Interval Type-2 Fuzzy Programming Method for Risky Multicriteria Decision-Making with Heterogeneous Relationship

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Abstract

We propose a new interval type-2 fuzzy (IT2F) programming method for risky multicriteria decision-making (MCDM) problems with IT2F truth degrees, where the criteria exhibit a heterogeneous relationship and decision-makers behave according to bounded rationality. First, we develop a technique to calculate the Banzhaf-based overall perceived utility values of alternatives based on 2-additive fuzzy measures and regret theory. Subsequently, considering pairwise comparisons of alternatives with IT2F truth degrees, we define the Banzhaf-based IT2F risky consistency index (BIT2FRCI) and the Banzhaf-based IT2F risky inconsistency index (BIT2FRII). Next, to identify the optimal weights, an IT2F programming model is established based on the concept that BIT2FRII must be minimized and must not exceed the BIT2FRCI using a fixed IT2F set. Furthermore, we design an effective algorithm using an external archive-based constrained state transition algorithm to solve the established model. Accordingly, the ranking order of alternatives is derived using the Banzhaf-based overall perceived utility values. Experimental studies pertaining to investment selection problems demonstrate the state-of-the-art performance of the proposed method, that is, its strong capability in addressing risky MCDM problems.

Keywords: risky multicriteria decision making, heterogeneous relationship, evolutionary computation, interval type-2 fuzzy set, 2-additive fuzzy measure, regret theory

1. Introduction

Multicriteria decision-making (MCDM) has been extensively applied to different fields, such as postgraduate course assessment [18], investment evaluation [13, 19], research and development project selection [20], and comprehensive logistics distribution center location selection [17]. The traditional linear programming technique for multidimensional analysis of preference (LINMAP), which was proposed by Srininvasan and Schcker [31], is currently one of the most well-known MCDM approaches in modern decision theory because of its two advantages:

- It not only adopts the evaluations of alternatives on multiple criteria, but also considers the preferences of decision-makers (DMs) on pairwise comparisons of alternatives.
- By incorporating mathematical programming, it can objectively determine the criteria weights and ideal solution.

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In the traditional LINMAP method, all the decision information is expressed by real values. Owing to the uncertainty and fuzziness of human thinking, some assessment values are insufficiently depicted by real values. In such cases, type-1 fuzzy sets (T1FSs) [43] and their extended versions, such as trapezoidal fuzzy sets (TrFSs), intuitionistic fuzzy sets (IFSs), interval-valued IFSs (IVIFSs), hesitant fuzzy sets (HFSs), probabilistic linguistic term sets (PLTSs), and Pythagorean fuzzy sets (PFSs) can be regarded as useful tools for modeling decision information. Therefore, Li and Wan [11], Li and Wan [38, 39], Wan and Dong [36], Wan *et al.* [40], Liao *et al.* [12], Wan *et al.* [37], and Chen [6] extended the LINMAP to TrFSs, IFSs, IVIFSs, HFSs, PLTSs, and PFSs, respectively, to solve fuzzy MCDM problems.

The extended LINMAP methods reported in [6, 11, 12, 36–40] appear to be well developed, as all fuzzy MCDM problems are effectively managed. However, four limitations remain to be addressed.

- In reality, for some decision-making problems, such as green supplier selection [15, 28], online review sentiment analysis [2], and overseas mineral investment [9], most of the decision information is unknown and many factors are affected by uncertainty. In such cases, T1FSs and their extensions are inadequate for depicting decision information because their membership function (MF) is uncertain. Hence, the extended LINMAP methods in [6, 11, 12, 36–40] are unsuitable for addressing MCDM problems in settings with high uncertainties. Therefore, an alternative method must be developed to express decision-making information by capturing the effects of uncertain MFs.

- In the actual MCDM, heterogeneous relationships often exist among the criteria. For example, an investment company can be assessed based on product substitutability, development potential, investment safety index and investment income. Product substitutability and development potential can be considered as negative synergetic interactive criteria, whereas development potential and investment income can be considered as positive synergetic interactive criteria. Hence, redundant and complementary relationships exist among the criteria [14]. However, the aforementioned LINMAP methods in [6, 11, 12, 36–40] assume that the criteria are independent; therefore, they fail to solve such cases.

- For some MCDM problems, the DMs typically encounter an uncertain condition. The criteria values of decision problems are random variables that can be changed according to the natural state. DMs are uncertain of the real state in the future; however, they can provide all possible states and quantify this randomness by establishing a probability distribution. This type of MCDM problem is known as risky MCDM [16], which has a wide range of practical backgrounds [27, 29]. To the best our knowledge, the extension of the LINMAP into a risky decision-making environment has not yet been emphasized.

- In risky decision-making activities, the DMs typically behave according to bounded rationality owing to cognition limitation and incomplete information. For example, during investment company selection, the DMs not only attend to the outcomes of the selected company, but also attend to the results of other companies and avoid selecting a regrettable company. However, the LINMAP methods in [6, 11, 12, 36–40] are based on the expected utility theory and assume that the DMs are completely rational, which would result in apparent differences between the real MCDM behavior and the predicted values of the expected utility theory [26].

The MCDM methods in [3, 42] based on type-2 fuzzy sets (T2FSs) can address the first limitation. T2FSs [44] can efficiently manage higher uncertainties because their MFs are three-dimensional and involve an uncertainty footprint. Thus far, interval T2FSs (IT2FSs) [24] are the most widely used T2FSs owing to their low computational complexity. Hence, Chen [5] and Haghighi *et al.* [8] proposed extensive LINMAPs to address MCDM problems in IT2F settings. In these models, although the criteria values are represented by IT2FSs, the comparisons of alternatives are still expressed by ordered pairs with numerical truth degrees of 0 or 1. In practice, however, the DMs might not be certain of all comparison values and thus might represent their opinions with an IT2F truth degree. Therefore, extending the LINMAP to fit the IT2F MCDM problems with the IT2F truth degrees of alternative comparisons is a popular topic worth investigating.

The MCDM methods in [14, 22, 33, 46] based on fuzzy measures (FMs) can overcome the second weakness. FMs introduced by Sugeno [32] only make monotonicity instead of the additivity property. It can capture complementary, redundant, or independent characteristics among criteria. For example, let $C = \{c_1, c_2, \cdots, c_n\}$ be a set of criteria and μ be a fuzzy measure of C. $\mu(S)(S \in C)$ can be considered as the importance of the decision criteria set S. Thus, in addition to the common weights on criteria taken separately, the weights for any combination of criteria are also defined. Bedsides, we can say the following about any pair of criteria sets $E, F \in C$, $E \cap F = \varnothing$: E and F are independent if $\mu(E \cup F) = \mu(E) + \mu(F)$; E and F exist a complementary relationship between them if $\mu(E \cup F) > \mu(E) + \mu(F)$, and E and F have redundant relationships if $\mu(E \cup F) < \mu(E) + \mu(F)$. Therefore, FMs are excellent for heterogeneous relationships among the criteria. To consider the heterogeneous relationships among the criteria, Zhang *et al.* [46] and Liu *et al.* [22] proposed an IVIF LINMAP model and a double-hierarchy hesitant fuzzy linguistic LINMAP model based on FMs, respectively. It is noteworthy that FMs are defined on a power set, rendering the methods in [22, 46] exponentially complex. Hence, it is difficult to derive FMs for each combination of criteria when the number of criteria is significant. To simplify the complexity of identifying FMs, two special cases of FMs are used: λ FMs [32] and 2-additive FMs (2AFMs) [7]. For λ FMs, only n coefficients are required to derive them for a decision-making problem with n criteria, whereas they can only depict the homogeneous relationship among the criteria [33]. For 2AFMs, $n(n+1)/2$ coefficients are required to derive them for a decision-making problem with n criteria. Meanwhile, they can describe the heterogeneous relationship between the two criteria [33]. Accordingly, 2AFMs can describe the heterogeneous relationships among criteria over λ FMs. Therefore, we use 2AFMs to solve the second weakness.

The MCDM methods in [27, 29, 41, 45] based on behavior decision theory can manage the last two issues. Two significant representatives of behavior decision theories are prospect theory and regret theory. Prospect theory, pioneered by Kahneman and Tversky [10], can capture DMs' reference dependence, loss aversion, and diminishing sensitivity. Regret theory, presented by Bell [1] and Loomes and Sugden [23], can reflect the anticipated regret and rejoice of DMs. To consider the psychological behaviors of DMs, Zhang *et al.* [41] and Zhang *et al.* [45] extended prospect theory and regret theory to the LINMAP, respectively, and constructed fuzzy mathematical programming models. However, these two models failed to address high-type fuzzy MCDM problems. Compared with prospect theory, regret theory has better expressive power, fewer parameters, and a simpler calculation process. Therefore, the combination of LINMAP with regret theory in IT2F settings requires further discussion and research.

The MCDM methods in [5, 8] can address the first issue, as IT2FSs have stronger capability and flexibility in describing uncertainties. Meanwhile, the MCDM methods in [33] can better manage the second issue, as 2AFMs can not only consider the heterogeneity among the criteria, but also simplify the complexity of identifying FMs. The MCDM methods in [29, 45] can better address the last two issues, as regret theory can simply and consistently describe the regret aversion of DMs. To the best of our knowledge, the study of risky MCDM problems capturing the heterogeneous relationship among the criteria and regret aversion of DMs in higher uncertain settings has not been reported hitherto. Based on the analysis above for addressing such MCDM problems, it is justifiable to extend 2AFMs and regret theory to the LINMAP in IT2F settings to establish an IT2F programming model. Additionally, because the established model involves a complex objective function and many constraints, an intelligent optimization algorithm is designed using an external archive-based constrained state transition algorithm (EA-CSTA) [47] to solve the established model. Therefore, the main objective of this study is to propose an IT2F programming method for addressing IT2F risky MCDM problems with heterogeneous relationships and bounded rationality. The main contributions of this study are as follows:

- IT2FSs are used to describe the decision information of truth degrees of the comparisons and criteria of alternatives to accurately and flexibly address MCDM problems that involve a high degree of uncertainty.

- A calculation formula of Banzhaf-based overall perceived utility values of alternatives is developed based on 2AFMs and regret theory to fully consider the heterogeneous relationship among the criteria and effectively reflect the DMs' regret aversion.

- A novel IT2F programming model is established based on the LINMAP; its solution algorithm is then designed using the EA-CSTA, which can objectively identify the optimal weights.

- An IT2F programming method that can overcome the drawbacks of existing IT2F MCDM methods in [5, 15, 27, 28] is proposed to manage risky MCDM problems. Compared with these existing methods, the proposed method not only flexibly describes higher uncertainties, but also fully considers the heterogeneous relationship among the criteria and effectively captures the DMs' regret aversion.

The remainder of this paper is organized as follows. In Section 2, we provide preliminaries regarding IT2FSs, 2AFMs, and regret theory. Section 3 formulates the risky MCDM problem with a heterogeneous relationship and bounded rationality and depicts a resolution procedure for it. In Section 4, according to the resolution procedure, an IT2F programming model is established, and its solution is proposed based on the EA-CSTA. In Section 5, the applicability and superiority of the proposed method are explained based on a numerical example. The conclusions are summarized in Section 6.

2. Preliminaries

In this section, we introduce some basic information regarding IT2FSs, 2AFMs, and regret theory, which will be utilized later in this article.

2.1. IT2FSs

Definition 1 [24]: It is assumed that X is the universe of discourse. A type-2 fuzzy set (T2FS) $\ddot{A}$ defined on X can be denoted as:

$$\ddot{A} = \{((x,u), \nu(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{1}$$

where x represents the primary variable, u is the secondary variable, and $J_x \subseteq [0,1]$ represents the primary membership function at x. Thus, (1) can be equivalently expressed as

$$\dot{A} = \int_{x\in X} \int_{u\in J_x} \nu(x,u)/(x,u) = \int_{x\in X} \left(\int_{u\in J_x} \nu(x,u)/u \right) /x \tag{2}$$

where $\int_{u\in J_x} \nu(x,u)/u$ represents the secondary membership function at x. The integral sign $\int\int$ represents the traversal for all available x and u.

Definition 2 [24]: For a T2FS $\ddot{A}$, if $\nu(x,u) = 1$ for all $x \in X$, then $\ddot{A}$ becomes an IT2FS $\dot{A}$, and it is denoted as:

$$\dot{A} = \int_{x\in X} \int_{u\in J_x} 1/(x,u) = \int_{x\in X} \left(\int_{u\in J_x} 1/u \right) /x \tag{3}$$

Definition 3 [5]: An IT2FS is called a trapezoidal IT2FS (Fig. 1) when the upper and lower MFs are both TrFSs, i.e.,

$$A = \left(A^u, A^l\right) = \left((a^u, b^u, c^u, d^u; h^u), (a^l, b^l, c^l, d^l; h^l)\right) \tag{4}$$

where A^u and A^l are TrFSs; a^u, b^u, c^u, d^u, a^l, b^l, c^l, and d^l denote the reference points of the trapezoidal IT2FS A, verifying $0 \le a^u \le b^u \le c^u \le d^u$, $0 \le a^l \le b^l \le c^l \le d^l$; h^l and h^u indicate the heights of A^u and A^l, respectively, verifying that $0 \le h^l \le h^u \le 1$. The upper MF $A^u(x)$ and lower MF $A^l(x)$ are represented by

$$A^u(x) = \begin{cases} \frac{(x-a^u)h^u}{b^u-a^u}, & a^u \le x < b^u \\ h^u, & b^u \le x \le c^u \\ \frac{(d^u-x)h^u}{d^u-c^u}, & c^u < x \le d^u \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$A^l(x) = \begin{cases} \frac{(x-a^l)h^l}{b^l-a^l}, & a^l \le x < b^l \\ h^l, & b^l \le x \le c^l \\ \frac{(d^l-x)h^u}{d^l-c^l}, & c^l < x \le d^l \\ 0. & \text{otherwise} \end{cases} \tag{6}$$

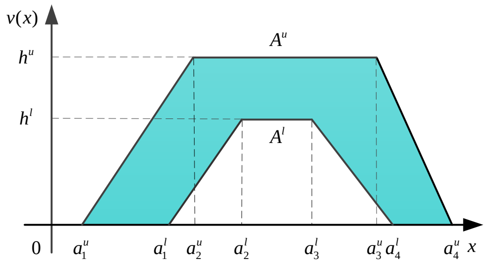

Fig. 1. A trapezoidal IT2FS A with geometrical interpretation.

To calculate the centroid interval $[B_A^l, B_A^u]$ of IT2FS A, the centroid calculation approach [21] was developed using the following formulas:

$$B_A^l = \int_{a^u}^{B_A^l} (B_A^l - x)A^u(x)dx + \int_{B_A^l}^{d^u} (B_A^l - x)A^l(x)dx, \tag{7}$$

$$B_A^u = -\int_{a^u}^{B_A^u} (B_A^u - x)A^l(x)dx - \int_{B_A^u}^{d^u} (B_A^u - x)A^u(x)dx. \tag{8}$$

Definition 4 [4]: It is assumed that A_1 and A_2 are two trapezoidal IT2FS. Their operations are as follows.

$$A_1 \oplus A_2 = ((a_1^u + a_2^u, b_1^u + b_2^u, c_1^u + c_2^u, d_1^u + d_2^u; \min\{h_1^u, h_2^u\}), (a_1^l + a_2^l, b_1^l + b_2^l, c_1^l + c_2^l, d_1^l + d_2^l; \min\{h_1^l, h_2^l\}));$$

$$kA_1 = ((ka_1^u, kb_1^u, kc_1^u, kd_1^u; h_1^u), (ka_1^l, kb_1^l, kc_1^l, kd_1^l; h_1^l)).$$

Definition 5 [25]: It is assumed that A is a trapezoidal IT2FS. Then, its expected value is

$$E_A = \frac{1}{16}(h^u + h^l)(a^u + a^l + b^u + b^l + c^u + c^l + d^u + d^l).$$

For any two IT2FSs A_1 and A_2, if $E_{A_2} > E_{A_2}$, then $A_1 > A_2$.

2.2. 2-Additive Fuzzy Measure

To derive a 2AFM, the following theorem was proven.

Theorem 1 [7]: For the set $C = \{c_1, c_2, \cdots, c_n\}$, μ is a 2AFM on C if the following restrictions are verified:

1) $\mu(\{c_j\}) \geq 0 \; (\forall c_j \in C)$;

2) $\sum_{\{c_j,c_q\}\subseteq C} \mu(\{c_j, c_q\}) - (n-2) \sum_{\{c_j\}\subseteq C} \mu(\{c_j\}) = 1$;

3) $\sum_{\{c_j\}\subseteq S\setminus c_q} (\mu(\{c_j, c_q\}) - \mu(\{c_j\})) \geq (|S|-2)\mu(\{c_q\}) \; (\forall S \subseteq C$ with $c_q \in S$ and $|S| \geq 2)$.

Although $\mu(S)(S \subseteq C)$ can be regarded as the importance of decision criteria set S, it can only model the heterogeneous relationship between two criteria, which may result in information loss. To capture the overall heterogeneous relationship among the criteria, the following generalized Banzhaf function with 2AFM is provided:

Theorem 2 [33]: It is assumed that μ is a 2AFM on set C. The generalized Banzhaf index with 2AFM μ is

$$\psi(B) = \sum_{\{c_j,c_q\}\subseteq B} \mu(\{c_j, c_q\}) + \sum_{c_j\in B, c_q\in C\setminus B} \frac{1}{2}\Big(\mu(\{c_j, c_q\}) - |B|\mu(c_j)\Big) - \frac{|C| + |B| - 4}{2} \sum_{c_j\in B} \mu(\{c_j\}), \tag{9}$$

where B represents any subset of C, $C \backslash B$ represents the difference set between C and B, and $|B|$ and $|C|$ are the cardinal numbers of the coalitions B and X, respectively. If only one element c_j exists in the set B, namely, $B = \{c_j\}$, then (9) becomes the Banzhaf function with 2AFM:

$$\psi(\{c_j\}) = \frac{3-n}{2}\mu(\{c_j\}) + \sum_{c_q \in C \backslash c_j} \frac{1}{2}\Big(\mu(\{c_j, c_q\}) - \mu(\{c_q\})\Big). \tag{10}$$

From *Theorem* 1 and (10), it is apparent that $\psi(\{c_j\}) \geq 0$ and $\sum_{i=1}^{n} \psi(\{c_j\}) = 1$. This implies that $\boldsymbol{\psi} = (\psi(\{c_1\}), \psi(\{c_2\}), \cdots, \psi(\{c_n\}))$ can be regarded as a weighting vector, called a Banzhaf-based weighting vector. The significant features of the Banzhaf-based weighting vector are that it not only provides the importance of the criteria, but also globally captures the heterogeneous relationships among them.

2.3. Regret Theory

As one of the most popular non-expected utility theories, regret theory [1, 23] depicts preferences using a bivariate utility function, which captures the DMs' feelings of regret and rejoice.

It is assumed that x_1 and x_2 are the possible results derived by selecting alternatives z_1 and z_2, respectively. Then, the DM's perceived utility for alternative z_1 is

$$V_1 = u(x_1) + R(u(x_1) - u(x_2)), \tag{11}$$

where $u(\cdot)$ denotes the utility function and verifies $u'(\cdot) > 0$ and $u''(\cdot) < 0$; $u(x_1)$ and $u(x_2)$ are the utilities that the DMs would obtain from alternatives A_1 and A_2 directly, respectively; $R(\cdot)$ denotes a regret–rejoice function and verifies $R(0) = 0$, $R'(\cdot) > 0$, and $R''(\cdot) < 0$; $R(u(x_1) - u(x_2))$ is a regret–rejoice function concerning $u(x_1) - u(x_2)$. According to the value of $R(u(x_1) - u(x_2))$, the different feelings of the DMs can be observed when selecting alternative A_1 instead of A_2. Specifically, if $R(u(x_1) - u(x_2)) > 0$, the DMs would rejoice, and if $R(u(x_1) - u(x_2)) < 0$, then the DMs would regret; if neither applies, then the DMs would neither rejoice nor regret.

Because real MCDM problems typically include more than two alternatives, regret theory based on two alternatives is extended to regret theory based on multiple alternatives, as described below.

It is assumed that the results x_1, x_2, $\cdots$, and x_m can be obtained by selecting alternatives z_1, z_2, $\cdots$, and z_m, respectively. Then, the DMs' perceived utility for alternative z_i $(i = 1, 2, \cdots, m)$ is

$$V_i = u(x_i) + R(u(x_i) - u(x^*)), \tag{12}$$

where $x^* = \max\{x_1, x_2, \cdots, x_m\}$ and $R(u(x_i) - u(x^*)) \leq 0$.

3. Framework for Risky MCDM Problem in IT2F Settings

In this section, we first illustrate the risky MCDM problem in IT2F settings and then present its resolution framework.

3.1. Description of Risky MCDM Problem in IT2F Settings

For convenience, the following notations are used to express the risky MCDM problem with heterogeneous relationships and incomplete weight information in IT2F settings.

$Z = \{z_1, z_2, \cdots, z_m\}$: the set of m alternatives, where $z_i (i = 1, 2, \cdots, m)$ represents the ith alternative.

$C = \{c_1, c_2, \cdots, c_n\}$: the set of n criteria, where $c_j (j = 1, 2, \cdots, n)$ represents the jth criterion. These criteria are classified into two categories: costs and benefits. It is assumed that C_d and C_b are the sets of cost and benefit criteria, respectively, verifying $C_d \cup C_b = C$ and $C_d \cap C_b = \emptyset$. Here, a heterogeneous relationship exists among the criteria.

$\omega = (\mu(\{c_1\}), \mu(\{c_2\}), \cdots, \mu(\{c_{n-1}, c_n\}))$: the vector of $n(n+1)/2$ 2AFMs on the criteria set, where $\mu(\{c_j\}) (j = 1, 2, \cdots, n)$ represents the weight of criterion c_j, such that $0 \leq \mu(\{c_j\}) \leq 1$, and $\mu(\{c_j, c_q\}) (j, q = 1, 2, \cdots, n; j \neq q)$ represents the weight of criteria set $\{c_j, c_q\}$, such that $0 \leq \mu\{c_j, c_q\} \leq 1$. Owing to the complexities and uncertainties of real MCDM problems and the DMs' limited experience in the problem, information regarding the weights of the criteria is often incomplete.

$\Theta = \{\theta_1, \theta_2, \cdots, \theta_o\}$: the set of o natural states, where $\theta_\tau(\tau = 1, 2, \cdots, o)$ represents the τth natural state.

$\mathbf{P} = (p_1, p_2, \cdots, p_o)$: the probability (weighting) vector of o natural states, where $p_\tau(\tau = 1, 2, \cdots, o)$ represents the probability that state θ_τ occurs, verifying $p_\tau \geq 0$ and $\sum_{\tau=1}^{o} p_\tau = 1$. In this study, information regarding the probability vector is partially known.

$\widetilde{\mathbf{A}}^\tau = [\widetilde{A}_{ij}^\tau]_{m\times n}(\tau = 1, 2, \cdots, o)$: risky decision matrices, where $\widetilde{A}_{ij}^\tau$ is the outcome of alternative z_i with respect to criterion c_j in state θ_τ.

$\Omega^\tau = \{< (k, i), t^\tau(k, i) > |z_k \geq_\tau z_i \text{ with } t^\tau(k, i)(k, i = 1, 2, \cdots, m)\}$: DM preferences derived using pairwise comparisons of the alternatives, where (k, i) represents an ordered pair of alternatives z_k and z_i that the DMs prefer z_k to z_i (expressed by $z_k \geq_\tau z_i$) with the truth degree $t^\tau(k, i)$ in state θ_τ.

The main issue addressed in this study is the method of selecting the optimal alternative(s) by utilizing three different types of decision information (criteria values, pairwise comparisons of alternatives, and weighting vectors) while capturing the heterogeneous relationships among the criteria and the DMs' regret revision.

3.2. Constructed Framework

To address this issue, a resolution framework was constructed, as shown in Fig. 2. The resolution framework consists of four parts. In the following section, we briefly describe each part.

- Acquire Information Phase. In this section, a group of DMs is formed. Subsequently, the alternative, criteria, and natural state sets are identified. In addition, linguistic terms and their associated IT2FSs are determined. Moreover, the DMs provide three different types of assessment information: information regarding the criteria values, pairwise comparisons of alternatives, and weighting vectors of criteria and natural states.

- Calculate Banzhaf-based Perceived Utility Value Phase. In this section, the IT2F utility value and regret/rejoice value with respect to the criteria values are proposed. Subsequently, a new technique is developed to calculate the Banzhaf-based overall perceived utility values of alternatives based on 2AFMs and regret theory to capture the heterogeneous relationship among the criteria and the DM's regret revision.

- Construct IT2F Programming Model Phase. In this section, the Banzhaf-based IT2F risky consistency index (BIT2FRCI) and the Banzhaf-based IT2F risky inconsistency index (BIT2FRII) are defined. Subsequently, a novel IT2F programming model is established based on LINMAP.

- Solve Model and Rank Alternative Phases. In this section, the established IT2F programming model is conveyed equivalently into a nonlinear mathematical programming model. Subsequently, the EA-CSTA-based optimization algorithm is designed to address the programming model and derive the optimal weights of the criteria and natural states. Accordingly, we can compute the optimal Banzhaf-based overall perceived utility values of the alternatives. Finally, the ranking order of the alternatives is derived.

4. The Developed IT2F Mathematical Programming Method

According to the resolution process depicted in Section 3, we provide a detailed description of the developed IT2F programming method.

4.1. Information Acquisition

A group of DMs is gathered for scientific assessment and decision. Subsequently, an alternative set Z, criteria set C, and natural state set Θ are identified. Furthermore, the group of DMs provides the following three types of decision information:

The DM provides the criteria values based on the linguistic term set L={"Very bad"(VB), "Bad"(B), "Medium Bad"(MB), "Medium"(M), "Medium Good"(MG), "Good"(G), "Very Good"(VG)}. Hence, the risky decision matrices $\widetilde{A}^\tau = [\widetilde{A}_{ij}^\tau]_{m\times n}(\tau = 1, 2, \cdots, o)$ are derived. Subsequently, the evaluation values are conveyed to the IT2Fs according to Table 1. The MFs of the IT2FSs linguistic terms are shown in Fig. 3. Additionally, Table 2 lists the complementary relations of the linguistic terms.

Fig. 2. Constructed Evaluation Framework.

Table 1
Linguistic Terms and their Corresponding IT2FSs.

Linguistic terms	IT2FSs
VB	$((0, 0, 0, 0.1; 1), (0, 0, 0, 0.05; 0.9))$
B	$((0, 0.1, 0.15, 0.3; 1), (0.05, 0.1, 0.1, 0.2; 0.9))$
MB	$((0.15, 0.3, 0.35, 0.5; 1), (0.2, 0.25, 0.3, 0.4; 0.9))$
M	$((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))$
MG	$((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))$
G	$((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))$
VG	$((0.9, 1, 1, 1; 1), (0.95, 1, 1, 1; 0.9))$

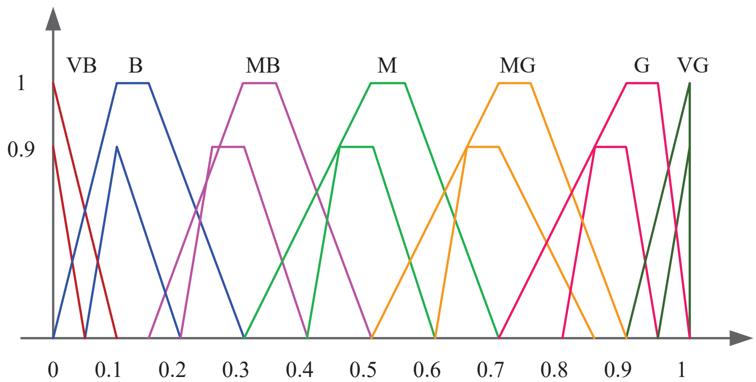

Fig. 3. MFs of IT2FSs Linguistic Terms.

Table 2
Complementary Relations.

Linguistic terms (L)	VB	B	MB	M	MG	G	VG
Complementary terms (L^c)	VG	G	MG	M	MB	B	VB

For pairwise comparisons of alternatives, the DMs represent their truth degrees based on Table 1. Using an ordered pair of the alternatives z_1 and z_2 as an example, the DMs prefer z_1 to z_2 with truth degree "VG" in state θ_1. Subsequently, the preference relation between alternatives z_1 and z_2 in state θ_1 is denoted by $\langle(1,2), t^1(1,2)\rangle$, where $t^1(1,2) = VG$. Therefore, all preference relations $\langle(k,i), t^\tau(k,i)\rangle$ between the ordered pairs of alternatives z_k and z_i in state θ_τ are derived, denoted by $\Omega^\tau = \{\langle(k,i), t^\tau(k,i)\rangle | z_k \geq_\tau z_i \text{ with } t^\tau(k,i)(k,i = 1,2,\cdots,m)\}$.

For information regarding the weights of the criteria and natural states, the DMs express them using interval numbers. For example, when assessing the weight of c_1, the DMs assume that their weights are 0.2, 0.25, 0.3 and 0.4, respectively. Subsequently, $0.2 \leq \mu(c_1) \leq 0.4$ denotes the value range of the weight of c_1. Therefore, all interval weights of criteria c_j and criteria set $\{c_j, c_q\}$ are derived. For simplicity, let Λ be the set of all interval weights of $c_j (j = 1,2,\cdots,n)$ and $\{c_j, c_q\}(j,q = 1,2,\cdots,n; j \neq q)$ provided by the DMs. Analogously, the set of all interval weights of natural states is derived and represented by Γ.

4.2. Calculation of Banzhaf-based Perceived Utility Value

4.2.1. Normalization of Decision-Making Information

Because the risky MCDM problem involves two types of criteria, i.e., the benefit and cost criteria, the effect of different criteria types must be removed by normalizing the decision matrix $[\widetilde{A}^\tau_{ij}]_{m\times n} (\tau = 1,2,\cdots,o)$. In this study, the cost criteria values are transformed into benefit criteria values based on Table 2, where the decision matrix $[\widetilde{A}^\tau_{ij}]_{m\times n}$ is conveyed into the normalized decision matrix $[A^\tau_{ij}]_{m\times n}$, where

$$A^\tau_{ij} = \begin{cases} \widetilde{A}^\tau_{ij}, & \text{if } c_j \in C_b \\ (\widetilde{A}^\tau_{ij})^c. & \text{if } c_j \in C_d \end{cases} \tag{13}$$

Here, $(\widetilde{A}^\tau_{ij})^c$ is the complement of $\widetilde{A}^\tau_{ij}$ such that $(\widetilde{A}^\tau_{ij})^c \in L^c$.

4.2.2. Derivation of Optimal Ideal Solutions

According to the expected value of IT2FS in *Definition* 5, we can derive the optimal ideal solution $\overline{\mathbf{A}}^\tau = (\overline{A}^\tau_1, \overline{A}^\tau_2, \cdots, \overline{A}^\tau_n)$, where

$$\overline{A}^\tau_j = \max_i(\widetilde{A}^\tau_{ij})(j = 1,2,\cdots,n). \tag{14}$$

4.2.3. Calculation of Centroid Interval of IT2FSs

To reduce tedious operations of IT2FSs and to simplify the risky MCDM problem, the centroid calculation approach [21] is used to derive the defuzzified value $B^\tau_{ij} = [B^l_{A^\tau_{ij}}, B^r_{A^\tau_{ij}}]$ of the IT2F criteria value $A^\tau_{ij}(i = 1,2,\cdots,m)$, where

$$B^l_{A^\tau_{ij}} = \int_{a^{\tau u}_{ij}}^{B^l_{A^\tau_{ij}}} (B^l_{A^\tau_{ij}} - x) A^{\tau u}_{ij}(x)dx + \int_{B^l_{A^\tau_{ij}}}^{d^{\tau u}_{ij}} (B^l_{A^\tau_{ij}} - x) A^{\tau l}_{ij}(x)dx, \tag{15}$$

$$B^u_{A^\tau_{ij}} = -\int_{a^{\tau u}_{ij}}^{B^u_{A^\tau_{ij}}} (B^u_{A^\tau_{ij}} - x) A^{\tau l}_{ij}(x)dx - \int_{B^u_{A^\tau_{ij}}}^{d^{\tau u}_{ij}} (B^u_{A^\tau_{ij}} - x) A^{\tau u}_{ij}(x)dx, \tag{16}$$

where $A^{\tau u}_{ij}(x)$ and $A^{\tau l}_{ij}(x)$ are the upper and lower MFs of A^τ_{ij}, respectively.

Similarly, we can derive the defuzzified value vector of the optimal ideal solution $\overline{\mathbf{A}}^\tau$, expressed by $\overline{\mathbf{B}}^\tau = (\overline{B}^\tau_1, \overline{B}^\tau_2, \cdots, \overline{B}^\tau_n)$.

4.2.4. Calculation of Utility Value Matrix

Because B^τ_{ij} is an interval value, it can be considered as a special random variable, denoted by $\widehat{B}^\tau_{ij}$, with the utility function $u(\widehat{B}^\tau_{ij})$, and the probability density function $f(\widehat{B}^\tau_{ij})$, where they verify $u'(\widehat{B}^\tau_{ij}) > 0$, $u''(\widehat{B}^\tau_{ij}) > 0$, $\int_{-\infty}^{+\infty} f(\widehat{B}^\tau_{ij})dB = 1$, and $f(\widehat{B}^\tau_{ij}) \geq 0$. Hence, the utility value of B^τ_{ij} is

$$U^\tau_{ij} = \int_{B^l_{A^\tau_{ij}}}^{B^u_{A^\tau_{ij}}} u(\widehat{B}^\tau_{ij}) f(\widehat{B}^\tau_{ij}) d\widehat{B}^\tau_{ij}. \tag{17}$$

Accordingly, the utility value matrix $[U^\tau_{ij}]_{m\times n}$ is derived.

In practice, we can express the utility of the DMs using the power function, that is, $u(\widehat{B}^\tau_{ij}) = (\widehat{B}^\tau_{ij})^\alpha$, where α represents the risk aversion coefficient of the DMs and verifies $0 < \alpha < 1$ [35]. The effect of α on the utility function $u(\widehat{B}^\tau_{ij})$ is shown in Supplementary Fig. 4 for a better demonstration. As shown in the figure, the greater the value of parameter α, the greater is the degree of risk aversion of the DMs. In this study, we set $\alpha = 0.88$, as suggested in [35]. In addition, we assume that $\widehat{B}^\tau_{ij}$ obeys a normal distribution or uniform distribution. Both of them are the most typical forms of distribution functions.

If $\widehat{B}^\tau_{ij}$ is normally distributed $\widehat{B}^\tau_{ij} \sim N(\nu^\tau_{ij}, (\sigma^\tau_{ij})^2)$, where ν^τ_{ij} and σ^τ_{ij} denote the mean and standard deviation, respectively, then according to the 3σ principle [30] in probability and statistics, the possibility of $\widehat{B}^\tau_{ij}$ is 99.73% in interval $[B^l_{A^\tau_{ij}}, B^u_{A^\tau_{ij}}]$, i.e., $\nu = \frac{B^l_{A^\tau_{ij}} + B^u_{A^\tau_{ij}}}{2}$ and $\sigma = \frac{B^u_{A^\tau_{ij}} - B^l_{A^\tau_{ij}}}{6}$. Hence, the probability density of $\widehat{B}^\tau_{ij}$ is

$$f(\widehat{B}^\tau_{ij}) = \frac{3\sqrt{2}}{\sqrt{\pi}(B^u_{A^\tau_{ij}} - B^l_{A^\tau_{ij}})} e^{-\frac{18(\widehat{B}^\tau_{ij} - \frac{B^l_{A^\tau_{ij}} + B^u_{A^\tau_{ij}}}{2})^2}{(B^u_{A^\tau_{ij}} - B^l_{A^\tau_{ij}})^2}}. \tag{18}$$

If $\widehat{B}^\tau_{ij}$ is uniformly distributed $\widehat{B}^\tau_{ij} \sim U(B^l_{A^\tau_{ij}}, B^u_{A^\tau_{ij}})$, the probability density of $\widehat{B}^\tau_{ij}$ is

$$f(\widehat{B}^\tau_{ij}) = \begin{cases} \frac{1}{B^u_{A^\tau_{ij}} - B^l_{A^\tau_{ij}}}, & B^l_{A^\tau_{ij}} \leq \widehat{B}^\tau_{ij} \leq B^u_{A^\tau_{ij}} \\ 0. & \text{otherwise} \end{cases} \tag{19}$$

Analogously, we can derive the utility vector of the optimal ideal solution $\overline{\mathbf{A}}^\tau$, denoted by $\overline{\mathbf{U}}^\tau = (\overline{U}^\tau_1, \overline{U}^\tau_2, \cdots, \overline{U}^\tau_n)$.

4.2.5. Calculation of Regret Value Matrix

To derive the regret value R^τ_{ij} of the alternative z_i relative to the optimal ideal solution A^τ concerning criterion c_j in state θ^τ, we must first determine the regret-rejoice function. The following function is used to denote the regret–rejoice function:

$$R^\tau_{ij} = 1 - e^{-\delta(\triangle U^\tau_{ij})}, \tag{20}$$

where $\triangle U^\tau_{ij}$ is the difference between U^τ_{ij} and $\overline{U}^\tau_j$, and δ represents the regret aversion coefficient with $\delta \geq 0$ [1]. The effect of δ on the regret–rejoice function R^τ_{ij} is depicted in Supplementary Fig. 5 for a better illustration. As shown in the figure, the greater the value of parameter δ, the greater the degree of the DMs' regret aversion. In this study, we set $\delta = 0.3$, as suggested in [45]. Accordingly, the regret value matrix $[R^\tau_{ij}]_{m\times n}$ is derived.

4.2.6. Calculation of Overall Perceived Utility Value

According to (11), (17), and (20), we derive the perceived utility value V_{ij}^{τ} of the alternative z_i concerning criterion c_j in state θ^{τ} as follows:

$$V_{ij}^{\tau} = U_{ij}^{\tau} + R_{ij}^{\tau}. \tag{21}$$

Accordingly, the perceived utility matrix $[V_{ij}^{\tau}]_{m\times n}$ is derived.

Subsequently, the perceived utility value V_i^{τ} of alternative z_i in state θ^{τ} is derived as follows:

$$V_i^{\tau} = \sum_{j=1}^{n} w_j V_{ij}^{\tau}, \tag{22}$$

where w_j is the weight of the criterion c_j. Considering the advantage of the Banzhaf-based weighting vector $\boldsymbol{\psi}$ in modeling the heterogeneous relationships among the criteria, we replace w_j with $\psi(\{c_j\})$ in (22) to obtain the Banzhaf-based perceived utility value as follows:

$$V_i^{\tau} = \sum_{j=1}^{n} \psi(\{x_j\}) V_{ij}^{\tau}, \tag{23}$$

where $\psi(\{c_j\})$ is the Banzhaf value on criterion c_j.

Finally, we derive the Banzhaf-based overall perceived utility value V_i of alternative z_i as follows:

$$V_i = \sum_{\tau=1}^{o} p_{\tau} V_i^{\tau} = \sum_{\tau=1}^{o} \sum_{j=1}^{n} p_{\tau} \psi(\{x_j\}) V_{ij}^{\tau}, \tag{24}$$

where p_{τ} represents the probability that state θ_{τ} occurs.

4.3. Construction of IT2F Mathematical Programming Model

4.3.1. Definitions of Banzhaf-based IT2F Risky Consistency and Inconsistency Indices

As stated above, the DMs provide subjective preference relations between alternatives, represented by $\Omega^{\tau} = \{\langle (k,i), t^{\tau}(k,i)\rangle | z_k \succeq_{\tau} z_i \text{ with } t^{\tau}(k,i)(k,i = 1,2,\cdots,m)\}(\tau = 1,2,\cdots,o)$, where $t^{\tau}(k,i)$ denotes an IT2FS expressed by $t^{\tau}(k,i) = ((a^u_{t^{\tau}(k,i)}, b^u_{t^{\tau}(k,i)}, c^u_{t^{\tau}(k,i)}, d^u_{t^{\tau}(k,i)}; h^u_{t^{\tau}(k,i)}), (a^l_{t^{\tau}(k,i)}, b^l_{t^{\tau}(k,i)}, c^l_{t^{\tau}(k,i)}, d^l_{t^{\tau}(k,i)}; h^l_{t^{\tau}(k,i)}))$. We assume that the support of Ω^{τ} is $\Omega_0^{\tau} = \{\langle (k,i)\rangle | E_{t^{\tau}(k,i)} > 0\}$. It is noteworthy that the preference relations provided by the DMs are pairwise comparisons of alternatives on all criteria instead of on each criterion, thereby capturing the DMs' view on ordered pairs of alternatives.

The ordered pair of alternatives $(k,i) \in \Omega_0^{\tau}$ captures the subjective preference of the DMs; therefore, it is a type of subjective criterion for determining the ranking order of alternatives. Nevertheless, from (23), the larger the Banzhaf-based perceived utility value V_i^{τ}, the better the alternative z_i in state θ^{τ}. Consequently, the Banzhaf-based perceived utility value V_i^{τ} is a type of objective criterion for determining the ranking order of alternatives. For a good decision, the subjective criterion should be consistent with the objective criterion as much as possible.

We assume that the criteria weighting vector $\boldsymbol{\psi}$ is provided by the DMs in advance. Subsequently, we derive all the Banzhaf-based perceived utility values V_k^{τ} and V_i^{τ} for each $(k,i) \in \Omega_0^{\tau}$ using (23) in state θ^{τ}. If $V_k^{\tau} \geq V_i^{\tau}$, then the Banzhaf-based perceived utility value of alternative z_k is not lower than that of alternative z_i in state θ^{τ}. Hence, the objective ranking order of alternatives z_k and z_i in state θ^{τ} obtained by V_k^{τ} and V_i^{τ} based on the criteria weighting vector $\boldsymbol{\psi}$ is consistent with the subjective preference relation provided by the DMs. In contrast, if $V_k^{\tau} < V_i^{\tau}$, then the objective ranking order based on the criteria weighting vector will be inconsistent with the subjective preference relation. Therefore, the criteria weighting vector $\boldsymbol{\psi}$ will not be selected appropriately.

To measure the degree of consistency between the ranking order of alternatives z_k and z_i in state θ^{τ} obtained by V_k and V_i, as well as the preferences provided by the DMs (who prefer z_k to z_i in state θ^{τ}), the following Banzhaf-based

IT2F consistency is provided:

$$(V_k^\tau - V_i^\tau)^+ = \begin{cases} t^\tau(k,i)(V_k^\tau - V_i^\tau), & V_k^\tau \geq V_i^\tau \\ ((0,0,0,0;1),(0,0,0,0;0.9)). & V_k^\tau < V_i^\tau \end{cases} \tag{25}$$

Clearly, the ranking order of alternatives z_k and z_i derived by Banzhaf-based perceived utility values V_k^τ and V_i^τ is consistent with the subjective preferences provided by the DMs if $V_k^\tau \geq V_i^\tau$. In such a case, $(V_k^\tau - V_i^\tau)^+$ is defined as $t^\tau(k,i)(V_k^\tau - V_i^\tau)$. On the other hand, the ranking order of alternatives z_k and z_i derived using Banzhaf-based perceived utility values V_k^τ and V_i^τ is inconsistent with the subject preferences provided by the DMs if $V_k^\tau < V_i^\tau$. In such a case, $(V_k^\tau - V_i^\tau)^+$ is defined to be $((0,0,0,0;1),(0,0,0,0;0.9))$. Therefore, the Banzhaf-based IT2F consistency index can be rewritten as

$$(V_k^\tau - V_i^\tau)^+ = t^\tau(k,i)\max\{0, V_k^\tau - V_i^\tau\}. \tag{26}$$

Consequently, the Banzhaf-based IT2F overall consistency index in state θ^τ is defined as

$$G^\tau = \sum_{(k,i)\in\Omega_0^\tau} (V_k^\tau - V_i^\tau)^+ = \sum_{(k,i)\in\Omega_0^\tau} t^\tau(k,i)\max\{0, V_k^\tau - V_i^\tau\}. \tag{27}$$

Definition 6: The BIT2FRCI is defined as

$$G = \sum_{\tau=1}^{o} p_\tau G^\tau = \sum_{\tau=1}^{o} p_\tau \sum_{(k,i)\in\Omega_0^\tau} (V_k^\tau - V_i^\tau)^+ = \sum_{\tau=1}^{o} \sum_{(k,i)\in\Omega_0^\tau} p_\tau t^\tau(k,i)\max\{0, V_k^\tau - V_i^\tau\}. \tag{28}$$

Similarly, to measure the degree of inconsistency between the ranking order of alternatives z_k and z_i in state θ^τ obtained by V_k and V_i as well as the preferences provided by the DMs (who prefer z_k to z_i in state θ^τ), the following Banzhaf-based IT2F risky inconsistency in state θ^τ is provided:

$$(V_k^\tau - V_i^\tau)^- = \begin{cases} t^\tau(k,i)(V_i^\tau - V_k^\tau), & V_k^\tau < V_i^\tau \\ ((0,0,0,0;1),(0,0,0,0;0.9)). & V_k^\tau \geq V_i^\tau \end{cases} \tag{29}$$

Subsequently, (29) can be equivalently transformed into

$$(V_k^\tau - V_i^\tau)^- = t^\tau(k,i)\max\{0, V_i^\tau - V_k^\tau\}. \tag{30}$$

Consequently, the Banzhaf-based IT2F overall inconsistency index in state θ^τ is defined as

$$B^\tau = \sum_{(k,i)\in\Omega_0^\tau} (V_k^\tau - V_i^\tau)^- = \sum_{(k,i)\in\Omega_0^\tau} t^\tau(k,i)\max\{0, V_i^\tau - V_k^\tau\}. \tag{31}$$

Definition 7: The BIT2FRII is defined as

$$B = \sum_{\tau=1}^{o} p_\tau B^\tau = \sum_{\tau=1}^{o} p_\tau \sum_{(k,i)\in\Omega_0^\tau} (V_k^\tau - V_i^\tau)^- = \sum_{\tau=1}^{o} \sum_{(k,i)\in\Omega_0^\tau} p_\tau t^\tau(k,i)\max\{0, V_i^\tau - V_k^\tau\}. \tag{32}$$

Remark 1: G and B are IT2FSs because their truth degrees are expressed by IT2FSs. Although a number of studies have extended consistency and inconsistency indices into various fuzzy environments, to the best our knowledge, IT2FSs have not been used to depict fuzzy truth degrees. This is the main difference between the studies in [5, 6, 8, 11, 12, 22, 36–41, 45, 46] and this study.

4.3.2. Establishment of IT2F Programming Model

In a typical LINMAP, a mathematical programming model is applied to determine the weight vector by minimizing the overall inconsistency index under the restriction that the overall inconsistency index does not exceed the overall consistency index minus a positive constant. Therefore, the following model is established using *Theorems* 1 and 2 to derive the optimal weighting vectors of the criteria $\boldsymbol{\omega}^*$ and natural states $\mathbf{P}^*$:

$$\min\left\{B=\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau}p_\tau t^\tau(k,i)\max\{0,V_i^\tau-V_k^\tau\}\right\}$$
$$s.t.\begin{cases}G-B\geq\rho;\\ \psi(\{c_j\})=\frac{3-n}{2}\mu(\{c_j\})+\sum\limits_{c_q\in C\setminus c_j}\frac{1}{2}(\mu(\{c_j,c_q\})-\mu(\{c_q\})\ (j=1,2,\cdots,n);\\ \mu(\{c_j\})\geq 0\ (\forall c_j\in C);\\ \sum\limits_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum\limits_{\{c_j\}\subseteq C}\mu(\{c_j\})=1;\\ \sum\limits_{\{c_j\}\subseteq S\setminus c_q}(\mu(\{c_j,c_q\})-\mu(\{c_j\}))\geq(|S|-2)\mu(\{c_q\})\ (\forall S\subseteq C \text{ with } c_q\in S \text{ and } |S|\geq 2);\\ \mu(\{c_j\}),\mu(\{c_j,c_q\})\in\Lambda\ (\forall\{c_j\},\{c_j,c_q\}\subseteq C);\\ p_\tau\geq 0, p_\tau\in\Gamma\ (\tau=1,2,\cdots,o);\\ \sum\limits_{\tau=1}^{o}p_\tau=1,\end{cases}\tag{33}$$

where $\rho=((a_\rho^u,b_\rho^u,c_\rho^u,d_\rho^u;h_\rho^u),(a_\rho^l,b_\rho^l,c_\rho^l,d_\rho^l;h_\rho^l))$ is an IT2FS, called the priori threshold.

For each $(k,i)\in\Omega_0^\tau$, let $\lambda_{ki}^\tau=\max\{0,V_i^\tau-V_k^\tau\}$. Subsequently, $\lambda_{ki}^\tau\geq 0$ and $\lambda_{ki}^\tau\geq V_i^\tau-V_k^\tau$. In addition, based on (26) and (30), we derive $(V_k^\tau-V_i^\tau)^+-(V_k^\tau-V_i^\tau)^-=t^\tau(k,i)(V_k^\tau-V_i^\tau)$. Moreover, based on (28) and (32), we derive $G-B=\sum_{\tau=1}^{o}p_\tau\sum_{(k,i)\in\Omega_0^\tau}t^\tau(k,i)(V_k^\tau-V_i^\tau)$. Therefore, (33) can be rewritten as

$$\min\left\{B=\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau}p_\tau t^\tau(k,i)\lambda_{ki}^\tau\right\}$$
$$s.t.\begin{cases}\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^\tau}p_\tau t^\tau(k,i)(V_k^\tau-V_i^\tau)\geq\rho;\\ V_k^\tau-V_i^\tau+\lambda_{ki}^\tau\geq 0\ ((k,i)\in\Omega_0^\tau;\tau=1,2,\cdots,o);\\ \lambda_{ki}^\tau\geq 0\ ((k,i)\in\Omega_0^\tau;\tau=1,2,\cdots,o);\\ \psi(\{c_j\})=\frac{3-n}{2}\mu(\{c_j\})+\sum\limits_{c_q\in C\setminus c_j}\frac{1}{2}(\mu(\{c_j,c_q\})\ -\mu(\{c_q\})\ (j=1,2,\cdots,n);\\ \mu(\{c_j\})\geq 0\ (\forall c_j\in C);\\ \sum\limits_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum\limits_{\{c_j\}\subseteq C}\mu(\{c_j\})=1;\\ \sum\limits_{\{c_j\}\subseteq S\setminus c_q}(\mu(\{c_j,c_q\})-\mu(\{c_j\}))\geq(|S|-2)\mu(\{c_q\})\ (\forall S\subseteq C \text{ with } c_q\in S \text{ and } |S|\geq 2);\\ \mu(\{c_j\}),\mu(\{c_j,c_q\})\in\Lambda\ (\forall\{c_j\},\{c_j,c_q\}\subseteq C);\\ p_\tau\geq 0, p_\tau\in\Gamma\ (\tau=1,2,\cdots,o);\\ \sum\limits_{\tau=1}^{o}p_\tau=1.\end{cases}\tag{34}$$

Remark 2: (34) can be known as an IT2F model because its constraints' coefficients, right-hand vector, and objective function include IT2FSs. Although the methods in [5, 8] are also based on IT2FSs, they are real mathematical programming models.

Remark 3: Because (34) is based on 2AFMs and regret theory, it not only considers the heterogeneous relationship among the criteria, but also reflects the DM's regret revision. Although the models in [22, 46] consider the heterogeneous relationship among the criteria, whereas the model in [45] reflects the DMs' regret revision, these existing models fail to capture these two aspects simultaneously. This is another prominent difference between the models in [22, 45, 46] and our model.

4.4. Solution Approach of IT2F Programming Model

Based on *Definition* 3, for (34), the objective function and left part of " $\geq$ " in the first constraint are IT2FSs, respectively, that is,

$$\begin{aligned}
&\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau t^\tau(k,i)\lambda_{ki}^\tau \\
&=\left(\left(\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau a^u_{t^\tau(k,i)}, \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau b^u_{t^\tau(k,i)}, \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau c^u_{t^\tau(k,i)}, \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau d^u_{t^\tau(k,i)}; \min_{\tau,(k,i)} h^u_{t^\tau(k,i)}\right),\right. \\
&\left.\left(\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau a^l_{t^\tau(k,i)}, \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau b^l_{t^\tau(k,i)}, \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau c^l_{t^\tau(k,i)}, \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau\lambda_{ki}^\tau d^l_{t^\tau(k,i)}; \min_{\tau,(k,i)} h^l_{t^\tau(k,i)}\right)\right). \qquad (35)
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau t^\tau(k,i)(V_k^\tau - V_i^\tau) \\
&=\left(\left(\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau \times a^u_{t^\tau(k,i)}(V_k^\tau - V_i^\tau), \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau b^u_{t^\tau(k,i)}(V_k^\tau - V_i^\tau), \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau c^u_{t^\tau(k,i)}(V_k^\tau - V_i^\tau),\right.\right. \\
&\left.\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau d^u_{t^\tau(k,i)} \times (V_k^\tau - V_i^\tau); \min_{\tau,(k,i)} h^u_{t^\tau(k,i)}\right), \left(\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau a^l_{t^\tau(k,i)} \times (V_k^\tau - V_i^\tau), \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau b^l_{t^\tau(k,i)}(V_k^\tau - V_i^\tau),\right. \\
&\left.\left.\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau c^l_{t^\tau(k,i)}(V_k^\tau - V_i^\tau), \sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^\tau} p_\tau d^l_{t^\tau(k,i)} \times (V_k^\tau - V_i^\tau); \min_{\tau,(k,i)} h^l_{t^\tau(k,i)}\right)\right). \qquad (36)
\end{aligned}$$

As no approach exists for addressing such an IT2F programming model, we herein propose an approach for addressing this model according to the expected value of IT2FS. Based on (35), (36), and *Definition* 5, (34) is equivalently expressed as

$$
\min\left\{B=\left(\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}a^{u}_{t^{\tau}(k,i)}+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}b^{u}_{t^{\tau}(k,i)}+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}c^{u}_{t^{\tau}(k,i)}+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}d^{u}_{t^{\tau}(k,i)}\right.\right.
$$

$$
\left.+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}a^{l}_{t^{\tau}(k,i)}+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}b^{l}_{t^{\tau}(k,i)}+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}c^{l}_{t^{\tau}(k,i)}+\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\lambda_{ki}^{\tau}d^{l}_{t^{\tau}(k,i)}\right)
$$

$$
\left.\times\left(\min_{\tau,(k,i)}h^{u}_{t^{\tau}(k,i)}+\min_{\tau,(k,i)}h^{l}_{t^{\tau}(k,i)}\right)\right\}
$$

$$
s.t.\left\{\begin{array}{l}
\left(\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}a^{u}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\times b^{u}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}c^{u}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})\right.\\
+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}d^{u}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\times a^{l}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}b^{l}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})\\
\left.+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}c^{l}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})+\sum\limits_{\tau=1}^{o}\sum\limits_{(k,i)\in\Omega_0^{\tau}}p_{\tau}\times d^{l}_{t^{\tau}(k,i)}(V_k^{\tau}-V_i^{\tau})\right)\times\left(\min\limits_{\tau,(k,i)}h^{u}_{t^{\tau}(k,i)}+\min\limits_{\tau,(k,i)}h^{l}_{t^{\tau}(k,i)}\right)\\
\geq(a^u_{\rho}+a^l_{\rho}+b^u_{\rho}+b^l_{\rho}+c^u_{\rho}+c^l_{\rho}+d^u_{\rho}+d^l_{\rho})(h^u_{\rho}+h^l_{\rho});\\
V_k^{\tau}-V_i^{\tau}+\lambda_{ki}^{\tau}\geq 0\ ((k,i)\in\Omega_0^{\tau};\tau=1,2,\cdots,o);\\
\lambda_{ki}^{\tau}\geq 0\ ((k,i)\in\Omega_0^{\tau};\tau=1,2,\cdots,o);\\
\mu(\{c_j\})\geq 0\ (\forall c_j\in C);\\
\psi(\{c_j\})=\frac{3-n}{2}\mu(\{c_j\})+\sum\limits_{c_q\in C\backslash c_j}\frac{1}{2}(\mu(\{c_j,c_q\})-\mu(\{c_q\})\ (j=1,2,\cdots,n);\\
\sum\limits_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum\limits_{\{c_j\}\subseteq C}\mu(\{c_j\})=1;\\
\sum\limits_{\{c_j\}\subseteq S\backslash c_q}(\mu(\{c_j,c_q\})-\mu(\{c_j\}))\geq(|S|-2)\mu(\{c_q\})\ (\forall S\subseteq C\text{ with }c_q\in S\text{ and }|S|\geq 2);\\
\mu(\{c_j\}),\mu(\{c_j,c_q\})\in\Lambda\ (\forall\{c_j\},\{c_j,c_q\}\subseteq C);\\
p_{\tau}\geq 0,p_{\tau}\in\Gamma\ (\tau=1,2,\cdots,o);\ \sum\limits_{\tau=1}^{o}p_{\tau}=1.
\end{array}\right. \tag{37}
$$

Remark 4: (37) includes $|\Omega_0|^{\tau}+(n^2+3n)/2$ unknown variables, $n+1$ equalities, and $n\times 2^{n-1}+|\Omega_0^{\tau}|+1$ inequalities (excluding the constraints in Λ and Γ), where $|\Omega_0|$ is the cardinality of set Ω_0. In general, the greater the value of $|\Omega_0^{\tau}|$, the more accurate the derived weighting vectors.

Because (37) involves a complex objective function and many constraints, it is difficult to solve using classical exact algorithms, and a significant amount of calculation time is required. To solve this model, an intelligent optimization algorithm is applied. One of the most typical intelligent optimization algorithms is the state transition algorithm (STA) [48]. Unlike most existing evolutionary algorithms, the STA is an individual-based optimization approach. It exhibits excellent local and global search abilities owing to its four types of state transformation operators: rotation, translation, expansion, and axesion. Recently, an improved algorithm called the EA-CSTA [47] has been presented. The EA-CSTA uses an external archive to save multiple potential solutions and adopts a preference trade-off strategy to select solutions. Hence, it improves the diversity of the solutions and the probability of investigating the global solution. Therefore, we design an EA-CSTA-based optimization algorithm to solve (37) and derive the optimal vector $\boldsymbol{\omega}^*=(\mu^*(\{c_1\}),\mu^*(\{c_2\}),\cdots,\mu^*(\{c_{n-1},c_n\}))$ of $(n^2+n)/2$ fuzzy measures on the criteria set and optimal possibility vector $\mathbf{P}^*=(p_1^*,p_2^*,\cdots,p_o^*)$ of o natural states, such that the objective value in (37) is minimal. The detailed procedure description is provided in Appendix II in the supplementary material. Algorithm 1 presents the pseudocode for (37).

Algorithm 1 Pseudocode of EA-CSTA-based optimization algorithm for (37)

Require: $miter$: maximum number of iterations; SE: number of samples; SA: capacity of external archive; ν: penalty factor; fc: decent efficient; α: rotation factor; β: translation factor; γ: expansion factor; δ: axesion factor; α_{max}: maximum value of α; α_{min}: minimum value of α; δ_{max}: maximum value of δ; δ_{min}: minimum value of δ.
Ensure: ω^*: optimal fuzzy measures on criteria set; $\mathbf{P}^*$: optimal possibility vector of natural states.

Generate SE samples $\mathbf{W}^1 = [\mathbf{W}_1^1, \mathbf{W}_2^1, \cdots, \mathbf{W}_{SE}^1]$, where $\mathbf{W}_\iota^1 = (\omega_\iota^1; \mathbf{P}_\iota^1)$ is ιth ($\iota = 1, 2, \cdots, SE$) sample, where $\boldsymbol{\omega}_\iota^1 = (\mu_\iota^1(\{c_1\}), \mu_\iota^1(\{c_2\}), \cdots, \mu_\iota^1(\{c_{n-1}, c_n\}))^T$ and $\mathbf{P}_\iota^1 = (p_{\iota 1}^1, p_{\iota 2}^1, \cdots, p_{\iota o}^1)^T$.
$[\mathbf{B}^1, \mathbf{D}^1] \leftarrow$ cal_objective_cons_violation($\mathbf{W}^1$).
$[\mathbf{W}_f^1, \mathbf{W}_{inf}^1] \leftarrow$ divide($\mathbf{W}^1, \mathbf{D}^1$).
$fp^1 \leftarrow SE_f^1/SE$.
if $0 < fp^1 < 1$ **then**
    $num_f^1 \leftarrow SA * (1 - fp^1)$.
    $num_{inf}^1 \leftarrow SA * fp^1$.
    $\bar{\mathbf{W}}_f^1 \leftarrow$ sort_fea_candidates($\mathbf{W}_f^1, num_f^1$).
    $T_{nor}(\mathbf{W}_{inf}^1) \leftarrow$ normalize_inf_candidates($\mathbf{W}_{inf}^1, \nu$).
    $\bar{\mathbf{W}}_{inf}^1 \leftarrow$ sort_inf_candidates($\mathbf{W}_{inf}^1, T_{nor}(\mathbf{W}_{inf}^1), num_{inf}^1$).
    $\bar{\mathbf{W}}^1 \leftarrow [\bar{\mathbf{W}}_f^1, \bar{\mathbf{W}}_{inf}^1]$.
else if $fp^1 = 0$ **then**
    $num_{inf}^1 \leftarrow SA$.
    $T_{nor}(\mathbf{W}_{inf}^1) \leftarrow$ normalize_inf_candidates($\mathbf{W}_{inf}^1, \nu$).
    $\bar{\mathbf{W}}_{inf}^1 \leftarrow$ sort_inf_candidates($\mathbf{W}_{inf}^1, T_{nor}(\mathbf{W}_{inf}^1), num_{inf}^1$).
    $\bar{\mathbf{W}}^1 \leftarrow \bar{\mathbf{W}}_{inf}^1$.
else
    $num_f^1 \leftarrow SA$.
    $\bar{\mathbf{W}}_f^1 \leftarrow$ sort_fea_candidates($\mathbf{W}_f^1, num_f^1$).
    $\bar{\mathbf{W}}^1 \leftarrow \bar{\mathbf{W}}_f^1$.
end if
for $iter = 1 : miter$ **do**
    if $\alpha < \alpha_{min}$ **then**
        $\alpha = \alpha_{max}$.
    end if
    if $\delta < \delta_{min}$ **then**
        $\delta = \delta_{max}$.
    end if
    $\bar{\mathbf{W}}^{e-iter} \leftarrow$ expansion ($\bar{\mathbf{W}}^{iter}, \gamma, SA, SE$).
    $\bar{\mathbf{W}}^{r-iter} \leftarrow$ rotation ($\bar{\mathbf{W}}^{e-iter}, \alpha, SA, SE$).
    $\bar{\mathbf{W}}^{a-iter} \leftarrow$ axesion ($\bar{\mathbf{W}}^{r-iter}, \delta, SA, SE$).
    $\bar{\mathbf{W}}^{iter+1} \leftarrow$ translation ($\bar{\mathbf{W}}^{a-iter}, \beta, SA, SE$).
    $\alpha \leftarrow \alpha \,/\, fc$.
    $\delta \leftarrow \delta \,/\, fc$.
end for
$[\mathbf{B}^{miter+1}, \mathbf{D}^{miter+1}] \leftarrow$ cal_objective_cons_violation($\bar{\mathbf{W}}^{miter+1}$).
$\mathbf{W}^* \leftarrow$ sort($\bar{\mathbf{W}}^{miter+1}, \mathbf{B}^{miter+1}, \mathbf{D}^{miter+1}$).

Based on (24) and the optimal weighting vectors $\boldsymbol{\omega}^*$ and $\mathbf{P}^*$, we compute the optimal Banzhaf-based overall perceived utility value V_i^* of each alternative $z_i (i = 1, 2, \cdots, m)$. Accordingly, the ranking order of the alternatives is derived. To summarize, the resolution process for the IT2F programming method is outlined as follows:

Phase 1. Information Acquisition

Step 1: A group of DMs is formed, which identifies the alternative set $Z = \{z_1, z_2, \cdots, z_m\}$, criteria set $C = \{c_1, c_2, \cdots, c_n\}$, and natural state set $\Theta = \{\theta_1, \theta_2, \cdots, \theta_o\}$.

Step 2: The DMs provide three types of decision information, i.e., the risky decision matrices $\widetilde{\mathbf{A}}^\tau = [\widetilde{A}_{ij}^\tau]_{m\times n} (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \tau = 1, 2, \cdots, o)$, truth degrees of alternatives' pairwise comparisons Ω^τ, and incomplete weight information of criteria Λ and natural states Γ.

Phase 2. Calculation of Banzhaf-based Perceived Utility Values

Step 3: Normalize the risky decision matrices $\widetilde{A}^\tau = [\widetilde{A}_{ij}^\tau]_{m\times n} (\tau = 1, 2, \cdots, o)$ into $A^\tau = [A_{ij}^\tau]_{m\times n}$ using (10).

Step 4: Derive the optimal ideal solution $\overline{\boldsymbol{A}}^{\tau} = (\overline{A}_1^{\tau}, \overline{A}_2^{\tau}, \cdots, \overline{A}_n^{\tau})$ in each state $\theta_\tau(\tau = 1, 2, \cdots, o)$ using (11).

Step 5: Calculate the centroid intervals of IT2FSs using (12)–(13).

Step 6: Calculate the utility value matrix $\mathbf{U}^{\tau} = [U_{ij}^{\tau}]_{m\times n}$ in each state $\theta_\tau(\tau = 1, 2, \cdots, o)$ using (14).

Step 7: Calculate the regret value matrix $\mathbf{R}^{\tau} = [R_{ij}^{\tau}]_{m\times n}$ in each state $\theta_\tau(\tau = 1, 2, \cdots, o)$ using (17).

Step 8: Calculate the perceived utility matrix $\mathbf{V}^{\tau} = [V_{ij}^{\tau}]_{m\times n}$ in each state $\theta_\tau(\tau = 1, 2, \cdots, o)$ using (18).

Phase 3. Construction of the IT2F Programming Model

Step 9: Compute the BIT2FRCI and BIT2FRCI using (25) and (29), respectively.

Step 10: Establish the IT2F programming model using (31).

Phase 4. Solution Approach for the IT2F Programming Model

Step 11: Transform the IT2F programming model into a nonlinear programming model using (34).

Step 12: Solve the nonlinear programming model using the EA-CSTA-based optimization algorithm to derive the optimal fuzzy measures $\boldsymbol{\omega}^* = (\mu^*(\{c_1\}), \mu^*(\{c_2\}), \cdots, \mu^*(\{c_{n-1}, c_n\}))$ on the criteria set and probability vector $\mathbf{P}^* = (p_1^*, p_2^*, \cdots, p_o^*)$ of natural states.

Step 13: Calculate the Banzhaf value $\psi(\{c_j\})$ on each criterion $c_j(j = 1, 2, \cdots, n)$ using (7).

Step 14: Compute the Banzhaf-based overall perceived utility value V_i of each alternative $z_i(i = 1, 2, \cdots, m)$ using (21).

Step 15: Rank the alternatives $z_1, z_2, \cdots,$ and z_m and select the optimal alternative based on $V_1, V_2, \cdots,$ and V_m.

5. Application Examples

In this section, a practical example is provided to illustrate the proposed IT2F programming method. In addition, a comparative analysis is presented to demonstrate the superiority of the proposed method.

5.1. Decision-Making Steps

Example 1. It is assumed that an international investment bank wishes to select a company to invest. Five candidate companies are to be assessed: travel company z_1, software company z_2, food company z_3, construction company z_4, and motorcar company z_5. It is assumed that four criteria are considered, i.e., product substitutability c_1, development potentials c_2, investment safety index c_3, and investment income c_4. Among these attributes, c_1 is the cost criterion, whereas c_2, c_3, and c_4 are the benefit criteria. The criteria are assumed to be independent, and their importance given by the DMs is $\Lambda = \{0.15 \le \mu(\{c_1\}) \le 0.2, 0.2 \le \mu(\{c_2\}) \le 0.3, 0.2 \le \mu(\{c_3\}) \le 0.3, 0.3 \le \mu(\{c_4\}) \le 0.4\}$. According to historical data, three natural states $\Theta = \{\theta_1, \theta_2, \theta_3\}$ exist for the criteria: high risk θ_1, medium risk θ_2, and low risk θ_3. It is assumed that the uncertain probability information is expressed as $\Gamma = \{0.4 \le p_1 \le 0.45, 0.35 \le p_2 \le 0.375, 0.175 \le p_3 \le 0.2\}$. The seven IT2FSs linguistic terms in Table 1 are used to assess the companies based on the criteria above. The risky decision matrices $\widetilde{\mathbf{A}}^{\tau} = [\widetilde{A}_{ij}^{\tau}]_{5\times 4}(\tau = 1, 2, 3)$ are listed in Table 3. With the DMs' comprehension and judgements, the seven IT2FSs linguistic terms in Table 1 are used to evaluate the preference relations between companies, i.e., $\Omega^1 = \{\langle(1,2), t^1(1,2)\rangle, \langle(2,5), t^1(2,5)\rangle, \langle(3,1), t^1(3,1)\rangle, \langle(3,2), t^1(3,2)\rangle, \langle(3,4), t^1(3,4)\rangle, \langle(5,4), t^1(5,4)\rangle\}$, $\Omega^2 = \{\langle(2,1), t^2(2,1)\rangle, \langle(2,5), t^2(2,5)\rangle, \langle(3,1), t^2(3,1)\rangle, \langle(3,2), t^2(3,2)\rangle, \langle(3,4), t^2(3,4)\rangle, \langle(5,4), t^2(5,4)\rangle\}$ and $\Omega^3 = \{\langle(2,1), t^3(2,1)\rangle, \langle(4,1), t^3(4,1)\rangle, \langle(5,1), t^3(5,1)\rangle, \langle(5,3), t^3(5,3)\rangle\}$, where the corresponding IT2F truth degrees are $t^1(1,2) = VG, t^1(2,5) = MG, t^1(3,1) = G, t^1(3,2) = G, t^1(3,4) = MG, t^1(5,4) = G; t^2(2,1) = G, t^2(2,5) = MG, t^2(3,1) = G, t^2(3,2) = M, t^2(3,4) = M, t^2(5,4) = M; t^3(2,1) = G, t^3(4,1) = VG, t^3(5,1) = G, t^3(5,3) = G$. Hence, the supports of Ω^1, Ω^2 and Ω^3 are $\Omega_0^1 = \{(1,2), (2,5), (3,1), (3,2), (3,4), (5,4)\}$, $\Omega_0^2 = \{(2,1), (2,5), (3,1), (3,2), (3,4), (5,4)\}$ and $\Omega_0^3 = \{(2,1), (4,1), (5,1), (5,3)\}$, respectively.

To select the optimal company, our proposed IT2F programming method is used, and the detailed process is provided in Appendix III in the supplementary material. Based on Appendix III, it is clear that if $\widehat{B}_{ij}^{\tau}(i = 1,2,3,4,5; j = 1,2,3,4; \tau = 1,2,3)$ is assumed to be uniformly distributed, then the ranking order of the companies is $z_3 > z_2 > z_5 > z_4 > z_1$, where the optimal company is z_3.

In addition, it is assumed that $\widehat{B}_{ij}^{\tau}(i = 1,2,3,4,5; j = 1,2,3,4; \tau = 1,2,3)$ obeys a normal distribution instead of a uniform distribution. Therefore, using our proposed IT2F programming method for the application example above, we derive the Banzhaf-based overall perceived utility value V_i of each company $z_i(i = 1, 2, \cdots, 5)$ as follows:

$$V_1 = 0.5700, V_2 = 0.6781, V_3 = 0.7118, V_4 = 0.5853, V_5 = 0.6569.$$

Table 3
Risky Decision-Making Matrices for Example 1.

	θ_1				θ_2				θ_3			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
z_1	G	MB	MG	VG	G	M	VG	G	MB	M	VG	MG
z_2	MB	G	B	MG	B	VG	MG	G	VB	G	G	MG
z_3	VG	VG	MG	VG	MB	G	G	G	MB	G	MB	G
z_4	VG	M	VG	MB	MB	VG	VG	MG	B	MG	VG	MB
z_5	VB	B	MG	M	MB	G	MG	VG	B	VG	MG	G

Because $V_3 > V_2 > V_5 > V_4 > V_1$, the ranking order of the companies is $z_3 > z_2 > z_5 > z_4 > z_1$, which is the same as that derived by our proposed IT2F programming method, where $\widetilde{B}^{\tau}_{ij}$ obeys a uniform distribution.

5.2. Comparative Analysis

To illustrate the feasibility and superiority of our proposed IT2F programming method, the experimental results obtained from our proposed method are compared with those of the following state-of-the-art methods: Qin *et al.*'s method [27] based on the vlsekriterijumska optimizacija i kompromisno resenje in serbian (VIKOR) and prospect theory, Qin *et al.*'s method [28] based on an acronym in Portuguese for interactive multicriteria decision making (TODIM), Chen *et al.*'s method [5] based on LINMAP, and Liu *et al.*'s method [15] based on a partitioned Bonferroni mean (PBM) operator. It is noteworthy that some minor errors were discovered in the definitions of Qin *et al.* [27], Qin *et al.* [28], and Liu *et al.*'s methods [15]. According to the formula of the rank-based distance function [28], *Definition* 7 on [page 119] of Qin *et al.*'s method [27], *Definition* 9 on [page 630] of Qin *et al.*'s method [28], and *Definition* 3.8 on [page 298] of Liu *et al.*'s method [15] should be revised as follows:

Definition 8: Let A_1 and A_2 be two IT2FSs, and let $\widetilde{1} = ((1,1,1,1;1),(1,1,1,1;1))$. Then, their ranking based on the rank-based distances $D(A_1 - \widetilde{1})$ and $D(A_2 - \widetilde{1})$ is as follows:

1) If $D(A_1 - \widetilde{1}) > R(A_2 - \widetilde{1})$, then A_1 is inferior to A_2, represented by $A_1 < A_2$.
2) If $D(A_1 - \widetilde{1}) = R(A_2 - \widetilde{1})$, then A_1 is indifferent to A_2, represented by $A_1 = A_2$.
3) If $D(A_1 - \widetilde{1}) < R(A_2 - \widetilde{1})$, then A_1 is superior to A_2, represented by $A_1 > A_2$.

For a fair comparison, it is assumed that for these existing methods, the possibility vector of natural states is $\mathbf{p} = (0.45, 0.375, 0.175)$, which is the same as that derived using our proposed method. Furthermore, these existing methods cannot directly address risky MCDM problems with incomplete criteria weights. In order to apply Qin *et al.*'s method [27] to address Example 1, we first use the deviation maximization method [9] to determine the weights of the criteria. In order to apply Qin *et al.* [28] and Liu *et al.*'s methods [15] to address Example 1, the application example is transformed into a group decision-making problem, where risky decision matrices are regarded as group decision matrices, and the experts' weights are regarded as the natural states' possibilities; subsequently, the deviation maximization method [9] is used to determine the weights of the criteria. To apply Chen *et al.*'s method [5] to address Example 1, Equation (62) in [5] is extended to the following form:

$$\min \left\{ \sum_{\tau=1}^{o} p_\tau \sum_{(k,i)\in\Omega_0^\tau} F^{\tau}_{k,i} \right\}$$
$$s.t. \begin{cases} \sum\limits_{\tau=1}^{o} \sum\limits_{(k,i)\in\Omega_0^\tau} \sum\limits_{j=1}^{n} p_\tau (CI^{\tau\beta}_{kj} - CI^{\tau\beta}_{ij}) w_j \geq \eta; \\ \sum\limits_{j=1}^{n} (CI^{\tau\beta}_{kj} - CI^{\tau\beta}_{ij}) + F^{\tau}_{k,i} \geq 0 \ ((k,i)\in\Omega_0^\tau); \\ w_j \geq 0 \ (j = 1,2,\cdots,n); \\ \sum\limits_{j=1}^{n} w_j = 1, \end{cases}$$

where $CI^{\tau\beta}_{kj}$ is the closeness-based index of the alternative $z_k (k = 1,2,\cdots,m)$ based on the Minkowski distance d_β with respect to criterion $c_j (j = 1,2,\cdots,n)$ in state $\theta_\tau (\tau = 1,2,\cdots,o)$, p_τ is the occurrence possibility of natural state θ_τ, w_j is the weight of criterion c_j, $\Omega_0^\tau = \{(k,i)|z_k \geq_\tau z_i \text{ for } k,i = 1,2,\cdots,m\}$ is provided by the DM a priori, and $F_{k,i}^\tau = 0 \vee \sum_{j=1}^{n}(CI_{kj}^{\tau\beta} - CI_{ij}^{\tau\beta})$. Moreover, for Qin *et al.*'s method [27], α, β, λ, γ and δ are set to 0.88, 0.88, 2.25, 0.61, and 0.69, respectively, as recommended by [35]; for Qin *et al.*'s method [28], the attenuation factor of loss θ is set to 1, as suggested in [28]; for Chen *et al.*'s method [5], η is set to 0.0178 according to the expected value of $\rho = ((0,0,0,0.1;1),(0,0,0,0.05;0.9))$ in Example 1; for Liu *et al.*'s method [15], p and q are set to 1 and 2, respectively, according to the characteristics presented in Example 1. Table 9 reports the ranking orders of the companies in Example 1 .

Table 9
Ranking results for different methods for example 1.

Method	Theoretical Foundation	Outcome	Ranking Order
Qin *et al.*'s method [27] (based on the VIKOR)	Prospect Theory	$\pi_1 = 0, \pi_2 = 0.9331,$ $\pi_3 = 1, \pi_4 = 0.334, \pi_5 = 0.7944$	$V_3 > V_2 > V_5 > V_4 > V_1$
Qin *et al.*'s Method [28] (based on the TODIM)	Prospect Theory	$S_1 = 0.5508, S_2 = 0.2708, S_3 = 0.2509,$ $S_4 = 0.5235, S_5 = 0.37; R_1 = 0.3, R_2 = 0.2,$ $R_3 = 0.136, R_4 = 0.3, R_5 = 0.1864; Q_1 = 1,$ $Q_2 = 0.2282, Q_3 = 0, Q_4 = 0.9544, Q_5 = 0.3522$	$V_3 = V_2 > V_5 > V_4 > V_1$
Chen *et al.*'s Method [5] (based on the LINMAP)	Expected Utility Theory	$\overline{WI}_1^2 = 0.4792, \overline{WI}_2^2 = 0.6247,$ $\overline{WI}_3^2 = 0.6661, \overline{WI}_4^2 = 0.4853, \overline{WI}_5^2 = 0.5884$	$V_3 > V_2 > V_5 > V_4 > V_1$
Liu *et al.*'s Method[15] (based on the PBM Operator)	Expected Utility Theory	$D(V_1 - \widetilde{1}) = 0.8109$, $D(V_2 - \widetilde{1}) = 0.7918,$ $D(V_3 - \widetilde{1}) = 0.7836, D(V_4 - \widetilde{1}) = 0.8080$, $D(V_5 - \widetilde{1}) = 0.7972$	$V_3 > V_2 > V_5 > V_4 > V_1$
The IT2F Programming Method (Obeying the Uniform Distribution)	Regret Theory	$V_1 = 0.5706$, $V_2 = 0.6794,$ $V_3 = 0.7118$, $V_4 = 0.5863$, $V_5 = 0.6596$	$V_3 > V_2 > V_5 > V_4 > V_1$
The IT2F Programming Method (Obeying the Normal Distribution)	Regret Theory	$V_1 = 0.5700$, $V_2 = 0.6781,$ $V_3 = 0.7118$, $V_4 = 0.5853$, $V_5 = 0.6569$	$V_3 > V_2 > V_5 > V_4 > V_1$

*__Note:__ π_i is the global prospect value of alternative z_i; S_i, R_i and Q_i are the group utility value, individual regret value and compromise value of alternative z_i, respectively; $\overline{I}_i$ is the comprehensive closeness-based value of alternative z_i; $D(V_i - \widetilde{1})$ is the ranked-based distance between V_i and $\widetilde{1}$.

As shown in Table 9, Qin *et al.*'s [27], Chen *et al.*'s [5], Liu *et al.*'s [15], and our proposed IT2F programming methods yield the same ranking result, that is, $V_3 > V_2 > V_5 > V_4 > V_1$, whereas Qin *et al.*'s method [28] yields a different ranking result, that is, $V_3 = V_2 > V_5 > V_4 > V_1$, where it could not distinguish the ranking order between companies z_3 and z_2. This verifies that our proposed method is suitable and can overcome the shortcomings of Qin *et al.*'s method [28]. Because the ranking results derived from Qin *et al.*'s [27], Chen *et al.*'s [5], Liu *et al.*'s [15], and our proposed IT2F programming methods are identical, the advantage of our proposed method compared with the other three approaches is not highlighted. Hence, we provide another example to further explain the superiority of our proposed IT2F programming method.

Example 2. In Example 1, to select the optimal investment company, four criteria were considered: product substitutability c_1, development potential c_2, investment safety index c_3, and investment income c_4, which were assumed to be independent. Nevertheless, in practice, these criteria exhibit a heterogeneous relationship, ranging from redundancy to complementarity. In general, the lower the product substitutability, the better the development potential; the higher the investment safety index, the lower the investment income. Consequently, it is unreasonable to assume that the criteria were independent. In such a situation, Example 1 is changed to presume that a heterogeneous relationship exists among the criteria on the following weight information of incomplete attributes:

$$0.15 \leq \mu(\{c_1\}) \leq 0.2, 0.2 \leq \mu(\{c_2\}) \leq 0.3, 0.2 \leq \mu(\{c_3\}) \leq 0.3, 0.3 \leq \mu(\{c_4\}) \leq 0.4,$$
$$0.2 \leq \mu(\{c_1,c_2\}) \leq 0.3, 0.2 \leq \mu(\{c_1,c_3\}) \leq 0.3, 0.2 \leq \mu(\{c_2,c_3\}) \leq 0.4, 0.3 \leq \mu(\{c_3,c_4\}) \leq 0.7.$$

The evaluation values remain identical to those shown in the risky decision matrix in Table 3. For a better comparison, we assume that the possibility vector of natural states is $\mathbf{p} = (0.45, 0.375, 0.175)$, which is the same as that in Example 1. To further explain the advantage of our proposed IT2F programming method, we use Qin *et al.* [27], Qin *et al.* [28], Chen *et al.* [5] and Liu *et al.*'s methods [15] to address Example 2. The experimental results from the existing approaches [5, 15, 27, 28] and our proposed method are listed in Table 10, where we assume that the values of the parameters in these methods are the same as those in Example 1.

Table 10
Ranking results for different methods for example 2.

Method	Theoretical Foundation	Outcome	Ranking Order
Qin *et al.*'s Method [27] (based on the VIKOR)	Prospect Theory	$\pi_1 = 0, \pi_2 = 0.9331$, $\pi_3 = 1, \pi_4 = 0.334, \pi_5 = 0.7944$	$V_3 > V_2 > V_5 > V_4 > V_1$
Qin *et al.*'s Method [28] (based on the TODIM)	Prospect Theory	$S_1 = 0.5508, S_2 = 0.2708, S_3 = 0.2509$, $S_4 = 0.5235, S_5 = 0.37$; $R_1 = 0.3, R_2 = 0.2$, $R_3 = 0.136, R_4 = 0.3, R_5 = 0.1864$; $Q_1 = 1$, $Q_2 = 0.2282, Q_3 = 0, Q_4 = 0.9544, Q_5 = 0.3522$	$V_3 = V_2 > V_5 > V_4 > V_1$
Chen *et al.*'s Method [5] (based on the LINMAP)	Expected Utility Theory	$\overline{WI}_1^2 = 0.4792, \overline{WI}_2^2 = 0.6247$, $\overline{WI}_3^2 = 0.6661, \overline{WI}_4^2 = 0.4853, \overline{WI}_5^2 = 0.5884$	$V_3 > V_2 > V_5 > V_4 > V_1$
Liu *et al.*'s Method [15] (based on the PBM Operator)	Expected Utility Theory	$D(V_1 - \tilde{1}) = 0.8109$, $D(V_2 - \tilde{1}) = 0.7918$, $D(V_3 - \tilde{1}) = 0.7836$, $D(V_4 - \tilde{1}) = 0.8080$, $D(V_5 - \tilde{1}) = 0.7972$	$V_3 > V_2 > V_5 > V_4 > V_1$
The IT2F Programming Method (Obeying the Uniform Distribution)	Regret Theory	$V_1 = 0.6364$, $V_2 = 0.7088$, $V_3 = 0.8028$, $V_4 = 0.5629$, $V_5 = 0.6348$	$V_3 > V_2 > V_1 > V_5 > V_4$
The IT2F Programming Method (Obeying the Normal Distribution)	Regret Theory	$V_1 = 0.5706$, $V_2 = 0.6794$, $V_3 = 0.7118$, $V_4 = 0.5863$, $V_5 = 0.6596$	$V_3 > V_2 > V_1 > V_5 > V_4$

***Note:** The meanings of π_i, S_i, R_i, Q_i and $\tilde{I}_i$ are the same as those in Table 9.

As shown in Table 10, when the independent criteria of the risky MCDM problem are changed to heterogeneous criteria, Qin *et al.*'s method [28] still maintains the same ranking order, that is, $V_3 = V_2 > V_5 > V_4 > V_1$, because it could not distinguish the ranking order between companies z_3 and z_2 nor consider the heterogeneous relationship among the criteria. In Example 2, it is known that the lower the product substitutability, the better the development potential, that is, product substitutability and development potential, are negatively interactive. Accordingly, the comprehensive weight of the two criteria considered simultaneously should be smaller than the sum of the weights of the two criteria when they are considered individually. Therefore, the ranking result derived using Qin *et al.*'s method [28] is not suitable because it disregards the heterogeneous relationship among the criteria and fails to distinguish the ranking order between companies z_3 and z_2. Additionally, one can observe that when the independent criteria of the risky MCDM problem are changed to heterogeneous criteria, Qin *et al.* [27], Chen *et al.* [5], and Liu *et al*'s methods [15] still yield the same rankings because they are based on the following equalities: $\mu(\{c_j, c_q\}) = \mu(\{c_j\}) + \mu(\{c_q\})(j, q = 1, 2, \cdots, n; j \neq q)$, which are invalid in Example 2. Therefore, the ranking result derived using Qin *et al.*[27], Chen *et al.*[5], and Liu *et al.*'s methods [15] are not suitable. Furthermore, when the independent criteria of the risky MCDM problem are changed to heterogeneous criteria, the preferred order derived using our proposed approach changes from $V_3 > V_2 > V_5 > V_4 > V_1$ to $V_3 > V_2 > V_1 > V_5 > V_4$, although that of the best company remains the same. Clearly, the ranking result of the companies using our proposed approach is reasonable because it is based on the following inequalities: $\mu(\{c_1, c_2\}) = 0.3 < \mu(\{c_1\}) + \mu(\{c_2\}) = 0.35$, $\mu(\{c_1, c_3\}) = 0.3 < \mu(\{c_1\}) + \mu(\{c_3\}) = 0.35$, $\mu(\{c_2, c_4\}) = 0.7053 > \mu(\{c_2\}) + \mu(\{c_4\}) = 0.5$. Hence, our proposed approach captures the negative synergetic interaction of criteria c_1 and c_2, the negative synergetic interaction of criteria c_1 and c_3, and the positive synergetic interaction of criteria c_2 and c_4. Therefore, our proposed approach yields a realistic ranking result, unlike the existing approaches [5, 15, 27, 28].

5.3. Further Analysis

Next, we discuss the characteristics of Qin *et al.*'s [27], Qin *et al.*'s [28], Chen *et al.*'s [5], Liu *et al.*'s [15], and our proposed IT2F programming methods, described as follows:

- With regard to the information regarding the pairwise comparisons of alternatives, Qin *et al.* [27], Qin *et al.* [28], and Liu *et al*'s methods [15] use only criteria values to rank alternatives and disregard information regarding the pairwise comparisons of alternatives. Chen *et al.*'s method [5] adopts the information regarding the pairwise comparisons of alternatives to make decisions; however, the information is in the form of ordered pairs with real truth degrees 0 or 1. Crisp values and type-1 fuzzy sets may be inadequate for practical cases owing to the increasing complexity of decision-making problems. In our proposed method, IT2FSs are used to denote the truth degrees for the comparison of alternatives, which are more accurate and effective for expressing vague and imprecise information. Therefore, our proposed method is more valid and suitable for practical decision-making problems with higher degrees of uncertainty.

- With regard to the heterogeneous relationship among the criteria, Qin *et al.*[27], Qin *et al.* [28], and Chen *et al.*'s methods [5] assume that the criteria are independent and fail to consider the heterogeneous relationships among the criteria. In Liu *et al.*'s method [15], the PBM operator presumes that the criteria are partitioned into several clusters, where the criteria in identical clusters are interrelated, whereas those in various clusters are irrelevant. However, Liu *et al.*'s method [15] could not consider the negative and positive synergetic interactions among the criteria simultaneously. Not considering the heterogeneous relationships among the criteria may result in less convincing outcomes. In our proposed method, 2AFMs are applied to model negative synergetic interactions, positive synergetic interactions, and the independence of the criteria. Consequently, our proposed method is more convincing and generic than existing methods [5, 15, 27, 28] because it can address both independent and dependent risky MCDM problems, whereas the methods in [5, 27, 28] can only address independent decision-making problems, and Liu *et al.*'s method [15] can only address partially heterogeneous decision-making problems.

- With regard to the theoretical foundation, Chen *et al.* [5] and Liu *et al.*'s methods [15] are based on the expected utility theory, which assumes that DMs are completely rational in the decision procedure. In actual risky MCDM problems, DMs typically behave in bounded rationality. Hence, these two methods may cause an irrational divergence between real decision-making behaviors and the predictable values of the expected utility theory owing to omitted emotions, such as regret, rejoice, and reward. Qin *et al.* [27] and Qin *et al.*'s methods [28] are based on prospect theory, which emphasizes the role of DMs' loss aversion in the decision-making procedure. Our proposed method is based on regret theory, which highlights the role of DMs' regret aversion in the decision-making process. Hence, the last three methods reflect the psychological behaviors of DMs and can overcome the limitations of Chen *et al.* [5] and Liu *et al.*'s methods [15]. Compared with prospect theory, regret theory involves fewer parameters and has better descriptive power. Accordingly, our proposed method can simplify the computational complexity and is more convenient to implement in actual applications.

- With regard to the weight information of criteria, Qin *et al* [27] and Qin *et al.*'s methods [28] assign weight information in advance; hence, subjective randomness is difficult to avoid. Liu *et al.*'s method [15] assumes that the weights of criteria are completely unknown and only relies on the decision matrix to determine them, which may be inconsistent with the actual weight information owing to the omitted valuable subjective judgments of the DMs. By employing LINMAP, Chen *et al.*'s method [5] can only derive the weights of independent criteria objectively. Utilizing subjective and objective information, our proposed method is based on the LINMAP and 2AFMs, which not only accurately determines the weights of independent criteria, but also reasonably determines the weights of the dependent criteria. Therefore, our proposed method is more comprehensive and applicable to real decision-making problems with incomplete weights.

Based on previous analyses, the characteristics of all the approaches involved are summarized in Table 11. It is clear that our proposed IT2F programming method outperforms comparative methods for risky MCDM problems. Furthermore, our proposed method can be a strong alternative to state-of-the-art approaches for solving IT2F MCDM problems, and its unique characteristics render it highly attractive for solving risky MCDM problems with IT2F truth degrees on alternatives' comparisons.

Table 11
Comparisons of Different Approaches.

Features / Methods	Can Use the Information on Pair-wise Comparisons	Can Consider the Heterogeneous Relationship among the Criteria	Can Reflect DM's Psychology Behavior	Can Determine the Weight Information
Qin *et al.*'s method [27]	No	No	Yes (Loss Aversion)	No
Qin *et al.*'s method [28]	No	No	Yes (Loss Aversion)	No
Chen *et al.*'s method [5]	Yes (0 or 1)	No	No	Yes (Independent)
Liu *et al.*'s method [15]	No	Yes (Partially)	No	Yes (Completely Unknown)
Our proposed method	Yes (IT2Fs)	Yes (Totally)	Yes (Regret Aversion)	Yes (Dependent and Independent)

6. Conclusion

In risky MCDM, three types of assessment information (criteria values, pairwise comparisons of alternatives, and weight information) are typically provided. The criteria often exhibit heterogeneous relationships, and the DMs commonly behave in bounded rationality. Herein, we proposed a novel IT2F programming method to manage such risky MCDM in an IT2F setting. Four phases were involved: an information acquisition phase, the calculation of the Banzhaf-based perceived utility value phase, the construction of an IT2F programming model phase, and the solution approach of the IT2F programming model phase.

- In the first phase, we used IT2FSs to depict the criteria values and the pairwise comparisons of alternatives. The prominent feature of IT2FSs is that they comprise primary and secondary MFs; as such, they are an excellent tool for addressing higher uncertainties.
- In the second phase, we introduced 2AFMs and regret theory into the risky MCDM procedure and developed a novel technique to calculate the Banzhaf-based IT2F perceived utility values of alternatives. This implies that the effects of regret aversion and heterogeneous relationships were quantified, while selection was performed from multiple alternatives in IT2F settings.
- In the third phase, based on the LINMAP method, we established an IT2F programming model, which can effectively combine three types of assessment information and objectively identify the optimal 2AFMs on the criteria set and the optimal probabilities of the natural states.
- In the fourth phase, using the EA-CSTA, we designed an algorithm to solve the established mathematical programming model.

To explain the validity and advantages of our proposed IT2F programming method, a comparative analysis was performed using our proposed method and previous MCDM methods in [5, 15, 27, 28]. From the experimental results, we conclude that our proposed method outperforms the MCDM methods presented in [5, 15, 27, 28]. In future work, we will focus on the following issues: (1) Because some parameters of our proposed method were provided by the DMs in advance, the identification of the related parameters using machine learning is a potential research topic. (2) It is desirable to explore our proposed method in various areas, such as project selection [20], online review sentiment analysis [2] and stock investment [34]. (3) Based on multi-time assessment information, the analysis of a dynamic IT2F programming method is a valuable research direction.

CRediT authorship contribution statement

Guolin Tang: Conceptualization, Methodology, Investigation, Validation, Software, Writing - original draft preparation. **Jianpeng Long:** Methodology, Formal analysis, Writing - review & editing. **Xiaowei Gu:** Investigation, Validation, Writing - review & editing. **Francisco Chiclana:** Investigation, Writing - review & editing, Supervision. **Peide Liu:** Methodology, Writing - review & editing, Supervision, Funding acquisition. **Fubin Wang:** Investigation, Validation, Supervision, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they do not have any commercial or associative interests that represent a conflict of interests in connection with this manuscript. There are no professional or other personal interests that can inappropriately influence their submitted work.

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Appendix I

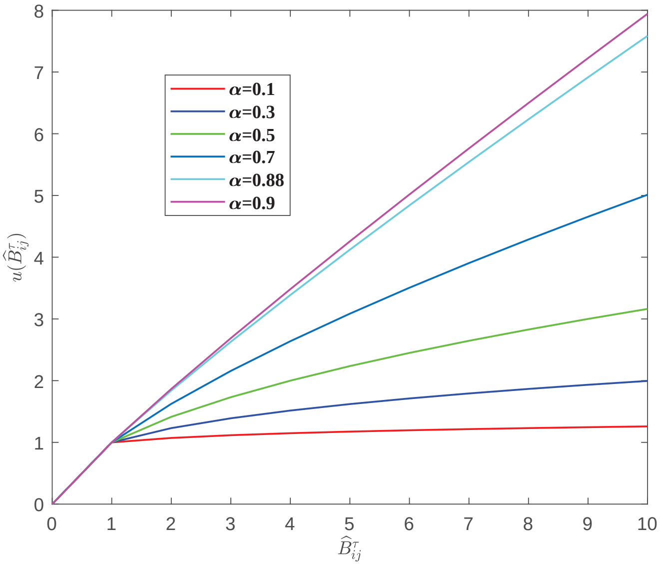

Fig. 4. Utility Function ($u(\widehat{B}_{ij}^{\tau}) = (\widehat{B}_{ij}^{\tau})^{\alpha}$)

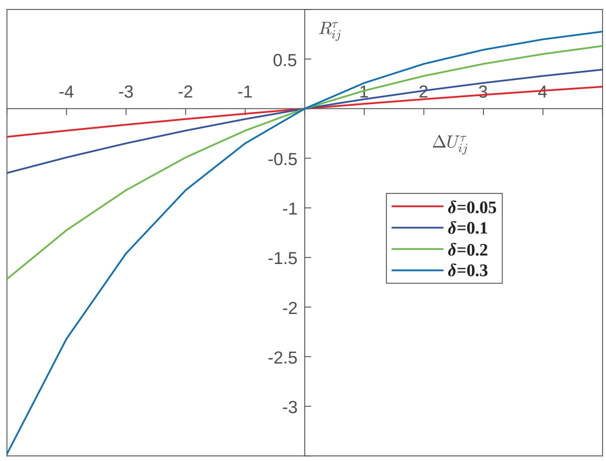

Fig. 5. Regret-rejoice Function ($R_{ij}^{\tau} = 1 - e^{-\delta(\triangle U_{ij}^{\tau})}$)

Appendix II

The detailed procedure description of the EA-CSTA-based optimization algorithm is provided as follows:

Step 1: Initialize samples $\mathbf{W}^1$, which comprise SE samples $\mathbf{W}^1 = [\mathbf{W}_1^1, \mathbf{W}_2^1, \cdots, \mathbf{W}_{SE}^1]$. The ιth ($\iota = 1, 2, \cdots, SE$) sample is denoted by $\mathbf{W}_\iota^1 = (\boldsymbol{\omega}_\iota^1; \mathbf{P}_\iota^1)$, where $\boldsymbol{\omega}_\iota^1 = (\mu_\iota^1(\{c_1\}), \mu_\iota^1(\{c_2\}), \cdots, \mu_\iota^1(\{c_{n-1}, c_n\}))^T$ and $\mathbf{P}_\iota^1 = (p_{\iota 1}^1, p_{\iota 2}^1, \cdots, p_{\iota o}^1)^T$, where $\mu_\iota^1(\{c_j\})(j = 1, 2, \cdots, n)$, $\mu_\iota^1(\{c_j, c_q\}(q = 1, 2, \cdots, n; j \neq q)$ and $p_{\iota\tau}^1(\tau = 1, 2, \cdots, o)$ are randomly generated, and they verify the following conditions: $\mu_\iota^1(\{c_j\}) \geq 0$, $\mu_\iota^1(\{c_j\}) \in \Lambda$, $\mu_\iota^1(\{c_j, c_q\} \in \Lambda$, $p_{\iota\tau}^1 \geq 0$ and $p_{\iota\tau}^1 \in \Gamma$. Additionally,

$\mu_\iota^1(\{c_n, c_{n-1}\})$ is determined by $\mu_\iota^1(\{c_n, c_{n-1}\}) = (n-2) \sum_{\{c_j\}\subseteq C} \mu_\iota^1(\{c_j\}) - 1 - \sum_{\{c_j, c_p\}\subseteq C\setminus\{c_n, c_{n-1}\}} \mu_\iota^1(\{c_j, c_p\})$, and $p_{\iota o}^1$ is determined by $p_{\iota o}^1 = 1 - p_{\iota 1}^1 - p_{\iota 2}^1 - \cdots - p_{\iota o-1}^1$. In this study, we set $SE = 30$.

Step 2: Calculate the objective value B_ι^1 and constraint violation degree D_ι^1 of each sample $\mathbf{W}_\iota^1(\iota = 1, 2, \cdots, SE)$, where B_ι^1 and D_ι^1 are determined by the objective function and constrained conditions in (34), respectively. According to the definition of constraint violation degree in [45], we observe that $D_\iota^1 \geq 0$. In general, if $D_\iota^1 > 0$, then $\mathbf{W}_\iota^1$ is an infeasible solution; otherwise, it is a feasible solution.

Step 3: Select SA potential solutions $\bar{\mathbf{W}}^1 = [\bar{\mathbf{W}}_1^1, \bar{\mathbf{W}}_2^1, \cdots, \bar{\mathbf{W}}_{SA}^1]$ from SE samples $\mathbf{W}^1$ and save them in an external archive. The SA potential solutions $\bar{\mathbf{W}}^1$ comprise num_f^1 potential feasible solutions $\bar{\mathbf{W}}_f^1$ and num_{inf}^1 potential infeasible solutions $\bar{\mathbf{W}}_{inf}^1$. To maintain diversity of the potential solutions and avoid falling into a local optimum, the tradeoff scheme is applied to determine the values of num_f^1 and num_{inf}^1, and the preference scheme is applied to select num_f^1 potential feasible solutions $\bar{\mathbf{W}}_f^1$ and num_{inf}^1 potential infeasible solutions $\bar{\mathbf{W}}_{inf}^1$. In this study, we set $SA = 20$. This mainly includes the following substeps.

Step 3.1: Classify SE samples $\mathbf{W}^1$ into SE_f^1 feasible candidates $\mathbf{W}_f^1 = [\mathbf{W}_{f1}^1, \mathbf{W}_{f2}^1, \cdots, \mathbf{W}_{fSE_f^1}^1]$ and SE_{inf}^1 infeasible candidates $\mathbf{W}_{inf}^1 = [\mathbf{W}_{inf1}^1, \mathbf{W}_{inf2}^1, \cdots, \mathbf{W}_{infSE_{inf}^1}^1]$ according to the values of $D_\iota^1(\iota = 1, 2, \cdots, SE)$. Subsequently, the ratio of feasible solutions fp^1 in the total solutions is derived using $fp^1 = SE_f^1/SE$.

Step 3.2: Based on the tradeoff scheme, the values of num_f^1 and num_{inf}^1 can be computed using as follows:

$$num_f^1 = \begin{cases} SA \times (1 - fp^1), & 0 < fp^1 < 1 \\ SA, & fp^1 = 1 \end{cases} \tag{35}$$

$$num_{inf}^1 = \begin{cases} SA \times fp^1, & 0 < fp^1 < 1 \\ SA. & fp^1 = 0 \end{cases} \tag{36}$$

Step 3.3: The preference scheme is applied to determine num_f^1 potential solutions $\bar{\mathbf{W}}_f^1$. Therefore, the feasible candidates $\mathbf{W}_f^1$ are sorted in ascending order of their objective values, and the first num_f^1 solutions $\bar{\mathbf{W}}_f^1$ are saved in the external archive as part of the potential solutions $\bar{\mathbf{W}}^1$.

Step 3.4: The normalized penalty function strategy and preference scheme are applied to determine num_{inf}^1 potential solutions $\bar{\mathbf{W}}_{inf}^1$. Specifically, we first derive the normalized objective value $B_{nor}(\mathbf{W}_{inf\chi}^1)$ and the normalized constraint violation $D_{nor}(\mathbf{W}_{inf\chi}^1)$ of infeasible candidate $\mathbf{W}_{inf\chi}^1(\chi = 1, 2, \cdots, SE_{inf}^1)$ as follows:

$$B_{nor}(\mathbf{W}_{inf\chi}^1) = \frac{B(\mathbf{W}_{inf\chi}^1) - \min(B(\mathbf{W}_{inf\chi}^1))}{\max(B(\mathbf{W}_{inf\chi}^1)) - \min(B(\mathbf{W}_{inf\chi}^1))}, \tag{37}$$

$$D_{nor}(\mathbf{W}_{inf\chi}^1) = \frac{D(\mathbf{W}_{inf\chi}^1) - \min(D(\mathbf{W}_{inf\chi}^1))}{\max(D(\mathbf{W}_{inf\chi}^1)) - \min(D(\mathbf{W}_{inf\chi}^1))}, \tag{38}$$

Subsequently, $T_{nor}(\mathbf{W}_{inf\chi}^1)$ is derived as follows:

$$T_{nor}(\mathbf{W}_{inf\chi}^1) = B_{nor}(\mathbf{W}_{inf\chi}^1) + \nu D_{nor}(\mathbf{W}_{inf\chi}^1), \tag{39}$$

where ν denotes the penalty factor. In this study, we set $\nu = 1.5$. Finally, the infeasible candidates $\mathbf{W}_{inf}^1$ are sorted in ascending order of the values $T_{nor}(\mathbf{W}_{inf\chi}^1)$, and then the first num_{inf}^1 solutions $\bar{\mathbf{W}}_{inf}^1$ are saved in the external archive as the other part of the potential solutions $\bar{\mathbf{W}}^1$.

Step 4: Initialize the values of parameters α, β, γ, and δ, where α is a rotation factor, β is a translation factor, γ is an expansion factor, and δ is an axesion factor. In this study, we set $\alpha = 2, \beta = 100, \gamma = 2$, and $\delta = 6$.

Step 5: Set the number of iterations $iter$ to 1. Update the values of α and δ, expand SA potential solutions $\bar{\mathbf{W}}^{iter}$ to $SA \times SE$ solutions $\hat{\mathbf{W}}^{iter}$ using expansion transformation, select SA solutions $\bar{\mathbf{W}}^{e-iter}$, and save them in the external archive. Similarly, rotation, axesion and translation are adopted sequentially. Subsequently, the number of iterations $iter$ is increased by one for each iteration until the number of iterations $iter$ reaches a predefined number $miter$ of iterations. This includes the following steps:

Step 5.1: Update the values of α and δ using $\alpha = \alpha/fc$ and $\delta = \delta/fc$, respectively. Subsequently, the values of α and δ are determined according to the following rules: if $\alpha < \alpha_{min}$, then $\alpha = \alpha_{max}$; if $\delta < \delta_{min}$, then $\delta = \delta_{max}$. In this study, we assume that $fc = 2$, $\alpha_{max} = 1$, $\alpha_{min} = 10^{-4}$, $\delta_{max} = 3$, and $\delta_{min} = 10^{-6}$.

Step 5.2: The process of expansion transformation is as follows: expand SA potential solutions $\bar{\mathbf{W}}^{iter}$ to $SA \times SE$ solutions $\hat{\mathbf{W}}^{iter} = [\hat{\mathbf{W}}_1^{iter}, \hat{\mathbf{W}}_2^{iter}, \cdots, \hat{\mathbf{W}}_{SA}^{iter}]$, where $\hat{\mathbf{W}}_\kappa^{iter} = [\hat{\mathbf{W}}_{\kappa_1}^{iter}, \hat{\mathbf{W}}_{\kappa_2}^{iter}, \cdots, \hat{\mathbf{W}}_{\kappa_{SE}}^{iter}] (\kappa = 1, 2, \cdots, SA)$. It is noteworthy that $\hat{\mathbf{W}}_\kappa^{iter}$ is derived using the following rules: (1) Set the number of iterations ite to 1. (2) The solution is repeatedly expanded using $\hat{\mathbf{W}}_{\kappa_{ite}}^{iter} = \bar{\mathbf{W}}_\kappa^{iter} + \gamma R_e \bar{\mathbf{W}}_\kappa^{iter}$, and $R_e \in R^{((n^2+n+2o)/2)\times((n^2+n+2o)/2)}$ is a random diagonal matrix whose elements obey the standard normal distribution. (3) If the number of iterations ite is smaller than SE, then ite is increased by one and proceeds to (1). Otherwise, derive SE solutions $\hat{\mathbf{W}}_\kappa^{iter}$. Next, the solutions $\hat{\mathbf{W}}^{iter}$ are combined with SA solutions $\bar{\mathbf{W}}^{iter}$ of the external archive and $(SA+1)\times SE$ solutions $\widetilde{\mathbf{W}}^{iter}$ are derived. Finally, similar to Steps 2 and 3, we derive SA solutions $\bar{\mathbf{W}}^{e-iter}$ from $(SA+1) \times SE$ solutions $\widetilde{\mathbf{W}}^{iter}$, and save them in the external archive.

Step 5.3: Similarly, rotation, axesion, and translation are adopted sequentially; therefore, SA solutions $\bar{\mathbf{W}}^{iter+1}$ are derived and then saved in the external archive. With respect to these three state transformation operators' formulas, please refer to references [45] and [52].

Step 5.4: If the number of iterations $iter$ is smaller than the predefined number $miter$ of iterations, then increase $iter$ by 1 and proceed to Step 5.1. Otherwise, the optimal solution $\mathbf{W}^*$ of the external archive in Step 5.3 is the optimal weighting vector.

Appendix III

To select the optimal company, the following steps are required:

Steps 1-2: See the detailed description of Example 1.

Step 3: Because c_1 is a cost criterion, and c_2, c_3 and c_4 are benefit criteria, we should normalize risky decision matrices $\widetilde{\mathbf{A}}^\tau = [\widetilde{A}_{ij}^\tau]_{5\times4} (\tau = 1, 2, 3)$ using (10) and Table II. The normalized decision matrices $\mathbf{A}^1$, $\mathbf{A}^2$, and $\mathbf{A}^3$ are listed in Table 4.

Table 4
Normalized Risky Decision-Making Matrices for Example 1.

	θ_1				θ_2				θ_3			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
z_1	B	MB	MG	VG	B	M	VG	G	MG	M	VG	MG
z_2	MG	G	B	MG	G	VG	MG	G	VG	G	G	MG
z_3	VB	VG	MG	VG	MG	G	G	G	MG	G	MB	G
z_4	VB	M	VG	MB	MG	VG	VG	MG	G	MG	VG	MB
z_5	VG	B	MG	M	MG	G	MG	VG	G	VG	MG	G

Step 4: Using (11) to compute the optimal ideal solution $\overline{\mathbf{A}}^\tau$ in each state $\theta_\tau (\tau = 1, 2, 3)$, we derive the following: $\overline{\mathbf{A}}^1 = (VG, VG, VG, VG)$, $\overline{\mathbf{A}}^2 = (G, VG, VG, VG)$, $\overline{\mathbf{A}}^3 = (VG, VG, VG, G)$.

Step 5: Using (12)–(13) to calculate the centroid intervals of IT2FSs, we derive the centroid interval matrices $\mathbf{B}^1$, $\mathbf{B}^2$, and $\mathbf{B}^3$, as shown in Table 5.

Similarly, we derive the following defuzzified value vector $\overline{\mathbf{B}}^\tau$ of each optimal ideal solution $\overline{\mathbf{A}}^\tau (\tau = 1, 2, 3)$:

$$\overline{\mathbf{B}}^1 = ([0.9644, 0.9841], [0.9644, 0.9841], [0.9644, 0.9841], [0.9644, 0.9841]),$$

$$\overline{\mathbf{B}}^2 = ([0.8485, 0.9020], [0.9644, 0.9841], [0.9644, 0.9841], [0.9644, 0.9841]),$$

$$\overline{\mathbf{B}}^3 = ([0.9644, 0.9841], [0.9644, 0.9841], [0.9644, 0.9841], [0.8485, 0.9020]).$$

Table 5
Centroid Interval Matrices for Example 1.

	θ_1				θ_2	
	c_1	c_2	c_3	c_4	c_1	c_2
z_1	[0.0982, 0.1602]	[0.2812, 0.3324]	[0.6803, 0.7323]	[0.9644, 0.9841]	[0.0982, 0.1602]	[0.4642, 0.5324]
z_2	[0.6803, 0.7323]	[0.8485, 0.9020]	[0.0982, 0.1602]	[0.6803, 0.7323]	[0.8485, 0.9020]	[0.9644, 0.9841]
z_3	[0.0159, 0.0356]	[0.9644, 0.9841]	[0.6803, 0.7323]	[0.9644, 0.9841]	[0.6803, 0.7323]	[0.8485, 0.9020]
z_4	[0.0159, 0.0356]	[0.4642, 0.5324]	[0.9644, 0.9841]	[0.2812, 0.3324]	[0.6803, 0.7323]	[0.9644, 0.9841]
z_5	[0.9644, 0.9841]	[0.0982, 0.1602]	[0.6803, 0.7323]	[0.4642, 0.5324]	[0.6803, 0.7323]	[0.8485, 0.9020]

	θ_2		θ_3			
	c_3	c_4	c_1	c_2	c_3	c_4
z_1	[0.9644, 0.9841]	[0.8485, 0.9020]	[0.6803, 0.7323]	[0.4642, 0.5324]	[0.9644, 0.9841]	[0.6803, 0.7323]
z_2	[0.6803, 0.7323]	[0.8485, 0.9020]	[0.9644, 0.9841]	[0.8485, 0.9020]	[0.8485, 0.9020]	[0.6803, 0.7323]
z_3	[0.8485, 0.9020]	[0.8485, 0.9020]	[0.6803, 0.7323]	[0.8485, 0.9020]	[0.2812, 0.3324]	[0.8485, 0.9020]
z_4	[0.9644, 0.9841]	[0.6803, 0.7323]	[0.8485, 0.9020]	[0.6803, 0.7323]	[0.9644, 0.9841]	[0.2812, 0.3324]
z_5	[0.6803, 0.7323]	[0.9644, 0.9841]	[0.8485, 0.9020]	[0.9644, 0.9841]	[0.6803, 0.7323]	[0.8485, 0.9020]

Step 6: It is assumed that $\widehat{B}_{ij}^{\tau}(i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4; \tau = 1, 2, 3)$ is uniformly distributed. Subsequently, (14) is used to derive the utility value matrices $\mathbf{U}^1$, $\mathbf{U}^2$, and $\mathbf{U}^3$, as shown in Table 6.

Table 6
Utility Value Matrices for Example 1.

	θ_1				θ_2				θ_3			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
z_1	0.1650	0.3535	0.7364	0.9773	0.1650	0.5417	0.9773	0.8893	0.7386	0.5417	0.9773	0.7364
z_2	0.7364	0.8893	0.1650	0.1650	0.8893	0.9773	0.7364	0.8893	0.9773	0.8893	0.8893	0.7364
z_3	0.0398	0.9773	0.7364	0.9773	0.7364	0.8893	0.8893	0.8893	0.7364	0.8893	0.3535	0.8893
z_4	0.0398	0.5417	0.9773	0.3535	0.7364	0.9773	0.9773	0.7364	0.8893	0.7364	0.9773	0.3535
z_5	0.9773	0.1650	0.7364	0.5417	0.7364	0.8893	0.7364	0.9773	0.8893	0.9773	0.7364	0.8893

Similarly, we derive the following optimal ideal solution utility vectors $\overline{\mathbf{U}}^1$, $\overline{\mathbf{U}}^2$, and $\overline{\mathbf{U}}^3$:

$$\overline{\mathbf{U}}^1 = (0.9773, 0.9773, 0.9773, 0.9773), \overline{\mathbf{U}}^2 = (0.8893, 0.9773, 0.9773, 0.9773), \overline{\mathbf{U}}^3 = (0.9773, 0.9773, 0.9773, 0.8893).$$

Step 7: (17) is used to calculate the regret value matrices $\mathbf{R}^1$, $\mathbf{R}^2$, and $\mathbf{R}^3$, as shown in Table 7.

Table 7
Regret Value Matrices for Example 1.

	θ_1				θ_2				θ_3			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
z_1	-0.2760	-0.2058	-0.0750	0	-0.2427	-0.1396	0	-0.0267	-0.0742	-0.1396	0	-0.0470
z_2	-0.0750	-0.0267	-0.2760	-0.0750	0	0	-0.0750	-0.0267	0	-0.0267	-0.0267	-0.0470
z_3	-0.3248	0	-0.0750	0	-0.0470	-0.0267	-0.0267	-0.0267	-0.0750	-0.0267	-0.2058	0
z_4	-0.3248	-0.1396	0	-0.2058	-0.0470	0	0	-0.0750	-0.0267	-0.0750	0	-0.1744
z_5	0	-0.2760	-0.0750	-0.1396	-0.0470	-0.0267	-0.0750	0	-0.0267	0	-0.0750	0.8893

Step 8: (18) is used to calculate the perceived utility matrices $\mathbf{V}^1$, $\mathbf{V}^2$, and $\mathbf{V}^3$, as shown in Table 8.

Table 8
Perceived Utility Value Matrices for Example 1.

	θ_1				θ_2				θ_3			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
z_1	-0.1110	0.1477	0.6614	0.9773	-0.0777	0.4021	0.9773	0.8626	0.6644	0.4021	0.9773	0.6894
z_2	0.6614	0.8626	-0.1110	0.6614	0.8893	0.9773	0.6614	0.8626	0.9773	0.8626	0.8626	0.6894
z_3	-0.2849	0.9773	0.6614	0.9773	0.6894	0.8626	0.8626	0.8626	0.6614	0.8626	0.1477	0.8893
z_4	-0.2849	0.4021	0.9773	0.1477	0.6894	0.9773	0.9773	0.6614	0.8626	0.6614	0.9773	0.1791
z_5	0.9773	-0.1110	0.6614	0.4021	0.6894	0.8626	0.6614	0.9773	0.8626	0.9773	0.6614	0.8893

Subsequently, using (21) to compute the Banzhaf-based overall perceived utility value V_i of each alternative $z_i(i = 1, 2, \cdots, 5)$, we derive the following:

$$\begin{aligned}
V_1 &= p1 \times (-0.1110\psi(\{c_1\}) + 0.1477\psi(\{c_2\}) + 0.6614\psi(\{c_3\}) + 0.9773\psi(\{c_4\})) + p2 \times (-0.0777\psi(\{c_1\}) + 0.4021\psi(\{c_2\}) \\
&\quad + 0.9773\psi(\{c_3\}) + 0.8626\psi(\{c_4\})) + p3 \times (0.6644\psi(\{c_1\}) + 0.4021\psi(\{c_2\}) + 0.9773\psi(\{c_3\}) + 0.6894\psi(\{c_4\})), \\
V_2 &= p1 \times (0.6614\psi(\{c_1\}) + 0.8626\psi(\{c_2\}) - 0.1110\psi(\{c_3\}) + 0.6614\psi(\{c_4\})) + p2 \times (0.8893\psi(\{c_1\}) + 0.9773\psi(\{c_2\}) \\
&\quad + 0.6614\psi(\{c_3\}) + 0.8626\psi(\{c_4\})) + p3 \times (0.9773\psi(\{c_1\}) + 0.8626\psi(\{c_2\}) + 0.8626\psi(\{c_3\}) + 0.6894\psi(\{c_4\})), \\
V_3 &= p1 \times (-0.2849\psi(\{c_1\}) + 0.9773\psi(\{c_2\}) + 0.6614\psi(\{c_3\}) + 0.9773\psi(\{c_4\})) + p2 \times (0.6894\psi(\{c_1\}) + 0.8626\psi(\{c_2\}) \\
&\quad + 0.8626\psi(\{c_3\}) + 0.8626\psi(\{c_4\})) + p3 \times (0.6614\psi(\{c_1\}) + 0.8626\psi(\{c_2\}) + 0.1477\psi(\{c_3\}) + 0.8893\psi(\{c_4\})), \\
V_4 &= p1 \times (-0.2849\psi(\{c_1\}) + 0.4021\psi(\{c_2\}) + 0.9773\psi(\{c_3\}) + 0.1477\psi(\{c_4\})) + p2 \times (0.6894\psi(\{c_1\}) + 0.9773\psi(\{c_2\}) \\
&\quad + 0.9773\psi(\{c_3\}) + 0.6614\psi(\{c_4\})) + p3 \times (0.8626\psi(\{c_1\}) + 0.6614\psi(\{c_2\}) + 0.9773\psi(\{c_3\}) + 0.1791\psi(\{c_4\})), \\
V_5 &= p1 \times (0.9773\psi(\{c_1\}) - 0.1110\psi(\{c_2\}) + 0.6614\psi(\{c_3\}) + 0.4021\psi(\{c_4\})) + p2 \times (0.6894\psi(\{c_1\}) + 0.8626\psi(\{c_2\}) \\
&\quad + 0.6614\psi(\{c_3\}) + 0.9773\psi(\{c_4\})) + p3 \times (0.8626\psi(\{c_1\}) + 0.9773\psi(\{c_2\}) + 0.6614\psi(\{c_3\}) + 0.8893\psi(\{c_4\})).
\end{aligned} \tag{40}$$

Steps 9-10: Because $\psi(\{c_1\})$, $\psi(\{c_2\})$, $\psi(\{c_3\})$, $\psi(\{c_4\})$, p_1, p_2, and p_3 are unknown, using (31), we set $\rho = ((0, 0, 0, 0.1; 1), (0, 0, 0, 0.05; 0.9))$ and establish (41) to determine them.

Step 11: Using (34), the IT2F programming model (41) is transformed into a nonlinear programming model (42).

Step 12: After applying the EA-CSTA-based optimization algorithm to solve (42) using MATLAB version R2017b, the values of the objective function in successive iterations are derived, as depicted in Fig. 6.

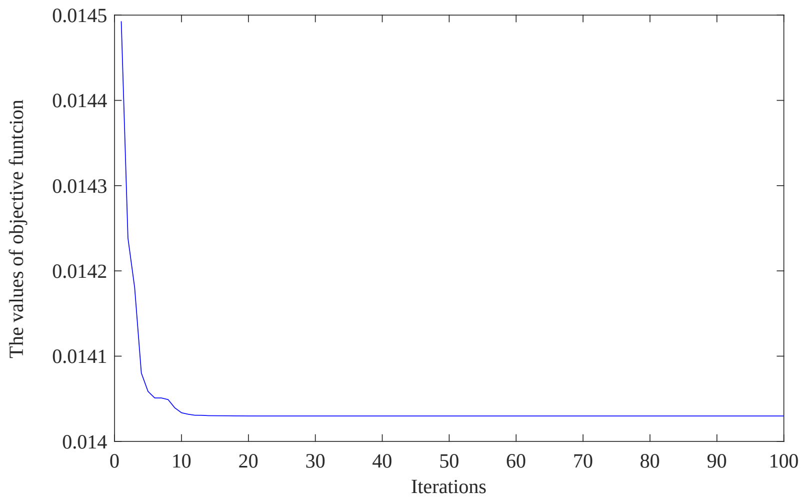

Fig. 6. Values of Objective Function in Successive Iterations of the EA-CSTA-based Optimization Algorithm.

Subsequently, the EA-CSTA-based algorithm yields the optimal fuzzy measures $\boldsymbol{\omega}^*$ on the criteria set and probability vector $\mathbf{P}^*$ of the natural states as follows:

$$\mu^*(\{c_1\}) = 0.1912, \mu^*(\{c_2\}) = 0.2306, \mu^*(\{c_3\}) = 0.2845, \mu^*(\{c_4\}) = 0.2937, \mu^*(\{c_1, c_2\}) = 0.4218,$$
$$\mu^*(\{c_1, c_3\}) = 0.4757, \mu^*(\{c_1, c_4\}) = 0.4849, \mu^*(\{c_2, c_3\}) = 0.5151, \mu^*(\{c_2, c_4\}) = 0.5243,$$
$$\mu^*(\{c_3, c_4\}) = 0.5782; p_1^* = 0.4500, p_2^* = 0.3750, p_3^* = 0.1750.$$

Step 13: Using (7) to calculate the Banzhaf value $\psi(\{c_j\})(j = 1, 2, 3, 4)$ on each criterion c_j, we derive the following:

$\psi(\{c_1\}) = 0.15, \psi(\{c_2\}) = 0.2715, \psi(\{c_3\}) = 0.2785, \psi(\{c_4\}) = 0.3.$

Step 14: Based on $\psi(\{c_j\})(j = 1, 2, 3, 4)$, using (40) to compute the Banzhaf-based overall perceived value V_i of each company $z_i(i = 1, 2, \cdots, 5)$, we derive the following:

$$V_1 = 0.5706, V_2 = 0.6794, V_3 = 0.7118, V_4 = 0.5863, V_5 = 0.6596.$$

Step 15: Because $V_3 > V_2 > V_5 > V_4 > V_1$, the ranking order of the companies is $z_3 > z_2 > z_5 > z_4 > z_1$, where the best company is z_3.

$$
\begin{aligned}
\min\Big\{B = {} & ((0.9, 1, 1, 1; 1), (0.95, 1, 1, 1; 0.9))p_1\lambda_{12}^1 + ((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))p_1\lambda_{25}^1 \\
& + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_1\lambda_{31}^1 + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_1\lambda_{32}^1 \\
& + ((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))p_1\lambda_{34}^1 + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_1\lambda_{54}^1 \\
& + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_2\lambda_{21}^2 + ((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))p_2\lambda_{25}^2 \\
& + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_2\lambda_{31}^2 + ((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))p_2\lambda_{32}^2 \\
& + ((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))p_2\lambda_{34}^2 + ((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))p_2\lambda_{54}^2 \\
& + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_3\lambda_{21}^3 + ((0.9, 1, 1, 1; 1), (0.95, 1, 1, 1; 0.9))p_3\lambda_{41}^3 \\
& + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_3\lambda_{51}^3 + ((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_3\lambda_{53}^3\Big\}
\end{aligned}
$$

$$
s.t.\left\{
\begin{array}{l}
((0.9, 1, 1, 1; 1), (0.95, 1, 1, 1; 0.9))p_1(-0.7724\psi(\{c_1\}) - 0.7149\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\})) \\
+((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))p_1(-0.3159\psi(\{c_1\}) + 0.9736\psi(\{c_2\}) - 0.7724\psi(\{c_3\}) + 0.2593\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_1(-0.1739\psi(\{c_1\}) + 0.8296\psi(\{c_2\}) + 0\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_1(-0.9463\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\})) \\
+((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))p_1(0\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.8296\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_1(1.2622\psi(\{c_1\}) - 0.5131\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.2544\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_2(0.9670\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+((0.5, 0.7, 0.75, 0.9; 1), (0.6, 0.65, 0.7, 0.85; 0.9))p_2(0.1999\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.1147\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_2(0.7671\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))p_2(-0.1999\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) + 0.2012\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))p_2(0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0.2012\psi(\{c_4\})) \\
+((0.3, 0.5, 0.55, 0.7; 1), (0.4, 0.45, 0.5, 0.6; 0.9))p_2(0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.3159\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_3(0.3129\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+((0.9, 1, 1, 1; 1), (0.95, 1, 1, 1; 0.9))p_3(0.1982\psi(\{c_1\}) + 0.2593\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.5103\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_3(0.1982\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.1999\psi(\{c_4\})) \\
+((0.7, 0.9, 0.95, 1; 1), (0.8, 0.85, 0.9, 0.95; 0.9))p_3(0.2012\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.5137\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
\geq ((0, 0, 0, 0.1; 1), (0, 0, 0, 0.05; 0.9)); \\
-0.7724\psi(\{c_1\}) - 0.7149\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\}) + \lambda_{12}^1 \geq 0; \\
-0.3159\psi(\{c_1\}) + 0.9736\psi(\{c_2\}) - 0.7724\psi(\{c_3\}) + 0.2593\psi(\{c_4\}) + \lambda_{25}^1 \geq 0; \\
-0.1739\psi(\{c_1\}) + 0.8296\psi(\{c_2\}) + 0\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{31}^1 \geq 0; \\
-0.9463\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\}) + \lambda_{32}^1 \geq 0; \\
0\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.8296\psi(\{c_4\}) + \lambda_{34}^1 \geq 0; \\
1.2622\psi(\{c_1\}) - 0.5131\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.2544\psi(\{c_4\}) + \lambda_{54}^1 \geq 0; \\
0.9670\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{21}^2 \geq 0; \\
0.1999\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.1147\psi(\{c_4\}) + \lambda_{25}^2 \geq 0; \\
0.7671\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{31}^2 \geq 0; \\
-0.1999\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) + 0.2012\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{32}^2 \geq 0; \\
0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0.2012\psi(\{c_4\}) + \lambda_{34}^2 \geq 0; \\
0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.3159\psi(\{c_4\}) + \lambda_{54}^2 \geq 0; \\
0.3129\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{21}^3 \geq 0; \\
0.1982\psi(\{c_1\}) + 0.2593\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.5103\psi(\{c_4\}) + \lambda_{41}^3 \geq 0; \\
0.1982\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.1999\psi(\{c_4\}) + \lambda_{51}^3 \geq 0; \\
0.2012\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.5137\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{53}^3 \geq 0; \\
\lambda_{12}^1, \lambda_{25}^1, \lambda_{31}^1, \lambda_{32}^1, \lambda_{34}^1, \lambda_{54}^1, \lambda_{21}^2, \lambda_{25}^2, \lambda_{31}^2, \lambda_{32}^2, \lambda_{34}^2, \lambda_{54}^2, \lambda_{21}^3, \lambda_{41}^3, \lambda_{51}^3, \lambda_{53}^3 \geq 0; \\
\psi(\{c_j\}) = -\frac{1}{2}\mu(\{c_j\}) + \sum\limits_{c_q \in C \setminus c_j} \frac{1}{2}(\mu(\{c_j, c_q\}) - \mu(\{c_q\})) \quad (j = 1, 2, 3, 4); \\
\sum\limits_{\{c_j, c_q\} \subseteq C} \mu(\{c_j, c_q\}) - 2\sum\limits_{\{c_j\} \subseteq C} \mu(\{c_j\}) = 1; \\
\sum\limits_{\{c_j\} \subseteq S \setminus c_q} (\mu(\{c_j, c_q\}) - \mu(\{c_j\})) \geq (|S| - 2)\mu(\{c_q\}) \quad (\forall S \subseteq C \text{ with } c_q \in S \text{ and } |S| \geq 2). \\
0.15 \leq \mu(\{c_1\}) \leq 0.2, 0.2 \leq \mu(\{c_2\}) \leq 0.3, 0.2 \leq \mu(\{c_3\}) \leq 0.3, 0.3 \leq \mu(\{c_4\}) \leq 0.4; \\
0.4 \leq p_1 \leq 0.45, 0.35 \leq p_2 \leq 0.375, 0.175 \leq p_3 \leq 0.2; p_1 + p_2 + p_3 = 1.
\end{array}
\right. \tag{41}
$$

$$
\begin{aligned}
\min \Big\{ B = {} & 0.9322 p_1 \lambda_{12}^1 + 0.8075 p_1 \lambda_{25}^1 + 0.8906 p_1 \lambda_{31}^1 + 0.8906 p_1 \lambda_{32}^1 + 0.8075 p_1 \lambda_{34}^1 + 0.8906 p_1 \lambda_{54}^1 \\
& + 0.8372 p_2 \lambda_{21}^2 + 0.7541 p_2 \lambda_{25}^2 + 0.8372 p_2 \lambda_{31}^2 + 0.6591 p_2 \lambda_{32}^2 + 0.6591 p_2 \lambda_{34}^2 + 0.6591 p_2 \lambda_{54}^2 \\
& + 0.8372 p_3 \lambda_{21}^3 + 0.8788 p_3 \lambda_{41}^3 + 0.8372 p_3 \lambda_{51}^3 + 0.8372 p_3 \lambda_{53}^3 \Big\}
\end{aligned}
$$

$$
s.t. \left\{
\begin{array}{l}
0.9322 p_1(-0.7724\psi(\{c_1\}) - 0.7149\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\})) \\
+0.8075 p_1(-0.3159\psi(\{c_1\}) + 0.9736\psi(\{c_2\}) - 0.7724\psi(\{c_3\}) + 0.2593\psi(\{c_4\})) \\
+0.8906 p_1(-0.1739\psi(\{c_1\}) + 0.8296\psi(\{c_2\}) + 0\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+0.8906 p_1(-0.9463\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\})) \\
+0.8075 p_1(0\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.8296\psi(\{c_4\})) \\
+0.8906 p_1(1.2622\psi(\{c_1\}) - 0.5131\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.2544\psi(\{c_4\})) \\
+0.8372 p_2(0.9670\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+0.7541 p_2(0.1999\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.1147\psi(\{c_4\})) \\
+0.8372 p_2(0.7671\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+0.6591 p_2(-0.1999\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) + 0.2012\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+0.6591 p_2(0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0.2012\psi(\{c_4\})) \\
+0.6591 p_2(0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.3159\psi(\{c_4\})) \\
+0.8372 p_3(0.3129\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\})) \\
+0.8788 p_3(0.1982\psi(\{c_1\}) + 0.2593\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.5103\psi(\{c_4\})) \\
+0.8372 p_3(0.1982\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.1999\psi(\{c_4\})) \\
+0.8372 p_3(0.2012\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.5137\psi(\{c_3\}) + 0\psi(\{c_4\})) \geq 0.0178; \\
-0.7724\psi(\{c_1\}) - 0.7149\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\}) + \lambda_{12}^1 \geq 0; \\
-0.3159\psi(\{c_1\}) + 0.9736\psi(\{c_2\}) - 0.7724\psi(\{c_3\}) + 0.2593\psi(\{c_4\}) + \lambda_{25}^1 \geq 0; \\
-0.1739\psi(\{c_1\}) + 0.8296\psi(\{c_2\}) + 0\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{31}^1 \geq 0; \\
-0.9463\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.7724\psi(\{c_3\}) + 0.3159\psi(\{c_4\}) + \lambda_{32}^1 \geq 0; \\
0\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.8296\psi(\{c_4\}) + \lambda_{34}^1 \geq 0; \\
1.2622\psi(\{c_1\}) - 0.5131\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.2544\psi(\{c_4\}) + \lambda_{54}^1 \geq 0; \\
0.9670\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{21}^2 \geq 0; \\
0.1999\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.1147\psi(\{c_4\}) + \lambda_{25}^2 \geq 0; \\
0.7671\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{31}^2 \geq 0; \\
-0.1999\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) + 0.2012\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{32}^2 \geq 0; \\
0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0.2012\psi(\{c_4\}) + \lambda_{34}^2 \geq 0; \\
0\psi(\{c_1\}) - 0.1147\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.3159\psi(\{c_4\}) + \lambda_{54}^2 \geq 0; \\
0.3129\psi(\{c_1\}) + 0.4605\psi(\{c_2\}) - 0.1147\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{21}^3 \geq 0; \\
0.1982\psi(\{c_1\}) + 0.2593\psi(\{c_2\}) + 0\psi(\{c_3\}) - 0.5103\psi(\{c_4\}) + \lambda_{41}^3 \geq 0; \\
0.1982\psi(\{c_1\}) + 0.5752\psi(\{c_2\}) - 0.3159\psi(\{c_3\}) + 0.1999\psi(\{c_4\}) + \lambda_{51}^3 \geq 0; \\
0.2012\psi(\{c_1\}) + 0.1147\psi(\{c_2\}) + 0.5137\psi(\{c_3\}) + 0\psi(\{c_4\}) + \lambda_{53}^3 \geq 0; \\
\lambda_{12}^1, \lambda_{25}^1, \lambda_{31}^1, \lambda_{32}^1, \lambda_{34}^1, \lambda_{54}^1, \lambda_{21}^2, \lambda_{25}^2, \lambda_{31}^2, \lambda_{32}^2, \lambda_{34}^2, \lambda_{54}^2, \lambda_{21}^3, \lambda_{41}^3, \lambda_{51}^3, \lambda_{53}^3 \geq 0; \\
\psi(\{c_j\}) = -\frac{1}{2}\mu(\{c_j\}) + \sum\limits_{c_q \in C \setminus c_j} \frac{1}{2}(\mu(\{c_j, c_q\}) - \mu(\{c_q\})) \quad (j = 1, 2, 3, 4); \\
\sum\limits_{\{c_j, c_q\} \subseteq C} \mu(\{c_j, c_q\}) - 2 \sum\limits_{\{c_j\} \subseteq C} \mu(\{c_j\}) = 1; \\
\sum\limits_{\{c_j\} \subseteq S \setminus c_q} (\mu(\{c_j, c_q\}) - \mu(\{c_j\})) \geq (|S| - 2)\mu(\{c_q\}) \quad (\forall S \subseteq C \text{ with } c_q \in S \text{ and } |S| \geq 2). \\
0.15 \leq \mu(\{c_1\}) \leq 0.2, 0.2 \leq \mu(\{c_2\}) \leq 0.3, 0.2 \leq \mu(\{c_3\}) \leq 0.3, 0.3 \leq \mu(\{c_4\}) \leq 0.4; \\
0.4 \leq p_1 \leq 0.45, 0.35 \leq p_2 \leq 0.375, 0.175 \leq p_3 \leq 0.2; p_1 + p_2 + p_3 = 1.
\end{array}
\right. \tag{42}
$$