# RESOURCE ALLOCATION FOR UPLINK CODE-DOMAIN NON-ORTHOGONAL MULTIPLE ACCESS 

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Science and Engineering

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## Abstract

Code domain Non-orthogonal Multiple Access (NOMA) is a promising technique for improving throughput performance in multi-user systems. The technique is expected to be implemented in future generations for wireless communications standards such as 5 G and 6 G . Resource allocation is essential due to the presence of interference when several users are transmitting simultaneously.
The aim of this research is to optimise resource allocation in code domain NOMA, focusing on Sparse Code Multiple Access (SCMA) and Pattern Domain Multiple Access (PDMA).

We focus on resource allocation for the transmission process of code domain NOMA due to its significant effect on the total transmission rate. We focus on subcarrier and power allocation, which are two main aspects of this problem, with maximisation of the total rate for the system as our main objective. To reduce the complexity of the joint solution for sum rate maximisation the problem will be divided into these sub-problems. In order to tackle the negative effects of the multi-user transmission SCMA system, the proposed solutions are based on iterative and convex optimisation. They are also compared with other methods for resource allocation. Our research also studies PDMA and its transmission characteristics in the code domain. The resource allocation problem in PDMA is tackled and decomposed into two sub-problems, as with SCMA, subcarrier and power allocation. For power allocation a closed form solution is obtained and a subgradient based solution is presented through optimisation tools, such as the Lagrange dual decomposition approach. Simulation results performed in this work show improvement in the total transmission rate and the individual user rates.

## Declaration

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## List of Abbreviations

| 1G | 1st Generation |
| :--- | :--- |
| 2G | 2nd Generation |
| 3G | 3rd Generation |
| 3GPP | The 3rd Generation Partnership Project |
| 4G | 4th Generation |
| 5G | 5th Generation |
|  |  |
| AMPS | Advanced Mobile Phone Service |
| AWGN | Additive White Gaussian Noise |
| BP | Belief Propagation |
| BS | Base Station |
| CD-NOMA | Code Domain Non-orthogonal Multiple Access |
| CDMA | Code Division Multiple Access <br> CF |
| Closed form  <br> CSI Channel State Information |  |
| D2D | Device to device Communication |
| eMBB | Enhanced Mobile Broadband |
| EP | Equal Power |
| FCC | Federal Communications Comission |
| FDD | Frequency Division Duplex |
| FRA | Future Radio Access |


| IoT | Internet of Things |
| :--- | :--- |
| ISI | Intersymbol Interference |
| ITU | International Telecommunications Union |
|  |  |
| KKT | Karush-Kuhn-Tucker |
|  |  |
| LDS | Low Density Signature |
| LTE | Long Term Evolution |
| MAI | Multiple Access Interference |
| MC | Mother Constellation <br> METIS |
|  | Mobile and wireless communications Enablers <br> for the Twenty-twenty Information Society |
| MIMO | Multiple Input Multiple Output |
| MISO | Multiple Input Single Output |
| mMTC | Massive Machine-Type Communications |
| MPA | Message Passing Algorithm |
| NOMA | Non-orthogonal Multiple Access |
| NR | New Radio |
| OFDMA | Orthogonal Frequency Division Multiple Access |
| OMA | Orthogonal Multiple Access |
| PARP | Peak to Average Power Ratio |
| PD-NOMA | Power Domain Non-orthogonal Multiple Access |
| PDMA | Pattern Division Multiple Access |

SNR Signal to Noise Ratio

TACS Total Access Communication System
TDD Time Division Duplex

URLLC Ultra-Reliable Low-Latency Communications

WCDMA Wideband Code Multiple Access
WRC World Radiocommunication Conference

# List of Mathematical Notation 

| L | Lagrange |
| :--- | :--- |
| $\mathrm{O}()$. | Complexity order |
| e | Exponential |
| $\frac{\partial F}{\partial(.)}$ | Partial derivative of a function |
| inf | Infimum of the function |
| $\log _{x}()$. | Logarithmic function to base x |
| $\max$ | Argument of the maximum |
| $\Sigma$ | Summation |

## List of Variables

$H_{k, j} \quad$ Complex channel with frequency selective fading of subcarrier $k$ and user j
$A_{z} \quad$ Swap matching function
A Set of codebook patterns
$B \quad$ Bandwidth
$F_{\phi} \quad$ Factor graph of rotation angles of K by J dimensions
$F \quad$ Factor graph of $K$ by $J$ elements
$I_{k, j} \quad$ Interference of user $j$ over subcarrier $k$
Id Identity matrix with unitary vectors of $K$ by $K$ size
$J \quad$ Number of users
$K \quad$ Number of subcarriers
MC Mother constellation
$M$ Length of the coding alphabet
$N_{o} \quad$ Additive white Gaussian noise
$N \quad$ Sparsity value
PL Path loss factor
$P^{\max } \quad$ Maximum transmission power per user
$P_{j} \quad$ Power per user $j$
$\operatorname{Pr}_{F V} \quad$ Probability of variable function connected to a node
$P r_{V F} \quad$ Probability of variable node connected to a function
$Q_{m} \quad$ Gray mapping element of size $M$
$R_{A} \quad$ Rate of section A
$R_{B} \quad$ Rate of section B
$R_{k, j} \quad$ Rate in subcarrier $k$ per user $j$
$R_{k_{\text {HML }}}$ Rate per subcarrier $k$ with decoding order from high to low
$R_{k_{L M H}}$ Rate per subcarrier $k$ with decoding order from low to high
$R_{k} \quad$ Rate per subcarrier $k$
$R_{\text {tot }}$ Total rate
$S C_{k} \quad$ Set of subcarriers k
$S_{N} \quad$ Rotation vector of $N$ dimension
$S_{e} \quad$ Even element of vector $S$
$S \quad$ Rotation vector
$U_{N} \quad$ Multidimensional phase rotation matrix
$U_{j} \quad$ Set of users j
$V_{j} \quad$ Dispersity matrix $V$ per user $j$
$V \quad K$ by $N$ dimensional matrix
$X^{i} \quad$ Codebook for user $i$
$X^{k} \quad$ Codebook for user $k$
$X_{j} \quad$ Codebook for user $j$
$X \quad$ Codebook
$\Delta R \quad$ Difference in total rate
$\Delta_{j} \quad$ Rotation operator per user $j$
$\varepsilon \quad$ Shadowing factor
$\gamma_{a 1}$ Lagrange multiplier for minimum rate constraint per subcarrier $a$
$\gamma_{a 2}$ Lagrange multiplier for minimum rate constraint for subcarrier $a$
$\gamma_{b 1} \quad$ Lagrange multiplier for minimum rate constraint per subcarrier $b$
$\gamma_{b 2} \quad$ Lagrange multiplier for minimum rate constraint for subcarrier $b$
$\hat{x}_{n}^{i} \quad$ Estimated value of $x$ for user $i$ in subcarrier $n$
$\hat{x}_{n}^{j} \quad$ Estimated value of $x$ for user $j$ in subcarrier $n$
$\hat{x}_{n}^{k} \quad$ Estimated value of $x$ for user $k$ in subcarrier $n$
$\lambda_{1} \quad$ Lagrange multiplier for power constraint of user 1
$\lambda_{2} \quad$ Lagrange multiplier for power constraint of user 2
$\lambda_{x a} \quad$ Lagrange multiplier for power constraint of user $x$ in subcarrier $a$
$\lambda_{x b} \quad$ Lagrange multiplier for power constraint of user $x$ in subcarrier $b$
$\lambda_{y a}$ Lagrange multiplier for power constraint of user $y$ in subcarrier $a$
$\lambda_{y b} \quad$ Lagrange multiplier for power constraint of user $y$ in subcarrier $b$
$\lambda j \quad$ Lagrange multiplier for power constraint for user j
$\mathbb{B} \quad$ Binary numbers
$\mathbb{C} \quad$ Complex numbers
$\mathbb{Z} \quad$ Integer numbers
$d f \quad$ Overloading factor
$d v \quad$ Spreading factor
$\mu_{o} \quad$ Mean
$\mu \quad$ Function of a set of users (players)
$\overline{P L} \quad$ Average Path Loss
$\phi_{u} \quad$ Angle phase rotation per user per subcarrier
$\sigma^{2} \quad$ Variance
$\tau_{1} \quad$ Lagrange multiplier for minimum rate constraint of user 1
$\tau_{2} \quad$ Lagrange multiplier for minimum rate constraint of user 2
$\tau_{x a} \quad$ Lagrange multiplier for minimum rate constraint of user $x$ in subcarrier $a$
$\tau_{x b} \quad$ Lagrange multiplier for minimum rate constraint of user $x$ in subcarrier $b$
$\tau_{y a} \quad$ Lagrange multiplier for minimum rate constraint of user $y$ in subcarrier $a$
$\tau_{y b} \quad$ Lagrange multiplier for minimum rate constraint of user $y$ in subcarrier $b$
$\tau j \quad$ Lagrange multiplier for minimum rate constraint for user $j$
$\theta \quad$ Factor of rotation of vector $S_{N}$
$\zeta \quad$ Mapping function of binary streams into a constellation
$a_{i} \quad$ Codebook pattern for user i. $a_{i} \in A$
$b \quad$ Bits
c Valued constellation point
$d_{0} \quad$ Reference distance
$d_{e} \quad$ Euclidean distance
d Distance
$e \quad$ Error threshold
$f_{\phi_{j}} \quad$ Condensed factor graph of rotation angles for user $j$
$f_{k, j} \quad$ Binary variable of user $j$ on subcarrier $k$
$g_{j, k} \quad$ Normalised channel gain in user $j$ subcarrier $k$
$h_{H, k} \quad$ Channel gain in user with high channel in subcarrier $k$
$h_{L, k} \quad$ Channel gain in user with low channel in subcarrier $k$
$h_{M, k} \quad$ Channel gain in user with medium channel in subcarrier $k$
$h \quad$ Channel matrix
$n_{k} \quad$ Noise in channel $k$
$p_{H, k} \quad$ Power in user with high channel in subcarrier $k$
$p_{L, k} \quad$ Power in user with low channel in subcarrier $k$
$p_{M, k} \quad$ Power in user with medium channel in subcarrier k
$p_{a 1} \quad$ Power of user 1 subcarrier $a$
$p_{a 2} \quad$ Power of user 2 subcarrier $a$
$p_{a x} \quad$ Power of user $x$ subcarrier $a$
$p_{a y} \quad$ Power of user $y$ subcarrier $a$
$p_{b 1} \quad$ Power of user 1 subcarrier $b$
$p_{b 2} \quad$ Power of user 2 subcarrier $b$
$p_{b x} \quad$ Power of user $x$ subcarrier $b$
$p_{b y} \quad$ Power of user $y$ subcarrier $b$
$p_{k, j} \quad$ Power per user $j$ over subcarrier $k$
rmin Minimim rate value
$t \quad$ Number of iterations
$w \quad$ Noise signal
$x_{j} \quad$ Codeword or codebook vector for user $i$
$x_{j} \quad$ Codeword or codebook vector for user $j$
$x_{j} \quad$ Codeword or codebook vector for user $k$
$x \quad$ codeword
$y \quad$ Transmitted signal

## Chapter 1

## Introduction

The ever-present demand for connectivity and requirements for high data transmission rates in mobile communications systems have strengthened the need to develop new technologies and techniques capable of fulfilling user and provider needs. All these technologies should be able to manage the amount of traffic produced by the Internet of Things (IoT) and its applications [1].

We have witnessed the continuous development of generations of wireless communication systems. The analogue Frequency Division Multiple Access (FDMA) transmission deployed in 1G, known in the United States as Advance Mobile Phone Service (AMPS), a technology where the total available spectrum was divided into two 25MHz bands. One band was assigned for the communication from mobile channels to the base station and the other band for the base station transmitting to mobile channels. Europe used a similar system called the Total Access Communication System(TACS). The Time Division Multiple Access (TDMA) combined with FDMA and Code Domain Multiple Access (CDMA) in 2G, introduced low data transmission systems and the voice and text services in their early stage [2] [3]. It was later on in this generation where the packet data transmission was increased, using adaptative modulation and coding schemes based on the channel conditions. 3G was able to achieve higher data transmission of up to 2 Mpbs in the best scenarios [4]. It also introduced multimedia messaging and streaming services [5]. North America used an enhanced version of coded multiple access: CDMA2000, whereas Europe used spread code sequences: Wideband Code Division Multiple Access (WCDMA). Long Term Evolution (LTE),
as the predecessor of 4G (Long Term Evolution advanced) was implemented based on Orthogonal Frequency Multiple Access (OFDMA) aided by cyclix prefix and offset frequency, in order to achieve higher data rates and enhanced video over Internet [6].

The upcoming generation of mobile communications technologies is expected to achieve higher data rates, better coverage, and greater capacity than previous generations. In the year 2022, downlink and uplink are expected to be able to manage an increase in traffic with expected peak rates of 20 Gbps and 10 Gbps respectively [7], which establishes further challenges for researchers in the industry.

It is expected that by the end of 2020, a complete framework of standards for the new generation will be published. However, there is no final agreement about these standards between academia and industry because even though the 5 G non-standalone network has already been deployed, the 5 G standalone is still under development [8].

We can say that the industry in 5 G is expected to be deployed over three main bases as presented in Figure 1.1. These include enhanced mobile broadband that includes higher data rates, a wider spectrum range and wide area of applications; ultra-reliable low latency communications for mission critical applications and specific quality of service; massive machine-type communications with scalable connectivity, wider area coverage and more indoor penetration signal [9], [10].


Figure 1.1: Pyramid deployment of 5G industry

The main technology components included for the deployment of the 5G radio access solution are: advanced multi-antenna technologies such as massive MIMO and beamforming, ultra-lean transmission to reduce interference in multiple access user systems and to maximise resource efficiency, access/backhaul integration and well-integrated device-to-device communication [11, 12].

In [11] the main techniques needed in order to achieve higher data rates and improvements in capacity are specified. These include broader spectrum utilisation, carrier aggregation, network densification and advancement in the physical layer by enhancing spectral efficiency. Examples of these improvements are advanced physical layer techniques (modulation, coding), advances in spatial processing (network MIMO and Massive MIMO) and advanced schemes in the media access layer [13], [14]. The communication between the user and the base station occurs in the access media layer, via two routes of communication; the uplink (communication from the mobile equipment to the base station) and the downlink (communication from the base station to the mobile equipment). These simultaneous connections work through the use of different frequencies, using Frequency Division Duplex (FDD) or using the same range of frequencies separated by different time slots with Time Division Duplex (TDD). Figure 1.2 exemplifies the uplink and the downlink in a mobile system.


Figure 1.2: Graphic representation of downlink and uplink

The air interface, described as the link of radio communication between the base station and the mobile equipment of the upcoming generation of radio access technology
and known in some literature as New Radio (NR), is an important aspect that is expected to deal with unused spectrum bands such as millimetre wave bands. However, researchers and technology leaders believe that, in contrast to previous generations, the new era of mobile communications technologies is oriented to operate based on a diversity of bands from low to very high. In the World Radiocommunication Conference (WRC-19) [15], experts focused on the usage of high frequency spectrum bands, although many leading companies in the industry have stated that use of low bands below 6 GHz will be required to ensure coverage and available bandwidth for the future applications. The WRC-19 vision for IMT-2020 and the Federal Communications Commission (FCC), have expressed the intention of releasing the 28 GHz and 39 GHz bands for the use of the radio link. The NR is expected to migrate to bands below 6 GHz in the short term, eventually occupying existing mobile bands below 3 GHz . Even though unused bands will be available to fulfil the requirements of the new mobile networks, adaptive resource management is required to enhance the performance of the system while ensuring the best use of the already available resources and those that will be released in the near future. In the specifications of the ITU-R [16], it was established that the upcoming systems are expected to support a higher user density while assuring a satisfactory experience at the service delivery point. When several devices per unit area are attempting to access the service, for instance, public transport or public entertainment events, there is the necessity for advanced techniques in the radio interface that allow multiple users to access the resources simultaneously. Consequently, techniques such as power based and coded non-orthogonal multiple access are promising for the future infrastructure of the mobile networks due to their ability to accommodate several devices in multiple subcarriers.

In conventional schemes such as Orthogonal Multiple Access (OMA), multiple users are assigned to radio resources orthogonally in time, frequency, or code. Ideally, there is no interference among the users due to the orthogonality, however in reality, OMA systems require the use of frequency offset, which implies usage of spectrum. It is known that OMA systems present limitations in the total data transmission and in the number of users transmitting at the same time [17]. NOMA was proposed to deal with these limitations of OMA and has been investigated since its inception. Unlike OMA, NOMA allows the presence of controlled interference and exploits the use of power difference and coding techniques [18]. The main advantages of NOMA are spectral efficiency, low latency, reduced signalling and massive connectivity [19]. Additionally,
the performance improves when the non-orthogonal system exploits the benefit of coding. Sparse Code Multiple Access (SCMA) [20] and Pattern Division Multiple Access (PDMA) [21] are examples of code domain NOMA (CD-NOMA), where the use of multidimensional codebooks and patterns allows the allocation of several users in a reduced number of radio resources. However, all these characteristics of NOMA impact the complexity of the detection process and the utilisation of the available resources such as power and subcarriers.

### 1.1 Motivation

Throughout the deployment of previous generations of mobile systems such as 4G, operators have expressed their concern about the increase in energy consumption and the lack of available frequency bands. Conversely, customers have demanded batteries that last for longer and a decrease in the service prices. In parallel, the development in the telecommunications industry has a direct ecological impact [22]. Therefore, we can say there is a high demand for greener communications and efficient use of all the energy resources such as power and wireless spectrum. The appropriate management of energy resources not only results in economic benefit for the industry and private sector, it also has low carbon foot print impact and makes a positive contribution towards fighting climate change.

The upcoming generation will require an infrastructure and software development. We have mentioned before the expected solutions 5 G will bring in the private and industrial services. However, both converge in the necessity for enhanced data rate transmission over a wider range of spectrum and massive communication of devices with low latency. As a result of the constantly increasing demand for high-speed and increased data transmission in communication systems with simultaneous users, improving network efficiency and optimisation of resources are key elements in the development of mobile networks. These aspects should be considered in the design of the systems and management of resource allocation, particularly in non-orthogonal multiple access systems where the spectrum is shared by multiple users at a time. Code domain NOMA helps to solve this problem by exploiting the advantage of multidimensional coding and interference diversity.

In terms of quality of service and technical requirements, there is a clear expectation in the forthcoming development of access techniques: the systems should be resilient
against random fluctuation of the channel conditions by adapting the allocation of resources in an efficient, fair, and scalable manner. The complication in applying all these exigencies lies mainly in dealing conveniently with the massive connectivity of simultaneous, autonomous and selfish users attempting to access a set of resources.

From the users' perspective, the power consumption should be optimised, achieving a maximum data rate transmission. Battery capacity is increasing only 1.5 x per decade and has always been a concern for the user [22]. In the short term, there will be unlimited access to information and data sharing with the ever increasing high energy consuming applications. Therefore, to satisfy users' demand for battery life and high data rate transmission, more energy efficient multiple access techniques in wireless communication are crucial.

### 1.2 Contributions

In this thesis, the resource allocation challenges for different code domain techniques such as SCMA and PDMA have been studied. Solutions to the formulated problems have been proposed. The main contributions of this work are summarised as follows:

- Analyse the NOMA system in the uplink and compare mathematically different SIC decoding orders and the effects on system performance.
- Extend and describe the framework for the SCMA system with multiple users over orthogonal frequencies as a specific example of code domain non-orthogonal access.
- Design and propose a low complexity subcarrier allocation solution for the SCMA uplink system based on the greedy principle.
- Provide a formulated approach to solve the power allocation problem. This is non-convex, therefore, a low complexity solution is proposed based on Lagrange Dual Decomposition and the water filling principle subgradient algorithm.
- Investigate PDMA as another example of a NOMA system. Extend the framework by implementing a PDMA system in the code domain and provide a resource allocation solution.
- Design and propose a solution for resource allocation in a PDMA system in
the uplink. This solves the subcarrier allocation problem based jointly on rate calculation and the greedy principle.
- Design a scheme for power allocation in a PDMA system. This may be approached in one of two ways. The first approach provides a closed form solution, and the second approach tackles the problem with constraints using optimisation tools such as Dual Lagrange Decomposition and the subgradient algorithm.


### 1.3 Thesis organization

The thesis is presented in 6 chapters as follows:
Chapter 1 provides an overview of this work. It introduces the background of multiple access systems and the reasons why radio resource optimisation is necessary. The motivation for and the objectives of this research are also included in this chapter.
Chapter 2 includes a description of the power based NOMA system and introduces the background to explore coded NOMA, which in turn provides the case to investigate further SCMA and PDMA.

Chapter 3 presents the characteristics of SCMA system uplink. It contains a detailed description of the elements involved in the construction of the SCMA system from codebook design to detection of the signal. Simulations in the link performance are included to support the implementation of the system.
Chapter 4 proposes a joint resource allocation solution for SCMA. The solution is presented in two stages; subcarrier and power allocation. The implemented algorithms are then compared with other resource allocation schemes.

Chapter 5 presents the PDMA system and its resource allocation solution. The problem is divided into two sub-problems; subcarrier and power allocation. There is a comparison between the implemented solution and other approaches to show its optimality. Finally, Chapter 6 concludes this work and presents plans for future research. Appendices include mathematical proof of the equations used in NOMA, SCMA and PDMA sum rate.

### 1.4 Publications

The following paper was produced from this research:

- (Chapter 4) Y. J. Licea, K. Shen and D. K. C. So, "Subcarrier and Power Allocation for Sparse Code Multiple Access," 2020 IEEE 91st Vehicular Technology Conference (VTC2020-Spring), Antwerp, Belgium, 2020, pp. 1-5, doi: 10.1109/VTC2020-Spring48590.2020.9128928.


## Chapter 2

## Non orthogonal multiple access

### 2.1 Introduction

Cellular systems have grown exponentially in recent years, and mobile equipments have become vital as the main way of communication in the development of business and daily life. Wireless communications have been assisting wired networks for a long period of time. Wired networks are continually evolving to provide higher data rates to support feature-rich modern applications, and wireless networks must evolve in turn to meet user expectations and provide comparable functionality [23]. Sometimes, wireless systems can reach extreme or remote locations where the deployment of fibre optic or copper is not feasible. Additionally, wired options can be expensive, while cellular communication can be relatively easy to implement and with a low cost of maintenance. However, the increase in possible solutions and the high demands of services based on wireless networks bring new challenges for the design of robust and resilient systems that deliver the necessary performance to support emerging high data rate applications. Improved wireless networks require enhanced techniques to obtain better throughput of data and efficient use of radio spectrum. An important aspect in the design of wireless systems is the characterisation of the system's wireless channels. This chapter provides a brief introduction to the theory behind wireless channels. It also explains the concept of NOMA and presents a literature review of power and code domain NOMA and its main characteristics.

### 2.2 Wireless channels theory

The impairments of wireless channels need to be studied in order to investigate resource allocation in the air interface. The random fluctuations of the wireless channels impact the performance of the system. In order to provide reliability and resilience against these rapid variations and their propagation effects, we need to consider channel models and study their behaviour. In cellular networks, it is very common to find obstacles such as buildings and vegetation between the transmitter and the receiver, which means there is no direct signal between them. This case is known in literature as Non-Line of Sight (NLOS) and the simplest channel is free space, Line of Sight (LOS). It occurs when there is direct communication between the transmitter and the receiver. Based on the effects on the wireless channels, there is a classification [24] of Large-scale and Small-scale propagation effects.

### 2.2.1 Path loss and shadowing

The presence of ground and obstacles can cause effects on the propagation channel. Path-loss and shadowing occur in long distances and are known as large-scale propagation effects [25], [26]. When the signal dissipates its power while travelling across the distance to the receiver, it is known as path loss (PL). In general we can express the average path loss in dB as shown in (2.1):

$$
\begin{equation*}
\overline{P L}(d)=\overline{P L}\left(d_{o}\right)+10 v \log \left(\frac{d}{d_{o}}\right) \tag{2.1}
\end{equation*}
$$

where $\bar{P} L$ is the mean path loss at a referenced distance $d_{o}, d$ depicts the distance from the mobile to the BS, $v$ represents the path loss exponent, which can vary depending on the characteristics of the environment (usually between 2 to 6) [26] [27]. Shadowing occurs when the signal is attenuated by passing through obstacles such as buildings, automobiles and trees. The signal attenuates between the transmitter and the receiver through effects such as: absorption, reflection, diffraction and scattering [27], [25]. The shadowing effect can be modelled as a log-normal distribution. In equation (2.2) we present the path loss at a distance $d$ considering shadowing. $\varepsilon$ depicts the shadowing effect with zero mean and standard deviation $\sigma$ (in dB )

$$
\begin{equation*}
P L(d)=\overline{P L}(d)+\varepsilon \tag{2.2}
\end{equation*}
$$

### 2.2.2 Fading and multipath effect

The power variations to the signal occur in short distances in a relatively short period of time due to the multiple replicas of the same signal causing constructive or destructive interference. These are considered as small scale propagation effects. The importance of the characterisation of the fading channel effect is to design adequate receivers. We can say that there are two different types of small scale fading channels: variant channels and multipath fading channels. The first category depends directly on the speed of the mobile and the Doppler spread. Fast fading occurs when the Doppler shift is high and the coherence time is smaller than the symbol period. It means that the channel variations are faster than the baseband signal variations. In contrast, in slow fading, the Doppler spread is low and the coherence time is greater than the symbol period.

Conversely, multipath fading results from the reflection of the transmitted signal with the surfaces of objects on the way, resulting in several replicas of the same signal at the receiver. The resultant signal can be affected by the constructive or destructive effect of the copies with different phase, amplitude and delays. This category is split into flat and frequency selective fading. When the coherence bandwidth is greater than the transmitted signal bandwidth, we can say there is a flat fading effect. However, frequency selective fading occurs when the channel has a coherence bandwidth that is smaller than the transmitted signal bandwidth. The delay spread is greater than the symbol period, causing time dispersion of the information symbols through the channel, distorting the received signal and producing inter-symbol interference (ISI).

Throughout this work, for the simulation of the system we use a six-path frequency selective fading channel with the ITU pedestrian model B, where the average power and the relative delays of the multipath are listed in Table 2.1, unless otherwise stated.

|  | Channel A |  | Channel B |  |
| :---: | :---: | :---: | :---: | :---: |
| Tap | Relative <br> delay (ns) | Average <br> power (dB) | Relative <br> delay (ns) | Average <br> power (dB) |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 110 | -9.7 | 200 | -0.9 |
| 3 | 190 | -19.2 | 800 | -4.9 |
| 4 | 410 | -22.8 | 1200 | -8.0 |
| 5 |  |  | 2300 | -7.8 |
| 6 |  |  | 3700 | -23.9 |

Table 2.1: ITU Pedestrian Model

### 2.3 Non orthogonal multiple access in the uplink

Non orthogonal multiple access is considered to be a promising technology for future radio access (FRA). It provides access to multiple users by obtaining the maximum usage efficiency of the available spectrum and is different from conventional orthogonal access. For instance Orthogonal Frequency Division Multiple Access (OFDMA) as a case of OMA, allocates one user per resource at a time. In particular, OFDMA presents some limitations for the future implementation of the 5 G network. Characteristics such as cyclic prefix and the carrier frequency offset affect the spectrum efficiency and orthogonality, however, it is more resilient against multiple access interference (MAI) [28]. It is well known that the orthogonal resources are limited, therefore, NOMA alleviates the problem of restricted resources by exploiting and managing the multiuser interference [19]. In fact, due to NOMA's characteristic of non orthogonal code domain multiplexing, it is able to increase the total transmission rate of the users connected at the same time [29]. However, this leads to intra cell interference. (This trait is shared with Power Domain Non-orthogonal Multiple Access (PD-NOMA)) [17, 18, 30]. Coded NOMA derives itself from Code Domain Multiple Access (CDMA), Low Density Signatures (LDS) and NOMA principles and aims to improve the data rate by including redundancy and the coding gain in the transmission and reducing the complexity at the receiver by the use of sparse codes or patterns. Figure 2.1 shows a graphic comparison of the techniques discussed previously.

Researchers have worked on NOMA systems, exploring the use of superposition, multiple antenna transmission, detection and resource allocation. Receiver and resource allocation optimisation in uplink NOMA have been explored in [31], showing that the
adequate allocation of resources can considerably improve the performance of the system. In [32] power allocation and game theory allocation have been presented as a solution to maximise the sum-rate. The main feature of PD-NOMA is that it exploits the multiuser effect by using the difference in the channel gain of the users multiplexed through different power allocations over the same resources. NOMA differs from orthogonal transmission by intentionally introducing inter-user interference in order to implement non-orthogonal transmission [33] [34]. At the receiver, power domain NOMA applies Successive Interference Cancellation (SIC) to separate the transmitted signals [17, 35]. In contrast, code domain NOMA uses SIC aided by Message Passing Algorithm (MPA) to decode the transmitted data.

It is vital to present the generalities of the uplink NOMA system because it is the pioneer of non-orthogonal access techniques and provides useful information in order to obtain a better understanding of coded NOMA. However, it is important to mention that power domain NOMA does not always require subcarrier allocation as often as coded NOMA systems. Furthermore, the operation of downlink NOMA differs from that of uplink NOMA. In the downlink, the source of power comes from the base station, while in the uplink the power comes from each user's mobile equipment. This characteristic impacts the effect of grouping users and managing of the power in multiuser techniques and optimisation methods.


Figure 2.1: Graphic representation of different multiple access techniques

### 2.3.1 Successive Interference Cancellation

PD-NOMA uses Successive Interference Cancellation (SIC), which is a detection scheme that deals with a compound signal built from combining the multiplexed users' signals. It is applied at the receiver to decode the transmitted signals as shown in Figure 2.2. SIC can work in two ways; by decoding the signals with the strongest channel conditions first in the presence of interference from weaker users, and removing its contribution from the received signal, or vice-versa. The principle of SIC in the downlink is that the signal with the greatest received power will treat the signal from other users as noise. The receiver should iteratively decode and subtract the signals until all signals are recovered [36] [37] [30].


Figure 2.2: Successive Interference Cancellation process

### 2.3.2 Sum rate and decoding order for NOMA uplink system

Downlink and uplink transmissions in NOMA apply SIC at the receiver to decode the users' signals. However, in the downlink, the decoding order is usually fixed. The user with the stronger channel must first decode the message for the weaker user, subtract the corresponding signal and then decode its own message. The weaker user will only be able to decode its own message. This decoding order is dependent on the fact that more power is allocated to the weak user's signal, which ensures that the signal power can overcome the degradations due to the channel. On the other hand, the decoding
order in uplink NOMA can be performed in either direction as the achievable sumrate will not be affected [32], [38], [39]. However, each individual user's rates will be affected by the uplink decoding order. Related works on uplink NOMA [40], [41] and [42] suggest that the signal from the stronger user should be decoded first, such that the weaker user's signal is free from interference. Works presented in [32], [35] and [43] have simulated scenarios in the uplink which assume perfect interference cancellation at the receiver. This assumption may lead to suboptimal rate results when evaluated in more realistic scenarios which consider non-ideal factors. Furthermore, in a practical NOMA system, hardware restrictions of the equipment may prevent SIC from being implemented perfectly.

In NOMA scenarios where imperfect SIC is studied, the performance is shown to degrade with increasing levels of SIC error when cancelling the interference [44]. In other words, there is residual interference in the system from the users whose signals were cancelled first. However, the residual interference caused by imperfect SIC is a complicated function of multiple factors such as coding/modulation related parameters, channel impairments, device/hardware limitations, etc. Additionally, due to the characteristics of error propagation of imperfect SIC, modelling the impact of imperfect SIC is challenging in the sum-rate analysis of the uplink NOMA system, which is a separate line of research. Therefore, with the focus of this thesis being on the evaluation and analysis of coded NOMA, we have assumed perfect SIC in this work, in line with other major NOMA literature [44] [45]. Coded NOMA is described in further detail in Section 2.4,

In this chapter, to describe NOMA in the uplink, we consider a scenario with a base station and $J$ number of users, and the spectrum of the system divided into $K$ subchannels with equal bandwidth $B$. Each user has a maximum available power denoted as $P_{j}^{\text {max }}$, where the transmission power of user $j$ allocated to subcarrier $k$ is expressed as $p_{k, j}$. The channel gain for the $j$-th user at subcarrier $k$ is expressed as $\left|h_{k, j}\right|^{2}=\frac{\varepsilon\left|H_{k, j}\right|^{2}}{P L}$. The users are uniformly distributed around the BS with the presence of path loss (PL), log-normal shadowing $\varepsilon$ and frequency selective fading expressed as the channel gain $\left|H_{k, j}\right|^{2}$. $N_{o}$ denotes the additive white Gaussian noise (AWGN) signal.

Therefore, assuming $\left|h_{k, 1}\right|^{2}>\left|h_{k, 2}\right|^{2}>\ldots\left|h_{k, J}\right|^{2}$, the achievable rate per subcarrier $k$ for a NOMA uplink system with $J$ users is presented as:

$$
\begin{equation*}
R_{k}=B \sum_{j=1}^{J} \log _{2}\left(1+\frac{p_{k, j}\left|h_{k, j}\right|^{2}}{B N_{o}+\sum_{i=(j+1)}^{J} p_{k, i}\left|h_{k, i}\right|^{2}}\right) . \tag{2.3}
\end{equation*}
$$

The summation of $R_{k, j}$ across the subcarriers can be depicted as the total rate:

$$
\begin{equation*}
R_{t o t}=B \sum_{k=1}^{K} \sum_{j=1}^{J} \log _{2}\left(1+\frac{p_{k, j}\left|h_{k, j}\right|^{2}}{B N_{o}+\sum_{i=(j+1)}^{J} p_{k, i}\left|h_{k, i}\right|^{2}}\right) . \tag{2.4}
\end{equation*}
$$

We can see in equation (2.5) that the rate per subcarrier is the summation of Signal to Interference and Noise Ratio (SINR) per user. The importance of the decoding order is that it will considerably affect the individual rate of the users even when the total rate of the system remains the same [32, 37].

$$
\begin{equation*}
R_{k}=B \sum_{j=1}^{J} \log _{2}\left(1+\operatorname{SINR}_{k, j}\right) \tag{2.5}
\end{equation*}
$$

Based on equation (2.5), we can perform the analysis of the rate per subcarrier. For instance, let us consider an uplink scenario with three users as presented in Figure 2.3


Figure 2.3: SCMA uplink transmission

The rate for subcarrier $k$ can be calculated as:

$$
\begin{equation*}
R_{k}=B \log _{2}\left(S I N R_{k, H}\right)+\log _{2}\left(1+\text { SINR }_{k, M}\right)+\log _{2}\left(1+\text { SINR }_{k, L}\right) \tag{2.6}
\end{equation*}
$$

Equation (2.6 is synonymous to:

$$
\begin{gather*}
R_{k}=B\left(\log _{2}\left(1+\frac{p_{k, H}\left|h_{k, H}\right|^{2}}{w+p_{k, M}\left|h_{k, M}\right|^{2}+p_{k, L}\left|h_{k, L}\right|^{2}}\right)+\log _{2}\left(1+\frac{p_{k, M}\left|h_{k, M}\right|^{2}}{w+p_{k, L}\left|h_{k, L}\right|^{2}}\right)+\right.  \tag{2.7}\\
\left.\log _{2}\left(1+\frac{p_{k, L}\left|h_{k, L}\right|^{2}}{w}\right)\right)
\end{gather*}
$$

where the subindexes $H, M$ and $L$ describe the order of users with respect to their channel gain; high, medium and low. the noise signal is represented by $w$. Therefore, for subcarrier $k$, equation (2.7) can be represented in terms of Signal to Noise Ratio (SNR) as:

$$
\begin{equation*}
R_{k}=\left(\log _{2}\left(1+\frac{S N R_{H, k}}{1+S N R_{M, k}+S N R_{L, k}}\right)+\log _{2}\left(1+\frac{S N R_{M, k}}{1+S N R_{L, k}}\right)+\log _{2}\left(1+S N R_{L, k}\right)\right) \tag{2.8}
\end{equation*}
$$

There are two possible scenarios. First, with a decreasing channel gain order, the strong user's signal is detected and cancelled first, and then the weak user's signal can be detected free from interference. Second, the weak user's signal is detected and cancelled first, and then the strong user's signal can be detected free from interference. However, in order to demonstrate the contribution and effect of each user on the overall rate, both scenarios should be considered. Equation (2.8) depicts the former scenario and equation (2.9) the latter:

$$
\begin{equation*}
R_{k}=\left(\log _{2}\left(1+\frac{S N R_{L, k}}{1+S N R_{M, k}+S N R_{H, k}}\right)+\log _{2}\left(1+\frac{S N R_{M, k}}{1+S N R_{H, k}}\right)+\log _{2}\left(1+S N R_{H, k}\right)\right) \tag{2.9}
\end{equation*}
$$

Lemma: The decoding order does not affect the overall rate.

$$
\begin{equation*}
\sum_{k=1}^{K} R_{\underline{k_{H M L}}}=\sum_{k=1}^{K} R_{k_{\underline{L M H}}} \tag{2.10}
\end{equation*}
$$

In order to prove this statement, we need to show the equivalence of both decoding orders. For this comparison, we assume that the users have perfect knowledge of the

Channel State Information (CSI) and perfect SIC is applied at the receiver. Therefore, we expect that when subtracted, the expanded terms for both variants cancel to zero. The comparison is shown as follows; we define the rate per subcarrier with decoding

$R_{\xrightarrow{k_{H M L}}}=\left(\log _{2}\left(1+\frac{S N R_{H, k}}{1+S N R_{M, k}+S N R_{L, k}}\right)+\log _{2}\left(1+\frac{S N R_{M, k}}{1+S N R_{L, k}}\right)+\log _{2}\left(1+S N R_{L, k}\right)\right)$
and rate per subcarrier with decoding order from low to high $(\underline{L M H})$ :
$R_{\underline{k_{L M M}}}\left(\log _{2}\left(1+\frac{S N R_{L, k}}{1+S N R_{M, k}+S N R_{H, k}}\right)+\log _{2}\left(1+\frac{S N R_{M, k}}{1+S N R_{H, k}}\right)+\log _{2}\left(1+S N R_{H, k}\right)\right)$.

Then, we show the difference is equal to zero in Proof 1 in Appendix B, based on the assumption of perfect SIC and the complete removal of the interference in both scenarios. From the above, we prove that the achievable rate of the system is not affected by the way the interference is treated in the detection process.

### 2.4 Coded Non-orthogonal Multiple Access

Coded NOMA and power domain NOMA are similar in allowing multiple users to use the resources in a non-orthogonal manner. However, in contrast to power domain NOMA, coded NOMA uses coding techniques to implement the multiplexing of signals. Similar to LDS-CDMA, where users are multiplexed in the same resources such as time and spectrum, users' separation of their own signals is done by assigning codes, known in the literature as spreading signatures, that are unique for each user [46]. The term low density signature relates to the action of reducing the weight of the codes. However, as previously mentioned, in systems where users share the resources, degrading effects such as multiple access interference (MAI) and inter-symbol interference (ISI) are present and lead to a degradation in performance [47].

Mitigating these undesirable effects is a heavily researched area. Some of the techniques used have included the optimisation of spreading, coding and detection, which
has led to the creation of new techniques such as coded multiple non-orthogonal access. This exploits the power difference between the users' signals in the multiplexing, the multilayer coding and the redundancy of the data transmission while aiding the process of the detection techniques by applying MPA.

Furthermore, MPA plays a crucial role in assisting SIC in the process of recovering the transmitted users' signals because once the superposed signal has been received at the base station, the signals are detected and decoded into symbols. The decoding is performed at the base station, with previous knowledge of the codebooks and the channel mapping, based on MPA. Once the interference from other users is removed by SIC, the resultant signal is processed and compared against the codewords in the codebooks to find the set of codewords with the maximum likelihood of being transmitted. After the comparison with the known codewords, the receiver selects the binary values corresponding to these codewords as the decoded signal. The accuracy of the detection process in SCMA is mainly based on the performance and efficiency of the MPA decoder [48-50].

With the intention of solving the issues previously discussed and including more features such as multidimensional coding and spatial diversity, various techniques have been developed, for instance, access schemes such as sparse code multiple access (SCMA), developed by Huawei in 2013 [51] and later pattern division multiple access (PDMA) based on the same principle [21].

SCMA is a technique that uses multidimensional codebooks and a non-orthogonal spreading approach to assign a group of users to a particular set of resources. QAM symbol mapping and spreading are combined together in SCMA in order to map bits into multidimensional codewords of codebook sets [20]. The multidimensional constellation allows shaping gain that provides enhancements in comparison to the QAM symbols in LDS. As in LDS, SCMA maintains an acceptable level of complexity in the reception techniques due to the sparsity present in SCMA codewords [52]. In PDMA, the element that is added is a pattern, which is introduced to differentiate signals of users sharing the same resources. The design of these patterns is made based on disparate diversity order and sparsity. Therefore, PDMA can take advantage of the joint design of transmitter and receiver to enhance the system performance while maintaining detection complexity at a reasonable level [53].

### 2.5 Chapter summary

In this chapter, power based NOMA has been described and in parallel the background of coded NOMA and its particularities. These techniques have been the focus of extensive research in academia and the telecommunications industry as a main key to improving the user experience while enhancing the performance of future mobile networks. This chapter explains the main characteristics of SIC and the decoding order for NOMA in the uplink. A novel proof of the independence of the decoding order on uplink sum rate is presented and developed in detail in Appendix B. We present SIC combined with MPA as a detection technique. In addition, special emphasis has been placed on the uplink transmission because this work is implemented in the uplink coded NOMA. We will find the description of the SCMA system in the next chapter and a description of PDMA in Chapter 5 . These are examples of coded NOMA.

## Chapter 3

## Sparse Code Multiple Access

SCMA is a novel non-orthogonal multiple access scheme. The technique is based on the CDMA principle, where Quadrature Amplitude Modulation (QAM) symbols are spread over OFDMA tones, and where a CDMA encoder spreads these into a predesigned sequence of complex symbols.

A particular case of CDMA and the predecessor of SCMA is Low Density Signature (LDS), where the symbols are spread in lower density sequences of bits. The sequences are known as spread signatures [52]. In contrast with LDS, in SCMA the processes of mapping and spreading QAM symbols are combined together, resulting in a direct mapping of bits into sparse codewords [54]. The codewords come from these multidimensional complex domain pre-designed codebooks [48]. Each codebook of a different user represents a layer; therefore, as there are many users many multidimensional layers will exist. The SCMA system benefits from coding gain, shaping gain and code sparsity due to the multidimensional modulation of the multiple users' signals. All these characteristics provide advantages to the receiver because the application of the multiple detection techniques enables the decoding of multiplexed codewords with low to moderate complexity [55]. The SCMA system includes a number of basic elements to set up the system, shown in Figure 3.1. In the transmission process, we can see that the data is mapped into SCMA codewords and multiplexed into a signal before it is sent through the air and affected by the channel impairments. Then, it is received at the base receptor, where MPA is applied in order to decode each user signal and recover the sent data.


Figure 3.1: Diagram of SCMA system components

### 3.1 SCMA codebook design

The codebook is an important part of the SCMA system because the users are multiplexed over the same resource. Although this work does not focus on the design of the codebook, it is crucial to describe the implementation of it, as this will allow us to describe the system and achieve an understanding of the encoding and transmission process [56].

Several papers have proposed codebook designs based on advanced mathematical methods such as: spherical, Star-QAM based constellations, constellation rotation and interleaving [57-59]. However, for the purposes of this research, the SCMA codebook is presented according to [60], which is a clear and practical example of codebook design. The codebook construction is needed because it plays a crucial role in the encoding process of a bitstream's users.

An SCMA encoder can be represented by a function $f: \mathbb{B}^{\log _{2}(M)} \mapsto X$ which maps a bitstream of length $\log _{2}(M)$ to a codebook vector or codeword $x$ of a codebook $\boldsymbol{X}$. A $K$-dimensional complex codeword, described as $\boldsymbol{x} \in \mathbb{C}^{K}$, is built as a sparse vector with $(K-N)$ zero-valued entries and the bitstream mapped to it. All the codebooks contain zeros in the same $(K-N)$ dimensions within each codebook and $(K-N)$ in all zero rows. We can assume that the codebooks have $M$ codewords, consisting of $K$ complex values when $N$ are non-zero values, where $N$ is the sparsity value.

The encoding process, where the bits are mapped to a codebook, can be split into a function, $\boldsymbol{c}=\zeta(b)$ mapping the bitstream onto a complex valued constellation point
$\boldsymbol{c} \in \mathbb{C}^{N}$ and a $K$ by $N$ dimensional matrix, $\boldsymbol{V} \in \mathbb{C}^{K \times N}$, mapping the $N$-dimensional complex constellation point onto a $K$-dimensional codeword $\boldsymbol{x}$. The constellation mapping function is defined as $\zeta: \mathbb{B}^{\log _{2}(M)} \mapsto \mathcal{C}^{N}$. The function representing the encoder can therefore be rewritten as $f:=\boldsymbol{V} \zeta[61,62]$.

We present the creation of the codebook as part of the encoding process, which is summarised as follows:

- The constellation rotation and interleaving design are obtained from the construction of a Mother Constellation (MC) based on Gray Mapping coding vectors.
- This structure is then rotated to create a multidimensional base constellation.
- Once the whole structure is obtained for one user, the base constellation is rotated again to obtain the set of codebooks for more users.

This process is presented as follows:
Phase I - Create the Mother Constellation.

1. Let $S_{1}$ be a subset of lattice $\mathbb{Z}^{2}$ defined as:

$$
\begin{equation*}
S_{1}=\left\{Q_{m}(1+i) \mid Q_{m}=2 m-1-M, m=1 \ldots M\right\} \tag{3.1}
\end{equation*}
$$

where $\mathbb{Z}$ is the set of integers.
2. Set the vector $S_{1}$ from equation (3.1) based on Gray mapping, considering $M=4$. Then, the $S_{1}$ vector, which is the starting point to build the $M C$, is represented as:

| $S_{11}$ | $S_{12}$ | $S_{13}$ | $S_{14}$ |
| :---: | :---: | :---: | :---: |
| $-3(1+\mathrm{i})$ | $-(1+\mathrm{i})$ | $(1+\mathrm{i})$ | $3(1+\mathrm{i})$ |
| 10 | 11 | 01 | 00 |

3. From the previous step, $S_{N}$ is derived and described as $S_{N}=S_{1} U_{N}$, where $U_{N}$ is a multidimensional phase rotation matrix.

$$
\begin{equation*}
U_{N}=\operatorname{diag}\left(1 e^{1 \theta_{l-1}}\right) \subset \mathbb{C}^{N \times M} \tag{3.2}
\end{equation*}
$$

4. Then, we present $\theta_{l-1}$, which is the factor that rotates the vector $S_{1}$ to create the
multi-dimension in the MC. In Figure 3.2, the MC is illustrated, using the base vector $S_{1}$ and the $N$ dimensions after rotating the vector, and it is defined as:

$$
\begin{equation*}
\theta_{l-1}=(l-1) \times \frac{\pi}{M N}, l=1, \ldots, N \tag{3.3}
\end{equation*}
$$



Figure 3.2: Rotation of vector $S_{1}$ to obtain the Mother Constellation [60]
5. After rotating the $S_{N}$ vector, the $N$-dimensional matrix MC with $M$ points can be expressed as:

$$
M C=\left(S_{1}, \ldots S_{N}\right)^{T}=\left[\begin{array}{cccc}
S_{11} & S_{12} & \ldots & S_{1 M} \\
\vdots & \vdots & \ldots & \vdots \\
\vdots & \vdots & \ldots & \vdots \\
S_{N 1} & S_{N 2} & \ldots & S_{N M}
\end{array}\right]
$$

6. The interleaving and reordering of the elements in the vector is needed to improve the performance of the codewords in the presence of fading. It is done as follows:

6a) Select the even dimensions (rows) in the MC and reorder them. For example, the $S_{e}$ vector after interleaving is $S_{e}^{\prime}$, where $e$ is an even element of $N$ and it can be written as:

$$
\begin{align*}
S_{e}^{\prime}=\left\{-S_{e, M / 2+1}, \ldots,-S_{e, 3 M / 4}, S_{e, 3 M / 4+1}, \ldots,\right. & -S_{e, M}, S_{e, M}, \ldots, \\
& \left.-S_{e, 3 M / 4}, S_{e, 3 M / 4}, \ldots, S_{e, M / 2+1}\right\} \tag{3.4}
\end{align*}
$$

6b) After this step, the MC is represented as:

$$
\begin{equation*}
M C=\left(S_{1}, \ldots, S_{e}^{\prime}, \ldots, S_{N}\right)^{T} \tag{3.5}
\end{equation*}
$$

7. For simplification and to calculate the MC, let us set $N=2$ and $M=4$. From Step 1 and 2 we have the elements of the $S_{1}$ vector as follows:
$S_{11}=-3(1+i), S_{12}=-(1+i), S_{13}=(1+i), S_{14}=3(1+i)$
8. We calculate the angle of rotation phases in the MC from equation (3.3) as follows:

For $l=1$ and for $l=2$, we have:
$\theta_{1-1}=\theta_{0}=\frac{(1-1) \pi}{(4)(2)}=0$ and $\theta_{1-2}=\theta_{1}=\frac{(2-1) \pi}{(4)(2)}=\frac{\pi}{8}$.
The angles $\theta_{0}=0$ and $\theta_{1}=\pi / 8$ ensure that the minimum square Euclidean distance has been maximized for the MC, as proved in the design presented in [60].
9. The angles rotate the initial vector and determine the dimensions of the MC. We obtain $S_{N}=U_{N} S_{1}$ from equation 3.2), then we have $U_{N}=\operatorname{diag}\left(1 e^{i \theta_{l-1}}\right)$.

9a) For $\theta_{0}=0$, we have

$$
S_{1}=\left[\begin{array}{cccc}
1+0 i & 0 & 0 & 0 \\
0 & 1+0 i & 0 & 0 \\
0 & 0 & 1+0 i & 0 \\
0 & 0 & 0 & 1+0 i
\end{array}\right]\left[\begin{array}{c}
-3-3 i \\
-1-1 i \\
1+1 i \\
3+3 i
\end{array}\right]=\left[\begin{array}{c}
-3-3 i \\
-1-1 i \\
1+1 i \\
3+3 i
\end{array}\right]
$$

as a result:

$$
\begin{aligned}
& S_{11}=-3-3 i \\
& S_{12}=-1-1 i \\
& S_{13}=1+1 i \\
& S_{14}=3+3 i
\end{aligned}
$$

9b) For $\theta_{1}=\frac{\pi}{8}$, we have

$$
S_{2}=\left[\begin{array}{cccc}
e^{\frac{\pi}{8}} & 0 & 0 & 0 \\
0 & e^{i \frac{\pi}{8}} & 0 & 0 \\
0 & 0 & e^{i \frac{\pi}{8}} & 0 \\
0 & 0 & 0 & e^{i \frac{\pi}{8}}
\end{array}\right]\left[\begin{array}{c}
-3-3 i \\
-1-1 i \\
1+1 i \\
3+3 i
\end{array}\right]=\left[\begin{array}{c}
-1.62-3.92 i \\
-0.54-1.31 i \\
0.54+1.31 i \\
1.62+3.92 i
\end{array}\right]
$$

as a result

$$
\begin{aligned}
& S_{21}=-1.62-3.92 i \\
& S_{22}=-0.54-1.31 i \\
& S_{23}=0.54+1.31 i \\
& S_{24}=1.62+3.92 i
\end{aligned}
$$

10. After we have obtained the $S$ vectors, we finish the interleaving process for the even numbered $S$ vectors. $S_{2}$, represented by $S_{e}$, will be modified as shown in equation (3.4) before the interleaving.

$$
\begin{aligned}
& S_{e 1}=S_{21}=-3(1+i) e^{i \frac{\pi}{8}}, \\
& S_{e 2}=S_{22}=-(1+i) e^{i \frac{\pi}{8}}, \\
& S_{e 3}=S_{23}=\quad(1+i) e^{i \frac{\pi}{8}}, \\
& S_{e 4}=S_{24}=3(1+i) e^{i \frac{\pi}{8}}
\end{aligned}
$$

As in this example, $e=2$, we have $S_{e}^{\prime}=S_{2}^{\prime}$ therefore after the interleaving, we have:

$$
\begin{aligned}
& S_{21}^{\prime}=S_{e 1}^{\prime}=S_{22}=-(1+i) e^{i \frac{\pi}{8}} \\
& S_{22}^{\prime}=S_{e 2}^{\prime}=S_{24}=3(1+i) e^{i \frac{\pi}{8}} \\
& S_{23}^{\prime}=S_{e 3}^{\prime}=S_{21}=-3(1+i) e^{i \frac{\pi}{8}} \\
& S_{24}^{\prime}=S_{e 4}^{\prime}=S_{23}=(1+i) e^{i \frac{\pi}{8}}
\end{aligned}
$$

11. Then, $M C=\left(S_{1}, S_{2}^{\prime}\right)^{T}$, where the final MC is represented as:

00

$$
\boldsymbol{M C}={ }_{S 1}^{S 1}\left[\begin{array}{cccc}
-3.00-3.00 i & -1.00-1.00 i & 1.00+1.00 i & 3.00+3.00 i \\
-0.54-1.31 i & 1.62+3.92 i & -1.62-3.92 i & 0.54-1.31 i
\end{array}\right]
$$

Phase II - Populate the complex Mother Constellation onto the sparse codewords. A codebook $X_{j}$ for a single user is generated by concatenating the codewords corresponding to different symbols transmitted by the user $j$. We represent each symbol in the codebook with a column vector containing $K$ rows. Two of the rows are populated with complex numbers and two contain the value 0 . Therefore, the SCMA codebook, known as $X_{j}$ for $j$-th user, is created as follows [61]:

$$
\begin{equation*}
X_{j}=V_{j} \Delta_{j} M C, j=1,2, \ldots, J \tag{3.6}
\end{equation*}
$$

1. The dispersion matrix for each user sparsely populated is $V_{j}$, where $V \in \mathbb{B}^{K \times N}$, mapping the $N$-dimensional complex constellation point onto a $K$-dimensional codeword $\boldsymbol{x}$.

For this case, when $K=4, N=2$ and $J=6$, the matrices $V_{j}, j=1 \ldots J$ are:

$$
\begin{array}{ll}
V_{1}=\left[\begin{array}{ll}
1 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{array}\right] & V_{2}=\left[\begin{array}{ll}
0 & 0 \\
1 & 0 \\
0 & 1 \\
0 & 0
\end{array}\right] \\
V_{3}=\left[\begin{array}{ll}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{array}\right] & V_{4}=\left[\begin{array}{ll}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{array}\right] \\
V_{5}=\left[\begin{array}{ll}
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1
\end{array}\right] & V_{6}=\left[\begin{array}{ll}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1
\end{array}\right]
\end{array}
$$

2. The codebook construction for different users requires that the Mother Constellation is rotated. The angles phase rotation are expressed:

$$
\begin{equation*}
\phi_{u}=(u-1) \frac{2 \pi}{M d f}+e_{u} \frac{2 \pi}{M}, \forall u=1, \ldots, d f \tag{3.7}
\end{equation*}
$$

where $e_{u}$ represents an integer number of $\mathbb{Z}$ and $d f$ is known as the overloading factor, or the number of elements present in the same subcarrier. $\phi_{u}$ are the minimum optimal rotation phase angles that keep the distance between the dimension per user over the $d f$ interfering layers. The calculated values from equation (3.7) when $M=4$ and $d f=3$ are: $\phi_{1}=0, \phi_{2}=\frac{\pi}{6}$ and $\phi_{3}=\frac{\pi}{3}$.

These are optimal values based on Latin squares that maintain the consistent Euclidean distance between codewords and the structure of the mother codebook [52] [60]. A factor graph of rotation angles that follows this principle is:

$$
F_{\phi}=\left[\begin{array}{cccccc}
\phi_{1} & 0 & \phi_{3} & 0 & \phi_{2} & 0  \tag{3.8}\\
\phi_{3} & 0 & 0 & \phi_{1} & 0 & \phi_{2} \\
0 & \phi_{3} & \phi_{2} & 0 & 0 & \phi_{1} \\
0 & \phi_{2} & 0 & \phi_{3} & \phi_{1} & 0
\end{array}\right]
$$

Depending on the codebook structure, there are multiple options to arrange the phase rotation as soon as it keeps the minimum dimensional distance between the layers. An approach can be based on the correlation between different codebooks from different users based on Latin squares mentioned in [43] or by creating a covariance matrix described in [62]. The functionality and structure of the connection between users and subcarriers will be described in further detail in Section 3.4.
3. The angles are condensed in a matrix $f_{\phi_{j}}$ as:

$$
f_{\phi_{j}}=\left[\begin{array}{llllll}
\phi_{1} & \phi_{3} & \phi_{3} & \phi_{1} & \phi_{2} & \phi_{2}  \tag{3.9}\\
\phi_{3} & \phi_{2} & \phi_{2} & \phi_{3} & \phi_{1} & \phi_{1}
\end{array}\right]
$$

4. The rotation operator $\Delta_{j}$ is obtained from the condensed angles from $f_{\phi_{j}}$ as follows:

$$
\begin{equation*}
\Delta_{j}=\operatorname{diag}\left(f_{\phi_{j}}\right) \forall j=1, \ldots, J \tag{3.10}
\end{equation*}
$$

For this case, when $J=6$, we have:

$$
\Delta_{1}=\left[\begin{array}{cc}
\phi_{1} & 0 \\
0 & \phi_{3}
\end{array}\right] \quad \Delta_{2}=\left[\begin{array}{cc}
\phi_{3} & 0 \\
0 & \phi_{2}
\end{array}\right]
$$

$$
\begin{array}{ll}
\Delta_{3}=\left[\begin{array}{cc}
\phi_{3} & 0 \\
0 & \phi_{2}
\end{array}\right] & \Delta_{4}=\left[\begin{array}{cc}
\phi_{1} & 0 \\
0 & \phi_{3}
\end{array}\right] \\
\Delta_{5}=\left[\begin{array}{cc}
\phi_{2} & 0 \\
0 & \phi_{1}
\end{array}\right] & \Delta_{6}=\left[\begin{array}{cc}
\phi_{2} & 0 \\
0 & \phi_{1}
\end{array}\right]
\end{array}
$$

Therefore, the SCMA codebook, known as $X_{j}$ for $j$-th user is expressed as:

$$
\begin{equation*}
X_{j}=V_{j} \Delta_{j} M C, j=1,2, \ldots, J \tag{3.11}
\end{equation*}
$$

For example, for $j=1$ :

$$
\begin{gathered}
X_{1}=\left[\begin{array}{ll}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{array}\right]\left[\begin{array}{cc}
\phi_{1} & 0 \\
0 & \phi_{3}
\end{array}\right]\left[\begin{array}{cccc}
S_{11} & S_{12} & S_{13} & S_{14} \\
S_{21}^{\prime} & S_{22}^{\prime} & S_{23}^{\prime} & S_{24}^{\prime}
\end{array}\right] \\
=\left[\begin{array}{ll}
1 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{cc}
e^{0} & 0 \\
0 & e^{\frac{\pi}{6}}
\end{array}\right]\left[\begin{array}{cccc}
-3.00-3.00 i & -1.00-1.00 i & 1.00+1.00 i & 3.00+3.00 i \\
-0.54-1.31 j & 1.62+3.92 i & -1.62-3.92 i & 0.54-1.31 i
\end{array}\right]
\end{gathered}
$$

then, after normalising the values we have $X_{1}$

$$
X_{1}=\begin{gathered}
00 \\
S C 1 \\
S C 2 \\
S C 3 \\
S C 4
\end{gathered}\left[\begin{array}{cccc}
-0.5741-0.5741 i & -0.2807-0.2807 i & 0.2807+0.2807 i & 0.6461+0.6461 i \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
0.0358-0.2688 i & -0.1563+1.1801 i & 0.1563-1.1801 i & 0.2418-0.1862 i \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000
\end{array}\right]
$$

For $j=2 \ldots J$, the codebooks are:

$$
\begin{gathered}
\\
S C 1 \\
X_{2}=\begin{array}{c}
00
\end{array}\left[\begin{array}{cccc}
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
S C 3 \\
S C 4
\end{array}\left[\begin{array}{c}
0.2101-0.7842 i
\end{array}\right]-0.1027-0.3834 i\right. \\
0.1654-0.2148 i \\
0.0000+0.0000 i
\end{gathered}
$$

$$
X_{3}=\begin{gathered}
00 \\
S C 1 \\
S C 2 \\
S C 3 \\
S C 4
\end{gathered}\left[\begin{array}{cccc}
01 & 10 & 11 \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
-0.2101-0.7842 i & -0.1027-0.3834 i & 0.1027+0.3834 i & 0.2365+0.8827 i \\
0.1654-0.2148 i & -0.7255+0.9438 i & 0.7255-0.9438 i & 0.3025-0.0404 i
\end{array}\right]
$$

$X_{4}=$| 00 |
| :---: |
| $S C 1$ |
| $S C 2$ |
| $S C 3$ |
| $S C 4$ |\(\left[\begin{array}{cccc}-0.5741-0.5741 i \& -0.2807-0.2807 i \& 0.2807+0.2807 i \& 0.6461+0.6461 i <br>

0.0358-0.2688 i \& -0.1563+1.1801 i \& 0.1563-1.1801 i \& 0.2418-0.1862 i <br>
0.0000+0.0000 i \& 0.0000+0.0000 i \& 0.0000+0.0000 i \& 0.0000+0.0000 i <br>
0.0000+0.0000 i \& 0.0000+0.0000 i \& 0.0000+0.0000 i \& 0.0000+0.0000 i\end{array}\right]\)

00
01
10
11

$$
X_{5}=\begin{gathered}
S C 1 \\
S C 2 \\
S C 3 \\
S C 4
\end{gathered}\left[\begin{array}{cccc}
0.2020-0.7538 i & 0.1131-0.4222 i & -0.1131+0.4222 i & -0.3862+1.4413 i \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
-0.0993-0.2410 i & 0.5007+1.2116 i & -0.5007-1.2116 i & 0.1899-0.4607 i
\end{array}\right]
$$

00

$$
X_{6}=\begin{gathered}
S C 1 \\
S C 2 \\
S C 3 \\
S C 4
\end{gathered}\left[\begin{array}{cccc}
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
0.2020-0.7538 i & 0.1131-0.4222 i & -0.1131+0.4222 i & -0.3862+1.4413 i \\
0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i & 0.0000+0.0000 i \\
-0.0993-0.2410 i & 0.5007+1.2116 i & -0.5007-1.2116 i & 0.1899-0.4607 i
\end{array}\right]
$$

01
10
11

### 3.2 SCMA transmission

We consider an SCMA uplink multiuser system, serving a group of $j \in(1, \ldots, J)$ users that are sharing $k \in(1, \ldots, K)$ subcarriers. Figure 3.3 shows the principle of SCMA users transmitting to the base station (BS). The BS will provide the users with information regarding codebook assignment as well as resource allocation, after the codebooks are generated, this process is visited in further detail in Section 3.1. This process is based on constellation rotation and the interleaving method. In particular, the first dimension is constructed using a Mother Constellation with Gray mapping vectors and interleaving. The other dimensions are then obtained by rotating the first dimension via a rotation operator and multiplying the resulting constellation by each unique user's sparse matrix [60]. Finally, a codeword (codebook pattern) will be selected from the user's codebook to represent a sequence of bits for each user.


Figure 3.3: SCMA system transmission representation

The codewords are multiplexed with the other users' codewords over the $k$ shared orthogonal resources, resulting in a $k$-th vector. The compound vector $y$ is the mathematical representation of the user's signal received at the base station, represented graphically in Figure 3.4 .


Figure 3.4: Graphic representation of the codebooks and multiplexed signals

The users will be located at different distances from the base station, and with the effect of path loss and fading, each of them will experience different channel gain. It is assumed that the BS and the users have perfect knowledge of the CSI. The channel gain for the $j$-th user at subcarrier $k$ is expressed as $\left|h_{k, j}\right|^{2}=\frac{\varepsilon\left|H_{k, j}\right|^{2}}{P L}$. The users are uniformly distributed around the BS with the presence of path loss (PL), log-normal shadowing $\varepsilon$ and a frequency selective fading gain of $\left|H_{k, j}\right|^{2}$ [25, 27]. The gain can be represented by the matrix $h$ such that:

$$
h=\left[\begin{array}{cccc}
h_{1,1} & h_{1,2} & \ldots & h_{k, 6}  \tag{3.12}\\
\vdots & \vdots & \ldots & \vdots \\
\vdots & \vdots & \ldots & \vdots \\
h_{4, j} & h_{4,2} & \ldots & h_{K, J}
\end{array}\right] \text { for } j=1 \ldots J \text { and } k=1 \ldots K
$$

Each of the $k$ elements of the noise vector $n_{k}$ satisfy the characteristic $n_{k} \sim \mathcal{C N}\left(0, \sigma^{2}\right)$, which means they follow a complex valued normal random variable with mean $\mu_{o}=0$ and variance $\sigma^{2}$. We can express the transmitted signal on the uplink channel, as received at the base station, as follows:

$$
\begin{equation*}
y_{k}=\sum_{j=1}^{J} h_{k, j} x_{k, j}+n_{k}, \forall k=1 \ldots K \tag{3.13}
\end{equation*}
$$

The parameter that will determine the sparsity in the codewords, expressed as $x_{k, j}$, is the number of subcarriers which are allocated to each user's codewords, better known as the spreading factor $d v$ over the total number of subcarriers. The number that represents the amount of users assigned to a subcarrier is known as the overloading factor $d f$. The overloading with respect to the number of symbols is determined by the ratio of the number of users sharing the same subcarrier $(d f=3)$ to the number of subcarriers per user $(d v=2)$, resulting in $\frac{d f}{d v}=1.5$. The non-zero $N$ elements on each codeword need to be fewer in number than the number of subcarriers in order to call the system sparse. These values will define the complexity and the characteristics of the codebook's design. The size of the codebook will be maximised when $N=K / 2[63]$. As an example, we present a numerical exercise to obtain the resultant signal $y$ in Appendix A

### 3.3 SCMA receiver and signal detection

After the superposed signal has been transmitted to the base station, the signal must be detected and decoded into symbols. This action is performed by the base station with previous knowledge of the codebooks for each user and channel mappings. The received signal is detected based on MPA, as shown in Figure 3.1 (Section 3). Once the interference from other users is removed, the resultant signal is processed and compared against the codewords in the codebooks to find the set of codewords with the highest likelihood of being transmitted. After the comparison with the known codewords, the receiver selects the binary values corresponding to these codewords as the decoded signal. The accuracy of the detection process in SCMA is affected by a variety of factors, including codebook design, channel performance and interference [48-50].

One effective way of performing the detection of the symbols sent by each user is Message Passing Algorithm (MPA) also known as belief propagation (BP) algorithm [64]. MPA operates by passing the message through the nodes along the edges in a factor graph until a maximum number of iterations is reached and uses the sum-product in each iteration to calculate joint probabilities of different codewords based on Bayesian principles. In order to obtain the probability calculation of the transmitted symbols, the
numerical values of the received vector and the probability of the codewords within the user's codebooks are required [64].

For practical purposes and to illustrate the detection process, we consider one of the techniques that implements MPA at the receiver based on [55,65]. In this case, each user presents their own codebook containing four codewords to choose from, assuming that all users will transmit a codeword within a data frame. As we have no knowledge of the data that is to be transmitted and assume that each codeword is selected by a user with equal likelihood (note that due to their nature, efficient compression, encryption and source coding/compression will produce data streams which use all symbols with roughly equal probability) we can set the initial probability of a user choosing a given codeword to be 1 divided by the cardinality of the set of codewords in a user's codebook, as shown in equation (3.14). Thus, all codewords will have an initial probability of $\frac{1}{4}$ as each user has four of them,

$$
\begin{equation*}
\operatorname{Pr}_{V_{k} F_{n}}^{(0)}\left(\hat{x}_{n}^{k}\right)=\frac{1}{\left|X_{n}^{k}\right|}, \operatorname{Pr}_{V_{i} F_{n}}^{(0)}\left(\hat{x}_{n}^{i}\right)=\frac{1}{\left|X_{n}^{i}\right|}, \operatorname{Pr}_{V_{j} F_{n}}^{(0)}\left(\hat{x}_{n}^{j}\right)=\frac{1}{\left|X_{n}^{j}\right|} \tag{3.14}
\end{equation*}
$$

where $\operatorname{Pr}_{V_{k} F_{n}}^{(0)}$ is the initial probability of a node connected to a function. $\hat{x}_{n}^{k}, \hat{x}_{n}^{i}$ and $\hat{x}_{n}^{j}$ are the estimated transmitted symbols for each user $(k, i, j)$ present in the node $n$ and $X_{n}^{k}, X_{n}^{i}$ and $X_{n}^{j}$ are the corresponding codebooks of user $k, i$ and $j .|\cdot|$ denotes codebook cardinality.

Subsequently, a connected graph is formed, containing nodes corresponding to the users and subcarriers in order to propagate the probability calculation, as depicted in Figure 3.5. For this example, we have six variable nodes representing users and four functional nodes (sometimes also known as factor or check nodes in some literature) representing subcarriers. Edges between functional and variable nodes represent that a user is carried by a given subcarrier or that a subcarrier is carrying a given user. Probabilities calculated at each step of the process can be said to be sent between connected nodes along these edges. From the connected graph in Figure 3.5, we can see that each user uses two subcarriers to transmit its signal (so each variable node has lines exiting it to connect to two functional nodes) and each subcarrier is shared by three users, therefore, it has three lines entering it.


Figure 3.5: Graph showing the connection of variable nodes with function nodes.

The initial probabilities of each codeword will be the entry value to a function which represents a functional node. The function uses these values and the difference from the received signal vector to calculate the probability of the codeword in the received vector corresponding to the subcarrier.

The functional node then supplies each variable node with the probability that each of its codewords corresponded to the received signal vector when combined with codewords from the other two nodes connected to that functional node. These probabilities are then normalised in order to make them sum to one at the variable node and passed to the functional node, in place of the initial probabilities, for the next iteration of the process. A higher number of iterations can achieve a greater level of accuracy in the decoded signal. Figure 3.6 illustrates the iterative process of MPA [66].


Figure 3.6: Probability functions diagram

Equation (3.15) depicts the calculation of the probability of the function connected to a node $\operatorname{Pr}_{F_{n} V_{k}}^{(l)}\left(\hat{x}_{n}^{k}\right)$, while equation 3.16 shows the calculation of the probability of a node connected to a function $\operatorname{Pr}_{V_{k} F_{n}}\left(\hat{x}_{n}^{k}\right)$. It is at this calculation where the probability is normalised before it is propagated to the next iteration between nodes and functions.

$$
\begin{gather*}
\operatorname{Pr}_{F_{n} V_{k}}^{(l)}\left(\hat{x}_{n}^{k}\right)=\sum_{\hat{x}_{n}^{i}} \sum_{\hat{x}_{n}^{j}} e^{-d_{e}\left(h, \hat{x}_{n}^{k}, \hat{x}_{n}^{i}, \hat{x}_{n}^{j}\right)} \operatorname{Pr}_{V_{i} F_{n}}^{(l-1)}\left(\hat{x}_{n}^{i}\right) \operatorname{Pr}{V_{j} F_{n}}_{(l-1)}^{\left(\hat{x}_{n}^{j}\right)}  \tag{3.15}\\
\operatorname{Pr}_{V_{k} F_{n}}^{(l)}\left(\hat{x}_{n}^{k}\right)=\frac{\operatorname{Pr}_{F_{n} V_{k}}^{(l)}\left(\hat{x}_{n}^{k}\right)}{\sum_{\hat{x}_{n}^{k}} \operatorname{Pr}_{F_{n} V_{k}}^{(l)}\left(\hat{x}_{n}^{k}\right)} \tag{3.16}
\end{gather*}
$$

From both equations, $l$ is the number of the iteration to decode the symbols of the users that are sharing the same resource.

$$
\begin{equation*}
d_{e}\left(h, \hat{x}_{n}^{k}, \hat{x}_{n}^{i}, x_{n}^{j}\right)=\left|y_{n}-\left(h_{n}^{k} \hat{x}_{n}^{k}+h_{n}^{i} \hat{x}_{n}^{i}+h_{n}^{j} \hat{x}_{n}^{j}\right)\right|^{2} \tag{3.17}
\end{equation*}
$$

Equation 3.17) obtains the Euclidean distance between the transmitted signal and the symbols for each user ( $k, i$ and $j$ ) per function node, expressed as:

$$
\begin{equation*}
\hat{x}_{n}^{k}, \hat{x}_{n}^{i}, \hat{x}_{n}^{j} \tag{3.18}
\end{equation*}
$$

The marginal probability of each codeword is calculated for each user, after the previous step, by multiplying the two functional node outputs relating to that user. This produces a resultant marginal probability value for each of a user's codewords. Next, the codeword with the highest marginal probability is selected for each user, and the bitstreams corresponding to the selected codewords are the data output for that user [65].

Having the received compound signal $y=\left[y_{1}, y_{2}, \ldots, y_{n}\right]$, the knowledge of the channel $h$ and the value of the joint maximum a-posteriori probability (MAP) the detection will be expressed as:

$$
\begin{equation*}
\hat{x}=\arg _{x \in X_{1} * \ldots * X_{j}} \max \operatorname{Pr}(X / y) \tag{3.19}
\end{equation*}
$$

where $\hat{x}$ is the expected codeword from the codebook and $X$ is the known value of the codebook.

### 3.4 Factor graph design and sparsity

We have explained how users will be spread over subcarriers and how the subcarriers will be connected to users in a connected graph, as presented in Figure 3.5. This will be represented by a matrix $F=\left[f_{k, j}\right]$ with entries $(0,1)$ that shows the mapping between users and subcarriers. If $f_{k j}=1$, it indicates that a user is allocated to a particular subcarrier and otherwise if it is 0 . The number of users contributing to each subcarrier will be defined by $d_{f k}=\left(d_{f 1}, \ldots, d_{f K}\right)^{T}=\sum_{j=1}^{J} f_{j}$

$$
\boldsymbol{F}=\left[\begin{array}{llllll}
1 & 0 & 1 & 0 & 1 & 0  \tag{3.20}\\
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0
\end{array}\right]
$$

### 3.5 Sparsity of codewords

The parameter that will determine the sparsity of the codewords is the number of available subcarriers $K$ minus the number of effective subcarriers (allocated subcarriers) represented by $d v$, also known as the spreading factor over the subcarriers. The nonzero $N$ elements on each codeword need to be fewer in number than the number of total subcarriers in order to call the system sparse. These values will determine the complexity and characteristics of the codebook, given by the calculation $\binom{N}{K}$. The size of the codebook is maximised when $N=K / 2$.

Having this combinatorial number, we can now calculate the size of the codebooks, which will be greatly increased when the number of subcarriers increases, while maintaining sparsity and its advantages. For instance, for $K=4$ and $N=2$, the number of codebooks generated will be 6 , corresponding to the number of users. However, for $K=12$ and $N=6$, the number of codebooks generated will be 924 [63].

### 3.6 Numerical results

In this section, we present link-level simulation results from the SCMA and PDMA systems, in order to illustrate the functionality and importance of each element involved in the transmission and detection processes of the coded NOMA systems. We have simulated SCMA and PDMA systems in MATLAB (PDMA is described in further detail in Section 5 ). The simulations include uplink scenarios with a single cell and $J$ users, which are uniformly distributed from the base station with total transmission power $P_{t}$ and bandwidth divided into $K$ number of subcarriers. The codebook for the transmitted signal has been generated based on the process presented in Section 3.1. For the receiver in the system the initial probability of each symbol is $1 / 4$. The wireless channel is modelled as a Rayleigh fading channel with Additive White Gaussian Noise (AWGN) as described in Section 3.2. The simulation results have been carried out using the Monte Carlo method with the parameters in Table 3.1 as follows:

| Parameters | SCMA Values | PDMA Values |
| :---: | :---: | :---: |
| Number of users | 6 | 6 |
| Number of codebooks | 6 | 6 |
| Subcarriers | 4 | 4 |
| Sparsity | 2 | - |
| Overloading of symbols | 1.5 | 1.5 |
| Factor overloading | 3 | 2 |
| Factor spread | 2 | - |
| Factor spread for overlay users | - | 2 |
| Factor spread for underlay users | - | 1 |
| Pedestrian model | $B$ | B |
| PL exponent | 3 | 3 |
| Bandwidth | 180 kHz | 180 kHz |
| Noise | -174 dBm | -174 dBm |
| Shadowing | 8 dB | 8 dB |
| Transmit power | $0-23 \mathrm{dBm}$ | $0-23 \mathrm{dBm}$ |
| Initial probability | $1 / 4$ | $1 / 4$ |
| Monte Carlo iterations | 10000 | 10000 |

Table 3.1: Parameters of link performance simulation for SCMA and PDMA

As part of the simulation of the SCMA system, we include BER simulations of the overall system in Figure 3.7 and BER per user in Figure 3.8. This provides an illustrative example of the performance of the codebook design and the detection process relative to the transmission bit error rate.


Figure 3.7: BER performance of SCMA system in the uplink for the purpose of comparison with PDMA

It must be noted that although this work does not investigate the codebook design or effectiveness of the detection process, it is important to simulate the average BER of the system in order to provide a framework for comparison with further code domain multiple access techniques such as PDMA.


Figure 3.8: BER performance of SCMA systems in the uplink per user

The average BER results presented in Figure 3.9 aim to compare the performance of SCMA and PDMA systems. We simulate a system with a similar codebook structure to the one presented in the SCMA system. It has been proved that the sparsity in the codebook is directly connected to the factor graph in coded based systems. However, the PDMA factor graph has lower density than the one presented for SCMA. We can see from the blue line in Figure 3.9 that the performance can be considerably degraded with a less dense codebook. The decoding process of the transmitted symbols in PDMA is applied in the same manner as the decoding process applied for the users in SCMA presented in Section 3.3. This is possible due to the similarity of structure with the pattern presented in PDMA. The differences are in the sparse codes for the users $(j=3 \ldots J)$. This does not impact the implementation of the decoder, however, higher diversity will impact the complexity of the detection process.


Figure 3.9: Comparison of average BER in SCMA vs PDMA systems

Figure 3.10 presents the BER results per user in order to compare those users with different pattern allocation and the direct impact on the performance. The users with lower density patterns present higher BER than those with higher density patterns.


Figure 3.10: BER per user in PDMA system. The users are ordered from low BER to high BER. User 1 shows the lowest BER and user 6 shows the highest BER.

### 3.7 Chapter summary

As in the case of SCMA, coded NOMA in the uplink has been of interest to researchers as the technique is able to accommodate several users by overloading the subcarriers. In this chapter, the system of SCMA has been described in detail in order to understand and implement the elements that are part of the system. We have focused special attention on the implementation of the codebook design and a particular decoding technique. Due to the complexity of the codebooks and detection techniques and for practical purposes, implementation examples are supplied for specific cases. Numerical results are provided to support the description of the systems and show the link performance evaluation. Results are also provided for the PDMA system described later in Chapter 5 of this work. BER comparisons per system and per user are presented. This chapter established the main principles of SCMA, which provides grounds for the next issue to investigate in order to enhance the system, namely resource allocation.

## Chapter 4

## Resource allocation for Uplink SCMA System

Resource allocation plays a vital role in multiple access techniques. Non-orthogonal access has many advantages over its predecessors and as shown in previous work, accurate resource allocation algorithms are essential to ensure the inherent capabilities of multiple access. Through this chapter, we will demonstrate how in order to exploit the full potential of code domain NOMA an efficient and practical subcarrier and power allocation solution is needed.

We have mentioned before that the assignment of resources can be approached in two stages. Previous work on SCMA has been implemented in a similar manner. Work in [67] has placed special emphasis on subcarrier allocation using matching theory. On the other hand, in that research power allocation was overlooked by using a simple waterfilling method and not taking interference from other users into account. Work in [62] has solved the subcarrier allocation problem by using an algorithm where in each iteration one user is allocated at a time per resource until all the possible codebook patterns are exhausted. However in the same work, power allocation is solved with a numerical calculation using the convex tools in MATLAB. The problem of interference is widely common in multiple user systems and has been of interest to optimisation experts. In the work presented in [32], in the NOMA uplink system a resource allocation solution has been explored, using matching theory and the water filling principle but taking the interference into account. This work has been very useful even when the subcarrier allocation constraints are not included and no minimum
rate is considered. The authors of [29] presented a system combining PD-NOMA and CD-NOMA techniques to support an increased number of users. Using power domain NOMA, additional low data rate users were superimposed onto an SCMA system. D2D transmission and matching theory in the uplink have been explored in [68], which provided deeper knowledge of MT and its complexity.

### 4.1 SCMA capacity and resource allocation

In the previous chapter, we have described the components of the SCMA system and its characteristics. We have witnessed the importance of maintaining the pattern of the codebook.

From the Shannon's capacity equation and based on power NOMA uplink, we can derive the SCMA rate total equation. The binary variable $f_{k, j}$ indicates whether a user $j$ is allocated to a particular subcarrier $k$, and the mobile equipment has a limited total power $P_{j}^{\text {max }}$, which is split between $d v$ subcarriers of equal bandwidth $B$. The transmission power of user $j$ allocated to subcarrier $k$ is expressed as $p_{k, j}$. The rate per subcarrier $k$ can be depicted as [32]:

$$
\begin{equation*}
R_{k}=B \sum_{j=1}^{J} \log _{2}\left(1+\frac{f_{k, j} p_{k, j}\left|h_{k, j}\right|^{2}}{B N_{o}+I_{k, j}}\right) \tag{4.1}
\end{equation*}
$$

where $I_{k j}$ is the interference caused by the other users over the subcarrier $k$, expressed as:

$$
\begin{equation*}
I_{k, j}=\sum_{i=1, i \neq j}^{J} f_{k, i} p_{k, i}\left|h_{k, i}\right|^{2} \tag{4.2}
\end{equation*}
$$

In previous sections, the SCMA system has been described and we have presented the challenge of this technique. The objective of the system is to transmit the highest amount of information by the best management of the available resources such as the ones optimised in this work; power and subcarriers. In order to solve this problem, it needs to be split into two sub-problems, which need to be solved independently, thus, a suboptimal joint solution is presented.

### 4.2 Subcarrier allocation for SCMA

SCMA benefits from allocating multiple users to the same resources by increasing the overall transmission rate, however, the codebook constraints can limit the effect of this feature due to the number of users that can be assigned to a specific subcarrier and the restriction of duplicated codebook patterns. These constraints and the presence of multiple users will generate high interference between them. Although this negative effect is partly mitigated by the sparsity of the codebook, subcarrier allocation will be required to improve the performance of the transmission rate.

In this research, different subcarrier allocation methods were carried out in order to compare method complexity and the effect of each method on the total rate performance [43], [67], [69], [70]. In this section, a proposed low complexity algorithm is described and presented to solve the problem of subcarrier allocation.

### 4.2.1 Greedy algorithm based methods

There are several approaches to solve heuristic problems. The greedy algorithm is known for being able to find solutions by solving the problem in local stages, not considering the global problem or whole picture. Additionally, algorithms based on this principle are usually capable of achieving good results with moderate complexity requirements. Whilst the optimal global solution may not be identified, greedy algorithm approaches often obtain a near-optimal or acceptable solution under specific purposes, using minimal computational complexity [71,72]. For this reason, it is worth exploring different subcarrier allocation schemes using this principle as minimising resource use is an important factor in mobile equipment applications. For this particular problem, exhaustive search will be used to find the global optimal solution in order to produce an upper bound indicator for the proposed algorithms. Random or fixed subcarrier allocation will be presented to establish a lower bound indicator for this comparison.

### 4.2.1.1 Proposed subcarrier allocation algorithm

User-subcarrier mapping $f_{k, j}, \in[0,1]$, from equation (4.1), is the discrete variable in the capacity equation of SCMA that requires optimisation. In order to solve this subproblem, we have designed a subcarrier algorithm that is presented as follows:

1: Let $A$ be a set of possible codebook pattern assignments $a_{i} \in A\{0,1\}$.

2: All the possible $K$-rowed binary column vectors from the codebook [57,60], which contain $d v$ number of ones and ( $K-d v$ ) number of zeros, are defined as codebook pattern assignments.

3: $F$ is defined as a matrix with dimensions $(J \times K)$ representing user-subcarrier allocations. We initialise it as a zero-value matrix $F=0_{K, J}$ and complete each column by using the following algorithm.

First, a random pattern is selected from all the possible patterns for user 1 , and the overall rate is calculated, assuming no interference as all the other users are not yet allocated. The vector that achieves the highest rate is allocated and is deleted from $A$, $A^{\prime}=A \backslash a$ as shown in Figure 4.1 .


Figure 4.1: Graphic representation of first user's subcarrier allocation

We proceed to the adjacent user and the opposite pattern $\hat{a}$ is selected, for instance, as shown in Figure 4.2. This is deleted from the set of available codebook pattern assignments $A^{\prime \prime}=A^{\prime} \backslash \hat{a}$.

For the odd numbered user $j=(2 n+1)$, the overall rate is calculated in order to select the best pattern allocation. We allocate even numbered users $j=(2 n)$ the opposite pattern compared to the previous odd user, as represented in Figure4.2. Used assignments are deleted from the set of available codebook pattern assignments at each stage.

User 1 User 2 User 3


Figure 4.2: Graphic example of odd and even numbered users' subcarrier allocation

This process should continue until all the users are allocated a codebook pattern as described in Algorithm 1 .

```
Algorithm 1 Proposed subcarrier algorithm
    Inputs:
        \(K, J, d v, h\)
    Initialize:
        \(n=0\) and \(F=0_{K, J}\)
    \(A=\{\) all possible user-subcarrier assignment vectors \(\}\)
    Select \(d v\) subcarriers with highest channel gain from \(h\)
    Set the corresponding row values in column 1 of \(F\) to 1
    Set column 2 of \(F\) to the binary inverse of column 1.
    Delete the vectors matching columns 1 and \(2 F\) from \(A\)
    do
        Let \(n=n+1\) and \(j=2 n+1\)
        Calculate the rate obtained when each vector from \(A\) is used in position \(j\) of \(F\)
        Select vector \(a_{i} \in A\) which gives the best performance.
        Assign the selected member of \(A\) to column \(j\) of \(F\)
        Set the values of column \(j+1\) to the inverse of those in column \(j\)
        Delete vectors matching \(F_{j}\) and \(F_{j+1}\) from \(A\), where \(F_{j}\) is the \(j\)-th column vector
    of \(F\)
    while \(n<(J / 2)\)
    Obtain \(F^{*}\)
```


### 4.2.1.2 Low complexity proposed algorithm

Computational complexity is an element that should be considered in subcarrier allocation methods. For this reason, a low complexity algorithm based on the greedy principle is proposed in this work (Algorithm1. Section 4.2.1.1) with the aim of satisfying the limitation of the power and processing of the mobile equipment with respect to the base station. The proposed algorithm is able to achieve a close to optimal solution while reducing the operations and the number of iterations. Thereby saving considerable computational resources such as time and computing processing, which can facilitate the practical implementation. The exhaustive search is affected by the number of permutations, the number of users to be allocated and the calculation of the total rate in every iteration, which results in $O(J!)$. The complexity of the proposed subcarrier allocation algorithm in SCMA can be obtained based on the following calculations:

- The complexity is determined by the rate calculation operations and the number of elements in the greedy algorithm $(J)$. Using the opposite pattern allocation for even numbered users reduces the number of operations required in the greedy process to $(J-d f)$.
- The number of rate calculation operations is given by $(d f)$.
- Therefore, we can express the complexity as $O\left((J-d f)^{2}\right)$.


### 4.2.2 Matching Theory

Matching theory is a tool used to generate a dynamic relation among a set of players competing for their own individual benefits. It has been widely used in different disciplines from economics to sociology since its first appearance with the work of Gale and Shapley on the stable marriage and college admission problems in 1962 [73]. It has recently been applied in wireless communications theory to deal with the interaction between rational and selfish resources [74] [75] [76]. For instance, in [77], MT is applied in OFDMA systems to assign the users to the available resources. In [78], the authors aim to optimize the sub-channel assignment to achieve a balance between the number of scheduled users and total sum rate maximization. The work presented in [79] proposes a Matching Theory resource allocation solution for MIMO cognitive radio links. There are variants of matching theory such as: one to one, many-to-one and many-to-many. The most well known and used is the one to one approach, which is
often explained using the analogy of a marriage. For instance, there is a game with two sets of players, a set of men and a set of women. A man offers marriage to a woman and she may accept or decline. The decision is based on which of the men makes the best offer in order to obtain a partner. Therefore, the game continues with players making individual offers to each other until one set of players obtains the highest advantages from the other set. However, conventional game theory can only deal with simpler scenarios, which is not the case for SCMA. The basic scenario cannot tackle resource allocation for SCMA systems due to the nature of the codebook and codeword constraints [60,61]. Many-to-many game theory is required to deal with more than one user connected to multiple subcarriers as in SCMA. The approach is capable of solving problems when a group of players (users) are competing with another group of players for the available resources (subcarriers). In this work, Matching Theory for SCMA (MTSCMA) is described and presented based on the codebook specifications presented in [32,67,80] to compare the improvement in the performance of the overall transmission rate in SCMA systems. In order to implement MT and compare it with other methods, we need to describe the main characteristics of this approach.

Definition 1: Let $\mu$ be a function from a set of players defined as users $U_{j}=\{1,2 \ldots J\}$ and subcarriers $S C_{k}=\{1,2 \ldots . K\}$. These sets are independent and they will compete individually for their own requirements. If a subset of users are connected to a subset of subcarriers, it means there is an entry in $f_{k j}=1$; then this connection is denominated a matching pair. $d f$ represents the number of subcarriers assigned to each user and $d v$ represents the number of users who may share a subcarrier. The connection of these two sets is determined by the following statements:

1. $\left|\mu\left(U_{j}\right)\right| \leq d f, \forall j:\{j \in 1,2 \ldots J\}$ and $\mu\left(U_{j}\right)=0$ if $U_{j}$ is not matched to any $S C_{k}$
2. $\left|\mu\left(S C_{k}\right)\right| \leq d \nu, \forall k:\{k \in 1,2 \ldots K\}$ and $\mu\left(S C_{k}\right)=0$ if $S C_{k}$ is not matched to any $U_{j}$
3. $S C_{k} \in \mu\left(U_{j}\right)$ iff $U_{j} \in \mu\left(S C_{k}\right)$

Condition 1 establishes that each user is assigned to a subset of subcarriers, and condition 2 states that each subcarrier is connected to a subset of users. These conditions are linked to the codebook design and the parameters are referenced by $d v=2$ and $d f=3$.

Definition 1 describes the main difference from the traditional matching theory method, where the preference lists are pre-established and the game is played by a set of users
with just one offer at a time. In the many-to-many approach adapted to resource allocation for SCMA uplink, the swap operation, known as any possible exchange between any set of players and its offers, continues until the rate conditions are accomplished. The swap matching is defined as:

$$
\begin{align*}
& A_{z}(\mu) i_{i^{\prime} j^{\prime}}^{i j}=\left\{\mu \notin\left\{(i, \mu(i)),\left(i^{\prime}, \mu\left(i^{\prime}\right)\right)\right\}\right\} \cup\left\{\left(i,\left\{\{\mu(i) \notin\{j\}\} \cup\left\{j^{\prime}\right\}\right\}\right),\right. \\
&\left.\left(i^{\prime},\left\{\left\{\mu\left(i^{\prime}\right) \notin\left\{j^{\prime}\right\}\right\} \cup\{j\}\right\}\right)\right\} \tag{4.3}
\end{align*}
$$

where $j \in \mu(i)$ and $j^{\prime} \in \mu\left(i^{\prime}\right)$ and $j^{\prime} \notin \mu(i)$ and $j \notin \mu\left(i^{\prime}\right)$. This statement means that a pair of users can exchange one of their matched resources with each other while keeping the other subcarrier unchanged.

Definition 2: The process defines a swap - blocking pair when one of the users identifies another user sharing the same resource (Figure 4.3). $\left(U_{j}, U_{j}^{\prime}\right)$ is defined as a swap - blocking pair if the statement is accomplished.

1) $\forall z \in\left\{i, i^{\prime}, j, j^{\prime}\right\}, A_{z}(\mu)_{i^{\prime} j^{\prime}}^{i j} \geq A_{z}(\mu)$
2) $\exists z \in\left\{i, i^{\prime}, j, j^{\prime}\right\}, A_{z}(\mu)_{i^{\prime} j^{\prime}}^{i j}>A_{z}(\mu)$
where $A_{z}(\mu)$ is the value of the function before and after the exchange between swap blocking pairs, which is the decision factor in the subcarrier allocation in MT.

The swap - blocking pairs are allowed to carry out swap operations, meaning that two users sharing a subcarrier can potentially become swap pairs. In other words, two users connected by a common shared subcarrier can exchange their allocation of their other assigned subcarriers without breaking the allocation rules. This swap may or may not increase the total sum rate, though the resulting sum rate can be compared against the original sum rate, and the swap is kept if there is an improvement.


Figure 4.3: Swap blocking pairs

Definition 3: A function matching $A_{z}(\mu)$ is a two sided exchange if there are no other possible swap - blocking pairs, which means that all the users have played and there is no other possible arrangement $A_{z}(\mu)_{i^{\prime} j^{\prime}}^{i j} \geq A_{z}(\mu)$. A graphic example of this exchange before and after the swap of pairs is show in Figure 4.4.

User $1 \quad$ User 3

| SC1 | (1) | 0 | (1) | 0 | 1 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SC2 | (1) | 0 | 0 | 1 | 0 | 1 |
| SC3 | ! | 1 | 0 | 1 | 1 | 0 |
| SC4 | ! | 1 | (1) | 0 | 0 | 1 |

Possible blocking pairs before swapping

User $3 \quad$ User 1


Figure 4.4: Graphic representation of a swap operation

### 4.2.2.1 Considerations of matching theory in SCMA

The MTSCMA process is divided into four main steps as follows:

1) Set up an initial matching state with random/fixed allocation.
2) Select a pivot or starting point for the swap - blocking pair to start the swap matching process (Figure 4.5).
3) Check the condition for each swap operation.
4) Select the output for the final state. The process ends when there are no more swap - blocking pairs and the final matching is done.

User 1


Figure 4.5: Example of a pivot element to start looking for possible blocking pairs

```
Algorithm 2 MTSCMA algorithm
    Inputs:
        \(J, K, d v, d f, h\)
    Initialize:
        1) Mobile users and subcarriers are randomly assigned based on
        the codebook constraints
        2) All the users have the same power allocation
    3:
    : for \(\mathrm{j}=1\) to J do
        for \(\mathrm{k}=1\) to K do
            Swap-matching process
            Select a user \(U_{j}\) as a pivot element and find another \(U_{j}^{\prime}\) element sharing the
        same resource to form a blocking pair.
            If \(U_{j}, U_{j}^{\prime}\) establish a possible swap blocking pair along with \(j \in \mu(i)\) and
    \(j \in \mu\left(i^{\prime}\right)\)
            Perform the swap between the pair and keep the remaining users the same.
        if \(A_{z}(\mu)>A_{z}\left(\mu^{\prime}\right)\)
    then
            update the actual matrix (the change benefits)
    else
            keep current state (no swap blocking pair )
    endif
            Move to the next possible blocking pair.
            Repeat until there are no swap blocking pairs and all the users have been
    played.
        end for
    end for
```

The elements that should be considered for MT implementation are

- Effectiveness

Every swap operation is only performed if the total sum rate in the system is increasing. Let us suppose every swap operation changes the state $A_{z}(\mu)_{i^{\prime} j^{\prime}}^{i j}$ to $A_{z}(\mu)$. This action is taken when $U_{j}(\mu)_{i^{\prime} j^{\prime}}^{i j} \geq U_{j}(\mu)$ and $U_{j}^{\prime}(\mu)_{i^{\prime} j^{\prime}}^{i j} \geq U_{j}^{\prime}(\mu)$.

- Stability

The stability of the process is reached, once the maximum gain is obtained and all the possible players have checked their possible swaps with all the blocking pairs defined as $\left(U_{j}, U_{j}^{\prime}\right)$.
Definition 4: We can define $A_{z}(\mu)$ as a stable state if the system is not looking for more swaps in the matrix.

- Convergence

Based on the previous statements, the sum rate increases after every swap operation. The upper bound is established due to the limited spectrum available for use. The process converges when there is no higher rate that can possibly be achieved. The systems stops making changes when there are no further arrangements that can increase the sum rate.

Definition 5: A pair - wise stability is a term used to describe when the system has converged to an stable point, avoiding infinite swaps between blocking pairs.

### 4.2.2.2 Complexity of Matching theory for SCMA

The complexity of matching theory in SCMA can be obtained based on the sum of the following calculations:

1: The sum of the number of links between the users per resource block $O(d f K)$.
2. Number of elements on the list of users that are checked in every swap - matching to find a swap - blocking pair is $O\left(2^{d f}\right)$.

3: Number of times that every user looks for open blocks in the subcarriers to exchange and form a swap - blocking pair, given by the maximum number of possible swaps and the calculation of the total rate every time a swap is required $O(J K)$.

### 4.3 Power allocation solution

In this section, with the objective of maximising the sum rate of the SCMA system, a power allocation scheme with minimum rate and maximum power constraints will be presented and compared with other power schemes. As the codebook has been assigned, we can assume that the elements of the binary variable $f_{k, j}$ have been set to 1 or 0 in the subproblem presented in Section 4.2.1.1, then the total rate $R_{\text {tot }}$ of the uplink SCMA system over $K$ subcarriers can be simplified from (4.1) as presented in [32, 35, 81] and based on the Proof 2 in Appendix C:

$$
\begin{equation*}
R_{t o t}=B \sum_{j=1}^{J} \sum_{k=1}^{K} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right) \tag{4.4}
\end{equation*}
$$

where $g_{k, j}$ is the channel gain normalised by noise (i.e., $\frac{\left|h_{k, j}\right|^{2}}{B N_{o}}$ ).

### 4.3.1 Problem formulation and solution

The optimisation problem can be formulated as:

$$
\begin{equation*}
\max _{p_{k, j}} R_{\text {tot }}, \tag{4.5}
\end{equation*}
$$

subject to:

$$
\begin{align*}
\sum_{k=1}^{K} p_{k, j} & \leq P_{j}^{\max },  \tag{4.6}\\
p_{k, j} & >0  \tag{4.7}\\
\sum_{k=1}^{K} R_{k, j} & \geq r_{\min } \tag{4.8}
\end{align*}
$$

where (4.6) represents the total transmission power constraint and (4.7) ensures that the power values are positive. Constraint (4.8) establishes that each user will achieve their minimum transmission rate.

Based on the objective function in 4.5 and using the Lagrange Dual Decomposition(LDD) technique to solve the problem, including the constraints (4.6), (4.7) and (4.8), the Lagrangian equation of the optimisation problem is presented as [82]:

$$
\begin{align*}
L\left(p_{k, j}, \lambda_{j}, \tau_{j}\right)=B \sum_{j \in J} \sum_{k \in K} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right) & -\sum_{j \in J} \lambda_{j}\left(P_{j}^{\max }-\right. \\
& \left.\sum_{k \in K} p_{k, j}\right)-\tau_{j}\left(r_{\min }-\sum_{k=1}^{K} R_{k, j}\right) . \tag{4.9}
\end{align*}
$$

In (4.9), $\lambda_{j}$ and $\tau_{j}$ are the Lagrange multipliers. From here, the Karush-Kuhn-Tucker (KKT) conditions can be established as:

$$
\begin{gather*}
\lambda_{j} \geq 0, j=1, \ldots, J  \tag{4.10}\\
\tau_{j} \geq 0, j=1, \ldots, J  \tag{4.11}\\
\lambda_{j}\left(\sum_{k \in K} p_{k, j}-P_{j}^{\max }\right)=0  \tag{4.12}\\
\tau_{j}\left(r_{\text {min }}-\sum_{k \in K} B \log _{2}\left(1+p_{k, j} g_{k, j}\right)\right)=0 \tag{4.13}
\end{gather*}
$$

Taking the derivative of 4.9 with respect to the power, the resulting equation is:

$$
\begin{equation*}
\frac{\partial L}{\partial p_{k, j}}=\lambda_{j}-\frac{f_{k, j} g_{k, j}\left(1+\tau_{j}\right)}{1+I_{k, j}+p_{k, j} g_{k, j}} . \tag{4.14}
\end{equation*}
$$

Where $I_{k, j}$ is the interference term in subcarrier $k$ defined as:

$$
\begin{equation*}
I_{k, j}=\sum_{i=(j+1)}^{J} p_{k, i} g_{k, i} \tag{4.15}
\end{equation*}
$$

As we can see from the resulting equation (4.9) with respect to the power, we cannot obtain a closed form solution as we cannot separate the term $p_{k, j}$. The equation will be non-convex and requires the use of complex numerical solutions [83].

### 4.3.2 Problem solution using Lagrange dual solution and subgradient algorithm in SCMA

We have proposed a simplified solution for power allocation. From equation (4.14), if we order the users with respect to the channel gain such that $\left|h_{k, 1}\right|^{2} \geq \ldots \geq\left|h_{k, J}\right|^{2}$, we can assume that the weakest user $j=J$ will receive no interference. Therefore, we can eliminate the interference term and we can present the problem as a water filling problem. Let us define the achievable rate for user $J$ as:

$$
\begin{equation*}
\sum_{k=1}^{K} R_{k, J}=B \log _{2}\left(1+\sum_{k=1}^{K} P_{k, J} g_{k, J}\right) \tag{4.16}
\end{equation*}
$$

Then, the optimisation problem for user $J$ can be presented as:

$$
\begin{equation*}
\max _{p_{k, J}} R_{k, J}, \tag{4.17}
\end{equation*}
$$

subject to:

$$
\begin{align*}
\sum_{k=1}^{K} p_{k, J} & \leq P_{J}^{\max },  \tag{4.18}\\
p_{k, J} & >0  \tag{4.19}\\
\sum_{k=1}^{K} R_{k, J} & \geq r_{\min } \tag{4.20}
\end{align*}
$$

(4.18) depicts the total transmission power constraint and (4.19) ensures that the power values are positive. Constraint (4.20) establishes that user J will achieve its minimum transmission rate.

From the objective function in (4.17) and using the Lagrange Dual Decomposition(LDD) technique to tackle the problem as presented in equation (4.9), the Lagrangian equation of the optimisation problem for user J is:

$$
\begin{align*}
L\left(p_{k, J}, \lambda_{J}, \tau_{J}\right)=B \sum_{k \in K} f_{k, J} \log _{2}\left(1+p_{k, J} g_{k, J}\right)-\lambda_{J}\left(P_{J}^{\max }\right. & \left.-\sum_{k \in K} p_{k, J}\right) \\
& -\tau_{J}\left(r_{\min }-\sum_{k=1}^{K} R_{k, J}\right) \tag{4.21}
\end{align*}
$$

Where $\lambda_{J}$ and $\tau_{J}$ are the Lagrange multipliers. From here, the Karush-Kuhn-Tucker (KKT) conditions can be established as:

$$
\begin{gather*}
\lambda_{J} \geq 0  \tag{4.22}\\
\tau_{J} \geq 0  \tag{4.23}\\
\lambda_{J}\left(\sum_{k \in K} p_{k, J}-P_{J}^{\max }\right)=0  \tag{4.24}\\
\tau_{J}\left(r_{\min }-\sum_{k \in K} B \log _{2}\left(1+p_{k, J} g_{k, J}\right)\right)=0 . \tag{4.25}
\end{gather*}
$$

We calculate the derivative of (4.21) with respect to the power and the resulting equation is:

$$
\begin{equation*}
\frac{\partial L}{\partial p_{k, J}}=\lambda_{J}-\sum_{k=1}^{K} \frac{g_{k, J}\left(1+\tau_{J}\right)}{1+p_{k, J} g_{k, J}} . \tag{4.26}
\end{equation*}
$$

We separate the term $p_{k, J}$ as follows:

$$
\begin{equation*}
P_{k, J}=\frac{1}{\lambda_{J}}-\frac{1}{\sum_{k=1}^{K} g_{k, J}\left(\frac{1}{1+\tau_{J}}\right)} . \tag{4.27}
\end{equation*}
$$

Therefore, the power allocation in (4.27) is synonymous with a single-user water filling problem, and it can be extended and solved by considering the interference $I_{k, j}$ from other users as noise [32]. However, by treating the users independently, the power will be allocated to each user in succession starting with the user $J$ and following with the remaining users $j=(J-1 \ldots 2,1)$. As a result of this, a mathematical method is required to obtain a solution. In this work, a subgradient based approach is proposed in order to obtain a suboptimal power scheme [84-86], where the interference is updated with each user's power calculation. Based on this analysis, the following iterative solution is proposed.

From the KKT conditions, by solving the Lagrange variable $\lambda_{J}$ using the subgradient
method, we obtain $p_{k, J}$. Next by solving 4.28) for the Lagrange variable $\lambda_{J-1}$ and taking the interference into account from user $J$, we solve equation (4.27), and we obtain $p_{k, J-1}$. This process is successively solved for the remaining Lagrange $\lambda_{j}$. The equation for the power for user $j$ is:

$$
\begin{equation*}
p_{k, j}=\frac{1}{\lambda_{j}}-\frac{1}{\sum_{k=1}^{K} g_{k, j}\left(\frac{1}{1+\tau_{j}}\right)}\left(1+\sum_{i=j+1}^{J} p_{k, i} g_{k, i}\right) . \tag{4.28}
\end{equation*}
$$

The power value in each iteration for user $j$ must be positive in order to ensure that the selected subcarriers are used. The variables $\mu_{J} \ldots \mu_{1}$ and $\lambda_{J} \ldots \lambda_{1}$ can be solved by the subgradient method given by:

$$
\begin{gather*}
\Delta \lambda_{J}=\sum_{k=1}^{K} p_{k, J}-P_{J}^{\max } \\
\vdots  \tag{4.29}\\
\Delta \lambda_{1}=\sum_{k=1}^{K} p_{k, 1}-P_{1}^{\max } \\
\Delta \tau_{J}=r_{\text {min }}-B\left(\log _{2}\left(1+\sum_{k=1}^{K} p_{k, J} g_{k, J}\right)\right) \\
\vdots  \tag{4.30}\\
\Delta \tau_{1}=r_{\text {min }}-B\left(\log _{2}\left(1+\sum_{k=1}^{K} p_{k, 1} g_{k, 1}\right)\right)
\end{gather*}
$$

In the subgradient method, the variables are initialised and updated in each iteration, based on the following equations:

$$
\begin{gather*}
\lambda_{J}(t+1)=\left[\lambda_{J}(t)+\alpha_{\lambda} \Delta \lambda_{J}\right]^{+}  \tag{4.31}\\
\tau_{J}(t+1)=\mu_{J}(t)+\alpha_{\tau} \Delta \mu_{J} \tag{4.32}
\end{gather*}
$$

From (4.31) and (4.32), $t$ represents the iteration index for the subgradient algorithm and $e$ the error threshold. The variables $\alpha_{\lambda}$ and $\alpha_{\mu}$ represent the values of the step sizes for each multiplier, which are set at $\frac{.001}{\sqrt{t}}$ and $\frac{P_{j}^{\max }}{100 t^{2}}$, respectively. The step sizes used in this work have been initialised based on the step size calculations presented in the subgradient method in [87]. The works in [86] [88] use similar step sizes to solve resource allocation problems. The value of the arbitrary constant was modified in order to find convergence efficiently in the solution for this problem. The steps of the proposed algorithm to implement the resource allocation solution are described in Algorithm 3 .

```
Algorithm 3 Subgradient power allocation algorithm
    Inputs:
        \(K, J, d v, d f, h, e\)
    Obtain design of graph \(F\) (subcarrier allocation)
    Initialize:
        \(j=J\)
    do
        Initialize:
        \(t_{j}=0, \lambda_{j}=\) init and \(\mu_{j}=\) init
        do
            Solve \(p_{k, j}\) based on eq. 4.28)
            Update \(I_{k, j}\) based on eq. (4.2)
            Update \(\lambda_{j}\left(t_{j}+1\right)\) based on eq. (4.31)
            Update \(\tau_{j}\left(t_{j}+1\right)\) based on eq. (4.32)
            \(t_{j}=t_{j}+1\)
            while \(\left|\lambda_{j}(t-1)-\lambda_{j}(t)\right| \leq e,\left|\tau_{j}(t-1)-\tau_{j}(t)\right| \leq e\)
            Obtain \(\lambda_{j}^{*}\) and \(p_{k, j}^{*}\) respectively
            Move to the next user \(j\)
            \(j=j-1\)
    while \(j \geq 1\)
```


### 4.4 Numerical results

In this section, the simulation results from the SCMA system are presented. The simulations for resource allocation in SCMA were implemented in MATLAB as follows:
we considered an uplink scenario with a single cell and $J$ users. The users are uniformly distributed from the base station. As presented in Section 3.2, the bandwidth is divided into $K$ subcarriers. The total power transmission is $P_{j}^{\max }$. For the resource allocation solution, the wireless channel is modelled as a six path frequency selective fading channel, using the ITU pedestrian model-B as depicted in Table 2.1. The channel we have considered is quasi-static, i.e. the channel is constant within a given transmission block but varies independently between blocks. The subcarrier allocation solution is implemented as depicted in the steps presented in Section4.2.1.1. Then, the resultant matrix $F$ is used to implement the solution for power allocation in SCMA. For the power allocation simulations, we have implemented equal power allocation per subcarrier regardless of the order of the users and the channel gain. In other words, if the total available power of a user is 20 dBm , then 17 dBm is assigned for transmission to one subcarrier and 17 dBm is assigned to the other. Water-filling is a method that is well-known as a solution for resource allocation. For practicality, we do not include the method, but the solution is implemented as presented in [32]. Our solution of power allocation is obtained through the subgradient method as presented in 4.3.2. The simulation results have been carried out using the Monte Carlo method with the parameters listed in Table 4.1 as follows:

| Parameters SCMA | Values |
| :---: | :---: |
| Number of users | 6 |
| Number of codebooks | 6 |
| Subcarriers | 4 |
| Sparsity | 2 |
| Overloading (symbols) | 1.5 |
| Factor overloading | 3 |
| Factor spread | 2 |
| Pedestrian model | B |
| PL exponent | 3 |
| Bandwidth | 180 kHz |
| Noise | -174 dBm |
| Shadowing | 8 dB |
| Transmit power | $0-23 \mathrm{dBm}$ |
| Cell diameter | 200 m |
| Monte Carlo iterations | 10000 |
| Error threshold | $10^{-5}$ |

Table 4.1: Parameters of simulation for SCMA


Figure 4.6: Subcarrier allocation in SCMA

From Figure 4.6, we can see that subcarrier allocation plays a significant role in the sum rate maximisation of SCMA and in adequately exploiting sparsity as random or fixed assignment of users to the subcarriers will considerably degrade the performance, as proved in [43], [67], [69] and [70]. Authors in [70], studied the joint impact of the SCMA constellation design and subcarrier allocation on the Peak to Average Power Ratio (PAPR). In the work presented in [43], an iterative algorithm for subcarrier allocation based on the average sum rate of an uplink system is presented. The authors focused their solution on the use of two main factor graphs and the probability function of the distance between the users. In [69], the subcarrier allocation solution is based on removing allocated users and updating the interference term. In [67], a Matching Theory method is introduced to solve the subcarrier allocation problem for SCMA.

Based on the above works and with the purpose of taking advantage of the SCMA characteristics such as sparsity and multidimensional codebooks, a subcarrier allocation algorithm has been designed and presented in Algorithm 1 (Section 4.2.1). The proposed algorithm is implemented based on the parameters presented in Table 4.1 and evaluated against other existing work conducted on subcarrier algorithms such as MT based approach and exhaustive search, which sets an optimal target. It is clear from the results that the proposed algorithm does not offer the closest performance to
the optimal, however, the complexity of implementation is lower in comparison with matching theory and exhaustive search. This provides practical implementation in the system as it is a simple mechanism to solve the subcarrier allocation problem.


Figure 4.7: Comparison of total rate of the classic greedy method and our proposed algorithm for subcarrier allocation in SCMA

From Figure 4.7 we can see the improvement on the rate on a classic version of a greedy algorithm implemented for the subcarrier allocation problem, where the rate calculation is not considered when making a decision on which user is allocated to a certain pattern. It is clear that our proposed algorithm increases the total rate of the system.

Computational complexity is a factor that should be considered for resource allocation algorithms. We have mentioned the importance of the factor graph design presented in 3.4 for SCMA and its impact on the system performance. Furthermore, practical implementation and a reduced use of computational resources are desirable in software based solutions. Exhaustive search is used to find the optimal factor graph. It is affected by the number of permutations, the number of users to be allocated and the calculation of the total rate in every iteration, which results in a complexity of $O(J!)$. When $J=6$ and $K=4$, the calculated value is 720 [43]. This method is not practical, however, it establishes an upper bound to enable comparison with other algorithms.

As presented in Section 4.2.2.2, the complexity of the Matching Theory method is determined by the sum of the complexity factors of the components involved in the allocation process. We calculate the complexity based on the number of links between the users per resource $O(d f K)=12$, the elements on the list of users that are checked in every swap $O\left(2^{d f}\right)=8$, and the calculation of the total rate every time a swap is required $O(J K)=24$. This results in a calculated value of 44 . Additionally, the average number of possible swaps (rate calculations before an after the swap) is 55. In contrast, in our proposed algorithm presented in Section 4.2.1.2, the complexity is considerably reduced by inverting the pattern allocation for odd numbered users. The complexity of the simple greedy algorithm is calculated based on the number of elements involved in the decision, which for the general case is $O(J)$ [72], resulting in a value of 6 . The complexity of the proposed algorithm is determined by the maximum number of iterations, which is the number of total rate calculation operations $(J-d f)=3$ and the complexity of the greedy algorithm. As a result, we can express the complexity as $O\left((J-d f)^{2}\right)$. The calculated value for our case is 9 . Therefore, although the overall sum rate is not improved in our proposed solution, as we can see in the Table 4.2, the complexity is clearly less than exhaustive search and matching theory, and the proposed algorithm outperforms exhaustive search and matching theory alternatives.

|  | Exhaustive search | Proposed algorithm | Matching Theory |
| :--- | :---: | :---: | :---: |
| Rate calculations | 720 | 3 | 55 |
| Elements evaluated in <br> every swap | - | 3 | 8 |

Table 4.2: Complexity comparison of subcarrier allocation schemes


Figure 4.8: Power allocation in SCMA

We have previously discussed that proper management of power can improve the performance of non-orthogonal multiple user systems. For instance, authors in [29] and [35] have proved this statement in power domain NOMA. However, limited work has been carried out on the resource allocation problem of the uplink SCMA system. The work presented in [43] has implemented a resource allocation solution by stabilising predetermined fixed proportional power assignment per user. In [69], the power allocation solution is obtained by updating the interference term using CVX toolbox MATLAB. Authors in [67] have proposed a solution based on simple water-filling method without taking into consideration either the interference term or the minimum rate constraints. Based on these works, a practical power allocation solution with minimum rate constraints for SCMA has been designed and presented in Algorithm 3 . This solution is jointly implemented with the subcarrier allocation algorithms previously presented; the matching theory approach and the proposed subcarrier allocation algorithm. The comparison has been carried out based on the parameters in Table 4.1 and the solution is compared with equal power allocation and simple water-filling power allocation, as depicted in Figure 4.8.

From the graph, we can see that the designed subgradient power allocation scheme improves the performance of the system because it maximises the total sum rate whilst
ensuring each user will achieve a minimum target rate. This approach guarantees a rate transmission for users dealing with the effect of the interference, as we can see from Figure 4.9. In contrast, equal power allocation treats all the users equally without any distinction and distributes the power regardless of the channel conditions. On the other hand, water filling power allocation tends to assign most of the available power to the strongest channels, with the disadvantage of under-utilising some strong channels. It is also important to mention that the simple water filling method does not take into consideration the undesired effect of the interference when assigning power to the active subcarriers.

It is quite clear in Figure 4.8 that the proposed power allocation scheme with the proposed subcarrier allocation solution provides a performance almost as effective as the MT approach. The performance gap between those approaches and equal power allocation is evident. Therefore, this demonstrates that the proposed power allocation scheme can overcome the suboptimality of the low complexity subcarrier allocation scheme while achieving a close to optimum performance. An improvement in the overall rate has been seen when compared to the equal power and water-filling methods while assuring a minimum transmission rate is achieved for each user and optimising the use of limited power within the mobile equipment. This is shown in Figure 4.10, where individual rates are presented at a maximum of 20 dBm power transmission per device.


Figure 4.9: Comparison of individual users' rates in SCMA with equal power allocation (EP) and a subgradient (SG) solution for power allocation. The users are ordered in decreasing order, user 1 is the user with strongest channel gain and user 6 is the user with the weakest channel gain. Power range per user $10-20 \mathrm{dBm}$.


Figure 4.10: Comparison of rates per user for equal power allocation and the subgradient method when the available power in the mobile equipment is 20 dBm . The users are ordered in decreasing order

Another way of evaluating the efficiency of a method and the performance of a system is with the simulation of users in outage. This parameter allows a comparison of the improvement when different methods are applied in a system. Figure 4.11 and Figure 4.12 present the results when we are comparing the outage probability of 6 users over 4 subcarriers when different power allocation is implemented in the system. In Figure 4.11, the target rate for the proposed algorithm is 500 kbps , this being the threshold to measure the percentage of users in outage. In Figure 4.12, the target for the proposed solution has been changed to 1 Mbps , this being the threshold to evaluate the users in outage.


Figure 4.11: Outage probability when 6 users are transmitting over 4 subcarriers. Comparison between equal power allocation and our proposed solution with different subcarrier allocation solutions with a minimum rate requirement of 500 kbps


Figure 4.12: Outage probability when 6 users are transmitting over 4 subcarriers. Comparison between equal power allocation and our proposed solution with different subcarrier allocation solutions with a minimum rate requirement of 1 Mbps

### 4.5 Chapter summary

Enhancing the performance of the system is desirable due to the high demand of data transmission caused by multiple users connected at the same time. In order to satisfy the demand, it is necessary to optimise the use of resources such as spectrum and power transmission. In this chapter, SCMA capacity is studied and we have proposed a full resource allocation scheme with the aim of maximising the overall rate. A low complexity subcarrier allocation has been proposed, based on greedy algorithm and compared to other methods. Although the proposed subcarrier allocation is suboptimal, the computational complexity of the compared subcarrier allocation methods has been presented in order to show the practical implementation of our solution. A power allocation solution has been proposed and implemented, based on the LDD approach, which has been solved by the subgradient algorithm. Equal power allocation and water filling approaches have been implemented in order to compare them with the proposed solution. Simulations of the individual rate of users and outage probability show the improvement in the performance of the system and the advantage of the solution, since this guarantees a minimum rate per user while maximising the sum rate.

## Chapter 5

## Pattern Division Multiple Access uplink system

Pattern Division Multiple Access is a versatile technique that can be implemented either in power, code or spatial domain. PDMA in code domain is derived from the previous technique, Low Density signature CDMA, where determined sequences are designed and implemented to allocate users to specific resources [89]. This technique is synonymous to SCMA described previously in Chapter 4, where users are allocated to a specific group of subcarriers, however, PDMA presents less dense codebooks or patterns [90]. In PDMA, the same symbol can be connected to a different number of subcarriers [91]. Another element where PDMA works differently from SCMA is where the users are strictly allocated to one subcarrier at a time, similar to the manner in which users are allocated in OFDMA systems. These orthogonal patterns are designed to maximise the diversity and reduce the interference caused by multiple users overlapping [92].

At the receiver in power domain, PDMA applies SIC, according to SINR difference among the multiplexed users' signals. MPA is applied at the receiver in code domain PDMA, to separate the signals from different users, similar to the receiver in SCMA. Multiplexing in space, can be compared with a multiple antenna scenario [53] [91] [93].

PDMA is comparable to cognitive radio, where users can be seen as underlay and overlay users. Underlay users are known in the literature as those users that transmit over a group of subcarriers, using a main channel of transmission and the others as a
second resource in presence of interference. Those users who transmit over a primary resource in the presence of no interference are denominated overlay users [94]. In order to exemplify, the simplest case of multiplexed users in PDMA is shown in Figure 5.1. where we can see them as two overlay users and one underlay user. This can also be a two user case of conventional NOMA.


Figure 5.1: PDMA transmission for three users

PDMA reduces the overloading factor by creating orthogonal patterns at the transmitter and improves the spectral efficiency by supporting massive connectivity based on a reasonable unequal diversity. This means the overlapping of users is considerably reduced, achieving near orthogonality in the transmitted signals, thus alleviating the error propagation [95] [96].

### 5.1 PDMA transmission

In the PDMA system, the encoding process maps a bit stream to a complex multidimensional codeword as presented in Section 3.2. The mapping function for the PDMA encoder is expressed as $f: \mathbb{B}^{\log _{2}(M)} \mapsto \boldsymbol{X}$ mapping $b_{n}^{k}$ to an $N$-dimensional complex codeword that is taken from the codebook. The channel mapping and superposition of users' codewords is defined by a dispersion matrix, $V_{j}$, where $V \in \mathbb{B}^{K \times N}$ such that $v_{j}=1$ if the $j$-th layer is mapped to the $k$-th element of a codeword $x_{k, j}$ [97]. This dispersion matrix presents a different sparsity, for some users, from the one used in SCMA in Section 3.1.

The users will be located at different distances from the base station, and with the effect of path loss and fading each of them will experience different channel gain.

It is assumed that the base station and the users have perfect knowledge of the CSI. The channel gain for the $j$-th user at subcarrier $k$ is expressed as $\left|h_{k, j}\right|^{2}$. The channel characteristics and the noise vector $n_{k}$ are described in detail in Section 3.2,

The transmitted signal on the uplink PDMA system as received at the base station can be represented graphically as in Figure 3.4 and the following mathematical expression:

$$
\begin{equation*}
y_{k}=\sum_{j=1}^{J} h_{k, j} x_{k, j}+n_{k}, \forall k=1 \ldots K \tag{5.1}
\end{equation*}
$$

As an example, we present a numerical exercise to obtain the resultant signal in the PDMA system. This is presented in Appendix F.

### 5.2 Factor graph design and sparsity in PDMA

We have explained how users will be spread over subcarriers and how the subcarriers will be connected to users in a connected graph as similarly presented in Section 3.4 for the SCMA scenario. A factor graph will be represented by a matrix $F=\left[f_{k, j}\right]$ with entries $(0,1)$ that shows the mapping between users and subcarriers. If $f_{k j}=1$, it indicates that a user is allocated to a particular subcarrier and if $f_{k j}=0$ it indicates that that user is not allocated to the subcarrier [98].


Figure 5.2: Graph showing the connection of variable nodes with function nodes in $P D M A_{2 \times 3}$.

The simplest case of a factor graph design, known as pattern in PDMA, is represented in Figure 5.2 and in a factor graph depicted as [96]:

$$
\boldsymbol{F}_{2 \times 3}=\left[\begin{array}{lll}
1 & 1 & 0  \tag{5.2}\\
1 & 0 & 1
\end{array}\right]
$$

We can present the sum rate of $P D M A_{2 \times 3}$ as:

$$
\begin{align*}
& R_{\text {tot }_{\text {PDMA }}^{2 \times 3}}=B\left[\log _{2}\left(1+\frac{p_{1,1} g_{1,1}}{1+p_{1,2} g_{1,2}}\right)+\log _{2}\left(1+p_{1,2} g_{1,2}\right)\right. \\
& \left.+\log _{2}\left(1+\frac{p_{2,1} g_{2,1}}{1+p_{2,3} p_{2,3}}\right)+\log _{2}\left(1+p_{2,3} g_{2,3}\right)\right] \tag{5.3}
\end{align*}
$$

Based on Proof 2 and 3 presented in Appendices C and D, the equation above is synonymous to equation (5.4):

$$
\begin{equation*}
R_{\text {totpDMA }_{2 X 3}}=B\left[\log _{2}\left(1+p_{1,1} g_{1,1}+p_{1,2} g_{1,2}\right)+\log _{2}\left(1+p_{2,1} g_{2,1}+p_{2,3} g_{2,3}\right)\right] \tag{5.4}
\end{equation*}
$$

Another pattern used in PDMA, is the factor graph $F$ derived from the connected graph in Figure 5.3. This graph presents similar connected edges to the design of the SCMA factor graph presented in Figure 3.5 (Section 3.3 ), but with a smaller overloading factor.


Figure 5.3: Graph showing the connection of variable nodes with function nodes in $P D M A_{4 \times 6}$.

The factor graph $F$ for $P D M A_{4 X 6}$ can be represented as the pattern shown in 5.5. This pattern is built with two underlay users (user 1 and 2 ) and $(J-2)$ overlay users.

$$
\boldsymbol{F}=\left[\begin{array}{llllll}
1 & 0 & 1 & 0 & 0 & 0  \tag{5.5}\\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1
\end{array}\right]
$$

The total rate equation of a PDMA is expressed as:

$$
\begin{equation*}
R_{t o t}=B \sum_{j=1}^{J} \sum_{k=1}^{K} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right) \tag{5.6}
\end{equation*}
$$

where the transmission power of user $j$ allocated to subcarrier $k$ is expressed as $p_{k, j}$ and $g_{k, j}$ is the channel gain normalised by noise (i.e., $\frac{\left|h_{k, j}\right|^{2}}{B N_{o}}$ ).

### 5.3 PDMA and resource allocation

Coded NOMA has the ability to handle multiple users in one resource block by making use of known information about the codebook pattern in the decoding process. Therefore, the receiver has assistance when trying to identify the sent codewords by pre-emptively populating the codebooks with the known information [53]. However, these patterns require assignment of users to a specific group of subcarriers, even when the channel gains are not as effective as other available subcarrier assignments that are not feasible due to the restriction on duplicated patterns. This constraint can affect the performance by dropping the total rate significantly. The transmission power of each user is an element that should be considered when evaluating the rate capacity performance of the system, as previous work in multiuser techniques has shown that an adequate management of this resource can lead to an improvement in the system with respect to data rate transmission [99], [100]. Similar to the factor graph in SCMA, the performance of PDMA is largely determined by the design of the subcarrier allocation, referred to as the PDMA pattern matrix [101].

### 5.3.1 Subcarrier allocation in PDMA

Multiple user systems such as PDMA benefit from allocating users to the same resources, in order to increase the overall transmission rate. Similar to SCMA, PDMA is restricted by the design, density and assignment of the pattern. These constraints can limit the effect of overloading the system because of the number of users that can be assigned to a specific subcarrier and the restriction of only assigning specific patterns to specific subcarriers. The presence of multiple users may result in high interference between them. For PDMA in particular, the undesirable effect of the interference is partly mitigated by reducing the overloading factor in the PDMA patterns. However, proper subcarrier allocation should be considered in order to achieve desirable transmission rate performance [101].

In this section, a low complexity algorithm for PDMA to solve the problem of subcarrier allocation is described and presented.

### 5.3.2 Proposed subcarrier allocation algorithm based on greedy algorithm

We have described in Section 4.2.1 the principle of the greedy algorithm and the feasibility of using the greedy algorithm to find close to optimal solutions while maintaining low complexity. As the algorithm was useful when investigating SCMA, we explore its use in PDMA for similar reasons. It is worth exploring subcarrier allocation based on this principle for PDMA as adequate and practical management of resources is desirable for multiuser systems. In order to produce an upper bound indicator for the proposed algorithm, exhaustive search will be used to find the global optimal solution and a random or fixed subcarrier allocation used to establish a lower bound indicator to set a point of reference for this comparison.

For PDMA, $f_{k, j}, \in[0,1]$ in equation 5.6 , is a discrete variable that represents the pattern of the user-subcarrier mapping. This variable should be optimised due to the impact of this relationship between users and subcarriers on the performance of the system.

1: Let $A$ be a set of possible codebook pattern assignments $a_{i} \in A\{0,1\}$.
2: Let $I d$ be an identity matrix with unitary vectors of $K$ by $K$ size.

3: All the possible $K$ column vectors from the codebook [97], which contains $d v$ number of one and $(K-d v)$ number of zeros, are defined as pattern assignments.

4: $F$ is defined as a matrix with dimensions $(J \times K)$ representing user-subcarrier allocations. We initialise it as a zero-value matrix. $F=0_{K, J}$

First, divide the set $A$ into pairs of patterns. Then a random pair pattern is selected from the possible pairs for user 1 and 2 , and the overall rate is calculated, considering no interference as all the other users are not yet allocated. The pair of vectors that achieves the highest rate is allocated and it is deleted from $A, a^{\prime}=A \notin a$. Therefore, we proceed to the remaining users, where the highest channel is selected for user $(J-(d v+1))$ to user $(J)$, as shown in Figure 5.4. This process should continue until all the remaining users are allocated to one subcarrier as presented in Algorithm4.

|  | ser | se | .... |  |  | er |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SC1 | 0 | 1 | 1 | 0 | 0 | 0 |
| SC2 | 0 | 1 | 0 | 1 | 0 | 0 |
| SC3 | 1 | 0 | 0 | 0 | 1 | 0 |
| SC4 | 1 | 0 | 0 | 0 | 0 | 1 |
| Pair of underlay users allocated by rate allocation |  |  | Users allocated based on greedy principle | Users allocated based on greedy principle |  |  |

Figure 5.4: Graphic example of subcarrier allocation in PDMA

```
Algorithm 4 Proposed subcarrier algorithm for \(P D M_{4} 46\)
    Inputs:
        \(K, J, d v, h\)
    Initialize:
        \(n=0\) and \(F=0_{K, J}\)
    \(A=\{\) all possible user-subcarrier assignment vectors \(\}\)
    \(I d=\{\) Identity matrix with unitary vectors of KxK\(\}\)
    do
        Let \(n=1\)
        Set the corresponding values in column 1 of \(F\) to 1 and column 2 to the inverse
    values.
        Calculate the rate obtained when each vector from \(A\) is used in position \(j\) and
    \(j+1\) of \(F\)
        Select the pair of patterns of \(A\) which gives the best performance.
        Delete vectors matching columns 1 and 2 of \(F\) from \(A\)
    while \(n<(d f)\)
    do
        Let \(n=d f\)
        Select the subcarrier with highest channel gain from \(h\)
        Delete unitary vectors matching from \(B\)
    while \(n<(J)\)
    Obtain \(F^{*}\)
```


### 5.4 Power allocation in PDMA

Similar to many other multiple user systems, PDMA, which carries more than one user per subcarrier, suffers from the effect of interference. In order to tackle this problem, optimal power allocation is required to achieve the maximum performance of the data transmission while ensuring the fairness for individual user transmission.

Based on the pattern presented in (5.5), we can see that the interaction between users and subcarriers can be split into two independent sections. The users in Section A will not be assigned to subcarriers in Section B and vice-versa. Thus, we can express the total rate of the pattern presented in (5.5) as:

$$
\begin{equation*}
R_{t o t_{4 \times 6}}=R_{A}+R_{B} \tag{5.7}
\end{equation*}
$$

where $R_{A}$ and $R_{B}$ are the sum rate of the users allocated in subcarriers respectively as shown in Figure 5.5:

| $S C_{1}$ | $\mathrm{U}_{1}$ | $\mathrm{U}_{2}$ | $U_{3}$ | $\mathrm{U}_{4}$ |  |  | Allocated usersto section A |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 0 | 1 | 0 | 0 | 0 |  |
| $S C_{2}$ | 1 | 0 | 0 | 1 | 0 | 0 |  |
| $S C_{3}$ | 0 | 1 | 0 | 0 | 1 | 0 |  |
| $\mathrm{SC}_{4}$ | 0 | 1 | 0 | 0 | 0 | 1 |  |

Figure 5.5: Example of PDMA pattern with users divided by sections
$R_{A}$ and $R_{B}$ are presented as:

$$
\begin{align*}
R_{A}=B & \sum_{j=1}^{J} \sum_{k=1}^{K-d v} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right)=B\left(\log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)+\right. \\
& \left.\log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)+\log _{2}\left(1+p_{a, x} g_{a, x}\right)+\log _{2}\left(1+p_{b, x} g_{b, x}\right)\right) \tag{5.8}
\end{align*}
$$

$$
\begin{align*}
R_{B}=B & \sum_{j=1}^{J} \sum_{k=K-d v+1}^{K} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right)=B\left(\log _{2}\left(1+\frac{p_{a, 2} g_{b, 2}}{1+p_{a, y} g_{a, y}}\right)+\right. \\
& \left.\log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)+\log _{2}\left(1+p_{a, y} g_{a, y}\right)+\log _{2}\left(1+p_{b, y} g_{b, y}\right)\right) \tag{5.9}
\end{align*}
$$

where $x$ and $y$ represent the number of the overlay users sharing subcarriers. $a$ and $b$ represent the active subcarriers. The derivations of the equations (5.9) and (5.8) are included in Appendix E, where we have proved that the rate in the PDMA uplink can be represented as the summation of the users' SNR.

### 5.4.1 Optimal power allocation for sum rate maximization in PDMA with power constraint

In this section, with the objective of maximising the sum rate of the PDMA system, a power allocation solution with maximum power constraints will be presented. The optimisation of the total rate problem can be managed in two parts, $R_{A}$ and $R_{B}$, sum rate for sections $A$ and $B$ respectively, and formulated as:

$$
\begin{equation*}
\max _{p_{k, j}} R_{t o t_{4 X 6}}=R_{A}+R_{B}, \tag{5.10}
\end{equation*}
$$

subject to:

$$
\begin{gather*}
\sum_{k=1}^{K} p_{k, j} \leq P_{j}^{\max }  \tag{5.11}\\
p_{k, j}>0 \tag{5.12}
\end{gather*}
$$

where (5.11) represents the total transmission power constraint and (5.12) ensures that the power values are positive. Based on the optimisation problem above, the Lagrangian function of the problem, divided into Sections A and B, including the constraints (5.11) and (5.12) can be presented as:

$$
\begin{align*}
& L_{A}\left(p_{a, 1}, p_{b, 1}, p_{a, x}, p_{b, x}, \lambda_{1}, \lambda_{x a}, \lambda_{x b}\right)=B\left(\log _{2}\left(1+p_{a, 1} g_{a, 1}+p_{a, x} g_{a, x}\right)\right. \\
& \left.\quad+\log _{2}\left(1+p_{b, 1} g_{b, 1}+p_{b, x} g_{b, x}\right)\right)- \\
& \lambda_{1}\left(p_{a, 1}+p_{b, 1}-P_{1}^{\max }\right)-\lambda_{x a}\left(p_{a, x}-P_{x a}^{\max }\right)-\lambda_{x b}\left(p_{b, x}-P_{x b}^{\max }\right) \tag{5.13}
\end{align*}
$$

$$
\begin{gather*}
L_{B}\left(p_{a, 2}, p_{b, 2}, p_{a, y}, p_{b, y}, \lambda_{2}, \lambda_{y a}, \lambda_{y b}\right)=B\left(\log _{2}\left(1+p_{a, 2} g_{a, 2}+p_{a, y} g_{a, y}\right)\right. \\
\left.+\log _{2}\left(1+p_{b, 2} g_{b, 2}+p_{b, y} g_{b, y}\right)\right)- \\
\lambda_{2}\left(p_{a, 2}+p_{b, 2}-P_{2}^{\max }\right)-\lambda_{y a}\left(p_{a, y}-P_{y a}^{\max }\right)-\lambda_{y b}\left(p_{b, y}-P_{y b}^{\max }\right) \tag{5.14}
\end{gather*}
$$

In (5.13) and (5.14, , $\lambda_{1}, \lambda_{2}, \lambda_{x a}, \lambda_{x b}, \lambda_{y a}, \lambda_{y b}$ represent Lagrange multipliers, respectively. From here, the Karush-Kuhn-Tucker (KKT) conditions can be established as:

$$
\begin{equation*}
\lambda_{1} \geq 0, \lambda_{x a} \geq 0 \text { and } \lambda_{x b} \geq 0 \tag{5.15}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{2} \geq 0, \lambda_{y a} \geq 0 \text { and } \lambda_{y b} \geq 0 \tag{5.16}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{1}\left(\left(p_{a, 1}+p_{a, 1}\right)-P_{1}^{\max }\right)=0, \lambda_{x a}\left(p_{a, x}-P_{x a}^{\max }\right)=0 \text { and } \lambda_{x b}\left(p_{b, x}-P_{x b}^{\max }\right)=0 \tag{5.17}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{2}\left(\left(p_{a, 2}+p_{a, 2}\right)-P_{2}^{\max }\right)=0, \lambda_{y a}\left(p_{a, y}-P_{y a}^{\max }\right)=0 \text { and } \lambda_{y b}\left(p_{b, y}-P_{y b}^{\max }\right)=0 \tag{5.18}
\end{equation*}
$$

If we calculate the derivatives in respect of the power per user, we obtain:

$$
\begin{align*}
& \frac{\partial L_{A}}{\partial p_{a, 1}}=-\lambda_{1}+\frac{B g_{a, 1}}{\left(1+p_{a, 1} g_{a, 1}+p_{a, x} g_{a, x}\right) \log (2)} \\
& \frac{\partial L_{A}}{\partial p_{b, 1}}=-\lambda_{1}+\frac{B g_{b, 1}}{\left(1+p_{b, 1} g_{b, 1}+p_{b, x} g_{b, x}\right) \log (2)} \\
& \frac{\partial L_{B}}{\partial p_{a, 2}}=-\lambda_{2}+\frac{B g_{a, 2}}{\left(1+p_{a, 2} g_{a, 2}+p_{a, y} g_{a, y}\right) \log (2)} \\
& \frac{\partial L_{B}}{\partial p_{b, 2}}=-\lambda_{2}+\frac{B g_{b, 2}}{\left(1+p_{b, 2} g_{b, 2}+p_{b, y} g_{b, y}\right) \log (2)} \\
& \frac{\partial L_{A}}{\partial p_{a, x}}=-\lambda_{x a}+\frac{B g_{a, x}}{\left(1+p_{a, x} g_{a, x}\right) \log (2)}+\frac{B g_{a, 1} p_{a, 1} g_{a, x}}{\left(p_{a, x} g_{a, x}\right)^{2}\left(1+p_{a, 1} g_{a, 1}+p_{a, x} g_{a, x}\right) \log (2)} \tag{5.23}
\end{align*}
$$

$$
\begin{equation*}
\frac{\partial L_{A}}{\partial p_{b, x}}=-\lambda_{x b}+\frac{B g_{b, x}}{\left(1+p_{b, x} g_{b, x}\right) \log (2)}+\frac{B g_{b, 1} p_{b, 1} g_{b, x}}{\left(p_{b, x} g_{b, x}\right)^{2}\left(1+p_{b, 1} g_{b, 1}+p_{b, x} g_{b, x}\right) \log (2)} \tag{5.24}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L_{B}}{\partial p_{a, y}}=-\lambda_{y a}+\frac{B g_{a, y}}{\left(1+p_{a, y} g_{a, y}\right) \log (2)}+\frac{B g_{a, 2} p_{a, 2} g_{a, y}}{\left(p_{a, y} g_{a, y}\right)^{2}\left(1+p_{a, 2} g_{a, 2}+p_{a, y} g_{a, y}\right) \log (2)} \tag{5.25}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L_{B}}{\partial p_{b, y}}=-\lambda_{y b}+\frac{B g_{b, y}}{\left(1+p_{b, y} g_{b, y}\right) \log (2)}+\frac{B g_{b, 2} p_{b, 2} g_{b, y}}{\left(p_{b, y} g_{b, y}\right)^{2}\left(1+p_{b, 2} g_{b, 2}+p_{b, y} g_{b, y}\right) \log (2)} \tag{5.26}
\end{equation*}
$$

$$
\frac{\partial L_{A}}{\partial \lambda 1}=-p_{a, 1}-p_{b, 1}+P_{1}^{\max }
$$

$$
\frac{\partial L_{B}}{\partial \lambda 2}=-p_{a, 2}-p_{b, 2}+P_{2}^{\max }
$$

$$
\begin{equation*}
\frac{\partial L_{A}}{\partial \lambda a, x}=-p_{a, x}+P_{x a}^{\max } \tag{5.29}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L_{A}}{\partial \lambda b, x}=-p_{b, x}+P_{x b}^{\max } \tag{5.30}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L_{B}}{\partial \lambda a, y}=-p_{a, y}+P_{y a}^{\max } \tag{5.31}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L_{B}}{\partial \lambda b, y}=-p_{b, y}+P_{y b}^{\max } \tag{5.32}
\end{equation*}
$$

If we solve the set of equations from (5.19) to (5.32), we can obtain a closed form solution. We can see from the objective function in (5.10) that the function is monotonically increasing if we assign the maximum available power to the overlay users by
making equations from (5.27) to (5.32) equal to zero. Therefore, we can reach the solution by solving the equations and we obtain:

$$
\begin{align*}
& p_{a, x}-P_{a x}^{\max }=0, p_{a, x}=P_{a x}^{\max }  \tag{5.33}\\
& p_{b, x}-P_{b x}^{\max }=0, p_{b, x}=P_{b x}^{\max }  \tag{5.34}\\
& p_{a, y}-P_{a y}^{\max }=0, p_{a, y}=P_{a y}^{\max }  \tag{5.35}\\
& p_{b, y}-P_{b y}^{\max }=0, p_{b, y}=P_{b y}^{\max } \tag{5.36}
\end{align*}
$$

After solving (5.19) to (5.32), the closed form solution can be obtained for the underlay users and presented as:

$$
\begin{align*}
& p_{a, 1}=\frac{\left(g_{a, 1}\right)-\left(g_{b, 1}\right)+\left(g_{b, 1} g_{a, x} p_{a, x}\right)+\left(g_{a, 1} g_{b, x} p_{b, x}\right)+\left(g_{a, 1} g_{b, 1} P_{1}^{\max }\right)}{\left(2 g_{a, 1} g_{b, 1}\right)}  \tag{5.37}\\
& p_{b, 1}=\frac{\left(-g_{a, 1}\right)+\left(g_{b, 1}\right)+\left(g_{b, 1} g_{a, x} p_{a, x}\right)-\left(g_{a, 1} g_{b, x} p_{b, x}\right)+\left(g_{a, 1} g_{b, 1} P_{1}^{\max }\right)}{\left(2 g_{a, 1} g_{b, 1}\right)}  \tag{5.38}\\
& p_{a, 2}=\frac{\left(g_{a, 2}\right)-\left(g_{b, 2}\right)+\left(g_{b, 2} g_{a, y} p_{a, y}\right)+\left(g_{a, 2} g_{b, y} p_{b, y}\right)+\left(g_{a, 2} g_{b, 2} P_{2}^{\max }\right)}{\left(2 g_{a, 2} g_{b, 2}\right)}  \tag{5.39}\\
& p_{b, 2}=\frac{\left(-g_{a, 2}\right)+\left(g_{b, 2}\right)+\left(g_{b, 2} g_{a, y} p_{a, y}\right)-\left(g_{a, 2} g_{b, y} p_{b, y}\right)+\left(g_{a, 2} g_{b, 2} P_{2}^{\max }\right)}{\left(2 g_{a, 2} g_{b, 2}\right)} \tag{5.40}
\end{align*}
$$

As we can see from the solution presented above, no constraints with respect to any target rates are considered. The closed form solution is presented in Algorithm 5 as follows:

```
Algorithm 5 Closed form power allocation algorithm for PDMA
    Inputs:
        \(K, J, h\)
    2: Obtain \(F\) (the pattern allocation)
    Initialize:
        \(p_{a, 1}=0, p_{b, 1}=0, p_{a, x}=0, p_{b, x}=0\) and
\(p_{a, 2}=0, p_{b, 2}=0, p_{a, y}=0, p_{b, y}=0\)
    Solve Section A and Section B simultaneously
    Solve \(p_{a, x}\) and \(p_{b, x}\) based on eq. (5.33) and (5.34)
    Solve \(p_{a, y}\) and \(p_{b, y}\) based on eq. (5.35) and (5.36)
    Solve \(p_{a, 1}\) and \(p_{b, 1}\) based on eq. (5.37) and (5.38)
    Solve \(p_{a, 2}\) and \(p_{b, 2}\) based on eq. 5.39 and (5.40
    Obtain \(p_{a, 1}, p_{b, 1}, p_{a, x}, p_{b, x}, p_{a, 2}, p_{b, 2}, p_{a, y}\) and \(p_{b, y}\)
```

Even though we have obtained a closed form solution, it is vital to take into consideration individual rates per user, pursuing fairness between them. Therefore, with the objective of maximising the sum rate of the PDMA system considering fairness, a power allocation scheme with minimum rate and maximum power constraints will be presented.

### 5.4.2 Problem solution using Lagrange dual solution and subgradient algorithm in PDMA for power allocation with minimum rate constraint

The optimisation problem can be split into two parts and formulated as before, but this time addressing the previously mentioned constraints:

$$
\begin{equation*}
\max _{p_{k, j}} R_{\text {tot } 4 \times 6}=R_{A}+R_{B}, \tag{5.41}
\end{equation*}
$$

subject to:

$$
\begin{gather*}
\sum_{k=1}^{K} p_{k, j} \leq P_{j}^{\max }, \\
p_{k, j}>0 \tag{5.43}
\end{gather*}
$$

$$
\begin{align*}
& \sum_{k=1}^{K} R_{k, j} \geq r_{m i n}  \tag{5.44}\\
& R_{k, j} \geq r_{m i n s c} \tag{5.45}
\end{align*}
$$

where (5.42) represents the total transmission power constraint and (5.43) ensures that the power values are positive. Constraint (5.44) establishes that each user will achieve their minimum transmission rate. Constraint (5.45) ensures the selected subcarriers are not given zero power in order to mantain the PDMA codeword pattern. This is done by achieving a minimum target rate per subcarrier.

Based on the optimisation problem above, the Lagrangian function of the problem, including the constraints (5.42), (5.43), (5.44) and (5.45), can be split into Sections A and $B$ and presented as:

Lagrange function for Section A

$$
\begin{gather*}
L_{A}\left(p_{a, 1}, p_{b, 1}, p_{a, x}, p_{b, x}, \lambda_{1}, \lambda_{x a}, \lambda_{x b}, \tau_{1}, \tau_{x a}, \tau_{x b}, \gamma_{a 1}, \gamma_{b 1}\right)=B\left(\log _{2}\left(1+p_{a, 1} g_{a, 1}+p_{a, x} g_{a, x}\right)\right. \\
\left.+\log _{2}\left(1+p_{b, 1} g_{b, 1}+p_{b, x} g_{b, x}\right)\right)-\lambda_{1}\left(p_{a, 1}+p_{b, 1}-P_{1}^{\max }\right) \\
-\lambda_{x a}\left(p_{a, x}-P_{x a}^{\max }\right)-\lambda_{x b}\left(p_{b, x}-P_{x b}^{\max x}\right)- \\
\tau_{1}\left(r_{\min }-B\left(\log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)+\log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)\right)\right) \\
\tau_{x a}\left(r_{\min }-B \log _{2}\left(1+p_{a, x} g_{a, x}\right)\right)-\tau_{x b}\left(r_{\min }-B \log _{2}\left(1+p_{b, x} g_{b, x}\right)\right) \\
\gamma_{a 1}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)\right)-\gamma_{b 1}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)\right) \tag{5.46}
\end{gather*}
$$

and Lagrange function for Section B

$$
\begin{gathered}
L_{B}\left(p_{a, 2}, p_{b, 2}, p_{a, y}, p_{b, y}, \lambda_{2}, \lambda_{y a}, \lambda_{y b}, \tau_{2}, \tau_{y a}, \tau_{y b}, \gamma_{a 2}, \gamma_{b 2}\right)=B\left(\log _{2}\left(1+p_{a, 2} g_{a, 2}+p_{a, y} g_{a, y}\right)\right. \\
\left.+\log _{2}\left(1+p_{b, 2} g_{b, 2}+p_{b, y} g_{b, y}\right)\right)-\lambda_{2}\left(p_{a, 2}+p_{b, 2}-P_{2}^{\max }\right) \\
-\lambda_{y a}\left(p_{a, y}-P_{y a}^{\max }\right)-\lambda_{y b}\left(p_{b, y}-P_{y b}^{\max }\right)-
\end{gathered}
$$

$$
\begin{gather*}
\tau_{2}\left(r_{\text {min }}-B\left(\log _{2}\left(1+\frac{p_{a, 2} g_{a, 2}}{1+p_{a, y} g_{a, y}}\right)+\log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)\right)\right) \\
\tau_{y a}\left(r_{\text {min }}-B \log _{2}\left(1+p_{a, y} g_{a, y}\right)\right)-\tau_{y b}\left(r_{\text {min }}-B \log _{2}\left(1+p_{b, y} g_{b, y}\right)\right) \\
\gamma_{a 2}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{a, 2} g_{a, 2}}{1+p_{a, y} g_{a, y}}\right)\right)-\gamma_{b 2}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)\right) \tag{5.47}
\end{gather*}
$$

where $\lambda_{1}, \lambda_{x a}, \lambda_{x b}, \tau_{1}, \tau_{x a}, \tau_{x b}, \gamma_{a 1}$ and $\gamma_{b 1}$ are the Lagrange multipliers for section A and $\lambda_{2}, \lambda_{y a}, \lambda_{y b}, \tau_{2}, \tau_{y a}, \tau_{y b}, \gamma_{a 2}$ and $\gamma_{b 2}$ for Section B, respectively. To solve the optimisation problem in 5.10, we obtain the KKT conditions as follows:

For Section A:

$$
\begin{align*}
\frac{\partial L_{A}}{\partial p_{a, 1}} & =-\lambda_{1}+\frac{B g_{a, 1}\left(1+\tau_{1}+\gamma_{a 1}\right)}{\left(1+g_{a, 1} p_{a, 1}+g_{a, x} p_{a, x}\right) \log (2)}=0  \tag{5.48}\\
\frac{\partial L_{A}}{\partial p_{a, 2}} & =-\lambda_{1}+\frac{B g_{b, 1}\left(1+\tau_{1}+\gamma_{b 1}\right)}{\left(1+g_{b, 1} p_{b, 1}+g_{b, x} p_{b, x}\right) \log (2)}=0 \tag{5.49}
\end{align*}
$$

$$
\frac{\partial L_{A}}{\partial p_{a, x}}=-\lambda_{x a}+\frac{B g_{a, x}\left(1+\tau_{x a}\right)}{\left(1+g_{a, x} p_{a, x}\right) \log (2)}
$$

$$
\begin{equation*}
+\frac{B g_{a, x} g_{a, 1}(1+\gamma a 1)}{\left(\left(1+g_{a, x} p_{a, x}\right)^{2}+\left(1+g_{a, x} p_{a, x}\right)\left(g_{a, 1} p_{a, 1}\right)\right) \log (2)}=0 \tag{5.50}
\end{equation*}
$$

$$
\begin{aligned}
\frac{\partial L_{A}}{\partial p_{b, x}}=-\lambda_{x b}+\frac{B g_{b, x}\left(1+\tau_{x b}\right)}{(1}+ & \left.g_{b, x} p_{b, x}\right) \log (2) \\
& \quad+\frac{B g_{b, x} g_{b, 1}(1+\gamma a 2)}{\left(\left(1+g_{b, x} p_{b, x}\right)^{2}+\left(1+g_{b, x} p_{b, x}\right)\left(g_{b, 1} p_{b, 1}\right)\right) \log (2)}=0
\end{aligned}
$$

$$
\lambda_{1}\left(p_{a, 1}+p_{b, 1}-P_{1}^{\max }\right)=0
$$

$$
\begin{equation*}
\lambda_{a x}\left(p_{a, x}-P_{a x}^{\max }\right)=0 \tag{5.53}
\end{equation*}
$$

$$
\begin{gather*}
\lambda_{b x}\left(p_{b, x}-P_{b x}^{\max }\right)=0  \tag{5.54}\\
\tau_{1}\left(r_{\text {min }}-B\left(\log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)+\log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)\right)\right)=0  \tag{5.55}\\
\tau_{a x}\left(r_{\text {min }}-B \log _{2}\left(1+p_{a, x} g_{a, x}\right)\right)=0  \tag{5.56}\\
\tau_{b x}\left(r_{\text {min }}-B \log _{2}\left(1+p_{b, x} g_{b, x}\right)\right)=0  \tag{5.57}\\
\gamma_{a 1}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)\right)=0  \tag{5.58}\\
\gamma_{b 1}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)\right)=0 \tag{5.59}
\end{gather*}
$$

and the KKT conditions for the Lagrange function of Section B

$$
\begin{align*}
\frac{\partial L_{B}}{\partial p_{a, 2}} & =-\lambda_{2}+\frac{B g_{a, 2}\left(1+\tau_{2}+\gamma_{a 2}\right)}{\left(1+g_{a, 2} p_{a, 2}+g_{a, y} p_{a, y}\right) \log (2)}  \tag{5.60}\\
\frac{\partial L_{B}}{\partial p_{b, 2}} & =-\lambda_{2}+\frac{B g_{b, 2}\left(1+\tau_{2}+\gamma_{b 2}\right)}{\left(1+g_{b, 2} p_{b, 2}+g_{b, y} p_{b, y}\right) \log (2)} \tag{5.61}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial L_{B}}{\partial p_{a, y}}=-\lambda_{y a}+\frac{B g_{a, y}\left(1+\tau_{y a}\right)}{\left(1+g_{a, y} p_{a, y}\right) \log (2)} \\
& \quad+\frac{B g_{a, y} g_{a, 2}\left(1+\gamma_{a 2}\right)}{\left(\left(1+g_{a, y} p_{a, y}\right)^{2}+\left(1+g_{a, y} p_{a, y}\right)\left(g_{a, 2} p_{a, 2}\right)\right) \log (2)} \tag{5.62}
\end{align*}
$$

$$
\frac{\partial L_{B}}{\partial p_{b, y}}=-\lambda_{y b}+\frac{B g_{b, y}\left(1+\tau_{y b}\right)}{\left(1+g_{b, y} p_{b, y}\right) \log (2)}
$$

$$
\begin{equation*}
+\frac{B g_{b, y} g_{a, 2}\left(1+\gamma_{a 2}\right)}{\left(\left(1+g_{b, y} p_{b, y}\right)^{2}+\left(1+g_{b, y} p_{b, y}\right)\left(g_{b, 2} p_{b, 2}\right)\right) \log (2)} \tag{5.63}
\end{equation*}
$$

$$
\begin{gather*}
\lambda_{2}\left(p_{a, 2}+p_{b, 2}-P_{2}^{\max }\right)=0  \tag{5.64}\\
\lambda_{a y}\left(p_{a, y}-P_{a y}^{\max }\right)=0  \tag{5.65}\\
\lambda_{b y}\left(p_{b, y}-P_{b y}^{\max }\right)=0  \tag{5.66}\\
\tau_{2}\left(r_{\text {min }}-B\left(\log _{2}\left(1+\frac{p_{a, 2} g_{a, 2}}{1+p_{a, y} g_{a, y}}\right)+\log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)\right)\right)=0  \tag{5.67}\\
\tau_{a y}\left(r_{\text {min }}-B \log _{2}\left(1+p_{a, y} g_{a, y}\right)\right)=0  \tag{5.68}\\
\tau_{b y}=\left(r_{\text {min }}-B \log _{2}\left(1+p_{b, y} g_{b, y}\right)=0\right.  \tag{5.69}\\
\gamma_{a 2}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{a, 2} g_{a, 2}}{1+p_{a, y} g_{a, y}}\right)\right)=0  \tag{5.70}\\
\gamma_{b 2}\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)\right)=0 \tag{5.71}
\end{gather*}
$$

In order to obtain an optimal solution, complex numerical calculations are required to solve the optimisation problem presented in equation (5.41) with power and minimum rate constraints. In the next section, we present a solution based on subgradient algorithm to obtain a suboptimal solution for power allocation [82].

From the obtained KKT conditions in (5.48) to (5.59) in Section A and (5.60) to (5.71) in Section B, the power allocation at each subcarrier can be expressed as:

$$
\begin{align*}
& p_{a, 1}=\frac{B\left(\left(g_{a, x} \lambda_{1}\right)-\left(g_{a, 1} \lambda_{a x}\right)-\left(g_{a, 1} \lambda_{a x} \tau_{1}\right)+\left(g_{a, x} \lambda_{1} \tau_{a x}\right)-\left(g_{a, 1} \lambda_{a x} \gamma_{a 1}\right)\right)}{\lambda_{1}\left(g_{a, x} \lambda_{1}-g_{a, 1} \lambda_{a x}\right) \log (2)}  \tag{5.72}\\
& p_{b, 1}=\frac{B\left(\left(g_{b, x} \lambda_{1}\right)-\left(g_{b, 1} \lambda_{b x}\right)-\left(g_{b, 1} \lambda_{b x} \tau_{1}\right)+\left(g_{b, x} \lambda_{1} \tau_{b x}\right)-\left(g_{b, 1} \lambda_{b x} \gamma_{b 1}\right)\right)}{\lambda_{1}\left(g_{b, x} \lambda_{1}-g_{b, 1} \lambda_{b x}\right) \log (2)} \tag{5.73}
\end{align*}
$$

$$
\begin{equation*}
p_{a, x}=\frac{B\left(-\left(g_{a, 1} g_{a, x} \tau_{1}\right)+\left(g_{a, 1} g_{a, x} \tau_{a x}\right)-\left(g_{a, 1} g_{a, x} \gamma_{a 1}\right)+\left(g_{a, x} \lambda_{1} \log (2)\right)-\left(g_{a, 1} \lambda_{a x} \log (2)\right)\right)}{g_{a, x}\left(g_{a, x} \lambda_{1}-g_{a, 1} \lambda_{x a}\right) \log (2)} \tag{5.74}
\end{equation*}
$$

$$
\begin{equation*}
p_{b, x}=\frac{B\left(-\left(g_{b, 1} g_{b, x} \tau_{1}\right)+\left(g_{b, 1} g_{b, x} \tau_{b x}\right)-\left(g_{b, 1} g_{b, x} \gamma_{b 1}\right)+\left(g_{b, x} \lambda_{1} \log (2)\right)-\left(g_{b, 1} \lambda_{b x} \log (2)\right)\right)}{g_{b, x}\left(g_{b, x} \lambda_{1}-g_{b, 1} \lambda_{b x}\right) \log (2)} \tag{5.75}
\end{equation*}
$$

$$
\begin{equation*}
p_{a, 2}=\frac{B\left(\left(g_{a, y} \lambda_{2}\right)-\left(g_{a, 2} \lambda_{a y}\right)-\left(g_{a, 2} \lambda_{a y} \tau_{2}\right)+\left(g_{a, y} \lambda_{2} \tau_{a y}\right)-\left(g_{a, 2} \lambda_{a y} \gamma_{a 2}\right)\right)}{\lambda_{2}\left(g_{a, y} \lambda_{2}-g_{a, 2} \lambda_{a y}\right) \log (2)} \tag{5.76}
\end{equation*}
$$

$$
\begin{equation*}
p_{b, 2}=\frac{B\left(\left(g_{b, y} \lambda_{2}\right)-\left(g_{b, 2} \lambda_{b y}\right)-\left(g_{b, 2} \lambda_{b y} \tau_{2}\right)+\left(g_{b, y} \lambda_{2} \tau_{b y}\right)-\left(g_{b, 2} \lambda_{b y} \gamma_{b 2}\right)\right)}{\lambda_{2}\left(g_{b, y} \lambda_{2}-g_{b, 2} \lambda_{b y}\right) \log (2)} \tag{5.77}
\end{equation*}
$$

$p_{a, y}=\frac{B\left(-\left(g_{a, 2} g_{a, y} \tau_{2}\right)+\left(g_{a, 2} g_{a, y} \tau_{a y}\right)-\left(g_{a, 2} g_{a, y} \gamma_{a 2}\right)+\left(g_{a, y} \lambda_{2} \log (2)\right)-\left(g_{a, 2} \lambda_{a y} \log (2)\right)\right)}{g_{a, y}\left(g_{a, y} \lambda_{2}-g_{a, 2} \lambda_{y a}\right) \log (2)}$
$p_{b, y}=\frac{B\left(-\left(g_{b, 2} g_{b, y} \tau_{2}\right)+\left(g_{b, 2} g_{b, y} \tau_{b y}\right)-\left(g_{b, 2} g_{b, y} \gamma_{b 2}\right)+\left(g_{b, y} \lambda_{2} \log (2)\right)-\left(g_{b, 2} \lambda_{b y} \log (2)\right)\right)}{g_{b, y}\left(g_{b, y} \lambda_{2}-g_{b, 2} \lambda_{y b}\right) \log (2)}$

Therefore, the problem defined in 5.41 becomes a dual problem, where the dual objective $D_{A}$ and $D_{B}$ are defined as the minimum value of the Lagrangian over the values of $p_{a, 1}, p_{b, 1}, p_{a, x}, p_{b, x}$ and $p_{a, 2}, p_{b, 2}, p_{a, y}, p_{b, y}$, which are concave even if the original problems are not convex because it is the pointwise infimum (inf) of a family of affine functions of $\lambda_{1}, \lambda_{x a}, \lambda_{x b}, \tau_{1}, \tau_{x a}, \tau_{x b}, \gamma_{a 1}, \gamma_{b 1}$ and $\lambda_{2}, \lambda_{y a}, \lambda_{y b}, \tau_{2}, \tau_{y a}, \tau_{y b}, \gamma_{a 2}, \gamma_{b 2}$ [84] and they can be presented as:

$$
\begin{align*}
& \max D_{A}\left(\lambda_{1}, \lambda_{x a}, \lambda_{x b}, \tau_{1}, \tau_{x a}, \tau_{x b}, \gamma_{a 1}, \gamma_{b 1}\right)= \inf _{p_{a, 1}, p_{b, 1}, p_{a, x}, p_{b, x}} \\
& L_{A}\left(p_{a, 1}, p_{b, 1}, p_{a, x}, p_{b, x}, \lambda_{1}, \lambda_{x a}, \lambda_{x b}, \tau_{1}, \tau_{x a}, \tau_{x b}, \gamma_{a 1}, \gamma_{b 1}\right) \tag{5.80}
\end{align*}
$$

subject to:

$$
\begin{equation*}
\lambda_{1}>0, \lambda_{x a}>0 \text { and } \lambda_{x b}>0 \tag{5.81}
\end{equation*}
$$

and

$$
\begin{align*}
\max D_{B}\left(\lambda_{2}, \lambda_{y a}, \lambda_{y b}, \tau_{2}, \tau_{y a}, \tau_{y b}, \gamma_{a 2}, \gamma_{b 2}\right)=\inf _{p_{a, 2}, p_{b, 2}, p_{a, y}, p_{b, y}} \\
L_{B}\left(p_{a, 2}, p_{b, 2}, p_{a, y}, p_{b, y}, \lambda_{2}, \lambda_{y a}, \lambda_{y b}, \tau_{2}, \tau_{y a}, \tau_{y b}, \gamma_{a 2}, \gamma_{b 2}\right) \tag{5.82}
\end{align*}
$$

subject to:

$$
\begin{equation*}
\lambda_{2}>0, \lambda_{y a}>0 \text { and } \lambda_{y b}>0 \tag{5.83}
\end{equation*}
$$

The Lagrange multipliers also termed as the dual variables $\lambda_{1}, \lambda_{x a}, \lambda_{x b}, \tau_{1}, \tau_{x a}, \tau_{x b}$, $\gamma_{a 1}, \gamma_{b 1}, \lambda_{2}, \lambda_{y a}, \lambda_{y b}, \tau_{2}, \tau_{y a}, \tau_{y b}, \gamma_{a 2}$ and $\gamma_{b 2}$ can be solved by the subgradient method. To implement the algorithm, it is necessary to obtain the subgradients, presented as follows [84-86]:

$$
\begin{gather*}
\nabla \lambda_{1}=p_{a, 1}+p_{b, 1}-P_{1}^{\max }  \tag{5.84}\\
\nabla \lambda_{a x}=p_{a, x}-P_{a x}^{\max }  \tag{5.85}\\
\nabla \lambda_{b x}=p_{b, x}-P_{b x}^{\max }  \tag{5.86}\\
\nabla \tau_{1}=\left(r_{\min }-B\left(\log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)+\log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)\right)\right)  \tag{5.87}\\
\nabla \tau_{a x}=\left(r_{\min }-B \log _{2}\left(1+p_{a, x} g_{a, x}\right)\right) \tag{5.88}
\end{gather*}
$$

$$
\begin{gather*}
\nabla \tau_{b x}=\left(r_{\text {min }}-B \log _{2}\left(1+p_{b, x} g_{b, x}\right)\right)  \tag{5.89}\\
\nabla \gamma_{a 1}=\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{a, 1} g_{a, 1}}{1+p_{a, x} g_{a, x}}\right)\right)  \tag{5.90}\\
\nabla \gamma_{b 1}=\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{b, 1} g_{b, 1}}{1+p_{b, x} g_{b, x}}\right)\right)  \tag{5.91}\\
\nabla \lambda_{2}=p_{a, 2}+p_{b, 2}-P_{2}^{\max } \tag{5.92}
\end{gather*}
$$

$$
\begin{equation*}
\nabla \lambda_{a y}=p_{a, y}-P_{a y}^{\max } \tag{5.93}
\end{equation*}
$$

$$
\begin{equation*}
\nabla \lambda_{b y}=p_{b, y}-P_{b y}^{\max } \tag{5.94}
\end{equation*}
$$

$$
\begin{equation*}
\nabla \tau_{2}=\left(r_{\min }-B\left(\log _{2}\left(1+\frac{p_{a, 2} g_{a, 2}}{1+p_{a, y} g_{a, y}}\right)+\log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)\right)\right) \tag{5.95}
\end{equation*}
$$

$$
\begin{equation*}
\nabla \tau_{a y}=\left(r_{\min }-B \log _{2}\left(1+p_{a, y} g_{a, y}\right)\right) \tag{5.96}
\end{equation*}
$$

$$
\begin{equation*}
\nabla \tau_{b y}=\left(r_{\min }-B \log _{2}\left(1+p_{b, y} g_{b, y}\right)\right. \tag{5.97}
\end{equation*}
$$

$$
\begin{equation*}
\nabla \gamma_{a 2}=\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{a, 2} g_{a, 2}}{1+p_{a, y} g_{a, y}}\right)\right) \tag{5.98}
\end{equation*}
$$

$$
\begin{equation*}
\nabla \gamma_{b 2}=\left(r_{\text {minsc }}-B \log _{2}\left(1+\frac{p_{b, 2} g_{b, 2}}{1+p_{b, y} g_{b, y}}\right)\right) \tag{5.99}
\end{equation*}
$$

The obtained subgradients are used to implement the subgradient method. In the algorithm, the dual variables are initialised and updated in each iteration through the following equations.

## Section A:

$$
\begin{gather*}
\lambda_{1}(t+1)=\left[\lambda_{1}(t)+\alpha_{\lambda_{1}} \nabla \lambda_{1}\right]^{+}  \tag{5.100}\\
\lambda_{a x}(t+1)=\left[\lambda_{a x}(t)+\alpha_{\lambda_{a x}} \nabla \lambda_{a x}\right]^{+}  \tag{5.101}\\
\lambda_{b x}(t+1)=\left[\lambda_{b x}(t)+\alpha_{\lambda_{b x}} \nabla \lambda_{b x}\right]^{+}  \tag{5.102}\\
\tau_{1}(t+1)=\left[\tau_{1}(t)+\alpha_{\tau_{1}} \nabla \tau_{1}\right]  \tag{5.103}\\
\tau_{a x}(t+1)=\left[\tau_{a x}(t)+\alpha_{\tau_{a x}} \nabla \tau_{a x}\right]  \tag{5.104}\\
\tau_{b x}(t+1)=\left[\tau_{b x}(t)+\alpha_{\tau_{b x}} \nabla \tau_{b x}\right]  \tag{5.105}\\
\gamma_{a 1}(t+1)=\left[\gamma_{a 1}(t)+\alpha_{\gamma_{a 1}} \nabla \gamma_{a 1}\right]  \tag{5.106}\\
\gamma_{b 1}(t+1)=\left[\gamma_{b 1}(t)+\alpha_{\gamma_{b 1}} \nabla \gamma_{b 1}\right] \tag{5.107}
\end{gather*}
$$

## Section B:

$$
\begin{gather*}
\lambda_{2}(t+1)=\left[\lambda_{2}(t)+\alpha_{\lambda_{2}} \nabla \lambda_{2}\right]^{+}  \tag{5.108}\\
\lambda_{a y}(t+1)=\left[\lambda_{a y}(t)+\alpha_{\lambda_{a y}} \nabla \lambda_{a y}\right]^{+}  \tag{5.109}\\
\lambda_{b y}(t+1)=\left[\lambda_{b y}(t)+\alpha_{\lambda_{b y}} \nabla \lambda_{b y}\right]^{+}  \tag{5.110}\\
\tau_{2}(t+1)=\left[\tau_{2}(t)+\alpha_{\tau_{2}} \nabla \tau_{2}\right] \tag{5.111}
\end{gather*}
$$

$$
\begin{align*}
& \tau_{a y}(t+1)=\left[\tau_{a y}(t)+\alpha_{\tau_{a y}} \nabla \tau_{a y}\right]  \tag{5.112}\\
& \tau_{b y}(t+1)=\left[\tau_{b y}(t)+\alpha_{\tau_{b y}} \nabla \tau_{b y}\right]  \tag{5.113}\\
& \gamma_{a 2}(t+1)=\left[\gamma_{a 2}(t)+\alpha_{\gamma_{a 2}} \nabla \gamma_{a 2}\right]  \tag{5.114}\\
& \gamma_{b 2}(t+1)=\left[\gamma_{b 2}(t)+\alpha_{\gamma_{b 2}} \nabla \gamma_{b 2}\right] \tag{5.115}
\end{align*}
$$

From the equations, $t$ is defined as the iteration index and the variables $\alpha$ are the positive step sizes respectively to each subgradient and [.] ${ }^{+}$denotes the projection onto the non-negative orthant [84].

The subgradient algorithm is presented in Algorithm 6as follows:

```
Algorithm 6 Subgradient power allocation algorithm for PDMA
```

    Inputs:
        \(K, J, d v, d f, h, e\)
    Obtain \(F\) (the pattern allocation)
    Initialize:
        \(t=0, \lambda_{1}=\) init,\(\lambda_{x a}=\) init,\(\lambda_{x b}=\) init,\(\tau_{1}=\) init, \(\tau_{x a}=\) init,\(\tau_{x b}=\)
        init,\(\gamma_{a 1}=\) init,\(\gamma_{b 1}=\) init,\(\lambda_{2}=\) init,\(\lambda_{y a}=\) init,\(\lambda_{y b}=\) init,\(\tau_{2}=\)
        init \(, \tau_{y a}=i n i t, \tau_{y b}=\) init,\(\gamma_{a 2}=\) init,\(\gamma_{b 2}=i n i t\)
    do
        Solve Section A and Section B simultaneously
        Solve \(p_{a, 1}, p_{b, 1}, p_{a, x}\) and \(p_{b, x}\) based on eq. (5.72), (5.73), (5.74) and (5.75)
        Solve \(p_{a, 2}, p_{b, 2}, p_{a, y}\) and \(p_{b, y}\) based on eq. (5.76), (5.77), (5.78) and (5.79)
        Update \(\lambda_{1}(t+1)\) based on eq. 5.100
        Update \(\lambda_{a x}(t+1)\) based on eq. (5.101)
        Update \(\lambda_{b x}(t+1)\) based on eq. (5.102)
        Update \(\lambda_{2}(t+1)\) based on eq. (5.103)
        Update \(\lambda_{a y}(t+1)\) based on eq. (5.104)
        Update \(\lambda_{b y}(t+1)\) based on eq. (5.105)
        Update \(\tau_{1}(t+1)\) based on eq. (5.106)
        Update \(\tau_{a x}(t+1)\) based on eq. (5.107)
        Update \(\tau_{b x}(t+1)\) based on eq. (5.108)
        Update \(\tau_{2}(t+1)\) based on eq. (5.109)
        Update \(\tau_{a y}(t+1)\) based on eq. 5.110
        Update \(\tau_{b y}(t+1)\) based on eq. (5.111)
        Update \(\gamma_{a 1}(t+1)\) based on eq. (5.112)
        Update \(\gamma_{b 1}(t+1)\) based on eq. (5.113)
        Update \(\gamma_{a 2}(t+1)\) based on eq. (5.114)
        Update \(\gamma_{b 2}(t+1)\) based on eq. (5.115)
        \(t=t+1\)
    while \(\left|\lambda_{1}(t-1)-\lambda_{1}(t)\right| \leq e,\left|\lambda_{a x}(t-1)-\lambda_{a x}(t)\right| \leq e,\left|\lambda_{b x}(t-1)-\lambda_{b x}(t)\right| \leq e\),
    \(\left|\tau_{1}(t-1)-\tau_{1}(t)\right| \leq e,\left|\tau_{a x}(t-1)-\tau_{a x}(t)\right| \leq e,\left|\tau_{b x}(t-1)-\tau_{b x}(t)\right| \leq e, \mid \gamma_{a 1}(t-\)
    1) \(-\gamma_{a 1}(t)\left|\leq e,\left|\gamma_{b 1}(t-1)-\gamma_{b 1}(t)\right| \leq e\right.\) and \(| \lambda_{1}(t-1)-\lambda_{1}(t)|\leq e,| \lambda_{a y}(t-1)-\)
    \(\lambda_{a y}(t)\left|\leq e,\left|\lambda_{b y}(t-1)-\lambda_{b y}(t)\right| \leq e,\left|\tau_{2}(t-1)-\tau_{2}(t)\right| \leq e,\left|\tau_{a y}(t-1)-\tau_{a y}(t)\right| \leq\right.\)
    \(e,\left|\tau_{b y}(t-1)-\tau_{b y}(t)\right| \leq e,\left|\gamma_{a 2}(t-1)-\gamma_{a 2}(t)\right| \leq e,\left|\gamma_{b 2}(t-1)-\gamma_{b 2}(t)\right| \leq e\)
    26: Obtain $\lambda_{1}^{*}, \lambda_{x a}^{*}, \lambda_{x b}^{*}, \tau_{1}^{*}, \tau_{x a}^{*}, \tau_{x b}^{*}, \gamma_{a 1}^{*}, \gamma_{b 1}^{*}, \lambda_{2}^{*}, \lambda_{y a}^{*}, \lambda_{y b}^{*}, \tau_{2}^{*}, \tau_{y a}^{*}, \tau_{y b}^{*}, \gamma_{a 2}^{*}, \gamma_{b 2}^{*}$ and $p_{a, 1}^{*}, p_{b, 1}^{*}, p_{a, x}^{*}, p_{b, x}^{*}, p_{a, 2}^{*}, p_{b, 2}^{*}, p_{a, y}^{*}, p_{b, y}^{*}$ respectively

### 5.5 Numerical results

For the system level simulation and the resource allocation simulations in PDMA, we considered an uplink scenario with a single cell and $J$ users, which are uniformly distributed from the base station. The bandwidth is divided into $K$ subcarriers. The total power transmission is $P_{t}$. The wireless channel is modelled as a six path frequency selective fading channel, using the ITU pedestrian model-B as depicted in Table 2.1. The channel we have considered is quasi-static. The subcarrier allocation has been implemented as presented in Section 5.3.2. The power allocation has been implemented using three different approaches: equal power allocation, which means that if the total power available is $20 \mathrm{dBm}, 17 \mathrm{dBm}$ is assigned for transmission to one subcarrier and 17 dBm is assigned to the other, power allocation based on the closed form solution as described in Section 5.4.1 and power allocation obtained through the subgradient method as presented in 5.4.2. The simulation results have been carried out using the Monte Carlo method with the parameters listed in Table 5.1 .

| Parameters for PDMA $_{4 \times 6}$ | Values |
| :---: | :---: |
| Number of users | 6 |
| Subcarriers | 4 |
| Factor overloading | 2 |
| Factor spread for overlay users | 2 |
| Factor spread for underlay users | 1 |
| Pedestrian model | B |
| PL exponent | 3 |
| Bandwidth | 180 kHz |
| Noise | -174 dBm |
| Shadowing | 8 dB |
| Transmit power | $0-23 \mathrm{dBm}$ |
| Monte Carlo iterations | 10000 |
| Error threshold | $10^{-5}$ |

Table 5.1: Parameters of simulation for $P D M_{4 X 6}$

In previous sections, the essential role of resource optimisation in improving system performance and individual user rates has been discussed. Furthermore, in work presented in [100], the authors design a joint solution with a different pattern, $P D M A_{3 X 7}$.

The subcarrier allocation scheme is based on removing the allocated users based on their performance and the power solution is a reversed version of the water-filling method. The works in [90] have presented a $P D M A_{4 X 6}$ pattern similar to our pattern under study, however, [90] presents a different higher density in the sparse pattern and it does not explore resource allocation. The authors in [96] evaluate the performance of $P D M A_{2 \times 3}$, which is clearly a lower density pattern with fewer users. They evaluate the achievable sum rate and the outage probability per user. Taking these elements into consideration, we can see that limited work has been produced in terms of resource allocation in PDMA. Therefore, we have designed resource allocation schemes for PDMA with the aim of maximising the system performance. Similar to the solution offered for SCMA in Chapter4 we have implemented a solution for PDMA, combining our subcarrier allocation method presented in Algorithm4 and the two proposed power allocation methods. The first scheme joints our subcarrier allocation method with the first power allocation method that considers power constraints and provides a closed form solution, presented in Section 5.4.1 and implemented in Algorithm 5. The second scheme joints our subcarrier allocation method with the second power allocation method that solves the problem taking into consideration minimum rate constraints, presented in Section 5.4.2 and implemented in Algorithm 6.

The results compared in Figure 5.6 are equal power allocation against our proposed solutions; closed form solution and subgradient method. Closed form solution is one of the contributions of this work. The result is marginally better than equal power allocation, as shown in Figure 5.6. Furthermore, in the rate results per user presented in Figure 5.7, although the best user's rate is slightly worse, the weakest user's rate is higher. This provides fairness to the users in the system. While the closed form solution can achieve an optimal result, there is a drawback because users allocated to one subcarrier are given the maximum amount of the available power. An additional limitation is that the solution does not provide valid power values in approximately $30 \%$ of the simulated attempts.


Figure 5.6: Comparison of total sum rate in PDMA

On the other hand, we can see from the Figure 5.6, that although the second contribution, subgradient method, does not outperform closed form solution and equal power allocation in the more generalised problem of sum rate, it has benefits over them when specific conditions are required. The user with the strongest channel has a considerably higher rate due to the controlled interference from other users, as shown in Figure 5.7 and 5.8, in comparison with the individual users' rates from the other two power allocation approaches. An optimised power control reduces interference from the overlay users while increasing the underlay users' rate and achieving a close to optimum performance. It needs to be considered that the reduced interference implies reducing the data transmission rate for some users with less favourable channels. Nevertheless, while the subgradient method may reduce the rate of weaker users, it guarantees those users a minimum target rate transmission.

Additionally, the subgradient method achieves minimum rates for those users allocated to one subcarrier and provides a solution where the average power consumption for the users is not fully used as shown in Figure 5.9 . We can say that the reduction of the average power usage can be considered as a positive result in comparison with equal power allocation and the close form solution where the total available power is completely used.


Figure 5.7: Comparison of individual users' rates in PDMA with a closed form solution power allocation (CF), equal power allocation (EP) and a subgradient (SG) solution for power allocation. The users are ordered in decreasing order, user 1 is the user with strongest channel gain and user 6 is the user with the weakest channel gain.


Figure 5.8: Comparison of individual users' rates in PDMA with 20 dBm available power transmission. The users are ordered in decreasing order, user 1 is the user with strongest channel gain and user 6 is the user with the weakest channel gain.


Figure 5.9: Average user power consumption for equal power allocation, closed form solution and subgradient schemes.

Equal power allocation splits evenly the total available power ( $0-23 \mathrm{dBm}$ standard range for a mobile equipment) between the allocated subcarriers $(d v=2)$. This means that if the total power available is $20 \mathrm{dBm}, 17 \mathrm{dBm}$ is assigned for transmission to one subcarrier and 17 dBm is assigned to the other. Equal power allocation can be an option and was presented as a benchmark, but the aim of this work was to propose an optimised solution without rate constraints and a second one that results in an improved method when there is a requirement for minimum rates to be achieved for all users.

### 5.6 Chapter summary

In this chapter, the PDMA system is explored and described. We have discussed in previous chapters the characteristics of multiple user systems and the undesirable effects of users sharing and competing for the same resources, as is the case with PDMA. As a result, resource allocation solutions have been designed for a particular scenario with similar characteristics to the SCMA system presented in Chapter 3. A low complexity subcarrier solution has been presented and two power allocation schemes have been propsed; a closed form solution with power constraint and a subgradient based solution with additional minimum rate constraints in order to reduce the effect of interference.

## Chapter 6

## Conclusions and future works

### 6.1 Conclusions

Clearly, the high demands of upcoming generations of mobile communications will require massive connectivity of multiple devices simultaneously in order to solve the expectations of multiple users within the public and industry sector. Through this work, we have studied the multiple access techniques that allocate users non-orthogonally. NOMA can theoretically support an unlimited number of simultaneous connections and thus can cause high levels of interference, degrading the performance of the system and affecting the rate transmission of individual devices. We have presented some of the schemes that operate on the code domain in non-orthogonal systems with the purpose of understanding their implementations, characteristics and similarities. Techniques such as SCMA and PDMA were evaluated because of NOMA's practical limitations in the number of users that can be assigned to a specific frequency. Through the study of coded domain schemes, we have seen that several users can transmit data simultaneously and exploit the effect of assigning them to a group of resources if they are assigned adequately.

SCMA and PDMA present similarities because they both work in the code domain and use factor graphs. Nevertheless, through this work we observed that in SCMA, even though multiple users can be allocated over the same resources, because of the restriction of codebooks, different approaches are needed to solve particular resource allocation problems.

A low complexity uplink resource allocation scheme for SCMA systems is proposed
with the objective of allocating subcarriers and power effectively over a set of users. According to the simulation results, the joint allocation scheme can achieve an increased total rate without the drawbacks of high computational complexity.

In SCMA, a power allocation solution was proposed based on water-filling and subgradient algorithm, where the interference term is updated through each iteration instead of considering the interference coming from all users at the same time. Furthermore, the power allocation scheme itself is versatile as there is an overall rate improvement regardless of the subcarrier allocation method in use. An improvement in the overall rate has been seen when compared to the equal power and typical water-filling methods, while ensuring a minimum transmission rate is achieved for each user and optimising the use of limited power within the mobile equipment.

In contrast, in PDMA we managed to obtain a closed form solution that offers an optimal solution to tackle the power allocation problem when there are no minimum rate requirements. Additionally, the second method is implemented considering the interference in the overlay users in the subgradient algorithm at the same time. This scheme manages to improve the strongest user's rate and keep control of the interference while achieving the minimum rate for the rest of the users.

### 6.2 Future works

The continuous increase in requirements and development of new technology solutions has created an open area of research in the mobile communications sector. This thesis studied the resource allocation problem for maximising the sum rate in code domain NOMA systems such as SCMA and PDMA. There are numerous other features and applications of non-orthogonal access in the uplink that have not been considered in this work, but would be interesting fields of study for additional research. Potential extensions to this work and future areas of research are elaborated further below.

### 6.2.1 Coded NOMA with multiple antennas

SCMA and PDMA as potential solutions for multi-user data transmission can be implemented jointly with other spatial access techniques such as SIMO, MISO and MIMO. In recent decades, mobile devices and base stations have been equipped with multiple antennas for transmission and reception. However, further implementation will be required, considering the characteristics of coded NOMA. In SCMA and PDMA uplink,
the allocation of more than one user at a time can be viewed as a case of MISO itself when the signals from different users are considered to behave as multiple transmitting antennas. Although it is common to find base stations with two or more antennas at the receiver, a scenario where SCMA or PDMA are combined with multiple uplink mobile devices can be implemented and evaluated as a special case of MIMO.

### 6.2.2 Massive connectivity and resource management

A common scenario for wireless communications in the uplink occurs when several devices are transmitting data at the same time with ultra reliable and low latency transmission. This requires multiple transmitters, allocated over a limited number of subcarriers. We have shown before that with the appropriate interference management and with accurate resource allocation the performance of the system can improve considerably. Regarding the issue of several users transmitting simultaneously, this work has presented a solution based on radio resource allocation. This should be developed and evaluated, considering practical scenarios which could accommodate a large number of users whilst maintaining the reliability and latency characteristics. Additionally, an increase in users can cause a change in codebook patterns, which may have a negative impact on the system. Therefore, further research on scenarios with these characteristics should be pursued.

### 6.2.3 Cache-aided D2D communication

D2D caching is widely studied and has the benefits of allowing multiple users to store popular content locally beforehand [102]. Since it is difficult to predict with full accuracy which data to cache locally, it would be interesting to cache a large variety of content at the user equipment. This cached content can then exploit the uplink channel, using SCMA or PDMA resource allocation to allow groups of users to communicate and exchange the cached content with each other during peak times. This further opens the pathway into content caching strategies, in addition to how coded NOMA could be implemented with D2D communications.

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## Appendix A

## Multiplexed signal for SCMA system

We provide a numerical example of the resultant signal $y$ when number of users is $J=6$ and the number of subcarriers is $K=4$.
First, we select the channel gain of the matrix $h$ for each user (column vector of $h$ ) and we expand it in a diagonal matrix to multiply
with the vector codebook $x$ and we add the noise $n$ as shown below:

$$
y=\sum_{j=1}^{J} h_{j} x_{j}+n
$$

Substituting in example values, we have:



| $h_{3}$ |
| :---: |
| $\left[\begin{array}{cccc}0.1125+0.5740 i & 0 & 0 & 0 \\ 0 & 0.4367-0.3672 i & 0 & 0 \\ 0 & 0 & 0.1330+0.5629 i & 0 \\ 0 & 0 & 0 & -1.4203+0.0462 i\end{array}\right]$ |

$\frac{x_{3}(00)}{\left[\begin{array}{c}0.0000+0.0000 i \\ 0.0000+0.0000 i \\ -0.2101-0.7842 i \\ 0.1654-0.2148 i\end{array}\right]}+$

|  | $h_{4}$ |  |  |  | $x_{4}(11)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left[\begin{array}{c}0.6695+0.0899 i \\ 0 \\ 0 \\ 0\end{array}\right.$ | 0 $-0.8171-0.1364 i$ 0 0 | 0 0 $1.2968-0.4847 i$ 0 | $\left.\begin{array}{c}0 \\ 0 \\ 0 \\ -0.3400+0.8590 i\end{array}\right]$ | $\left[\begin{array}{l}0.6461+0.6461 i \\ 0.2418-0.1862 i \\ 0.0000+0.0000 i \\ 0.0000+0.0000 i\end{array}\right]+$ |
|  |  | $h_{5}$ |  |  | $x_{5}(10)$ |
| t | $\left[\begin{array}{c}-1.5593-0.9230 i \\ 0 \\ 0 \\ 0\end{array}\right.$ | 0 $-0.3528+1.0807 i$ 0 0 | iccole $\begin{gathered}0 \\ \\ \\ 1.1343+0.1525 i \\ \\ \\ 0\end{gathered}$ | $\left.\begin{array}{cc}0 \\ 0 \\ 0 \\ 0.0545-0.7113 i\end{array}\right]$ | $\left[\begin{array}{c}-0.1131+0.4222 i \\ 0.0000+0.0000 i \\ 0.0000+0.0000 i \\ -0.5007-1.2116 i\end{array}\right]$ |
|  |  | $h_{6}$ |  |  | $x_{6}(11)$ |
|  | $\left[\begin{array}{c}0.2236-0.8103 i \\ 0 \\ 0 \\ 0\end{array}\right.$ | 0 $0.2886+0.8725 i$ 0 0 | 0 0 $0.4336+1.0664 i$ 0 | $\left.\begin{array}{c}0 \\ 0 \\ 0 \\ -0.2731-0.6498 i\end{array}\right]$ | $\left[\begin{array}{c}0.0000+0.0000 i \\ -0.3862+1.4413 i \\ 0.0000+0.0000 i \\ 0.1899-0.4607 i\end{array}\right]+$ |

$$
\frac{n}{\left[\begin{array}{c}
-0.9461+0.9908 i \\
-0.2166-0.2770 i \\
0.0596-0.0493 i \\
0.3083-1.5869 i
\end{array}\right]}
$$

Once each channel gain per user has been multiplied by the column vector of the codebook we add the resultant values and the noise column vector. We then obtain the resultant multiplexed signal as follows:

|  | User 1 | User 2 | User3 | User 4 | User 5 | User 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pm$ | $y=\left[\begin{array}{l}0.2933+0.1011 i \\ 0.0000+0.0000 i \\ 0.4853+0.1278 i \\ 0.0000+0.0000 i\end{array}\right]$ | $+\left[\begin{array}{c}0.0000+0.0000 i \\ 0.0676+0.1337 i \\ -0.1797-0.5524 i \\ 0.0000+0.0000 i\end{array}\right]$ | $+\left[\begin{array}{c}0.0000+0.0000 i \\ 0.0000+0.0000 i \\ 0.4135-0.2226 i \\ -0.2250+0.3127 i\end{array}\right]$ | $+\left[\begin{array}{c}0.3745+0.4906 i \\ -0.2230+0.1192 i \\ 0.0000+0.0000 i \\ 0.0000+0.0000 i\end{array}\right]$ | $+\left[\begin{array}{c}0.5661-0.5540 i \\ 0.0000+0.0000 i \\ 0.0000+0.0000 i \\ -0.8892+0.2901 i\end{array}\right]$ | $+\left[\begin{array}{c}0.0000+0.0000 i \\ -1.3689+0.0790 i \\ 0.0000+0.0000 i \\ -0.3512+0.0024 i\end{array}\right]$ |

$$
\frac{n}{\left[\begin{array}{c}
-0.9461+0.9908 i \\
-0.2166-0.2770 i \\
0.0596-0.0493 i \\
0.3083-1.5869 i
\end{array}\right]}=\frac{y}{\left[\begin{array}{c}
0.2878+1.0286 i \\
-1.7409+0.0548 i \\
0.7787-0.6965 i \\
-1.1571-0.9817 i
\end{array}\right]}
$$

## Appendix B

## Proof 1: Independence of decoding order on uplink sum rate

To prove, in theory, the decoding order does not affect the total rate in the uplink system, we need to show the equivalence of both decoding orders. For this comparison, we assume that the users have perfect knowledge of the CSI, and perfect SIC is applied at the receiver. Therefore, we expect that when subtracted, the expanded terms for both variants cancel to zero. The mathematical comparison is shown as follows:

Define $\Delta R$ as the rate difference between the total rates with different decoding order. i.e.

$$
\Delta R=\sum_{k=1}^{K} R_{\underline{\text { HML }}}-\sum_{k=1}^{K} R_{k_{L M H}}
$$

Then:

$$
\begin{gathered}
\Delta R=\left(\log _{2}\left(1+\frac{S N R_{H, k}}{1+S N R_{M, k}+S N R_{L, k}}\right)+\log _{2}\left(1+\frac{S N R_{M, k}}{1+S N R_{L, k}}\right)+\log _{2}\left(1+S N R_{L, k}\right)\right)- \\
\left(\log _{2}\left(1+\frac{S N R_{L, k}}{1+S N R_{M, k}+S N R_{H, k}}\right)+\log _{2}\left(1+\frac{S N R_{M, k}}{1+S N R_{H, k}}\right)+\log _{2}\left(1+S N R_{H, k}\right)\right)
\end{gathered}
$$

Simplifying:

$$
\begin{aligned}
& \Delta R=\left(\log _{2}(1\right.\left.\left.+\frac{S N R_{H, k}}{1+S N R_{M, k}+S N R_{L, k}}\right)\left(1+\frac{S N R_{M, k}}{1+S N R_{L, k}}\right)\left(1+S N R_{L, k}\right)\right)- \\
&\left(\log _{2}\left(1+\frac{S N R_{L, k}}{1+S N R_{M, k}+S N R_{H, k}}\right)\left(1+\frac{S N R_{M, k}}{1+S N R_{H, k}}\right)\left(1+S N R_{H, k}\right)\right)
\end{aligned}
$$

then, we calculate the difference:

$$
\begin{aligned}
& \Delta R=\left(\left(1+\frac{S N R_{H, k}}{1+S N R_{M, k}+S N R_{L, k}}\right)\left(1+\frac{S N R_{M, k}}{1+S N R_{L, k}}\right)\left(1+S N R_{L, k}\right)\right)- \\
& \left(\left(1+\frac{S N R_{L, k}}{1+S N R_{M, k}+S N R_{H, k}}\right)\left(1+\frac{S N R_{M, k}}{1+S N R_{H, k}}\right)\left(1+S N R_{H, k}\right)\right) \\
& =\left(\left(1+\frac{S N R_{H, k}}{1+S N R_{M, k}+S N R_{L, k}}\right)\left(\frac{S N R_{M, k}}{1+S N R_{L, k}}\right)\left(\frac{\left(S N R_{H, k}\right)\left(S N R_{M, k}\right)}{\left(1+S N R_{M, k}+S N R_{L, k}\right)\left(1+S N R_{L, k}\right)}\right)\right) \\
& \left(1+S N R_{L, k}\right)- \\
& \left.\left(1+\frac{S N R_{L, k}}{1+S N R_{H, k}+S N R_{M, k}}\right)\left(\frac{S N R_{M, k}}{1+S N R_{H, k}}\right)\left(\frac{\left(S N R_{L, k}\right)\left(S N R_{M, k}\right)}{\left(1+S N R_{H, k}+S N R_{M, k}\right)\left(1+S N R_{H, k}\right)}\right)\right) \\
& \left(1+S N R_{H, k}\right) \\
& =\left(\left(1+S N R_{L, k}\right)+\frac{\left(S N R_{H, k}\right)\left(1+S N R_{L, k}\right)}{1+S N R_{M, k}+S N R_{L, k}}+S N R_{M, k}+\frac{\left(S N R_{H, k}\right)\left(S N R_{M, k}\right)}{1+S N R_{M, k}+S N R_{L, k}}\right)- \\
& \left(\left(1+S N R_{H, k}\right)+\frac{\left(S N R_{L, k}\right)\left(1+S N R_{H, k}\right)}{1+S N R_{H, k}+S N R_{M, k}}+S N R_{M, k}+\frac{\left(S N R_{L, k}\right)\left(S N R_{M, k}\right)}{1+S N R_{H, k}+S N R_{M, k}}\right) \\
& =\left(1+S N R_{L, k}\right)+\frac{\left(S N R_{H, k}\right)+\left(\left(S N R_{H, k}\right)\left(S N R_{L, k}\right)\right)}{1+S N R_{M, k}+S N R_{L, k}}+\frac{\left(S N R_{H, k}\right)\left(S N R_{M, k}\right)}{1+S N R_{M, k}+S N R_{L, k}}-1 \\
& -S N R_{H, k}-\frac{\left(S N R_{L, k}\right)+\left(\left(S N R_{L, k}\right)\left(S N R_{H, k}\right)\right)}{1+S N R_{H, k}+S N R_{M, k}}-\frac{\left(S N R_{L, k}\right)\left(S N R_{M, k}\right)}{1+S N R_{H, k}+S N R_{M, k}} \\
& =S N R_{L, k}-S N R_{H, k}+\frac{\left(S N R_{H, k}\right)\left(1+S N R_{M, k}+S N R_{L, k}\right)}{1+S N R_{M, k}+S N R_{L, k}}-
\end{aligned}
$$

$$
\begin{array}{r}
\frac{\left(S N R_{L, k}\right)\left(1+S N R_{H, k}+S N R_{M, k}\right)}{1+S N R_{H, k}+S N R_{M, k}} \\
=S N R_{L, k}-S N R_{H, k}+S N R_{H, k}-S N R_{L, k}=0
\end{array}
$$

$$
\therefore \Delta R=0
$$

From the previous derivations and algebraic operations, we have shown that $\Delta R=0$, therefore, in theory, the rates with different decoding orders are equal, and we prove that the achievable rate of the system is not affected by the decoding order and the way the interference is treated in the detection process.

## Appendix C

## Proof 2: Derivation of the SINR for the SCMA rate equation

The equation of the rate per subcarrier shown above can be represented as the summation of the SINR, however it can be simplified as the summation of the SNR.
We define $g_{k, j}$ as the channel gain normalised by noise (i.e., $\frac{\left|h_{k, j}\right|^{2}}{B N_{o}}$ ), the rate per subcarrier $k$ and the number of users is $J=3$.

$$
\begin{aligned}
& R_{k}=B \sum_{j=1}^{J} f_{k, j} \log _{2}\left(1+\frac{p_{k, j} g_{k, j}}{1+\sum_{i=, i \neq j}^{J} p_{k, i} g_{k, i}}\right) \\
& =B\left(\log _{2}\left(1+\frac{p_{k, 1} g_{k, 1}}{1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(1+\frac{p_{k, 2} g_{k, 2}}{1+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(1+p_{k, 3} g_{k, 3}\right)\right) \\
& =B\left(\log _{2}\left(1+\frac{p_{k, 1} g_{k, 1}}{1+p_{k, 2} 2 g_{k, 2}+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(\frac{1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}}{1+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(1+p_{k, 3} g_{k, 3}\right)\right) \\
& =B\left(\log _{2}\left(1+\frac{p_{k, 1} g_{k, 1}}{1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}\right)-\log _{2}\left(1+p_{k, 3} g_{k, 3}\right)+\right. \\
& \left.\log _{2}\left(1+p_{k, 3} g_{k, 3}\right)\right)=B\left(\log _{2}\left(1+\frac{p_{k, 1} g_{k, 1}}{1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}\right)\right) \\
& =B\left(\log _{2}\left(\frac{1+p_{k, 1} g_{k, 1}+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}}{1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}}\right)+\log _{2}\left(1+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}\right)\right) \\
& \quad=B\left(\log _{2}\left(1+p_{k, 1} g_{k, 1}+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}\right)\right.
\end{aligned}
$$

$$
=B\left(\log _{2}\left(1+p_{k, 1} g_{k, 1}+p_{k, 2} g_{k, 2}+p_{k, 3} g_{k, 3}\right)\right)=B\left(\log _{2}\left(1+\sum_{j}^{J} p_{k, j} g_{k, j}\right)\right)
$$

## Appendix D

## Proof 3: Derivation of the SNR for the SCMA rate equation

The equation of the rate per subcarrier shown above can be represented as the summation of the SNR, however, it can be expanded as the summation of the SINR.

Let us define $\beta_{k, j}=f_{k, j} p_{k, j}\left|h_{k, j}\right|^{2}$ and $w=B N_{o}$. The rate in subcarrier $k$ :

$$
\begin{aligned}
& \log _{2}\left(1+\frac{\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w}\right) \\
= & \log _{2}\left(\left(\frac{w}{w}\right)+\left(\frac{\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w}\right)\right) \\
= & \log _{2}\left(\frac{w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w}\right) \\
= & \log _{2}\left(\left(\frac{w+\beta_{k, 6}}{w+\beta_{k, 6}}\right)\left(\frac{w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w}\right)\right) \\
= & \log _{2}\left(\left(\frac{w+\beta_{k, 6}}{w}\right)\left(\frac{w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w+\beta_{k, 6}}\right)\right) \\
= & \log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w+\beta_{k, 6}}\right) \\
= & \log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{\left(w+\beta_{k, 6}+\beta_{k, 5}\right)\left(w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}\right)}{\left(w+\beta_{k, 6}+\beta_{k, 5}\right)\left(w+\beta_{k, 6}\right)}\right) \\
= & \log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right) \\
& +\log _{2}\left(\frac{w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w+\beta_{k, 6}+\beta_{k, 5}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& =\log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right) \\
& +\log _{2}\left(\frac{\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}\right)\left(w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}\right)}{\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}\right)\left(w+\beta_{k, 6}+\beta_{k, 5}\right)}\right) \\
& =\log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}{w+\beta_{k, 6}+\beta_{k, 5}}\right) \\
& +\log _{2}\left(\frac{w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}\right) \\
& =\log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}{w+\beta_{k, 6}+\beta_{k, 5}}\right) \\
& +\log _{2}\left(\frac{\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}\right)\left(w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}\right)}{\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}\right)\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}\right)}\right) \\
& =\log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}{w+\beta_{k, 6}+\beta_{k, 5}}\right) \\
& +\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}+\beta_{k, 6}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}}\right) \\
& =\log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}{w+\beta_{k, 6}+\beta_{k, 5}}\right) \\
& +\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}\right) \\
& +\log _{2}\left(\frac{\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}\right)\left(w+\beta_{k, 1}+\beta_{k, 2}+\beta_{k, 3}+\beta_{k, 4}+\beta_{k, 5}+\beta_{k, 6}\right)}{\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}\right)\left(w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}\right)}\right) \\
& =\log _{2}\left(\frac{w+\beta_{k, 6}}{w}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}}{w+\beta_{k, 6}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}{w+\beta_{k, 6}+\beta_{k, 5}}\right) \\
& +\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}\right)+\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}}\right) \\
& +\log _{2}\left(\frac{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}+\beta_{k, 1}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}}\right) \\
& =\log _{2}\left(1+\frac{\beta_{k, 6}}{w}\right)+\log _{2}\left(1+\frac{\beta_{k, 5}}{w+\beta_{k, 6}}\right)+\log _{2}\left(1+\frac{\beta_{k, 4}}{w+\beta_{k, 6}+\beta_{k, 5}}\right) \\
& +\log _{2}\left(1+\frac{\beta_{k, 3}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}}\right)+\log _{2}\left(1+\frac{\beta_{k, 2}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}}\right) \\
& +\log _{2}\left(1+\frac{\beta_{k, 1}}{w+\beta_{k, 6}+\beta_{k, 5}+\beta_{k, 4}+\beta_{k, 3}+\beta_{k, 2}}\right) \\
& =\sum_{j=1}^{J} \log _{2}\left(1+\frac{\beta_{k, j}}{w+\sum_{i=1, i \neq j}^{J} I_{k, i}}\right)
\end{aligned}
$$

## Appendix E

## Proof 4: Derivation of the rate per section in PDMA

The total rate for the PDMA system can be split into rate $R_{A}$ and rate $R_{B}$, and they are graphically presented as follows:


Figure E.1: PDMA pattern divided by sections A and B

We can represent the above subcarrier allocation as the following $F_{4 \times 6}$ pattern:

$$
\boldsymbol{F}=\left[\begin{array}{llllll}
1 & 0 & 1 & 0 & 0 & 0  \tag{E.1}\\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1
\end{array}\right]
$$

The rate of Section A is derived as follows:
$R_{A}=B \sum_{j=1}^{J} \sum_{k=1}^{K-d v} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right)=$
$B\left(f_{1,1} \log _{2}\left(1+\frac{p_{1,1} g_{1,1}}{1+p_{1,2} g_{1,2}+p_{1,3} g_{1,3}+p_{1,4} g_{1,4}+p_{1,5} g_{1,5}+p_{1,6} g_{1,6}}\right)+\right.$
$f_{1,2} \log _{2}\left(1+\frac{p_{1,2} g_{1,2}}{1+p_{1,3} g_{1,3}+p_{1,4} g_{1,4}+p_{1,5} g_{1,5}+p_{1,6} g_{1,6}}\right)+$
$f_{1,3} \log _{2}\left(1+\frac{p_{1,3} g_{1,3}}{1+p_{1,4} g_{1,4}+p_{1,5} g_{1,5}+p_{1,6} g_{1,6}}\right)+f_{1,4} \log _{2}\left(1+\frac{p_{1,4} g_{1,4}}{1+p_{1,5} g_{1,5}+p_{1,6} g_{1,6}}\right)+$
$f_{1,5} \log _{2}\left(1+\frac{p_{1,5} g_{1,5}}{1+p_{1,6} g_{1,6}}\right)+f_{1,6} \log _{2}\left(1+p_{1,6} g_{1,6}\right)+$
$f_{2,1} \log _{2}\left(1+\frac{p_{2,1} g_{2,1}}{1+p_{2,2} g_{2,2}+p_{2,3} g_{2,3}+p_{2,4} g_{2,4}+p_{2,5} g_{2,5}+p_{2,6} g_{2,6}}\right)+$
$f_{2,2} \log _{2}\left(1+\frac{p_{2,2} g_{2,2}}{1+p_{2,3} g_{2,3}+p_{2,4} g_{2,4}+p_{2,5} g_{2,5}+p_{2,6} g_{2,6}}\right)+$
$f_{2,3} \log _{2}\left(1+\frac{p_{2,3} g_{2,3}}{1+p_{2,4} g_{2,4}+p_{2,5} g_{2,5}+p_{2,6} g_{2,6}}\right)+f_{2,4} \log _{2}\left(1+\frac{p_{2,4} g_{2,4}}{1+p_{2,5} g_{2,5}+p_{2,6} g_{2,6}}\right)+$
$\left.f_{2,5} \log _{2}\left(1+\frac{p_{2,5} g_{2,5}}{1+p_{2,6} g_{2,6}}\right)+f_{2,6} \log _{2}\left(1+p_{2,6} g_{2,6}\right)\right)$.

Due to the sparse nature of the PDMA factor graph, the rate $R_{A}$ can be simplified based on the variable $f_{k, j}$ and the users that are not allocated to subcarriers $k=1$ and $k=2$. Then, we obtain:

$$
\begin{aligned}
& R_{A}=B\left(f_{1,1} \log _{2}\left(1+\frac{p_{1,1} g_{1,1}}{1+p_{1,6} g_{1,6}}\right)+f_{1,3} \log _{2}\left(1+p_{1,3} g_{1,3}\right)+\right. \\
& \left.f_{2,1} \log _{2}\left(1+\frac{p_{2,1} g_{2,1}}{1++p_{2,4} g_{2,4}}\right)+f_{2,4} \log _{2}\left(1+p_{2,4} g_{2,4}\right)\right) .
\end{aligned}
$$

Similarly, $R_{B}$ can also be derived as follows:

$$
\begin{aligned}
& R_{B}=B \sum_{j=1}^{J} \sum_{k=K-d v+1}^{K} f_{k, j} \log _{2}\left(1+p_{k, j} g_{k, j}\right)= \\
& B\left(f_{3,1} \log _{2}\left(1+\frac{p_{3,1} g_{3,1}}{1+p_{3,2} g_{3,2}+p_{3,3} g_{3,3}+p_{3,4} g_{3,4}+p_{3,5} g_{3,5}+p_{3,6} g_{3,6}}\right)+\right. \\
& f_{3,2} \log _{2}\left(1+\frac{p_{3,2} g_{3,2}}{1+p_{3,3} g_{3,3}+p_{3,4} g_{3,4}+p_{3,5} g_{3,5}+p_{3,6} g_{3,6}}\right)+ \\
& f_{3,3} \log _{2}\left(1+\frac{p_{3,3} g_{3,3}}{1+p_{3,4} g_{3,4}+p_{3,5} g_{3,5}+p_{3,6} g_{3,6}}\right)+ \\
& f_{3,4} \log _{2}\left(1+\frac{p_{3,4} g_{3,4}}{1+p_{3,5} g_{3,5}+p_{3,6} g_{3,6}}\right)+f_{3,5} \log _{2}\left(1+\frac{p_{3,5} g_{3,5}}{1+p_{3,6} g_{3,6}}\right)+ \\
& f_{3,6} \log _{2}\left(1+p_{3,6} g_{3,6}\right)+f_{4,1} \log _{2}\left(1+\frac{p_{4,1} g_{4,1}}{1+p_{4,2} g_{4,2}+p_{4,3} g_{4,3}+p_{4,4} g_{4,4}+p_{4,5} g_{4,5}+p_{4,6} g_{4,6}}\right)+ \\
& f_{4,2} \log _{2}\left(1+\frac{p_{4,2} g_{4,2}}{1+p_{4,3} g_{4,3}+p_{4,4} g_{4,4}+p_{4,5} g_{4,5}+p_{4,6} g_{4,6}}\right)+ \\
& f_{4,3} \log _{2}\left(1+\frac{p_{4,3} g_{4,3}}{1+p_{4,4} g_{4,4}+p_{4,5} g_{4,5}+p_{4,6} g_{4,6}}\right)+f_{4,4} \log _{2}\left(1+\frac{p_{4,4} g_{4,4}}{1+p_{4,5} g_{4,5}+p_{4,6} g_{4,6}}\right)+ \\
& \left.f_{4,5} \log _{2}\left(1+\frac{p_{4,5} g_{4,5}}{1+p_{4,6} g_{4,6}}\right)+f_{4,6} \log _{2}\left(1+p_{4,6} g_{4,6}\right)\right) .
\end{aligned}
$$

Again, the rate equation can be simplified by removing some terms based on the variable $f_{k, j}$ and the users that are not allocated to subcarriers $k=3$ and $k=4$

$$
\begin{aligned}
& R_{B}=B\left(f_{3,2} \log _{2}\left(1+\frac{p_{3,2} g_{3,2}}{1+p_{3,5} g_{3,5}}\right)+f_{3,5} \log _{2}\left(1+p_{3,5} g_{3,5}\right)+\right. \\
& \left.f_{4,2} \log _{2}\left(1+\frac{p_{4,2} g_{4,2}}{1+p_{4,6} g_{4,6}}\right)++f_{4,6} \log _{2}\left(1+p_{4,6} g_{4,6}\right)\right) .
\end{aligned}
$$

## Appendix F

## Multiplexed signal for PDMA system

We provide a numerical example of the resultant signal $y$ in PDMA when number of users is $J=6$ and the number of subcarriers is $K=4$.

First, we select the channel gain of the matrix $h$ for each user (column vector of $h$ ) and we expand it in a diagonal matrix to multiply with the codebook vector $x$ and we add the noise vector $n$ as follows:

$$
y=\sum_{j=1}^{J} h_{j} x_{j}+n
$$

Substituting in example values, we have:



| $h_{3}$ |
| :---: |
| $\left[\begin{array}{cccc}0.1125+0.5740 i & 0 & 0 & 0 \\ 0 & 0.4367-0.3672 i & 0 & 0 \\ 0 & 0 & 0.1330+0.5629 i & 0 \\ 0 & 0 & 0 & -1.4203+0.0462 i\end{array}\right]$ |

$$
\frac{x_{3}(00)}{\left[\begin{array}{c}
-0.5741-0.5741 i \\
0.0000+0.0000 i \\
0.0000+0.0000 i \\
0.0000+0.0000 i
\end{array}\right]}+
$$

|  |  | $h_{4}$ |  |  | $x_{4}(01)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left[\begin{array}{c}0.6695+0.0899 i \\ 0 \\ 0 \\ 0\end{array}\right.$ | 0 $-0.8171-0.1364 i$ 0 0 | 0 0 $1.2968-0.4847 i$ 0 | $\left.\begin{array}{c}0 \\ 0 \\ 0 \\ -0.3400+0.8590 i\end{array}\right]$ | $\left[\begin{array}{l}0.0000+0.0000 i \\ 0.1131-0.4222 i \\ 0.0000+0.0000 i \\ 0.0000+0.0000 i\end{array}\right]$ |
|  |  | $h_{5}$ |  |  | $x_{5}(10)$ |
| $\stackrel{\square}{t}$ | $\left[\begin{array}{c}-1.5593-0.9230 i \\ 0 \\ 0 \\ 0\end{array}\right.$ | 0 $-0.3528+1.0807 i$ 0 0 | iccole $\begin{gathered}0 \\ \\ \\ 1.1343+0.1525 i \\ \\ \\ 0\end{gathered}$ | $\left.\begin{array}{cc} \\ & 0 \\ 0 \\ 0 \\ 0.0545-0.7113 i\end{array}\right]$ | $\left[\begin{array}{l}0.0000+0.0000 i \\ 0.0000+0.0000 i \\ 0.7255-0.9438 i \\ 0.0000+0.0000 i\end{array}\right]$ |
|  |  | $h_{6}$ |  |  | $x_{6}(11)$ |
|  | $\left[\begin{array}{c}0.2236-0.8103 i \\ 0 \\ 0 \\ 0\end{array}\right.$ | $\begin{array}{cc}0 \\ 0.2886+0.8725 i \\ 0 & 0 . \\ 0\end{array}$ | 0 0 $0.4336+1.0664 i$ 0 | $\left.\begin{array}{c}0 \\ 0 \\ 0 \\ -0.2731-0.6498 i\end{array}\right]$ | $\left[\begin{array}{l}0.0000+0.0000 i \\ 0.0000+0.0000 i \\ 0.0000+0.0000 i \\ 0.1899-0.4607 i\end{array}\right]$ |

$$
\frac{n}{\left[\begin{array}{c}
-0.9461+0.9908 i \\
-0.2166-0.2770 i \\
0.0596-0.0493 i \\
0.3083-1.5869 i
\end{array}\right]}
$$

Once each channel gain per user has been multiplied by the column vector of the codebook we add the resultant values and the noise column vector. We then obtain the resultant signal as follows:


