NETWORKS, OPTIONS AND PREEMPTION

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Networks, Options and Preemption*

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Abstract

This paper examines the irreversible adoption of a technology whose returns are uncertain, when there is an advantage to being the first adopter, but a network advantage to adopting when others also do so. Two patterns of adoption emerge: sequential, in which the leader aggressively preempts its rival; and a more accommodating outcome in which the firms adopt simultaneously. There are two main results. First, conditional on adoption being sequential, the follower adopts at the incorrect point, compared to the co-operative solution. The leader adopts at the co-operative point when there is no preemption, and too early if there is preemption. Secondly, there is insufficient simultaneous adoption in equilibrium. The paper examines the effect of uncertainty, network effects and preemption on these inefficiencies. Standard results do not always hold. For example, the analysis raises the unusual possibility that an increase in uncertainty may cause the first mover to adopt the technology earlier.

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1. Introduction

This paper examines the irreversible adoption of a technology whose returns are uncertain, when there is an advantage to being the first adopter, but a network benefit to adopting when others also do so. The paper asks: does adoption occur 'too fast' or 'too slow' in equilibrium? Does the 'right kind' of adoption occur? How do the various factors—network effects, uncertainty and preemption—interact?

These questions can be seen most clearly in a simple location entry model. Consider a street with three possible locations for shops: two sites at the ends of the street, and one in the middle. Consumers live all the way (e.g. are uniformly distributed) along the street. Two entrepreneurs are considering when and where to open a shop. There are sunk costs (such as fitting) in opening a shop. If both entrepreneurs open a shop, they compete in prices against each other for custom. There are, however, positive externalities to being located on the same street; for example, because a common cost can be shared (such as a fixed cost of delivery of goods), or because aggregate demand is increased by lowering consumer search costs. Finally, total demand (the mass of consumers) is growing over time but is uncertain.

Three factors affect the location decision of an entrepreneur who enters before its rival (the leader): profit before the entry of the rival; profit after the entry of the rival; and the expected time to entry of the rival. Suppose that the leader decides to locate in the middle of the street. In doing so, while it has the only shop on the street its profit is higher than if it had located at one end of the street. Once its rival (the follower) enters, its profit is lower than if it had located at one end of the street, since price competition is more intense.[1] But this fact also means that the follower will delay opening its shop until the level of demand is higher, and so the leader will enjoy monopoly profits for longer.

There are two patterns of entry and location. In the first, the leader locates 'aggressively' in the middle of the street and the follower enters (relatively) late. In the second,

[1] Its rival will, given standard assumptions, locate as far away as possible to minimise price competition. The furthest away it can locate from a shop in the middle of the street is half the street length, rather than the whole street length if the leading entrepreneur locates at one end.

the leader locates 'accommodatingly' at the end of the street and the follower enters (relatively) soon. In fact, in the model that is developed here, this contrast is particularly stark: when the leader is aggressive, it preempts its rival and entry occurs sequentially; but when the leader accommodates, entry is simultaneous.

In order to study in detail these economic forces, the continuous time analysis of Fudenberg and Tirole (1985) is adapted: the firms' profit functions are modelled in order to parameterise the first mover advantage and network effects, and to include uncertainty. Entry is interpreted as adoption of a technology by two firms. Then an aggressive leader preempts by adopting 'early' (i.e., when the level of instantaneous return is low), with the follower adopting 'later' (at a higher level of instantaneous return). Accommodation means that simultaneous adoption occurs at a point which is after the leader's adoption point in the corresponding sequential equilibrium, but before the follower's. (These facts will be established during the analysis.)

Two types of externalities arise in the model: a backward externality, since the follower does not take into account the effect of its adoption on the leader; and a forward externality, since the leader does not consider its effect on the follower. (The terms 'backward' and 'forward' externality are borrowed from Choi (1994).) Hence there are two types of inefficiencies in adoption decisions. First, when the leader adopts aggressively, the firms adopt at the incorrect times relative to the co-operative solution. Secondly, the leader acts aggressively too often: adoption may be sequential when the co-operative solution involves simultaneous adoption. (The analysis will also show that the converse—simultaneous adoption in equilibrium when co-operation requires it to be sequential—does not occur.)

The analytical task is to determine the adoption times, patterns and inefficiencies, and examine their dependence on the factors in the problem: preemption, network effects and the degree of uncertainty. Simple intuition suggests the following. The first mover advantage should make preemption more attractive; and preemption should lead to earlier adoption of the technology by the leader. Network effects should encourage accommodation, so that when the effects are sufficiently large, preemption does not occur and the technology is adopted simultaneously. They should also increase inefficiencies in adoption behaviour. Uncertainty should delay adoption, due to the option values that are created by the irreversibility of the investment.

In fact, the analysis makes several qualifications to this intuition. Preemption does hasten adoption. Network effects do indeed increase adoption inefficiencies. But they do so in several ways, through: (i) decreasing (relative to the co-operative benchmark) the amount of simultaneous adoption that occurs in equilibrium; (ii) changing the follower's adoption point when adoption is sequential; and (iii) bringing forward the adoption point of the preempting firm when adoption is sequential. Uncertainty does not always delay adoption: for sufficiently large network effects, the introduction of a small amount of uncertainty into the model decreases the adoption point of the preemptor in an equilibrium with sequential adoption. Overall, therefore, the effects and interactions of these three factors are rather subtle and surprising.

There are many cases of technology adoption in which uncertainty, network effects and first mover advantages are important. Two examples are mentioned briefly (in addition to the shop location story discussed earlier). The first example concerns two firms deciding whether to set up sites on the World Wide Web. There is some benefit to having a Web site; but the exact size of the benefit is uncertain.[2] Sunk costs are incurred in setting up a site: skilled labour is required to design and write the pages, a domain name must be purchased, marketing expenditures incurred etc..[3] An increasingly important source of first mover advantage is the ability of early adopters to buy their preferred domain names cheaply.[4] Generic Web addresses (such as `business.com` and `internet.com`), generally perceived to be the most valuable, are a limited resource. In 1997, `business.com` was

[2]A recent study found that one-third of the small businesses that use the Internet increased their revenues by at least 10 per cent over the previous year. However, in the first nine months of 1999, consumer e-commerce in the U.S. initially fell and then plateaued; participation in online auctions has followed the same pattern. See InternetNews.Com (1999). Recent bankruptcies have emphasised the high degree of uncertainty facing internet-based businesses.

[3]Estimates of the cost of setting up the most basic web site range between US$225–1050, with an annual maintenance cost of between US$200–350; the most complex sites may cost several hundreds of thousands of dollars. See PC World (1999). Since its inception, marketing expenditure has been 25% of `amazon.com`'s revenues.

[4]Before 1994, Internic, the primary international authority for registration of domain names, did not charge; after this date, registration fees were instituted (in September 1999, US$70 per address for first 2 years, with a renewal fee thereafter). See Radin and Wagner (1996) for details.

sold for US$150,000, `consumers.com` and `internet.com` both sold for US$100,000.[5] In the words of one industry newspaper, the "Internet equivalent of an uptown address just got a little bit pricier" (see CNET News.Com (1997)). A first mover advantage may also arise because the firm that acts first to set up its Web site may face lower staff costs—site designers being relatively abundant—than later firms who have to hire when designers are more scarce. Finally, network effects arise since a firm setting up a Web site benefits from the efforts of other firms, both directly (e.g. by being able to learn from the design of other sites) and indirectly (e.g. consumers already being accustomed to buying online).

As a second example, consider competing satellite systems for global communications. (The following discussion relies on Vu (1996).) Initially, there were two competing types of system: geosynchronous earth orbit (GEO) satellites and low earth orbit (LEO) satellites.[6] There are large sunk costs to implementing either system: the GEO system was estimated to cost around US$4 billion, while the cheapest LEO proposal costs US$9 billion. Wireless communications systems such as satellites use frequencies within the radio spectrum. If two users employ the same frequency at the same time, interference is created. A broader range of lower frequencies is more desirable and a first mover advantage arises through the allocation of these frequencies.[7] Network effects arise naturally, since the availability of satellite systems stimulates demand for global communications that all system operators benefit from. Finally, the industry faced considerable uncertainty, about both the cost of satellite technology and the total demand for wireless communication services.

The paper closest to this one is Choi (1994) who examines a model in which there are network effects, uncertainty and the possibility of delay. Choi identifies the two

[5]It might be argued that the most famous web addresses, such as `amazon.com` and `yahoo.com`, are non-generic. The point is, however, that generic web addresses are advantageous in attracting uninformed consumers who are unaware of specific brands.

[6]The former orbit approximately 35,000 km above the equator, and require between three and fifteen satellites to deliver worldwide service. LEOs orbit at about 1,350 km above the earth's surface, and require a much larger network of satellites to cover the entire world. More recently, a third system—global stratospheric telecommunications system (GSTS)—has been proposed. GSTS involves floating communication platforms suspended 12 km above the earth by helium balloons.

[7]Two aspects of frequency are important. The first is amount: the bandwidth made available to a wireless operator determines the total demand that it can serve. Secondly, the range of the frequency has important consequences for transmission. Very high frequencies can be blocked by tree leaves, windows and even very heavy rain storms, causing loss of signal.

externalities mentioned above. In Choi's model, users are exogenously asymmetric: user 1 is able to choose which of two technologies (with random returns) to adopt in either of two periods, while user 2 is able to adopt only in the second period. Implicitly, he includes a first mover advantage—the forward externality creates an early adopter's commitment power, similar to that identified by Farrell and Saloner (1985). Choi finds that the forward externality outweighs the backward, and there is excess momentum: the option to delay is not held for long enough in equilibrium compared to the social optimum.

This paper departs from Choi's in several respects. Most importantly, it does not impose exogenously an asymmetry between players, but rather allows the first mover to be determined endogenously. To show the consequence of this, two versions of the model are presented. In the first, the roles of leader and follower are preassigned exogenously, so that (as in Choi) preemption is not an issue; see section 3.1. In the second version, the roles are determined endogenously: the leader adopts at the point at which it is indifferent between leading and following; see section 3.2.[8] The fact that adoption by the leader is determined by indifference, rather than optimally (for the leader), makes an important difference to adoption behaviour. As in Choi's model, there is excess momentum, in the sense that sequential adoption occurs too often in equilibrium compared to the co-operative solution. A second aspect of the forward externality is identified: endogenous determination of the leading firm (i.e., preemption) causes the leader to adopt too early. This endogenous determination of roles also lies behind the surprising comparative static result that an increase in uncertainty may cause the first mover to adopt the technology earlier.[9]

Three strands of literature are related to this paper. Real options models have been used to explain delay and hysteresis arising in a wide range of contexts. McDonald and Siegel (1986) and Pindyck (1988) consider irreversible investment opportunities available to a single firm. Dixit (1989) and Dixit (1991) considers product market entry and exit in monopolistic and perfectly competitive settings respectively. The second strand of literature concerns timing games of entry or exit in a deterministic setting. Papers

[8] This is the rent equalisation principle identified in Fudenberg and Tirole (1985).

[9] There are other differences between our model and Choi's: for example, we choose a continuous time setting rather than a two period model so that dynamics can be examined in detail.

analysing preemption games include Fudenberg, Gilbert, Stiglitz, and Tirole (1983) and Fudenberg and Tirole (1985), while wars of attrition have been modelled by e.g. Fudenberg and Tirole (1986). Finally, technology adoption in the presence of network effects has been analysed by many papers, including Farrell and Saloner (1986) and Katz and Shapiro (1986). Existing real option models typically assume a monopolistic or perfectly competitive framework, and do not include network effects. Preemption models allow for incomplete information about the types of players, but not for common uncertainty about payoffs or network benefits. Network papers have not (with the exception reviewed above) analysed explicitly the effect of 'option values'—created when there is exogenous uncertainty, adoption is irreversible, and agents are able to choose the time of adoption.

Three further papers, though less closely related, are also relevant. Smets (1991) examines irreversible market entry in a duopoly facing stochastic demand. Simultaneous investment may arise only when the leadership role is exogenously preassigned. Consequently, he does not consider fully forward externalities. Weeds (1999) presents a model in which two firms may invest in competing research projects with uncertain returns. She does not impose an asymmetry between the firms, but allows the leader to emerge endogenously. She does not include, however, network effects. Finally, Hoppe (2000) analyses a timing game of new technology adoption in an uncertain environment. She considers second, rather than first, mover advantages and models uncertainty in a different way to this paper.

The rest of the paper is structured as follows. Section 2 presents the set-up. Section 3 analyses the non-co-operative equilibria of two versions of the model—the first where the roles of leader and follower are exogenously preassigned, the second where they are endogenous. Section 4 determines the co-operative solution. Various inefficiencies in the model are analysed in section 5, which also reviews the main results and comparative statics of the model. Section 6 concludes. The appendix covers certain technical aspects of the micro-foundation for the reduced-form model and contains lengthier proofs.

2. The Model

This section develops a simple model to capture the three effects that are the focus of this paper: (i) uncertainty, irreversibility and the possibility of delay in adoption; (ii) network effects, where the return to adoption of the technology depends on the number of other adopters; and (iii) preemption, where early adopters have a first mover advantage. The purpose is to provide a micro-foundation for the reduced-form instantaneous return functions that are used for the analysis later in the paper. It should be noted, however, that the subsequent analysis is not specific to the particular micro-model chosen. A number of alternative frameworks could have been used as a foundation for the same reduced-form equations. Section 2.1 develops a spatial model that underlies the reduced form in section 2.2.

2.1. A Spatial Model of Adoption

The adoption of a new technology by two firms is modelled as Hotelling-style entry into a horizontally differentiated market. In the spatial model, a first mover advantage arises because the early adopter can locate so as to attract more demand than the later adopter. This is intended to capture the possibility that an early adopter can, for example, obtain better access to scarce resources such as a generic Web site address. The entry game is treated quite informally in this section; see section 2.2 for a more formal statement of the firms' strategies, equilibrium etc.. The purpose of this section is to establish instantaneous return functions that are amenable to further analysis.

Consumers are uniformly distributed on the unit interval. A consumer located at $x \in [0, 1]$ gains a utility from purchasing a unit of the good located at $y \in [0, 1]$ given by $U(x, y) = V - l(|x - y|)^2 - p$, where V is a constant that is the same for all consumers, $l > 0$ is the transport cost, or measure of horizontal differentiation, and p is the price that is charged for the good. Each consumer buys one or zero units of any good. It is assumed that V is 'sufficiently large'; exactly how large and the role of this assumption is explained below and in the appendix. Time is continuous and labelled by $t \in [0, \infty)$.

The mass of consumers is time-varying and is described further in section 2.2.

Two risk neutral firms, labelled $i = 1, 2$ can each enter the industry. There is a cost K to doing so, which is the same for both firms. Entry is irreversible (the cost K is entirely sunk), and can be delayed indefinitely. Once a firm has entered, it can sell its product at zero marginal cost. There are three possible locations at which the firms can enter: at $x = 0$, $x = \frac{1}{2}$ and $x = 1$. The restriction on locations is made to keep the analysis clear.

There are three factors to consider: (i) the timing of entry; (ii) the location of entry; and (iii) the prices set by the firms. The first question is tackled in later sections; in this section, flow returns are calculated conditional on entry. Notice that the firms will not enter at the same location on the line, since Bertrand competition would drive flow profits to zero; with the sunk cost of entry K, entry would not be profitable in this case. Two configurations are possible: one of the firms locates at $x = \frac{1}{2}$ while the other locates at $x = 0$ (or, equivalently, $x = 1$); or both firms locate at the ends of the line, at $x = 0$ and $x = 1$. In the static Hotelling model with endogenous locations and a quadratic transport cost function, there is maximum differentiation. Therefore, when (a) entry is simultaneous, the firms locate at the ends of the line; (b) entry is sequential, the second entrant, or follower, locates as far away from the first entrant, or leader, as possible. In the latter case, there are two possibilities: (i) the leader locates at $x = \frac{1}{2}$, or (ii) the leader locates at one end. In either case, the follower locates at the (other) end. Only the first possibility is considered in this paper; the conditions required for this to be the optimal choice of the leader are derived in the appendix. The purpose of this restriction is to focus attention on the choice of interest: whether a firm considering adoption before its rival should be aggressive or accommodating. If it is aggressive, by locating in the middle, then the conditions ensure that the other firm delays adopting. If it is accommodating, by locating at one end, then the other firm locates at the other end simultaneously.

Consider first the outcome when the firms enter the market sequentially. The extensive form of the game is as follows: one firm enters and, having entered at its preferred location, sets its price to maximise profit. The second firm then enters at its preferred location; once it has entered, the firms compete in prices. Without loss of generality (wlog), let firm 1 be the firm that enters first. Let the mass of consumers in the market at time t in this case be θ_t. Once the firm has entered (at location $x = \frac{1}{2}$) and before firm 2 has entered, it chooses its price to maximise its profit $\pi_1 = \theta p_1(1 - 2x^*)$, where $(x^*, 1 - x^*)$ are the locations of the marginal consumers who are indifferent between buying and not: i.e., for whom $V - l(\frac{1}{2} - x^*)^2 - p_1 = 0$, when the firm sets a price p_1. A straightforward calculation shows that, when V is sufficiently large, the firm chooses to sell to all consumers, setting its price so that the consumers located at $x = 0$ and 1 are indifferent between buying and not.[10] Therefore the firm's profit maximising price and maximum profit are

$$p_1^I = V - \frac{l}{4}, \qquad \pi_1^I = \left(V - \frac{l}{4}\right)\theta.$$

Now consider the outcome once the second firm has entered, wlog at $x = 0$. The assumption that V is sufficiently large (greater than $\frac{3}{2}l$) ensures that in equilibrium all consumers buy from one of the firms. The usual calculations show that the Nash equilibrium prices are

$$p_1^{II} = \frac{7}{12}l, \qquad p_2^{II} = \frac{5}{12}l.$$

$V \geq \frac{3}{2}l$ means that these prices are lower than p_1^I, and so certainly all consumers receive a greater net surplus in this case.

A positive network effect is introduced by letting the market size at time t when two firms have adopted be $(1+\alpha)\theta_t$, where $\alpha \geq 0$. There are several ways in which this feature can be justified. First, with two firms in the industry, competition is more intense and consumer surplus is greater. Consequently, more consumers will be willing to buy the firms' goods. Alternatively, there may be a positive externality arising from the presence of search costs for consumers that ensures an increase in aggregate demand when two firms are located on the line.[11] Finally, it could be that the firms must cover jointly a fixed

[10] The alternative is that $x^* > 0$. In this case, the firm's profit maximising price would be $p = \frac{2V}{3}$ and $x^* = \frac{1}{2} - \sqrt{\frac{V}{3l}}$. In order for $x^* > 0$, therefore, it must be that $V < \frac{3}{4}l$. It is assumed below, however, that $V \geq \frac{3}{2}l$, so this case does not occur.

[11] In this story, there is some 'outside' good that consumers can buy when they do not find their preferred good at a firm on the line. When there is only one firm on the line, expected search costs are higher and so fewer consumers are willing to buy than when there are two firms on the line.

cost for delivery of inputs from a perfectly competitive supplier. When there is one firm in the industry, it pays the entire delivery cost; when there are two firms in the industry, the fixed cost can be shared. This decrease in fixed cost is represented as a multiplicative increase in the profit function of the firms. The firms' instantaneous profits are therefore

$$\pi_1^{II} = \left(\frac{49l}{144}\right)(1+\alpha)\theta_t, \qquad \pi_2^{II} = \left(\frac{25l}{144}\right)(1+\alpha)\theta_t.$$

Finally, consider the outcome when the firms enter simultaneously at either end of the line. The standard calculation gives the Nash equilibrium prices as $p_1^{III} = p_2^{III} = l$. Since $V \geq \frac{3}{2}l$, these prices are below p_1^I, but they are above both p_1^{II} and p_2^{II}. There is again a positive network effect, but smaller than previously: it is $\gamma(1+\alpha)\theta_t$ where $\gamma \in (0,1)$. Depending on which justification is used, this is because price competition is less fierce in this case, and so consumers are less willing to buy the firms' goods; or because consumers' expected search costs are higher because the firms on the line are farther apart; or because the fixed cost of delivery to two firms that are further apart is bigger. The firms' symmetric profits are then (each)

$$\pi^{III} = \frac{l}{2}\gamma(1+\alpha)\theta_t.$$

2.2. A Reduced Form

The micro-model has provided the following expressions for the instantaneous profits at time t of the 'leader' and 'follower' when entry—or adoption of the technology—is sequential, normalised by the leader's profit before the follower adopts:

$$\pi_1^I = \theta_t, \tag{1}$$

$$\pi_1^{II} = \left(\frac{49l}{36(4V-l)}(1+\alpha)\right)\theta_t \;\equiv\; (1+\delta_L)\theta_t, \tag{2}$$

$$\pi_2^{II} = \left(\frac{25l}{36(4V-l)}(1+\alpha)\right)\theta_t \;\equiv\; (1+\delta_F)\theta_t. \tag{3}$$

The instantaneous profits of the firms at time t when adoption is simultaneous is

$$\pi^{III} = \left(\frac{8l}{4V-l}\gamma(1+\alpha)\right)\theta_t \equiv (1+\delta_S)\theta_t, \tag{4}$$

again normalised by π_1^I. (Note that these reduced-form expressions are not unique to the micro-model that has been used—the micro-model is provided to aid interpretation.)

Many configurations of the parameters δ_L, δ_F and δ_S are possible. The following assumption is made:

Assumption 1:

$$-\left(\frac{\beta}{\beta+1}\right) \leq \delta_F \leq 0,$$
$$\delta_F \leq \delta_S \leq 0,$$
$$\frac{\delta_F}{\beta} \leq \delta_L \leq -\delta_F,$$

where $\beta \in (1,\infty)$ *(and will be defined later).*

This assumption captures the features of interest in this paper, and in particular a first mover advantage ($\delta_L \geq \frac{\delta_F}{\beta} \geq \delta_F$). The other restrictions are fairly reasonable and simplify the analysis. The role of particular aspects of assumption 1 will be pointed out as the analysis progresses.

It is worth contrasting the instantaneous profit functions implied by assumption 1 with those used by Fudenberg and Tirole (1985). There are two key differences. First, we have introduced more modelling than Fudenberg and Tirole, who use quite general functions. This is deliberate: the interest of this paper is to study in detail how equilibrium outcomes and inefficiencies depend on certain features of the problem. For this purpose, parameters have been introduced explicitly to allow comparative static analysis (see section 5). Secondly, here the leader receives a higher instantaneous profit than the follower once the latter has adopted. In contrast, in Fudenberg and Tirole (1985), firms who have adopted receive the same flow payoff. This emphasises that this paper is concerned with situations in which there is a preferred 'location', or a persistent first mover advantage.

θ_t was interpreted in section 2.1 as the mass of consumers in the market; more generally it is referred to as the stand-alone benefit from adopting the technology (the instantaneous return received by a firm that is the sole adopter of the technology). This stand-alone benefit is assumed to be exogenous and stochastic, evolving according to a geometric Brownian motion (GBM) with drift:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t \tag{5}$$

where $\mu \in [0, r)$ is the drift parameter, measuring the expected growth rate of θ, r is the continuous time discount rate,[12] $\sigma > 0$ is the instantaneous standard deviation or volatility parameter, and dW is the increment of a standard Wiener process, $dW_t \sim N(0, dt)$. The parameters μ, σ and r are common knowledge and constant over time. The choice of continuous time and this representation of uncertainty is motivated by the analytical tractability of the value functions that result.

The strategies of the firms in the adoption game are now defined. If firm i has not adopted the technology at any time $\tau < t$, its action set is $A_t^i = \{\text{adopt, don't adopt}\}$. If, on the other hand, firm i has adopted at some $\tau < t$, then A_t^i is the null action 'don't move'. The firm therefore faces a control problem in which its only choice is when to choose the action 'adopt'. After taking this action, the firm can make no further moves.

A strategy for firm i is a mapping from the history of the game H_t (the sample path of the stochastic variable θ and the actions of both firms up to time t) to the action set A_t^i. Firms are assumed to use stationary Markovian strategies: actions depend on only the current state and the strategy formulation itself does not vary with time. Since θ follows a Markov process, Markovian strategies incorporate all payoff-relevant factors in this game. Furthermore, if one player uses a Markovian strategy, then its rival has a best response that is Markovian as well. Hence, a Markovian equilibrium remains an equilibrium when history-dependent strategies are also permitted, although other non-Markovian equilibria

[12] The restriction that $\mu < r$ ensures that there is a positive opportunity cost to holding the 'option' to adopt, so that the option will not be held indefinitely.

may then also exist. (For further explanation see Maskin and Tirole (1988) and Fudenberg and Tirole (1991).)

The formulation of the firms' strategies is complicated by the use of a continuous time model. Fudenberg and Tirole (1985) point out that there is a loss of information inherent in representing continuous time equilibria as the limits of discrete time mixed strategy equilibria. To correct for this, they extend the strategy space to specify not only the cumulative probability that player i has adopted, but also the 'intensity' with which each player adopts at times 'just after' the probability has jumped to one.[13] Although this formulation uses mixed strategies, the equilibrium outcomes are equivalent to those in which firms employ pure strategies. (See section 2 of Fudenberg and Tirole (1985).) Consequently, the analysis will proceed as if each firm uses a pure Markovian strategy i.e., a stopping rule specifying a critical value or 'trigger point' for the exogenous variable θ at which the firm adopts. Note, however, that this is for convenience only: underlying the analysis is an extended space with mixed strategies.

The possible states of each firm are denoted $n_i \in \{0, 1\}$ when the firm has not adopted and has adopted the technology, respectively. The following assumptions are made:

Assumption 2: *If $n_i(\tau) = 1$, then $n_i(t) = 1$ for all $t \geq \tau$, $i = 1, 2$.*

Assumption 3: $\mathbb{E}_0 \left[\int_0^\infty \exp(-rt)\theta_t dt\right] - K < 0$.

Assumption 2 formalises the irreversibility of adoption: if firm i has adopted by date τ, it then remains active at all dates subsequent to τ. Assumption 3 states that the initial value of the technology is sufficiently low that the expected return from adoption is negative, thus ensuring that immediate adoption is not worthwhile. (The operator $\mathbb{E}_0$ denotes expectations conditional on information available at time $t = 0$.)

[13] In Fudenberg and Tirole (1985), a firm's strategy is a *collection* of *simple strategies* satisfying an *intertemporal consistency condition*. A simple strategy for firm i in a game starting at a positive level θ of the state variable is a pair of real-valued functions $(G_i(\theta), \epsilon_i(\theta)) : (0, \infty) \times (0, \infty) \to [0, 1] \times [0, 1]$ satisfying certain conditions (see definition 1 in their paper) ensuring that G_i is a cumulative distribution function, and that when $\epsilon_i > 0$, $G_i = 1$ (so that if the intensity of atoms in the interval $[\theta, \theta + d\theta]$ is positive, the firm is sure to adopt by θ). A collection of simple strategies for firm i, $(G_i^\theta(.), \epsilon_i^\theta(.))$, is the set of simple strategies that satisfy Bayes rule.

3. Non-Co-operative Equilibrium

Two models are studied. In the first, there are no preemption effects: one firm is assigned exogenously the role of adopting the technology first. This model is well-suited to cases in which one firm has a clear advantage in the adoption of the technology—it may be technically more literate, have a more flexible organisation, or be less dependent on the existing technology. More importantly, by ignoring the possibility of preemption (the first mover always moves first), it isolates the option and network effects (and so allows a comparison with Choi (1994)). In the second model, firms are *ex ante* symmetric, but may be *ex post* asymmetric; which firm adopts first and which second is determined endogenously.

3.1. Without Preemption

Start by assuming that the preassigned leader and follower adopt at different points. The possibility of simultaneous adoption is considered below. As usual in dynamic games the stopping time game is solved backwards. Thus the first step is to consider the optimisation problem of the follower who adopts strictly later than the leader. Given that the leader has adopted irreversibly, the follower's payoff on adopting has two components: the flow benefit from the technology, $(1 + \delta_F)\theta_t$; and the cost of adoption, $-K$. The follower's value function at time t given a level θ_t of the state variable is therefore

$$F(\theta_t) = \max_{T_F} \mathbb{E}_t \left[\int_{T_F}^{\infty} \exp\left(-r(\tau - t)\right)(1 + \delta_F)\theta_\tau d\tau - K \exp\left(-r(T_F - t)\right) \right] \tag{6}$$

where T_F is the random adoption time for the follower, and the operator $\mathbb{E}_t$ denotes expectations conditional on information available at time t. The value function F has two components, holding over different ranges of θ: one relating to the value of adoption before the follower has adopted, the other to after adoption. Let these value functions be denoted F_0 and F_1, respectively.

Prior to adoption, the follower holds an option to adopt but receives no flow payoff.

In this 'continuation' region, in any short time interval dt starting at time t the follower experiences a capital gain or loss dF_0. The Bellman equation for the value of the adoption opportunity is therefore

$$F_0 = \exp(-rdt)\mathbb{E}_t\left[F_0 + dF_0\right]. \tag{7}$$

Itô's lemma and the GBM equation (5) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 F_0''(\theta) + \mu\theta F_0'(\theta) - rF_0(\theta) = 0. \tag{8}$$

From equation (5), it can be seen that if θ ever goes to zero, then it stays there forever. Therefore the option to adopt has no value when $\theta = 0$, and must satisfy the boundary condition $F_0 = 0$. Solution of the differential equation subject to this boundary condition gives $F_0 = b_F\theta^\beta$, where b_F is a positive constant and $\beta > 1$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$; i.e., $\beta = \frac{1}{2}\left(1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right)$.

Now consider the value of the firm in the 'stopping' region, in which the value of θ is such that it is optimal to adopt at once. Since adoption is irreversible, the value of the firm in the stopping region is given by the expected value alone with no option value terms: when the level at time t of the state variable is θ_t, this is

$$F_1(\theta_t) = \mathbb{E}_t\left[\int_t^\infty \exp(-r(\tau - t))(1 + \delta_F)\theta_\tau d\tau - K\right].$$

θ is expected to grow at rate μ, so that

$$F_1(\theta) = \frac{(1 + \delta_F)\theta}{r - \mu} - K. \tag{9}$$

The boundary between the continuation region and the stopping region is given by a trigger point θ_F of the stochastic process such that continued delay is optimal for $\theta < \theta_F$ and immediate adoption is optimal for $\theta \geq \theta_F$. The optimal stopping time T_F is then defined as the first time that the stochastic process θ hits the interval $[\theta_F, \infty)$ from below.

Putting together the two regions gives the follower's value function:

$$F(\theta) = \begin{cases} b_F \theta^{\beta} & \theta < \theta_F, \\ \frac{(1+\delta_F)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \quad (10)$$

given that the leader adopts at $\theta_L < \theta_F$.

By arbitrage, the critical value θ_F must satisfy a value-matching condition; optimality requires a second condition, known as 'smooth-pasting', to be satisfied. (See Dixit and Pindyck (1994) for an explanation.) This condition requires the two components of the follower's value function to meet smoothly at θ_F with equal first derivatives, which together with the value matching condition implies that

$$\theta_F = \frac{\beta}{\beta-1}\left(\frac{K}{1+\delta_F}\right)(r-\mu), \quad (11)$$

$$b_F = \frac{(1+\delta_F)\theta_F^{-(\beta-1)}}{\beta(r-\mu)}. \quad (12)$$

Equation (11) for the follower's trigger point can be interpreted as the effective flow cost of adoption with an adjustment for uncertainty. The sunk adoption cost is K, but this yields a flow payoff of $(1+\delta_F)\theta$; hence the effective sunk cost is $\frac{K}{1+\delta_F}$. With an effective interest rate of $r-\mu$ (i.e., the actual interest rate r minus the expected proportional growth in the flow payoff μ), this gives an instantaneous cost of $\left(\frac{K}{1+\delta_F}\right)(r-\mu)$. If a Marshallian rule were used for the adoption decision, the trigger point would be simply this cost. But with uncertainty, irreversibility and the option to delay adoption, the Marshallian trigger point must be adjusted upwards by the factor $\frac{\beta}{\beta-1} > 1$.

There are three components to the leader's value function holding over different ranges of θ. The first L_0 describes the value of adoption before the leader (and so the follower) has adopted; the second L_1 after the leader has adopted, but before the follower has done so; and the third L_2, after the follower has adopted. The first and third components are equivalent to those of the follower, determined previously. The second component is new, and so is derived first.

After the leader has adopted, it has no further decision to take and its payoff is given by the expected value of its adoption. This payoff is affected, however, by the action of the follower adopting later at θ_F. Taking account of subsequent adoption by the follower, the leader's post-adoption payoff is given by

$$L_1(\theta_t) = \mathbb{E}_t \left[\int_t^{T_F} \exp\left(-r(\tau - t)\right)\theta_\tau d\tau + \int_{T_F}^{\infty} \exp\left(-r(\tau - t)\right)(1+\delta_L)\theta_\tau d\tau - K \right]. \quad (13)$$

The Bellman equation for the leader is

$$L_1 = \theta dt + \exp\left(-rdt\right)\mathbb{E}_t \left[L_1 + dL_1\right]. \quad (14)$$

Using Itô's lemma and equation (5) gives

$$\frac{1}{2}\sigma^2\theta^2 L_1''(\theta) + \mu\theta L_1'(\theta) - rL_1(\theta) + \theta = 0. \quad (15)$$

As before, adoption has no value when $\theta = 0$, and so $L_1 = \frac{\theta}{r-\mu} + b_{L1}\theta^\beta$, where b_{L1} is a constant. The first part of the value function L_1 gives the expected value of adoption before the follower adopts, while the second is an option-like term reflecting the value (due to the network benefit α) to the leader of future adoption by the follower.

The other components of the leader's value function follow immediately from the calculations of the previous section:

$$L(\theta) = \begin{cases} b_{L0}\theta^\beta & \theta < \theta_L, \\ \frac{\theta}{r-\mu} + b_{L1}\theta^\beta - K & \theta \in [\theta_L, \theta_F), \\ \frac{(1+\delta_L)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \quad (16)$$

given the leader's trigger point θ_L and adoption by the follower at the higher θ_F.

The value of the unknown constant b_{L1} is found by considering the impact of the follower's adoption on the payoff to the leader. When θ_F is first reached, the follower adopts and the leader's expected flow payoff is altered. Since value functions are forward-looking, L_1 anticipates the effect of the follower's action and must therefore meet L_2 at θ_F. Hence, a value-matching condition holds at this point (for further explanation see Harrison (1985)); however, there is no optimality on the part of the leader, and so no corresponding smooth-pasting condition. This implies that

$$b_{L1} = \frac{\delta_L}{r-\mu}\theta_F^{-(\beta-1)}. \tag{17}$$

The usual value matching and smooth pasting conditions at the optimally-chosen θ_L determine the other unknown variables:

$$\theta_L = \frac{\beta}{\beta-1}K(r-\mu), \tag{18}$$

$$b_{L0} = \frac{\theta_L^{-(\beta-1)}}{\beta(r-\mu)} + b_{L1}. \tag{19}$$

Assumption 1 (specifically $\delta_F \leq 0$) ensures that $\theta_L \leq \theta_F$, so that the leader does indeed adopt before the follower. If $\delta_F > 0$, then adoption would occur as a cascade: the 'leader' would adopt at θ_L, and the 'follower' would adopt immediately afterwards.

So, in the model without preemption when equilibrium adoption is sequential, the leader adopts at $\theta_L = \frac{\beta}{\beta-1}K(r-\mu)$ and the follower at $\theta_F = \frac{\beta}{\beta-1}\left(\frac{K}{1+\delta_F}\right)(r-\mu)$. Uncertainty and network effects have very simple effects. (Of course, there is no preemption incentive, since the roles are preassigned.) Uncertainty leads to delay (higher trigger points for both firms), since $\frac{\beta}{\beta-1}$ is increasing in σ. Network effects decrease the cost of being the follower, since δ_F is decreasing in α, but have no effect on the leader's adoption point. These findings are discussed further in section 5.

Now consider the alternative case, in which adoption is simultaneous at the trigger point θ_S. The previous analysis indicates that the value function of each firm is then

$$S(\theta) = \begin{cases} b_S\theta^\beta & \theta < \theta_S, \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_S. \end{cases} \tag{20}$$

(This value function can be derived from the appropriate Bellman equation, following the steps shown above.) There is a continuum of simultaneous solutions; it is straightforward to show that they can be Pareto ranked, with higher trigger points yielding higher value functions. In this case, it seems reasonable that the firms will adopt at the Pareto optimal point, given by both value matching and smooth pasting. So

$$\theta_S = \frac{\beta}{\beta-1}\left(\frac{K}{1+\delta_S}\right)(r-\mu), \tag{21}$$

$$b_S = \frac{(1+\delta_S)\theta_S^{-(\beta-1)}}{\beta(r-\mu)}. \tag{22}$$

Note that $\theta_L \leq \theta_S \leq \theta_F$: when adoption is sequential, the leader adopts earlier and the follower later than when adoption is simultaneous. This is quite reasonable: when adoption is viewed within the location entry story, this simply says that aggressive entry by the leader ($\theta_L \leq \theta_S$) deters entry by the follower ($\theta_S \leq \theta_F$).

The following lemma describes when simultaneous adoption is an equilibrium.

LEMMA 1: *The necessary and sufficient condition for simultaneous adoption to occur in equilibrium in the model without preemption is*

$$(1+\delta_S)^\beta \geq 1 + \beta\delta_L(1+\delta_F)^{\beta-1}. \tag{23}$$

A necessary condition is $\delta_S \geq \delta_L$.

PROOF: For equilibrium simultaneous adoption, it must be that $S(\theta) \geq L(\theta)$ for $\theta \in [\theta_L, \theta_S]$. Due to the convexity of the value functions, this requires that $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_L]$, and so that $b_S \geq b_{L0}$. Therefore from equations (19) and (22), the necessary and sufficient condition is

$$\theta_S^{-\beta}\left(\frac{(1+\delta_S)\theta_S}{r-\mu} - K\right) \geq \frac{\theta_L^{-(\beta-1)}}{\beta(r-\mu)} + \frac{\delta_L}{r-\mu}\theta_F^{-(\beta-1)}.$$

When the expressions for θ_S and θ_L are substituted, this reduces to equation (23). The necessary condition follows immediately from rearrangement of equation (23). □

Whether simultaneous adoption occurs in equilibrium is determined by whether the leader wishes to adopt before the follower, or at the same time (i.e., by the comparison of

$S(\theta)$ and $L(\theta)$). The lemma shows the reasonable condition that, in order for simultaneous adoption to occur in equilibrium, it must be the case that δ_S is sufficiently large and/or δ_L and δ_F sufficiently small. In terms of the location entry story in section 2.1, the first firm to adopt wishes to be accommodating whenever the benefits from the other firm adopting the technology are sufficiently large relative to the first mover advantage. Note that the simultaneous adoption equilibrium, when it exists, Pareto dominates the sequential outcome; this is an immediate consequence of the condition for existence of the simultaneous adoption equilibrium: $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_S]$.

3.2. With Preemption

Instead of preassigning roles to the two firms, suppose that the leader is determined endogenously. This supplements the option and network effects with a preemption incentive (c.f. Fudenberg and Tirole (1985) and Weeds (1999)). The applied motivation is that this set-up reflects firms' concerns with network technologies: they want to wait until the market is developed, but are concerned that they will be at some disadvantage at that stage relative to firms that have adopted earlier. Nevertheless, firms are symmetric before moving—no firm has an intrinsic (dis-)advantage from the start. Analytically, this allows the preemption incentive to be studied.

As before, start by supposing that one firm (the preemptor) adopts strictly before the other. The follower's value function and trigger point is the same as for the model without preemption; so

$$\theta_F = \frac{\beta}{\beta - 1} \left(\frac{K}{1 + \delta_F} \right) (r - \mu).$$

The preemptor's value function is as described in the previous section. As before, value matching at θ_F determines the unknown variable $b_{L1} = \frac{\delta_L}{r-\mu} \theta_F^{-(\beta-1)}$. But the preemptor can no longer choose its adoption point optimally, as it could when roles were preassigned. Instead, the first firm to adopt does so at the point at which it prefers to lead rather than follow, not the point at which the benefits from leading are largest. Clearly, it cannot be that the first firm adopts when the value from following is greater than the value from leading—if this were the case, the firm would do better by waiting. Likewise, it cannot be that the first firm adopts when the value from leading is strictly greater than the value from adopting, since in this case without preassigned roles, the other firm could preempt it and still gain. Hence the adoption point is defined by indifference between leading and following. Whereas in the model without preemption, θ_L was determined by value matching and smooth pasting, the trigger point θ_P in the preemption model is given by indifference: $L(\theta_P) = F(\theta_P)$.

The first step is to show that there is such a trigger point.

LEMMA 2: *There exists a unique $\theta_P < \theta_L$ such that $L(\theta_P) = F(\theta_P)$ and $L(\theta) < F(\theta)$ for $\theta < \theta_P$, $L(\theta) > F(\theta)$ for $\theta > \theta_P$.*

PROOF: See the appendix.

The indifference relation $L(\theta_P) = F(\theta_P)$ gives the following non-linear equation for θ_P

$$\frac{\theta_P}{r-\mu} - K - \frac{K}{\beta - 1}\left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\left(\frac{\theta_P}{\theta_F}\right)^{\beta} = 0. \tag{24}$$

The solution for simultaneous adoption in the preemption model is the same as in the model without preemption: the trigger point is the same, θ_S, and the necessary and sufficient condition for simultaneous adoption to occur in equilibrium is given by equation (23). The conditions of simultaneous adoption are unaltered because the value (function) from being the first to adopt is the same regardless of whether the roles are preassigned or determined endogenously. This feature of the model arises because the follower's trigger point θ_F is independent of trigger point of the other firm.

4. Co-operative Solution

This section analyses the co-operative solution, in which the firms' adoption trigger points are chosen to maximise the sum of their two value functions. The objective is to provide a benchmark to identify inefficiencies in the next section. Notice that there is only one co-operative solution—the previous distinction between preassigned and endogenous leader/follower roles is not relevant.

Consider first the co-operative solution when adoption is sequential. Two trigger points, $\theta_1 < \theta_2$, are chosen to maximise the sum of the leader's and follower's value functions. Call the co-operative value function in this case C_{L+F}; using the same steps as before,

$$C_{L+F}(\theta) = \begin{cases} b_0\theta^{\beta} + b_1\theta^{\beta} & \theta < \theta_1, \\ \frac{\theta}{r-\mu} + b_2\theta^{\beta} - K + b_3\theta^{\beta} & \theta \in [\theta_1, \theta_2), \\ \frac{(2+\delta_L+\delta_F)\theta}{r-\mu} - 2K & \theta \geq \theta_2, \end{cases} \tag{25}$$

where $b_i,\ i = 0, 1, 2, 3$ are constants. The co-operative trigger points are determined by value matching and smooth pasting conditions at both points. Therefore

$$\theta_1 = \left(\frac{\beta}{\beta-1}\right) K(r-\mu) = \theta_L, \tag{26}$$

$$\theta_2 = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_L+\delta_F}\right)(r-\mu). \tag{27}$$

Assumption 1 ensures that $\theta_2 > \theta_1$, since $\delta_L \leq -\delta_F$.

Now consider the co-operative solution with simultaneous adoption at the trigger point θ_3. The co-operative value function in this case is

$$C_S(\theta) = \begin{cases} b_4\theta^{\beta} & \theta < \theta_3, \\ \frac{2(1+\delta_S)\theta}{r-\mu} - 2K & \theta \geq \theta_3. \end{cases} \tag{28}$$

Again, value matching and smooth pasting determine θ_3:

$$\theta_3 = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_S}\right)(r-\mu) = \theta_S. \tag{29}$$

A similar analysis to those undertaken with the non-co-operative equilibria shows when co-operation will involve simultaneous adoption.

LEMMA 3: *The necessary and sufficient condition for simultaneous adoption to be a co-operative solution is*

$$2(1+\delta_S)^{\beta} \geq 1 + (1+\delta_L+\delta_F)^{\beta}. \tag{30}$$

A necessary condition is $\delta_S \geq \frac{\delta_L+\delta_F}{2}$.

PROOF: The necessary and sufficient condition is that the value function for simultaneous adoption $C_S(\theta) \geq C_{L+F}(\theta)$, for all $\theta \in [\theta_1, \theta_3]$. The strict convexity of the value functions means, however, that this requires that $C_S(\theta) > C_{L+F}(\theta)$ for all $\theta \in [0, \theta_1]$ i.e., $b_4 \geq b_0 + b_1$. From above,

$$\begin{aligned} b_0 + b_1 &= \left(\frac{1+(1+\delta_L+\delta_F)^{\beta}}{\beta-1}\right)\left(\left(\frac{\beta-1}{\beta}\right)\frac{1}{K(r-\mu)}\right)^{\beta} K, \\ b_4 &= \left(\frac{2}{\beta-1}\right)\left(\left(\frac{\beta-1}{\beta}\right)\frac{1+\delta_S}{K(r-\mu)}\right)^{\beta} K. \end{aligned}$$

It is immediate that $b_4 \geq b_0 + b_1$ iff condition (30) holds, and that a necessary condition is $\delta_S \geq \frac{\delta_L+\delta_F}{2}$. □

The condition (30) is very similar to condition (23)—accommodation occurs when network effects are sufficiently large.

5. INEFFICIENCIES AND COMPARATIVE STATICS

The previous two sections have established the conditions under which adoption is sequential or simultaneous, and the trigger points for adoption, for equilibrium with and without preemption, and for the co-operative solution. This section first analyses inefficiencies that arise in the non-co-operative equilibria;[14] and then assesses how the conditions for simultaneous adoption and the trigger points vary as the parameters of the model change.

[14]Note that the inefficiencies analysed are relative to the co-operative benchmark. The analysis does not consider the consumer welfare aspects of the problem.

5.1. Inefficiencies

The next two propositions compare equilibrium outcomes with the co-operative solution, identifying two types of inefficiency. First, when both non-co-operative equilibrium and the co-operative solution involve sequential adoption, the non-co-operative trigger points differ from the co-operative points (proposition 1). Secondly, the non-co-operative equilibria and the co-operative solution involve different amounts of simultaneous adoption (proposition 2).

PROPOSITION 1: $\theta_P < \theta_L = \theta_1$ *and* $\theta_F > (<)\ \theta_2$ *when* $\delta_L > (<)\ 0$. *In words, conditional on both equilibrium and the co-operative solution involving sequential adoption, the non-co-operative leader adopts at the co-operative point when there is no preemption, and too early if there is preemption. The non-co-operative follower adopts too late (early) when* δ_L *is greater (less) than zero.*

PROOF: All comparisons are immediate from equations (11), (18), (27) and (26) and lemma 2. □

PROPOSITION 2: *Compared to the co-operative solution, there is insufficient simultaneous adoption in equilibrium.*

PROOF: See the appendix.

The backward and forward externalities emphasised by Choi (1994) are in this paper also, but they take a particular form. A backward externality arises when adoption is sequential. In the model without preemption, the leader adopts at the correct (i.e., co-operative) point, but the follower adopts at the wrong point. For the leader, the network effect causes a constant proportional change (an increase if $\delta_L > 0$, a decrease otherwise) to its value function; this change has no marginal effect on the leader, and so its trigger point is unaffected. (In terms of the calculation, any term in δ_L or θ_F drops out of the value matching and smooth pasting conditions that determine θ_L.) The follower does not consider the effect on the leader of its adoption, and consequently adopts either too soon (when $\delta_L < 0$) or too late (when $\delta_L > 0$). A forward externality arises through inefficient simultaneous adoption. In equilibrium, whether adoption is sequential or simultaneous is determined by the leader's incentive to adopt. Proposition 2 shows that the leader wishes to adopt before the follower too often, compared to the co-operative solution. In short: the leader is too aggressive in equilibrium; and the follower adopts inefficiently when it has been preempted.

5.2. Comparative Statics I: Factors in Isolation

Preemption causes the leader to adopt earlier, but does not alter the follower's adoption behaviour. This can be seen by comparing the trigger points of the non-co-operative equilibria in the models with and without preemption when adoption is sequential. Without preemption, the leader adopts at the point $\left(\frac{\beta}{\beta-1}\right) K(r-\mu)$; with preemption, the trigger point θ_P is strictly less, from proposition 1. The follower in both cases adopts at the trigger point $\left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_F}\right)(r-\mu)$.

When adoption is sequential, network effects in isolation affect the follower's equilibrium θ_F and co-operative θ_2 trigger points, but not the leader's.

Proposition 3: *θ_F and θ_2 are decreasing in α. If $\delta_L < (>)0$, then the gap between θ_F and θ_2 decreases (increases) as α increases.*

Proof: The proposition follows from the facts that $\partial\theta_F/\partial\alpha \leq 0$, $\partial\theta_2/\partial\alpha \leq 0$ and $\partial/\partial\alpha\,(\theta_F/\theta_2) > 0$. □

Differentiation of the conditions (23) and (30) with respect to (wrt) α shows that simultaneous adoption is more likely to occur in equilibrium and the co-operative solution when network effects are larger.[15] In addition, network effects impact the relative prevalence of

[15] To be precise: if the conditions hold for a value $\hat{\alpha}$, then they also hold for any $\alpha > \hat{\alpha}$.

simultaneous adoption in the co-operative solution compared with equilibrium. The next proposition shows that network effects exacerbate the forward externality.

PROPOSITION 4: *The insufficiency of equilibrium simultaneous adoption increases with the size of network effects.*

PROOF: See the appendix.

Although greater network effects cause the leader to act less aggressively, nevertheless it is still more aggressive than is required by the co-operative solution. This is a result of the forward externality.

Turning now to the volatility parameter σ, greater uncertainty usually leads to adoption delay (in the sense of higher trigger points). The intuition is that delay allows for the possibility that the random process (5) will go up; if it goes down, then the firm need not adopt. The greater the variance of the process, the more valuable is the option created by this asymmetric situation, and so the more delay occurs. All but one of the trigger points in the paper have this general feature: $\theta_L, \theta_F, \theta_1, \theta_2$ and θ_3 are all increasing in σ. The exception, θ_P, is analysed below.

In addition, greater uncertainty increases the occurrence of simultaneous adoption, in both equilibria and the co-operative solution. Two observations are relevant. First, the necessary and sufficient conditions for simultaneous adoption, given in equations (23) and 30, are both easier to satisfy when σ is large. Secondly, simultaneous adoption becomes more attractive to the leader as uncertainty increases (given that both simultaneous and sequential adoption are possible). Once the leader has adopted at θ_L, θ_P or $\theta_1 = \theta_L$, it must wait until $\theta_2 = \theta_F$ before the follower adopts. When network effects are large relative to the preemption incentive (i.e., δ_S is large relative to δ_L, as is necessary for simultaneous adoption to occur in equilibrium), this delay is costly to the leader. As uncertainty increases, the follower's adoption is delayed further (θ_2 increases), and the cost to the leader of lost network effects rises. Therefore simultaneous adoption becomes more attractive.[16]

[16]Unfortunately, there is no counterpart to proposition 4 with respect to uncertainty: the extent of

There are two interactions between the network and preemption effects. First, proposition 5 shows that (conditional on sequential adoption) the preemptor's trigger point θ_P, which is below the co-operative level due to preemption, is decreasing in the size of the network effect measured by the parameter α.

PROPOSITION 5: *θ_P is decreasing in α.*

PROOF: See the appendix.

Proposition 5 shows that the preemptor adopts earlier when network effects are larger. This result is not immediately obvious: at its trigger point, the preemptor is indifferent between leading and following; but both the leader's and follower's returns increase as α increases; and so it must be shown that the increase in the leader's return is the stronger effect. To gain an intuition for the result, rewrite the preemptor's indifference condition as $L(\theta_P;\alpha) - F(\theta_P;\alpha) = 0$. Then

$$\frac{\partial \theta_P}{\partial \alpha} = -\left(\frac{\frac{\partial L}{\partial \alpha} - \frac{\partial F}{\partial \alpha}}{\frac{\partial L}{\partial \theta_P} - \frac{\partial F}{\partial \theta_P}} \right).$$

Since $L < (>)F$ for $\theta < (>)\theta_P$, it must be that $\frac{\partial L}{\partial \theta_P} > \frac{\partial F}{\partial \theta_P}$. Hence the sign of $\frac{\partial \theta_P}{\partial \alpha}$ is determined by whether $\frac{\partial L}{\partial \alpha}$ is greater or less than $\frac{\partial F}{\partial \alpha}$. (The partial derivatives here hold θ_P constant, but allow θ_F to vary.)

There are two effects as α increases. First, the firms' flow returns once both firms have adopted increase. Secondly, the follower adopts earlier (θ_F decreases); when $\delta_L > 0$, this benefits both firms, while when $\delta_L < 0$, it benefits the follower but not the leader. For the leader, both effects are important. For the follower, only the first effect is of first-order significance: since the follower chooses θ_F optimally, any variation in the trigger point due to a small change in α induces only a second-order variation in returns. When

insufficient simultaneous adoption in the equilibrium without preemption is a non-monotonic function of the uncertainty parameter σ.

$\delta_L > 0$, this suggests qualitatively that the leader's return increases more when α rises. The comparison is more complicated when $\delta_L < 0$; note, however, that the second effect for the leader is limited by assumption 1, which requires that $\delta_L > -\frac{1}{\beta+1}$. In fact, proposition 5 shows that the first effect dominates for the leader, and to such an extent that the leader's value function increases with α by more than does the follower's.

Secondly, network effects and preemption interact in the way in which uncertainty affects the preemptor's trigger point θ_P, raising the possibility that an increase in uncertainty lowers the trigger point.

PROPOSITION 6: *Sufficient conditions for* $\frac{\partial \theta_P}{\partial \sigma} > 0$ *are: either*

1. $1 - \beta\delta_L + \delta_F > 0$ *and* $\phi_1 \vee \phi_2 < \epsilon$*; or*
2. $1 - \beta\delta_L + \delta_F < 0$ *and* $\phi_1 \wedge \phi_2 > \epsilon$*.*

Sufficient conditions for $\frac{\partial \theta_P}{\partial \sigma} < 0$ *are: either*

1. $1 - \beta\delta_L + \delta_F > 0$ *and* $\phi_1 \wedge \phi_2 > \epsilon$*; or*
2. $1 - \beta\delta_L + \delta_F < 0$ *and* $\phi_1 \vee \phi_2 < \epsilon$*.*

$\phi_1 \equiv -\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right)$, $\phi_2 \equiv \frac{\theta_P}{\theta_F}$, *and* ϵ *is the solution to the equation* $\epsilon + \ln \epsilon = 0$ *(i.e.,* $\epsilon \approx 0.57$*).*

PROOF: See the appendix.

The result therefore raises the unusual possibility that greater uncertainty lowers the preemptor's trigger point. This is counter to the usual comparative static. The difference arises from the lack of optimality in the choice of the preemption trigger point. An optimal trigger point is such that the marginal benefit from delaying adoption for a period equals the marginal cost. The marginal benefit is the interest saved on the adoption cost plus the expected gain from the possibility that the flow payoff increases. The marginal cost is the flow payoff foregone by not adopting the technology. In this marginal calculation, the firm does not consider the effect of its delay on the adoption decision of the other firm, since in the models considered in this paper, each firm's trigger point (with the exception of θ_P) does not depend on the other's. Increased uncertainty raises the expected gain from delay, causing the (optimally chosen) trigger point to increase. This reasoning does not apply in the case of θ_P, however: it is not chosen according to a marginal equality, but an absolute equality between the value from leading and the value from following. The proposition shows that this difference in the determination of the trigger point can lead to θ_P decreasing as uncertainty increases.

There are two cases in which this can occur. In the first, $1 - \beta\delta_L + \delta_F > 0$ and $\phi_1 \wedge \phi_2 > \epsilon$. This sufficient condition requires that $\delta_L < 0$ and that $\beta < \frac{1}{\epsilon}$. In the second case, the sufficient conditions $1 - \beta\delta_L + \delta_F < 0$ and $\phi_1 \vee \phi_2 < \epsilon$ require that $\beta < \frac{1+\delta_F}{\delta_L} + \frac{1}{\epsilon} < \frac{1}{\epsilon}$ if $\delta_L < 0$, and $\beta > \frac{1+\delta_F}{\delta_L} + \frac{1}{\epsilon} > \frac{1}{\epsilon}$ if $\delta_L > 0$. In summary, therefore, there are two situations that are conducive to a rise in uncertainty increasing the preemptor's trigger point (i.e., must obtain for the sufficient conditions to hold): either network effects are sufficiently small and uncertainty is sufficiently large; or the converse.

In order for this unusual comparative static to hold, it must be that the leader's value function increases by more than the follower's when uncertainty rises, holding constant the preemptor's trigger point θ_P. (This statement follows directly from using the implicit function theorem on the non-linear equation (24) defining θ_P.) The leader's value function depends on uncertainty due to the option-like term that anticipates adoption by the follower: $b_{L1}\theta^\beta$, where $b_{L1} \equiv \frac{\delta_L \theta_F^{-(\beta-1)}}{r-\mu}$. Hence this option-like term is positive if δ_L is positive, and negative otherwise. As σ increases, two factors are important. First, holding the trigger point of the follower constant, there is a change in the value of the option-like term: if $\delta_L < 0$, it becomes more negative; if $\delta_L > 0$, it becomes more positive. Secondly, the follower's trigger point θ_F increases. When $\delta_L < 0$, this increases the option-like term (makes it less negative), since adoption by the follower reduces the preemptor's instantaneous return; when $\delta_L > 0$, the option-like term decreases. Hence the first, direct effect always works in the opposite direction to the second, indirect effect. It is straightforward to show that the direct effect dominates the indirect effect at low levels of uncertainty, but at higher levels of σ, the indirect effect dominates. The follower's value function increases with uncertainty since the firm holds a standard (call) option relating to its future irreversible adoption. The same two factors are relevant, but the effect of a change in the follower's trigger point is of second-order, by the envelope theorem, so that only the direct effect is of first-order significance.

The favourable conditions identified above ($\delta_L < 0$ and σ large, or $\delta_L > 0$ and σ small) arise immediately from these observations. When $\delta_L < 0$, the direct effect is negative and the indirect effect positive. When σ is small, the former dominates; and so it is only when uncertainty is large that the leader's value function is increasing in σ. In the other case ($\delta_L > 0$), the direct effect is positive and the indirect effect negative. When σ is large, the latter dominates; and so it is only when uncertainty is small that the leader's value function is increasing in σ.

The case $\delta_L > 0$ and σ small is of particular interest, since it implies that for sufficiently large network effects (such that $\delta_L > 0$), the introduction of a small amount of uncertainty into the model increases all trigger points except the preemptor's, which decreases. More precisely, large network effects mean that $\frac{\partial \theta_P}{\partial \sigma}\big|_{\sigma=0} < 0$.

6. Conclusions

This paper has analysed adoption of a technology when adoption is irreversible, the returns are uncertain, when there is an advantage to being the first adopter, but a network advantage to adopting when others also adopt. It departs from previous work in this area by the combination of factors analysed. The combination has proved interesting: the interaction of preemption with both network effects and uncertainty generates novel results.

In future work, we hope to examine other types of uncertainty and so make the network effects endogenous. Currently, uncertainty is completely exogenous: the stand-alone benefit follows a random process whose parameters are common knowledge. In another setting, it may be that there is uncertainty about the drift of the process (i.e. growth rate of the industry). Beliefs about the unknown parameter are then important and would be updated as industry profitability is observed. If that were the end of the story, there would be little change from the current analysis: beliefs about the drift could be the state variable, rather than the level of stand-alone benefit, and much the same analysis would go through. (Of course, the solutions for value functions and trigger points would be different.) More interesting questions arise when the act of adoption by one of the firms reveals information about the unknown parameter. The positive network effect could then be seen as an informational externality. This possibility of preemption with strategic experimentation (see e.g. Bergemann and Välimäki (1996) and Bolton and Harris (1999)) raises interesting questions for further research.

APPENDIX

A.1. The Sequential Location Model

This section derives the conditions for the assumption made in section 2.1 that when entry is sequential, the leader locates at $x = \frac{1}{2}$ and the follower locates at the end. Suppose that the leader locates at the end, $x = 0$ say. Then before the follower enters, a straightforward calculation shows that the leader maximises profit by setting a price $p_0 = \frac{2V}{3}$, earning a profit of $\pi_0 = \frac{2V}{3}\sqrt{\frac{V}{3l}}\theta_t$. Note two things. First, this solution holds while $V < 3l$; the modification if $V \geq 3l$ is straightforward and so omitted. Secondly, the mass of consumers is taken to be θ_t i.e., the same as when the leader locates at $x = \frac{1}{2}$. The calculations can easily be revised to alter this aspect; little would change. Normalising by π_1^I, this means that

$$\pi_0 = (1 + \delta_0)\theta_t \equiv \left(\frac{8V\sqrt{\frac{V}{3l}}}{3(4V - l)} \right) \theta_t.$$

If $\frac{V}{l} \in [0.3196, 6.2180]$, then $\delta_0 < 0$. This assumption is maintained in the paper.

Therefore the value function of the leader when it locates at $x = 0$ and adoption is sequential is

$$L(\theta) = \begin{cases} b_{L0}^0 \theta^\beta & \theta < \theta_0, \\ \frac{(1+\delta_0)\theta}{r-\mu} + b_{L1}^0 \theta^\beta - K & \theta \in [\theta_0, \theta_F^0), \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_F^0, \end{cases}$$

given the leader's trigger point θ_0 and adoption by the follower at the later point θ_F^0. The value function of the follower in this case is

$$F(\theta) = \begin{cases} b_F^0 \theta^\beta & \theta < \theta_F^0, \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_F^0. \end{cases}$$

Using the techniques of section 3.1, the trigger points when the firms' roles are preassigned are

$$\begin{aligned} \theta_0 &= \frac{\beta}{\beta - 1} \left(\frac{K}{1 + \delta_0} \right) (r - \mu) > \theta_L, \\ \theta_F^0 &= \theta_S < \theta_F. \end{aligned}$$

It is clear that the leader will prefer to locate at $x = \frac{1}{2}$ (and hence adopt at θ_L) rather than at $x = 0$ (θ_0) iff $b_{L0} \geq b_{L0}^0$ i.e., iff $\delta_0 \leq \overline{\delta}_0$, where

$$(1+\overline{\delta}_0)^\beta - \beta\overline{\delta}_0(1+\delta_S)^{\beta-1} = 1 + \beta\delta_L(1+\delta_F)^\beta - \beta\delta_S(1+\delta_S)^{\beta-1}.$$

Given that the preemptor's value function is strictly below the preassigned leader's, $\delta_0 \leq \overline{\delta}_0$ is also sufficient to ensure that the preemptor will prefer to locate at $x = \frac{1}{2}$.

A.2. Proof of Lemma 2

Define

$$\Delta(\theta) \equiv \frac{\theta}{r-\mu} - K - \left(\frac{\theta}{\theta_F}\right)^\beta \left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\frac{K}{\beta-1}$$

i.e., $L(\theta) - F(\theta)$, where $L(\theta)$ is conditional on the preemptor having invested, and $F(\theta)$ is conditional on the preemptor having invested but not the follower. Then the following facts can be shown by straightforward manipulations, using assumption 1 throughout: (i) $\Delta(0) = -K < 0$; (ii) $\Delta(\theta_L) = \frac{K}{(\beta-1)(1+\delta_F)}\left(\left(\frac{\theta_L}{\theta_F}\right)^\beta \beta\delta_L + (1-\left(\frac{\theta_L}{\theta_F}\right)^\beta)(1+\delta_F)\right) > 0$; (iii) $\Delta(\theta_F) = \frac{\beta(\delta_L-\delta_F)K}{(\beta-1)(1+\delta_F)} > 0$; (iv) $\Delta'(0) = \frac{1}{r-\mu} > 0$; (v) $\Delta'(\theta_F) = \frac{\beta\delta_L-\delta_F}{r-\mu} > 0$; (vi) hence there exists a $\tilde{\theta} < \theta_L$ such that $\Delta(\tilde{\theta}) = 0$; (vii) for any $\hat{\theta}$ such that $\Delta(\hat{\theta}) = 0$ and $\Delta'(\hat{\theta}) < 0$, then it must be that for another $\overline{\theta} > \hat{\theta}$ such that $\Delta(\overline{\theta}) = 0$, $\Delta'(\overline{\theta}) < 0$; (viii) since $\Delta(\theta_F) > 0$ (point (iii)), it must be therefore that there is no $\hat{\theta}$ such that $\Delta(\hat{\theta}) = 0$ and $\Delta'(\hat{\theta}) < 0$. Hence there is a unique $\tilde{\theta} = \theta_P < \theta_L$ such that $\Delta(\theta_P) = 0$, and $\Delta(\theta) \gtrless 0$ as $\theta \gtrless \theta_P$.

A.3. Proof of Proposition 2

The proof requires a comparison of the necessary and sufficient conditions (23) and (30). First note that if $\delta_L > \delta_S$, then simultaneous adoption cannot occur in the no-preemption equilibrium but may occur in the co-operative solution. Conditional on $\delta_L \leq \delta_S$, let $\Delta \equiv \left[1+2\beta\delta_L(1+\delta_F)^{\beta-1}\right] - \left[(1+\delta_L+\delta_F)^\beta\right]$. $\Delta < 0$ iff $\delta_F \geq \delta_F^*$ and $\delta_L < \delta_L^*$, where $\delta_F^* = (2\beta)^{-\frac{1}{\beta}} - 1$ and $\delta_L^* = (2\beta)^{-\frac{1}{\beta}}$. But from assumption 1, $\beta\delta_L > \delta_F$; and so if $\delta_F \geq \delta_F^*$, then $\delta_L \geq \delta_L^*$. Hence $\Delta \geq 0$. Consequently, the necessary and sufficient condition for simultaneous adoption is stricter in the equilibrium of the model without preemption than in the co-operative solution.

A.4. Proof of Proposition 4

The proof involves a comparison of the necessary and sufficient conditions (23) and (30) as α increases. Rewriting these conditions,

$$
\begin{aligned}
2(1+\delta_S)^\beta - 1 &\geq 1 + 2\beta\delta_L(1+\delta_F)^{\beta-1}, \\
2(1+\delta_S)^\beta - 1 &\geq (1+\delta_L+\delta_F)^\beta.
\end{aligned}
$$

Note that a necessary condition for the first inequality to hold is $\delta_L \leq 0$. Let the right hand side of the first equation be NP and the right hand side of the second equation be C. Then

$$
\begin{aligned}
\frac{\partial NP}{\partial \alpha} &= 2(1+\delta_F)^{\beta-1} + 2\beta\delta_L(1+\delta_F)^{\beta-1}, \\
\frac{\partial C}{\partial \alpha} &= (1+\delta_L+\delta_F)^\beta + (1+\delta_L+\delta_F)^{\beta-1}.
\end{aligned}
$$

The sign of $\left(\frac{\partial NP}{\partial \alpha} - \frac{\partial C}{\partial \alpha}\right)$ is determined by whether $\lambda_1 \equiv 2(1-\delta_F)^{\beta-1} + 2\beta\delta_L(1-\delta_F)^{\beta-1}$ is greater or less than $\lambda_2 \equiv (1+\delta_L+\delta_F)^\beta + (1+\delta_L+\delta_F)^{\beta-1}$. Note that (i) both λ_1 and λ_2 are increasing in δ_L, for any given value of δ_F; (ii) when $\delta_L = 0$, $\lambda_1 = 2(1+\delta_F)^{\beta-1}$ and $\lambda_2 = (1+\delta_F)^\beta + (1+\delta_F)^{\beta-1} < 2(1+\delta_F)^{\beta-1}$; (iii) $\lambda_1 > \lambda_2$ for all $\delta_L < 0$ and δ_F, for values of δ_L and δ_F that satisfy assumption 1. Therefore when $\delta_L < 0$, $\frac{\partial NP}{\partial \alpha} > \frac{\partial C}{\partial \alpha} > 0$.

A.5. Proof of Proposition 5

Rewrite equation (24) as

$$
-\psi\theta_P^\beta + \frac{\theta_P}{r-\mu} - K = 0, \tag{A1}
$$

$$
\psi \equiv \frac{K}{\beta-1}\left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\theta_F^{-\beta}. \tag{A2}
$$

Total differentiation gives

$$\frac{\partial \theta_P}{\partial \alpha} = \left(\frac{\theta_P^{\beta}}{\frac{1}{r-\mu} - \beta \psi \theta_P^{\beta-1}} \right) \frac{\partial \psi}{\partial \alpha}.$$

The denominator is positive from equation (A1). Hence Sign$\frac{\partial \theta_P}{\partial \alpha}$ = Sign$\frac{\partial \psi}{\partial \alpha}$. Differentiation gives

$$\frac{\partial \psi}{\partial \alpha} = -\left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\alpha}\right)\left(\frac{\beta\delta_L - \delta_F}{1+\delta_F}\right)\theta_F^{-\beta} < 0,$$

where the inequality follows from assumption 1.

A.6. Proof of Proposition 6

Differentiation of equation (24) gives

$$\frac{\partial \theta_P}{\partial \beta} = \frac{\psi \theta_P^{\beta}\left(-\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right) + \ln\left(\frac{\theta_P}{\theta_F}\right)\right)}{\frac{1}{r-\mu} - \beta\psi\theta_P^{\beta-1}}, \tag{A3}$$

where ψ was defined earlier. Hence Sign$\frac{\partial \theta_P}{\partial \sigma}$ = $-$Sign$\left[\psi\left(-\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right) + \ln\left(\frac{\theta_P}{\theta_F}\right)\right)\right]$. (This statement uses the fact that $\frac{\partial \beta}{\partial \sigma} < 0$.) Let $\phi_1 \equiv -\left(\frac{\delta_L}{1-\beta\delta_L+\delta_F}\right)$ and $\phi_2 \equiv \frac{\theta_P}{\theta_F}$. When $1-\beta\delta_L+\delta_F > 0$, $\psi > 0$. Hence Sign$\frac{\partial \theta_P}{\partial \sigma}$ = $-$Sign$[\phi_1 + \ln \phi_2]$. A sufficient condition for $\phi_1 + \ln \phi_2 > 0$ is that $\phi_1 \wedge \phi_2 > \epsilon$. Conversely, a sufficient condition for $\phi_1 + \ln \phi_2 < 0$ is that $\phi_1 \vee \phi_2 < \epsilon$. An equivalent argument holds when $1 - \beta\delta_L + \delta_F < 0$.

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