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Razors and lasers: an undergraduate experiment to determine thermal expansion coefficients using single slit diffraction measurements

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Abstract

Single slit diffraction and the thermal expansion of materials are common components of an undergraduate physics course, though these topics are often taught independently in both lectures and laboratory based courses. Higher levels of cognitive domains can be achieved by building on these established topics and combining them into a single experiment that also introduces new tools and techniques for data handling and analysis. Here we describe an experiment where the thermal expansion of a metal bar shifts an attached razor blade such that the separation between this blade and another fixed blade decreases. This decrease in blade separation allows for changes in the peak separation for single slit diffraction to be measured and the expansion coefficient of three metal bars (copper, steel and aluminium)

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is determined to good agreement with accepted values. The use of multiple peak fitting allows students to develop more sophisticated analysis techniques that are applicable beyond this experiment and are transferable to different areas of experimental physics.

Keywords: experimental design, thermal expansion, diffraction, teaching laboratory

1. Introduction

As educators we aspire for our graduates to access the higher levels of the cognitive and psychomotor domains [1] and yet often find it challenging to provide structured opportunities for students to synthesise and combine different subject areas to solve a problem. Ambrose [2] proposes a model of how students master a subject using different levels of hierarchy and interconnection between individual knowledge 'units', however many university courses teach both didactic and experimental courses in isolation. To support students through their journey towards mastery we must scaffold their learning to develop the connections between subjects and skills over the course of their studies.

A logical midpoint is to introduce students to a problem that requires them to use theory and techniques from two distinct fields within a scaffolded experimental investigation. Optics and thermodynamics are both integral parts of most undergraduate physics courses however many institutions often teach these topics in isolation. A number of different approaches have already been used to measure thermal expansion in undergraduate physics including a strain gauge [3] and reflection of a laser beam [4]. Single slit diffraction experiments have long been used in undergraduate laboratories [5] but are still being enhanced to utilise modern technologies [6] or to apply the techniques to characterise physical systems such as human hair [7] or even cat whiskers [8]. Within these large topics we make use of two common concepts, namely single slit diffraction and thermal expansion of solids, and provide an experimental setup that uses analysis of the former to quantify an example of the latter. The experiment and analysis methods developed here are based on the existing work proposed by Fakhruddin [9] and subsequently used by others [10, 11]. The two key developments we implemented as part of our own use of the experimental setup were the simplification of the experimental equipment, and introducing a new analysis technique to reduce the number of measured variables required.

This modified experiment was designed for delivery at the University of Sheffield (United Kingdom) as part of a second year undergraduate degree programme. Students had already been taught the individual optics and thermal theories in their first year and so the cognitive load is focused on learning new experimental and analysis methods is reduced as well as giving students the opportunity to synthesise two seemingly distinct topics from the previous studies. In this paper we first review the two core topics in thermal physics and optics in order to define variables and equations in preparation for combining the two. We then describe the experimental setup and analysis techniques utilised by students, and finally present results from this experiment and compare them to accepted values.

2. Theory

2.1. Thermal expansion

When a thin, uniform solid rod of initial length L_0 is heated the change in length ΔL is described by:

$$\Delta L = L(T) - L_0 = L_0 \alpha \Delta T, \tag{1}$$

where α is the coefficient of linear thermal expansion of the material and ΔT is the change in the temperature. Similarly the volume expansion of a uniform three dimensional material is:

$$\Delta V = V(T) - V_0 = V_0 \beta \Delta T,$$

where β is the coefficient of volumetric thermal expansion. For an isotropic material it is assumed that an increase in temperature leads to an equal expansion in all three dimensions and thus,

$$\alpha = \frac{\beta}{3}. \tag{2}$$

2.2. Single slit diffraction

The Fraunhofer diffraction equation for constructive interference from a single slit describes the angular separation θ of diffraction peak maxima relative to the central maximum [12]:

$$d \sin\theta_n = \left(n + \frac{1}{2}\right)\lambda, \tag{3}$$

where d is the slit separation, n is the order number of the diffraction pattern ($n = 1, 2, 3, \ldots$), θ_n is the angle between central maximum and nth maxima and λ is the wavelength of light incident on the slit.

When the diffraction pattern is incident on a perpendicular screen a distance D from the slits the on-screen distance between the central maximum and the nth maximum y_n is:

$$y_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}. \tag{4}$$

Therefore, for two adjacent maxima the on-screen distance between adjacent maxima is:

$$\Delta y = y_{n+1} - y_n = \frac{\lambda D}{d}. \tag{5}$$

2.3. Combining the two theories

The system used here comprises of two razor blades acting as the single slit, with one blade attached to a metal bar on a hotplate (see figure 1). As the metal bar is heated and expands the attached blade moves towards the fixed blade thus decreasing the slit width. Using equations (1) and (2) the dependence of slit width on temperature is given by:

$$\begin{aligned} \Delta d &= -\Delta L \\ &= -3L_0\alpha\Delta T. \end{aligned} \tag{6}$$

By taking an arbitrary temperature T_0 as a reference point (for example room temperature) the slit width at different temperatures is defined in terms of the slit width at the reference temperature, namely:

$$d(T) = d(T_0) - 3L_0\alpha(T - T_0). \tag{7}$$

Therefore using equations (5) and (7):

$$\frac{\Delta y(T_0)}{\Delta y(T)} = \frac{d(T)}{d(T_0)} = 1 - \frac{3L_0\alpha}{d(T_0)}(T - T_0), \tag{8}$$

where taking the ratio of the peak separation at the reference temperature T_0 to that at the measurement temperature T ensures that D, $d(T)$ and λ are not required. This method of analysis also removes the need to calibrate on-screen pixel distance to physical units. Finally, equation (8) is in a form suitable for performing linear regression analysis if $\frac{\Delta y(T_0)}{\Delta y(T)}$ is plotted against $(T - T_0)$, a technique that is already familiar to most undergraduate students.

3. Method

The experimental setup is shown in figure 1. It comprises of two steel razor blades, one fixed to a clamp stand and the other attached to the end of a 20 cm long flat metal bar (approximately 2 cm wide and 0.5 cm thick). In this work three different metal bars were used: copper, aluminium and steel (RS Components, UK). The metal bar was laid flat on top of a standard laboratory hotplate (Fisher Scientific, UK) with an additional thermocouple probe fixed to the side of the bar to provide a second temperature reading in addition to the hot plate display. The razor blade fixed to the clamp stand was moved into position to create a single slit. A short length of thin copper wire of diameter 0.6 mm was temporarily placed between the blades such that the length of the wire aligned normal to the plane of the blades. The flat metal bar was then moved slowly to decrease the distance between blade edges until the copper wire met resistance when moved up and down along the slit formed between the two blades. This process was used to provide a uniform slit of known separation equal to the diameter of the wire, after which the copper wire was removed and discarded before measurements.

A 650 nm diode laser (3B Scientific Ltd, United Kingdom) was used to produce the interference pattern onto a paper sheet positioned a fixed distance from and coplanar to the razor blades. A camera (Hero 4, GoPro) was placed

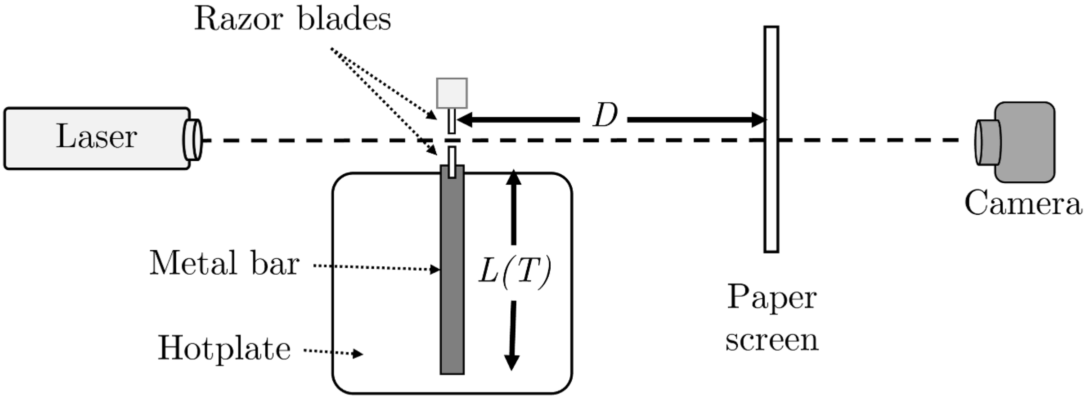

Figure 1. Birds eye view of the setup. The dotted line represents the laser path, and shows that the position of the camera should be inline with the laser beam in the absence of any slit.

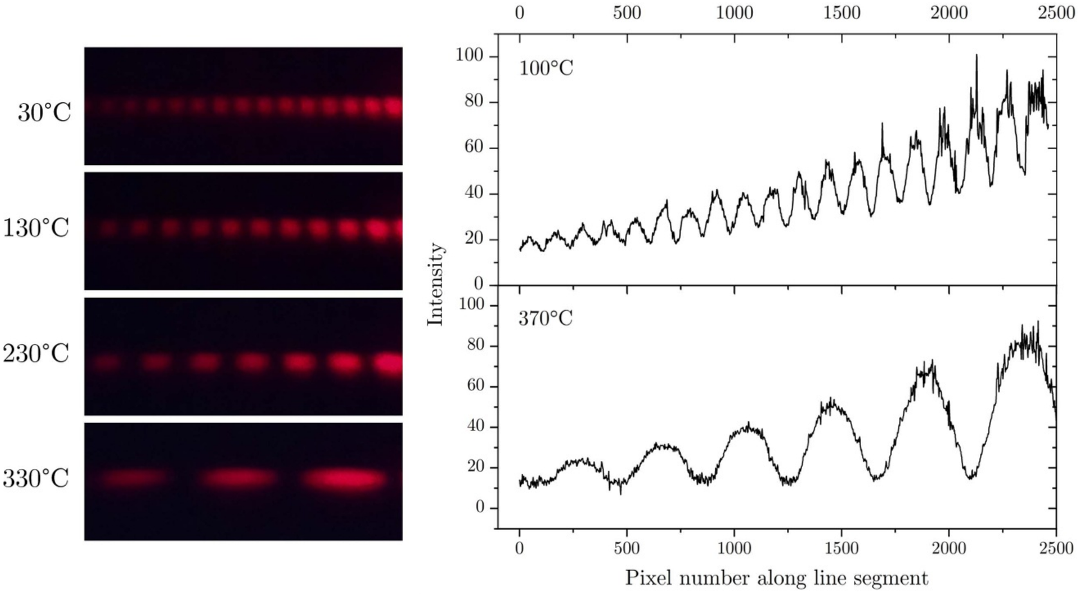

Figure 2. (Left) Images taken of the diffraction patterns for a copper bar heated over a 300 °C range. (Right) Intensity spectra for two different temperatures as extracted using the line profile tool in Fiji.

behind the screen to reduce image skew from parallax. An image was taken of the interference pattern at room temperature to provide the reference measurement. The temperature of the hotplate was then increased and the bar was left for approximately 5 min to reach thermal equilibrium before another image was taken. No thermal insulation was used on top of the bar as the thickness was sufficiently small such that the temperature was assumed to be uniform across the entire bar.

3.1. Peak analysis

Intensity profiles of the diffraction pattern (see figure 2) was extracted from each image using Fiji [13]. Multiple Gaussian peaks were fitted using OriginPro with an increasing baseline that was initially autodetected and subsequently refined manually by the student. The pixel position of the central maxima for each peak were used for the position coordinates in equation (4) and full width at half maximum of the fitted Gaussian was

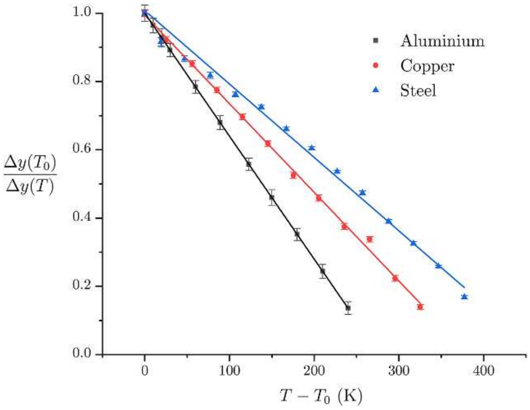

Figure 3. The difference in peak position relative to the reference value at room temperature for three different metal bars. Linear regression allows students to determine the gradient, from which the expansion coefficient can be determined.

used for the associated error. All data sets produced multiple peaks and so peak-to-peak distance between adjacent peaks were found for each pair in the dataset, and a final weighted average was calculated for each diffraction pattern. No calibration from pixel to physical distance was required as the final values used for analysis were the ratio of distances (in equation (8)) and as such the conversion from pixel to physical distances was not required.

A simplified analysis method can be used if multiple Gaussian fitting is too complicated. Students can identify the turning points in their data, either by using Maxima/Minima tools in analysis software or by manually identifying peaks using if statements in Excel to find changes from increasing to decreasing values for consecutive cells. We found that students are able to perform the more sophisticated multiple Gaussian peak fitting method with very little additional support and training, and that the ability to create and modify multiple peak functions is a skill that they use in future experimental work.

4. Results

The measurements of changes in the separation of maxima (Δy) based on equation (8) are shown in figure 3. Linear regression analysis on each data set determined the gradient which equals $\frac{3L_0\alpha}{d(T_0)}$, and measurements of the initial bar length and slit separation enabled α to be determined. The values for the determined expansion coefficients are shown in table 1 alongside accepted values from the literature [14].

Table 1. Measured and accepted coefficients of linear expansion for copper, aluminium and steel. Associated errors are determined from the FWHM of the diffraction peaks. Accepted values taken from Kaye and Laby [14].

	α_{measured} $(\times 10^{-6}\ \text{K}^{-1})$	α_{accepted} $(\times 10^{-6}\ \text{K}^{-1})$
Copper	15.9 ± 0.9	16.5–18.3
Aluminium	20.9 ± 1.5	23.1–26.4
Steel	11.3 ± 0.7	10.7–13.7

4.1. Additional analysis

Most thermal physics courses taken as part of undergraduate physics programmes introduce the thermal expansion coefficient as a constant, and even though typical textbooks [15] mention that this is only an approximation they rarely give additional information. A polynomial has been proposed [14] and including this as a non-linear fit provides an additional degree of complexity for more advanced students to utilise. By using the modified equation:

$$\frac{\Delta y(T_0)}{\Delta y(T)} = 1 - \frac{3L_0}{d(T_0)}\left(\alpha + \kappa T + \frac{\gamma}{T}\right)(T - T_0), \tag{9}$$

values for constants α, β, and γ are determined in table 2. It is worth emphasising that κ is used here instead of β used in the literature [14], as β has already been defined in equation (2) as the volume expansion coefficient.

5. Discussion and conclusion

The experimental and analysis procedures presented here demonstrate a simple but accurate way to determine the expansion coefficients of different metals to within an acceptable range of literature values. Despite the complexity of the data analysis technique compared to what students

Table 2. Non-linear fit of equation (9) gives values for α, κ and γ. The values of α for all three materials are in agreement with the accepted values stated in table 1.

	$\alpha\,(\times 10^{-6}\ \mathrm{K}^{-1})$	$\kappa\,(\mathrm{K}^{-2})$	$\gamma\,(\mathrm{K}^{2})$
Copper	17.8 ± 1.0	$(-2.4 \pm 0.8) \times 10^{-9}$	$(-4.1 \pm 2.5) \times 10^{-4}$
Aluminium	21.9 ± 0.3	$(-3.2 \pm 0.1) \times 10^{-10}$	$(-6.7 \pm 0.2) \times 10^{-5}$
Steel	11.8 ± 0.1	$(2.5 \pm 0.6) \times 10^{-9}$	$(-3.5 \pm 2.2) \times 10^{-4}$

had previously experienced, and the more sensitive experimental setup required, students reported that tackling more challenging techniques was balanced by the relative ease of the underlying physics; the use of first year physics reduced their cognitive load and allowed them to focus on developing their skills and techniques rather than also needing to learn new physical concepts.

In addition, even though the topics of single slit diffraction and thermal expansion are well established in their first year course there were still some interesting and sometimes unintended learning points for students to deepen their knowledge of their first year courses. Students reporting expansion coefficients that were a factor of 3 out from the expected value was common but allowed for students to reflect on volume and linear thermal expansions, and that even though their metal bar expanded in three dimensions the effect ‘seen’ by the single slit is akin to linear expansion (i.e. expansion in the two dimensions perpendicular to the bar length occur but do not contribute to changes in the slit separation). Other students were initially fixated on ensuring the central peak of the diffraction pattern was included in their images because ‘all the problems we solved in first year used the central maximum and usually the first peak’. In the experimental setup students quickly discovered that including the central maximum would saturate the image and obscure all other peaks. At this point staff would discuss and introduce the concept of taking adjacent peaks (equation (5)) or ratios to avoid calibrations (equation (8)) and explain that these are useful ‘tricks’ that students can utilise in future experiments where certain physical limitations of the equipment necessitate modifications, so long as the students understand the use and limitations of them.

The inclusion of a polynomial model in equation (9) also allows for students to reflect on the appropriateness of using a simple model even when a more sophisticated and complex one exists. For the fit parameters found in this work there is only a difference between models of 1% for copper across the 300 K measurement range, whereas for steel a 14% difference occurs at $\Delta T = 400$ K. Building the confidence to understand the limitations of a particular module but knowing that under certain conditions (in this case, within a certain temperature range) the simple model will provide a good approximation is a worthwhile skill for students when tasked with experimental design.

This experiment has been successful in our teaching laboratory courses in that both students and staff note that the structured complexity allows students to not only develop their experimental techniques but also supports their development of transferable skills and forces them to revisit physical concepts from previous studies. Even though it is more challenging than a traditional scripted experiment we believe the benefits significantly outweigh the potential issues and we have adopted this experiment within our core teaching laboratory course.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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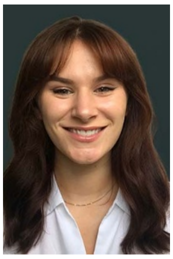
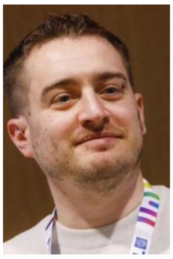


Molly Vokes was an undergraduate student in the Department of Physics and Astronomy at the University of Sheffield. She studied an integrated master's degree in physics and graduated in 2021. The work presented in this article is the result of her self-directed physics project undertaken in her second year experimental physics course.

Chloe McDaid is a University Teacher in the Department of Physics and Astronomy at the University of Sheffield. She currently runs the second year experimental physics course and leads the development of undergraduate teaching spaces, introducing elements on industrial collaboration into final year projects.



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