The measurement problem in Quantum Mechanics

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(Dated: February 5, 2021)

Abstract: The measurement problem in quantum mechanics is the problem of how to conciliate the deterministic evolution of the Schrödinger equation with the random collapse experienced when a measurement is performed. This mystery is one of the most remarkable enigmas in the theory that successfully constitutes the foundation of much of modern physics. In this work we aim to expose the problem and briefly discuss the main proposed alternatives that have given rise to the so-called interpretations of quantum mechanics.

I. INTRODUCTION

As stated in the standard postulates of quantum mechanics (QM) any state of a quantum system $|\psi\rangle$ evolves deterministically according to the Schrödinger equation $i\hbar\partial_t|\psi(t)\rangle = \mathcal{H}|\psi(t)\rangle$ but when a destructive measurement of a certain observable $\mathcal{A}$ of spectrum $\{\alpha_i\}_{i\in I}$ is performed by an observer, the measurement outcome will randomly be some eigenvalue with probability given by the Born rule $\mathcal{P}_{|\psi\rangle}(\mathcal{A}:\alpha_i) = ||\Pi_i|\psi\rangle||^2$, where Π_i is the projector into the subspace of eigenvalue α_i. The state of the system is destroyed in the process (in the sense that we will later precise). On the other hand, if the measurement is filtering for a subset of eigenvalues $\{\alpha_i\}_{i\in J\subset I}$, the system will either evolve into the projected state $\Pi_J|\psi\rangle/||\Pi_J|\psi\rangle||$ leaving the measurement device without displaying any detection, with probability $||\Pi_J|\psi\rangle||^2$ (where $\Pi_J = \sum_{i\in J}\Pi_i$), or it will be destructively measured on one of the remaining eigenvalues. In any case, we say that a measurement causes a collapse.

The collapse postulate endows the act of measurement with an uncomfortable prominent role, distinguishing the action of observers from the rest of physical processes with its own evolution, which, moreover, is nondeterministic, instantaneous and nonlocal. It may seem artificial and to be hidden some physics we yet do not understand, but, furthermore, as we are going to discuss, it ends up being paradoxical.

II. THE MEASUREMENT PROBLEM

QM is meant to be (in principle) a universal theory, that is, applicable to all physical systems. Hence, if we assume a reductionist hypothesis, i.e., that the observer and the measurement apparatus are just a part of the physical universe and consequently many-particle systems and nothing more, then all measurement processes should be explained in terms of quantum interacting particles following the Schrödinger equation.

We consider the observation on a quantum system of a certain observable $\mathcal{A}$ with eigenbasis $\{|\alpha_i\rangle\}_{i\in I}$ (nondegenerate, for simplicity), and the measurement device used, filtering for $\{\alpha_i\}_{i\in J\subset I}$, as a many-particle system. Notice that a completely destructive measurement will just be the particular case of $J=\emptyset$. If the observed system is initially prepared in one of the eigenstates and the device is in an independent state ready for measure $|R\rangle$, then the initial global state will be the tensor product:

$$|\psi_i(t_0)\rangle = |\alpha_i\rangle \otimes |R\rangle \tag{1}$$

If we assume the measurement to be destructive for the considered eigenstate ($i \notin J$), Born's rule implies that the only possible result is the detection of the corresponding eigenvalue. Therefore, after a time t such that the measurement is completed, the global state will have evolved following the Schrödinger equation into an expression of the form:

$$|\psi_i(t)\rangle = |D^u_{\alpha_i}\rangle := \sum_{j\in I,k\in K} u_{jk}|\alpha_j\rangle \otimes |M_{\alpha_i},k\rangle \tag{2}$$

where $\{|M_{\alpha_i},k\rangle\}_{k\in K}$ are the different microscopic states of the measurement device indicating that the result of the observation is α_i through a macroscopic reading variable. The device gets entangled with the system but remains with a definite reading variable yielding a definite result. The state of the system is said to be destroyed because in practice it becomes inaccessible due to the impossibility of knowing exactly the initial microscopic state of the device and the subsequent interaction. On the other hand, if the measurement is filtering for the considered eigenvalue ($i \in J$), according to the measurement postulates the state of the system remains unaltered and the measurement device will not exhibit any detection, so its state will evolve into a microscopically different state $|R,i\rangle$ macroscopically equivalent to $|R\rangle$, resulting in:

$$|\psi_i(t)\rangle = |\alpha_i\rangle \otimes |R,i\rangle \tag{3}$$

Now we suppose that the observed system is initially in a general superposition of eigenstates $\sum_{i\in I} c_i|\alpha_i\rangle$. Then, the initial global state will be:

$$|\psi(t_0)\rangle = \sum_{i\in I} c_i|\alpha_i\rangle \otimes |R\rangle = \sum_{i\in I} c_i|\psi_i(t_0)\rangle \tag{4}$$

and the linear evolution given by the Schrödinger equation results in the global state when the measurement is completed being:

$$|\psi(t)\rangle = \sum_{i\in I} c_i|\psi_i(t)\rangle = \sum_{k\in I\setminus J} c_k|D^u_{\alpha_k}\rangle + c_J \sum_{j\in J} c'_j|\alpha_j\rangle \otimes |R,j\rangle \tag{5}$$

where $c_J = || \sum_{j\in J} c_j|\alpha_j\rangle||$ and $c'_j = c_j/c_J$ are introduced for normalization. This final state does not yield any definite measurement result, as the device gets entangled with the system in a superposition each term of which describes either the measurement device having detected a different definite result α_k destroying the state of the system ($|D^u_{\alpha_k}\rangle$), or the observed system projected into the filtering eigenspace together with the device macroscopically unaltered ($\sum_{j\in J} c'_j|\alpha_j\rangle \otimes |R,j\rangle$). However, contrary to expectations, no random collapse into one of these terms emerges from the Schrödinger equation. One might think that the collapse must happen after, for example through the human observation of the measurement device, but if we stick to the reductionist hypothesis the human observer would be again a quantum system and we would get the same result. This argument will hold *ad infinitum*. Consequently, the assumption of universality of QM and the reductionist hypothesis lead to a contradiction, known as the measurement problem. We are left with two main interpretative alternatives (and some others, cited in [1]):

1. Assuming that the standard formulation given by quantum states does not represent a complete description of an ontological reality. Such a description may exist in the form of hidden-variables (section VI), or not, and quantum states may not describe a reality at all (section III).
2. Assuming that the standard formulation is not completely right and the evolution needs a reformulation in order to be consistent. This reformulation can be intended to induce the collapse rather than postulate it (section V) or to cope with the suppression of it (section IV).

Otherwise, we would be forced to refuse the reductionist hypothesis assuming that there is some kind of supra-physical entity in the observer, the consciousness for instance, that causes the collapse. This bold claim, a part from having no evidence and going beyond the materialism in which natural science is grounded, it is neither free of conceptual ill-definition, because the level of consciousness needed to cause the collapse by observing would be unclear. In addition, it would imply a universe in superposition until the first conscious being appeared.

III. NON-REALIST INTERPRETATIONS

Non-realist interpretations, headed by the original Copenhagen interpretation developed by Bohr, Born and Heisenberg among others in 1927 (which still remains as the most widely taught), assume an epistemological instrumentalism. Quantum states represent all possible dynamical knowledge we can obtain of a system (there are no hidden-variables) and the formalism is just a set of rules that describe the behaviour of these states and how we interact with them when we measure. The collapse is taken to be phenomenological and responds to the fact that our knowledge is updated after a measurement. Under this epistemological approach, the theory is meant to be a pragmatic tool to construct physical models and make calculations, rather than providing a complete fundamental description of an ontological physical reality. Embracing logical empiricism, questions such as if superposition and reality between measurements exists or not are meaningless and beyond physics. The concepts of observer and measurement are not well-defined as physical entities even though they play a crucial role in the formalism, and neither is the boundary between the microscopic world quantum described and the macroscopic world classically described.

From this point of view, the measurement problem is not even considered, as the theory is not intended to be a fundamental description, accounting for all processes including measurements. Nevertheless, if QM is not the fundamental theory that explains the phenomenology of quantum measurements, then the question of how we can explain what measurements and results are, arises as a goal of quantum physics.

IV. MANY-WORLDS INTERPRETATION

In 1957, Hugh Everett attempted to solve the measurement problem by proposing a reformulation of the theory in which the collapse postulate, which seems to be the focus of all issues, is removed and the Schrödinger equation holds all the time everywhere. Quantum states are meant to be a complete description of an ontological reality rather than observers knowledge. Since Everett's work, many other similar formalisms with the same approach appeared, constituting the many-worlds interpretation or MWI.

As discussed in section II, the final state obtained from the Schrödinger equation after a measurement (Eq.5) is given by a superposition of terms, each representing one of the different definite outcomes that we actually observe experimentally. Thus, according to the MWI, the measurement problem is solved by removing the collapse postulate and concluding that the result of a measurement process is the superposition, meaning that all possible measurement outcomes are simultaneously realized and reality is split into branches. Each of these branches described by the corresponding normalized terms in Eq.5 where an apparent collapse and a definite result is obtained is called a world.

The entanglement between the system and the measurement device will be spread across the environment and the nearby universe, using the same argument for the system-device as a new system and the environment as a device, eventually resulting in the state of the universe $|\Psi\rangle$ split into worlds $|\Psi\rangle = \sum_{k\in I\setminus J} c_k|\psi_{w_k}\rangle + c_J|\psi_{w_J}\rangle$, each of which $|\psi_{w_i}\rangle$ describing a definite macroscopic reality [2]. In fact, as the measurement process has no longer the preferred role it had in the standard formulation, this reality branching will happen not only when a measurement is performed, but whenever a system in superposition gets entangled with the environment. This mechanism, which is extremely fast and results in the universe constantly branching out, is also responsible for decoherence, that is, the loss of interference phenomena of macroscopic objects, which explains why is highly difficult for these worlds to interfere and why we can describe them independently as if a collapse occurred. The reality superposition is reconciled with our experience by arguing that we are also entangled with the the environment, in superposition of observing all the different outcomes relative to each world, but our subjective experience corresponds to the definite reality of just one of these worlds, where an illusion of collapse is perceived.

The MWI is fully deterministic, since the Schrödinger equation holds all the time, and recovers the locality lost with the collapse, as the instantaneous action at a distance would just be an illusion of the branching of the worlds, that are nonlocal entities. However, it is not exempt of other issues. The branching of worlds is constructed from the spread of a superposition in which a subsystem is initially found across the environment. Nonetheless, in QM the decomposition of a state as a superposition is not unique. Therefore, the splitting of worlds would depend on the choice of a certain basis in which superpositions are expressed. This is called the preferred basis problem. It is argued that, since we experience worlds with localized position objects, the position eigensbasis should be the one used as a preferred basis, but this assumption brings back the special role of the measurement that MWI tried to avoid, as the choice is made according to our observations. However, it has been proposed that the problem is solved when introducing decoherence, as the preferred basis would emerge as the basis stable under environmental decoherence, i.e., the basis the states of which, even if expressed as a superposition of another basis, do not get entangled with the environment (splitting the reality) for a significantly long time, due to the nature of the interactions which in most cases are position dependent.

The main issue with the MWI is that, since it is a deterministic theory and all measurement outcomes are simultaneously realized in different worlds, then associating a probability to them may seem pointless. Nevertheless, it is possible to recover the probabilistic nature of our experience by reintroducing it as an illusion of probability arose from the observers ignorance of which world is experiencing. It is in this sense that the following probability postulate completely equivalent to Born's rule should be introduced: *an observer should set his subjective probability of the outcome of a quantum experiment proportional to the sum of the module squared of the coefficients of all worlds with that outcome.* However, although some attempts have been made, as discussed in [2], the fact that this extra postulate can not be deduced from the theory even though it is in principle fully deterministic may seem to reestablish the random collapse postulate that Everett was trying to avoid, since it postulates the probability of experiencing one branch or another, which could be understood as the probability of our experience collapsing into one branch or another, but can not explain why we experience our particular branch.

V. OBJECTIVE-COLLAPSE THEORIES

Another different approach towards the measurement problem is to reformulate the Schrödinger evolution in a way in which the collapse emerges naturally from it without the need of any extra postulate, solving directly the paradox. The theories that explores this possibility are called spontaneous or objective-collapse theories. The first one, and one of the most well-known is the GRW theory, proposed by Giancarlo Ghirardi, Alberto Rimini and Tullio Weber in 1986.

We consider a quantum system of many (distinguishable) particles described by a state $|\psi\rangle$, that is taken to be a complete description as in the MWI. The GRW theory states that each particle is subjected to random (in space and time) sudden spontaneous localization processes, known as jumps or hittings. A jump localising the i-th particle around the position a is described by the global state $|\psi\rangle$ instantaneously turning into $|\psi_a^i\rangle/\sqrt{\langle\psi_a^i|\psi_a^i\rangle}$, where $|\psi_a^i\rangle = \mathcal{L}_a^i|\psi\rangle$ and $\mathcal{L}_a^i$ is the i-th particle localization operator around the position a, given by:

$$\mathcal{L}_a^i = \left(\frac{1}{\pi r^2}\right)^{\frac{3}{4}} exp\left[-\frac{(\mathcal{X}_i - a)^2}{2r^2}\right] \tag{6}$$

where $\mathcal{X}_i$ is the position operator of the i-th particle and r the localization distance. For each particle, jumps are randomly distributed in time according to a Poisson distribution with mean rate λ and the probability distribution in space is given by $\mathcal{P}_i(a) = \langle\psi_a^i|\psi_a^i\rangle$. Hence, jumps occur with higher probability at those places where Born's rule gives a higher probability of finding the system when measuring. Between two jumps the system evolves according to the Schrödinger equation.

The key feature of the GRW theory that really solves the measurement problem and explains why macroscopic superpositions are not observed is that the hitting mechanism is enhance by a large number of particles, which is known as the amplification mechanism. For one particle initially delocalized over a distance greater than r it is easy to see that, multiplying its wave function by the gaussian $\mathcal{L}_a$, a jump localizes it. Similarly, if the particle is in a highly localized state $|\phi_b\rangle$ around a position b (meaning that $\phi_b(x) \simeq 0$ for $|x-b| > \sigma \ll r$) then:

$$\mathcal{L}_a|\phi_b\rangle \simeq \begin{cases} c|\phi_b\rangle & \text{if } |b-a| \simeq 0 \\ 0 & \text{if } |b-a| \gg 0 \end{cases} \tag{7}$$

Therefore, considering for instance a N-particle system in superposition of being around many distant positions

$\{x_j\}_{j\in J}$, meaning that all the particles are highly localized around positions $x_j^1, ..., x_j^N$ near x_j on each of the terms and $|x_j^i - x_l^i| \gg r \; \forall j \neq l \; \forall i$, then:

$$\mathcal{L}^i_{x^i_k}\left(\sum_{j\in J} c_j |\phi^1_{x^1_j}\rangle \cdots |\phi^N_{x^N_j}\rangle\right) \simeq c'_k |\phi^1_{x^1_k}\rangle \cdots |\phi^N_{x^N_k}\rangle \tag{8}$$

Thus, a localization of just one particle around one of its positions triggers the collapse of the whole system around it and since any jump localizing the i-th particle in a significantly different position than $\{x_j^i\}_{j\in J}$ will lead to an almost zero state, the probability of occurring is almost zero. Consequently, the probability of the whole system collapsing in time is the sum of probabilities of each of the particles being localized. In fact, using the density matrix formalism [3], it can be shown that, in general, the center of masses of a system of N particles collapses with a time rate $\Lambda = N\lambda$.

The usual values proposed for the parameters in order to preserve the well-tested quantum behaviour in microscopic system, besides tiny deviations, but induce a rapid collapse in macroscopic superpositions are $\lambda = 10^{-16}s^{-1}$ and $r = 10^{-7}m$ [4]. With this choice, a particle will be spontaneously localized on average every hundred million years, but macroscopic superpositions, as the obtained after a measurement process, will collapse in time scales of $10^{-7}s$. Born's rule is preserved because of the space probability distribution chosen for the jumps.

It is clear that in the GRW theory the position basis has a preferred role, but, as we mentioned, it can be justified claiming that is the basis stable under environmental decoherence. What constitutes a major drawback for the theory is that it is unable to describe systems of identical particles, because localizations do not respect the symmetry conditions of such systems. However, the problem is overcome by the extension known as the QMUPL model or other collapse models such as the CSL model, in which the discrete jumps are replaced by a continuous stochastic evolution, or the Diósi-Penrose model, in which is gravity or, more precisely, a critical superposition in the curvature of space-time what causes the collapse.

Nonetheless, all collapse models violate the energy conservation principle for isolated systems, increasing it at small constant rate due to the acceleration of particles caused by a diffusion process induced by the collapse jumps noise, similarly to a Brownian motion. However, there are some reformulations that try to recover energy conservation by including dissipative effects in the theory. In addition, unlike the standard quantum theory, no collapse model is able to conciliate the nonlocal hitting processes with the relativistic casual locality, though some proposals for a Lorentz-covariant extension are under research [4]. It is also criticized the fact that localizations are not absolute and leave always an infinite region of space with a really small but non-zero wave function, meaning that particles could be infinitely far apart after a measurement. However, according to the objective-collapse supporters, this tails problem would be a problem of the standard description as well, and it is within the objective-collapse framework that the square module of wave functions can be interpreted as an actual particle matter density rather than a potential measurement probability distribution for a point-like particle. Hence, the tails would just represent an insignificant small amount of spread out matter of the particles that can be considered point-like for all practical purposes.

VI. HIDDEN-VARIABLE THEORIES

The hidden-variable approach to avoid the measurement problem consists in assume that quantum states do not represent a complete description of a physical reality, but claim that such a description exists in the form of classical well-defined variables that evolve deterministically and the value of which is not simultaneously accessible for us due to technical impossibility. From this point of view, uncertainty and the probabilistic nature of our experience would just respond to our ignorance about the real value of these variables and measurements will represent the revelation of them. However, in 1964 John Bell proved that a local hidden-variable assumption, i.e., without instantaneous action at a distance, should satisfy an inequality of expectation values in conflict with the standard QM when performing a certain experiment. The realization of the experiment proved the violation of Bell's inequality rejecting local hidden-variable theories. In addition, a similar result known as the Bell-Kochen-Specker theorem ruled out the possibility of a general noncontextual hidden-variable theory, meaning that it is impossible for all quantum observables to have a well-defined hidden value simultaneously, but just a chosen set of compatible ones, while the rest have to remain undetermined until a measurement is performed.

Nevertheless, nonlocal and contextual hidden-variable theories would be still allowed. The most well-known was proposed by David Bohm in 1952, recovering the idea first suggested by De Broglie of a pilot wave theory. Assuming for simplicity a universe with a fixed number of particles N without spin, Bohmian mechanics (also known as De Broglie-Bohm theory) states that their positions are always classically defined as a point in a configuration space $Q = (Q_1, \ldots, Q_N) \in \mathcal{Q}$ and the wave function of the universe $\Psi(q,t)$, which is a function of all possible configurations $q \in \mathcal{Q}$, evolves all the time following the Schrödinger equation. This wave function acts as a pilot wave, guiding the motion of all particles according to the guiding equation:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k} Im\left(\frac{\nabla_k \Psi}{\Psi}\right)(Q,t) \tag{9}$$

which can be thought as a generalization of the free particle case, where from its wave function $\psi(x,t) = Ae^{ikx-\omega t}$ it is clear that $ik = imv/\hbar = \nabla\psi/\psi$, though some more formal derivations exist [5]. Other properties such as charge or mass, although linked to the particles, are taken to be spread out in the wave function, and it is the wave function itself that account for all interactions and determines the dynamical evolution of the system; particles do not act back upon the wave function nor interact directly between them. The nonlocality in the theory is clearly reflected in the fact that, at a given time, the motion of a certain particle depends on the position of all particles in that time. It is possible to extend the theory to deal with spin particles and to handle creation and destruction operators, but due to the crucial role that this nonlocal equation plays, a many-particle relativistic extension has not been achieved yet.

According to Bohmian mechanics, position is the only observable intrinsic to particles and the only one that is directly measurable. The other observables, such as spin, are considered properties of the wave function, and are always measured indirectly by measuring particle positions. Like in the MWI, the wave function never collapses, so a measurement will just split it into components. However, since different components are extremely difficult to interfere, just the component corresponding to the measured result will guide with high accuracy the actual evolution of the particles of the system and the measurement device. Therefore, the use of the collapsed wave function is justified as a legitimate simplification for practical purposes. For example, measuring the spin of a 1/2-spin particle with a Stern-Gerlach will separate the wave function into two wave packets moving in different directions, the term of spin up and spin down, but the particle, depending on its initial position, will end up in one of these packets being guided by it and eventually being detected by a screen detector revealing which spin branch has followed. The measurement problem is solved in the same way that in the MWI but with the difference that particles act as pointers that select the branch that we experience. Moreover, defining the known as the conditional wave function of a subsystem it collapses exactly as stated in the standard theory [5].

As mentioned, our probabilistic perception emerges from the fact that positions, although determined, are hidden and unknown. Born's rule is recovered thanks to the quantum equilibrium hypothesis, that is, that for a given wave function Ψ particles tend to be statistically distributed in $\mathcal{Q}$ according to $\mathcal{P}_\Psi(q) = |\Psi(q)|^2$. The reason is that it is an equivariant distribution in the sense that its temporal variation because of the evolution of the wave function is equal to the variation due to the actual flow of particles: $\mathcal{P}_{\Psi(t)}(q) = \mathcal{P}_\Psi(q)(t)$ [5]. Thus, if particles are initially distributed according to $|\Psi|^2$ the evolution will preserve the distribution and, furthermore, even for initial conditions out of this equilibrium the system would tend to evolve towards it [6]. Therefore, in Bohmian mechanics Born's rule is not a postulate but is deduced from the theory, and although it could in principle be violated, it will hold in the vast majority of cases.

VII. CONCLUSIONS

In this paper we have shown the apparent contradictory nature of the two types of evolution postulated in QM and presented the main theoretical approaches that tackle it. In order to discern the validity of these different proposals, they have to be experimentally tested. By modifying the dynamical evolution, collapse models make slightly different predictions than the standard QM that are susceptible to be testable in a near future. As discussed in [4], the main experiments proposed consist either in detecting the energy increase and the diffusion process predicted or in interferometric experiments with considerable large systems. On the other hand, Bohmian mechanics and the MWI may be in principle empirically equivalent to the standard formulation. However, the hypothetical non-equilibrium regime in Bohmian mechanics may provide some potentially testable deviations and the experimental capability to interfere hypothetical macroscopic superpositions to test the existence of different worlds, although extremely difficult because of decoherence, could be achievable in a far future. Only time will tell if any of the presented proposals (or any other) is proven to be empirically valid putting an end to the enduring discussion about the measurement problem.

Acknowledgments

I would like to thank my advisor Dr. Lluís Garrido for his guidance, and my parents, without whom and their unconditional support during all these years it would have been impossible.

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