THE DUMMIES' GUIDE TO LOTTERY DESIGN

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NO: 572

WARWICK ECONOMIC RESEARCH PAPERS

THE UNIVERSITY OF WARWICK

DEPARTMENT OF ECONOMICS

The Dummies’ Guide to Lottery Design

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Wednesday, 4 October 2000

Abstract

This paper outlines the issues relevant to the operation of lottery games: it attempts to put some science into the art of lottery design. Our research suggests that lottery tickets sales depend positively on: the average return (i.e the proportion of revenue returned as prizes) because punters like better bets; the *skewness* in the prize distribution (e.g how much of the prize money goes to the jackpot) and we find that the more the better; and negatively on the *variance* in the prize distribution (which is a measure of the riskiness of the return – so the less the better). The sizes of these effects are important: our statistics suggest that the effect of the mean is small, as is the effect of the skewness, while the (negative) variance effect is quite important.

The work suggests that good causes revenue might be higher if: the game were meaner (less of the stakes used as prize money) because, although sales would fall a little, the good causes would getter a larger share of the smaller revenue; more of the prize money was used for the jackpot, or the variance in the expected prizes were reduced. BUT, in practice, it is difficult to change one aspect of the design of the game without having a counterveiling effect on another aspect. Thus, it is difficult to make judgements about the merits of alternative game designs without looking at ALL of the parameters being proposed.

However, the research suggests that there is no obvious case for not increasing the take-out (the revenue that is not returned as prizes) if the current game design is kept. If the game were changed to make the odds longer then our research suggests that other parameters of the design of the game may have to be changed to stop sales falling.

While we feel that there should be a lottery (because people enjoy playing and it does little harm), we also feel that lotteries are not good vehicles for taxation because they are a larger part of the spending of the poor than the rich. Moreover, we find no compelling empirical evidence to suggest that there is any merit in having much of the take-out dedicated to good-causes – hypothecation is bad for sound investment decision-making, the best causes are already the recipients of taxpayer largesse, and adding lottery funds to these causes simply displaces Treasury dollars. That is not to say that (some, perhaps most) of the good causes are deserving – rather that they should be funded in some other way.

Finally, the current “beauty contest” process of choosing an operator is fraught with risk (for the Commissioners but not the bidders) and we suggest that, if the aim is to raise good causes funds, then the license should be auctioned.

* This paper draws on Ian Walker’s research with colleagues at Keele University that was funded by ESRC under research grant R000236821, and on Juliet Young’s MSc dissertation at the University of Warwick. We exploit data that has been provided by the Lottoshop website, the National Lottery Commissioners, the Consumers’ Association, and the Family Expenditure Survey and the British Social Attitudes Survey which were provided by ESRC Data Archive at the University of Essex with the permission of the Office of National Statistics. We are grateful for the cooperation that we have received from these agencies. The views expressed here are those of the authors alone and do not represent the views of our employers.

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Introduction

1. The UK National Lottery has been in existence for almost 6 years[1]. It is operated by Camelot PLC, a consortium which was awarded the license in competition against other bidders, including a consortium bid headed by Richard Branson's company, Virgin. The license is due to expire in one year and both Virgin and Camelot have been bidding for this second license which gives the right to operate lottery games in the UK for a further seven years.

2. While both the current and the 1993/4 competitions between Virgin and Camelot have been the subject of some controversy, the intention of this paper is to step back from the controversy and to consider how the UK game **ought** to be designed, operated, taxed and regulated. Little attention has so far been given to the considerations raised here and yet they are central to both the objectives that government have set for the operator and for wider objectives such as the welfare of society as a whole. Unfortunately, there has also been little analytical research into how lottery games work, or how they should be operated and designed. This paper attempts to draw together the little analytical work that has been done and uses it to address the UK lottery market.

3. There are statistical issues concerned with how to structure the game to generate sales and this involves choosing the number of combinations of numbers that can be bought so as to make the game attractive to players. The statistical design affects how hard it is to win, and this affects how attractive the game is on a draw-by-draw basis, and so also affects how sales might be expected to behave in the long run. Our evidence suggests that the current Camelot on-line game may be "too easy" but it may be difficult to design a harder game that is not also "meaner".

4. There are economic issues concerned with the sizes of the prize pools for different winners. The bigger the overall prize pool the better the bet being offered and the more attractive the game will be. The market structure is also important: on-line parimutuel games exhibit economies of scale – bigger games are more "efficient" than small ones so regulating entry into the market is likely to be very important. Moreover, the stability of

[1] See R. Munting, *An economic and social history of gambling in Britain and the USA*, Manchester UP, 1994 for the history of UK lotteries.

sales is likely to be adversely affected by a competitive market structure. Of course, monopolistic supply may imply a need for effective regulation – and this will be true even if the licensee is operating on a not-for-profit basis. The license competition is currently organised as a "beauty contest" rather than as an auction and it is not obvious that the present arrangements are optimal.

5. Further economic considerations arise from the taxation of the games – the existing games are taxed at a rate that is thought to be approximately the rate levied on other forms of expenditure so as to be revenue neutral. But, the good causes levy is also a "tax" – albeit one which is "hypothecated" for particular forms of expenditure. Thus, the overall tax rate is around 40% at present. In general taxes on goods and services should be designed with two things in mind: to minimise the "distortions" to the choices that people make, and to impose a smaller burden on the poor than the rich. Our evidence is that lottery taxation fares badly on both these counts.

6. Game design can be fine tuned to exploit the psychological weaknesses of players so as to improve how the game operates. For example, the distribution of the overall prize pool between jackpot winners and lesser winners also affect the attractiveness of the game. For example, the ability for players to choose their own numbers, as opposed to being forced to buy tickets with randomly generated numbers is thought to be important for sales.

7. The distribution of play across individuals may also be amenable to manipulation through intelligent game design.

8. We expand on these issues below and summarise our views as to what the UK game *should* look like in the concluding section[2].

[2] There are three areas where we have little to say: First, technology affects both how games can be presented to players and the kind of game that it is possible to organise. Parimutuel games that allow players to choose their numbers require sophisticated computer systems. But new technology also offers the prospect of internet-based games and games operated via mobile phones using SMS or WAP. The technological possibility of international competition also imposes constraints on the domestic market as well as offering further market possibilities. Secondly, gambling can have adverse social consequences and intelligent game design can be used to minimise these. However, imposing constraints on game design because of a concern over adverse social consequences will generally has adverse consequences for sales so a trade-off may be involved. For example, it might be regarded as better to have a large number of small players than a small number of large ones.

How to Design a Lottery

Statistical Considerations in On-line Parimultuel Lottery Game Design

9. On-line[3] games usually feature players buying a ticket where they choose *n* numbers from a possible *N* available numbers. Such games are usually *parimutuel* in design – that is winners, whose tickets match the winning combination (or some part of it), receive a *share* of the prize pool with any other players who chose the same numbers. The chances of winning depend on *n* and *N* – the bigger is n the harder it is to match the winning numbers since you have to match more of them, and the bigger is *N* the more possible combinations there are to be matched.

10. The problem of designing such a game is one of choosing the right vales of *n* and *N* for the market circumstances. The bigger is *N* the less likely it is that anyone will hold the winning combination of the *n* numbers drawn – so if *n*=6 and *N*=49 then the probability of holding the winning combination is (approximately) 1 in 14 million, while if *n*=6 and *N*=53 then the chance of buying the winning combination is (approximately) 1 in 23 million.

11. Thus *n* and *N* affect the likely number of winners: with *n*=6 and *N*=49 and 60 million tickets sold then the number of jackpot winners to be expected is more than 4[4], but if *N*=53 then the expected number of winners is less than 3. These are just the numbers of winners that we would expect on average – there is a variance around these numbers and the implications of *N*=53 rather than 49 is an increase in the chance of there being NO winners, i.e the rollover probability. In the event of a rollover the jackpot is added to the jackpot of the next draw.

12. The dynamic behaviour of sales over time rests largely on the choice of *n* and *N*, which determine the probabilities of winning the different prizes and the likelihood of a rollover. If the game is easy to win, then rollovers are infrequent and double rollovers very

Finally, scratchcards are a part of the portfolio of the UK game and we have little to say about this since we do not have good data for them.

[3] In the lottery industry "on-line" means games where ticket sales are recorded electronically at a dedicated terminal.

[4] That is, it is 60m divided by 14m.

infrequent so each draw is much the same as the next. The danger of the game being too easy to win is that players become bored with the monotony of playing. Estimates in an earlier paper[5] suggested that the "half-life" of the UK game would have been approximately 150 draws – sales would halve every 3 years (of weekly draws) – if there had been no rollovers. What rollovers do is enhance the attractiveness of the **next** draw so that players are enticed to then play more, come back to the game, or join the game for the first time, and this effect takes some time to decay. Thus, rollovers are an essential ingredient of an attractive game to overcome it's essential monotony.

13. However, a game that is too hard to win will also be bad for sales in the long run. In the extreme case imagine a game that was almost impossible to win, it would rollover almost forever since sales would be very low and hence few of the available combinations on sales in each draw would be bought. But the size of the rollover would very slowly accumulate and hence so would sales. This is an example of "intertemporal substitution" - players sit on their hands waiting for the jackpot to grow sufficiently large for the draw to become attractive and only then play heavily. Even in less extreme cases, rollovers give rise to intertemporal substitution since rollover draws are more attractive that regular draws. While it is true that extra sales occur when there is a rollover, this is, in part, at the expense of sales in regular draws. Thus, designing the game to maximise sales is a balancing act of making it hard enough to win to overcome the tedium but easy enough to win to avoid significant intertemporal substitution[6].

14. It is important that the game design matches the likely size of the market. A game that is sensible for the UK is likely to be too hard for Israel whose population is just 10% of that of the UK[7].

[5] See L. Farrell, E. Morgenroth and I. Walker, "A Time Series Analysis of UK Lottery Sales: Long and Short Run Price Elasticities", **Oxford Bulletin of Economics and Statistics,** 61, 1999.

[6] The problem is made more complex where there are other substitution possibilities - for example, in the US it is possible that cross-state substitution takes place. This gives rise to incentives for neighbouring states to collude and share the proceeds of a single large game rather than have two competing games.

[7] In fact, the Israeli on-line lotto game has just been redesigned from 6/49 (1 in 14m) to 6/45 (1 in 8.1m) precisely because the operators felt that it was too difficult to win and rollovers were to frequent – it is being promoted as "Less numbers, bigger chances"). In contrast the game in Ireland (population 3.8m) has twice been redesigned to make it harder to win to induce more rollovers. Indeed, the redesigns followed organised attempts to "buy the pot" because large jackpots had

15. The prize pool is defined by the take-out rate, t, which is the proportion of sales (i.e the stakes) that is not returned as prizes. Thus, the overall prize pool is *(1-t)S*, where *S* is sales revenue (in many games the cost of a ticket is fixed at a unit of currency so *S* is both the number of tickets sold and the level of sales revenue). It is common for the take-out rate to be in the range 40-50% so that the pay-out rate is 60-50% [8].

16. Smaller prizes are usually awarded for matching fewer than n numbers so it is common for the prize fund to be split into separate pools. More complex designs are possible – for example, in the Camelot game there is a seventh "bonus" ball that is also used to define a prize pool for matching 5 of the first 6 numbers drawn plus the bonus number. Thus, the overall prize pool is usually divided into separate pools for funding players who match all n numbers in the winning combination, match *n-1*, *n-2*, etc. This set of prize pools might be characterised by $s = s_1, s_2, ..., s_n$. In the Camelot game $s_1 = s_2 = 0$ and s_3 is not a share at all but a fixed payout and these match-3 prizes are awarded first and the shares of the other prizes is defined out of the residual[9].

17. The odds of matching fewer than all n numbers also depends on *N*: thus the odds of matching 3 in a 6/49 design is 1 in 57, while the odds of matching 3 in 6/53 is 1 in 71[10]. Thus both n and N affect the number of prize winners for each prize pool, and it is the shares that affect the amount of money in each prize pool. Thus, the average amount won by each type of prize winner depends on all of the design parameters of the game.

accrued. Under the new design, a 6/42 game so that the odds of winning are 1 in 5.25m, there are more frequent but smaller jackpots. In California (population 34 m) the game began as 6/49, went to 5/53 and then to 6/51 (1 in 18 million) but, since June, has a complex 5/47+1/27 design that gives extremely long jackpot odds of 1 in 41.4m. In Florida (population 15 m) the game has also recently become more difficult, going from 6/49 to 6/53.

[8] Care must be taken when comparing across games to recognise that some games pay prizes as a lump sum (in the UK, for example) while others (most US states) pay an annuity (or some heavily discounted lump sum). Moreover, in some countries prizes (the USA) are liable for income tax while in other countries (UK) they are not.

[9] That is $s_i = p_i.\left[(1-t)S - 10.N_3\right]$, where $i = 4,5,5+b,6$, N_3 is the number of players that match 3 of the numbers drawn, and p_i is a fraction. For example $p_6 = 0.52$.

[10] Gerry Quinn in Ireland provides a helpful website, http://indigo.ie/~gerryq/Lotodds/lotodds.htm , that allows probabilistically-challenged readers to compute the odds for many common game designs.

18. So, for any specific n and N, the design of the distribution of the prize money, through choice of t and the s_i shares, affects the mean return from buying a ticket (which is less than $1-t$ because of the rollover probability – the higher the rollover probability the lower the return to the current draw), the variance in returns around this mean, and the degree to which the prizes are skewed towards large or small prizes. The larger the share given to the jackpot the more positively skewed is the distribution of prizes and the larger the share given to the lower prizes the more negatively skewed is the distribution[11]. The variance, depends of how much weight is given to middle as opposed to extreme prizes. Note that, these "moments" of the prize distribution are not independent of each other: for example, reallocating the prize money away from the easy-to-win prizes and towards the hard-to-win prizes increases the skewness but also increases the variance and lowers the mean (because there is a chance that the jackpot rolls over).

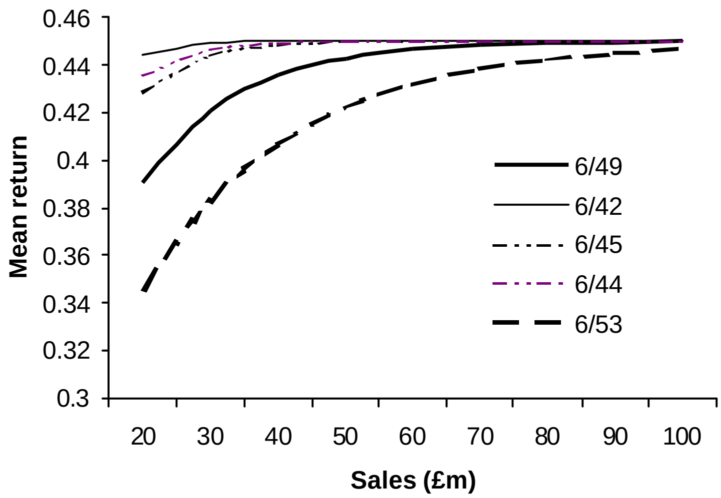
19. One way of summarising the complications of how all the various aspects of game design impacts on sales is through the mean, variance and skewness of the prize distribution. To estimate the impact of these three moments of the prize distribution on sales we would, ideally, like to conduct experiments where the design features were changed and sales recorded – better still, it would be useful to offer one group of individuals one game design and a control group another design. In practice such experiments are not available to us. In practice we observe either no variation in game design over the history of sales or, at best, changes in game design that the operator has chosen with a view to increasing sales. That is, any variation in game design that we may observe in ANY dataset is unlikely to tell us anything useful about, for example, how sales would change if a policymaker wanted to change the tax rate levied on the game.

20. The best we can do is to try to make inferences about how sales would be affected by game design changes, from the *random* variation in the terms on which people

[11] Games that are hard to win often feature large jackpot shares. For example, in the Florida on-line twice weekly lotto draw the odds of matching 5 of the 6/53 has a (relatively) high chance but it has such a small share of the overall prize pool that it is only, on average, worth approximately $5000. That is, the Florida lotto game is both hard to win and highly skewed. It is the large jackpot that entices people to play in regular draws even though there is a high chance that it will be rolled over and won by someone in subsequent weeks.

participate that we can typically observe – and that is through the effect of variations in the size of the jackpot on sales. The size of the jackpot is a random variable in our data because rollovers are statistically random events. The value of each of the moments of the prize distribution depend on the game design parameters and on the level of sales – for example, for any given design the mean return on a ticket is higher the higher is sales. Thus rollovers cause there to be exogenous variation in the nature of the prize distribution.

Figure 1 Lotto's Peculiar Economies of Scale



21. Figure 1 shows the "expected value" of a lottery ticket for common types of design in a regular (non-rollover) draw. Expected value is the average return on buying a £1 ticket. In the figure the take-out rate t is set at 0.55 which is a typical value and approximately the value used in the UK Camelot lotto game. The shape of this figure has given rise to what has been called lotto's "peculiar economies of scale" since it shows that the game gets cheaper to play (in the sense that the expected loss is smaller) the higher is sales[12]. This is because: the higher is sales the smaller is the chance of a rollover occurring because more of the possible combinations are sold[13]; this makes the return higher in the current draw because rollovers take money from the current draw and add it to the next

[12] See Cook, P.J. and C. T. Clotfelter, "The Peculiar Scale Economies of Lotto" **American Economic Review** 83, 1993.

[13] The rollover probability is $(1-p_6)^S$ where p_6 the jackpot odds (1 /14m in the 6/49 case) and S is the level of sales.

draw; and your ticket in this draw gives you a possible claim on prizes in this draw but not the next. So the higher the chances that a jackpot rolls-over the less a ticket for the current draw is worth. Note that at very large levels of sales all games have the same mean return which simply equals the $1-t$, because the chance of a rollover is small when ticket sales are large since most possible combinations will be sold. Notice also that at any given level of sales easier games offer better value in regular draws since the rollover chance is smaller.

22. Figure 1 shows the situation for regular draws. However, when a rollover occurs, the mean, variance and skewness all change and the way in which they change can be calculated from a knowledge of the determinants of these moments. In Figure 2 we show, for a 6/49 design, how mean, variance and skewness vary with sales and how these relationships are shifted when there is a small rollover (£4m) and a large rollover (£8m). Rollovers make a difference in kind to the relationship between the moments and the level of sales for the following reason. In regular draws players simply play against each other for a slice of the overall prize pool which comes from stakes in the current draw – since players play against each other any addition to the prize pool is matched by additional potential winners. But in rollover draws players are also playing for the jackpot pool from the previous draw[14] and the value of this extra gets spread more thinly as more tickets are sold. Thus, the relationship between sales and the mean of the prize distribution (also known as the expected value) is made up of what would happen in a regular draw (as shown in Figure 1) that would have the upward sloping economies of scale characteristic *plus* the value of the previous jackpot which falls as sales rise because its value gets spread more thinly the more players are competing for this fixed sum.

23. Thus, overall, as the top panel of Figure 2 shows, the expected value first rises (as the economies of scale effect dominates) and then falls (as the competition for the fixed rolled-over amount takes over and the economies of scale effect flattens out).

[14] In principle, lower prize pools could also roll over but we have no evidence that this has ever occurred in practice.

24. The probability distribution implied by the Camelot 6/49 prize structure has a large spike at zero, since mostly players lose, and a further smaller spike at £10, where 1 in 57 tickets match 3. For the larger prizes however, it is not possible to describe the distribution as further spikes because the amount won depends on the number of people who also win a share in each prize pool. Instead of a spike, there is a small peak with a (local) maximum in the distribution for each prize type, which corresponds to the most probable number of winners for that type, but around this is a distribution that arises because there may be fewer winners each getting a larger share of the pool or more winners than expected each getting a smaller share. Successive peaks, corresponding to the mean winnings of bigger prizes are lower (as the chance of winning is smaller) and wider (because the variance in the number of prize winners is higher for the more difficult to win prizes). The overall distribution is thus left skewed. The bottom two panels in Figure 2 show how the variance and skewness of the prize distribution vary with sales and the rollover size. A rollover decreases the left skew (i.e increases (right) skewness since it increases the size of the jackpot pool.

25. Figure 3 shows the effects of rollover size on the mean, variance and skewness for two levels of sales, typical of Wednesday and Saturday draw levels. A rollover affects only the top prize increasing (right) skewness. Increasing ticket sales has no impact on the two mass points corresponding to winning nothing or ten pounds but increases the prize pool for the other prizes and also the likely number of winners. With no rollover, the first effect dominates[15] and the increase in sales increases the expected value and the peaks of the distribution corresponding to the higher value prizes move rightward. With a rollover, however, the second effect dominates for high sales, and, although the expected amount won for the 4, 5 and 5+bonus prizes increases, the expected amount won in the jackpot prize may decrease.

[15] See Clotfelter, C. T. and P. J. Cook, **Selling Hope: State Lotteries in America**, Harvard University Press, 1991.

Figure 2 The relationship between moments and sales for different rollovers

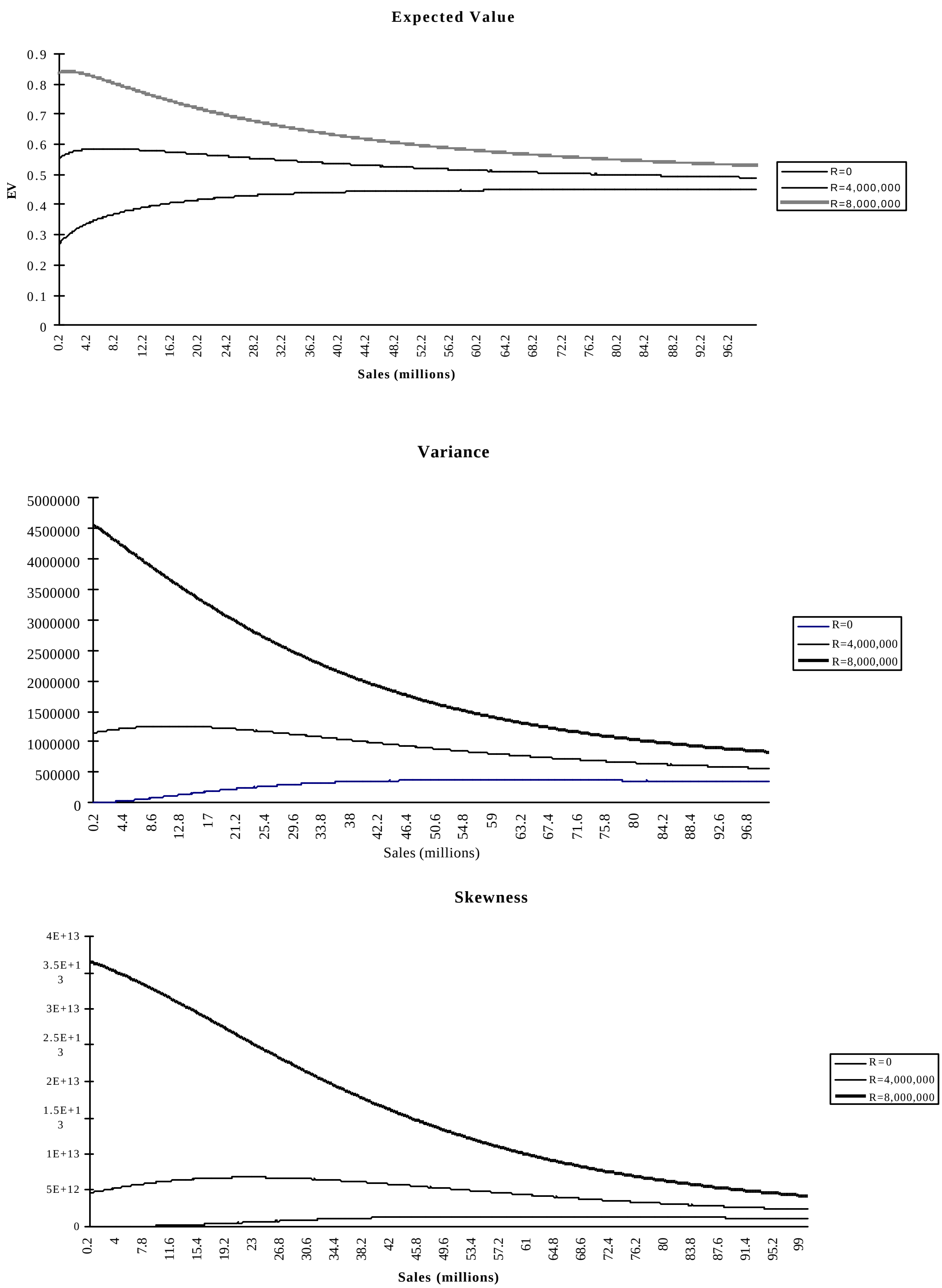
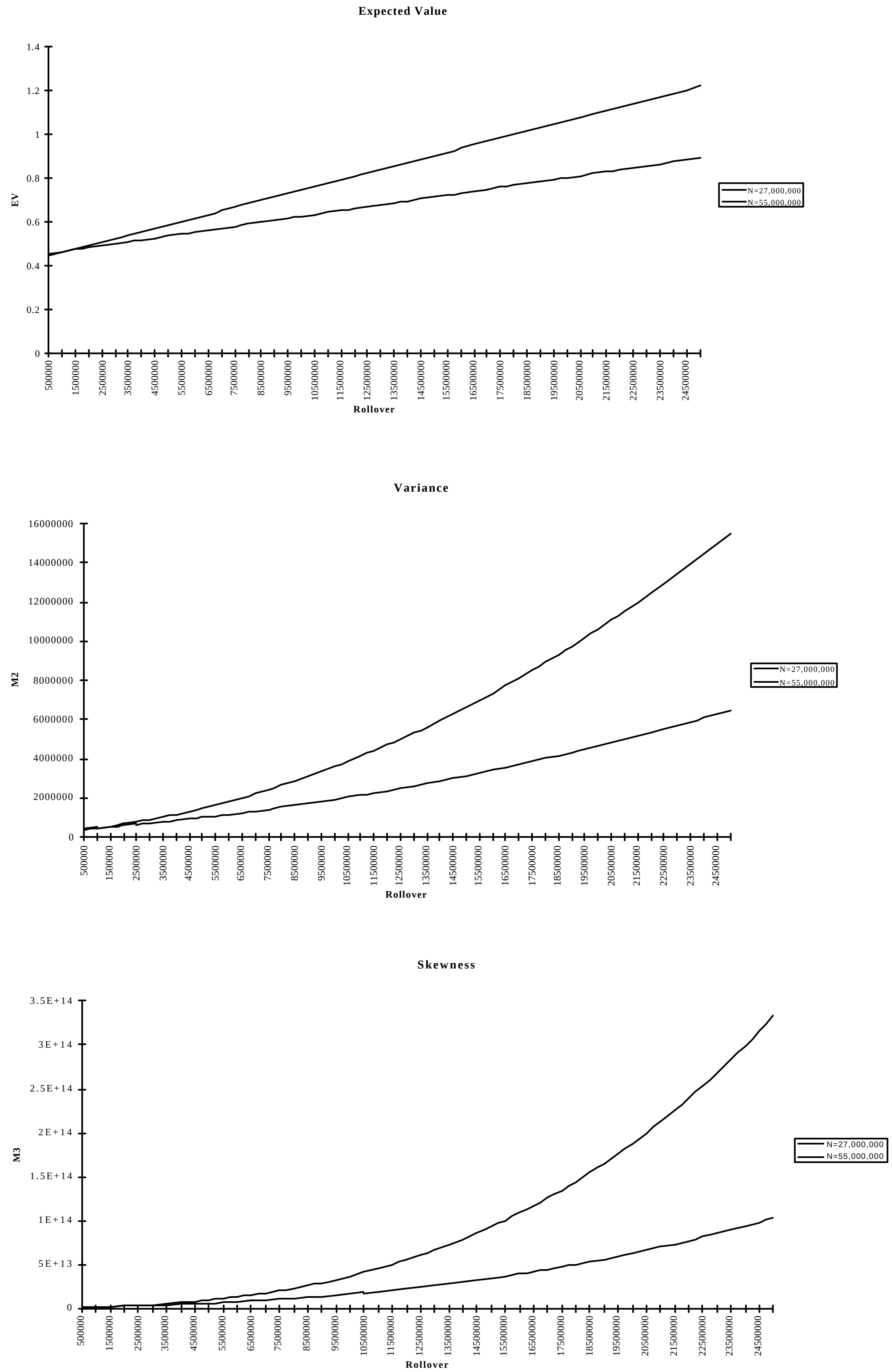


Figure 3   The relationship between moments and rollover size



26. Table 1 shows the actual values of the moments for typical examples. The message is that rollovers have a large effect on the mean, variance and skewness of the prize distribution, especially at low levels of sales, while the effects of variation in sales (for a given rollover size) is relatively small, especially at large levels of sales.

27. Few previously published papers have looked at the modelling of lotto sales. One US example[16] suggests that **decreasing** the takeout rate for the jackpot prize could **increase** revenues (for the Florida state lottery)[17]. The decrease in takeout rate would have two opposing effects: it would decrease revenue since, ceteris paribus, less money is taken as profits, but the larger prizes made possible would, however, increase sales and thus increase the "tax" revenue raised. While the increase in sales would also decrease the probability of a rollover, the increase in the probable size of any rollover which does occur more than makes up for this.

Table 1 Variation of moments with changes in sales and rollover values

a) Typical Saturday sales

Sales (millions)	60	80	60	80
rollover (millions)	0	0	10	10
Mean	0.4480	0.4495	0.6124	0.5741
Variance	373817	362262	1738480	1262490
Skewness	1.4126×10^{12}	1.2896×10^{12}	14.3768×10^{12}	8.5044×10^{12}

b) Typical Wednesday sales

Sales (millions)	30	40	30	40
rollover (millions)	0	0	10	10
Mean	0.4333	0.4418	0.7276	0.6775
Variance	303475	350841	3324420	2623430
Skewness	0.9136×10^{12}	1.2257×10^{12}	33.8585×10^{12}	25.4434×10^{12}

[16] J.F. Scoggins, "The lotto and expected net nevenue" **National Tax Journal** , 48, 1995.

[17] See D Forrest, D Gulley and R Simmons, "Elasticity of demand for UK National Lottery tickets" forthcoming in **National Tax Journal** ,(2000) for UK work that follows this line but does not support the proposition.

28. To consider the overall effect of a change in the takeout rate, Scoggins statistically estimates two equations: one showing how sales depend on the size of the jackpot; and the other showing how the rollover probability is affected by any change in sales. The simplest approach to this would be to say, for a probability of winning the jackpot of p_6, then, if sales are at level S, the probability of no-one winning the jackpot must be $(1-p_6)^S$. There is, however, a tendency for players to choose numbers in a non-random fashion (a phenomenon termed "conscious selection") so that some combinations of numbers are chosen more often than others. This implies that the 'coverage' of numbers is lower and thus the rollover probability would be under-estimated by the expression above. Instead Scoggins assumes that the rollover probability is can be represented by the form $(1-p_6)^{a+bS}$ and estimates the coefficients a and b to uncover the rollover probability. Using the rollover probability equation and the ticket sales equation, average sales and average tax revenue can be calculated as a function of the takeout rate, and the optimal takeout rate can be determined[18].

29. A further paper[19], using Israeli data, also considers the effects of changing the takeout rate. However, this work stresses the importance of rollovers in creating additional excitement and publicity which they refer to as 'lottomania'. The paper suggests that "the optimal strategy consists of a delicate balancing act between increasing the incidence of rollover, since big money is made when lottomania takes possession of the public, and making it sufficiently attractive to play in the early rounds. If it were too difficult to win in the first round there would be less money to be rollover over, and less lottomania". Again, they use the size of the jackpot rather than the expected value as the main determinant of sales.

30. The majority of the literature has thus been based on either the jackpot size or the expected value of the lottery. Simple thought experiments, however, suggest that these do not capture the full effect that the distribution of prizes may have on demand. For

[18] By using the size of the jackpot rather than the expected value as the determinant of lottery sales, Scoggins overlooks the fact that a rational player would realise that on a rollover week, higher sales imply a smaller likely share in the jackpot if the winning number is chosen. The relationship between the jackpot and expected value therefore differs according to whether it is a rollover week or not.

[19] Beenstock, M., E. Goldin and Y. Haitovsky (1999), "What Jackpot? The Optimal Lottery Tax", The Hebrew University of Jerusalem, mimeo (1999).

example, if the *probability* of winning the jackpot and the existence of smaller prizes have no effect, then the use of the jackpot size as the sole explanatory variable begs the question of why do lottery operators the world over bother with smaller prizes and why they fail to further decrease the odds of winning the jackpot in order to increase profits?

31. Moreover, empirical evidence of a preference for skewness in the distribution of prizes has been suggested by Golec and Tamarkin[20] for racetracks, who suggest that the long shot bias, a commonly observed race-track phenomenon in which low-probability, high-variance bets (long shots) are over-bet and favourites are underbet, can be consistent with risk averse behaviour if bettors have a preference for skewness. Garrett and Sobel[21] take a similar approach to explain demand for (not necessarily parimutuel) lotteries. Both papers assume that preferences can be expressed as a function with prize money as the argument but the way in which the prize money enter the model is via three powers of the prize fund which, in their work, represent mean, variance and skewness respectively. In both of these papers the authors find that the coefficient on the first power and third power of the prize were significantly negative and the power on the second term was significantly negative, indicating a positive preference for money, an aversion to risk, but a preference for skewness.

32. In their work, Garrett and Sobel assume that the lottery players welfare depends only upon the top prize payouts of each lottery game in each state and where the prize structure was parimutuel, so that the top prize varied according to the number of tickets sold and the number of winners, the average top prize was estimated using annual sales and the takeout rate. Thus the effects of game design on the skewness and variance of parimutuel lotteries are essentially ignored. In contrast to Garret and Sobel in the work reported below the mathematical mean, variance and skewness of the distributions are calculated and all used as explanatory variables. Although the dominant term in each of these variables are, respectively, the jackpot, the jackpot squared and the jackpot

[20] Golec, J and M. Tamarkin, "Bettors love skewness, not Risk, at the horse track", **Journal of Political Economy** 106, 1998.

[21] Garrett, T. A and R. S. Sobel, "Gamblers favor skewness, not risk: Further evidence from United States' Lottery games", **Economics Letters** 63, 1999.

cubed, (the variables used by Garret and Sobel) our approach also enables the effect of the other prizes and the level of sales to be captured.

33. Three recent UK papers[22] present evidence on the determinants of sales in the form of statistical estimates of the extent to which sales increase for every £1 addition to the jackpot due to a rollover or superdraw[23]. These papers uses a variety of datasets but are all couched in terms of rollovers affecting sales *only through the mean* return to buying a ticket. This work is extended and updated in recent work by Young[24] which shows how the effect of rollovers on sales can be decomposed into effects via the *mean, variance and skewness* in the prize distribution. Thus, Young calculates the level of the mean, variance and skewness for each draw, allowing for the effect of the rollover size, and includes these three variables into a statistical model of sales. The results show that:

- Sales are a statistically significant increasing function of the mean of the prize distribution – so better bets are more attractive ones.
- Sales are a statistically significant but decreasing function of the variance in the prize distribution – so riskier bets are less attractive.
- Sales are a statistically significant and increasing function of the skewness of the prize distribution – so players exhibit a preference for skewness.
- Sales exhibit statistically significant positive correlation across time which means that changes in sales in one draw persist over time – thus a rollover which raises sales in the current draw will also raise sales a little in subsequent draws. This is termed the "halo effect" in the industry.

34. While it is true that we cannot distinguish between our interpretation of the results and a model which simply says that rollovers have a highly non-linear effect of sales[25], these

[22] L Farrell, G lanot, R Hartley and I Walker, "The Demand for Lotto and the Role of Rollovers and Conscious Selection", **Journal of Business and Economic Statistics** Fall 2000; L Farrell, E Morgenroth, and I Walker, "A Time Series Analysis of UK Lottery Sales: Long and Short Run Price Elasticities", **Oxford Bulletin of Economics and Statistics,** 61, 1999; and L Farrell and I Walker, "The Welfare Effects of Lotto: Evidence from the UK", **Journal of Public Economics**, April 1999.

[23] We also deal with the distinction between superdraws that add to the jackpot and those that guarantee a minimum.

[24] See J. Young, "The effect of higher moments on the demand for lottery tickets", MSc Dissertation, University of Warwick, 2000. The data used consists of 486 draws for both Wednesday and Saturdays, starting from the first draw, including 45 Superdraws, 64 rollovers and 5 double rollovers

[25] Camelot, on their website, state that "after months of extensive research amongst the British Public it was found that the chance (of winning millions) was the most motivating strategy for potential British players." No further details are given on the methodology. Of course, if they really

same effects of mean, variance and skewness have been estimated before in the context of gambling on horse races in the USA and by comparing sales across US lottery games, and exactly the same pattern emerged. Thus, in the absence of truly experimental evidence we feel that our interpretation of the estimates of the way in which sales vary with rollover size is the best available[26].

35. In principle, the estimates in Young imply that we can make inferences about game design changes. Table 2 shows the theoretical distribution of prizes under the current

believed this then the lower prize pools would be better spent on the jackpot and they should redesign the pool sharing rule. However, in our analysis the lower prizes have a role in promoting sales by reducing the variance that players dislike.

[26] Several other explanatory variables were also used in the modelling. A variable was included to allow for differences in sales on Saturdays and Wednesdays. A further variable was included to allow for a shift in Saturday sales once the Wednesday draw had been introduced to allow for possible substitution effects. The number of retailer terminals has also been included. This grew steadily for the first 110 draws and is intended to pick up a ease of availability. However, the change in the number of terminals is unlikely to account entirely for the rise in sales over the initial period and this variable may also be picking up the natural growth in sales following the introduction of a new product before equilibrium is established. This point is reinforced by the fact that the Wednesday draw also exhibits low sales initially. In the longer run, however, time tends to have a negative effect, as people become bored and loose interest in the game. In order to capture these effects a trend representing the weeks since the introduction of the lottery was included. A separate linear and quadratic trend for the Wednesday draw were also included to capture change in interest for the Wednesday draw. Variables were also included to correspond to the introduction of the "Thunderball" game in June 1999. This game has a far less skewed distribution, with a one in 4 million chance of winning the top-prize of £250,000, and a one in 33 chance of winning the bottom prize of £5. Unlike the standard draw however, prizes are fixed in size and there are no rollovers. The game is drawn on a Saturday and generally averages sales of between 4 and 5 million. These sales may, in part, be taken from the main game, although it is notable that in previous empirical work, little substitution between draws has been found, so it may be that the Thuderball draws in new sales, or alternatively, increases sales, by creating additional interest in the lottery.

Lagged variables to capture the effect of the previous draw were also included and are likely to reflect habit, or possibly addiction. They are also intended to pick up the "halo effect" which can be seen clearly in the graphs of sales; after a rollover has occurred, sales continue to be higher for the subsequent regular draws. The lag structure is likely to change following the introduction of the Wednesday draw so separate variables were used for lags before and after the introduction of the Wednesday draw. After the introduction of the Wednesday draw, lags may have a further importance, since it is possible to buy a ticket for the following Saturday draw when buying a ticket on a Wednesday and vice versa. Thus higher sales on a Wednesday may imply higher sales on a Saturday because of the convenience of being able to but tickets for Saturdays' draw on Wednesday. Alternatively, there may be a negative effect if a Wednesday rollover induces people to substitute Saturday sales for Wednesday sales. Since the effect of Saturday draw on the following Wednesday draw is not necessarily the same as the effect of a Wednesday draw on the following Saturday, lags of Saturday draws were entered as a separately from lags of Wednesday draws. A variable was also included which took account of the death of Diana, Princess of Wales. The television show was cancelled and many retail outlets were also closed. Finally, quarterly variables were included to represent any seasonal effects that might be present. For example, during the winter sales may be higher, as TV viewing goes up and more people watch the lottery show. Alternatively, during the summer, when there are fewer major news stories, the lottery may receive more media attention. Further details of the results are in Appendix B.

arrangements and, for one particular draw, the actual distribution. The expectation in this particular draw was there to be 4.5 jackpot winners sharing the jackpot pool of close to £8m. In fact, by chance, there were just two winners who each received close to £4m. Other prizes were distributed as shown according to the shares that define the prize pools: .22 for 4-ball, .10 for 5 ball, .16 for 5+bonus, and 0.52 for 6-ball.

Table 2 The allocation of prize money for the 62,476,486 tickets sold in the week ending 18.3.95

Prize types	Odds	Allocation of prize money	Expected no. of winners	Prize money allocated (£)	Actual no. of winners	Actual prizes awarded (£)
3 main numbers	1 in 57	£10 per ticket	1,096,131	12,778,130	1,277,813	10
4 main numbers	1 in 1033	22% of remainder	60,484	3,339,867	71,061	47
5 main numbers	1 in 55492	10% of remainder	1,126	1,533,070	1,210	167
5 + bonus number	1 in 2330636	16% of remainder	27	2,454,000	25	98,160
All 6 main numbers	1 in 13983816	52% of remainder	4.5	7,975,572	2	3,987,786

Source: J. Moore 1997.

36. Table 3 shows two suggested alternative ways of distributing the prize pool that has been suggested[27]. These represent an attempt to reduce the jackpot size by increasing the share of the 5+bonus pool (Scheme C), or increasing the share of the 5-ball pool (Scheme B), keeping the other prize pools constant. Both of these schemes would reduce the skewness of the prize distribution and this would be expected to reduce sales according to our estimates. However, they also reduce the variance and this, according to our estimates, should increase sales. The distribution of the number of winners remains the same since n and N that determine the game design is being kept at 6/49. Moore helpfully calculates the effect of this change in the expected levels of prizes for each prize type and his calculations are reproduced in Table 3. The expected number of winners is calculated assuming that sales are 60m, a typical Saturday figure. Scheme B reduces the average jackpot win by £267,000 which allows the 4-ball prizes to be

[27] Moore, P.G., "The Development of the National Lottery: 1992-96" **Journal of the Royal Statistical Society**, 160, 1997.

approximately £1000 larger, while scheme C reduces the typical jackpot by more than £450,000 and this allows the typical 5+bonus winner to win more than £75k more.

Table 3 *Expected monetary prizes for winners under three schemes of allocation*

Prize type	Expected no. of winners	Expected prizes (£) under the following schemes: Scheme A (52-16-10-22%)	Scheme B (45-16-17-22%)	Scheme C (40-28-10-22%)
Jackpot	4.3	1,988,749	1,721,032	1,529,807
5 + bonus number	25.7	101,987	101,987	178,477
5 numbers	1080	1,518	2,580	1,518
4 numbers	58050	62	62	62
3 numbers	1,057,800	10	10	10

Source: J. Moore 1997.

37. The suggested alternatives have several effects on the characteristics of the prize distribution. Firstly, as money is taken away from the large jackpot and moved to the smaller prizes, the mean increases. This is because the smaller prizes are extremely unlikely to roll over, so the total amount of money likely to be paid out on the current draw is higher. Thus basis A gives the lowest mean and basis C the highest. Moving money away from the jackpot also decreases both the variance and the right skewness of the distribution. Compared to basis A, both B and C are less skewed and have smaller variance. Although it is harder to compare B to C, as C has a smaller jackpot but a larger 5+bonus prize than B, calculations of the moments of these distribution for plausible sales and rollover size imply C is less skewed and has smaller variance than B.

38. Decreasing the amount of money allocated to the jackpot prize also decreases the size of the rollovers, so that rollover draws under basis B and C will have a lower mean (offsetting the increase in mean for the regular draws) and have a smaller variance and skewness in comparison to rollover draws under basis A. In summary, for regular draws, a move from A to B to C increases the mean but decreases the variance and skewness, and decreases the value of all three moments for rollover draws.

39. In order to examine the overall effect on demand, sales for the first 200 lottery draws were simulated for each of the alternative prize allocations suggested above[28] compared

[28] Since the mean, variance and skewness depend on the level of sales, forecasting sales amounts to solving a highly complex non-linear equation in sales. This proved hard to solve analytically and, instead, a recursive algorithm was used. There is, however, no guarantee that the solution will tend to a limit, or that any limit that does exist will be unique. In the examples shown here, the system converged very quickly and seemed invariant to the initial value of sales used.

to the simulated sales for the actual distribution of prizes. The results are presented in Table 4. According to the results of the simulation, basis C which is the least skewed, sells the most tickets. For non-rollover draws the sales increase from A through to C, implying the increase in mean and decrease in variance outweigh the effect of the decrease in skewness. Predictably, the size of the rollovers are smaller from A to C but the higher sales for regular draws also imply fewer rollovers under basis B and C than A. Interestingly, despite the smaller rollovers, average sales for rollover draws are still higher under basis B and C than under A. This is because the larger rollover and the large jackpot imply an increase in skewness and decrease in variance which is not outweighed by the increase in mean.

Table 4: Simulation Summary

	total sales over 200 draws (m.)	Total amount rolled over (£ m.)	number of rollovers	average rollover size (£ m.)	total rollover sales (m.)	average rollover sales (m)	total non rollover sales (m)	average non rollover sales (m)
Basis A	11,096	139.9	15	9.32	994	66.32	10,101	54.6
Basis B	11,912	85.8	10	8.58	709	70.92	11,203	59.0
Basis C	12,358	78.7	10	7.87	731	73.15	11,626	61.2

40. The implications are that the attractions of the lower variance and higher mean in the alternative schemes outweigh the detraction of their lower skewness: this generates higher sales in regular draws; this higher level of sales depresses the rollover probability so there are fewer rollovers; and while rollover sales are higher than sales in regular draws this is not enough of a difference to outweigh the higher sales that occur under B and C.

41. Thus, despite what Camelot finds about what the public says that it wants, our evidence, based on what the public does when asked to dig into its collective pocket, suggests that it is quite possible to tweak the game to promote sales at the same time as reducing the typical jackpot size.

42. Attempts to simulate the sales for different take out rates failed, as the iterations did not converge. Instead, the impact a change in takeout would have on sales was assessed by examining the slope of the sales function with respect to the takeout rate. This only captures the static effect of a change in takeout - the dynamic effects which are transmitted through rollovers are not accounted for.

43. Our results of this exercise are only suggestive: they suggest that an **increase** in the take-out rate would **decrease** sales but **increase** tax revenue. The higher frequency of rollovers caused by the lower sales would counteract this conclusion (but the rollovers would be smaller and so there would also be a loss of sales due to this). Thus, it would appear that the current game is "too generous" – it would be worth increasing the take-out rate despite the reduction in sales that might ensue. But we cannot say quite how mean the game should be.

44. The final change in game design considered was the effect of changing the format of the game from 6/49 design currently used to a 6/53 design. The latter format is that proposed by the People's Lottery.

45. Our attempt to evaluate the possible impacts of this change once again did not converge[29]. Neither was it possible to examine how sales would vary with a small change in these design parameters because of their discrete nature. However, in attempt to draw at least tentative conclusions, Table 5 compares the value of each of the moments for the two different game designs evaluated at typical values of sales. The final column gives the "predicted level of sales" relative to the base of 6/49 with sales of £60m which is simply obtained by setting the values of the moments at the levels relevant to the assumed sales levels for each game design[30]. The moments are computed using the present arrangements for sharing the prize pool (we assume that there is also a bonus ball in the 6/53 design). Figures 2 and 3 above suggested that, at least at high levels of sales the effects of sales on the moments of the prize distribution is quite small – most of the variation in these variables arise from rollovers. Thus, in Table 5 we are assuming that the effect of sales on the moments is small enough to be ignored and we compute the predicted sales at the calculated levels of the moments corresponding to the chosen figures for sales and rollover size. We base the predictions at sales of 60m in a regular draw using the 6/49 design.

[29] Several other attempts were made to resolve this, for example, by using the 'findroot' command in *Mathematica*, which solves non-polynomial expressions using the Jenkins-Traub algorithm. However, even with considerably simplified expressions for the moments, this also failed to produce a solution.

[30] Allowing for the typical Wednesday draw to be less popular than the typical regular Saturday draw by the estimated value (-29.2m in Appendix B).

Table 5 Effect of Change in Game Format

6/49 format

Sales (£m)	Rollover (£m)	Mean	Variance	Skewness	Predicted sales relative to base
60	0	0.4480	3.74×10^5	1.41×10^{12}	-
80	10	0.5741	1.26×10^6	8.50×10^{12}	11.8
30	0	0.4333	3.03×10^5	9.14×10^{13}	-31.4
40	10	0.6775	2.62×10^6	2.54×10^{13}	-17.3

6/53 format

Sales (£m)	Rollover (£m)	Mean	Variance	Skewness	Predicted sales relative to base
60	0	0.4382	6.94×10^5	4.34×10^{12}	–11.5
80	10	0.5662	2.39×10^6	2.94×10^{13}	–3.6
30	0	0.4065	4.07×10^5	1.58×10^{12}	–42.5
40	10	0.6298	3.46×10^6	4.36×10^{13}	–35.6

46. Increasing the number of balls in the draw makes all prizes more difficult to win and the mean for any draw therefore decreases. However, since fewer people are expected to win a share in each of the pari-mutuel prizes, those that do win can expect to win more. The variance and skewness therefore increase. These effects can be seen by comparing any row of panel (a) with the corresponding row in panel (b). The implications for sales are shown in the final column. Comparing, for example, 6/49 at a regular Saturday draw level of sales of 60m with the same draw under 6/53 the model predicts that sales would be lower by 11.5m. Comparing a regular Wednesday sales in 6/49 we find that sales would be 31.4m lower than the Saturday, but under 6/53 sales would be even lower than this – by a further 10.1m. Thus, for each of the points where the sales function was evaluated, the effect of the increased variance and decreased mean outweigh the effect of the increased skewness – sales would be about £21m lower per week under 6/53 compared to 6/49 **if** the shares and the take-out rate were kept fixed.

47. Against this lower level of sales in any particular draw has to be set the higher probability of a rollover. As Table 6 suggests rollovers imply that sales would be higher: about 14m higher[31] on a Wednesday rollover draw and about 11.8m on a Saturday rollover. The hypothetical probability of a rollover for each game design can be computed assuming that individuals choose their numbers randomly. Note that these hypothetical numbers are considerably different from actual experience of the 6/49 game: the **theory** suggests

[31] That is, 31.4-17.3.

that Saturday (Wednesday) rollovers should occur about once every 70 (9) draws, **in practice** it is more like every 7 (3). Thus, these figures drastically underestimate the likely number of rollovers. However, they are probably a good guide to the **change** in the frequency of rollovers if we moved from 6/49 to 6/53[32]: Saturday rollovers should be about 4 times more frequent and Wednesday rollovers would be about twice as frequent. Since, Wednesday sales are predicted to be about 14m higher in a rollover and Saturday sales are predicted to be about 12m higher in a rollover draw, weighting the predicted sales figures together by the rollover probabilities suggests that, on average, the 6/49 game would generate weekly (Wednesday plus Saturday) sales of around £95m while the 6/53 game would generate weekly sales of around £85m.

48. Thus, the higher rollover probabilities only compensate for about half of the loss in sales due to the change in the moments. Thus, over the course of the license, 6/53 might sacrifice £3.5b relative to 6/49. However, this is not to say that 6/53 **could** not be **more** successful that 6/49. But it would have to be combined with other changes: the take-out rate would need to be dropped a little to stop the mean return under 6/53 falling too far to depress sales in regular draws, and the higher variance in the prize distribution under 6/53 would need to be addressed, perhaps by dropping the bonus ball (and hence the 5+bonus prize pool) and adding it, instead, to the 3-ball prize pool.

49. Moreover, it would need to be marketed in such a way that sales in regular draws were encouraged: the danger with high rollover probability games is that the loss in sales that occurs in regular draws does not get made good by the occasional bout of lottomania that occurs when multiple rollovers have accumulated[33].

[32] In fact, this comparison might even underestimate the rollover probability under 6/53 in practice because 53 has more numbers about 31 than does 49 so a higher proportion of the available combinations lie outside the range within which birthdays lie. Thus, 6/53 may experience a higher degree of conscious selection that does 6/49 and hence an even higher number of rollovers that we would expect.

[33] Sales may also be affected by the change in design because of behavioural considerations, which would not be picked up in our modelling. For example, getting two numbers right may lead the player to feel some measure of success and encourage him to play again, even though he won no prize. With a 51 or 53 board, the likelihood of getting 2 numbers is smaller and may leave the player feeling discouraged or bored. To give another example, a common pattern of play is to "reinvest" small winnings, for example from getting 3 balls correct, in further play. Since the likelihood of getting 3 balls right is decreased with a 51 or 53 board game, this may again contribute to reduced sales. Moreover, the 3-ball prize pool not only serves to reduce variance it also serves as an

50. However, our results need to be qualified. The simulations assume that sales respond to variations in mean, variance and skewness from design changes in the same way as they respond to these variables when rollovers occur. However, it is plausible that people may respond differently to these two types of changes. Firstly, changes induced by occasional rollovers allows for the possibility of substitution between draws, but this possibility does not exist for changes coming through the game design rather than rollovers. This suggests that ticket sales are higher when changes come from rollovers than from game design. Rollovers are rather like sales promotions – they induce people to change their behaviour quite differently to a temporary difference in the offer than they would to a permanent change. However, in the absence of a well-designed social experiment we cannot overcome this problem.

51. Secondly, our analysis is based on an econometric model of sales that, while it explains a high proportion of the variation in sales over time (as do many aggregate time series models), may not be good at forecasting the effects of structural changes (as is the case for many aggregate time series models). Thirdly, the econometric model itself fails some of the diagnostic tests that were applied to it suggesting that it is misspecified is some way.

52. Finally, the model has not been validated by investigating how well it predicts structural breaks since none have occurred. Thus, one avenue for further research would be to apply the methodology to other places where changes have occurred: Israel, Ireland and several US states spring to mind.

53. Thus, we feel that our analysis is suggestive rather than predictive at this stage: the suggestions are as follows:

- 6/49 with a take-out rate of around 50% is possibly too difficult and too generous a game for the UK;
- to make 6/53 attractive it would probably need to be tuned to make it less mean and would have to have a different prize distribution – perhaps a larger 3-ball share and no 5+bonus pool.

advertisement – with 1 in 57 winning a 3-ball prize under 6/49 players are likely to know someone who has won in the recent past. Under 6/53 there would be fewer 3-ball winners.

Wider Issues in Game Design

Regulation

54. Lotteries are products that necessarily exhibit economies of scale and are therefore candidates for being treated as "natural" monopolies. It is still the case that most lotteries are operated by agencies that are essentially government departments. In a few cases, such as the UK, the operator is a private company that is licensed to operate the game for some period. In either case, there is an argument for having some independent agency that protects consumers from the policies that the operator might wish to pursue. The players interests are best served by having a reliable and fair game on the best possible terms – that is, with the lowest take-out compatible with the operator receiving a market return on its investment. A private sector operator may wish to pursue policies that maximise its return on capital and this is likely to entail a higher take-out rate and a correspondingly inferior product in the eyes of the consumer. In practice public sector operators are also motivated to maximise the surplus over costs which is then used to fund government expenditure of some kind. The weakness of the public sector model is that the incentives for cost control are limited, while the weakness of the private sector model is that the operator has to be chosen and then prevented from abusing its monopoly market position.

55. Camelot, a private sector operator, can legitimately claim to have operating costs that are amongst the lowest in the world, even among those operators that also operate large games. However, it is difficult to say whether a different company might have done better. A not-for-profit operator may well have higher costs than a profit seeking operator and there is no guarantee that one or the other would have produced greater revenue and hence greater good causes income. Moreover, there is little quantitative support for the suggestion that players would prefer a not-for-profit operator. Consumers' Association survey data, a random sample that included both players and non-players, shows in Figure 4 that those that disagree or strongly disagree with the view that they would buy more if there was a not-for-profit operator out weight those that agreed/strongly agreed.

Figure 4 *Buy More if a Not-for-profit Operator?*

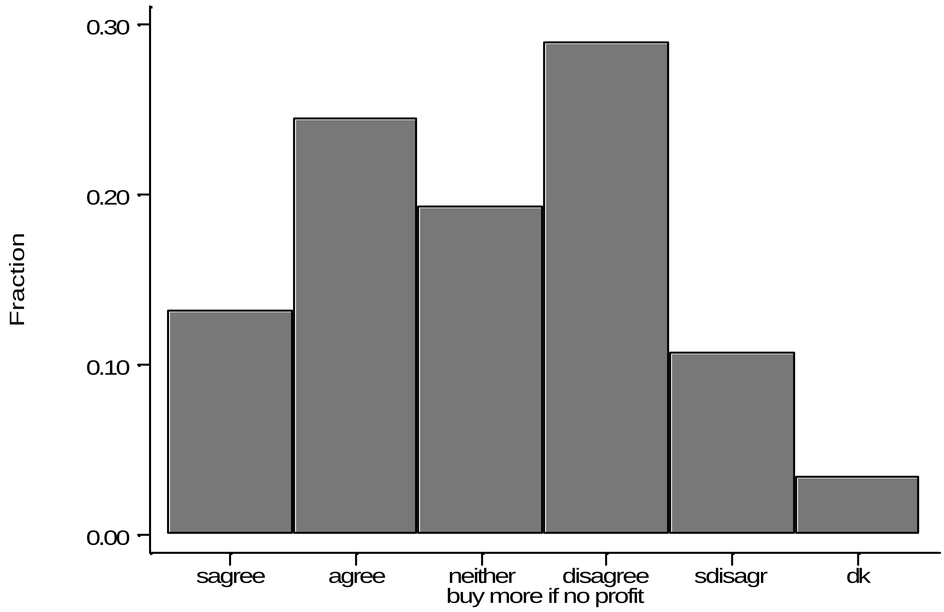
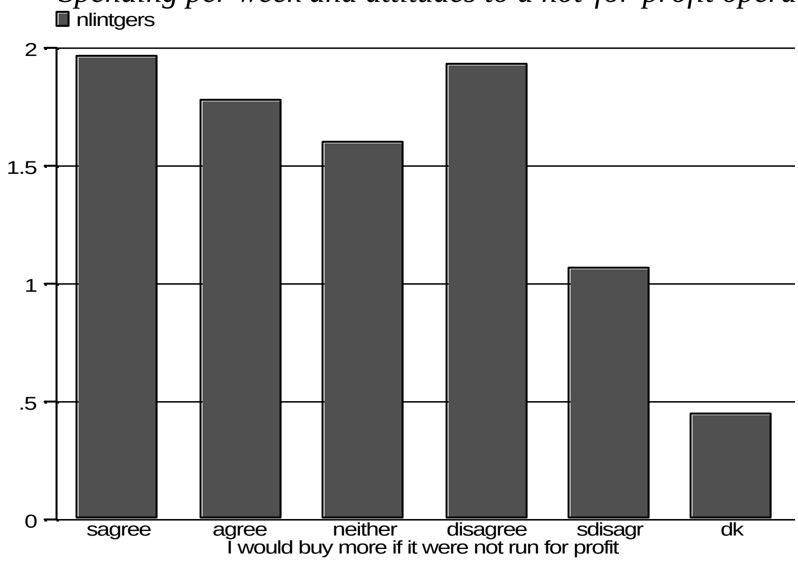


Figure 5 *Spending per week and attitudes to a not-for-profit operator.*



56. Moreover, the same data in Figure 5 shows that those who feel strongest about having a not-for-profit operator already seem to spend a little more than those that express satisfaction with the existing arrangements.

57. The mechanism for deciding which company should be chosen to operate the lottery in the UK has been controversial. The principal difficulty is that the choice is made through a "beauty contest": companies make proposals which aim to demonstrate their competence to run the game, their probity, and offer a portfolio of games together with a forecast of the likely revenue. The trouble with beauty is that it is in the eye of the

beholder and is frequently skin deep. In this case the beholders are intelligent and hard working individuals who have been successful in their pasts and who offer some of their valuable time, for token payments, as an act of public service. They have little industry expertise and their ability to discern the bone structure behind the gloss is limited. The bids themselves do not bind the operator to actually deliver what they forecast. Thus, there is little to stop bidders from widely exaggerating the likely revenues and no way for the Commissioners to make scientific quantitative evaluations of what is being proposed. The Commissioners, rightly, attach a considerable amount of weight to the probity of the operator since evaluating the forecasts and merits of the game designs proposed is more difficult. Not surprisingly, they were unable to convince the courts that they their recent decision, based essentially on probity, should stand.

58. The decision that the Commissioners are being asked to make is an extremely risky one and one they are not well equipped to make. However, it does not have to be like this: if the purpose is to raise good causes revenue then a straightforward solution is available: as at present, bidders should list the nature of their organisation so that probity checks can be made; those that pass this hurdle could then invited to enter an auction for the right to run whatever games they choose at whatever prices they choose. The highest bidder is the winner of this right. This effectively makes the good causes revenue essentially risk free for the Commissioners. The downside of such an arrangement is that it transfers the risk to the operator. Currently, the risks of running a lottery game, at least a pari-mutuel one, is close to zero: in a pari-mutuel game players play against each other, the operator simply takes a commission off the top[34]. Under an auction process there would be a risk for the operator that sales will be less than anticipated. Thus, operators would require a larger share of the revenue to compensate them for carrying this risk. Thus, such an auction mechanism should offer lower, but more certain, good causes

[34] It is possible to design games that do entail risk for the operator: ones that offer fixed prizes for example run the risk of not selling enough tickets to pay for the prizes. This is especially a problem for games that offer players the ability to choose their own numbers, rather than receiving a random number. This is because the number of winners then has much higher variance than would otherwise be the case. Nevertheless it should be easy to insure against such risks since it is straightforward to compute the variance.

revenue than the current beauty contest. Such a mechanism has been used successfully in allocating frequencies to cellular phone operators and in other industries.

59. The use of an auction may have further merit. The operator would not then face a "distortionary" tax. In principle, the lump sum payment for the license would not affect the pricing and output decisions of the firm relative to a competitive supplier.

Inappropriate Play

60. The regulator is required to ensure that the games are run so as to not invite inappropriate play. The Commissioners have put some effort into ensuring that under-age play does not occur and has commissioned survey evidence to measure the extent to which it might be occurring. However, the evidence that it has cannot be used to answer the important policy question of what the effects would be of raising the age limit on the age distribution of play and on the overall level of sales. Nor is there any research which sows how pricing affects the age distribution of play. As far as we are aware there has been no attempt to quantify these issues.

61. The Commissioners themselves rely on occasional "omnibus" surveys to inform them of patterns of play. These datasets were originally data on individual play but subsequently became surveys that asked individuals about the level of play in their households. The data on on-line draw expenditure seems to be reasonably accurate and "grosses-up" to figures which are close to Camelot's sales figures. However, the income data is poor with a very high proportion of observations with missing data and the income that is recorded is only grouped into a few wide bands. This data is not, by itself, of high enough quality to support an analysis of the distribution of lottery play by income.

62. Moreover, the scratchcard data, as in the FES data, is badly deficient and does not gross-up well.

63. The early data on individual play indicated that, although 99% of individuals surveyed spent less than £10 on the on-line draw in the survey week, a very small proportion (just 0.1%) spent more than £50. But even such a tiny minority constitutes a large number of individuals in the population. It would be sensible for the Commissioners to have a "panel" dataset that they could use to track players behaviour – to track the extent to which heavy play persists.

Good causes revenue

64. In addition to our more formal analysis of the statistics and economics of game design our wider thinking has prompted us to question the original rationale for the game: raising money for “good causes”.

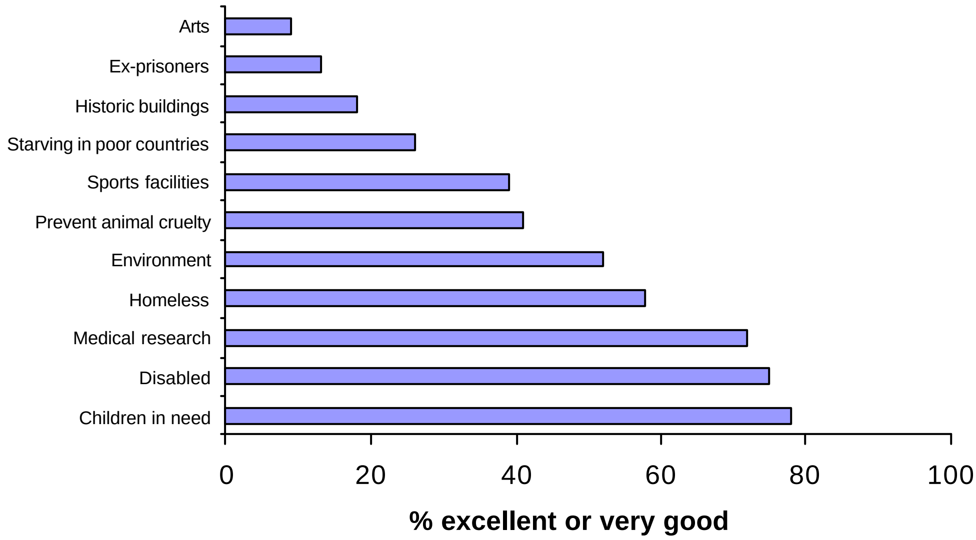
65. We have four main reservations for using lotteries to finance the provision of services and facilities. First, the good causes levy is a “hypothecated” tax and suffers from the common shortcoming that, since the money **has** to be spent for specific purposes the normal criteria for investment decisions are not relevant. Thus, it is likely that good causes funding will be spent on projects that have a lower return to society than either private sector investments, or public sector investments funded through general revenue. Where normal commercial criteria get suspended funding can be diverted to pursue other objectives and there have been some highly visible examples of this. Quantitative analysis of this issue would require that the benefits of lottery funded projects be compared with non-lottery funded projects and this would be a difficult exercise.

66. Secondly, the large amounts of additional funding targeted on quite specific areas gives rise to a large increase in the demand for culture, heritage, etc. without addressing their “supply”. In some cases, supply can only be increased by lowering the quality so that less and less worthwhile activities or projects get undertaken. In other cases, supply can be increased but the market is not competitive and the supply price has to rise to meet the additional demand. In this case, the subsidy that is provided to the demand side gets, in part, captured by the supply side. Thus, part of the subsidy gets appropriated by the suppliers of culture, heritage, etc. We have no data on this issue but it may be possible to investigate such things as the cost of UK heritage items in auctions, the costs of hiring an good tenor, and the cost of architectural services.

67. Thirdly, there is a paradox that the public wants the good causes spending to support the most worthwhile areas but, to a large extent, these areas are already being funded from regular tax revenue. Figure 6 shows data from the 1995 British Social Attitudes Survey, another random sample, where individuals were asked to declare the extent to which they favoured spending lottery funds on different causes. Thus, lottery funding may crowd out government funding so that the overall provision is little changed or it is spent

on projects that the public place little value on. It is very difficult to quantify this: the best example is Florida where all of the surplus revenue from the state lottery games is used for educational funding. Educational funding has grown no faster in Florida than it has in non-lottery states, nor than neighbouring states – lottery funding has simply displaced state government funding[35].

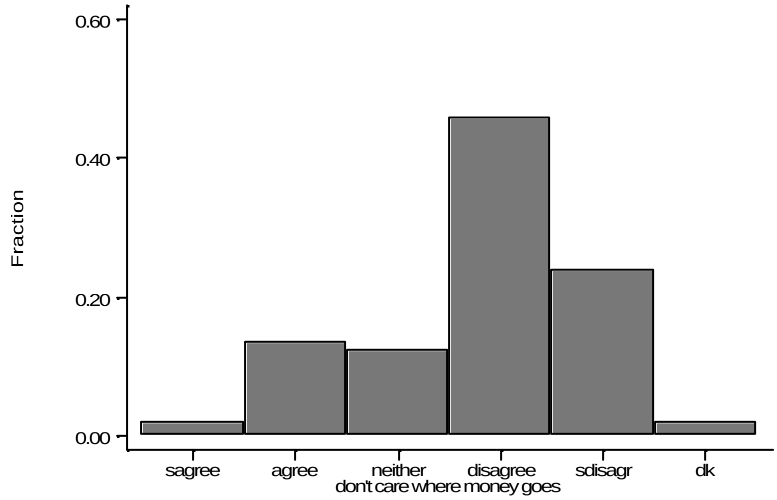
Figure 6 Preferred lottery spending



68. Indeed, our Consumers' Association data shows in Figure 7 that while there is widespread disagreement with the view that people do not care where the money goes, there is no evidence that those who are unhappy with the existing distribution of funding play any less than those who are happy with existing arrangements. Thus, there is no evidence to suggest that play would be sensitive to the distribution of funds even though individuals may express disapproval over it.

69. Fourthly, the incidence of the lottery-funded expenditures is unlikely to be distributionally neutral. It is extremely difficult to allocate the benefits of government expenditure programmes, say on health, education, police, etc. across individuals and the beneficiaries of lottery spending are probably no easier to deduce. It seems likely that the consumers of many cultural and heritage programmes are richer than average but, on the other hand, much of the expenditure is of direct benefit to young people. It is extremely unlikely that this would be an easy question for research to quantify.

Figure 7 Don't care where the money goes?



70. In addition to these issues we have two further reservations about the use of lottery to fund public expenditure. Firstly, one general rule is that good candidates for taxation are those commodities where the tax does not change the behaviour of the consumers very much. So the question this raises is how sensitive is demand for lottery tickets to the take-out rate. Our research suggested that lottery sales was quite sensitive to the take-out so that a high tax on this commodity would therefore be "inefficient". Thus, irrespective of the merits of good causes expenditure, using the lottery as a way of financing this expenditure is inefficient in the sense that it significantly distorts peoples choices.

71. While, we normally expect there to be a trade-off in the design of taxes between efficiency and equity: goods which are "efficient" to tax tend to be essentials and that is why they are good vehicles for taxation. But because they are essentials, their consumption is a larger part of the expenditures of poor individuals than rich individuals. Thus, policymakers have to balance the efficiency case against the equity one. However, lottery spending is both price inelastic, and hence it is inefficient to tax highly, **and** we find that it is quite highly concentrated amongst poorer individuals.

72. The evidence that we have on lottery spending and how it relates to income comes largely from the Family Expenditures Survey (FES): a continuous survey that records individual spending within households in great detail including the purchase of lottery tickets and records income and sources of income with some accuracy. Moreover, the

FES data on on-line draw expenditure seems to be surprisingly accurate[36]. The FES data has been used to argue that the game is played by rich and poor alike and this is approximately true in the aggregate data: if you group households into gross income deciles the average expenditure per household is broadly constant. However, this is a rather misleading way of using the data – in particular, richer households tend to be larger ones and contain more adults who can play; poorer households often contain old people who are less likely to play, not because they are poor, but because they feel that they will not live long enough to enjoy spending any winnings. Thus, here we use the data, for the post November 1994 period, at the individual level, rather than household. At the individual level the proportion of players (among those 16+) is 48% and the amount spent by players averages £2.43 per week. The **raw** FES data is depicted in Figure 8 and shows that the relationship between expenditure per week and income follows an inverted U shape. However, when we use multivariate statistical methods to control for all of the other characteristics[37] associated with playing, so as to isolate the relationship between play and income we find that *for players* expenditure on lottery tickets rises with income but by only a small, albeit statistically significant, degree. But, we found that the likelihood of being a player **fell** as income rises – and to such an extent that a rise in income reduces the overall expected amount of spending[38]. This is consistent with overall spending on lottery tickets falling fast as income rises.

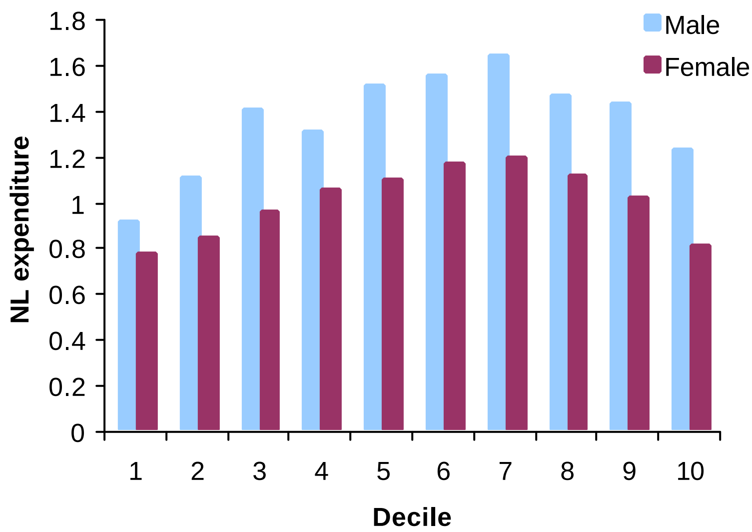
73. The implication is that, it is both inefficient and inequitable to tax lottery spending highly. Thus the case for high taxation would have to be made on social grounds: as we do with petrol, cigarettes and alcohol. However, it is difficult to find strong evidence that lottery participation does have widespread adverse social consequences.

[36] The same cannot be said for spending on scratchcards which is considerably under-recorded. This we do no analysis of scratchcards.

[37] Control variables included were other forms income besides earnings, gender, age, marital status, and employment status. The probability of playing was determined by the same variables plus indicators of whether the individuals were cigarette and newspaper purchaser (and so would frequent shops that were often lottery ticket vendors. The results further suggest that the demand for tickets is not independent of the sources of income. For example, we find that the effect of £1 extra child benefit on lottery spending is much higher than £1 of extra earnings.

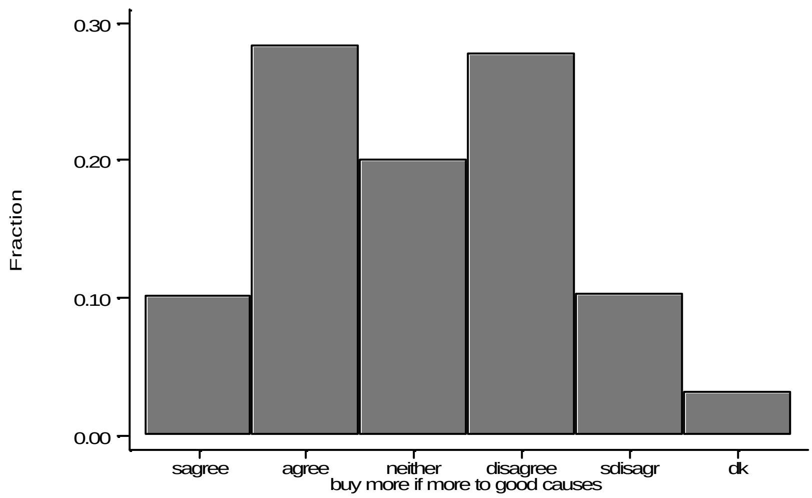
[38] That is, the probability of playing times the amount spent if a player.

Figure 8 Lottery spending and individual incomes



74. This rather begs the question: that if the good causes expenditure does no good, because it crowds out government expenditure, and if, in any case, it is inefficient and inequitable to raise the money from taxing the lottery; should we have a lottery? The answer, of course, is yes, on standard consumer sovereignty grounds: players happily part with the cash to participate, the operator gladly receives it, and it does little harm to anyone else. However, the case for taxing it to fund extensive expenditures seems weak.

Figure 9 Should more go to good causes?



75. One possible role that the good causes levy might have is actually to promote play if players attach some value to contributing to such causes. Survey evidence provides no clear support for this idea: respondents to the survey conducted by the Consumers Association in 1996 were divided on the question of whether more should go to good

causes or not: as Figure 9 shows about the same number agreed with the view that they would buy more if more went to good causes as disagreed.

Summary and Conclusion

76. Our analysis has considered most of the important questions relevant to running a lottery. Our methodology for analysing the implications of game design is, as far as we are aware, the most analytically rigorous yet to be applied to this issue, and yet it reflects the informal received wisdoms that dominate industry debate. Thus, it probably captures many of the important features of realities of the game but provides a degree of abstraction from reality to allow counterfactual changes to be analysed in a formal and quantitative way.

77. The impacts of changing the allocation of prize money, the takeout rate and the game format were investigated using an empirical model of sales. Simulations performed for different allocations of prize money seemed to indicate that the current format would sell less well than alternative designs where **less** money was devoted to the jackpot.

78. Our analysis is computationally very complex and relies on numerical methods to simulate the effects of reforms – there is no guarantee that the method will always produce a solution. Thus, our analyses of changes in the take-out rate and in the format of the game are more speculative. They do *suggest* that the take-out rate is too low – more revenue could be raised if the take-out rate were increased despite the drop in sales. They also suggest that 6/53, while it has some attractions, would, without other changes, lower the mean return to playing, raise the variance, as well as raise the skewness. The first two effects would lower sales and only the latter would raise them and, without other changes, the overall result would be lower sales.

Appendix A Calculation of Moments

Consider a lottery with a total of p +q prizes, the first p of which are pari-mutuel prizes and the next q are fixed-value prizes. The i^{th} prize is won with a probability π_i. The amount won for each of the fixed value prizes is given by W_i, For the i^{th} pari-mutuel prize, the jackpot is given by J_i but the amount won depends on the number of other people who have correctly guessed the numbers. Since S is large, the number of winners can be approximated by a Poisson distribution so that the probability of j winners for prize i if S tickets are bought is given by:

$$P_{ij} = e^{-S\pi_i} (S\pi_i)^j \ / j!$$

The expected value of a lottery ticket can then be approximated by:

$$\mu_1 = \sum_i \sum_j \pi_i P_{ij} J_i/(j+1) + \sum_i \pi_i W_i$$

$$= \sum_i (\sum_j \pi_i (e^{-S\pi_i} (S\pi_i)^j /j!) J_i/(j+1) + \sum_i \pi_i W_i$$

$$= \sum_i \pi_i J_i e^{-S\pi_i} (\sum_j (S\pi_i)^j /(j+1)!) + \sum_i \pi_i W_i$$

$$= \sum_i \pi_i J_i e^{-S\pi_i} (\sum_j (S\pi_i)^{j+1} /(j+1)!) / S\pi_i + \sum_i \pi_i W_i$$

Approximating S by infinity, the second summation can be simplified by using the exponential series, so that

$$\mu_1 \cong \sum_i \pi_i J_i e^{-S\pi_i} (e^{S\pi_i} -1)/ S\pi_i + \sum_i \pi_i W_i$$

$$= \sum_i J_i (1 - e^{-S\pi_i})/ S + \sum_i \pi_i W_i$$

The second and third moments are given by:

$$\mu_2 = \sum_i \sum_j \pi_i P_{ij} (J_i/(j+1))^2 + \sum_i \pi_i W_i^2$$

$$= \sum_i \sum_j \pi_i (e^{-S\pi_i} (S\pi_i)^j /j!) (J_i/(j+1))^2 + \sum_i \pi_i W_i^2$$

$$= \sum_i \pi_i J_i^2 e^{-S\pi_i} \sum_j (S\pi_i)^j /((j+1)(j+1)!) + \sum_i \pi_i W_i^2$$

$$\mu_3 = \sum_i \sum_j \pi_i P_{ij} (J_i/(j+1))^3 + \sum_i \pi_i W_i^3$$

$$= \sum_i \sum_j \pi_i (e^{-S\pi_i} (S\pi_i)^j /j!) (J_i/(j+1))^3 + \sum_i \pi_i W_i^3$$

$$= \sum_i \pi_i J_i^3 e^{-S\pi_i} \sum_j (S\pi_i)^j /((j+1)^2(j+1)!) + \sum_i \pi_i W_i^3$$

These last two series can be shown to converge by using a simple ratio test which states given an infinite series of positive terms $a_1 + a_2 + a_3 + a_4 + \ldots + a_n + a_{n+1} + \ldots$ if the ratio a_{n+1}/a_n tends to a limit A as n tends to infinity then if A<1 the series converges (absolutely) and when A>1 the series diverges. If A=1 the test gives no information.

The variance and skewness (central moments) can then be derived using the expansions below, although in practice these barely differ from the moments because the mean is so small (of order 1) compared to the second moment and third moments, which were of order 10^5 and 10^12 respectively.

$$E(X-\mu_1)^2 = \mu_2^2 - \mu_1^2$$

$$E(X-\mu_1)^3 = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3$$

The size of the jackpots were calculated by approximating the amount paid out in three ball prizes as $S^*\pi_3$ and then dividing the remaining prize money as laid out in section 3.1. The moments were then evaluated by truncating each of the infinite series to the first fifty terms, which ensured the calculations were accurate to five significant figures. In order to ensure numerical precision during estimation the second moment was scaled by 10^5 and the third moment by 10^{12}.

Appendix B Modelling Results

The results of the regressions give a high R squared (possibly because a large amount of the variation in sales is explained by the Wednesday dummy) although other signs are less encouraging. Inspection of the residuals plots reveal that the regression seems to consistently underpredict sales for double rollovers and the first two double rollovers in particular. Although the AR test is passed, indicating absence of autocorrelation, as does the ARCH test, the χ^2 for heteroscedasticity and the normality and Reset tests all fail.

The heteroscedasticity test may be picking up genuine heteroscedasticity it can also fail because of some form of misspecification, which would mean that the coefficients are biased. However, plots of the squared residuals against the size of the jackpot enhancement seem to confirm that the regression is genuinely heteroscedastic and variance increases with the size of the rollover, which seems plausible. The normality test probably fails because of the large outliers caused by the first rollover. The failure of the Reset indicates the possibility of a misspecified functional form (although the log regressions does no better) or an omitted variable, either of which implies that the coefficients will be biased. Parameter constancy tests for both models were also carried out on the remaining observations and passed, although the forecast sample did not contain any double rollovers. The forecasts also systematically under predicted although this may be due to inadequate modelling of the time trend.

A Wald test was also carried out to see whether dummies allowing for a different slopes on a Wednesday should have been included. This gave an F statistic of 164.56 [0.0000] which has a 0% probability level, indicating that they should have been included. However, including slope dummies gave coefficients of very different signs and magnitudes for the moments for Saturday and Wednesday which is somewhat difficult to justify under the hypothesis that mean, variance and skewness are the driving forces of demand for lottery tickets. Although this could be a bias due to an omitted variable, this seems to lend little support to the hypothesis. This interaction could be the reason the Reset test failed. It could also be argued that the introduction of the Thunderball may have changed the response to the moment distributions, since the consumers with less of a taste for skewness and a greater dislike of variance might have stopped buying the normal lottery tickets and instead bought tickets for the Thunderball draw. The hypothesis that the response to the moments was different after the Thunderball draw was introduced was tested by applying a Wald test to slope dummies corresponding to the introduction of the Thunderball draw. Although the hypothesis that these dummies were zero was rejected with the regression in its original form, producing an F statistic of 19.083 [0.0000], once Wednesday slope dummies had been introduced the hypothesis could not be rejected, producing an F statistic of 0.12067 [0.9479].

We also include a variable to capture the draw when the operator added £20m to the 5+b prize pool.

It is possible some of the problems arise due to the failure to include a variable relating to advertising and media coverage. Double rollovers attracted particularly large amounts of coverage during the early days of the lottery and this may explain the large outliers associated with the double rollovers. Modelling the inevitable rise and decline of interest in the lottery with a simple quadratic or linear time trend is almost certainly inadequate, although if this is uncorrelated with any of the other variables in the regression, this should not cause any bias.

Finally, although numbers are chosen non-randomly, the moments have been calculated on the basis of randomly chosen numbers, and so the expected value calculations will be biased downwards and, strictly speaking, differ with the choice of numbers. The results in Farell, Hartley, Lanot and Walker (2000) suggest that conscious selection have only a small effect on the mean. Similarly if tickets are bought by syndicates, then consumers face a different, less skewed, distribution.

OLS results: dependent variable lottery sales (millions)

	Coefficient	t-value
constant term	-91.916**	-4.93619
Mean	294.34**	4.334649
Variance (/10^5)	-4.1716**	-2.45793
Skewness (/10^{12})	1.6333*	1.825793
Wednesday	-29.154**	-4.702865
Dummy for Sat. after intro. of Wed. draw	4.2761	0.770468
No. of terminals (1000s)	1.2562**	12.09629
weeks since first draw	-0.06403**	-10.2617
No. of Wed. draws	0.16522**	6.961615
Square of the Weds. Draw No.	-0.00051**	-4.20045
Thunderball dummy for Saturday draws	-0.92387**	-2.31291
Thunderball dummy for Wednesday draws	0.097721	0.161774
Dummy for delayed Diana draw	-11.664**	-15.2299
Dummy for superdraw in which 5b prize topped up	-68.49**	-4.64875
1st lag of Sat. sales before intro. of Wed. draw	0.20469**	2.415734
2nd lag of Sat sales before intro. of Wed. draw	0.065805	1.471785
1st lag of Sat. sales after intro. of Wed. draw	0.14921**	2.014147
2nd lag of Sat. sales after intro. of Wed. draw	0.023499	0.630068
1st lag of Wed. sales	0.027889	0.526575
2nd lag of Wed. sales	0.068137**	2.143549
quarter two dummy	2.4337**	4.198641
quarter three dummy	1.1806**	4.114449
quarter one dummy	1.2756**	5.023234
R^2	0.973825	
Diagnostic Tests	AR 1- 2 F(2,447) = 0.87574 [0.4173]	
	ARCH 1 F(1,447) = 0.20186 [0.6534]	
	Normality $\chi^2(2)$= 2607.2 [0.0000]	
	χ^2 F(34,414) = 17.403 [0.0000]	
	RESET F(1,448) = 111.36 [0.0000]	
Parameter Constancy	Forecast χ^2 (14)= 10.99 [0.6868]	
	Chow F(14,449) = 0.5078 [0.9289]	

Note: t statistics calculated using HCSEs.