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# Direct integration of the third-order two point and multipoint Robin type boundary value problems

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**Abstract**

This numerical study exclusively focused on the direct two point diagonally multistep block method of order four (2DDM4) in the form of Adams-type formulas. The proposed predictor–corrector scheme was applied in this study to compute two equally spaced numerical solutions for the third-order two point and multipoint boundary value problems (BVPs) subject to Robin boundary conditions concurrently at each step. The optimization of the computational costs was taken into consideration by not resolving the equation into a set of first-order differential equations. Instead, its implementation involved the use of shooting technique, which included the Newton divided difference formula employed for the iterative part, for the estimation of the initial guess. Apart from studying the local truncation error, the study also included the method analysis, including the order, stability, and convergence. The results of eight numerical problems demonstrated and highlighted competitive computational cost attained by the scheme, as compared to the existing method.


*Keywords:* Boundary value problems; Linear multistep method; Robin boundary conditions; Shooting method; Third-order differential equations

## 1. Introduction

A wide range of applications in science and engineering leads to the mathematical formulation of higher order differential equations mainly applied to model a physical phenomenon. In fluid flow studies, as discussed in [21] and [17], these differential equations have commonly occurred. Concerning that, the third-order differential equation was emphasized in the current study given by

$$y'''(x) = f(x, y, y', y''), \quad a_1 \leq x \leq a_3 \tag{1}$$

subject to the set of three Robin type boundary conditions

$$c_{1,1}y''(a_1) + c_{1,2}y'(a_1) + c_{1,3}y(a_1) = B_1, \tag{2}$$
$$c_{2,1}y''(a_2) + c_{2,2}y'(a_2) + c_{2,3}y(a_2) = B_2, \tag{3}$$
$$c_{3,1}y''(a_3) + c_{3,2}y'(a_3) + c_{3,3}y(a_3) = B_3, \tag{4}$$

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