Title:
A Unified Approach for Airfoil Parameterization Using Bezier Curves

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Introduction:

Parametrization is at the core of optimization, as it defines the design space that the optimizing algorithm explores. The success of any shape optimization methodology depends extensively on the type of parameterization technique employed [10]. One straightforward route which results in the most flexible parametrization strategy is to use the nodes of the computational mesh as the design variables. One major drawback for this parameterization strategy is that, as all surface mesh nodes can move independently, the implementation of a smoothing algorithm is required to prevent the appearance of non-smooth shapes during the optimization process. In this regard, the Free-form deformation (FFD) techniques have been successfully implemented for aerodynamic shape optimization problems [Ref]. The benefit of this approach is that it imparts smooth deformations to the analysis mesh and enables the parameterization to alter the thickness, sweep, twist, etc. for the design of an aerospace system. However, in either of these parameterization strategies it is only the mesh which reaches the optimum, and must be translated into a CAD model before it can be used for further analysis or manufacturing assessments. Thus, to align with the industrial ambition of having a more integrated design workflow, the compatibility of design parameterization with Computer-Aided Design (CAD) software has become very important. In the recent past, some authors have attempted to develop optimization processes based on parameterization developed with CAD systems. These include, parameterization based on non-uniform rational B-splines (NURBS) [8], B-Splines in the Open-Cascade Technology [6], parameters defining CAD features [3] and Bezier curves [4] within CATIA V5. But there has been no standard approach which can be followed to parameterize different airfoil geometries and can also be used within CAD systems. In this research, a unified approach is presented to obtain the Bezier parameterizations for different airfoil geometries obtained from the UIUC Airfoil Data Site [2].

Bezier Parameterization:

The Bezier curve is a parametric curve named after Pierre Bezier who developed and used them for designing the outer surfaces of Renault cars. The basic mathematical formulation of the Bezier curves consists of two parts:

- A set of points which define the geometry of the Bezier curve also known as control points.
- The basis function, also known as Bernstein polynomial.

Any point on the Bezier curve is parametrically defined as:

$$P(t) = \sum_{i=0}^{k} P_i B_{i,k}(t) \tag{1}$$

where, k corresponds to the degree of Bernstein's polynomial and the value of t ranges from 0 to1. Further, the Bernstein polynomial can be described as:

$$B_{i,k}(t) = \binom{k}{i}(1-t)^{k-i}t^{i} \tag{2}$$

i=0,1,2,3,...,k, where *k* is the degree of polynomial and,

$$\binom{k}{i} = \frac{k!}{(k-i)!\,i!} \tag{3}$$

The vector P_i represents the *k+1* vertices of the control polygon and these vertices are further specified as control points. The Bezier curve need not pass through every control point but they alter the shape of the Bezier curve by attracting the curve towards themselves. This gives a generalized control on the Bezier curve and it further aids in the optimization of the Bezier curve in fitting it to complex geometries.

Airfoil Parameterization:

Airfoil also known as aerofoil is the cross-sectional area of any wing, propeller blades, rotor and turbine blades which is mainly responsible for producing an aerodynamic force when it moves through a fluid. The shape of an airfoil mainly depends on the application for which they are employed. Airfoils can be symmetric or asymmetric in shape as per their symmetry about the centerline. An asymmetric airfoil is more efficient at producing lifts as compared to the symmetric airfoils. There are a number of methods that can be used to define the geometry of airfoils. The type of parameterization used to define these airfoils play a vital role, when the geometry is optimized to achieve certain objectives. Since Bezier curves possess good geometrical capabilities (as explained earlier) in terms of providing smooth airfoil geometries and allowing the exploration of a larger design space during optimization. The parameterization of airfoils should ideally possess following characteristics:

- Able to represent a wide range of airfoil shapes.
- Smooth geometries with curvature continuity at the leading edge of the airfoil.
- Be computationally efficient in terms of runtime and memory usage for storing the parametrization.
- Able to produce CAD geometry which can be used for downstream applications.

Sohn and Lee [11] presented a semi-analytical approach to determine the Bezier control points which can accurately approximate the initial airfoil shape. They used the Bezier curves to define the geometry of an airfoil by using two different methodologies: (i) considering *y*-coordinates of the Bezier control points defining the upper and lower surfaces as design variables, (ii) consider *x*- and *y*-coordinates of the control points of the Bezier defining the camber and the scale factor of the thickness variation as the design variables. All the control points were analytically obtained, except the first control point on either side of y-axis (found experimentally) which would impose C1 continuity at the leading edge. Balu and Selvakumar [5] used genetic algorithm to optimize the *x*- and *y*- coordinates of the Bezier control points (eight for both upper and lower surface) and applied the methodology on the RAE 2822 airfoil. and the concept of degree elevation to obtain the required profile of airfoil. Jaiswal [7] used the fourth-order, sixth-order and eighth-order Bezier curves to approximate the airfoil shapes by firstly using a linear least square functional problem to obtain initial set of Bezier control points and subsequently used them in a nonlinear least square algorithm to obtain the final optimized control points. The approach was applied to obtain the Bezier curves for NACA0012 and RAE2822. Wei et al. [12] used the basic Bezier parameterization approach to define the initial airfoil and subsequently used direct search algorithms to obtain the optimized control point locations to precisely fit the upper and lower surfaces of low-Reynolds number airfoil E387.

The research mentioned above covered various research areas from aircrafts, and enriched the application of Bezier curves for defining the shapes of the airfoils, which is of great significance to engineering application. But, as per the authors understanding there is no standard approach that can be followed to obtain the Bezier control points for defining general airfoil geometry. This paper is an effort in this direction, and presents a standard methodology (see Fig. 1) to obtain the set of Bezier control points (random initialization) that fits the data points on the airfoils (from database) or the geometry of any airfoil specified as per the requirement.

In the field of computer graphics, the problem of fitting a parametric polynomial to approximate a set of data points have been tackled using error metric based on Least-squares [9]. An iterative approach using Newton-Raphson method was followed to obtain parametric values t for the cubic B-Spline polynomial that minimizes the distance between the data points and the polynomial curve. In this paper, a slightly different approach is followed to fit a sixth degree (user dependent) Bezier curve on the data points defining an airfoil section as described below:

Two different Bezier-curves are used for defining the upper and lower airfoil surfaces. The curve is then divided into m sections (m = 251) using parametric variable t and the Bezier control point locations. At this point, the parametric data points on Beziers are added to an efficient data structure known as Kd-tree (K-dimensional tree). It is a type of data structure that is used for efficient storage of information that is to be retrieved in subsequent searches. Two different Kd-tree are constructed for the upper and lower surface respectively. The Kd-tree facilitates efficient nearest neighbor search for a test point not in the tree to obtain closest points in the tree. For a data point on the airfoil, P_a (upper or lower), a query is performed with the respective Kd-tree (K_B) to return nearest two points (P_1 and P_2) on the respective Bezier curve. The nearest distance of the point P_a from P_1 and P_2 is obtained by projecting the point (P_a) on the line-segment P_1P_2 and subsequently computing the perpendicular distance D_{pi}. This distance D_{pi} is computed for all the points on the airfoil (N), and subsequently used in a Least Square optimization framework defined as

$$Minimize\ f(x) = \sum_{i=0}^{N} \left(D_{pi}\right)^2 \tag{4}$$

This is the first step in the direction to create automated geometry parameterization methodology for 3D wing models. These Bezier control points can be directly used within a CAD system to construct two- or three-dimensional airfoil or wings and define a CAD-based optimization approach with a number of geometrical constraints, which are only possible within a CAD environment. These constraints may include the presence of structural elements like fuel-tank or spar or other structural elements. The basic flowchart of the proposed methodology is represented as below:

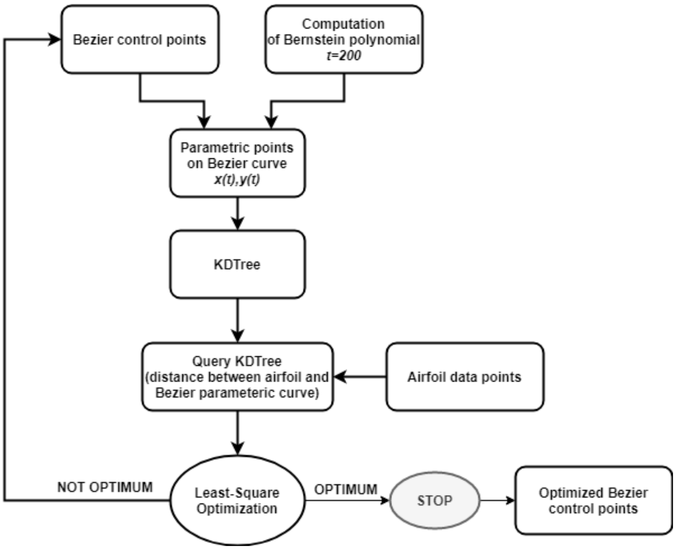

Fig. 1: Flow chart of the airfoil Parameterization methodology

<u>Results</u>

In order to demonstrate the applicability of the developed methodology, three test cases have been considered, (i) NACA0012, (ii) RAE2822, and (iii) NACA66$_4$021.

<u>*NACA0012 airfoil*</u>

NACA0012 is a symmetrical airfoil with zero camber and have been used as a benchmark problem for the AIAA CFD drag prediction workshop in 2016 [1]. The developed methodology is then used to obtain the optimized Bezier control point locations to approximately fit the data points on the airfoil. The results are shown in Fig. 2, where it can be seen that the control points on the lower surface is the mirror image of the control points for upper surface, which is expected for the symmetrical airfoil.

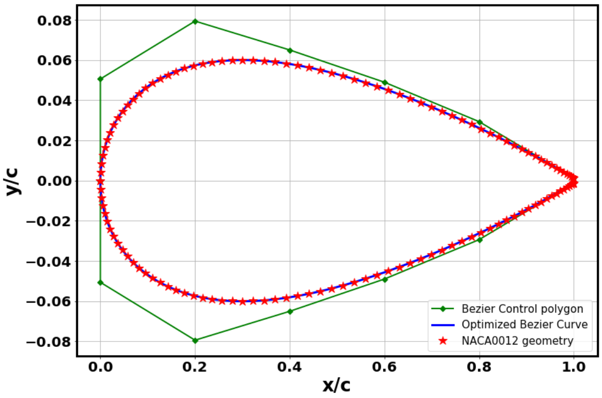

Fig. 2: Optimized Bezier curve for NACA0012 airfoil.

RAE2822 airfoil
The second test case considered is the asymmetric RAE2822 supercritical airfoil, which has been mainly analyzed in the transonic flow regime to model shockwaves in two-dimensional flows. Here the initial Bezier control points are chosen to be the same for the upper and lower surfaces, and subsequently optimized to fit the airfoil data points on both upper and lower surfaces, as shown in Fig. 3.

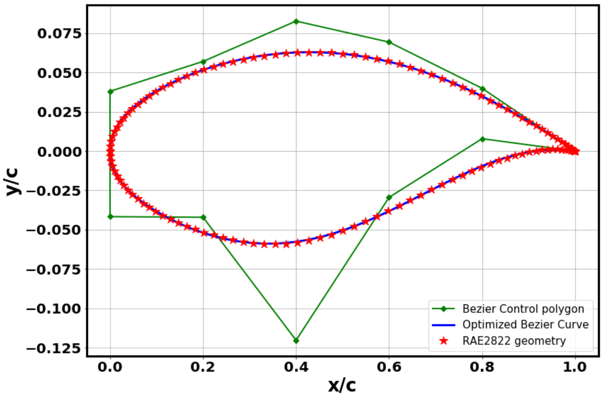

Fig. 3: Optimized Bezier curve for RAE2822 airfoil.

NACA66(4)-021 airfoil
The third test case considered is the NACA 66(4)-021 airfoil which have been designed for laminar flow applications. NACA 66(4)-021 represents a symmetrical airfoil with zero camber and maximum thickness of 21% at 45% chord.

Conclusions:
In this paper, a unified approach is presented to obtain the locations of Bezier control points to fit the data points on the upper and lower surfaces of three two-dimensional airfoils, namely NACA0012, RAE2822 and NACA66(4)-021. The developed methodology has shown promising results to approximate the airfoils with no noticeable visual difference between the approximated curve and data points. The obtained Bezier control points can be varied to obtain different airfoil configurations, providing a faster and efficient approach to create databases for executing machine learning approaches. Moreover, a

gradient or gradient-free shape optimization can be established using these Bezier control points as the design variables. As part of the future work, this methodology will be used to parameterize three-dimensional wing models.

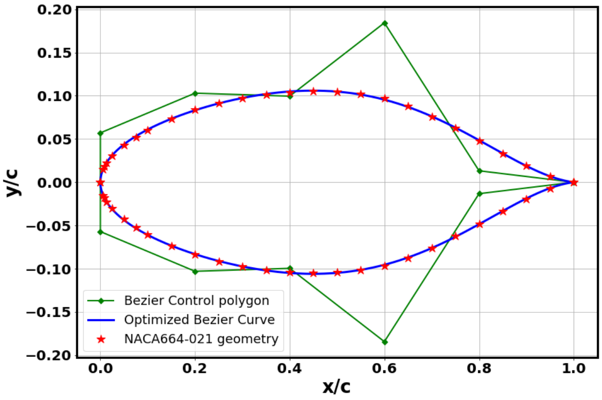

Fig. 4: Optimized Bezier curve for NACA66(4)-021 airfoil.

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