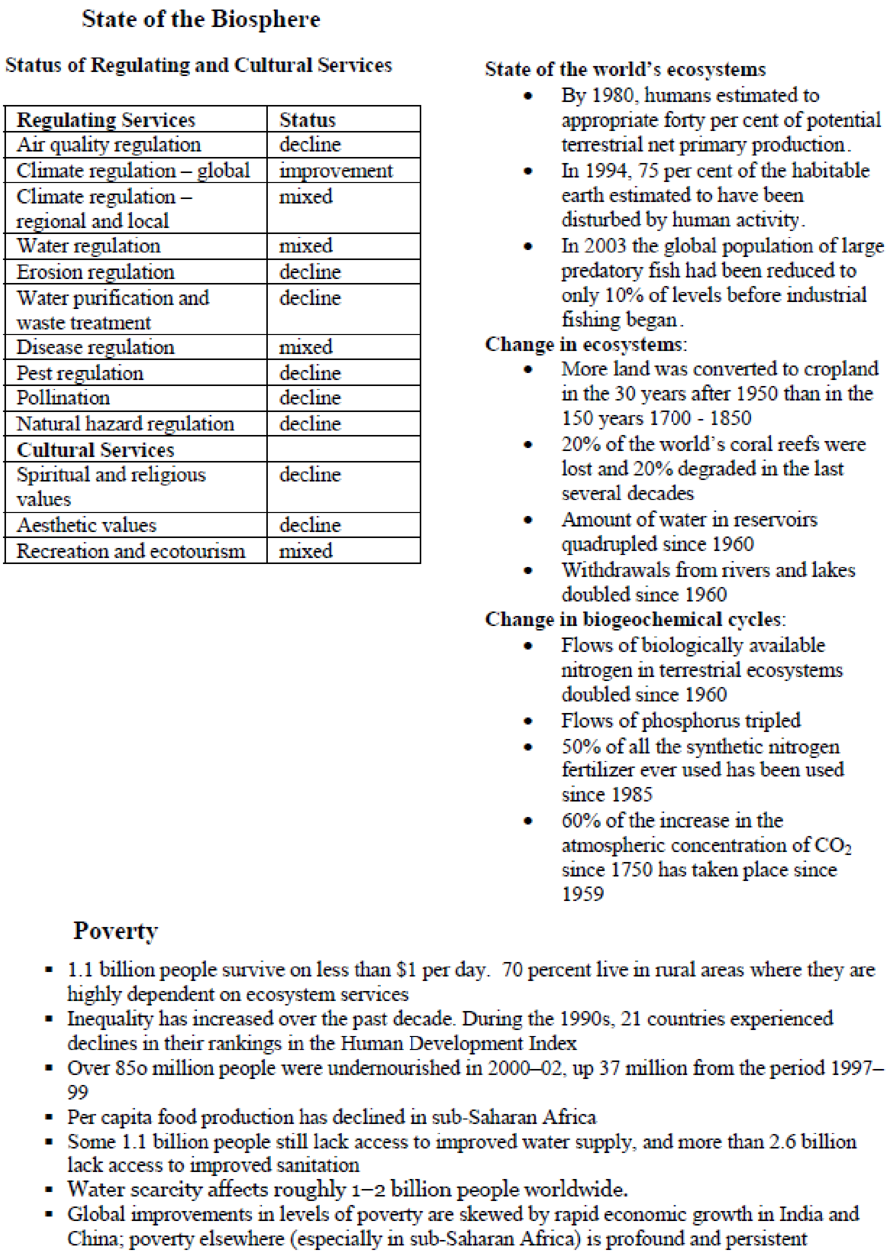
Appendix A

UN Report Findings



State of the Biosphere

Status of Regulating and Cultural Services

Regulating Services	**Status**
Air quality regulation	decline
Climate regulation – global	improvement
Climate regulation – regional and local	mixed
Water regulation	mixed
Erosion regulation	decline
Water purification and waste treatment	decline
Disease regulation	mixed
Pest regulation	decline
Pollination	decline
Natural hazard regulation	decline
Cultural Services	
Spiritual and religious values	decline
Aesthetic values	decline
Recreation and ecotourism	mixed

State of the world's ecosystems

- By 1980, humans estimated to appropriate forty per cent of potential terrestrial net primary production.
- In 1994, 75 per cent of the habitable earth estimated to have been disturbed by human activity.
- In 2003 the global population of large predatory fish had been reduced to only 10% of levels before industrial fishing began.

Change in ecosystems:

- More land was converted to cropland in the 30 years after 1950 than in the 150 years 1700 - 1850
- 20% of the world's coral reefs were lost and 20% degraded in the last several decades
- Amount of water in reservoirs quadrupled since 1960
- Withdrawals from rivers and lakes doubled since 1960

Change in biogeochemical cycles:

- Flows of biologically available nitrogen in terrestrial ecosystems doubled since 1960
- Flows of phosphorus tripled
- 50% of all the synthetic nitrogen fertilizer ever used has been used since 1985
- 60% of the increase in the atmospheric concentration of CO_2 since 1750 has taken place since 1959

Poverty

- 1.1 billion people survive on less than \$1 per day. 70 percent live in rural areas where they are highly dependent on ecosystem services
- Inequality has increased over the past decade. During the 1990s, 21 countries experienced declines in their rankings in the Human Development Index
- Over 850 million people were undernourished in 2000–02, up 37 million from the period 1997–99
- Per capita food production has declined in sub-Saharan Africa
- Some 1.1 billion people still lack access to improved water supply, and more than 2.6 billion lack access to improved sanitation
- Water scarcity affects roughly 1–2 billion people worldwide.
- Global improvements in levels of poverty are skewed by rapid economic growth in India and China; poverty elsewhere (especially in sub-Saharan Africa) is profound and persistent

Figure A.1: Overview of the progress of development in 2006 from the IUCN report "The Future of Sustainability". Figure sourced from [1].

Sustainability	*Development*
Nature Anthropogenic interference with one-half of the terrestrial ecosystems and one-quarter of the freshwater supply. Biodiversity continues to decrease at rates 100 to 1,000 times their pre-human levels. Global CO_2 emissions from fossil-fuel burning, cement manufacture, and gas flaring have increased at an accelerated rate. They increased from 24.8 $GtCO_2$ in 2000 to 35.1 $GtCO_2$ in 2012 - the largest increase in any decade in human history. 41 per cent of the oceans showed high human-induced impacts on marine ecosystems in 2012.	**People** World population reached 7 billion people, 80 million added each year. Life expectancy extended by 22 years with persistent gaps between regions and a widening gap between men and women and since 1950. Better global health and shifting disease, but more years in injury and illness. The 2000s were the first decade since 1980 when both the absolute numbers and the proportion of people in absolute poverty declined. However, the number of relative poor in the developing world has continued to increase ever since 1980. 850 million people suffer from hunger which is slightly more than in 1990 but 150 million less than in 1970. Universal primary education achieved in most parts of the world. The literacy rate of 15- to 24-year-olds in developing countries reached 88 per cent in 2011. In stark contrast to twenty years earlier, today women dominate tertiary education in most parts of the world. 740 million people lack access to safe drinking water (i.e. 500 million fewer than in 1990) and 2.4 billion people lack access to basic sanitation (650 million more than in 1990). Water pollution continues to claim the lives of millions. Great improvements in modern energy access since 1990, but in 2010 there were still 1.27 billion people without access to electricity and 2.59 billion without access to clean cooking fuels. Increased aging including in many developing countries. 810 million people are now older than 60 years. In 2010: 215 million international migrants (59 million more than in 1990) and 740 million internal migrants. 383 million employed people getting by on less than US$1.25 per day – half the number of 1990, but no reduction in LDCs, LLDCs and SIDS. Intergenerational social mobility earning, wage and educational mobility varied widely across countries Mixed progress on human security and human rights. Overall well-being of people – as measured by HDI - has substantially improved since 1950.
Life support Human settlements now cover 7% of the world's ice-free land cover and their croplands another 21%. The protected terrestrial and marine areas have been greatly expanded in developed and developing countries. Loss of half of the world's forests historically to domestication. Tropical forests declined at around 12-14 million hectares per year in both the 1990s and 2000s, and a similar amount was degraded. In contrast, temperate and boreal forests were reforesting since the 1980s. Global arable land and permanent crops expanded by 160 million ha since 1961, due to expansion in developing economies, but the world likely reached peak farmland by 2010. Humanity claims about 24 per cent of the global terrestrial net primary production, more than ever before. Local and regional freshwater shortages, and water stress was widespread in one-third of the world. The proportion of overexploited fish stocks tripled from 10% in 1970 to 30% in 2012. Many concentrations of local air pollutants have decreased, but the health burden of local air pollution remains large, especially in megacities of developing countries. Ozone layer on a long-term path to stabilization by 2020/2030. Degraded coastal zones where half the world population lives.	**Economy** Affluence has increased amidst persistent poverty. The world economy doubled since 1990 to US$69 trillion in 2012. The Genuine progress index per capita has slightly decreased since 1978. Consumption remains grossly inadequate for the poorest. Greater material consumption and less per unit of value, but progress in technology access and performance has fallen far short of the requirements for sustainability. From 1988 to 2008, all gains in real income have been reaped by the super-rich in all countries and the rising middle-class in developing countries. Growing income inequality in many parts of the world. Trade has grown at more than twice the rate of economic growth since 1950. Total assistance to developing countries more than doubled since 2000 to US$126 billion in 2012. The proportion of net ODA to donors' gross national income regained their 1990 levels of 0.32% in 2010, up from 0.22% in 2002. Estimates for 2012 are 0.29%. Energy almost tripled between 1970 and 2010 – reaching 493EJ. Renewable energy share increased from 5.4% in 1970 to 7.0% in 2000 and 8.2% in 2010. Growing but slowing water withdrawals.
Community More State-based armed conflicts than in the cold war. Greatly reduced number of deaths from non-State armed conflicts, including terrorism. Diversity of cultural heritage, traditions, and traditional knowledge and 90% of indigenous languages threatened, but also indications of some revivals.	**Society** Extraordinary changes in developed and developing countries alike, in terms of values, attitudes, and actual behaviour, in particular the attitudinal and behavioral shifts in sex and reproduction, the role of women, the environment, and human rights. Fewer stable families in most developed and developing countries than in past decades. In developed countries, crude marriage rate halved since 1970 and divorce rate increased. The average duration of marriages has stayed constant at 10-15 years. Widening governance and globalization. Power has shifted from the nation State upward to the global level and downward to the local level, and at all levels from the public to the private. Crisis of multilateralism. In most countries where a high level of societal consensus existed on intergenerational equity, it has been lost or come under pressure.

Note: red colour coding indicates trends that scientists have expressed concerns about, green indicates what is typically considered a trend toward sustainable development, and black indicates a neutral or mixed trend.

Figure A.2: Overview of the progress of development in 2014 from the UN report "Prototype". Figure sourced from [174].

Appendix B

Standard Deviation of Input Variables

Table B.1 of this appendix summarises the standard deviations assigned to each variable, as a percentage of its nominal value. These standard deviations were pertinent to the research conducted in Chapter 2.

Variable	$SD_{input}\%$	Page
Population age bracket 1/2/3/4 Initial	5	138
Lifetime Multiplier from Food Table	2.5	67
Health Services Allocations Per Capita Table	5.8	69
Health Services Impact Delay	15	71
Lifetime Multiplier from Health Services 1/2 Table	2.7	72
Fraction of Population Urban Table*	7.8	89
Crowding Multiplier from Industrialization Table*	5	90
Lifetime Multiplier from persistent Pollution Table*	6.4	94
Maximum Total Fertility Normal	5	99
Compensatory Multiplier from Perceived Life Expectancy Table	5	112
Lifetime Perception Delay	15	111
Desired Completed Family Size Normal	7.5	113
Social Adjustment Delay	15	120
Fraction of Services Allocated to Fertility Control Table	10	132
Fertility Control Effectiveness Table*	2.1	134
Industrial Capital-Output Ratio 1/2	8.3	218
Industrial Capital Initial	15	221
Average Lifetime of Industrial Capital 1/2	6.7	222
Fraction of Industrial Output Allocated to Consumption 1/2*	5.3	224
Indicated Service Output Per Capita 1/2 Table	21.3	227
Fraction of Industrial Output Allocated to Services 1/2 Table	10	229
Service Capital Initial	10	230
Average Lifetime of Service Capital 1/2	6.7	231
Service Capital-Output Ratio 1/2	15	232
Jobs Per Industrial Capital Unit Table	14.9	235
Jobs Per Service Capital Unit Table	16.2	236
Jobs Per Hectare Table	70.9	241
Labour Force Participation Fraction*	5	241
Labour Utilization Fraction Delayed Table	15	241
Capital Utilization Fraction Table*	16.9	242
Potentially Arable Land Table	3	278
Arable Land Initial	5	279
Land Fraction Harvested	12.7	281
Processing Losses	20	281

Table B.1: Standard deviation assigned to each input variable given as a percentage of variables nominal value. Asterisk (*) indicates variables assigned a Normal PDF. All other variables received a Log Normal PDF. Page number indicates the location of relevant information in the book "Dynamics of Growth in a Finite World" [112].

Variable	SD_{input}%	Page
Indicated Food Per Capita 1/2 Table	7.4	285
Fraction of Industrial Output Allocated to Agriculture 1/2 Table	5	288
Development Cost Per Hectare Table	10	291
Agricultural Inputs	15	293
Average Life of Agricultural Inputs 1/2	15	292
Land Yield Multiplier from Capital Table	5	305
Land Yield Multiplier from Air Pollution 1/2 Table*	6.2	310
Industrial Output in $_{19}70$	5	309
Fraction of Inputs Allocated to Land Development Table*	8.7	311
Social Discount	15	312
Average Life of Land Normal	20	315
Land Life Multiplier from Yield 1/2	15	316
Urban-Industrial Land Per Capita Table	10	320
Urban-Industrial Land Development Time	15	321
Urban-Industrial Land Initial	5	322
Land Fertility Degradation Rate Table	10	325
Initial Land Fertility	10	329
Land Fertility Regeneration Time Table	10	330
Fraction of inputs Allocated to Land Maintenance Table	10	332
Subsistent Food Per Capita	5	332
Food Shortage Perception Delay	15	332
Non-renewable Resources Initial	15	389
Non-renewable Resources Usage Fraction 1/2	15	390
Per Capita Resource Usage Multiplier Table	9.5	390
Fraction of Capital Allocated to Obtaining Resources 1/2 Table*	14.4	394
Persistent Pollution Generation Factor 1/2	5	428
Fraction of Resources as Persistent Materials	10	430
Industrial Materials Emission Factor	10	431
Industrial Material Toxicity Index	20	432
Fraction of Inputs as Persistent Materials	15	434
Persistent Pollution Transmutation Delay	15	440
Persistent POLlution	5	441
Persistent POLlution in $_{19}70$	5	441
Assimilation Half-Life Multiplier Table	10	453
Assimilation Half-Life in $_{19}70$	10	452

Table B.2: Table B.1 continued.

Appendix C

Modification of World3 Model Relationships

The calculation for the industrial capital investment rate ($ICIR$) variable in the World3 model [111] was initially arranged such that all output that did not go into consumption, services or agriculture was redirected into industrial capital (see Figure C.1). This is evident in equation C.13. To manage industrial capital investment according to the desired consumption rates of the population (desired industrial output per capita consumed ($IOPCCD$), desired service output per capita ($SOPCD$), and desired food per capita), as opposed to automatic reinvestment, the code was modified.

Original equation from the World3 model are: C.1 (L 52)[1], C.5 (R 53), C.6 (A 50), C.13 (A 56).

To begin, we know that industrial capital (IC) at the current time step i changes with respect to the investment rate ($ICIR$) and decay rate ($ICDR$) and time step size (DT).

$$IC_{i+1} = IC_i + DT \cdot (ICIR - ICDR) \tag{C.1}$$

$$\frac{IC_{i+1} - IC_i}{DT} = ICIR - ICDR \tag{C.2}$$

We want industrial capital to reach it's steady state, i.e. the investment rate equals the decay rate.

$$\frac{IC_{i+1} - IC_i}{DT} = 0 \tag{C.3}$$

$$ICIR = ICDR \tag{C.4}$$

The industrial depreciation rate is dependent on the average lifetime of industrial capital ($ALIC$).

$$ICDR = \frac{IC}{ALIC} \tag{C.5}$$

---

[1] The bracketed number, e.g. (X 99), refers to the equations position noted in the published code in "Dynamics of Growth in a Finite World" [112].

Industrial output (IO) is given by C.6. (Note: $FCAOR =$ Fraction of Capital Allocation to Obtaining Resources - table variable based off resource stock levels; CUF = Capital Utilization Fraction - table function based off labour force employment; and $ICOR$ = Industrial Capital-Output Ratio.)

$$IO = IC \cdot \frac{(1 - FCAOR) \cdot CUF}{ICOR} \tag{C.6}$$

$$IC = IO \cdot \frac{ICOR}{(1 - FCAOR) \cdot CUF} \tag{C.7}$$

Given C.4 and C.5 we get C.8.

$$ICIR = \frac{IC}{ALIC} \tag{C.8}$$

Given C.8 and C.7 we get C.9.

$$ICIR = IO \cdot \frac{ICOR}{ALIC \cdot (1 - FCAOR) \cdot CUF} \tag{C.9}$$

Because we define a new term, fraction of industrial output going into industrial reinvestment (IOF), as

$$ICIR = IO \cdot IOF \tag{C.10}$$

C.9 becomes C.11,

$$IOF = \frac{ICOR}{ALIC \cdot (1 - FCAOR) \cdot CUF} \tag{C.11}$$

Industrial output goes to four different streams: industrial output per capita consumed ($IOPCC$); industrial capital investment rate ($ICIR$); service capital investment rate ($SCIR$); and total agricultural investment (TAI). Note: POP = Population.

$$IO = IOPCC \cdot POP + ICIR + SCIR + TAI \tag{C.12}$$

The fraction of industrial output allocated to industry ($FIOAI$) can be defined as the remaining fraction left over from agriculture ($FIOAA$), services ($FIOAS$), and consumption ($FIOAC$).

$$FIOAI = 1 - FIOAA - FIOAS - FIOAC \tag{C.13}$$

The desired industrial output investment rate ($ICIRD$) can be found by combining C.10 with C.12 once converted into desired investment rates. Note: $SCIRD$ = desired service capital investment rate; and $TAID$ = desired total agricultural investment.

$$ICIRD = (IOPCCD \cdot POP + ICIRD + SCIRD + TAID) \cdot IOF \tag{C.14}$$

By simplifying the $ICIRD$ terms we get C.15.

$$ICIRD = (IOPCCD \cdot POP + SCIRD + TAID) \cdot \frac{IOF}{1 - IOF} \tag{C.15}$$

We create a term, desired fraction (DF), to represent the ratio of how much output can be given to the sectors that are not industry (i.e. consumption, services and agriculture), and these sectors desired investment rates.

$$DF = \frac{IO - ICIR}{IOPCCD \cdot POP + SCIRD + TAID} \tag{C.16}$$

The desired investment rates of each sector is calculated via the following equations. The variable service output per capita desired plays a similar roll as indicated service output per capita in the original model, however this variable is a fixed amount rather than a table function based off industrial output per capita. This is likewise for indicated food per capita ($IFPC$)[2]. We can see that if the desired service output per capita is not matching the desired level the investment rate will be altered accordingly. Note: $SOPC$ = service output per capita; and FPC = food per capita.

$$SCIRD = SCIR \cdot \frac{SOPCD}{SOPC} \tag{C.17}$$

$$TAID = TAI \cdot \frac{IFPC}{FPC} \tag{C.18}$$

The investment rates are then calculated as such.

$$IOPCC = IOPCCD \cdot DF \tag{C.19}$$

$$SCIR = SCIRD \cdot DF \tag{C.20}$$

$$TAI = TAID \cdot DF \tag{C.21}$$

Industrial capital investment rate is capped at 50% of industrial output and must be greater than zero, i.e. there cannot be negative investment.

$$ICIR = max(min(ICIRD, 0.5 \cdot IO), 0) \tag{C.22}$$

Equations C.11, C.15, C.16, C.17, C.18, C.19, C.20, C.21, and C.22, were implemented in the modified code.

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[2] $IFPC$ can be essentially thought of as desired food per capita.

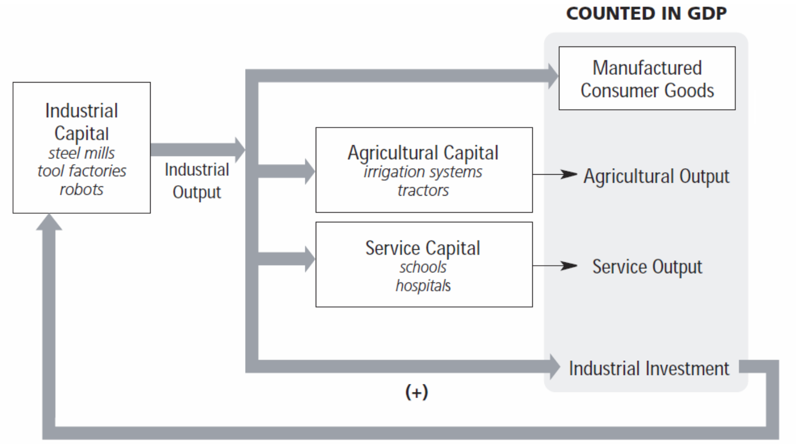


Figure C.1: The flows of industrial output in the original World3 model. Figure has been slightly modified from the source image found in “Limits to Growth: The 30-year update” [113].

Appendix D

Formal Mathematics used in Optimisation Analysis

The following Pareto front and indicator definitions have been derived primarily from these papers: [193, 68].

The hypervolume definitions have been derived from these papers: [185, 143, 157].

Definitions of symbols used in this chapter can be found in Table D.1.

D.1 Set-up

Lets consider a function f (or a model) which maps X of an input space to Y of an output space. There are m inputs and n outputs.

$$x = (x_1, x_2, ..., x_m) \in X \subseteq \mathbb{R}^m \land X \neq \emptyset$$

$$y = f = (y_1, y_2, ..., y_n) = (f_1(x), f_2(x), ..., f_n(x)) \in Y \subseteq \mathbb{R}^n_{>0} \land Y \neq \emptyset$$

We will first loosely define a and b as:

$$a \in A \subset Y$$

$$b \in B \subset Y$$

We will also define i and j as:

$$i \in \{1, 2, \ldots, n\}$$

$$j \in \{1, 2, \ldots, n\}$$

We will define all functions f as objectives to be maximised.

$$\forall i, maximise\, f_i(x)$$

Not to be confused with $maximise\, ||f(x)||$.

D.2 Defining Domination Types

See figure D.1 for a graphical representation of the various domination types.

Strictly Dominates

$$a \succ\!\succ b \iff \forall i, a_i > b_i \tag{D.1}$$

$$a \prec\!\prec b \iff \forall i, a_i < b_i \tag{D.2}$$

Dominates

$$a \succ b \iff \forall i, a_i \geq b_i \wedge \exists j, a_j > b_j \tag{D.3}$$

$$a \prec b \iff \forall i, a_i \leq b_i \wedge \exists j, a_j < b_j \tag{D.4}$$

Weakly Dominates

$$a \succeq b \iff \forall i, a_i \geq b_i \tag{D.5}$$

$$a \preceq b \iff \forall i, a_i \leq b_i \tag{D.6}$$

Incomparable

$$a||b = b||a \iff a \not\succeq b \wedge b \not\succeq a \tag{D.7}$$

D.3 Defining the Pareto Optimal Set

We will now define A as the Pareto optimal set taken from Y, B as the set of Y minus A (the solutions in Y that were dominated), and C as the set dominated by A (from the entire output space). See figure D.2 for a graphical representation.

Pareto Optimal Set

$$\begin{aligned} A = \mathrm{P}(Y) = \{ & A \subseteq Y \\ & : \nexists b \succeq a, \nexists a' \succ a \\ & | B = Y \setminus A, a \in A, \\ & a' \in A, b \in B \} \end{aligned} \tag{D.8}$$

Dominated Set

$$
\begin{aligned}
C = \Pi(A) = \{ & C \subseteq \mathbb{R}^n_{>0} \\
& : \forall c \exists a : a \succeq c \\
& \wedge \forall d \nexists a : a \succeq d \\
& | c \in C \, d \notin C \}
\end{aligned}
\tag{D.9}
$$

D.4 Epsilon Dominance

Given $\epsilon \in \mathbb{R}_{>0}$ we can define the following dominance relationships. Note: $\epsilon y = (\epsilon y_1, \epsilon y_2, ..., \epsilon y_n)$, $\epsilon + y = (\epsilon + y_1, \epsilon + y_2, ..., \epsilon + y_n)$, $\frac{y}{\epsilon} = (\frac{y_1}{\epsilon}, \frac{y_n}{\epsilon}, ..., \frac{y_n}{\epsilon})$.

ϵ-Dominates

$$a \succeq_\epsilon b \iff a \succeq \epsilon b \tag{D.10}$$

$$a \preceq_\epsilon b \iff a \preceq \epsilon b \tag{D.11}$$

Additive ϵ-Dominates

$$a \succeq_{\epsilon +} b \iff a \succeq \epsilon + b \tag{D.12}$$

$$a \preceq_{\epsilon +} b \iff a \preceq \epsilon + b \tag{D.13}$$

ϵ-Box Dominates

$$
\begin{aligned}
& a \succ_{\lceil \epsilon \rceil} b \iff \lceil \frac{a}{\epsilon} \rceil \succ \lceil \frac{b}{\epsilon} \rceil \vee \\
& (\lceil \frac{a}{\epsilon} \rceil = \lceil \frac{b}{\epsilon} \rceil \wedge \| \epsilon \lceil \frac{a}{\epsilon} \rceil - a \| < \| \epsilon \lceil \frac{b}{\epsilon} \rceil - b \|)
\end{aligned}
\tag{D.14}
$$

D.5 Epsilon Indicators

ϵ-Indicator

The indicator is the minimum value of ϵ such that A is dominated by ϵB. This can be formally expressed as $A \subset \Pi(\epsilon B)$. See figure D.3 for a graphical representation.

$$I_\epsilon(A, B) = inf\{\epsilon : \forall a \exists b : a \preceq_\epsilon b\} \tag{D.15}$$

This can be calculated using:

$$I_\epsilon(A, B) = max(min(max(\frac{a_i}{b_i}, \forall i | a, b), \forall b | a), \forall a)$$

Additive ϵ-Indicator

The indicator is the minimum value of ϵ such that A is dominated by $\epsilon + B$. This can be formally expressed as $A \subset \Pi(\epsilon + B)$.

$$I_{\epsilon+}(A, B) = inf\{\epsilon : \forall a \exists b : a \preceq_{\epsilon+} b\} \tag{D.16}$$

D.6 Epsilon Progress

Description of ϵ-progress. See figure D.4 for further explanation.

$$\epsilon-\text{progress} \iff \lceil \frac{a}{\epsilon} \rceil \succ \lceil \frac{b}{\epsilon} \rceil \tag{D.17}$$

D.7 Average Minimum Euclidean Distance

Description of generational distance D, i.e. the average distance from a point in B to the closest point in A. We will say that there are q elements in B and that b^i is the ith element of B.

$$D(B, A) = \frac{1}{q} \sum_{i=1}^{q} min(||a - b^i||, \forall a) \tag{D.18}$$

D.8 Hypervolume

Description of Hyper-volume.

If $F = \mathrm{P}(F)$ (that is F is a Pareto optimal set) then the hypervolume of F can be found using the following recursive function. The hypervolume is the area encapsulated by the set C.

$$\begin{aligned}
\vartheta(F) = & \begin{cases} \sum_{i=1}^{n} g_i \cdot \vartheta(\mathrm{P}(F_i^*)) & \text{if } d > 1 \\ f_1^1 & \text{if } d = 1 \end{cases} \\
& \text{given that} \\
F = & \{(f_1^1, f_2^1, \ldots, f_d^1), (f_1^2, f_2^2, \ldots, f_d^2), \ldots, \\
& \quad (f_1^n, f_2^n, \ldots, f_d^n)\} \\
& \text{and} \\
& f_d^1 \leq f_d^2 \leq \cdots \leq f_d^n \\
& \text{and} \\
g_i = & \begin{cases} f_d^i & \text{if } i = 1 \\ f_d^i - f_d^{i-1} & \text{if } i > 1 \end{cases} \\
& \text{and} \\
F_i^* = & \{(f_1^i, f_2^i, \ldots, f_{d-1}^i), (f_1^{i+1}, f_2^{i+1}, \ldots, f_{d-1}^{i+1}), \ldots, \\
& \quad (f_1^n, f_2^n, \ldots, f_{d-1}^n)\}
\end{aligned} \tag{D.19}$$

Definition	Symbol
if and only if	$\Longleftrightarrow$
maps to	$\Rightarrow$
element of	$\in$
subset of	$\subset$
proper subset of	$\subseteq$
minus set	$\setminus$
for all	$\forall$
there exist	$\exists$
there is no	$\nexists$
and	$\wedge$
or	$\vee$
Euclidean distance	$\|\| \cdot \|\|$
ceiling	$\lceil \cdot \rceil$
Infimum	inf
Such that	$:$
Given that	$\mid$
Set	$\{\cdot\}$
Empty Subset	$\emptyset$

Table D.1: List of symbol definitions.

A fast approach to calculation hypervolume is to randomly sample the output space and record the ratio of dominated to non-dominated samples.

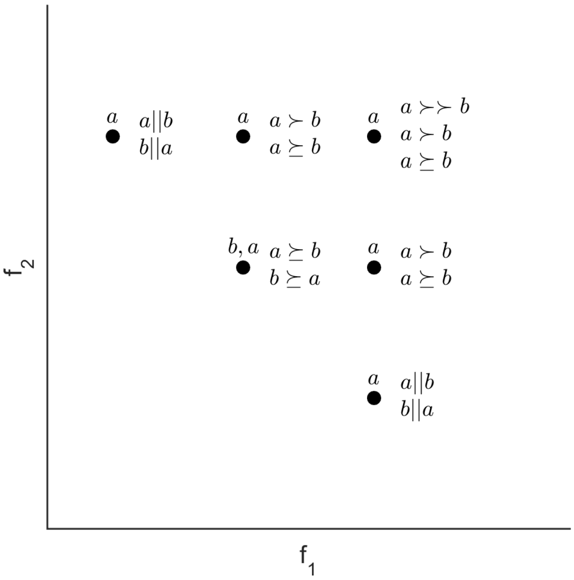


Figure D.1: Graphic showing the various domination types.

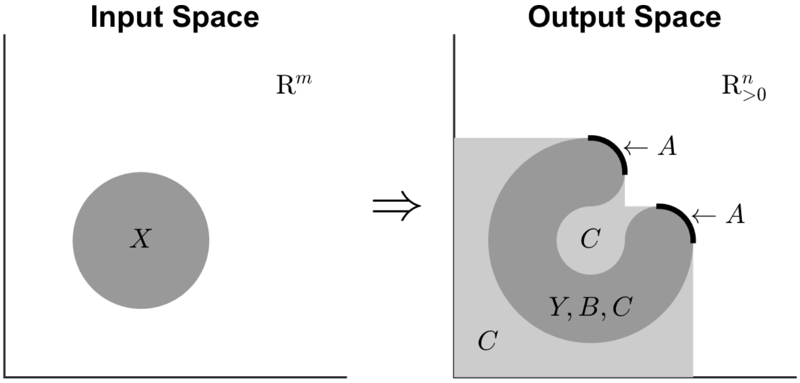


Figure D.2: Set definitions for a fictional optimisation problem and model.

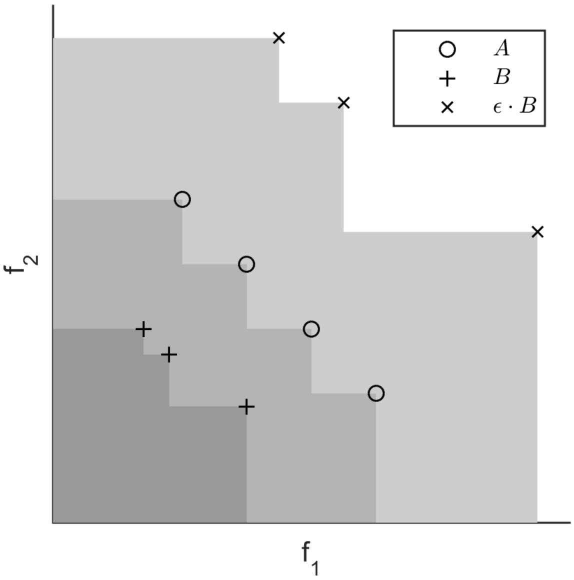
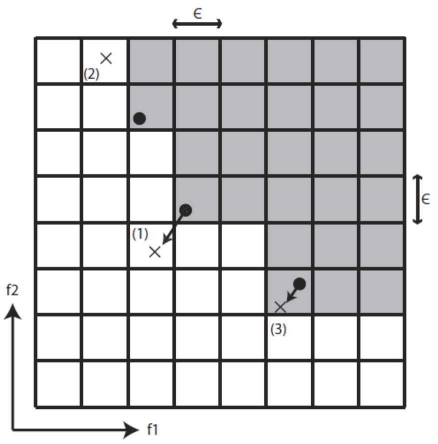


Figure D.3: Demonstration of $\epsilon \cdot B$ dominating A.



2D example depicting how ϵ-progress is measured. Existing archive members are indicated by •, and the ϵ-boxes dominated by these members are shaded gray. New solutions being added to the archive are indicated by ×. Cases (1) and (2) depict occurrences of ϵ-progress. The new solutions reside in previously unoccupied ϵ-boxes. Case (3) shows the situation in which the new solution is accepted into the archive, but since it resides in an occupied ϵ-box it does not count toward ϵ-progress—the improvement is below the threshold ϵ.

Figure D.4: Explanation of ϵ-progress. Figure sourced from [68].

Appendix E

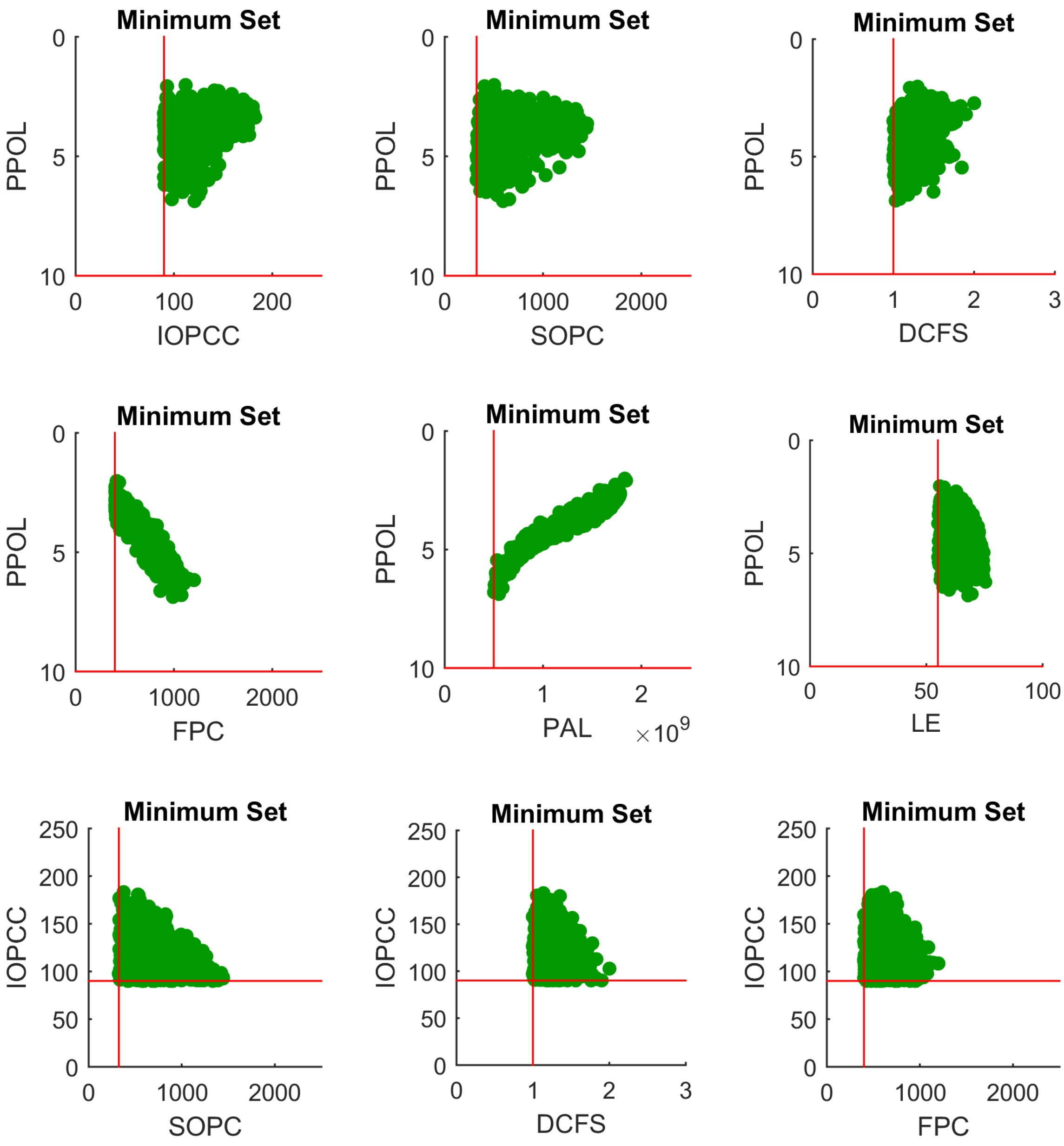
Potential Explanations for Each Observed Pairwise Relationship

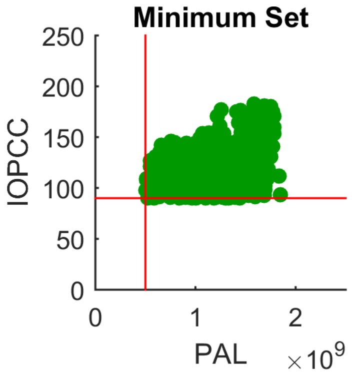
- **PPOL - IOPCC** (Zero Trade-off, Strongly Affected by Others): The relationship between these two variables could be explained by the relative magnitudes of the two components that make up persistent pollution in the model, i.e. industrial pollution and agricultural pollution. The correlation seen between FPC and PPOL suggests that agricultural pollution is the dominant factor and hence we see a very minimal trade-off between the two variables with no correlation. The absence of solutions in the lower right corner indicates another factor of the model. As more resources are funnelled toward industrial output, less are channelled into agricultural production, which in turn reduces the pollution generated.
- **PPOL – SOPC** (Zero Trade-off, Strongly Affected by Others): As service output does not directly contribute to pollution generation in the model it has a minimal trade-off with pollution with no correlation. The same absence of solutions can be observes as in PPOL – IOPCC for much the same reason.
- **PPOL – DCFS** (Zero Trade-off, Strongly Affected by Others): The relationship between these two variables is hard to explain. The minimal pollution and high birth rate solution can be achieved if many sacrifices are made in other objectives. Thus a zero trade-off can be achieved between these two variables at the expense of other objectives.
- **PPOL – FPC** (Linear Trade-off, Strong Correlation): This relationship indicates that FPC has a major impact on PPOL. In the model both industrial output and agriculture capital (fertilisers and pesticides) contribute to the production of pollution, and this suggests that the majority of the pollution comes from agricultural activities. Variance in PPOL for a given FPC can be attributed to two factors, i.e. different consumption levels of industrial capital, and varying ratios of agricultural capital (increasing fertiliser) to land development efforts (sewing larger areas).
- **PPOL – PAL** (Zero Trade-off, Strong Correlation, Strongly Affected by Others): The relationship shown here is explained by the intensity to which food is produced. If food is produced in small quantities, then only small amounts of land need to be cultivated (i.e. high PAL) and thus only small amounts of pollution are produced. If food consumption is increased then more of the PAL must be consumed.

- **PPOL – LE** (Small Trade-off, Strongly Affected by Others): Life expectancy is influenced by levels of food consumption, urbanisation, industrialisation and pollution. It appears that while greater food, services, and industrial capital per capita increase pollution, the negatives of pollution are out-weighed by the positives of the other factors. The combined effects produce a minimal trade-off outcome.
- **IOPCC – SOPC** (Linear Trade-off, Strongly Affected by Others): The linear correlation is a consequence of the division of industrial output. Industrial output is directed to four areas; industrial output consumption, services capital, agricultural capital, and industrial capital. This gives the linear trade-off between IOPCC and SOPC, as well as the dependence on another factor (agricultural production).
- **IOPCC - DCFS** (Linear Trade-off, Strongly Affected by Others): As DCFS increases, so too does the total population. This then causes the IOPCC to be diminished because the industrial output is finite.
- **IOPCC – FPC** (Small Trade-off, Strongly Affected by Others): Similar to IOPCC – SOPC.
- **IOPCC – PAL** (Zero Trade-off, Strongly Affected by Others): The diversion of industrial output to industrial consumption appears to have little effect on PAL. This is probably due to a lack of direct links between the two variables.
- **IOPCC – LE** (Small Trade-off, Strongly Affected by Others): Similar to IOPCC – PAL.
- **SOPC – DCFS** (Linear Trade-off, Strongly Affected by Others): Similar to IOPCC – DCFS.
- **SOPC – FPC** (Linear Trade-off, Strongly Affected by Others): Similar to IOPCC – SOPC.
- **SOPC – PAL** (Zero Trade-off, Strongly Affected by Others): Similar to IOPCC – PAL.
- **SOPC – LE** (Small Trade-off, Strongly Affected by Others): Funds spent on health increase with increases in SOPC. Thus we have no trade-off between the two variables, and the absence of solutions in the top left section of the graph.
- **DCFS – FPC** (Linear Trade-off, Strongly Affected by Others): Similar to IOPCC – DCFS.
- **DCFS – PAL** (Zero Trade-off, Strongly Affected by Others): The depletion of PAL is largely dependent on the level of food production in total and only slightly affected by the total population size (controlled by DCFS). There is a minor trade-off if food consumption levels are set quite low.
- **DCFS – LE** (Linear Trade-off, Strongly Affected by Others): Increasing the population reduce SOPC which intern reduces health care expenditure which leads to lower life expectancies, hence the linear relationship. The funnelling of resources from services to industrial or agricultural consumption produces the dependence on other objectives.
- **FPC – PAL** (Linear Trade-off, Strong Correlation): To achieve higher levels of food production PAL must be converted into productive farm land, hence the linear relationship. The spread of solutions is indicative of crop yields.
- **FPC – LE** (Zero Trade-off, Strongly Affected by Others): The relationship is controlled

by two factors: increased FPC increases LE; and that diverting resources from agriculture (decreasing FPC) to SOPC also increases life expectancy. This has the combined effect of producing an approximate zero trade-off relationship.

- **PAL – LE** (Small Trade-off, Strongly Affected by Others): Increasing FPC increases LE but decreases PAL. In total a minimal trade-off is seen between PAL and LE.

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IOPCC
PAL
×10^9
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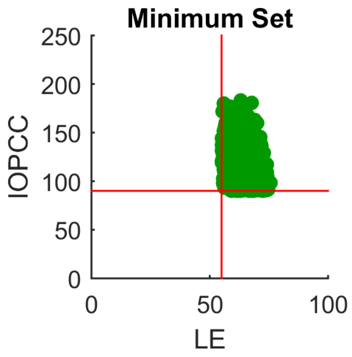
2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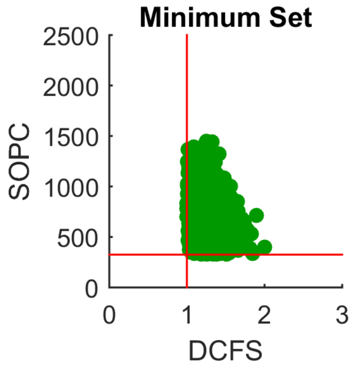
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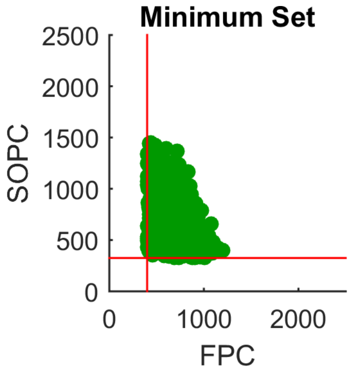
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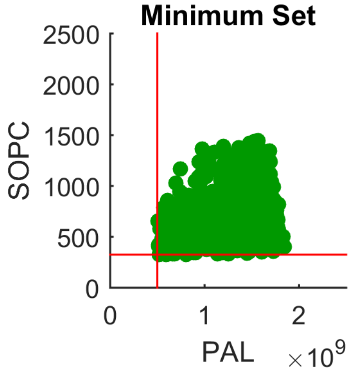
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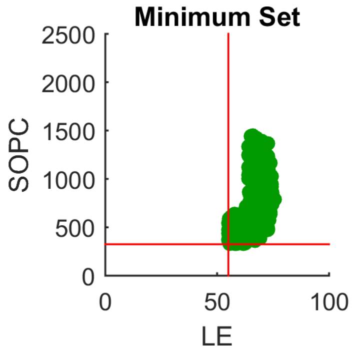
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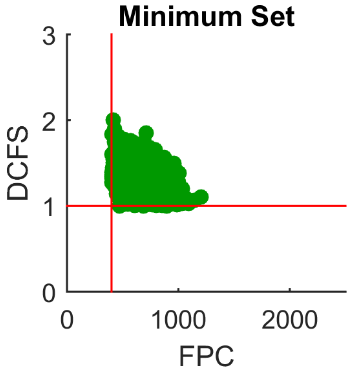
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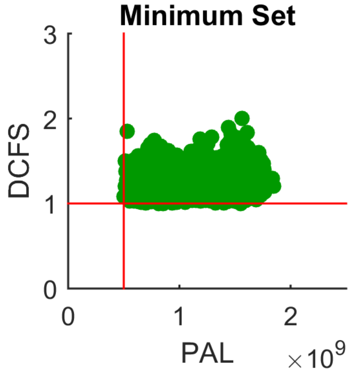
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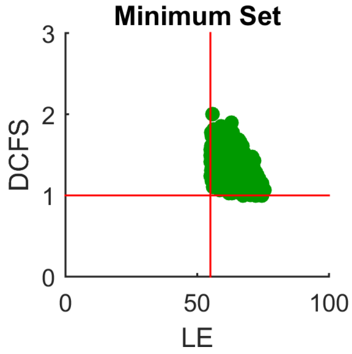
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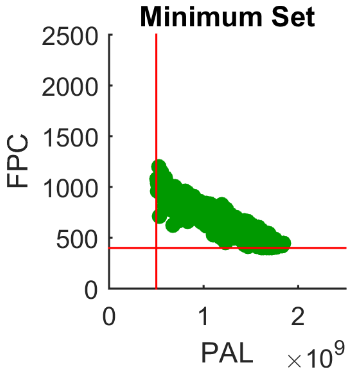
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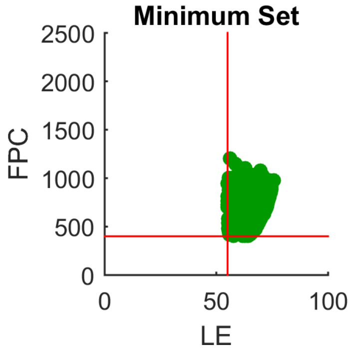
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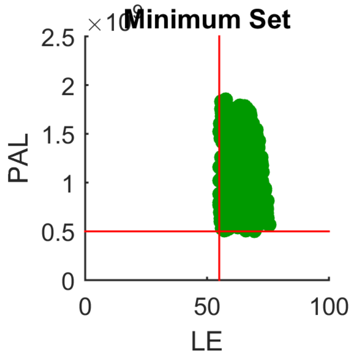
×10^9
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2.5
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