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“Detecting multiple level shifts in bounded time series”

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Abstract

The paper proposes a sequential statistical procedure to test for the presence of level shifts affecting bounded time series, regardless of their order of integration. The paper shows that bounds are relevant for the statistic that assume that the time series are integrated of order one, whereas they do not affect the limiting distribution of the statistic that is defined for time series that are integrated of order zero. The paper proposes a union rejection statistic for bounded processes that does not require information about the order of integration of the stochastic processes. The model specification is general enough to consider the existence of structural breaks that can affect either the level of the time series and/or the bounds that limit its evolution. Monte Carlo simulations indicate that the procedure works well in finite samples. An empirical application that focuses on the Swiss franc against the euro exchange rate evolution illustrates the usefulness of the proposal.

JEL Classification: C12, C22.

Keywords: Structural breaks, Bounded processes, Changing bounds.

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Detecting multiple level shifts in bounded time series*

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July 22, 2021

Abstract

The paper proposes a sequential statistical procedure to test for the presence of level shifts affecting bounded time series, regardless of their order of integration. The paper shows that bounds are relevant for the statistic that assume that the time series are integrated of order one, whereas they do not affect the limiting distribution of the statistic that is defined for time series that are integrated of order zero. The paper proposes a union rejection statistic for bounded processes that does not require information about the order of integration of the stochastic processes. The model specification is general enough to consider the existence of structural breaks that can affect either the level of the time series and/or the bounds that limit its evolution. Monte Carlo simulations indicate that the procedure works well in finite samples. An empirical application that focuses on the Swiss franc against the euro exchange rate evolution illustrates the usefulness of the proposal.

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1 Introduction

The presence of structural breaks affecting non-trending time series macroeconomic variables is an interesting issue since evidence on popular economic theories might be affected by unattended structural breaks. However, the estimation of structural breaks depends on the stochastic properties of the time series. For model specifications in which the recurrent shocks have transitory effects – i.e., the error term of the model is an integrated of order zero, I(0), stochastic process – consistent estimates of the break dates can be obtained considering that level shifts have a fixed magnitude.

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However, this is not the case for models specifications in which the recurrent shocks have permanent effects – i.e., the disturbance term is an integrated of order one, I(1), stochastic process. In this case the magnitude of the level shifts have to be of the same order of magnitude as the stochastic trend to be asymptotically non-negligible so that the structural break dates can be consistently estimated.

The analysis is more involved if we realize that some non-trending macroeconomic time series have the characteristic of being bounded, either by construction, by definition or by institutional/political constraints. Variables such as interest rates, unemployment rates, labour participation rates, current account over GDP ratio, exchange rates under non-floating regimes, flow volume of rivers, among others, are examples of time series bounded above, below or both. This feature implies that these time series can not have infinite support on the real line. As noted by Granger (2010), if the limitations of a limited process are activated quite often this will bias the standard test results. In this regard, there are some proposals in the literature that show that the presence of bounds needs to be considered, for instance, when testing the order of integration using either standard unit root tests – see Cavaliere (2005a), Carrion-i-Silvestre and Gadea (2013) and Cavaliere and Xu (2014) – or unit root tests with structural breaks – see Carrion-i-Silvestre and Gadea (2016) – and when working with fractional integrated stochastic processes – Trokic (2013). From an empirical point of view, this characteristic is also relevant in analyses that focus on current account sustainability – see Herwartz and Xu (2008) – mean reversion of exchange rates – Cavaliere (2005b) – and unemployment rate persistence – see Carrion-i-Silvestre, Gadea and Montañés (2020) – among others.

To the best of our knowledge, there are no proposals in the literature that specifically test for the presence of structural breaks affecting bounded stochastic processes. The aim of this paper is to fill this gap with the design of a robust statistical procedure to detect multiple level shifts for bounded processes regardless of their order of integration. This is of great importance since, for instance, knowledge about the presence of structural breaks when assessing the order of integration of bounded time series is relevant if misleading conclusions are to be avoided – see Perron (1989) and Carrion-i-Silvestre and Gadea (2016). The approach that is designed in the paper contributes and extends the robust structural analysis literature powered in Perron and Yabu (2009), Harvey, Leybourne and Taylor (2010) and Kejriwal and Perron (2010) to bounded stochastic processes. To be specific, the proposal relies on the generalized fluctuation statistics in Harvey, Leybourne and Taylor (2010), who define statistics that can be easily implemented to detect multiple level shifts for unbounded time series. The paper shows that the limiting distribution of the generalized fluctuation statistics depends on the bounds when they are computed for bounded I(1) non-stationary stochastic processes. However, the effect is asymptotically negligible for bounded I(0) stationary stochastic processes of the type that are considered in the paper. The model specification is flexible enough to include the possibility of two different sets of structural breaks, depending on whether the structural breaks affect either the level and/or the bounds. This adds more complexity to the analysis, although it increases the ability of the model to accommodate this type of situations that might arise in empirical applications. The definition of a union rejection statistic is also suggested in the paper to conduct statistical inference that is robust to the order of integration of the time series. The presence of structural breaks that affect the bounds turns out to be a crucial feature for the validity of the union rejection rule. In this case, it is possible to implement a modification that will restore the implementation of the robust test statistic.

The rest of the paper is organized as follows. Section 2 describes the model and the various possibilities that can be accounted for when modelling bounded stochastic processes. Section 3 describes the statistics that are used, derives the corresponding limiting distribution, and computes asymptotic and finite sample (response-surfaces-based) critical values. Section 4 discusses the estimation of the long-run variance that is required in the implementation of the statistics. Further, it derives the limiting distribution of the statistics when the wrong order of integration of the time series is assumed, which justifies the implementation of the union rejection rule. Section 5 details the sequential testing strategy that is suggested to detect multiple level shifts. Section 6 conducts an extensive simulation experiment to analyse the finite sample performance of the test statistics that are proposed. Section 7 provides an empirical illustration that analyses the recent evolution of the Swiss franc versus euro exchange rate. Finally, Section 8 concludes. The proofs and additional simulation results are collected in the appendix.

2 The multiple level breaks and breaking bounds model

Let x_t be a stochastic process with data generating process (DGP) given by:

$$x_t = \mu + \sum_{i=1}^{m} \gamma_i DU_{i,t} + y_t \quad (1)$$

$$y_t = \alpha y_{t-1} + u_t, \quad (2)$$

$t = 1, \ldots, T$, with $x_t \in \left[\underline{b}_t, \overline{b}_t\right]$ almost surely for all t, where $\left[\underline{b}_t, \overline{b}_t\right]$ denote the known boundaries, and $y_0 = O_p(1)$.[1] The bounded integrated of order one (BI(1)) case for x_t is implemented setting $\alpha = \exp(-\kappa/T) \approx 1 - \kappa/T$, with $\kappa \geq 0$ being the non-centrality parameter, so that the model specification covers both the case in which the time series is a near-bounded-integrated process – i.e., a NBI(1) process with $\kappa > 0$ – and a BI(1) non-stationary process – when $\kappa = 0$. The bounded integrated of order zero (BI(0)) case for x_t is considered setting $|\alpha| < 1$. The model specification is general enough to allow for the presence of multiple structural breaks that might act in two areas: (i) structural breaks that cause level shifts and (ii) structural breaks that change the boundaries $\left[\underline{b}_t, \overline{b}_t\right]$.

The first set of structural breaks concerns the deterministic component $D_t = \mu + \sum_{i=1}^{m} \gamma_i DU_{i,t}$, which is defined by the step dummy regressors $DU_{i,t} = 1\left(t > T_i^0\right)$, where $1(\cdot)$ is the indicator function, $T_i^0 = \lfloor \lambda_i^0 T \rfloor$ is the i-th break date, $i = 1, \ldots, m$ – $\lfloor \cdot \rfloor$ denotes the integer part – $\lambda_i^0 \in \Lambda =$

[1] The model can accommodate the cases of stochastic processes that are only limited below – i.e., $x_t \in [\underline{b}_t, \infty]$ – or only limited above – i.e., $x_t \in \left[-\infty, \overline{b}_t\right]$ – but also covers the case of unbounded processes – i.e., $x_t \in [-\infty, \infty]$.

$[\epsilon, 1-\epsilon]$ is the break fraction parameter, and ϵ is the trimming parameter. The superscript 0 in T_i^0 and λ_i^0 is used to denote the true break date and break fraction, respectively, $i = 1, \ldots, m$. It is worth noting that the assumption of known structural breaks affecting the level is relaxed below.

The changing boundaries can be specified as $\left[\underline{b}_t, \overline{b}_t\right] = \left[\underline{b}_j, \overline{b}_j\right]$ for $\tau_{j-1}^0 < t \leq \tau_j^0$, where τ_j^0, $j = 1, \ldots, n+1$, are the break date that affect the boundaries of the time series, with the convention that $\tau_0^0 = 0$ and $\tau_{n+1}^0 = T$. The corresponding bounds break fractions are labelled as $\pi_j^0 = \tau_j^0/T$, $j = 1, \ldots, n$, and it is assumed that $\pi_1^0 < \cdots < \pi_n^0$. It is worth stressing that, as in Cavaliere (2005a) and Cavaliere and Xu (2014), throughout the paper the boundaries $\left[\underline{b}_t, \overline{b}_t\right]$ are assumed to be known, which implies in turn that if there are structural breaks that affect the bounds, they are known.[2] Consequently, the set of structural breaks that change the boundaries are assumed to be known.

Note that in general $m \neq n$ and, even if $m = n$, it might be the case that for $i = j$, $T_i^0 \neq \tau_j^0$ for some i, j. From an empirical point of view, this configuration makes sense since it would be possible that time series boundaries do not change – for instance, the boundaries that limit the unemployment rate are defined by construction – but the level of the time series might change. This defines a model specification where the structural breaks affect the level, but not the time series boundaries. However, it would be also possible that the boundaries change at a given point in time – an example would be the changes in the exchange rate fluctuation boundaries that experienced some European currencies during the late 90s – which might have also affected the level of the time series. The combination of these situations define interesting specifications for empirical analyses, which have led the paper to distinguish between two different cases depending on whether the bounds remain stable (Case A, $n = 0$) or they are affected by structural breaks (Case B, $n > 0$).

The disturbance term u_t is defined as:

$$u_t = \varepsilon_t + \underline{\xi}_t - \overline{\xi}_t, \tag{3}$$

and satisfies the following assumptions:

Assumption 1: $\varepsilon_t = C(L)\upsilon_t$, where $C(L) = \sum_{j=0}^{\infty} C_j L^j$ with $\sum_{j=0}^{\infty} j^s |C_j| < \infty$ for some $s \geq 1$, and υ_t is a martingale difference sequence adapted to the filtration $F_t = \sigma - field\{\upsilon_{t-j}; j \geq 0\}$. The long-run variance (LRV) of ε_t is given by (a) $\sigma_1^2 = \lim_{T\to\infty} E[T^{-1}(\sum_{t=1}^{T} \varepsilon_t)^2] = \sigma_\upsilon^2 C(1)^2$, (b) $\sigma_\upsilon^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E\left(\upsilon_t^2\right) < \infty$ $\forall t$, and (c) $E\left|\upsilon_t^r\right| < \infty$ for some $r > 4$.

Assumption 2: $\{\underline{\xi}_t\}_{t=1}^{T}$and $\left\{\overline{\xi}_t\right\}_{t=1}^{T}$ satisfy restrictions to ensure that $\max_{t=1,\ldots,T} |\underline{\xi}_t| = o_p\left(T^{1/2}\right)$ and $\max_{t=1,\ldots,T} |\overline{\xi}_t| = o_p\left(T^{1/2}\right)$.

Assumption 3: $(\underline{b}_t - D_t) = \underline{c}_j \sigma_1 T^{1/2} + o(1)$, $(\overline{b}_t - D_t) = \overline{c}_j \sigma_1 T^{1/2} + o(1)$ for $\tau_{j-1}^0 < t \leq \tau_j^0$, $j = 1, \ldots, n+1$, with $\tau_0^0 = 0$, $\tau_{n+1}^0 = T$, and $\underline{c}_j \leq 0 \leq \overline{c}_j$, $\underline{c}_j \neq \overline{c}_j$.

The order of integration of the bounded stochastic process $x_t \sim BI(d)$ is assumed to be either

[2] Cavaliere and Xu (2014) argue that when a time series is known to be regulated but the bounds are unknown, it might be possible that "a reasonable range of bounds can be inferred from historical observations and/or from the relevant economic theory" or from economic policy implemented by institutions – e.g., specification of target values for some macroeconomic variables. As it has been previously mentioned, this assumption does not need to be imposed on the structural breaks that change the level of the time series, which can be estimated as described below.

$d = 0$ or $d = 1$, which in turn can be used to define the magnitude of the structural breaks as in Harvey, Leybourne and Taylor (2010):

$$\gamma_i = \gamma_i^* T^{d-1/2}, \tag{4}$$

being $T^{d-1/2}$, $d \in \{0, 1\}$, the Pitman's drift and $|\gamma_i^*| < \infty$, $i = 1, \ldots, m$. This implies that the structural breaks are non-negligible when $d = 1$, whereas we deal with shrinking structural breaks when $d = 0$. In addition, it is also possible to consider the case of structural breaks with fixed magnitude if we set $\gamma_i = \gamma_i^*$. Therefore, there are up to three scenarios depending on the definition of the magnitude of the structural break that is used, all of them considered in the simulation exercise that assess the finite sample properties of the proposed statistics.

3 The fluctuation test statistic with bounds

The contribution of the paper is to design a statistical strategy to detect the presence of multiple level shifts on bounded time series. The null and alternative hypotheses that are specified are given by:

$$\begin{cases} H_0: & \gamma_i = 0 \quad \forall i \\ H_1: & \gamma_i \neq 0 \quad \text{for some } i \end{cases},$$

which can be tested using the generalized fluctuation tests in Harvey, Leybourne and Taylor (2010):

$$S_d = \sigma_d^{-1} T^{1/2-d} \max_{t \in \Lambda_T} \left| \frac{\sum_{j=1}^{\lfloor \frac{w}{2}T \rfloor} x_{t+j} - \sum_{j=1}^{\lfloor \frac{w}{2}T \rfloor} x_{t-j+1}}{\lfloor \frac{w}{2}T \rfloor} \right|; \quad d \in \{0, 1\}, \tag{5}$$

$\Lambda_T = T\Lambda$ with $\Lambda = [\epsilon, 1-\epsilon]$. S_0 denotes the test statistic that is computed assuming that $x_t \sim BI(0)$ and S_1 is the statistic that assumes that $x_t \sim BI(1)$ – in the $BI(0)$ case the LRV of u_t is given by $\sigma_0^2 = \lim_{T\to\infty} E[T^{-1}(\sum_{t=1}^T u_t)^2] = \sigma_\varepsilon^2 C(1)^2 / (1-\alpha)^2$. These fluctuation statistics are based on the difference between the mean of the $\lfloor \frac{w}{2}T \rfloor$ observations $x_{t+1}, \ldots, x_{t+\lfloor \frac{w}{2}T \rfloor}$ and the mean of the $\lfloor \frac{w}{2}T \rfloor$ observations $x_t, x_{t-1}, \ldots, x_{t-\lfloor \frac{w}{2}T \rfloor+1}$, where w is the bandwidth of the window of observations that are used. The proposal assumes that at most only one potential level shift lay inside a given window of observations, so that for $\epsilon < \lambda_1^0 < \cdots < \lambda_m^0 < 1-\epsilon$, it is required that $|\lambda_i^0 - \lambda_{i+1}^0| \geq w$ – if desired, we could impose $w = \epsilon$. The limiting distribution of the statistics in (5) is presented in the following theorem.

Theorem 1 *Let $\{x_t\}_{t=1}^T$ the stochastic process with DGP given by (1) to (3). Under the null hypothesis that there is no structural breaks affecting the level of the time series – $\gamma_i = 0$ $\forall i$ in (1) – the S_d test statistics, $d \in \{0, 1\}$, given in (5) converge as $T \to \infty$ to:*
(a) For the $BI(0)$ case:

$$S_0 \Rightarrow \sup_{\lambda \in \Lambda} \left| 2w^{-1} \left(J^\kappa(\lambda + w/2) - 2J^\kappa(\lambda) + J^\kappa(\lambda - w/2) \right) \right|.$$

(b) For the BI (1) case:

$$S_1 \Rightarrow \sup_{\lambda\in\Lambda_j}\left|2w^{-1}\left(\int_{\lambda}^{\lambda+w/2} J_{\underline{c}_j^{+}}^{\overline{c}_j^{+},\kappa}(s)\,ds - \int_{\lambda-w/2}^{\lambda} J_{\underline{c}_j^{-}}^{\overline{c}_j^{-},\kappa}(s)\,ds\right)\right|; \quad j=1,\ldots,n+1,$$

where $\Rightarrow$ denotes weak convergence to the associated measure of probability, $\lambda = t/T$, $\Lambda_j = (\pi_{j-1}, \pi_j]$, $j = 1, \ldots, n+1$, $\pi_0 = \epsilon$, $\pi_{n+1} = 1-\epsilon$, $J^{\kappa}(\lambda)$ is a standard Ornstein-Uhlenbeck (OU) process, and $J_{\underline{c}_j^{+}}^{\overline{c}_j^{+},\kappa}(\lambda)$ and $J_{\underline{c}_j^{-}}^{\overline{c}_j^{-},\kappa}(\lambda)$ are standard regulated OU processes.

The proof is given in the appendix. There are some interesting features that deserve attention. First, the limiting distribution of S_0 does not depend on the bounds, so that in the limit we can use the standard critical values obtained in Harvey, Leybourne and Taylor (2010). Second, the limiting distribution of S_1 depends on the bounds $[\underline{c}_j, \overline{c}_j]$, $j = 1, \ldots, n+1$ – i.e., it depends both on the number and position of the structural breaks affecting the bounds, as well as on the value of the bounds. The case of stable bounds ($n = 0$) is covered if $[\underline{c}_j, \overline{c}_j] = [\underline{c}, \overline{c}]$ $\forall j$. Note that for $n > 0$, it might be the case that, for observations next to the border that is defined between two consecutive bounds regimes, some observations of the right-half window – i.e., $x_{t+1}, \ldots, x_{t+\lfloor \frac{w}{2}T\rfloor}$ – might end up on one of the bounds-defined-regimes and/or some observations of the left-half window – i.e., $x_{t-1}, \ldots, x_{t-\lfloor \frac{w}{2}T\rfloor+1}$ – might end up on the other bounds-defined-regime. This implies that $J_{\underline{c}_j}^{\overline{c}_j,\kappa}(\lambda)$ will be (partially) limited by different bounds for values of λ next to π_j. In order to highlight this issue, we have included the superscripts $-/+$ in the notation of $J_{\underline{c}_j}^{\overline{c}_j,\kappa}(\lambda)$. It should be understood that when λ is not close to the extremes of Λ_j, then $\overline{c}_j^{+} = \overline{c}_j^{-} = \overline{c}_j$ and $\underline{c}_j^{+} = \underline{c}_j^{-} = \underline{c}_j$. Finally, the limiting distribution of both statistics depends on the bandwidth (w) of the window that specifies the amount of observations that is used.

Harvey, Leybourne and Taylor (2010) proposed also the use of a union of rejection statistic to obtain robust conclusions regarding of the order of integration of the time series. The union of rejections decision rule establishes that:

$$U : \text{Reject } H_0 \qquad \text{if } \max\left\{S_1, \left(\frac{cv_{\xi}^{1}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})}{cv_{\xi}^{0}}\right)S_0\right\} > \kappa_{\xi}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})\, cv_{\xi}^{1}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}), \tag{6}$$

$\underline{\boldsymbol{c}} = (\underline{c}_1, \ldots, \underline{c}_{n+1})$ and $\overline{\boldsymbol{c}} = (\overline{c}_1, \ldots, \overline{c}_{n+1})$, where $cv_{\xi}^{1}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})$ and cv_{ξ}^{0} are the critical values of S_1 and S_0 statistics, respectively, at the ξ level of significance, and $\kappa_{\xi}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})$ is a positive scaling constant whose role is to avoid any size distortions of the S_1 and S_0 statistics due to the violation of the order of integration that is assumed in each case – henceforth, the union test statistic is denoted as S_U. Note that we use $cv_{\xi}^{1}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})$ and $\kappa_{\xi}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})$ in (6) to stress the idea that the critical values that are used for the S_1 statistic depends on the bounds, so does the value of $\kappa_{\xi}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})$.

The asymptotic critical values for S_1, S_0 and the value of the constant $\kappa_{\xi}(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}})$ for the symmetric bounds case are computed by Monte Carlo simulations using 1,000 steps to approximate the Brownian motions involved in the limiting distributions and 10,000 replications. The simulation considers the pair of values $[\underline{c}, \overline{c}]$ given by $-\underline{c} = \overline{c} \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.5\}$

and uses the algorithm in Cavaliere (2005a). Although the limiting distribution of the S_0 statistic does not depend on the bounds and the critical values computed in Harvey, Leybourne and Taylor (2010) apply, it might be the case that bounds that define a tiny range of variation might have some effects on the computation of the critical values, since they might be defining *path-divergent* Brownian motions. This is the reason why critical values for S_0 are also computed.

The first panel of Table 1 reports the asymptotic critical values for the S_d statistics, $d \in \{0, 1\}$ and the $\kappa_\xi\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$ constant at the 5% level of significance for $n = 0$ (Case A) with $w \in \{0.1, 0.3\}$. As suggested by the theory, the asymptotic critical values for S_0 do not depend on the bounds. Although there is a small difference between the critical values computed for $\bar{c} = 0.1$ and the other values for $\bar{c}$, this difference might be due to the path-controlled divergence that imposes small values of the bounds. In addition, the difference is small when compared to the critical values computed by Harvey, Leybourne and Taylor (2010) for unlimited time series – for instance, 23.4 (12.392) in Table 1 of Harvey, Leybourne and Taylor (2010) versus 22.8337 (12.3046) in Table 1 for $w = 0.1$ (0.3). The effect of $[\underline{c}, \bar{c}]$ on the asymptotic critical values for the S_1 statistic depends on the range of fluctuation defined by $[\underline{c}, \bar{c}]$. The asymptotic critical values do not change significantly when the range of fluctuation is wide, although this is not the case for narrower ranges. Note that these features are found regardless of w. Finally, it is worth pointing out that we have also estimated response surfaces to approximate the finite sample critical values at the 1, 2.5, 5 and 10% levels of significance for the S_d statistics, $d \in \{0, 1\}$ and the $\kappa_\xi\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$ constant for the symmetric bounds case when $n = 0$ – see Tables B.1 to B.3 in the companion appendix.[3]

The second panel of Table 1 presents asymptotic critical values for the S_d statistics, $d \in \{0, 1\}$ and the $\kappa_\xi\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$ constant at the 5% level of significance for Case B allowing for one structural break ($n = 1$) with $\pi = 0.5$ and $w \in \{0.1, 0.3\}$. The potential combinations of number of structural breaks, break locations, values of bounds, direction of the change affecting the bounds and the bandwidth of the window have led us to report a small set of critical values for this case with an illustrative purpose. As for Case A, the critical values for S_0 do not depend on the bounds in the limit, which is not the case for S_1 and $\kappa_\xi\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$. A Matlab program to compute the critical values for any desired combination of parameters, for both Cases A and B, is available upon request.

4 Estimation of the long-run variance

The computation of the statistics defined above requires a consistent estimator of the long-run variance. Harvey, Leybourne and Taylor (2010) suggest a parametric estimation procedure which implementation relies on two important features. First, the equations that are used to estimate the long-run variance depends on the order of integration that is assumed for the time series. Second, the estimation procedure considers the maximum number of potential structural breaks that admits the trimming parameter and the window bandwidth that are specified – i.e., $m_{\max} = 1 + \lfloor \left((1-\epsilon) - \epsilon \right) / w \rfloor$. Allowing for the maximum number of structural breaks avoids obtaining

[3] The response surfaces can also be used for the statistics in Harvey, Leybourne and Taylor (2010) for unbounded time series if an arbitrarily large value for the bounds is specified.

a biased long-run variance estimate due to unaccounted breaks when performing the (sequential) testing procedure that is described below.

Conditional on $m_{\max}$, the break locations are obtained from the ordinary least square (OLS) estimation of the model in (1) in first differences:

$$\Delta x_t = \sum_{i=1}^{m_{\max}} \theta_i D\left(T_i\right)_t + v_t \qquad t = 2, \ldots, T, \tag{7}$$

with $D\left(T_i\right)_t = 1$ for $t = T_i + 1$ and 0 otherwise, $i = 1, \ldots, m_{\max}$, and estimate the break dates as the argument that minimizes the sum of squared residuals (SSR) of the model in (7) over all possible combinations of $m_{\max}$ break dates – the estimated break dates are denoted as $\hat{T}_{B,m_{\max}} = (\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_{m_{\max}})'$, with $\hat{\lambda}_{B,m_{\max}} = \hat{T}_{B,m_{\max}}/T$. Harvey, Leybourne and Taylor (2010) show that this procedure provides consistent estimates of the break fraction parameters iff $\gamma_i = \gamma_i^* T^{1/2}$. For coherence and as far as for the LRV estimation is concerned, the vector of estimated break dates $\hat{T}_{B,m_{\max}}$ is used in the estimation of the LRV regardless of the assumed d.

For the S_1 statistic, the residuals of (7) are used to estimate the augmented Dickey-Fuller (ADF) type regression equation:

$$\Delta \hat{v}_t = \rho \hat{v}_{t-1} + \sum_{j=1}^{k-1} \psi_j \Delta \hat{v}_{t-j} + e_t, \tag{8}$$

$t = k+2, \ldots, T$, with $\hat{\sigma}_e^2 = (T - 2k - 1)^{-1} \sum_{t=k+2}^{T} \hat{e}_t^2$ and k selected so that it satisfies that as $T \to \infty$, $1/k + k^3/T \to 0$ – Harvey, Leybourne and Taylor (2010) suggest using the Bayesian information criterion (BIC) to choose k, although other criteria such as the modified information criteria in Ng and Perron (2001) and Perron and Qu (2008) might be applied. A consistent estimate of the long-run variance is obtained as $\hat{\sigma}_1^2 = \hat{\sigma}_e^2/\hat{\rho}^2$, where the subscript indicates that it has been assumed that $x_t \sim I\left(1\right)$.

The estimation of the long-run variance for the S_0 statistic starts with the OLS estimation of:

$$y_t = \mu + \sum_{i=1}^{m_{\max}} \gamma_i DU_{i,t} + u_t \qquad t = 1, \ldots, T, \tag{9}$$

which residuals are in turn used to estimate a Perron's ADF-type regression equation:

$$\Delta \hat{u}_t = \rho \hat{u}_{t-1} + \sum_{j=1}^{k-1} \psi_j \Delta \hat{u}_{t-j} + \sum_{i=1}^{m_{\max}} \sum_{j=0}^{k-1} \psi_{i,j} D(\hat{T}_i)_{t-j} + e_t \qquad t = k+1, \ldots, T. \tag{10}$$

A consistent estimator of long-run variance is then given by $\hat{\sigma}_0^2 = \hat{\sigma}_e^2/\hat{\rho}^2$ with $\hat{\sigma}_e^2 = (T - (2 + m_{\max})\, k)^{-1} \sum_{t=k+1}^{T} \hat{e}_t^2$.

In practice, the order of integration of the time series is not known a priori, so that it would be interesting to derive the expressions towards which each statistic converge in the limit when the wrong order of integration is assumed. This analysis is conducted in the following lemma.

Lemma 1 *Let $\{x_t\}_{t=1}^{T}$ the stochastic process with DGP given by (1) to (3). Under the null hypothesis that there is no structural breaks affecting the level of the time series – $\gamma_i = 0$ $\forall i$ in (1) – the S_d test statistics, $d \in \{0,1\}$, given in (5) computed under a wrong order of integration assumption converge as $T \to \infty$ to:*
(a) When $x_t \sim BI(1)$:

$$S_0 \Rightarrow \frac{\sup_{\lambda \in \Lambda_j} \left| 2w^{-1} \left(\int_{\lambda}^{\lambda+w/2} J_{\underline{c}_j^{+}}^{\overline{c}_j^{+},\kappa}(s)\, ds - \int_{\lambda-w/2}^{\lambda} J_{\underline{c}_j^{-}}^{\overline{c}_j^{-},\kappa}(s)\, ds \right) \right|}{Q^{1/2}(\kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})}; \quad j = 1, \ldots, n+1,$$

where $Q(\kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})$ is a function of standard regulated OU processes and break fractions defined in the appendix.
(b) When $x_t \sim BI(0)$:

$$S_1 = O_p\left(kT^{-1}\right).$$

The proof is outlined in the appendix. As can be seen, S_1 converges towards zero when $x_t \sim BI(0)$, so that the empirical size of S_1 is under control regardless of d. The case of S_0 is more interesting since the limiting distribution of S_0 is bounded when $x_t \sim BI(1)$ – note that the numerator of the limiting distribution coincides with the limiting distribution of the S_1 statistic when $x_t \sim BI(1)$. Simulations available upon request show that the percentiles of the limiting distribution of S_0 when $x_t \sim BI(1)$ with constant bounds (Case A) are smaller or similar to the ones obtained for S_0 when $x_t \sim BI(0)$. Therefore, the empirical size of S_0 for Case A is also under control regardless of d. The features shown S_0 and S_1 for Case A when the wrong d is assumed motivate the use of the union statistic as a way to obtain robust conclusions about the presence of level shifts for bounded stochastic processes, regardless of d.

The picture changes for Case B, since, in general, the percentiles of S_0 when $x_t \sim BI(1)$ might be larger than the ones that are obtained when $x_t \sim BI(0)$, which increase with $\delta_j = |\overline{c}_j - \overline{c}_{j-1}|$, $j = 2, \ldots, n+1$. This would invalidate the strategy in which the union statistic is based on since the empirical size of S_0 would not be under control when $x_t \sim BI(1)$. Intuitively, the changing bounds might lead S_0 to detect false level shifts when $x_t \sim BI(1)$. To illustrate this issue, let us suppose that there is one change in the (symmetric) bounds at time τ_1^0 so that $\delta = |\overline{c}_2 - \overline{c}_1| > 0$, with $\mu = m = 0$ and $\alpha = 1$ in (1) and (2). Under this situation, it might be the case that $x_{\tau_1^0}$ equals the upper limit defined by the boundaries of the first regime – i.e., $x_{\tau_1^0} = \sigma_1 T^{1/2}\overline{c}_1$ – and that $x_{\tau_1^0+1}$ equals the lower limit defined by the boundaries of the second regime – i.e., $x_{\tau_1^0+1} = -\sigma_1 T^{1/2}\overline{c}_2$. Therefore, the change that might be observed between the two regimes would be of magnitude $-\sigma_1 T^{1/2}\overline{c}_2 - \sigma_1 T^{1/2}\overline{c}_1 = -\sigma_1 T^{1/2}(\overline{c}_2 + \overline{c}_1) = -\sigma_1 T^{1/2}((\overline{c}_1 + \delta) + \overline{c}_1) = -\sigma_1 T^{1/2}(2\overline{c}_1 + \delta)$. Conversely, it might be the case that $x_{\tau_1^0} = -\sigma_1 T^{1/2}\overline{c}_1$ and $x_{\tau_1^0+1} = \sigma_1 T^{1/2}\overline{c}_2$, so that the change that might be observed between the two regimes is $\sigma_1 T^{1/2}\overline{c}_2 + \sigma_1 T^{1/2}\overline{c}_1 = \sigma_1 T^{1/2}(\overline{c}_2 + \overline{c}_1) = \sigma_1 T^{1/2}((\overline{c}_1 + \delta) + \overline{c}_1) = \sigma_1 T^{1/2}(2\overline{c}_1 + \delta)$. Consequently, the apparent change that might experience the time series between the two consecutive regimes lay inside the set $\sigma_1 T^{1/2}[-|2\overline{c}_1 + \delta|, |2\overline{c}_1 + \delta|]$. This implies that $x_t \sim BI(1)$ at t near τ_1^0 might be stuck on either the lower or upper limit, on either regime, for a sufficient number of periods so that S_0 would detect a false level shift. This situation is investigated in the simulations section.

To alleviate the consequences of this phenomenon, we suggest the computation of S_0 and S_1 in (5) but with x_t replaced by $\hat{x}_t$, where $\hat{x}_t$ denotes the demeaned time series that is obtained when x_t is projected against a constant and the set of dummy variables $DU_{j,t} = 1\left(t > \tau_j^0\right)$, $j = 1, \ldots, n$. This modification does not apply to the estimation procedure of the long-run variance estimation described above. This procedure defines the modified S_0, S_1 and S_U statistics, which are denoted as S_0^*, S_1^* and S_U^*. The limiting distribution of S_0^* and S_1^* is derived in the following lemma.

Lemma 2 *Let $\{x_t\}_{t=1}^T$ the stochastic process with DGP given by (1) to (3). Under the null hypothesis that there is no structural breaks affecting the level of the time series – $\gamma_i = 0\ \forall i$ in (1) – the S_d^* test statistics, $d \in \{0, 1\}$, converge as $T \to \infty$ to:*
(a) For the $BI(0)$ case:

$$S_0^* \Rightarrow \sup_{\lambda \in \Lambda_j} \left| 2w^{-1} \left(V^\kappa(\lambda + w/2) - 2V^\kappa(\lambda) + V^\kappa(\lambda - w/2) \right) \right|,$$

where $V^\kappa(\lambda) = J^\kappa(\lambda) - J^\kappa(\pi_{j-1}^0) - (\lambda - \pi_{j-1}^0)/(\pi_j^0 - \pi_{j-1}^0)(J^\kappa(\pi_j^0) - J^\kappa(\pi_{j-1}^0))$, $j = 1, \ldots, n+1$.
(b) For the $BI(1)$ case:

$$S_1^* \Rightarrow \sup_{\lambda \in \Lambda_j} \left| 2w^{-1} \left(\int_\lambda^{\lambda + w/2} W_{\underline{c}_j^+}^{\overline{c}_j^+, \kappa}(s)\, ds - \int_{\lambda - w/2}^{\lambda} W_{\underline{c}_j^-}^{\overline{c}_j^-, \kappa}(s)\, ds \right) \right|,$$

where $W_{\underline{c}_j^+}^{\overline{c}_j^+, \kappa}(s) = J_{\underline{\mathbf{c}}}^{\overline{\mathbf{c}}, \kappa}(s) - \int_0^{\pi_{j-1}^0} J_{\underline{\mathbf{c}}}^{\overline{\mathbf{c}}, \kappa}(u)\, du - (s - \pi_{j-1}^0)/(\pi_j^0 - \pi_{j-1}^0) \int_{\pi_{j-1}^0}^{\pi_j^0} J_{\underline{\mathbf{c}}}^{\overline{\mathbf{c}}, \kappa}(u)\, du$, $j = 1, \ldots, n+1$.

The proof is given in the appendix. The limiting distribution of S_0^* and S_1^* involves the projected OU stochastic processes onto the space spanned by $(1, 1(r > \pi_1^0), \ldots, 1(r > \pi_n^0))$ so that misleading signals of level shifts that might be induced by the changing bounds are removed. It is to be expected that this modification comes at the cost of power reductions if there are true level shifts that exactly coincide with the change in the bounds. The asymptotic critical values at the 5% significance level for S_0^* and S_1^*, as well as, $\kappa_\xi^*(\underline{\mathbf{c}}, \overline{\mathbf{c}})$ for a small set of parameter combinations are reported in the third panel of Table 1.

5 The sequential testing strategy

In practice neither the number nor the position of the structural breaks affecting the level of the time series might be to be known a priori. To address this issue, Harvey, Leybourne and Taylor (2010) design an iterative estimation procedure that allows the estimation of both elements. Let us centre the discussion on the S_1 statistic, although the strategy is similar for the other statistics that have been proposed. The first stage consists of testing the null hypothesis of no structural break (no level shift) against one structural break (one level shift), considering all time observations $t = 1, \ldots, T$. If we denote by $cv_{\xi}^{1}\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$ the critical value at the $\xi\%$ significance level, evidence against the null hypothesis is found if $\max_{t \in \Lambda_T} S_{1,t,\lfloor wT \rfloor} > cv_{\xi}^{1}\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$. On the contrary, the null of no structural break is not rejected if $\max_{t \in \Lambda_T} S_{1,t,\lfloor wT \rfloor} \leq cv_{\xi}^{1}\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right)$. In the former case, the first break date is estimated as:

$$\tilde{T}_1 = \arg\max_{t \in \Lambda_T} S_{1,t,\lfloor wT \rfloor} > cv_{\xi}^{1}\left(\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}\right).$$

Although theoretical limits $\underline{b}_t$ and $\overline{b}_t$ are assumed to be known, in practice we need to estimate $[\underline{c}_j, \overline{c}_j]$ using:

$$\left[\hat{\underline{c}}_j, \hat{\overline{c}}_j\right] = \left[\frac{\underline{b}_t - \hat{D}_t}{\hat{\sigma}_1 T^{1/2}}, \frac{\overline{b}_t - \hat{D}_t}{\hat{\sigma}_1 T^{1/2}}\right] \qquad \tau_{j-1}^{0} < t \leq \tau_j^{0}, \tag{11}$$

$j = 1, \ldots, n+1$ which, besides the long-run variance $\left(\sigma_1^2\right)$ that can be estimated as described above, it requires an estimation of the deterministic component (D_t). Following Cavaliere and Xu (2014), D_t is estimated under the null hypothesis of unit root so that, at this initial stage, $\hat{D}_t = x_0$. This defines $\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right)$ and the corresponding critical value $cv_{\xi}^{1}\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right)$ that is used to conduct the statistical inference.

The second stage proceeds with the definition of an exclusion area of searching for potential additional breaks given by the set $\Lambda_{1,T} = \left[\tilde{T}_1 - \lfloor wT \rfloor + 1, \tilde{T}_1 + \lfloor wT \rfloor + 1\right]$, so that the sequential testing procedure looks for an additional break in the range of observations $t = 1, 2, 3, \ldots, \tilde{T}_1 - \lfloor wT \rfloor, \tilde{T}_1 + \lfloor wT \rfloor + 2, \ldots, T$ – i.e., the eligible break dates are inside the set $t \in \Lambda_T - \Lambda_{1,T}$. Evidence against the null hypothesis of an additional structural break is found when $\max_{t \in \Lambda_T - \Lambda_{1,T}} S_{1,t,\lfloor wT \rfloor} > cv_{\xi}^{1}\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right)$ and the estimated break date is obtained as:

$$\tilde{T}_2 = \arg\max_{t \in \Lambda_T - \Lambda_{1,T}} S_{1,t,\lfloor wT \rfloor} > cv_{\xi}^{1}\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right).$$

As above, D_t is estimated under the null hypothesis of unit root so that, in this second stage, $\hat{D}_t = x_0 + \hat{\theta}_1 DU_t$ where $\hat{\theta}_1$ are the OLS estimates of θ_1 in (7) with one structural break placed at $\tilde{T}_1$. This defines a new set of estimates $\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right)$ – this *continuous updating* is a characteristic of each sequential step – so that the corresponding critical value $cv_{\xi}^{1}\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right)$ that is used to conduct the statistical inference at the second stage might be different from the ones used in the first stage.

The procedure continues until we find that $\max_{t \in \Lambda_T - \Lambda_{1,T} - \Lambda_{2,T} - \cdots \Lambda_{m,T}} S_{1,t,\lfloor wT \rfloor} \leq cv_{\xi}^{1}\left(\hat{\underline{\boldsymbol{c}}}, \hat{\overline{\boldsymbol{c}}}\right)$, in which case the null hypothesis of no (additional) structural break is not rejected. Note that now $\hat{D}_t = x_0 + \sum_{i=1}^{m} \hat{\theta}_i DU_t$ where $\hat{\theta}_i$ are the OLS estimates of θ_i in (7) with m structural breaks located at $(\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_m)$. The estimated number of structural breaks from the use of the S_1 statistic is denoted by $\tilde{m}_1$.

A similar approach can be applied using the S_0 statistic and obtain $\tilde{m}_0$ structural breaks. In general, $\tilde{m}_1$ has not to be equal to $\tilde{m}_0$. If $\tilde{m}_1 \geq \tilde{m}_0$, the $\tilde{m}_0$ breaks are simply a subset of the $\tilde{m}_1$ breaks. Similarly, if $\tilde{m}_0 \geq \tilde{m}_1$, the $\tilde{m}_1$ breaks are simply a subset of the $\tilde{m}_0$ breaks. If $\tilde{m}_1 = \tilde{m}_0$, both sets of break locations are identical. Note that the number of breaks that is estimated with the S_U statistic will simply be $\tilde{m}_1 = \max(\tilde{m}_1, \tilde{m}_0)$. Finally, the same sequential testing procedure applies to the modified statistics S_0^*, S_1^* and S_U^*, which will deliver $\tilde{m}_0^*$ and $\tilde{m}_1^*$. A Matlab code is available upon request to implement the whole statistical inference that is proposed in this paper.

6 Finite sample performance

In this section we investigate the empirical size and power of the statistics that have been proposed in the paper. The DGP is given by (1) to (3), where $x_t \in [\underline{b}_t, \overline{b}_t]$ and $\varepsilon_t \sim iid\ N(0,1)$. The deterministic component is specified as:

$$D_t = \mu + \sum_{i=1}^{m} \gamma_i DU_{i,t}, \tag{12}$$

with $\mu = 0$ and three different ways of defining the break magnitude depending on whether γ_i varies with T – i.e., $\gamma_i = \gamma^* T^{d-1/2}$, $\gamma^* \in \{1, 5, 10\}$, $d \in \{0, 1\}$ – or is fixed – $\gamma_i = \gamma^*$. The simulation experiment considers up to four structural breaks, $m \in \{1, 2, 3, 4\}$. When $m = 1$ the level shift is located at $\lambda^0 = 0.5$, whereas $\lambda^0 = (0.3, 0.7)'$ for $m = 2$, $\lambda^0 = (0.25, 0.5, 0.75)'$ for $m = 3$, and $\lambda^0 = (0.2, 0.4, 0.6, 0.8)'$ for $m = 4$. In all cases, different values of the window parameter are essayed setting $w \in \{0.1, 0.15, 0.2, 0.25, 0.3\}$, although some of these values are incompatible with the assumption that there can be only one break inside the window – for instance, when $w = 0.3$ the maximum number of structural breaks that satisfy the requirements is three, so that we are clearly violating the assumption that, at most, there is one structural break inside the window for $m = 4$. The lower and upper bounds are defined in a general way as:

$$[(\underline{b} - D_t) T^{-1/2}, (\overline{b} - D_t) T^{-1/2}] = [\underline{c}_j, \overline{c}_j] \quad \text{for} \quad \tau_{j-1}^0 < t \le \tau_j^0,\ j = 1, \ldots, n+1, \tag{13}$$

with $\tau_0^0 = 0$, $\tau_j^0 = \pi_j^0 T$ and $\tau_{n+1}^0 = T$, $j = 1, \ldots, n$, and are estimated following the procedure described above, unless otherwise indicated. The simulations consider Cases A and B, dealing with $x_t \sim BI(1)$ – by setting $\alpha = 1$ in (2) – and with $x_t \sim BI(0)$ – for which $\alpha \in \{0.95, 0.9, 0.7, 0.5\}$ in (2). Throughout this section the value of k used in the estimation of parametric long-run variance in (8) and (10) is based on the modified Akaike's information criterion (AIC) proposed in Ng and Perron (2001) and Perron and Qu (2007) with a maximum of $\lfloor 4(T/100)^{1/4} \rfloor$ lags. Further, the break dates of the level shifts and the values of the bounds are estimated as described in Section 5. The sample sizes are $T \in \{50, 150, 300\}$, 1,000 replications are conducted using Matlab, and the nominal size is set at the 5% level of significance.

6.1 Case A. Constant bounds

The set of bound values that is used in this section is given by $[\underline{c}, \overline{c}]$, $-\underline{c} = \overline{c} \in \{0.3, 0.5, 0.8, 1, 1.5\}$. The empirical size of no structural breaks implies $\gamma_i = 0\ \forall i$ in (12), whereas the empirical power is assessed allowing for up to four unknown structural break as described above. Throughout this section, the simulations use the finite sample critical values that are obtained from the estimated response surfaces.

Table 2 presents the empirical size of the three statistics for the combination of $\bar{c}$ and w parameters. In general, the empirical size is under control when the assumption concerning the order of integration of the stochastic process is met for the respective S_d, $d \in \{0,1\}$, statistics. However, it is worth noting that under-rejection distortions are observed for S_1, especially when $\bar{c} = 0.3$. In general, the empirical size tends towards the nominal one as T increases for a given $\bar{c}$. These under-rejection distortions disappear if we assume known $\bar{c}$ – these results are available in the companion appendix. Similar features are found for S_0 when $|\alpha| < 1$, with the empirical size being smaller than the nominal one. Notwithstanding, in this case the mild under-size distortions do not disappear when $\bar{c}$ is assumed to be known.

Let us now focus on the situation in which the order of integration of the time series does not match the one assumed by the S_d, $d \in \{0,1\}$, statistics. As predicted by the theory, S_1 becomes very conservative, with a rejection rate that tends to zero as T increases. The rejection rate of S_0 is bounded with values that are slightly larger than the assumed nominal size. These are desirable features since in this case the assumed d is wrong. Consequently, incorrect d does not seem to cause too much harm on S_d, $d \in \{0,1\}$, which in turn defines the basis for the computation of the union statistic defined above. As can be seen, the empirical size of S_U is close ($\alpha = 1$) or below ($|\alpha| < 1$) the nominal size as T increases, regardless of $\bar{c}$.

In all, the empirical size of the statistics tends towards the nominal one as T increases when the assumed d is correct, and the rejection rates tend either to zero (S_1) or take values around 5% (S_0) when the wrong d is imposed. Note that these results hold especially for values of $w = 0.1$ and $w = 0.15$, which coincide with the recommended values in Harvey, Leybourne and Taylor (2010).

Table 3 collects the empirical power that is based on a T-increasing break magnitude ($\gamma_i = \gamma^* T^{1/2}$) for the one structural break case ($m = 1$). The discussion centres on the results that specify $w = 0.15$, since similar conclusions are reached for other values of w – see the companion appendix. The statistics show non-negligible power when the experiment setup matches the theoretical requirements that impose each statistic. Some remarks are in order. First, the empirical power of S_1 reduces as $\bar{c}$ increases for a given T when $\gamma_i = T^{1/2}$ and $\alpha = 1$, which indicates that the break date is easier to detect for narrow ranges. This is not surprising since in this case a small value of γ^* represents a big effect on time series bounded by small $\bar{c}$'s. Notwithstanding, the empirical power tends to one as γ^* increases, regardless of $\bar{c}$. Second, the empirical power of S_0 reduces as $\bar{c}$ increases for a given value of T when $\gamma_i = T^{1/2}$ and α is close to one, although S_0 shows good power for large T. As α moves away from one, the power tends to one as T increases, regardless of $\bar{c}$. For values of $\gamma^* > 1$ the empirical power of S_0 equals one in all cases. Third, it is worth noting that both S_1 and S_0 retain some ability to detect the presence of a level shift under the wrong assumption of d – the minimum value of 0.042 is achieved for the S_1 when $\alpha = 0.5$, $\bar{c} = 1.5$ and $T = 300$. Finally, the empirical power of S_U is very good in all cases, with a minimum value of 0.62 that is found when $\gamma^* = 1$, $\alpha = 1$, $\bar{c} = 1.5$ and $T = 50$. This indicates that S_U

leads to robust conclusions concerning the presence of level shifts. Simulation results for up to four structural breaks are qualitatively similar (if not better), so that they are not reported here to save space – see the companion appendix for further details.

The empirical power analysis for the shrinking break magnitude $\left(\gamma_i = \gamma^* T^{-1/2}\right)$ setup is summarized in Table 4 for $m = 1$ and $w = 0.15$. As can be seen, the rejection rates of S_1 tend either to the nominal size ($\alpha = 1$) or to zero ($|\alpha| < 1$), since in this case the level shift is asymptotically negligible. Similarly, S_0 shows a limited ability to detect the level shift for small T, with rejection rates that approaches the nominal size as T increases. This behaviour is replicated by S_U. These results are somewhat to be expected, since the effect of the structural break disappears in the limit – the same outcome is obtained if we allow for additional shrinking structural breaks.

Table 5 reports the empirical power for the fixed break magnitude ($\gamma_i = \gamma^*$) case with $m = 1$ and $w = 0.15$. Not surprisingly, when $\gamma^* = 1$ the power tends to the nominal size as T increases given the small magnitude of the level shift. As γ^* increases, the power of the statistics increases, showing qualitatively similar features as the ones discussed above for the $\gamma_i = \gamma^* T^{1/2}$ case. It should be born in mind that in this setup the break magnitude is asymptotically negligible for S_1 when $\alpha = 1$, which explains the lower power values that are found when $\alpha = 1$ compared to the ones obtained when $\gamma_i = \gamma^* T^{1/2}$. The S_U statistic has non-negligible power, which suggests its use in empirical applications to attain robust conclusions about the presence of level shifts in bounded stochastic processes.

The specification of $m > 1$ structural breaks leads to an improvement of the empirical power of the statistics in those cases where the level shifts are more difficult to detect – i.e., when $\gamma^* = 1$; see the companion appendix for further details. However, it is interesting to analyse the case in which there are some structural breaks that are not possible to detect due to the definition of w. These situations appear, for instance, when $w = 0.3$ and $m \in \{3, 4\}$ with the locations defined above – the results are presented in the companion appendix. When $\gamma_i = \gamma^* T^{1/2}$, S_1 shows good power, whereas the rejection rates for the S_0 statistic are virtually zero. There is not a clear pattern for S_U. In general and when $\alpha = 1$, the power of S_U increases with T and γ^*, on the one hand, but decreases with $\bar{c}$, on the other hand. When $\alpha < 1$, the power decreases with T as γ^* increases, reaching values around 0.5. For the shrinking breaks case, the rejection rates tend to the nominal size when the correct order of integration is assumed, and either tend to zero (S_1) or take values around the nominal size (S_0) otherwise. The S_U statistic lies in between, with power figures that are slightly below the S_0 ones. Finally, for the fixed break magnitudes case, S_1 shows good power with values that resemble the ones obtained for $m \in \{1, 2\}$ with $w = 0.3$, situations in which the detection of the structural breaks is feasible. The power of S_0 decreases towards zero as the magnitude of the structural break increases, which evidences the effect of unattended structural breaks. This misspecification error is also evident on the performance of S_U, with power values that decrease as γ^*, $\bar{c}$ and T increase.

The simulation experiment has also focused on the performance of the sequential testing strategy to estimate the number of structural breaks. For the $\gamma_i = \gamma^* T^{1/2}$ case, results available upon request evidence that S_U the frequency of correct estimation of m tends to one as γ^* and T increase for a given $\bar{c}$. For the shrinking break magnitude case, S_U points towards the absence of structural breaks in most cases as T increases, which is to be expected given the asymptotically negligible effect of the structural breaks. Finally, for the fixed magnitude structural breaks case, the ability of S_U to estimate m depends on γ^* and α. Thus, when $\alpha = 1$ the impact of the structural breaks disappears in the limit, and S_U tends to select $m = 0$ in most cases. However, as α moves away from one, the frequency of correct detection of the number of structural breaks approaches one as γ^* and T increase.

6.2 Case B. Breaking bounds

This section illustrates the finite sample performance of the statistics when there is a known structural break that affect the bounds. In order to rely on a manageable framework, $[\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}]$ are assumed to be known. Relaxing this assumption would imply the computation of critical values for each set of $[\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}]$ for S_1 on each replication of the simulation experiment. Consequently, the finite sample critical values that assume known $[\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}]$ are used throughout this section. The DGP is defined as above but with one bounds structural break at $\pi^0 = 0.5$, with two sets of bounds regimes defined as $-\underline{\boldsymbol{c}} = \overline{\boldsymbol{c}} \in \{(0.3, 0.5), (0.5, 1)\}$.

In general, the empirical size analysis offers similar features regardless of $\overline{\boldsymbol{c}}$. Table 6 evidences that the empirical size of S_1 is close to the nominal one when the correct d is assumed regardless of T. The S_0 statistic tends to over-reject for high persistent processes when $T = 50$, although these size distortions disappear as T increases, leading towards a mildly conservative statistic. When the wrong d is assumed, the rejection rates of S_1 tends to zero, as predicted by the theory. However, in most cases the rejection rates of S_0 are larger than 5%, which generates a worrisome situation. Thus and although they reduce as T increases for a given w, the rejection rates of S_0 for the $\overline{\boldsymbol{c}} = (0.5, 1)$ case are large enough to pose in doubt the use of the union statistic. Hopefully, note that values near 5% are reached for the combination of $\overline{\boldsymbol{c}} = (0.3, 0.5)$, $w = 0.1$ and $T = 300$. This illustrates the discussion raised above regarding the spurious detection of level shifts by S_0 when $x_t \sim BI(1)$ with changing bounds, since the increase of the variation range might lead to conclude on the presence of non-existent level shifts. This problem is more evident as the range of variation increases. The computation of the modified statistic S_0^* implies important reductions on the rejection rates when $x_t \sim BI(1)$, without minimum effects on the empirical size that is obtained when $x_t \sim BI(0)$, especially when $w = 0.1$ or $w = 0.15$.

Table 7 offers the empirical power of the original and modified statistics when the break magnitude is defined by $\gamma_i = \gamma^* T^{1/2}$, $\gamma^* \in \{1, 5\}$, with $m = 1$ structural break placed at $\lambda \in \{0.25, 0.5, 0.75\}$, $w = 0.15$ and $\alpha \in \{1, 0.95\}$ – results for $\gamma^* = 10$ and for other values of w and α are available upon request. The statistics S_0, S_1 and S_U show good power, with values that increase with T and γ^* regardless of α. Notwithstanding, the empirical power for $\gamma^* = 1$ decreases as $\overline{\boldsymbol{c}}$ moves from $(0.3, 0.5)$ to $(0.5, 1)$, although this feature disappears for $\gamma^* = 5$ and $\gamma^* = 10$. Interestingly, the modified statistics show better power than the original ones, except when $\lambda = 0.5$. This result is not surprising since we have to bear in mind that in this case both the change in the level and in the bounds occurs at the same time, and the modification simply removes the real level shift. Consequently, the power figures that are obtained here approach the nominal size. Finally, it is observed that, for a given α and T values, the empirical power of the modified statistics is slightly higher for $\lambda = 0.25$ than for $\lambda = 0.75$. This is due to the relative magnitude of the structural break over the range of variation, since in the first bounds regime the effect of the structural break is more noticeable than in the second bounds regime.

The empirical power of the statistics for the shrinking break case decreases with T and increases with γ^* – see Table 8. This is to be expected since in the limit the structural break effect disappears. The modified statistics are more powerful than the original ones, except for $\lambda = 0.5$. As above, we realize that for a given α and T values, the empirical power of the modified statistics is higher for $\lambda = 0.25$ than for $\lambda = 0.75$, which is related to the relative impact of the structural break compared to the range of variation of the different regimes. Finally, it is worth highlighting that S_0 and all modified statistics retain some power ability when $\alpha = 1$.

The last scenario that is investigated is based on structural breaks with a fixed magnitude. Table 9 indicates that for a given T, α and $\overline{\boldsymbol{c}}$ combination values, the empirical power of all statistics increase as γ^* increases – with the obvious exception of the modified statistics when $\lambda = 0.5$. For a given γ^*, the empirical power is lower for $\overline{\boldsymbol{c}} = (0.3, 0.5)$ than for $\overline{\boldsymbol{c}} = (0.5, 1)$, a situation that is a consequence of the relative importance of γ^* over the range of variation defined by each set of $\overline{\boldsymbol{c}}$ values. Finally, although for a given combination of γ^* and $\overline{\boldsymbol{c}}$ values the empirical power of S_0 and S_0^* reduces as T increases when $\alpha = 0.95$, it approaches to one for lower values of α – see the companion appendix for the results that uses $\alpha \in \{0.7, 0.5\}$.

7 Empirical illustration. The recent story of the Swiss franc

The recent evolution of the exchange rate of the Swiss franc (CHF) against the euro (EUR) serves to illustrate the effect of bounds on the detection of structural changes in time series. In this case, there are no defined boundaries except for the fact that the exchange rate cannot take negative values. However, there are forces that have constrained the evolution of the exchange rate.

The Swiss franc has traditionally been one of the so-called safe-haven currencies with low political and inflation risk, used by investors in times of financial or foreign exchange market turmoil. This occurred when the global financial crisis, that began in the United States in 2008, hit the developed world and, in a second round, the countries of the euro zone provoking the sovereign debt crisis that even endangered the survival of the euro. In this scenario, the CHF/EUR exchange rate passed from 1.65 to 1.11 between December 2007 and August 2011. This appreciation, and its negative consequences on Swiss exports, tried to be contained by the Swiss National Bank (SNB), which began to intervene heavily in the currency markets and in September 2011 decided to anchor its currency to the euro with an exchange rate of 1.2 CHF/EUR. On January 15, 2015, the SNB stopped anchoring the value of the CHF to the EUR. During this period, the expansive European monetary policy, quantitative easing, depreciated the euro significantly and the SNB was unable to contain appreciation tensions although it introduced other regulatory capital control measures (penalizing foreign deposits to scare off foreign investors) and implemented an aggressive policy of low interest rates. During 2019, the CHF will once again act as a safe-haven currency due to market uncertainties (US-China trade war) and the SNB cut interest rates again. In 2020 it made a strong intervention (acquired more than 100,000 MCHF of foreign currencies) to curb the appreciation of the CHF and reduced interest rates.

In short, there are two opposing interventions. On one side, that of the SNB trying to contain the appreciation of the franc and avoid jeopardizing the foreign sector. The SNB has actively engaged in currency market interventions to help cap the strong CHF, and also keeps interest rates low or negative to dissuade strong speculative buying of the franc. On the other side, that of the European Central Bank (ECB) exemplified by Draghi's famous phrase pronounced on July 26, 2012 with the opposite effect of depreciating the euro: "The ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough". Accordingly, the ECB has pursued a very accommodative monetary policy to guarantee the liquidity of the euro zone and ward off the risks that were looming over it, with the collateral effect of putting downward pressure on the euro exchange rate.

From the previous narrative we can identify several important dates: (i) May 2009, the start of informal interventions, (ii) September 2011, the formal declaration of anchoring to the euro, and (iii) January 2015, the formal elimination of this commitment. Figure 1 clearly shows these counter interventions on reducing the variability of the CHF exchange rate by trying to provoke its depreciation (SNB through reserve movements) or its appreciation (ECB through monetary policy). These elements might justify the specification of a model that allows for changing bounds in the analysis (Case B).

We use monthly data of the exchange rate of the CHF against the EUR – denoted by E_t – from January of 2000 to June of 2021 by crossing the exchange rate of the CHF with respect to the US dollar (USD) and the exchange rate of USD with respect to EUR. The source is Fred database (Federal Reserve Bank of St. Louis). The EONIA interest rate series (ECB) and the volume of reserves of the SNB have been used as a complement.

The empirical analysis starts with the computation of the original S_U statistic in Harvey, Leybourne and Taylor (2010) – i.e., omitting the bounded nature of the time series – which does not find any structural breaks at the 5% significance level. Next, we proceed with the consideration the three structural breaks in the boundaries described above – i.e., May 2009, September 2011 and January 2015 – as these dates mark different milestones in interventions on the CHF exchange rate and supports opposing forces that restrict its variability. Whereas the S_U statistic does not detect any structural breaks affecting the level of E_t, the (more powerful) S_U^* statistic pinpoints different structural breaks – see Table 10. As a robustness check and following Herwartz and Xu (2008), Table 10 also presents the results with different sizes of limits starting from $\overline{b} = max(E_t)$, $\underline{b} = min(E_t)$ and gradually increasing according to $\overline{b} * (1 + \delta/100)$ and $\underline{b} * (1 - \delta/100)$

with $\delta \in \{0, 1, 2, 5, 10, 20, 50, 100\}$ – see Figure 2.[4] As can be seen, the increase in the range of variation leads to reduce the evidence of structural breaks, which is consistent with the results that are obtained if bounds are not accounted for. However, even for a range of variation five times the range defined by the minimum and maximum values, the evidence of structural breaks is noticeable.

Chronologically, the first two structural breaks in April 2003 and July 2006 are related to episodes of strong appreciation of the euro against the dollar. The following ones, in June 2010 and May 2014, correspond to the strong appreciation trends of the CHF. Finally, the structural break in July 2017 is explained by the stabilization of the exchange rate. Figure 4 represents estimated mean for the different regimes.

Summing up, the recent evolution of the CHF exchange rate against the euro illustrates the relevance of considering the limits between which a time series moves in order to correctly detect structural changes. The exercise of gradually widening the bands also shows that the boundaries affecting many economic time Harvey, Leybourne and Taylor (2010) series are often unobservable, since they depend on economic policy interventions, while theoretical boundaries are not valid.

8 Conclusions

The paper has developed statistics to test for the presence of multiple level shifts affecting bounded stochastic processes without knowledge about whether the time series is BI(0) or BI(1). The paper shows that bounds do not affect the limiting distribution of the statistic that assumes that the stochastic process is BI(0), whereas they have an important effect on the test statistic that is valid for BI(1) non-stationary processes. Therefore, bounds play an asymmetric role depending on the order of integration (d) of the stochastic process, something that needs to be taken into account when trying to obtain robust conclusions about the presence of multiple level shifts. This issue is addressed through the design of a union rejection statistic that allows testing for the presence of structural breaks regardless of d. The model specification is general enough to allow for the possibility of changing bounds in the framework of analysis, a situation that might be found in empirical applications. This generates the definition of two sets of (potential) structural breaks, depending on whether they affect the level and/or the bounds. The paper conducts an extensive simulation exercise to assess the finite sample properties of the statistics that have been proposed, which leads to suggest the use of the statistics that have been designed in empirical analyses. The usefulness of the proposal is illustrated with the analysis of the Swiss franc exchange rate, for which no structural breaks are detected if (well known/declared) bounds are not considered.

A Appendix

Lemma 3 *Let $\{y_t\}_{t=1}^T$ be a stochastic process generated according to (2) and (3) with $\alpha = \exp(-\kappa/T)$, $\kappa \geq 0$, and satisfying Assumptions A1 to A4 in Cavaliere (2005a). As $T \to \infty$, $\sigma^{-1}T^{-1/2}y_t \Rightarrow$*

[4] In order to guarantee the robustness of the results we have considered a second option in which the series is not bounded before May 2009 – see Figure 3 – obtaining identical results.

$J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r)$, *with* $\underline{\boldsymbol{c}} = (\underline{c}_1, \ldots, \underline{c}_{n+1})$, $\overline{\boldsymbol{c}} = (\overline{c}_1, \ldots, \overline{c}_{n+1})$, $\underline{c}_j \leq 0 \leq \overline{c}_j$, $\underline{c}_j \neq \overline{c}_j$, $j = 1, \ldots, n+1$, *where* $J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r) = J^{\kappa}(r) + L(r) - U(r)$ *denotes a standard regulated Ornstein-Uhlenbeck (OU) process being* $J^{\kappa}(r) = \int_0^r \exp(-\kappa(r-s))\, dB(s)$ *a standard OU process,* $B(r)$ *a standard Brownian motion on* $r \in [0,1]$, *and* $L(r) = -\{0 \wedge \inf_{\pi_{i-1}^0 \leq r' \leq r}(J^{\kappa}(r') - \underline{c}_i)\}$ *and* $U(r) = \{0 \wedge \inf_{\pi_{i-1}^0 \leq r' \leq r}(\overline{c}_i - J^{\kappa}(r'))\}$, $r \in \left[\pi_{j-1}^0, \pi_j^0\right]$, $j = 1, \ldots, n+1$, *the two side regulator processes.*

See Theorems 1 and 4 in Cavaliere (2005a) for the proof.

A.1 Proof of Theorem 1

Part (a), the $BI(0)$ case. Let us focus on the first mean that appears in (5), $A_{1,t} = \left\lfloor \frac{w}{2}T \right\rfloor^{-1} \sum_{i=1}^{\lfloor \frac{w}{2}T \rfloor} x_{t+i} = \left\lfloor \frac{w}{2}T \right\rfloor^{-1} \left(\sum_{j=1}^{t+\lfloor \frac{w}{2}T \rfloor} x_j - \sum_{j=1}^{t} x_j \right)$. Note that $x_t \in \left[\underline{b}_t, \overline{b}_t\right]$ implies that $\sum_{j=1}^t x_j \in \left[\underline{b}_t t, \overline{b}_t t\right]$, so that the re-scaled partial sum is given by $\sigma_0^{-1} T^{-1/2} \sum_{j=1}^t x_j \in \left[\sigma_0^{-1} \underline{b}_t T^{-1/2} t, \sigma_0^{-1} \overline{b}_t T^{-1/2} t\right]$. Similarly, under the null hypothesis of no structural breaks affecting the level of the time series we have $\sigma_0^{-1} T^{-1/2} \sum_{j=1}^t (x_j - \mu) = \sigma_0^{-1} T^{-1/2} \sum_{j=1}^t y_j \in \left[\sigma_0^{-1} (\underline{b}_t - \mu) T^{-1/2} t, \sigma_0^{-1} \left(\overline{b}_t - \mu\right) T^{-1/2} t\right]$. It is worth noting that:

$$\lim_{(t,T)\to\infty} [\sigma_0^{-1} (\underline{b}_t - \mu) T^{-1/2} t, \sigma_0^{-1} \left(\overline{b}_t - \mu\right) T^{-1/2} t] = [-\infty, \infty],$$

regardless of $\left[\underline{b}_t, \overline{b}_t\right]$, $\underline{b}_t \neq \overline{b}_t$ – the same result is obtained if constant boundaries are assumed. Thus and using the Functional Central Limit Theorem (FCLT), $\sigma_0^{-1} T^{-1/2} \sum_{j=1}^t (x_j - \mu) = \sigma_0^{-1} T^{-1/2} \sum_{j=1}^t y_j \Rightarrow J^{\kappa}(r)$, where $J^{\kappa}(r) \in [-\infty, \infty]$ is a standard OU process.

Using these elements, it is straightforward to see that the re-scaled mean is:

$$\begin{aligned} \sigma_0^{-1} T^{1/2} A_{1,t} &= \sigma_0^{-1} T^{1/2} \left\lfloor \frac{w}{2} T \right\rfloor^{-1} \left(\sum_{j=1}^{t+\lfloor \frac{w}{2}T \rfloor} x_j - \sum_{j=1}^{t} x_j \right) \\ &\in \sigma_0^{-1} T^{1/2} \left\lfloor \frac{w}{2} T \right\rfloor^{-1} \left[\underline{b}_t \left\lfloor \frac{w}{2} T \right\rfloor, \overline{b}_t \left\lfloor \frac{w}{2} T \right\rfloor\right] = \left[\sigma_0^{-1} \underline{b}_t T^{1/2}, \sigma_0^{-1} \overline{b}_t T^{1/2}\right]. \end{aligned}$$

If we use the demeaned values of the process instead, $x_t - \mu = y_t$, we have $\sigma_0^{-1} T^{1/2} \bar{A}_{1,t} \in [\sigma_0^{-1} (\underline{b}_t - \mu) T^{1/2}, \sigma_0^{-1} \left(\overline{b}_t - \mu\right) T^{1/2}]$. Therefore, the first re-scaled mean converges to:

$$\sigma_0^{-1} T^{1/2} A_{1,t} \Rightarrow \frac{w}{2} \left[J^{\kappa}(r + w/2) - J^{\kappa}(r)\right].$$

The difference between the two means defines the statistic:

$$\begin{aligned}
T^{1/2}M_{t,\lfloor wT\rfloor} &= T^{1/2}\frac{\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} x_{t+j}-\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} x_{t-j+1}}{\lfloor \frac{w}{2}T\rfloor}\\
&= T^{1/2}\frac{\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} (x_{t+j}-\mu)-\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} (x_{t-j+1}-\mu)}{\lfloor \frac{w}{2}T\rfloor}\\
&= T^{1/2}\frac{\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} y_{t+j}-\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} y_{t-j+1}}{\lfloor \frac{w}{2}T\rfloor},
\end{aligned}$$

or, equivalently, $T^{1/2}M_{t,\lfloor wT\rfloor}=\lfloor \frac{w}{2}T\rfloor^{-1}T^{1/2}[(\sum_{j=1}^{t+\lfloor \frac{w}{2}T\rfloor} y_j-\sum_{j=1}^{t}y_j)-(\sum_{j=1}^{t}y_j-\sum_{j=1}^{t-\lfloor \frac{w}{2}T\rfloor} y_j)]$, so that it follows from the Continuous Mapping Theorem (CMT):

$$T^{1/2}M_{t,\lfloor wT\rfloor}\Rightarrow \sigma_0 2w^{-1}\left(J^{\kappa}(r+w/2)-2J^{\kappa}(r)+J^{\kappa}(r-w/2)\right).$$

The use of the FCLT allows us to establish the following result:

$$\begin{aligned}
S_0 &= \hat{\sigma}_0^{-1}T^{1/2}\max_{t\in\Lambda_T}\left|M_{t,\lfloor wT\rfloor}\right|\\
&\Rightarrow \sup_{\lambda\in\Lambda}\left|2w^{-1}\left(J^{\kappa}(\lambda+w/2)-2J^{\kappa}(\lambda)+J^{\kappa}(\lambda-w/2)\right)\right|,
\end{aligned}$$

provided that $\hat{\sigma}_0^2\xrightarrow{p}\sigma_0^2$, where "$\xrightarrow{p}$" denotes convergence in probability.

Part (b), the $BI(1)$ case. Now we have $T^{-1/2}y_t\Rightarrow\sigma_1 J_{\underline{c}}^{\overline{c},\kappa}(r)$ and

$$\begin{aligned}
T^{-1/2}M_{t,\lfloor wT\rfloor} &= T^{-1/2}\frac{\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} y_{t+j}-\sum_{j=1}^{\lfloor \frac{w}{2}T\rfloor} y_{t-j+1}}{\lfloor \frac{w}{2}T\rfloor}\\
&\Rightarrow 2w^{-1}\sigma_1\left(\int_r^{r+w/2} J_{\underline{c}_j^+}^{\overline{c}_j^+,\kappa}(s)\,ds-\int_{r-w/2}^{r} J_{\underline{c}_j^-}^{\overline{c}_j^-,\kappa}(s)\,ds\right),
\end{aligned}$$

$\tau_{j-1}^0<t\le\tau_j^0$, where τ_j^0, $j=1,\ldots,n+1$. Note that when $n>0$ it might be possible that, for observations next to the border defined by two consecutive bound-regimes, some observations that are used in the computation of the statistic lie in different bound-regimes. In this case, $J_{\underline{c}_j}^{\overline{c}_j,\kappa}(r)$ would be (partially) limited by different bounds for values of r next to π_j. The use of the superscripts $-/+$ in the notation of $J_{\underline{c}_j}^{\overline{c}_j,\kappa}(\lambda)$ aims at highlighting this possibility. It should be understood that when r is not close to the extremes of the Λ_j set, then $\overline{c}_j^+=\overline{c}_j^-=\overline{c}_j$ and $\underline{c}_j^+=\underline{c}_j^-=\underline{c}_j$. The application of the FCLT gives:

$$\begin{aligned}
S_1 &= \hat{\sigma}_1^{-1}T^{-1/2}\max_{t\in\Lambda_T}\left|M_{t,\lfloor wT\rfloor}\right|\\
&\Rightarrow \sup_{\lambda\in\Lambda_j}\left|2w^{-1}\left(\int_\lambda^{\lambda+w/2} J_{\underline{c}_j^+}^{\overline{c}_j^+,\kappa}(s)\,ds-\int_{\lambda-w/2}^{\lambda} J_{\underline{c}_j^-}^{\overline{c}_j^-,\kappa}(s)\,ds\right)\right|,
\end{aligned}$$

with $\Lambda_j = (\pi_{j-1}, \pi_j]$, $j = 1, \ldots, n+1$, $\pi_0 = \epsilon$, $\pi_{n+1} = 1 - \epsilon$, and provided that $\hat{\sigma}_1^2 \xrightarrow{p} \sigma_1^2$.

A.2 Proof of Lemma 1

The proof follows the one of Theorem 3 in Harvey, Leybourne and Taylor (2010). To prove Statement (a) we first note that $y_t = \mu + \sum_{i=1}^{m_{\max}} \gamma_i DU_{i,t} + u_t$, which implies that $T^{-1/2}\hat{u}_{\lfloor rT \rfloor} \Rightarrow \sigma_1 H(r, \kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})$, where $H(r, \kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})$ is the residual of the orthogonal projection of $J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r)$ onto the space spanned by $(1, 1(r > \hat{\lambda}_1), \ldots, 1(r > \hat{\lambda}_{m_{\max}}))$. Harvey, Leybourne and Taylor (2010) distinguish among three different cases depending on (i) $0 < m = m_{\max}$, (ii) $m = 0$ with $m_{\max} > 0$, and (iii) $0 < m < m_{\max}$. In the first case, $\hat{\lambda}_{B,m_{\max}} \xrightarrow{p} \lambda$, so that $\hat{\lambda}_{B,m_{\max}}$ is a non-stochastic argument of H. In the second case, $\hat{\lambda}_{B,m_{\max}}$ is a $m_{\max}$-vector of dependent random variables, but whose distribution is independent of $J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r)$. Finally, the third case defines an intermediate situation in which m elements of $\hat{\lambda}_{B,m_{\max}}$ converge towards the corresponding true break fraction, whereas the $m_{\max} - m$ remaining elements are dependent random variables, but whose distribution is independent of $J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r)$. Due to the random characteristic of (part of) the elements in $\hat{\lambda}_{B,m_{\max}}$ in two out of three cases, the notation that defines H uses $\hat{\lambda}_{B,m_{\max}}$ instead of λ.

The OLS estimator of ρ in (10) converges towards:

$$T\hat{\rho} \Rightarrow \frac{\sigma_e}{\sigma_1} \frac{\int_0^1 H(r, \kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}}) dJ_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r)}{\int_0^1 H(r, \kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})^2 dr},$$

and, since $\hat{\sigma}_e^2 \xrightarrow{p} \sigma_e^2$, we have that:

$$\begin{aligned} T^{-2}\hat{\sigma}_0^2 &= \hat{\sigma}_e^2 / (T\hat{\rho})^2 \\ &\Rightarrow \sigma_1^2 \left(\frac{\int_0^1 H(r, \kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})^2 dr}{\int_0^1 H(r, \kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}}) dJ_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r)} \right)^2 \\ &\equiv \sigma_1^2 Q(\kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}}). \end{aligned}$$

Using the results from the previous proof, it is easy to see that:

$$\begin{aligned} S_0 &= \hat{\sigma}_0^{-1} T^{1/2} \max_{t \in \Lambda_T} \left| M_{t,\lfloor wT \rfloor} \right| \\ &= \left(T^{-1}\hat{\sigma}_0\right)^{-1} T^{-1/2} \max_{t \in \Lambda_T} \left| M_{t,\lfloor wT \rfloor} \right| \\ &\Rightarrow \frac{\sup_{\lambda \in \Lambda_j} \left| 2w^{-1} \left(\int_{\lambda}^{\lambda + w/2} J_{\underline{c}_j^+}^{\overline{c}_j^+,\kappa}(s)\, ds - \int_{\lambda - w/2}^{\lambda} J_{\underline{c}_j^-}^{\overline{c}_j^-,\kappa}(s)\, ds \right) \right|}{Q^{1/2}(\kappa, \underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}, w, \hat{\lambda}_{B,m_{\max}})}; \quad j = 1, \ldots, n+1. \end{aligned}$$

The proof of Statement (b) follows from the fact that $\hat{\sigma}_1^2 = O_p\left(k^{-2}\right)$ as shown in Harvey, Leybourne and Taylor (2010), so that $S_1 = \hat{\sigma}_1^{-1} T^{-1/2} \max_{t \in \Lambda_T} \left| M_{t,\lfloor wT \rfloor} \right| = \hat{\sigma}_1^{-1} T^{-1} T^{1/2} \max_{t \in \Lambda_T} \left| M_{t,\lfloor wT \rfloor} \right| = O_p\left(kT^{-1}\right)$.

A.3 Proof of Lemma 2

Let us first deal with the $BI(0)$ case and define $\hat{x} = M_D x = M_D y = y - D(D'D)^{-1} D'y$, with $D = diag(\iota_1, \ldots, \iota_{n+1})$ a $(T \times (n+1))$ block-orthogonal diagonal matrix built by $(T_j - T_{j-1})$-vector of ones, $j = 1, \ldots, n+1$, and the convention that $T_0 = 1$ and $T_{n+1} = 1$. Provided that:

$$T^{-1} D'D = \begin{bmatrix} \pi_1^0 & & & 0 \\ & \pi_2^0 - \pi_1^0 & & \\ & & \ddots & \\ 0 & & & 1 - \pi_n^0 \end{bmatrix}; \quad T^{-1/2} D'y \Rightarrow \sigma_0 \begin{pmatrix} J^{\kappa}(\pi_1^0) \\ J^{\kappa}(\pi_2^0) - J^{\kappa}(\pi_1^0) \\ \vdots \\ J^{\kappa}(1) - J^{\kappa}(\pi_n^0) \end{pmatrix},$$

we have that for a generic $\tau_{j-1}^0 < t \leq \tau_j^0$ – suppose, for instance, $j > 2$:

$$\begin{aligned} T^{1/2} \sum_{j=1}^{t} \hat{x}_j &\Rightarrow \sigma_0 \Bigg(J^{\kappa}(r) - \frac{1}{\pi_1^0} J^{\kappa}(\pi_1^0) \pi_1^0 - \frac{1}{\pi_2^0 - \pi_1^0} \left(J^{\kappa}(\pi_2^0) - J^{\kappa}(\pi_1^0) \right) (\pi_2^0 - \pi_1^0) \\ &\quad - \cdots - \frac{1}{\pi_j^0 - \pi_{j-1}^0} \left(J^{\kappa}(\pi_j^0) - J^{\kappa}(\pi_{j-1}^0) \right) (r - \pi_{j-1}^0) \Bigg) \\ &= \sigma_0 \left(J^{\kappa}(r) - J^{\kappa}(\pi_{j-1}^0) - \left(J^{\kappa}(\pi_j^0) - J^{\kappa}(\pi_{j-1}^0) \right) \frac{r - \pi_{j-1}^0}{\pi_j^0 - \pi_{j-1}^0} \right) \\ &\equiv \sigma_0 V^{\kappa}(r), \end{aligned}$$

for $\pi_{j-1}^0 < r \leq \pi_j^0$, $j = 1, \ldots, n+1$. Then, the re-scaled mean difference statistic is:

$$\begin{aligned} T^{1/2} M_{t,\lfloor wT \rfloor}^* &= T^{1/2} \frac{\sum_{j=1}^{\lfloor \frac{w}{2} T \rfloor} \hat{x}_{t+j} - \sum_{j=1}^{\lfloor \frac{w}{2} T \rfloor} \hat{x}_{t-j+1}}{\lfloor \frac{w}{2} T \rfloor} \\ &\Rightarrow 2w^{-1} \sigma_0 \left(V^{\kappa}(r + w/2) - 2V^{\kappa}(r) + V^{\kappa}(r - w/2) \right). \end{aligned}$$

The application of the FCLT gives:

$$\begin{aligned} S_0^* &= \hat{\sigma}_0^{-1} T^{1/2} \max_{t \in \Lambda_T} \left| M_{t,\lfloor wT \rfloor}^* \right| \\ &\Rightarrow \sup_{\lambda \in \Lambda} \left| 2w^{-1} \left(V^{\kappa}(\lambda + w/2) - 2V^{\kappa}(\lambda) + V^{\kappa}(\lambda - w/2) \right) \right|, \end{aligned}$$

provided that $\hat{\sigma}_0^2 \xrightarrow{p} \sigma_0^2$.

Let us now center on the $BI(1)$ case, with $\hat{x} = M_D x$ and $T^{-1} D'D$ defined, and:

$$T^{-3/2} D'y \Rightarrow \sigma_1 \begin{pmatrix} \int_0^{\pi_1^0} J_{\underline{c}}^{\bar{c},\kappa}(s)\, ds \\ \int_{\pi_1^0}^{\pi_2^0} J_{\underline{c}}^{\bar{c},\kappa}(s)\, ds \\ \vdots \\ \int_{\pi_n^0}^{1} J_{\underline{c}}^{\bar{c},\kappa}(s)\, ds \end{pmatrix},$$

which for a generic $\tau_{j-1}^0 < t \leq \tau_j^0$ – suppose, for instance, $j > 2$ – leads to:

$$\begin{aligned} T^{-1/2}\hat{x}_t &\Rightarrow \sigma_1 \left(J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r) - \int_0^{\pi_1^0} J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(s)\,ds - \int_{\pi_1^0}^{\pi_2^0} J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(s)\,ds - \cdots - \int_{\pi_{j-1}^0}^{\pi_j^0} J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(s)\,ds \frac{r-\pi_{j-1}^0}{\pi_j^0 - \pi_{j-1}^0} \right) \\ &= \sigma_1 \left(J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r) - \int_0^{\pi_{j-1}^0} J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(s)\,ds - \int_{\pi_{j-1}^0}^{\pi_j^0} J_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(s)\,ds \frac{r-\pi_{j-1}^0}{\pi_j^0 - \pi_{j-1}^0} \right) \\ &\equiv \sigma_1 W_{\underline{\boldsymbol{c}}}^{\overline{\boldsymbol{c}},\kappa}(r). \end{aligned}$$

We can define the modified statistic as:

$$\begin{aligned} T^{-1/2} M_{t,\lfloor wT \rfloor}^* &= T^{-1/2} \frac{\sum_{j=1}^{\lfloor \frac{w}{2}T \rfloor} \hat{x}_{t+j} - \sum_{j=1}^{\lfloor \frac{w}{2}T \rfloor} \hat{x}_{t-j+1}}{\lfloor \frac{w}{2}T \rfloor} \\ &\Rightarrow 2w^{-1}\sigma_1 \left(\int_r^{r+w/2} W_{\underline{c}_j^+}^{\overline{c}_j^+,\kappa}(s)\,ds - \int_{r-w/2}^{r} W_{\underline{c}_j^-}^{\overline{c}_j^-,\kappa}(s)\,ds \right). \end{aligned}$$

As pointed out above, it should be understood that when r is not close to the extremes of the Λ_j set, then $\overline{c}_j^+ = \overline{c}_j^- = \overline{c}_j$ and $\underline{c}_j^+ = \underline{c}_j^- = \underline{c}_j$, $j = 1, \ldots, n+1$. Then:

$$\begin{aligned} S_1^* &= \hat{\sigma}_1^{-1} T^{-1/2} \max_{t \in \Lambda_T} \left| M_{t,\lfloor wT \rfloor}^* \right| \\ &\Rightarrow \sup_{\lambda \in \Lambda_j} \left| 2w^{-1} \left(\int_\lambda^{\lambda+w/2} W_{\underline{c}_j^+}^{\overline{c}_j^+,\kappa}(s)\,ds - \int_{\lambda-w/2}^{\lambda} W_{\underline{c}_j^-}^{\overline{c}_j^-,\kappa}(s)\,ds \right) \right|, \end{aligned}$$

with $\Lambda_j = (\pi_{j-1}, \pi_j]$, $j = 1, \ldots, n+1$, $\pi_0 = \epsilon$, $\pi_{n+1} = 1-\epsilon$, and provided that $\hat{\sigma}_1^2 \xrightarrow{p} \sigma_1^2$.

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Table 1: Asymptotic critical values for the test statistics at the 5% level of significance for Cases A and B

		Case A ($n = 0$) $[\underline{c}, \overline{c}]$, $\underline{c} = -\overline{c}$										
	w	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.5
S_0	0.1	22.7698	22.8337	22.8337	22.8337	22.8337	22.8337	22.8337	22.8337	22.8337	22.8337	22.8337
	0.3	12.2785	12.3046	12.3046	12.3046	12.3046	12.3046	12.3046	12.3046	12.3046	12.3046	12.3046
S_1	0.1	0.1202	0.2790	0.4041	0.4799	0.5086	0.5236	0.5362	0.5452	0.5505	0.5554	0.5711
	0.3	0.0787	0.2397	0.3930	0.5255	0.6314	0.6968	0.7308	0.7559	0.7758	0.7927	0.8557
κ_ξ	0.1	1.0403	1.0350	1.0390	1.0493	1.0539	1.0546	1.0533	1.0533	1.0543	1.0559	1.0534
	0.3	1.0654	1.0542	1.0466	1.0517	1.0571	1.0649	1.0675	1.0649	1.0658	1.0626	1.0656

		Case B ($n = 1$) $[\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}]$, $\underline{\boldsymbol{c}} = -\overline{\boldsymbol{c}}, \overline{\boldsymbol{c}} = (\overline{c}_1, \overline{c}_2)$, $\pi = 0.5$									
	w	$(0.3, 0.5)$	$(0.3, 0.8)$	$(0.3, 1)$	$(0.3, 1.5)$	$(0.5, 0.8)$	$(0.5, 1)$	$(0.5, 1.5)$	$(0.8, 1)$	$(0.8, 1.5)$	$(1, 1.5)$
S_0	0.1	22.7927	22.7927	22.7927	22.7927	22.7927	22.7927	22.7927	22.7927	22.7927	22.7927
	0.3	12.2298	12.2298	12.2298	12.2298	12.2298	12.2298	12.2298	12.2298	12.2298	12.2298
S_1	0.1	0.5767	0.9485	1.1574	1.3836	0.6801	1.0080	1.6191	0.5775	1.0910	0.6971
	0.3	0.6144	0.8562	1.0490	1.3989	0.7632	0.9192	1.4325	0.7914	0.9589	0.8510
κ_ξ	0.1	1.0612	1.0495	1.0503	1.0554	1.0728	1.0641	1.0751	1.0482	1.0938	1.1030
	0.3	1.0656	1.0642	1.0614	1.0651	1.0715	1.0797	1.0965	1.0633	1.0904	1.0614

		Case B ($n = 1$). Modified statistics $[\underline{\boldsymbol{c}}, \overline{\boldsymbol{c}}]$, $\underline{\boldsymbol{c}} = -\overline{\boldsymbol{c}}, \overline{\boldsymbol{c}} = (\overline{c}_1, \overline{c}_2)$, $\pi = 0.5$									
	w	$(0.3, 0.5)$	$(0.3, 0.8)$	$(0.3, 1)$	$(0.3, 1.5)$	$(0.5, 0.8)$	$(0.5, 1)$	$(0.5, 1.5)$	$(0.8, 1)$	$(0.8, 1.5)$	$(1, 1.5)$
S_0^*	0.1	22.7601	22.7601	22.7601	22.7601	22.7601	22.7601	22.7601	22.7601	22.7601	22.7601
	0.3	12.0219	12.0219	12.0219	12.0219	12.0219	12.0219	12.0219	12.0219	12.0219	12.0219
S_1^*	0.1	0.5622	0.6683	0.6980	0.7500	0.6809	0.7346	0.8227	0.7006	0.8366	0.8404
	0.3	0.6022	0.7105	0.7303	0.7662	0.7127	0.7335	0.7713	0.7617	0.7937	0.8108
κ_ξ^*	0.1	1.0633	1.0825	1.0824	1.0849	1.0771	1.0771	1.0778	1.0758	1.0771	1.0772
	0.3	1.0690	1.0722	1.0763	1.0785	1.0697	1.0742	1.0772	1.0669	1.0668	1.0632

Table 2: Case A. Empirical size

			$w = 0.10$			$w = 0.15$			$w = 0.20$			$w = 0.25$			$w = 0.30$		
$\bar{c}$	T	α	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
0.3	50	1	0.022	0.199	0.157	0.016	0.178	0.129	0.017	0.168	0.132	0.015	0.123	0.101	0.005	0.146	0.114
		0.95	0.033	0.201	0.151	0.023	0.159	0.127	0.013	0.150	0.106	0.012	0.136	0.099	0.004	0.137	0.103
		0.9	0.024	0.204	0.175	0.021	0.172	0.130	0.013	0.151	0.120	0.008	0.144	0.107	0.004	0.147	0.119
		0.7	0.019	0.179	0.141	0.008	0.135	0.093	0.005	0.123	0.089	0.007	0.102	0.076	0.003	0.105	0.079
		0.5	0.009	0.157	0.112	0.005	0.094	0.054	0.004	0.071	0.048	0.002	0.061	0.043	0.001	0.064	0.044
	150	1	0.015	0.069	0.053	0.006	0.085	0.068	0.004	0.084	0.068	0.001	0.083	0.063	0.001	0.093	0.075
		0.95	0.012	0.073	0.057	0.000	0.076	0.052	0.000	0.074	0.059	0.000	0.091	0.062	0.000	0.095	0.072
		0.9	0.006	0.051	0.039	0.002	0.049	0.035	0.002	0.050	0.035	0.002	0.049	0.041	0.001	0.054	0.042
		0.7	0.000	0.026	0.014	0.000	0.026	0.011	0.000	0.023	0.012	0.000	0.031	0.019	0.000	0.018	0.006
		0.5	0.000	0.029	0.018	0.000	0.035	0.014	0.000	0.027	0.013	0.000	0.037	0.020	0.000	0.034	0.016
	300	1	0.021	0.031	0.030	0.012	0.056	0.047	0.005	0.055	0.045	0.001	0.070	0.052	0.000	0.072	0.054
		0.95	0.007	0.034	0.019	0.000	0.035	0.027	0.000	0.051	0.032	0.000	0.067	0.051	0.000	0.074	0.051
		0.9	0.001	0.024	0.010	0.000	0.024	0.019	0.000	0.027	0.023	0.000	0.033	0.023	0.000	0.033	0.020
		0.7	0.000	0.020	0.010	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.022	0.009
		0.5	0.000	0.025	0.010	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010
0.5	50	1	0.055	0.177	0.143	0.047	0.187	0.152	0.038	0.205	0.162	0.040	0.201	0.162	0.033	0.235	0.196
		0.95	0.051	0.172	0.137	0.024	0.166	0.131	0.023	0.155	0.128	0.023	0.153	0.122	0.018	0.190	0.148
		0.9	0.032	0.184	0.150	0.023	0.163	0.124	0.020	0.165	0.120	0.019	0.149	0.123	0.011	0.194	0.154
		0.7	0.024	0.188	0.151	0.015	0.118	0.081	0.008	0.100	0.070	0.006	0.089	0.072	0.003	0.085	0.060
		0.5	0.013	0.154	0.117	0.007	0.083	0.054	0.004	0.078	0.049	0.002	0.065	0.053	0.000	0.067	0.046
	150	1	0.054	0.064	0.064	0.035	0.091	0.074	0.035	0.102	0.092	0.025	0.134	0.105	0.026	0.141	0.112
		0.95	0.018	0.055	0.041	0.009	0.055	0.041	0.013	0.067	0.052	0.008	0.063	0.053	0.006	0.074	0.054
		0.9	0.001	0.040	0.023	0.001	0.038	0.031	0.001	0.038	0.027	0.000	0.046	0.035	0.000	0.043	0.033
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.016	0.000	0.030	0.015	0.000	0.036	0.019	0.000	0.033	0.016
	300	1	0.046	0.023	0.040	0.051	0.061	0.062	0.054	0.075	0.073	0.042	0.103	0.094	0.037	0.114	0.091
		0.95	0.003	0.016	0.011	0.000	0.024	0.017	0.001	0.026	0.018	0.001	0.042	0.029	0.001	0.040	0.028
		0.9	0.000	0.021	0.010	0.000	0.023	0.018	0.000	0.025	0.020	0.000	0.028	0.020	0.000	0.029	0.015
		0.7	0.000	0.020	0.009	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.008
		0.5	0.000	0.025	0.010	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010
0.8	50	1	0.059	0.174	0.153	0.054	0.142	0.130	0.059	0.151	0.132	0.059	0.152	0.123	0.051	0.178	0.161
		0.95	0.055	0.173	0.141	0.046	0.150	0.126	0.047	0.158	0.137	0.042	0.135	0.118	0.033	0.165	0.136
		0.9	0.047	0.180	0.155	0.036	0.140	0.109	0.031	0.128	0.101	0.022	0.119	0.097	0.019	0.148	0.122
		0.7	0.028	0.188	0.146	0.014	0.104	0.078	0.009	0.101	0.073	0.006	0.087	0.068	0.000	0.086	0.062
		0.5	0.004	0.155	0.118	0.002	0.088	0.056	0.000	0.083	0.058	0.000	0.067	0.053	0.000	0.067	0.047
	150	1	0.081	0.049	0.068	0.038	0.068	0.066	0.044	0.085	0.086	0.033	0.103	0.094	0.040	0.124	0.110
		0.95	0.038	0.041	0.035	0.011	0.045	0.037	0.009	0.051	0.037	0.005	0.051	0.042	0.005	0.063	0.042
		0.9	0.007	0.044	0.028	0.002	0.036	0.029	0.000	0.035	0.025	0.000	0.041	0.031	0.000	0.039	0.028
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.018	0.000	0.030	0.015	0.000	0.036	0.019	0.000	0.033	0.016
	300	1	0.054	0.018	0.038	0.048	0.050	0.052	0.043	0.066	0.066	0.038	0.085	0.089	0.036	0.089	0.076
		0.95	0.004	0.017	0.012	0.001	0.024	0.016	0.000	0.025	0.017	0.000	0.043	0.027	0.000	0.041	0.028
		0.9	0.000	0.021	0.010	0.000	0.023	0.018	0.000	0.025	0.019	0.000	0.028	0.019	0.000	0.029	0.015
		0.7	0.000	0.020	0.010	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.007
		0.5	0.000	0.025	0.010	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010

continues ...

			$w = 0.10$			$w = 0.15$			$w = 0.20$			$w = 0.25$			$w = 0.30$		
$\bar{c}$	T	α	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	50	1	0.032	0.150	0.125	0.043	0.137	0.120	0.048	0.147	0.129	0.047	0.142	0.125	0.051	0.197	0.178
		0.95	0.036	0.166	0.139	0.033	0.136	0.113	0.035	0.147	0.117	0.037	0.129	0.118	0.032	0.164	0.140
		0.9	0.025	0.176	0.147	0.032	0.133	0.105	0.021	0.127	0.096	0.019	0.115	0.093	0.016	0.142	0.118
		0.7	0.007	0.188	0.146	0.006	0.104	0.076	0.001	0.102	0.075	0.001	0.088	0.069	0.000	0.086	0.063
		0.5	0.001	0.155	0.118	0.001	0.088	0.056	0.000	0.083	0.058	0.000	0.067	0.053	0.000	0.067	0.048
	150	1	0.117	0.040	0.088	0.056	0.055	0.064	0.062	0.074	0.083	0.050	0.105	0.112	0.049	0.119	0.118
		0.95	0.056	0.043	0.048	0.017	0.045	0.039	0.013	0.051	0.039	0.004	0.051	0.042	0.004	0.063	0.042
		0.9	0.014	0.044	0.028	0.004	0.036	0.031	0.001	0.035	0.025	0.000	0.041	0.031	0.000	0.039	0.029
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.018	0.000	0.030	0.015	0.000	0.036	0.020	0.000	0.033	0.016
	300	1	0.088	0.012	0.069	0.074	0.041	0.069	0.063	0.063	0.080	0.061	0.080	0.090	0.052	0.088	0.092
		0.95	0.007	0.017	0.013	0.001	0.024	0.017	0.000	0.025	0.017	0.000	0.043	0.028	0.000	0.041	0.029
		0.9	0.000	0.021	0.010	0.000	0.023	0.018	0.000	0.025	0.019	0.000	0.028	0.019	0.000	0.029	0.015
		0.7	0.000	0.020	0.010	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.007
		0.5	0.000	0.025	0.011	0.000	0.033	0.018	0.000	0.027	0.013	0.000	0.023	0.010	0.000	0.028	0.010
1.5	50	1	0.000	0.150	0.112	0.002	0.133	0.105	0.004	0.132	0.101	0.005	0.125	0.099	0.007	0.171	0.143
		0.95	0.001	0.160	0.134	0.001	0.132	0.101	0.002	0.140	0.107	0.004	0.120	0.103	0.002	0.154	0.122
		0.9	0.000	0.176	0.145	0.001	0.133	0.098	0.001	0.126	0.099	0.002	0.114	0.086	0.000	0.142	0.113
		0.7	0.000	0.187	0.145	0.000	0.104	0.077	0.000	0.102	0.074	0.000	0.088	0.071	0.000	0.086	0.064
		0.5	0.000	0.154	0.117	0.000	0.089	0.059	0.000	0.084	0.059	0.000	0.067	0.054	0.000	0.067	0.048
	150	1	0.011	0.040	0.034	0.017	0.039	0.034	0.024	0.058	0.061	0.025	0.081	0.080	0.030	0.103	0.096
		0.95	0.003	0.043	0.032	0.000	0.045	0.034	0.000	0.051	0.035	0.000	0.051	0.041	0.000	0.063	0.041
		0.9	0.001	0.044	0.026	0.000	0.036	0.029	0.000	0.035	0.025	0.000	0.041	0.031	0.000	0.039	0.030
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.018	0.000	0.030	0.015	0.000	0.036	0.021	0.000	0.033	0.016
	300	1	0.020	0.008	0.021	0.026	0.035	0.040	0.034	0.046	0.052	0.034	0.072	0.066	0.042	0.081	0.083
		0.95	0.000	0.017	0.010	0.000	0.024	0.016	0.000	0.025	0.017	0.000	0.043	0.028	0.000	0.041	0.029
		0.9	0.000	0.021	0.010	0.000	0.023	0.019	0.000	0.025	0.019	0.000	0.028	0.020	0.000	0.029	0.015
		0.7	0.000	0.020	0.011	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.008
		0.5	0.000	0.025	0.011	0.000	0.033	0.018	0.000	0.027	0.013	0.000	0.023	0.010	0.000	0.028	0.010

Table 3: Case A. Empirical power with one structural break, $\gamma = \gamma^* T^{1/2}$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.981	0.944	0.990	0.983	0.963	0.985	0.983	0.972	0.993	0.993	0.976	0.990	0.994	0.993	0.994
		150	1.000	0.966	0.999	1.000	0.974	0.999	1.000	0.988	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.961	1.000	1.000	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.909	0.792	0.918	0.909	0.827	0.929	0.903	0.817	0.932	0.895	0.891	0.936	0.906	0.976	0.970
		150	0.979	0.770	0.971	0.980	0.812	0.978	0.986	0.925	0.986	0.997	1.000	1.000	0.982	1.000	1.000
		300	0.982	0.733	0.976	0.996	0.920	0.993	0.999	0.996	0.999	1.000	1.000	1.000	0.939	1.000	1.000
	0.8	50	0.782	0.673	0.844	0.770	0.701	0.837	0.784	0.707	0.836	0.790	0.880	0.914	0.784	0.973	0.966
		150	0.954	0.622	0.935	0.958	0.749	0.941	0.970	0.915	0.980	0.959	1.000	1.000	0.842	1.000	1.000
		300	0.961	0.615	0.946	0.987	0.908	0.984	0.997	0.997	0.999	0.946	1.000	1.000	0.575	1.000	1.000
	1.0	50	0.651	0.635	0.775	0.646	0.672	0.782	0.659	0.704	0.799	0.659	0.880	0.900	0.625	0.973	0.963
		150	0.936	0.572	0.919	0.952	0.748	0.940	0.960	0.915	0.979	0.896	1.000	1.000	0.658	1.000	1.000
		300	0.962	0.568	0.943	0.989	0.908	0.986	0.995	0.997	0.999	0.840	1.000	1.000	0.317	1.000	1.000
	1.5	50	0.261	0.582	0.621	0.268	0.655	0.670	0.271	0.702	0.712	0.234	0.880	0.860	0.160	0.974	0.958
		150	0.617	0.511	0.717	0.617	0.748	0.841	0.587	0.915	0.938	0.396	1.000	1.000	0.173	1.000	1.000
		300	0.785	0.509	0.807	0.815	0.908	0.953	0.786	0.997	0.998	0.371	1.000	1.000	0.042	1.000	1.000
5	0.3	50	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.996	0.999	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	0.993	0.999	1.000	0.994	0.999	1.000	0.996	1.000	1.000	0.982	1.000	1.000	0.972	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	0.969	1.000	1.000
		300	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.907	1.000	1.000
	1.0	50	0.972	0.999	1.000	0.978	0.999	1.000	0.981	1.000	1.000	0.956	1.000	1.000	0.942	1.000	1.000
		150	1.000	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.986	1.000	1.000	0.919	1.000	1.000
		300	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.976	1.000	1.000	0.755	1.000	1.000
	1.5	50	0.856	0.995	1.000	0.858	0.999	1.000	0.838	1.000	1.000	0.772	1.000	1.000	0.710	1.000	1.000
		150	0.997	0.992	1.000	0.994	1.000	1.000	0.985	1.000	1.000	0.899	1.000	1.000	0.662	1.000	1.000
		300	1.000	0.992	1.000	0.999	1.000	1.000	0.998	1.000	1.000	0.840	1.000	1.000	0.311	1.000	1.000
10	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	0.997	1.000	1.000	0.997	1.000	1.000	0.998	1.000	1.000	0.995	1.000	1.000	0.986	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	1.000	1.000
		300	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.975	1.000	1.000
	1.0	50	0.988	1.000	1.000	0.993	1.000	1.000	0.994	1.000	1.000	0.979	1.000	1.000	0.968	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	1.000	1.000	0.959	1.000	1.000
		300	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	0.878	1.000	1.000
	1.5	50	0.947	0.999	1.000	0.935	1.000	1.000	0.923	1.000	1.000	0.881	1.000	1.000	0.834	1.000	1.000
		150	0.999	0.998	1.000	0.999	1.000	1.000	0.995	1.000	1.000	0.954	1.000	1.000	0.804	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.927	1.000	1.000	0.503	1.000	1.000

Table 4: Case A. Empirical power with one structural break, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.019	0.181	0.126	0.011	0.160	0.128	0.012	0.173	0.129	0.003	0.131	0.088	0.001	0.093	0.053
		150	0.003	0.085	0.071	0.000	0.077	0.053	0.001	0.047	0.033	0.000	0.027	0.011	0.000	0.035	0.017
		300	0.000	0.056	0.046	0.000	0.035	0.026	0.000	0.024	0.019	0.000	0.020	0.015	0.000	0.033	0.016
	0.5	50	0.047	0.191	0.143	0.025	0.171	0.129	0.020	0.166	0.124	0.006	0.112	0.079	0.001	0.086	0.057
		150	0.027	0.091	0.077	0.013	0.056	0.042	0.001	0.039	0.034	0.000	0.028	0.017	0.000	0.038	0.019
		300	0.040	0.060	0.059	0.000	0.024	0.017	0.000	0.023	0.019	0.000	0.020	0.015	0.000	0.033	0.017
	0.8	50	0.021	0.147	0.118	0.015	0.153	0.120	0.011	0.141	0.102	0.002	0.104	0.072	0.000	0.090	0.061
		150	0.038	0.070	0.070	0.013	0.045	0.036	0.003	0.037	0.031	0.000	0.028	0.018	0.000	0.038	0.019
		300	0.053	0.050	0.058	0.001	0.024	0.016	0.000	0.023	0.019	0.000	0.020	0.015	0.000	0.033	0.017
	1.0	50	0.006	0.138	0.109	0.006	0.137	0.107	0.007	0.135	0.103	0.000	0.104	0.074	0.000	0.090	0.061
		150	0.039	0.056	0.058	0.012	0.045	0.037	0.001	0.037	0.031	0.000	0.028	0.018	0.000	0.038	0.019
		300	0.073	0.041	0.069	0.001	0.024	0.017	0.000	0.023	0.019	0.000	0.020	0.015	0.000	0.033	0.018
	1.5	50	0.002	0.133	0.102	0.001	0.131	0.101	0.001	0.137	0.103	0.000	0.104	0.076	0.000	0.092	0.061
		150	0.012	0.043	0.035	0.001	0.045	0.034	0.000	0.037	0.032	0.000	0.028	0.018	0.000	0.038	0.019
		300	0.022	0.035	0.035	0.000	0.024	0.016	0.000	0.023	0.020	0.000	0.020	0.015	0.000	0.033	0.018
5	0.3	50	0.014	0.181	0.137	0.012	0.168	0.139	0.011	0.174	0.135	0.004	0.127	0.088	0.002	0.085	0.057
		150	0.003	0.086	0.066	0.000	0.070	0.055	0.001	0.048	0.035	0.000	0.033	0.014	0.000	0.041	0.018
		300	0.000	0.057	0.046	0.000	0.035	0.026	0.000	0.023	0.019	0.000	0.021	0.015	0.000	0.036	0.020
	0.5	50	0.050	0.190	0.158	0.028	0.179	0.131	0.020	0.162	0.125	0.007	0.116	0.085	0.001	0.092	0.063
		150	0.028	0.092	0.079	0.012	0.057	0.046	0.002	0.038	0.035	0.000	0.032	0.014	0.000	0.044	0.019
		300	0.041	0.062	0.057	0.000	0.023	0.016	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
	0.8	50	0.022	0.149	0.122	0.017	0.152	0.127	0.012	0.137	0.098	0.001	0.112	0.083	0.000	0.093	0.065
		150	0.038	0.072	0.073	0.013	0.048	0.037	0.003	0.037	0.033	0.000	0.032	0.015	0.000	0.044	0.020
		300	0.053	0.051	0.059	0.001	0.022	0.015	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
	1.0	50	0.007	0.139	0.110	0.006	0.137	0.112	0.007	0.126	0.091	0.000	0.112	0.084	0.000	0.093	0.066
		150	0.042	0.054	0.058	0.012	0.048	0.038	0.001	0.037	0.032	0.000	0.032	0.015	0.000	0.044	0.022
		300	0.074	0.042	0.068	0.001	0.022	0.016	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
	1.5	50	0.002	0.140	0.114	0.001	0.134	0.108	0.001	0.127	0.093	0.000	0.112	0.086	0.000	0.093	0.066
		150	0.010	0.041	0.037	0.001	0.048	0.036	0.000	0.037	0.032	0.000	0.032	0.015	0.000	0.044	0.022
		300	0.021	0.035	0.035	0.000	0.022	0.016	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
10	0.3	50	0.033	0.192	0.155	0.033	0.205	0.161	0.022	0.205	0.158	0.009	0.142	0.103	0.005	0.114	0.089
		150	0.003	0.089	0.067	0.000	0.074	0.053	0.001	0.047	0.032	0.000	0.035	0.017	0.000	0.052	0.028
		300	0.000	0.057	0.042	0.000	0.034	0.024	0.000	0.023	0.019	0.000	0.022	0.016	0.000	0.040	0.021
	0.5	50	0.060	0.211	0.177	0.046	0.196	0.153	0.030	0.181	0.136	0.011	0.132	0.096	0.003	0.112	0.080
		150	0.024	0.090	0.075	0.010	0.054	0.048	0.002	0.042	0.031	0.000	0.037	0.018	0.000	0.055	0.028
		300	0.041	0.063	0.058	0.000	0.023	0.014	0.000	0.021	0.018	0.000	0.022	0.016	0.000	0.040	0.022
	0.8	50	0.038	0.160	0.130	0.024	0.170	0.130	0.018	0.151	0.111	0.004	0.128	0.090	0.000	0.113	0.083
		150	0.039	0.075	0.076	0.012	0.046	0.035	0.002	0.040	0.030	0.000	0.037	0.019	0.000	0.055	0.028
		300	0.056	0.052	0.060	0.001	0.022	0.013	0.000	0.021	0.018	0.000	0.022	0.017	0.000	0.040	0.023
	1.0	50	0.014	0.158	0.130	0.013	0.161	0.125	0.011	0.148	0.107	0.002	0.129	0.091	0.000	0.113	0.083
		150	0.042	0.055	0.060	0.012	0.047	0.037	0.002	0.040	0.029	0.000	0.037	0.019	0.000	0.055	0.029
		300	0.076	0.042	0.066	0.001	0.022	0.014	0.000	0.021	0.018	0.000	0.022	0.017	0.000	0.040	0.023
	1.5	50	0.002	0.159	0.133	0.003	0.159	0.127	0.002	0.150	0.107	0.000	0.129	0.092	0.000	0.114	0.084
		150	0.009	0.045	0.037	0.001	0.047	0.033	0.000	0.040	0.029	0.000	0.037	0.019	0.000	0.055	0.029
		300	0.022	0.038	0.035	0.000	0.022	0.013	0.000	0.021	0.018	0.000	0.022	0.018	0.000	0.040	0.023

Table 5: Case A. Empirical power with one structural break, $\gamma = \gamma^*$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.019	0.187	0.137	0.020	0.174	0.145	0.015	0.175	0.132	0.004	0.129	0.091	0.002	0.093	0.057
		150	0.003	0.086	0.066	0.000	0.075	0.057	0.001	0.049	0.035	0.000	0.034	0.022	0.000	0.058	0.028
		300	0.001	0.052	0.040	0.000	0.031	0.024	0.000	0.025	0.020	0.000	0.031	0.016	0.000	0.080	0.046
	0.5	50	0.056	0.202	0.152	0.033	0.184	0.132	0.021	0.166	0.124	0.008	0.116	0.091	0.001	0.096	0.076
		150	0.027	0.088	0.071	0.010	0.053	0.050	0.002	0.045	0.033	0.000	0.036	0.020	0.000	0.061	0.034
		300	0.043	0.060	0.056	0.000	0.023	0.015	0.000	0.024	0.020	0.000	0.031	0.016	0.000	0.080	0.048
	0.8	50	0.028	0.148	0.118	0.017	0.158	0.129	0.013	0.134	0.097	0.002	0.112	0.085	0.000	0.097	0.074
		150	0.043	0.075	0.076	0.013	0.043	0.039	0.003	0.043	0.032	0.000	0.036	0.020	0.000	0.061	0.035
		300	0.054	0.049	0.062	0.001	0.022	0.015	0.000	0.024	0.020	0.000	0.031	0.016	0.000	0.080	0.048
	1.0	50	0.008	0.151	0.119	0.005	0.146	0.117	0.006	0.126	0.093	0.001	0.112	0.087	0.000	0.097	0.074
		150	0.045	0.056	0.062	0.015	0.045	0.041	0.002	0.043	0.031	0.000	0.036	0.020	0.000	0.061	0.035
		300	0.078	0.042	0.067	0.001	0.022	0.016	0.000	0.024	0.020	0.000	0.031	0.016	0.000	0.080	0.048
	1.5	50	0.001	0.145	0.123	0.002	0.144	0.115	0.002	0.127	0.094	0.000	0.112	0.088	0.000	0.097	0.074
		150	0.010	0.048	0.039	0.001	0.045	0.037	0.000	0.043	0.031	0.000	0.036	0.020	0.000	0.061	0.036
		300	0.022	0.037	0.037	0.000	0.022	0.015	0.000	0.024	0.020	0.000	0.031	0.016	0.000	0.080	0.048
5	0.3	50	0.767	0.760	0.827	0.784	0.792	0.844	0.788	0.820	0.848	0.738	0.828	0.829	0.718	0.909	0.874
		150	0.237	0.324	0.315	0.199	0.382	0.368	0.196	0.392	0.351	0.141	0.827	0.777	0.156	0.973	0.960
		300	0.108	0.139	0.133	0.100	0.170	0.156	0.050	0.259	0.210	0.003	0.955	0.939	0.027	0.996	0.992
	0.5	50	0.582	0.561	0.639	0.586	0.590	0.653	0.554	0.591	0.645	0.516	0.666	0.657	0.532	0.826	0.792
		150	0.168	0.205	0.225	0.117	0.191	0.193	0.092	0.242	0.207	0.053	0.813	0.766	0.091	0.972	0.969
		300	0.095	0.109	0.131	0.030	0.108	0.092	0.002	0.221	0.177	0.001	0.955	0.942	0.000	0.996	0.993
	0.8	50	0.432	0.453	0.513	0.403	0.466	0.501	0.400	0.492	0.502	0.403	0.648	0.621	0.415	0.822	0.781
		150	0.175	0.143	0.191	0.112	0.165	0.166	0.080	0.238	0.200	0.032	0.813	0.767	0.008	0.972	0.969
		300	0.110	0.077	0.113	0.020	0.106	0.088	0.004	0.221	0.177	0.000	0.955	0.942	0.000	0.996	0.993
	1.0	50	0.297	0.417	0.437	0.292	0.448	0.449	0.278	0.488	0.480	0.262	0.648	0.597	0.257	0.822	0.775
		150	0.170	0.124	0.170	0.107	0.165	0.163	0.079	0.238	0.199	0.012	0.813	0.767	0.002	0.972	0.970
		300	0.135	0.069	0.116	0.025	0.106	0.088	0.002	0.221	0.177	0.000	0.955	0.942	0.000	0.996	0.993
	1.5	50	0.081	0.356	0.331	0.081	0.434	0.395	0.076	0.489	0.439	0.043	0.649	0.574	0.031	0.823	0.771
		150	0.060	0.101	0.101	0.034	0.165	0.141	0.016	0.238	0.188	0.001	0.813	0.768	0.000	0.972	0.970
		300	0.046	0.055	0.060	0.003	0.106	0.084	0.000	0.221	0.178	0.000	0.955	0.943	0.000	0.996	0.994
10	0.3	50	0.997	0.993	1.000	0.993	0.996	0.999	0.995	0.998	1.000	0.998	1.000	1.000	0.999	1.000	0.999
		150	0.975	0.894	0.970	0.980	0.915	0.981	0.987	0.945	0.983	0.999	1.000	1.000	1.000	1.000	1.000
		300	0.748	0.536	0.714	0.767	0.690	0.765	0.803	0.889	0.883	0.974	1.000	1.000	0.997	1.000	1.000
	0.5	50	0.988	0.938	0.998	0.990	0.954	1.000	0.995	0.953	0.997	0.992	0.982	0.999	0.983	1.000	1.000
		150	0.863	0.624	0.868	0.863	0.654	0.858	0.871	0.794	0.895	0.970	1.000	1.000	0.969	1.000	1.000
		300	0.460	0.328	0.460	0.450	0.477	0.550	0.410	0.850	0.813	0.588	1.000	1.000	0.791	1.000	1.000
	0.8	50	0.932	0.884	0.989	0.936	0.885	0.987	0.948	0.895	0.991	0.929	0.980	0.997	0.910	1.000	1.000
		150	0.807	0.472	0.773	0.787	0.567	0.779	0.819	0.780	0.863	0.904	1.000	1.000	0.792	1.000	1.000
		300	0.435	0.226	0.396	0.369	0.465	0.503	0.336	0.850	0.810	0.435	1.000	1.000	0.285	1.000	1.000
	1.0	50	0.854	0.854	0.974	0.868	0.858	0.974	0.868	0.896	0.980	0.835	0.980	0.994	0.777	1.000	1.000
		150	0.768	0.415	0.742	0.785	0.563	0.769	0.809	0.780	0.865	0.826	1.000	1.000	0.595	1.000	1.000
		300	0.462	0.212	0.394	0.395	0.465	0.503	0.364	0.850	0.811	0.295	1.000	1.000	0.070	1.000	1.000
	1.5	50	0.520	0.815	0.888	0.493	0.852	0.912	0.471	0.895	0.931	0.401	0.980	0.983	0.345	1.000	0.999
		150	0.398	0.360	0.485	0.393	0.563	0.615	0.361	0.780	0.783	0.218	1.000	1.000	0.078	1.000	1.000
		300	0.215	0.170	0.241	0.141	0.465	0.442	0.075	0.850	0.808	0.001	1.000	1.000	0.000	1.000	1.000

Table 6: Case B. Empirical size, $\pi = 0.5$, known $\bar{\boldsymbol{c}}$

			Original statistics									Modified statistics								
			$w = 0.10$			$w = 0.15$			$w = 0.30$			$w = 0.10$			$w = 0.15$			$w = 0.30$		
$(\bar{c}_1, \bar{c}_2)$	T	α	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*
$(0.3, 0.5)$	50	1	0.049	0.224	0.187	0.045	0.225	0.181	0.058	0.245	0.186	0.058	0.163	0.124	0.051	0.172	0.131	0.061	0.176	0.140
		0.95	0.038	0.233	0.184	0.045	0.198	0.160	0.040	0.208	0.166	0.038	0.154	0.121	0.048	0.152	0.121	0.047	0.140	0.116
		0.9	0.034	0.222	0.179	0.028	0.188	0.144	0.032	0.193	0.159	0.030	0.153	0.116	0.031	0.156	0.115	0.036	0.140	0.118
		0.7	0.012	0.172	0.135	0.009	0.129	0.093	0.010	0.102	0.069	0.015	0.118	0.082	0.010	0.105	0.068	0.014	0.087	0.060
		0.5	0.004	0.155	0.104	0.003	0.099	0.067	0.003	0.070	0.045	0.005	0.092	0.072	0.005	0.074	0.050	0.003	0.058	0.041
	150	1	0.060	0.101	0.093	0.053	0.092	0.092	0.052	0.160	0.152	0.056	0.085	0.081	0.057	0.069	0.074	0.052	0.098	0.098
		0.95	0.013	0.074	0.058	0.019	0.055	0.049	0.021	0.086	0.069	0.017	0.073	0.056	0.025	0.050	0.048	0.021	0.078	0.064
		0.9	0.000	0.046	0.032	0.002	0.030	0.021	0.002	0.057	0.037	0.002	0.041	0.025	0.005	0.028	0.021	0.002	0.050	0.033
		0.7	0.000	0.028	0.017	0.000	0.017	0.007	0.000	0.021	0.007	0.000	0.027	0.012	0.000	0.016	0.006	0.000	0.022	0.008
		0.5	0.000	0.036	0.019	0.000	0.025	0.011	0.000	0.039	0.018	0.000	0.032	0.018	0.000	0.024	0.011	0.000	0.039	0.016
	300	1	0.048	0.057	0.064	0.052	0.082	0.091	0.050	0.155	0.150	0.044	0.042	0.048	0.047	0.061	0.071	0.047	0.099	0.104
		0.95	0.000	0.023	0.011	0.001	0.029	0.023	0.001	0.051	0.035	0.000	0.020	0.010	0.001	0.024	0.020	0.001	0.045	0.031
		0.9	0.000	0.017	0.007	0.000	0.022	0.017	0.000	0.029	0.017	0.000	0.013	0.003	0.000	0.020	0.014	0.000	0.029	0.015
		0.7	0.000	0.015	0.007	0.000	0.021	0.015	0.000	0.020	0.008	0.000	0.014	0.005	0.000	0.021	0.012	0.000	0.022	0.008
		0.5	0.000	0.020	0.007	0.000	0.033	0.018	0.000	0.028	0.010	0.000	0.017	0.005	0.000	0.033	0.016	0.000	0.022	0.006
$(0.5, 1)$	50	1	0.035	0.281	0.224	0.041	0.262	0.219	0.052	0.281	0.232	0.041	0.133	0.105	0.050	0.147	0.115	0.041	0.145	0.135
		0.95	0.012	0.201	0.149	0.011	0.194	0.140	0.016	0.214	0.154	0.019	0.135	0.102	0.023	0.144	0.113	0.034	0.141	0.111
		0.9	0.001	0.178	0.134	0.001	0.151	0.098	0.005	0.168	0.120	0.011	0.125	0.088	0.014	0.127	0.077	0.023	0.133	0.098
		0.7	0.000	0.183	0.137	0.000	0.118	0.079	0.000	0.093	0.057	0.007	0.122	0.078	0.008	0.098	0.065	0.004	0.079	0.052
		0.5	0.000	0.153	0.111	0.000	0.089	0.055	0.000	0.071	0.045	0.000	0.095	0.073	0.001	0.066	0.042	0.002	0.061	0.046
	150	1	0.049	0.176	0.159	0.047	0.148	0.132	0.042	0.232	0.206	0.050	0.103	0.086	0.054	0.077	0.083	0.050	0.104	0.105
		0.95	0.000	0.049	0.031	0.000	0.034	0.022	0.000	0.076	0.049	0.000	0.049	0.021	0.000	0.037	0.024	0.006	0.066	0.045
		0.9	0.000	0.046	0.029	0.000	0.034	0.025	0.000	0.044	0.033	0.000	0.043	0.023	0.001	0.028	0.025	0.000	0.040	0.026
		0.7	0.000	0.025	0.014	0.000	0.022	0.009	0.000	0.020	0.005	0.000	0.024	0.010	0.000	0.020	0.009	0.000	0.022	0.007
		0.5	0.000	0.037	0.019	0.000	0.026	0.010	0.000	0.039	0.016	0.000	0.033	0.017	0.000	0.025	0.010	0.000	0.039	0.015
	300	1	0.053	0.129	0.116	0.051	0.167	0.140	0.047	0.197	0.172	0.032	0.043	0.043	0.035	0.068	0.059	0.048	0.088	0.089
		0.95	0.000	0.013	0.009	0.000	0.025	0.015	0.000	0.039	0.025	0.000	0.013	0.009	0.000	0.023	0.013	0.000	0.039	0.023
		0.9	0.000	0.016	0.009	0.000	0.023	0.017	0.000	0.029	0.014	0.000	0.013	0.007	0.000	0.020	0.013	0.000	0.029	0.012
		0.7	0.000	0.015	0.006	0.000	0.021	0.012	0.000	0.020	0.006	0.000	0.014	0.005	0.000	0.021	0.010	0.000	0.022	0.007
		0.5	0.000	0.021	0.007	0.000	0.033	0.014	0.000	0.028	0.009	0.000	0.018	0.005	0.000	0.033	0.014	0.000	0.022	0.006

Table 7: Case B. Empirical power, one structural break, $\gamma = \gamma^* T^{1/2}$, $w = 0.15$, $\pi = 0.5$, known $\overline{\boldsymbol{c}}$

			$\lambda = 0.25$						$\lambda = 0.50$						$\lambda = 0.75$					
			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$		
γ^*	$(\bar{c}_1, \bar{c}_2)$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	$(0.3, 0.5)$	50	0.898	0.784	0.886	0.907	0.811	0.910	0.898	0.813	0.896	0.918	0.849	0.925	0.897	0.908	0.922	0.919	0.924	0.944
		150	0.962	0.748	0.955	0.981	0.817	0.970	0.952	0.826	0.954	0.982	0.877	0.982	0.929	0.880	0.934	0.991	0.953	0.996
		300	0.964	0.800	0.957	0.994	0.944	0.996	0.945	0.820	0.938	0.999	0.962	0.998	0.938	0.875	0.938	0.996	0.983	0.997
	$(0.5, 1)$	50	0.509	0.611	0.606	0.491	0.628	0.588	0.504	0.645	0.630	0.484	0.705	0.659	0.511	0.730	0.698	0.511	0.808	0.764
		150	0.499	0.532	0.566	0.476	0.694	0.672	0.490	0.597	0.621	0.496	0.748	0.740	0.495	0.674	0.658	0.506	0.818	0.805
		300	0.527	0.599	0.638	0.547	0.916	0.896	0.520	0.600	0.645	0.580	0.912	0.897	0.524	0.657	0.653	0.555	0.922	0.914
5	$(0.3, 0.5)$	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$(0.5, 1)$	50	1.000	0.999	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	0.997	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

			$\lambda = 0.25$						$\lambda = 0.50$						$\lambda = 0.75$					
			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$		
γ^*	$(\bar{c}_1, \bar{c}_2)$	T	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*
1	$(0.3, 0.5)$	50	0.935	0.804	0.923	0.927	0.824	0.934	0.046	0.154	0.128	0.024	0.157	0.119	0.924	0.916	0.942	0.945	0.930	0.963
		150	0.983	0.764	0.974	0.990	0.823	0.982	0.053	0.061	0.065	0.020	0.046	0.040	0.959	0.891	0.957	0.993	0.956	0.996
		300	0.981	0.810	0.972	0.998	0.946	0.997	0.047	0.057	0.066	0.001	0.022	0.016	0.963	0.886	0.958	1.000	0.984	0.998
	$(0.5, 1)$	50	0.779	0.635	0.783	0.804	0.659	0.804	0.043	0.146	0.122	0.014	0.146	0.103	0.714	0.741	0.764	0.798	0.815	0.854
		150	0.832	0.565	0.808	0.907	0.701	0.897	0.053	0.061	0.071	0.001	0.034	0.025	0.748	0.688	0.780	0.940	0.826	0.946
		300	0.817	0.622	0.821	0.970	0.918	0.975	0.034	0.067	0.059	0.000	0.019	0.009	0.728	0.664	0.743	0.956	0.923	0.969
5	$(0.3, 0.5)$	50	1.000	1.000	1.000	1.000	1.000	1.000	0.046	0.154	0.128	0.024	0.157	0.119	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	0.053	0.061	0.065	0.020	0.046	0.040	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	0.047	0.057	0.066	0.001	0.022	0.016	1.000	1.000	1.000	1.000	1.000	1.000
	$(0.5, 1)$	50	1.000	0.999	1.000	1.000	0.999	1.000	0.038	0.138	0.114	0.014	0.146	0.103	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	0.997	1.000	1.000	1.000	1.000	0.051	0.063	0.072	0.001	0.034	0.025	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	0.032	0.066	0.056	0.000	0.019	0.009	1.000	1.000	1.000	1.000	1.000	1.000

Table 8: Case B. Empirical power, one structural break, $\gamma = \gamma^* T^{-1/2}$, $w = 0.15$, $\pi = 0.5$, known $\overline{\boldsymbol{c}}$

			$\lambda = 0.25$						$\lambda = 0.50$						$\lambda = 0.75$					
			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$		
γ^*	$(\bar{c}_1, \bar{c}_2)$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	(0.3, 0.5)	50	0.051	0.183	0.129	0.038	0.145	0.113	0.043	0.189	0.149	0.049	0.170	0.130	0.067	0.223	0.183	0.051	0.169	0.124
		150	0.046	0.069	0.076	0.023	0.050	0.042	0.053	0.094	0.091	0.017	0.055	0.048	0.053	0.113	0.109	0.009	0.058	0.043
		300	0.055	0.087	0.078	0.002	0.028	0.019	0.051	0.084	0.088	0.001	0.029	0.024	0.037	0.109	0.087	0.001	0.027	0.023
	(0.5, 1)	50	0.015	0.181	0.118	0.002	0.141	0.090	0.046	0.236	0.175	0.013	0.160	0.107	0.073	0.299	0.237	0.020	0.177	0.128
		150	0.014	0.078	0.063	0.000	0.033	0.024	0.043	0.146	0.133	0.000	0.034	0.022	0.068	0.223	0.190	0.000	0.036	0.021
		300	0.014	0.100	0.079	0.000	0.026	0.014	0.050	0.167	0.140	0.000	0.026	0.014	0.080	0.234	0.212	0.000	0.026	0.014
5	(0.3, 0.5)	50	0.058	0.193	0.149	0.054	0.168	0.136	0.063	0.212	0.171	0.056	0.180	0.138	0.072	0.246	0.203	0.049	0.185	0.149
		150	0.048	0.070	0.074	0.023	0.050	0.044	0.052	0.097	0.099	0.016	0.057	0.047	0.055	0.111	0.115	0.013	0.060	0.044
		300	0.054	0.090	0.080	0.001	0.028	0.017	0.054	0.089	0.091	0.001	0.028	0.023	0.036	0.113	0.091	0.001	0.030	0.024
	(0.5, 1)	50	0.022	0.169	0.122	0.004	0.144	0.093	0.054	0.224	0.172	0.015	0.163	0.114	0.084	0.299	0.238	0.021	0.178	0.126
		150	0.015	0.079	0.056	0.000	0.034	0.024	0.042	0.149	0.128	0.000	0.039	0.024	0.073	0.223	0.189	0.000	0.033	0.020
		300	0.012	0.104	0.080	0.000	0.026	0.011	0.048	0.169	0.139	0.000	0.024	0.014	0.078	0.232	0.208	0.000	0.027	0.014

			$\lambda = 0.25$						$\lambda = 0.50$						$\lambda = 0.75$					
			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$		
γ^*	$(\bar{c}_1, \bar{c}_2)$	T	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*
1	(0.3, 0.5)	50	0.053	0.178	0.128	0.057	0.145	0.120	0.052	0.176	0.136	0.045	0.152	0.119	0.082	0.226	0.191	0.063	0.186	0.134
		150	0.052	0.057	0.074	0.028	0.046	0.046	0.057	0.068	0.074	0.025	0.050	0.049	0.072	0.120	0.127	0.011	0.055	0.041
		300	0.067	0.079	0.081	0.002	0.023	0.017	0.047	0.061	0.071	0.001	0.024	0.021	0.051	0.110	0.101	0.002	0.025	0.020
	(0.5, 1)	50	0.107	0.192	0.176	0.031	0.151	0.111	0.050	0.145	0.119	0.023	0.142	0.115	0.220	0.315	0.302	0.054	0.192	0.160
		150	0.117	0.090	0.130	0.000	0.033	0.024	0.054	0.077	0.084	0.000	0.038	0.024	0.219	0.245	0.260	0.000	0.040	0.023
		300	0.104	0.115	0.128	0.000	0.023	0.013	0.035	0.068	0.060	0.000	0.023	0.013	0.236	0.236	0.275	0.000	0.023	0.013
5	(0.3, 0.5)	50	0.069	0.188	0.152	0.074	0.163	0.139	0.052	0.174	0.133	0.038	0.153	0.113	0.094	0.249	0.218	0.066	0.203	0.164
		150	0.057	0.056	0.072	0.029	0.047	0.046	0.057	0.069	0.075	0.026	0.052	0.049	0.071	0.123	0.128	0.014	0.060	0.042
		300	0.067	0.082	0.083	0.002	0.023	0.016	0.047	0.061	0.071	0.001	0.024	0.021	0.049	0.114	0.101	0.002	0.028	0.022
	(0.5, 1)	50	0.111	0.181	0.174	0.040	0.156	0.119	0.051	0.136	0.107	0.022	0.151	0.111	0.217	0.320	0.308	0.060	0.189	0.155
		150	0.122	0.093	0.122	0.000	0.035	0.023	0.052	0.076	0.084	0.000	0.037	0.023	0.214	0.242	0.259	0.000	0.038	0.021
		300	0.106	0.119	0.125	0.000	0.023	0.012	0.035	0.068	0.061	0.000	0.022	0.013	0.236	0.233	0.264	0.000	0.025	0.012

Table 9: Case B. Empirical power, one structural break, $\gamma = \gamma^*$, $w = 0.15$, $\pi = 0.5$, known $\bar{\boldsymbol{c}}$

			$\lambda = 0.25$						$\lambda = 0.50$						$\lambda = 0.75$					
			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$		
γ^*	$(\bar{c}_1, \bar{c}_2)$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	$(0.3, 0.5)$	50	0.069	0.203	0.164	0.062	0.185	0.143	0.079	0.229	0.187	0.063	0.202	0.148	0.084	0.268	0.211	0.056	0.210	0.161
		150	0.057	0.077	0.081	0.025	0.050	0.044	0.059	0.104	0.108	0.017	0.052	0.043	0.064	0.127	0.121	0.018	0.071	0.048
		300	0.057	0.096	0.091	0.001	0.029	0.017	0.057	0.097	0.102	0.001	0.026	0.022	0.041	0.112	0.101	0.001	0.033	0.027
	$(0.5, 1)$	50	0.029	0.177	0.126	0.004	0.135	0.090	0.062	0.232	0.179	0.019	0.169	0.125	0.087	0.307	0.243	0.020	0.191	0.140
		150	0.019	0.074	0.059	0.000	0.034	0.024	0.041	0.155	0.135	0.000	0.039	0.026	0.067	0.217	0.193	0.000	0.034	0.020
		300	0.011	0.105	0.084	0.000	0.026	0.012	0.044	0.164	0.135	0.000	0.023	0.014	0.079	0.231	0.206	0.000	0.024	0.015
5	$(0.3, 0.5)$	50	0.593	0.528	0.608	0.616	0.576	0.642	0.599	0.607	0.643	0.621	0.639	0.682	0.610	0.724	0.711	0.602	0.729	0.716
		150	0.287	0.226	0.295	0.181	0.163	0.214	0.327	0.290	0.352	0.201	0.261	0.253	0.343	0.362	0.401	0.206	0.338	0.327
		300	0.159	0.169	0.196	0.024	0.121	0.088	0.187	0.200	0.235	0.032	0.134	0.111	0.192	0.248	0.259	0.026	0.174	0.138
	$(0.5, 1)$	50	0.212	0.396	0.347	0.154	0.388	0.328	0.269	0.480	0.435	0.177	0.481	0.409	0.294	0.572	0.514	0.176	0.589	0.508
		150	0.060	0.148	0.130	0.000	0.097	0.079	0.104	0.218	0.203	0.002	0.155	0.123	0.124	0.321	0.293	0.002	0.191	0.153
		300	0.039	0.144	0.116	0.000	0.107	0.072	0.074	0.180	0.165	0.000	0.106	0.075	0.117	0.264	0.236	0.000	0.112	0.081

			$\lambda = 0.25$						$\lambda = 0.50$						$\lambda = 0.75$					
			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 1$			$\alpha = 0.95$		
γ^*	$(\bar{c}_1, \bar{c}_2)$	T	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*	S_1^*	S_0^*	S_U^*
1	$(0.3, 0.5)$	50	0.082	0.201	0.169	0.080	0.179	0.152	0.058	0.177	0.136	0.040	0.159	0.116	0.116	0.276	0.231	0.079	0.225	0.176
		150	0.074	0.068	0.082	0.030	0.048	0.045	0.057	0.080	0.076	0.026	0.047	0.042	0.091	0.138	0.138	0.019	0.072	0.053
		300	0.070	0.091	0.093	0.003	0.029	0.016	0.047	0.058	0.068	0.001	0.023	0.019	0.052	0.115	0.108	0.001	0.032	0.026
	$(0.5, 1)$	50	0.114	0.193	0.173	0.043	0.155	0.116	0.051	0.136	0.109	0.019	0.155	0.114	0.217	0.326	0.305	0.065	0.200	0.173
		150	0.114	0.096	0.131	0.000	0.034	0.024	0.052	0.080	0.085	0.001	0.036	0.024	0.206	0.235	0.251	0.002	0.036	0.023
		300	0.101	0.117	0.122	0.000	0.025	0.011	0.034	0.068	0.060	0.000	0.020	0.010	0.230	0.232	0.264	0.000	0.022	0.014
5	$(0.3, 0.5)$	50	0.670	0.550	0.683	0.684	0.593	0.698	0.048	0.157	0.127	0.024	0.155	0.118	0.659	0.741	0.738	0.642	0.744	0.739
		150	0.344	0.235	0.340	0.241	0.180	0.252	0.057	0.063	0.069	0.020	0.045	0.038	0.378	0.376	0.424	0.241	0.353	0.352
		300	0.191	0.179	0.221	0.034	0.138	0.106	0.049	0.059	0.067	0.001	0.022	0.016	0.225	0.259	0.276	0.038	0.187	0.143
	$(0.5, 1)$	50	0.496	0.426	0.499	0.464	0.420	0.474	0.042	0.151	0.121	0.015	0.141	0.099	0.485	0.580	0.578	0.468	0.603	0.590
		150	0.228	0.176	0.227	0.065	0.107	0.108	0.051	0.062	0.072	0.001	0.035	0.025	0.273	0.336	0.353	0.077	0.205	0.175
		300	0.159	0.168	0.186	0.004	0.122	0.084	0.032	0.063	0.055	0.000	0.019	0.009	0.247	0.281	0.302	0.002	0.123	0.086

Table 10: Estimated structural break dates with the S_U^* statistic

$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 5$	$\delta = 10$	$\delta = 20$	$\delta = 50$	$\delta = 100$
June 2010	June 2010	June 2010	June 2010	-	-	-	-
April 2003	April 2003	April 2003	April 2003	-	-	-	-
July 2006	July 2006	July 2006	July 2006	-	-	-	-
July 2017	July 2017	July 2017	July 2017	-	-	-	-
May 2014	May 2014	May 2014	-	-	-	-	-

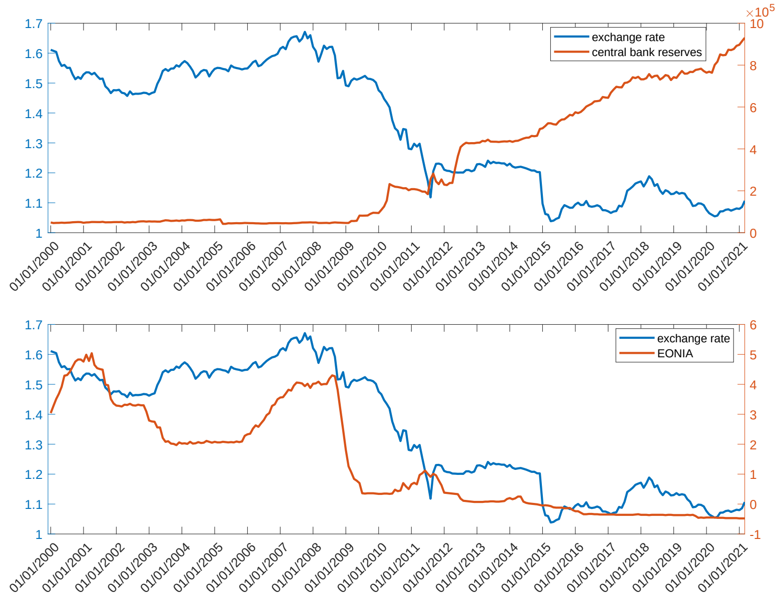

Figure 1: Interventions on the recent evolution of Swiss franc vs. Euro exchange rate

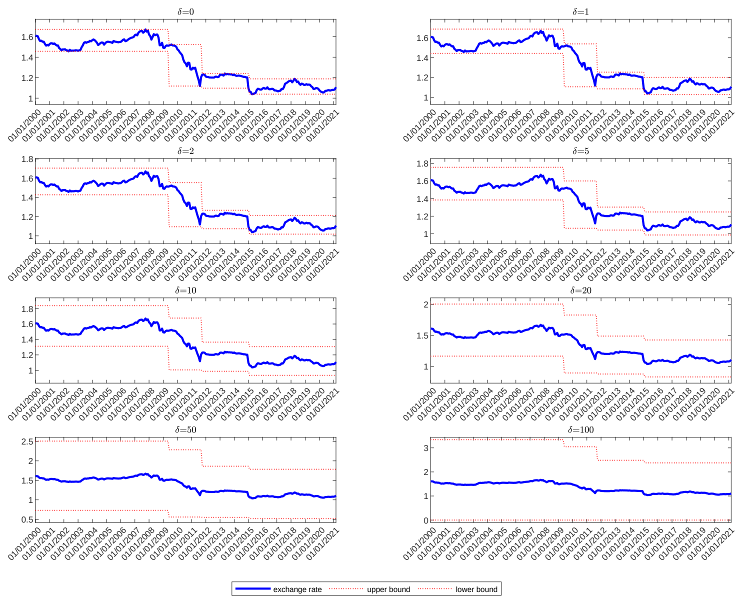

Figure 2: Different boundaries evolution of CHF vs. EUR exchange rate

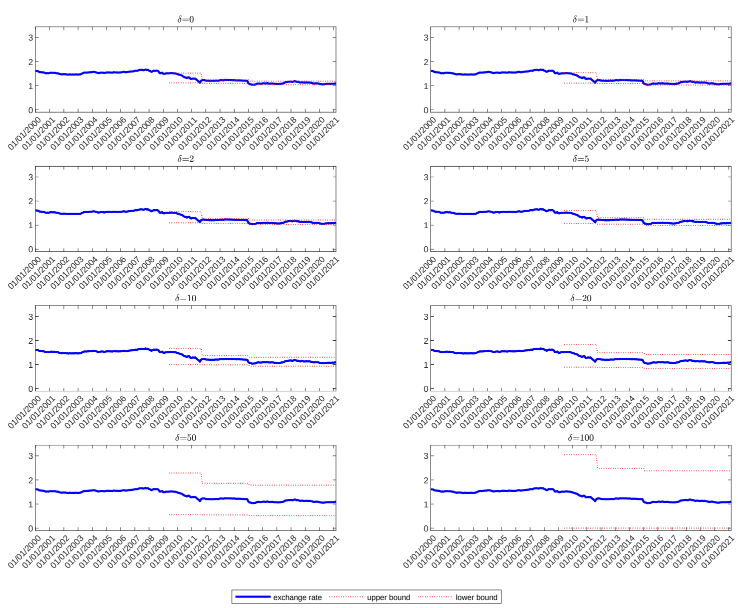

Figure 3: Different boundaries evolution of CHF vs. EUR exchange rate

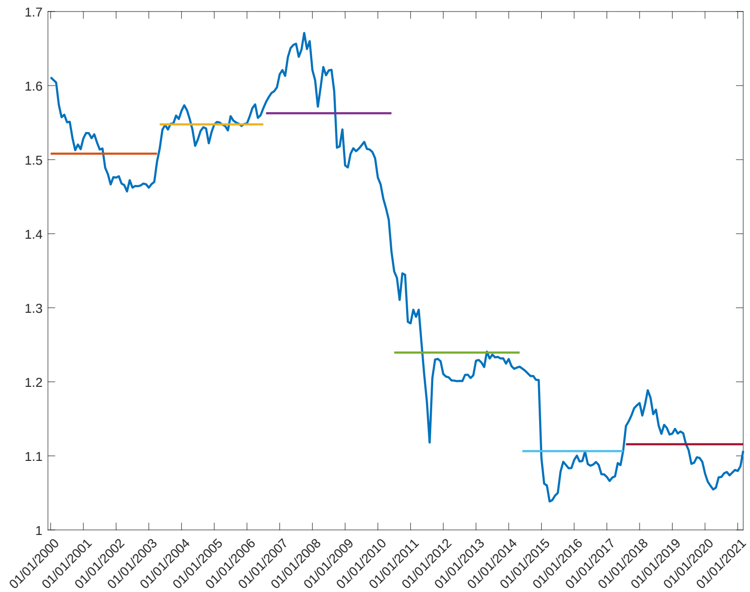

Figure 4: Structural changes in the mean of CHF vs. EUR exchange rate

B Supplementary material

This section provides the estimated response surfaces to approximate finite sample critical values for the S_1 and S_0 test statistics, along with the κ_ξ constant that is required to compute the S_U test statistic. The estimation of these response surfaces bases on simulations conducted for $T \in \{50, 100, 200, 300, 500, 1000\}$ and following the setup described in the paper.

We also include tables with additional simulation results for the known $\bar{c}$ case. Tables report empirical size and power using the DGP described in the paper.

Table B.1: Response surfaces to approximate the finite sample critical values for the S_1 statistic with symmetric bounds

	$w = 0.10$				$w = 0.15$				$w = 0.20$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%	10%	5%	2.5%	1%
constant	-0.0272	-0.0382	-0.0484	-0.0613	-0.0780	-0.0889	-0.0996	-0.1096	-0.1155	-0.1272	-0.1330	-0.1360
T^{-1}	32.4089	36.6912	41.2042	48.7145	24.9894	28.5286	32.9623	38.5357	23.3007	27.2884	30.4611	33.4288
T^{-2}	-741.0389	-866.5405	-1001.9015	-1254.6402	-425.4140	-504.4253	-637.1026	-761.4661	-383.3252	-490.9470	-570.6621	-581.2185
c	1.9204	2.0707	2.1987	2.3457	2.2031	2.3457	2.4669	2.5834	2.3728	2.5096	2.5906	2.6445
$\bar{c}^2$	-2.1234	-2.2604	-2.3614	-2.4547	-2.3352	-2.4284	-2.4968	-2.5293	-2.4167	-2.4780	-2.4632	-2.3683
$\bar{c}^3$	0.7315	0.7726	0.7987	0.8138	0.7867	0.8046	0.8137	0.8057	0.7964	0.7984	0.7713	0.7041
$\bar{c}^4$	-0.0007	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0007
$\bar{c} * T^{-1}$	-74.9451	-86.2788	-96.3263	-116.8884	-40.0891	-44.8251	-52.7967	-59.2761	-35.3101	-41.0690	-38.4749	-25.4911
$\bar{c} * T^{-2}$	3361.5814	4184.5057	4925.6884	6382.3568	1428.8268	1834.8853	2585.3694	3059.0907	1213.1917	1743.8886	1957.5294	1680.9062
$\bar{c}^2 * T^{-1}$	71.5370	81.2767	85.5469	94.8280	37.0525	35.6861	36.3486	29.0293	28.0775	28.2073	11.2318	-26.0660
$\bar{c}^2 * T^{-2}$	-3566.4012	-4551.4692	-5242.1248	-6553.1509	-1735.2868	-2078.5800	-2829.5447	-2901.1846	-1368.3257	-1944.9633	-1801.0323	-875.5949
$\bar{c}^3 * T^{-1}$	-22.6420	-25.6328	-25.8253	-25.2551	-12.0480	-10.2112	-8.4445	-2.8602	-8.1339	-7.1069	2.0782	20.3434
$\bar{c}^3 * T^{-2}$	1199.4488	1564.5051	1778.7938	2117.8758	650.7444	746.1038	967.9464	881.0565	487.1894	694.8194	536.0106	53.5662
$\bar{c}^4 * T^{-1}$	0.0226	0.0256	0.0257	0.0252	0.0120	0.0102	0.0084	0.0028	0.0081	0.0071	-0.0021	-0.0203
$\bar{c}^4 * T^{-2}$	-1.1959	-1.5600	-1.7736	-2.1113	-0.6490	-0.7440	-0.9651	-0.8782	-0.4858	-0.6929	-0.5342	-0.0527
R^2	0.9841	0.9862	0.9880	0.9874	0.9918	0.9934	0.9945	0.9937	0.9954	0.9965	0.9969	0.9960
$\bar{R}^2$	0.9806	0.9832	0.9853	0.9846	0.9900	0.9919	0.9933	0.9923	0.9944	0.9957	0.9962	0.9951

	$w = 0.25$				$w = 0.30$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%
constant	-0.1372	-0.1445	-0.1485	-0.1493	-0.1562	-0.1647	-0.1640	-0.1634
T^{-1}	19.3205	20.9835	23.7976	25.6175	19.8808	22.0289	22.6587	25.5017
T^{-2}	-236.9602	-245.3901	-299.7381	-269.0444	-396.3552	-458.3672	-421.7474	-461.7986
c	2.4422	2.5472	2.6102	2.6512	2.4774	2.5919	2.6207	2.6539
$\bar{c}^2$	-2.3951	-2.3910	-2.3384	-2.2243	-2.3342	-2.3372	-2.2115	-2.0875
$\bar{c}^3$	0.7722	0.7462	0.7030	0.6300	0.7365	0.7125	0.6349	0.5611
$\bar{c}^4$	-0.0008	-0.0007	-0.0007	-0.0006	-0.0007	-0.0007	-0.0006	-0.0006
$\bar{c} * T^{-1}$	-20.6071	-10.7639	-8.1930	8.6823	-22.1302	-17.0795	-1.2069	7.5776
$\bar{c} * T^{-2}$	598.7341	447.9544	702.9974	347.1309	1099.5463	1365.6730	1045.5857	1247.0678
$\bar{c}^2 * T^{-1}$	15.8265	-8.0288	-21.0506	-58.5172	12.7372	-3.1765	-38.5112	-62.3610
$\bar{c}^2 * T^{-2}$	-951.7825	-454.3881	-633.5831	142.9941	-1132.3950	-1276.3968	-484.9603	-554.5455
$\bar{c}^3 * T^{-1}$	-4.5428	6.4313	13.1342	30.3679	-3.0092	4.7811	21.3655	32.4825
$\bar{c}^3 * T^{-2}$	384.7144	133.5712	161.9679	-206.1955	397.5533	395.9821	-3.8064	-11.2354
$\bar{c}^4 * T^{-1}$	0.0045	-0.0064	-0.0131	-0.0303	0.0030	-0.0048	-0.0213	-0.0324
$\bar{c}^4 * T^{-2}$	-0.3838	-0.1331	-0.1613	0.2061	-0.3964	-0.3947	0.0043	0.0118
R^2	0.9971	0.9977	0.9978	0.9975	0.9978	0.9980	0.9978	0.9973
$\bar{R}^2$	0.9965	0.9972	0.9973	0.9969	0.9973	0.9976	0.9974	0.9967

Table B.2: Response surfaces to approximate the finite sample critical values for the S_0 statistic with symmetric bounds

	$w=0.10$				$w=0.15$				$w=0.20$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%	10%	5%	2.5%	1%
constant	22.1183	23.5280	24.7063	26.4647	17.3921	18.6688	19.6912	21.0319	14.6851	15.7431	16.7090	17.7656
T^{-1}	-615.4416	-690.0226	-706.6354	-914.6751	-360.2750	-471.5112	-473.6891	-540.6454	-276.3849	-287.4300	-313.2832	-363.9001
T^{-2}	70175.0512	81519.6740	86869.8692	126654.1197	25477.1759	39925.6281	41166.2919	54415.0341	23702.3555	23371.6754	24350.2673	35582.3512
T^{-3}	-1654736.6369	-1663161.0846	-1472815.8921	-2574383.3575	-391192.4189	-759242.8191	-703011.4861	-1003192.2646	-505469.5510	-401563.3884	-308034.6511	-601759.6037
$\bar{c}^{-1}$	0.0113	0.0084	0.0040	0.0067	0.0028	0.0060	0.0093	0.0124	-0.0003	0.0020	-0.0018	0.0088
$\bar{c}^{-2}$	-0.0057	-0.0047	-0.0021	-0.0047	-0.0015	-0.0027	-0.0043	-0.0059	0.0001	-0.0012	0.0008	-0.0045
$\bar{c}^{-3}$	0.0005	0.0004	0.0002	0.0006	0.0001	0.0002	0.0003	0.0005	-0.0000	0.0001	-0.0000	0.0004
$\bar{c}^{-1}*T^{-1}$	-20.3990	-16.2260	-10.1497	-14.2320	-6.0144	-10.0985	-16.6283	-21.5677	-0.5348	-3.7036	1.1685	-14.1231
$\bar{c}^{-1}*T^{-2}$	5514.7314	4792.3887	3476.5078	4006.7306	1718.5963	2206.0382	3854.4262	5413.3956	282.3400	1205.9696	695.7891	4088.7614
$\bar{c}^{-1}*T^{-3}$	-249015.5667	-232953.6433	-188931.1323	-186987.7938	-61814.4556	-65314.3783	-131234.7064	-199020.7855	-3176.2010	-37712.9676	-18564.6233	-159208.9628
$\bar{c}^{-2}*T^{-1}$	10.2085	8.8699	5.2202	9.1647	3.1847	4.6194	7.8143	10.2386	0.3693	2.0686	-0.5576	7.0573
$\bar{c}^{-2}*T^{-2}$	-2688.4894	-2480.5564	-1657.2554	-2372.3265	-853.3105	-1000.5324	-1797.8275	-2510.7533	-147.3491	-622.2003	-297.2645	-1994.8591
$\bar{c}^{-2}*T^{-3}$	119416.1058	116563.2253	86926.8660	106216.7086	28729.2368	28537.7855	60594.3002	91148.7926	1132.3909	19456.8569	7440.1688	78327.7008
$\bar{c}^{-3}*T^{-1}$	-0.8306	-0.7871	-0.4811	-1.0082	-0.2813	-0.3040	-0.5854	-0.8444	-0.0212	-0.1894	0.0163	-0.5881
$\bar{c}^{-3}*T^{-2}$	202.9138	195.4675	125.4327	220.3967	65.8993	62.0296	130.0948	191.8439	7.2588	49.3331	25.5518	155.8430
$\bar{c}^{-3}*T^{-3}$	-8833.4448	-8878.3445	-6317.1753	-9233.6582	-2144.1547	-1585.5522	-4261.0970	-6806.3877	94.1813	-1510.4469	-652.5883	-6018.9030
R^2	0.9986	0.9990	0.9997	0.9997	0.9657	0.9829	0.9937	0.9916	0.9939	0.9938	0.9941	0.9974
$\bar{R}^2$	0.9983	0.9988	0.9996	0.9996	0.9575	0.9788	0.9921	0.9896	0.9924	0.9924	0.9927	0.9967

	$w=0.25$				$w=0.30$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%
constant	12.8107	13.7580	14.6620	15.7372	11.5211	12.4431	13.2386	14.3031
T^{-1}	-181.6447	-190.8138	-226.3521	-277.7108	-185.5749	-215.1431	-202.0122	-332.4581
T^{-2}	7834.3435	9093.4746	12105.5994	17822.0847	16203.6691	19306.3862	16333.5326	34835.5547
T^{-3}	59069.2979	47082.9403	35386.5417	-3607.8094	-438857.8134	-495795.9351	-341389.4684	-871066.9079
$\bar{c}^{-1}$	-0.0021	0.0001	-0.0044	-0.0048	-0.0024	-0.0032	-0.0011	0.0039
$\bar{c}^{-2}$	0.0011	0.0001	0.0019	0.0023	0.0011	0.0015	0.0006	-0.0017
$\bar{c}^{-3}$	-0.0001	-0.0000	-0.0002	-0.0002	-0.0001	-0.0001	-0.0000	0.0001
$\bar{c}^{-1}*T^{-1}$	3.3029	-1.3970	4.5540	5.2747	3.2582	4.3367	0.2087	-7.7257
$\bar{c}^{-1}*T^{-2}$	-679.7848	892.1475	-305.5635	-163.1674	-406.2941	-573.3426	490.2260	2305.5217
$\bar{c}^{-1}*T^{-3}$	32405.0076	-33680.2239	21613.7262	13926.7784	17742.1313	24053.2819	-18781.5712	-108774.5673
$\bar{c}^{-2}*T^{-1}$	-1.7550	0.4869	-1.8789	-2.4693	-1.5561	-2.0275	-0.2580	3.4695
$\bar{c}^{-2}*T^{-2}$	374.4021	-356.6461	85.3406	87.0534	199.1167	265.8003	-155.1851	-1017.3553
$\bar{c}^{-2}*T^{-3}$	-18276.5763	11977.1951	-8630.8062	-7005.8087	-8990.1854	-11802.8873	4669.6647	45752.7950
$\bar{c}^{-3}*T^{-1}$	0.1607	-0.0277	0.1840	0.2392	0.1330	0.1861	-0.0089	-0.2405
$\bar{c}^{-3}*T^{-2}$	-34.1456	23.3767	-14.5527	-18.4790	-17.1061	-24.7536	12.9979	69.2457
$\bar{c}^{-3}*T^{-3}$	1599.7859	-730.0903	986.0084	1002.0653	743.8567	1041.2153	-339.9041	-3082.3539
R^2	0.9811	0.9880	0.9948	0.9963	0.9789	0.9928	0.9784	0.9769
$\bar{R}^2$	0.9766	0.9852	0.9936	0.9954	0.9738	0.9910	0.9733	0.9714

Table B.3: Response surfaces to approximate the finite sample values for the κ_ξ coefficient with symmetric bounds

	$w = 0.10$				$w = 0.15$				$w = 0.20$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%	10%	5%	2.5%	1%
constant	1.0587	1.0510	1.0527	1.0476	1.0611	1.0509	1.0523	1.0507	1.0628	1.0566	1.0441	1.0477
T^{-1}	-0.4274	0.0462	-1.8108	-0.8536	-1.7112	1.3830	-0.8048	-2.6441	0.3502	0.4057	2.0960	-0.7231
T^{-2}	163.1169	73.6730	237.3213	245.2911	305.6696	-35.0351	163.4576	346.1212	-8.1313	-153.2986	-88.2278	39.0247
T^{-3}	1433.5565	3182.2463	-2428.2226	-5450.8414	-6580.8633	2920.4301	-3058.9360	-8971.7706	3006.7370	10343.2633	3332.0077	2747.7975
$\bar{c}^{-1}$	0.0029	-0.0089	-0.0039	-0.0215	-0.0095	-0.0124	-0.0212	0.0151	-0.0185	-0.0153	-0.0047	-0.0007
$\bar{c}^{-2}$	-0.0076	0.0179	0.0047	0.0401	0.0230	0.0306	0.0397	-0.0156	0.0476	0.0359	0.0238	0.0069
$\bar{c}^{-3}$	0.0075	-0.0094	-0.0019	-0.0277	-0.0143	-0.0214	-0.0276	0.0033	-0.0344	-0.0272	-0.0216	-0.0101
$\bar{c}^{-4}$	-0.0031	0.0015	-0.0000	0.0081	0.0035	0.0060	0.0080	-0.0000	0.0101	0.0083	0.0071	0.0039
$\bar{c}^{-5}$	0.0005	-0.0000	0.0001	-0.0011	-0.0004	-0.0007	-0.0010	-0.0000	-0.0013	-0.0011	-0.0010	-0.0006
$\bar{c}^{-6}$	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
$T^{-1} * \bar{c}^{-1}$	0.0519	1.5487	0.2922	3.5371	1.2477	-1.4519	1.0830	-4.5537	-0.4274	0.9705	-1.8319	-0.8491
$T^{-1} * \bar{c}^{-2}$	0.6926	-2.4316	0.1860	-3.7355	-1.4662	1.4598	-0.8513	4.8618	-0.2656	-0.9256	1.4445	1.8237
$T^{-1} * \bar{c}^{-3}$	-0.4004	1.4005	-0.1248	1.4539	0.6591	-0.4799	0.5770	-1.4908	0.4713	0.5441	-0.2515	-0.6638
$T^{-1} * \bar{c}^{-4}$	0.0804	-0.2623	0.0404	-0.2043	-0.1028	0.0739	-0.1187	0.1922	-0.1110	-0.1007	0.0072	0.1038
$T^{-1} * \bar{c}^{-5}$	-0.0047	0.0145	-0.0030	0.0093	0.0051	-0.0039	0.0069	-0.0087	0.0067	0.0055	0.0005	-0.0054
$T^{-2} * \bar{c}^{-1}$	-4.5316	-56.4399	8.7181	-240.5697	-13.8664	52.3586	28.3546	233.7589	47.7554	-72.4813	60.1471	67.1695
$T^{-2} * \bar{c}^{-2}$	-41.4003	88.6664	-28.3510	268.8853	-3.1843	-56.1748	-46.1022	-235.1384	-28.1607	84.0789	-52.9356	-125.0184
$T^{-2} * \bar{c}^{-3}$	27.0199	-53.0476	19.0321	-102.6584	5.9494	24.1353	10.4687	77.3198	0.8929	-38.8670	14.4454	57.9338
$T^{-2} * \bar{c}^{-4}$	-5.3945	10.2572	-4.6396	14.6229	-1.7812	-4.4022	-0.5732	-10.6558	0.8884	6.4271	-1.5957	-10.2508
$T^{-2} * \bar{c}^{-5}$	0.3092	-0.5807	0.2992	-0.6813	0.1220	0.2485	-0.0057	0.5020	-0.0757	-0.3328	0.0620	0.5625
R^2	0.9961	0.9827	0.9749	0.9210	0.9755	0.9804	0.9647	0.8795	0.9643	0.9700	0.9573	0.9434
$\bar{R}^2$	0.9949	0.9773	0.9669	0.8960	0.9677	0.9741	0.9534	0.8412	0.9529	0.9605	0.9437	0.9255

	$w = 0.25$				$w = 0.30$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%
constant	1.0620	1.0563	1.0510	1.0433	1.0672	1.0567	1.0511	1.0489
T^{-1}	1.4213	1.2973	-0.2531	0.0046	-0.0586	3.7175	0.0506	-1.0475
T^{-2}	-28.0854	-127.0049	69.1418	111.0926	35.7747	-415.4720	206.9892	269.9337
T^{-3}	808.8494	6215.8208	1403.4631	-3095.7012	1148.9176	14450.7426	-7070.8301	-9253.4064
$\bar{c}^{-1}$	-0.0155	-0.0206	0.0029	-0.0229	-0.0257	-0.0086	0.0073	0.0058
$\bar{c}^{-2}$	0.0461	0.0511	0.0138	0.0513	0.0622	0.0388	0.0046	0.0124
$\bar{c}^{-3}$	-0.0358	-0.0389	-0.0187	-0.0402	-0.0451	-0.0355	-0.0112	-0.0224
$\bar{c}^{-4}$	0.0111	0.0121	0.0072	0.0128	0.0136	0.0121	0.0047	0.0095
$\bar{c}^{-5}$	-0.0015	-0.0016	-0.0011	-0.0017	-0.0018	-0.0017	-0.0007	-0.0015
$\bar{c}^{-6}$	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001
$T^{-1} * \bar{c}^{-1}$	-1.6808	0.8417	-3.5619	-4.1274	0.1428	-3.4316	-3.6735	-9.5773
$T^{-1} * \bar{c}^{-2}$	1.4147	-2.0622	4.6755	4.0771	-0.4531	2.6056	2.9810	12.0882
$T^{-1} * \bar{c}^{-3}$	-0.1992	1.1891	-1.8788	-1.0152	0.4106	-0.5724	-0.8106	-4.6712
$T^{-1} * \bar{c}^{-4}$	-0.0125	-0.2168	0.2993	0.0885	-0.0876	0.0421	0.0934	0.6988
$T^{-1} * \bar{c}^{-5}$	0.0020	0.0118	-0.0154	-0.0023	0.0051	-0.0007	-0.0038	-0.0343
$T^{-2} * \bar{c}^{-1}$	96.6397	-78.3401	135.2895	258.5915	-25.8508	135.8483	146.7149	477.1892
$T^{-2} * \bar{c}^{-2}$	-94.8552	151.4570	-195.3165	-251.2513	46.0066	-86.6500	-96.7326	-587.5070
$T^{-2} * \bar{c}^{-3}$	27.0779	-73.5687	87.3082	75.6819	-23.1578	16.9439	20.5466	234.5597
$T^{-2} * \bar{c}^{-4}$	-2.9046	12.4501	-14.7836	-9.0062	3.9332	-0.8583	-1.4191	-35.9697
$T^{-2} * \bar{c}^{-5}$	0.1045	-0.6535	0.7865	0.3674	-0.2066	-0.0100	0.0188	1.7902
R^2	0.9569	0.9713	0.9706	0.9541	0.9397	0.9503	0.9430	0.9369
$\bar{R}^2$	0.9432	0.9622	0.9613	0.9396	0.9205	0.9346	0.9249	0.9169

Table B.4: Case A. Empirical size, assuming known $\bar{c}$

$\bar{c}$	T	α	$w = 0.10$ S_1	S_0	S_U	$w = 0.15$ S_1	S_0	S_U	$w = 0.20$ S_1	S_0	S_U	$w = 0.25$ S_1	S_0	S_U	$w = 0.30$ S_1	S_0	S_U
0.3	50	1	0.064	0.194	0.163	0.062	0.183	0.141	0.060	0.169	0.139	0.063	0.126	0.108	0.051	0.147	0.119
		0.95	0.073	0.196	0.163	0.067	0.164	0.139	0.044	0.158	0.118	0.047	0.139	0.111	0.044	0.138	0.110
		0.9	0.064	0.200	0.180	0.063	0.181	0.140	0.050	0.154	0.131	0.048	0.148	0.113	0.042	0.152	0.127
		0.7	0.039	0.177	0.143	0.027	0.138	0.097	0.022	0.125	0.091	0.025	0.103	0.082	0.022	0.105	0.079
		0.5	0.022	0.154	0.116	0.014	0.094	0.058	0.012	0.071	0.050	0.009	0.063	0.046	0.007	0.066	0.042
	150	1	0.091	0.071	0.080	0.060	0.085	0.084	0.050	0.084	0.089	0.051	0.084	0.078	0.042	0.094	0.092
		0.95	0.053	0.073	0.072	0.046	0.076	0.069	0.040	0.075	0.073	0.041	0.091	0.075	0.035	0.095	0.089
		0.9	0.042	0.051	0.051	0.030	0.049	0.047	0.021	0.050	0.044	0.017	0.049	0.046	0.020	0.054	0.047
		0.7	0.001	0.026	0.016	0.000	0.026	0.012	0.000	0.024	0.012	0.000	0.031	0.019	0.000	0.018	0.006
		0.5	0.000	0.029	0.018	0.000	0.035	0.020	0.000	0.027	0.013	0.000	0.037	0.019	0.000	0.034	0.015
	300	1	0.091	0.031	0.067	0.072	0.056	0.072	0.063	0.055	0.064	0.051	0.070	0.068	0.043	0.073	0.068
		0.95	0.042	0.034	0.031	0.031	0.035	0.040	0.027	0.052	0.038	0.021	0.067	0.055	0.014	0.074	0.051
		0.9	0.002	0.024	0.011	0.001	0.024	0.019	0.000	0.027	0.023	0.000	0.033	0.022	0.000	0.033	0.020
		0.7	0.000	0.020	0.012	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.022	0.009
		0.5	0.000	0.025	0.012	0.000	0.033	0.018	0.000	0.027	0.013	0.000	0.023	0.010	0.000	0.028	0.011
0.5	50	1	0.036	0.177	0.140	0.048	0.187	0.150	0.046	0.206	0.161	0.050	0.201	0.164	0.057	0.235	0.197
		0.95	0.036	0.172	0.129	0.027	0.166	0.132	0.033	0.156	0.127	0.032	0.153	0.120	0.032	0.190	0.154
		0.9	0.026	0.183	0.146	0.020	0.164	0.121	0.029	0.166	0.121	0.025	0.149	0.125	0.025	0.195	0.155
		0.7	0.019	0.187	0.146	0.012	0.118	0.080	0.009	0.100	0.070	0.007	0.089	0.072	0.008	0.084	0.060
		0.5	0.007	0.154	0.115	0.006	0.086	0.053	0.004	0.079	0.049	0.002	0.065	0.051	0.000	0.067	0.046
	150	1	0.071	0.064	0.072	0.054	0.091	0.088	0.065	0.102	0.105	0.060	0.134	0.118	0.069	0.141	0.127
		0.95	0.032	0.055	0.045	0.022	0.055	0.044	0.022	0.067	0.058	0.020	0.063	0.057	0.021	0.074	0.061
		0.9	0.003	0.040	0.023	0.002	0.038	0.032	0.002	0.038	0.027	0.003	0.046	0.034	0.001	0.043	0.033
		0.7	0.000	0.023	0.011	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.016	0.000	0.030	0.015	0.000	0.036	0.020	0.000	0.033	0.016
	300	1	0.056	0.023	0.050	0.068	0.061	0.076	0.067	0.075	0.091	0.065	0.103	0.108	0.066	0.114	0.105
		0.95	0.004	0.016	0.011	0.001	0.024	0.017	0.001	0.026	0.018	0.001	0.042	0.030	0.001	0.040	0.028
		0.9	0.000	0.021	0.010	0.000	0.023	0.018	0.000	0.025	0.020	0.000	0.028	0.020	0.000	0.029	0.015
		0.7	0.000	0.020	0.009	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.008
		0.5	0.000	0.025	0.010	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010
0.8	50	1	0.049	0.174	0.150	0.040	0.142	0.122	0.050	0.150	0.126	0.046	0.152	0.119	0.049	0.179	0.160
		0.95	0.033	0.173	0.139	0.027	0.150	0.120	0.038	0.158	0.133	0.032	0.135	0.112	0.033	0.165	0.136
		0.9	0.035	0.180	0.153	0.029	0.139	0.102	0.020	0.128	0.101	0.017	0.119	0.094	0.018	0.148	0.119
		0.7	0.024	0.189	0.149	0.015	0.104	0.077	0.007	0.101	0.073	0.006	0.087	0.068	0.002	0.086	0.060
		0.5	0.008	0.155	0.118	0.005	0.088	0.057	0.004	0.083	0.057	0.002	0.067	0.053	0.000	0.067	0.047
	150	1	0.050	0.049	0.056	0.033	0.068	0.063	0.037	0.085	0.085	0.034	0.103	0.094	0.040	0.124	0.111
		0.95	0.018	0.041	0.030	0.006	0.045	0.037	0.008	0.051	0.036	0.005	0.051	0.042	0.005	0.063	0.042
		0.9	0.002	0.044	0.026	0.002	0.036	0.028	0.000	0.035	0.025	0.000	0.041	0.031	0.000	0.039	0.028
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.016	0.000	0.030	0.015	0.000	0.036	0.018	0.000	0.033	0.016
	300	1	0.043	0.018	0.034	0.047	0.050	0.052	0.042	0.066	0.065	0.038	0.085	0.092	0.042	0.089	0.077
		0.95	0.003	0.017	0.011	0.001	0.024	0.016	0.000	0.025	0.017	0.000	0.043	0.027	0.000	0.041	0.028
		0.9	0.000	0.021	0.010	0.000	0.023	0.018	0.000	0.025	0.019	0.000	0.028	0.019	0.000	0.029	0.015
		0.7	0.000	0.020	0.010	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.007
		0.5	0.000	0.025	0.010	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010

Table B.5: Case A. Empirical size, assuming known $\bar{c}$

			$w = 0.10$			$w = 0.15$			$w = 0.20$			$w = 0.25$			$w = 0.30$		
$\bar{c}$	T	α	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	50	1	0.079	0.150	0.144	0.066	0.137	0.129	0.067	0.147	0.137	0.067	0.142	0.130	0.072	0.196	0.180
		0.95	0.056	0.166	0.147	0.056	0.136	0.116	0.053	0.147	0.124	0.036	0.129	0.119	0.039	0.164	0.140
		0.9	0.055	0.176	0.155	0.042	0.132	0.103	0.031	0.127	0.099	0.026	0.115	0.093	0.020	0.142	0.117
		0.7	0.035	0.189	0.153	0.019	0.104	0.080	0.013	0.102	0.076	0.008	0.088	0.071	0.002	0.086	0.062
		0.5	0.014	0.155	0.119	0.007	0.088	0.059	0.005	0.083	0.058	0.002	0.067	0.053	0.000	0.067	0.048
	150	1	0.097	0.040	0.074	0.053	0.055	0.060	0.056	0.074	0.084	0.048	0.105	0.112	0.049	0.119	0.118
		0.95	0.049	0.043	0.039	0.012	0.045	0.036	0.010	0.051	0.038	0.004	0.051	0.042	0.004	0.063	0.042
		0.9	0.007	0.044	0.028	0.002	0.036	0.029	0.000	0.035	0.025	0.000	0.041	0.031	0.000	0.039	0.029
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.017	0.000	0.030	0.015	0.000	0.036	0.019	0.000	0.033	0.016
	300	1	0.076	0.012	0.056	0.064	0.041	0.063	0.059	0.063	0.078	0.060	0.080	0.087	0.052	0.088	0.092
		0.95	0.004	0.017	0.012	0.001	0.024	0.016	0.000	0.025	0.017	0.000	0.043	0.028	0.000	0.041	0.029
		0.9	0.000	0.021	0.010	0.000	0.023	0.018	0.000	0.025	0.019	0.000	0.028	0.019	0.000	0.029	0.015
		0.7	0.000	0.020	0.010	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.007
		0.5	0.000	0.025	0.010	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010
1.5	50	1	0.056	0.150	0.126	0.046	0.133	0.113	0.052	0.131	0.115	0.056	0.125	0.114	0.059	0.171	0.152
		0.95	0.040	0.160	0.143	0.031	0.131	0.108	0.031	0.140	0.112	0.021	0.120	0.109	0.023	0.154	0.124
		0.9	0.037	0.176	0.151	0.027	0.133	0.102	0.020	0.126	0.098	0.013	0.114	0.088	0.013	0.142	0.116
		0.7	0.024	0.188	0.150	0.015	0.104	0.079	0.007	0.102	0.076	0.006	0.088	0.071	0.001	0.086	0.064
		0.5	0.008	0.155	0.118	0.005	0.088	0.058	0.003	0.083	0.058	0.002	0.067	0.054	0.000	0.067	0.048
	150	1	0.064	0.040	0.053	0.036	0.039	0.040	0.047	0.058	0.073	0.045	0.081	0.088	0.055	0.103	0.106
		0.95	0.019	0.043	0.034	0.003	0.045	0.035	0.004	0.051	0.035	0.001	0.051	0.041	0.001	0.063	0.041
		0.9	0.002	0.044	0.026	0.001	0.036	0.029	0.000	0.035	0.025	0.000	0.041	0.031	0.000	0.039	0.029
		0.7	0.000	0.023	0.012	0.000	0.028	0.018	0.000	0.028	0.015	0.000	0.030	0.018	0.000	0.016	0.005
		0.5	0.000	0.030	0.018	0.000	0.038	0.018	0.000	0.030	0.015	0.000	0.036	0.020	0.000	0.033	0.016
	300	1	0.051	0.008	0.040	0.051	0.035	0.051	0.054	0.046	0.066	0.046	0.072	0.078	0.052	0.081	0.094
		0.95	0.002	0.017	0.011	0.000	0.024	0.016	0.000	0.025	0.017	0.000	0.043	0.029	0.000	0.041	0.029
		0.9	0.000	0.021	0.010	0.000	0.023	0.019	0.000	0.025	0.019	0.000	0.028	0.020	0.000	0.029	0.015
		0.7	0.000	0.020	0.011	0.000	0.020	0.015	0.000	0.020	0.009	0.000	0.016	0.009	0.000	0.021	0.008
		0.5	0.000	0.025	0.011	0.000	0.033	0.018	0.000	0.027	0.012	0.000	0.023	0.010	0.000	0.028	0.010

Table B.6: Case A. Empirical power with one structural break, $\gamma = \gamma^* T^{1/2}$ and $w = 0.30$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.987	0.988	0.994	0.987	0.997	0.996	0.988	0.995	0.995	0.994	1.000	1.000	0.994	1.000	1.000
		150	1.000	0.993	1.000	1.000	0.994	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.798	0.819	0.885	0.828	0.868	0.919	0.812	0.875	0.914	0.847	0.982	0.974	0.906	0.999	0.999
		150	0.893	0.822	0.917	0.925	0.867	0.938	0.966	0.974	0.981	0.998	1.000	1.000	0.962	1.000	1.000
		300	0.929	0.819	0.943	0.984	0.972	0.990	0.999	1.000	1.000	0.999	1.000	1.000	0.890	1.000	1.000
	0.8	50	0.655	0.737	0.785	0.630	0.730	0.779	0.632	0.768	0.801	0.690	0.977	0.969	0.707	0.999	0.999
		150	0.788	0.676	0.821	0.809	0.815	0.870	0.881	0.969	0.966	0.952	1.000	1.000	0.758	1.000	1.000
		300	0.828	0.718	0.855	0.910	0.970	0.972	0.981	1.000	1.000	0.925	1.000	1.000	0.472	1.000	1.000
	1.0	50	0.523	0.683	0.732	0.525	0.702	0.733	0.527	0.758	0.769	0.556	0.977	0.968	0.534	0.999	0.999
		150	0.724	0.635	0.763	0.780	0.811	0.861	0.848	0.969	0.966	0.878	1.000	1.000	0.553	1.000	1.000
		300	0.802	0.660	0.809	0.887	0.970	0.969	0.967	1.000	1.000	0.812	1.000	1.000	0.227	1.000	1.000
	1.5	50	0.229	0.606	0.609	0.206	0.693	0.669	0.194	0.759	0.733	0.135	0.977	0.964	0.082	0.999	0.999
		150	0.452	0.559	0.622	0.457	0.811	0.817	0.444	0.969	0.961	0.282	1.000	1.000	0.096	1.000	1.000
		300	0.587	0.583	0.671	0.608	0.970	0.964	0.626	1.000	1.000	0.270	1.000	1.000	0.018	1.000	1.000
5	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.998	0.999	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
	0.8	50	0.995	0.998	1.000	0.994	0.999	1.000	0.996	1.000	1.000	0.990	1.000	1.000	0.968	1.000	1.000
		150	1.000	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	0.948	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	0.850	1.000	1.000
	1.0	50	0.982	0.997	1.000	0.988	0.998	1.000	0.985	0.998	1.000	0.969	1.000	1.000	0.917	1.000	1.000
		150	1.000	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	1.000	1.000	0.872	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.976	1.000	1.000	0.671	1.000	1.000
	1.5	50	0.905	0.993	1.000	0.902	0.998	1.000	0.885	0.998	1.000	0.820	1.000	1.000	0.705	1.000	1.000
		150	0.997	0.977	1.000	0.992	1.000	1.000	0.988	1.000	1.000	0.904	1.000	1.000	0.581	1.000	1.000
		300	1.000	0.992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.836	1.000	1.000	0.269	1.000	1.000
10	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	0.999	1.000	1.000	0.998	0.999	1.000	0.998	1.000	1.000	0.996	1.000	1.000	0.986	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.979	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.963	1.000	1.000
	1.0	50	0.992	0.998	1.000	0.995	1.000	1.000	0.995	1.000	1.000	0.986	1.000	1.000	0.960	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	0.938	1.000	1.000
		300	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	0.810	1.000	1.000
	1.5	50	0.966	0.995	1.000	0.962	1.000	1.000	0.953	1.000	1.000	0.913	1.000	1.000	0.830	1.000	1.000
		150	1.000	0.996	1.000	1.000	1.000	1.000	0.998	1.000	1.000	0.950	1.000	1.000	0.740	1.000	1.000
		300	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.915	1.000	1.000	0.449	1.000	1.000

Table B.7: Case A. Empirical power with one structural break, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.30$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.001	0.143	0.109	0.000	0.143	0.102	0.000	0.152	0.122	0.000	0.100	0.066	0.000	0.069	0.045
		150	0.000	0.092	0.075	0.000	0.097	0.072	0.001	0.052	0.041	0.000	0.017	0.009	0.000	0.034	0.017
		300	0.000	0.072	0.055	0.000	0.075	0.052	0.000	0.034	0.020	0.000	0.022	0.009	0.000	0.029	0.010
	0.5	50	0.027	0.235	0.196	0.012	0.194	0.151	0.008	0.194	0.153	0.000	0.091	0.061	0.000	0.071	0.051
		150	0.011	0.139	0.110	0.003	0.074	0.055	0.000	0.044	0.032	0.000	0.014	0.006	0.000	0.033	0.016
		300	0.021	0.116	0.087	0.000	0.042	0.028	0.000	0.030	0.015	0.000	0.022	0.008	0.000	0.029	0.010
	0.8	50	0.028	0.182	0.158	0.019	0.164	0.131	0.011	0.144	0.113	0.000	0.090	0.061	0.000	0.072	0.052
		150	0.040	0.129	0.115	0.004	0.061	0.043	0.000	0.039	0.027	0.000	0.014	0.007	0.000	0.033	0.016
		300	0.034	0.087	0.080	0.000	0.043	0.029	0.000	0.030	0.015	0.000	0.022	0.008	0.000	0.029	0.010
	1.0	50	0.019	0.193	0.162	0.010	0.161	0.130	0.004	0.139	0.109	0.000	0.090	0.063	0.000	0.072	0.052
		150	0.042	0.123	0.116	0.002	0.061	0.043	0.000	0.039	0.027	0.000	0.014	0.007	0.000	0.033	0.016
		300	0.051	0.087	0.090	0.000	0.043	0.029	0.000	0.030	0.015	0.000	0.022	0.008	0.000	0.029	0.010
	1.5	50	0.005	0.172	0.142	0.002	0.149	0.119	0.001	0.138	0.107	0.000	0.090	0.063	0.000	0.072	0.052
		150	0.024	0.106	0.093	0.000	0.061	0.042	0.000	0.039	0.027	0.000	0.014	0.007	0.000	0.033	0.016
		300	0.029	0.079	0.078	0.000	0.043	0.029	0.000	0.031	0.015	0.000	0.022	0.009	0.000	0.029	0.010
5	0.3	50	0.002	0.149	0.118	0.000	0.154	0.118	0.001	0.175	0.140	0.000	0.122	0.086	0.000	0.089	0.061
		150	0.000	0.097	0.079	0.000	0.093	0.068	0.001	0.055	0.044	0.000	0.022	0.014	0.000	0.041	0.021
		300	0.000	0.074	0.054	0.000	0.070	0.052	0.000	0.037	0.021	0.000	0.027	0.009	0.000	0.042	0.021
	0.5	50	0.034	0.217	0.190	0.010	0.189	0.149	0.010	0.191	0.158	0.000	0.102	0.074	0.000	0.089	0.067
		150	0.011	0.137	0.112	0.004	0.075	0.054	0.000	0.045	0.032	0.000	0.019	0.010	0.000	0.041	0.021
		300	0.021	0.113	0.093	0.000	0.042	0.028	0.000	0.033	0.017	0.000	0.027	0.008	0.000	0.042	0.021
	0.8	50	0.034	0.192	0.164	0.025	0.169	0.134	0.012	0.139	0.116	0.000	0.103	0.077	0.000	0.092	0.071
		150	0.042	0.129	0.111	0.004	0.058	0.044	0.000	0.042	0.028	0.000	0.019	0.010	0.000	0.041	0.021
		300	0.034	0.092	0.082	0.000	0.043	0.029	0.000	0.033	0.016	0.000	0.027	0.008	0.000	0.042	0.021
	1.0	50	0.018	0.205	0.170	0.010	0.161	0.130	0.005	0.137	0.110	0.000	0.103	0.078	0.000	0.092	0.071
		150	0.044	0.116	0.119	0.002	0.058	0.042	0.000	0.042	0.028	0.000	0.019	0.010	0.000	0.041	0.021
		300	0.052	0.090	0.090	0.000	0.043	0.029	0.000	0.033	0.016	0.000	0.027	0.008	0.000	0.042	0.021
	1.5	50	0.004	0.194	0.156	0.003	0.154	0.120	0.001	0.136	0.107	0.000	0.103	0.078	0.000	0.092	0.072
		150	0.024	0.104	0.090	0.000	0.058	0.041	0.000	0.042	0.028	0.000	0.019	0.010	0.000	0.041	0.022
		300	0.029	0.081	0.078	0.000	0.043	0.029	0.000	0.033	0.016	0.000	0.027	0.008	0.000	0.042	0.021
10	0.3	50	0.009	0.189	0.152	0.006	0.180	0.137	0.003	0.218	0.175	0.004	0.180	0.129	0.000	0.176	0.131
		150	0.000	0.094	0.074	0.000	0.098	0.071	0.001	0.063	0.047	0.000	0.034	0.024	0.000	0.066	0.040
		300	0.000	0.074	0.049	0.000	0.073	0.059	0.000	0.036	0.024	0.000	0.036	0.020	0.000	0.081	0.048
	0.5	50	0.039	0.244	0.217	0.023	0.221	0.187	0.015	0.199	0.165	0.001	0.139	0.107	0.000	0.144	0.112
		150	0.016	0.140	0.115	0.004	0.080	0.056	0.000	0.051	0.036	0.000	0.031	0.023	0.000	0.066	0.041
		300	0.021	0.114	0.095	0.000	0.043	0.028	0.000	0.031	0.018	0.000	0.036	0.018	0.000	0.081	0.047
	0.8	50	0.043	0.217	0.197	0.031	0.181	0.158	0.013	0.169	0.136	0.000	0.140	0.107	0.000	0.145	0.117
		150	0.042	0.132	0.116	0.003	0.065	0.048	0.000	0.045	0.030	0.000	0.031	0.023	0.000	0.066	0.041
		300	0.034	0.089	0.082	0.000	0.043	0.029	0.000	0.030	0.017	0.000	0.036	0.018	0.000	0.081	0.048
	1.0	50	0.028	0.221	0.193	0.018	0.186	0.151	0.009	0.171	0.130	0.000	0.140	0.109	0.000	0.145	0.117
		150	0.045	0.120	0.117	0.002	0.065	0.047	0.000	0.045	0.031	0.000	0.031	0.023	0.000	0.066	0.041
		300	0.051	0.089	0.091	0.000	0.043	0.029	0.000	0.030	0.017	0.000	0.036	0.018	0.000	0.081	0.048
	1.5	50	0.010	0.207	0.183	0.007	0.181	0.145	0.002	0.170	0.129	0.000	0.140	0.109	0.000	0.145	0.118
		150	0.021	0.105	0.093	0.000	0.065	0.046	0.000	0.045	0.032	0.000	0.031	0.023	0.000	0.066	0.041
		300	0.029	0.082	0.078	0.000	0.043	0.030	0.000	0.031	0.017	0.000	0.036	0.018	0.000	0.081	0.049

Table B.8: Case A. Empirical power with one structural break, $\gamma = \gamma^*$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.003	0.169	0.128	0.001	0.160	0.121	0.002	0.192	0.153	0.000	0.146	0.099	0.000	0.119	0.088
		150	0.000	0.093	0.074	0.000	0.100	0.072	0.001	0.063	0.046	0.000	0.046	0.028	0.000	0.085	0.055
		300	0.000	0.070	0.057	0.000	0.073	0.057	0.000	0.040	0.032	0.000	0.073	0.037	0.000	0.271	0.181
	0.5	50	0.038	0.220	0.186	0.013	0.196	0.157	0.011	0.192	0.149	0.000	0.115	0.088	0.000	0.109	0.076
		150	0.016	0.139	0.121	0.004	0.075	0.057	0.000	0.049	0.037	0.000	0.041	0.026	0.000	0.085	0.055
		300	0.022	0.115	0.100	0.000	0.045	0.032	0.000	0.036	0.023	0.000	0.073	0.036	0.000	0.271	0.182
	0.8	50	0.036	0.209	0.177	0.029	0.176	0.132	0.013	0.147	0.124	0.000	0.115	0.091	0.000	0.111	0.081
		150	0.044	0.134	0.122	0.003	0.065	0.046	0.000	0.042	0.031	0.000	0.041	0.026	0.000	0.085	0.056
		300	0.037	0.092	0.084	0.000	0.045	0.033	0.000	0.035	0.022	0.000	0.073	0.036	0.000	0.271	0.184
	1.0	50	0.024	0.205	0.177	0.012	0.175	0.132	0.006	0.145	0.116	0.000	0.114	0.091	0.000	0.111	0.081
		150	0.048	0.122	0.125	0.002	0.065	0.047	0.000	0.042	0.032	0.000	0.041	0.026	0.000	0.085	0.056
		300	0.050	0.089	0.091	0.000	0.045	0.033	0.000	0.035	0.022	0.000	0.073	0.036	0.000	0.271	0.184
	1.5	50	0.006	0.198	0.161	0.005	0.166	0.121	0.002	0.145	0.116	0.000	0.114	0.091	0.000	0.111	0.081
		150	0.020	0.109	0.098	0.000	0.065	0.046	0.000	0.042	0.033	0.000	0.041	0.026	0.000	0.085	0.056
		300	0.031	0.085	0.077	0.000	0.045	0.033	0.000	0.035	0.023	0.000	0.073	0.036	0.000	0.271	0.184
5	0.3	50	0.667	0.899	0.866	0.680	0.909	0.900	0.675	0.939	0.921	0.663	0.968	0.934	0.651	0.989	0.975
		150	0.219	0.502	0.447	0.176	0.568	0.516	0.165	0.619	0.557	0.045	0.967	0.951	0.042	0.964	0.957
		300	0.104	0.287	0.250	0.085	0.373	0.322	0.021	0.565	0.507	0.000	0.984	0.977	0.000	0.970	0.967
	0.5	50	0.430	0.646	0.627	0.417	0.661	0.643	0.396	0.663	0.651	0.347	0.844	0.794	0.341	0.960	0.940
		150	0.158	0.348	0.333	0.092	0.317	0.290	0.039	0.442	0.390	0.004	0.958	0.940	0.000	0.964	0.961
		300	0.085	0.237	0.213	0.012	0.265	0.223	0.000	0.519	0.455	0.000	0.984	0.978	0.000	0.970	0.966
	0.8	50	0.300	0.562	0.539	0.278	0.538	0.520	0.264	0.571	0.548	0.224	0.830	0.785	0.179	0.958	0.941
		150	0.131	0.263	0.267	0.054	0.281	0.251	0.015	0.440	0.378	0.000	0.958	0.940	0.000	0.964	0.961
		300	0.075	0.194	0.193	0.003	0.257	0.215	0.000	0.519	0.453	0.000	0.984	0.978	0.000	0.970	0.966
	1.0	50	0.221	0.523	0.505	0.206	0.512	0.477	0.188	0.564	0.535	0.135	0.830	0.786	0.075	0.958	0.942
		150	0.124	0.243	0.245	0.049	0.278	0.249	0.013	0.440	0.381	0.000	0.958	0.941	0.000	0.964	0.961
		300	0.091	0.182	0.179	0.002	0.257	0.215	0.000	0.519	0.453	0.000	0.984	0.977	0.000	0.970	0.966
	1.5	50	0.081	0.452	0.425	0.064	0.503	0.451	0.055	0.565	0.518	0.017	0.830	0.787	0.001	0.958	0.943
		150	0.064	0.214	0.194	0.013	0.278	0.245	0.002	0.440	0.382	0.000	0.958	0.943	0.000	0.964	0.961
		300	0.046	0.174	0.163	0.000	0.257	0.215	0.000	0.519	0.456	0.000	0.984	0.978	0.000	0.970	0.966
10	0.3	50	0.998	1.000	1.000	0.998	1.000	1.000	0.998	1.000	1.000	0.998	1.000	1.000	0.995	1.000	1.000
		150	0.955	0.959	0.964	0.958	0.975	0.981	0.977	0.988	0.987	1.000	1.000	1.000	1.000	1.000	1.000
		300	0.702	0.770	0.766	0.729	0.896	0.884	0.803	0.986	0.976	0.952	1.000	1.000	0.990	1.000	1.000
	0.5	50	0.988	0.942	0.996	0.985	0.968	0.999	0.992	0.975	0.997	0.994	0.997	1.000	0.985	1.000	1.000
		150	0.710	0.714	0.781	0.727	0.756	0.814	0.760	0.914	0.913	0.939	1.000	1.000	0.942	1.000	1.000
		300	0.354	0.487	0.501	0.315	0.723	0.684	0.205	0.969	0.953	0.181	1.000	1.000	0.474	1.000	1.000
	0.8	50	0.920	0.885	0.971	0.912	0.889	0.966	0.929	0.922	0.972	0.943	0.998	1.000	0.885	1.000	1.000
		150	0.582	0.558	0.646	0.536	0.685	0.709	0.555	0.907	0.893	0.688	1.000	1.000	0.607	1.000	1.000
		300	0.273	0.372	0.393	0.128	0.710	0.663	0.042	0.968	0.952	0.007	1.000	1.000	0.000	1.000	1.000
	1.0	50	0.852	0.854	0.946	0.858	0.864	0.949	0.865	0.917	0.967	0.853	0.998	1.000	0.766	1.000	1.000
		150	0.524	0.512	0.582	0.501	0.681	0.698	0.506	0.907	0.891	0.559	1.000	1.000	0.370	1.000	1.000
		300	0.264	0.332	0.350	0.106	0.710	0.663	0.030	0.968	0.952	0.000	1.000	1.000	0.000	1.000	1.000
	1.5	50	0.504	0.791	0.857	0.499	0.849	0.905	0.494	0.915	0.944	0.441	0.998	0.997	0.331	1.000	1.000
		150	0.290	0.454	0.472	0.219	0.681	0.663	0.150	0.907	0.884	0.021	1.000	1.000	0.001	1.000	1.000
		300	0.148	0.291	0.287	0.017	0.710	0.660	0.001	0.968	0.952	0.000	1.000	1.000	0.000	1.000	1.000

Table B.9: Case A. Empirical power with two structural breaks, $\gamma = \gamma^* T^{1/2}$ and $w = 0.15$

			$\alpha=1$			$\alpha=0.95$			$\alpha=0.9$			$\alpha=0.7$			$\alpha=0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.995	0.994	1.000	0.994	0.998	0.999	0.994	0.998	1.000	0.998	1.000	0.999	0.998	1.000	1.000
		150	1.000	0.992	1.000	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.986	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
	0.5	50	0.973	0.953	0.993	0.977	0.963	0.992	0.975	0.966	0.990	0.973	0.982	0.990	0.970	0.998	0.998
		150	1.000	0.918	1.000	1.000	0.945	1.000	1.000	0.983	0.999	0.996	1.000	1.000	0.977	1.000	1.000
		300	1.000	0.897	1.000	1.000	0.973	0.999	1.000	1.000	1.000	0.999	1.000	1.000	0.908	1.000	1.000
	0.8	50	0.896	0.902	0.959	0.894	0.919	0.958	0.897	0.932	0.965	0.899	0.978	0.985	0.887	0.998	0.998
		150	0.997	0.845	0.992	0.998	0.914	0.998	0.999	0.975	0.996	0.966	1.000	1.000	0.802	1.000	1.000
		300	0.998	0.813	0.991	0.999	0.971	0.999	1.000	1.000	1.000	0.948	1.000	1.000	0.531	1.000	1.000
	1.0	50	0.807	0.894	0.933	0.825	0.903	0.948	0.820	0.930	0.957	0.798	0.978	0.982	0.747	0.998	0.996
		150	0.992	0.817	0.982	0.993	0.910	0.993	0.989	0.975	0.994	0.899	1.000	1.000	0.611	1.000	1.000
		300	0.996	0.769	0.983	0.999	0.971	0.999	0.998	1.000	1.000	0.854	1.000	1.000	0.282	1.000	1.000
	1.5	50	0.426	0.862	0.878	0.430	0.902	0.910	0.416	0.929	0.929	0.350	0.978	0.974	0.266	0.998	0.997
		150	0.794	0.770	0.908	0.772	0.911	0.965	0.726	0.975	0.990	0.451	1.000	1.000	0.185	1.000	1.000
		300	0.912	0.726	0.920	0.903	0.971	0.994	0.858	1.000	1.000	0.382	1.000	1.000	0.035	1.000	1.000
5	0.3	50	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.996	0.999	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	0.993	0.999	1.000	0.994	0.999	1.000	0.996	1.000	1.000	0.982	1.000	1.000	0.972	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	0.969	1.000	1.000
		300	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.907	1.000	1.000
	1.0	50	0.972	0.999	1.000	0.978	0.999	1.000	0.981	1.000	1.000	0.956	1.000	1.000	0.942	1.000	1.000
		150	1.000	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.986	1.000	1.000	0.919	1.000	1.000
		300	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.976	1.000	1.000	0.755	1.000	1.000
	1.5	50	0.856	0.995	1.000	0.858	0.999	1.000	0.838	1.000	1.000	0.772	1.000	1.000	0.710	1.000	1.000
		150	0.997	0.992	1.000	0.994	1.000	1.000	0.985	1.000	1.000	0.899	1.000	1.000	0.662	1.000	1.000
		300	1.000	0.992	1.000	0.999	1.000	1.000	0.998	1.000	1.000	0.840	1.000	1.000	0.311	1.000	1.000
10	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	0.997	1.000	1.000	0.997	1.000	1.000	0.998	1.000	1.000	0.995	1.000	1.000	0.986	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	1.000	1.000
		300	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.975	1.000	1.000
	1.0	50	0.988	1.000	1.000	0.993	1.000	1.000	0.994	1.000	1.000	0.979	1.000	1.000	0.968	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	1.000	1.000	0.959	1.000	1.000
		300	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	0.878	1.000	1.000
	1.5	50	0.947	0.999	1.000	0.935	1.000	1.000	0.923	1.000	1.000	0.881	1.000	1.000	0.834	1.000	1.000
		150	0.999	0.998	1.000	0.999	1.000	1.000	0.995	1.000	1.000	0.954	1.000	1.000	0.804	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.927	1.000	1.000	0.503	1.000	1.000

Table B.10: Case A. Empirical power with two structural breaks, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.015	0.176	0.131	0.013	0.154	0.129	0.010	0.166	0.128	0.005	0.132	0.092	0.001	0.093	0.052
		150	0.003	0.086	0.071	0.000	0.074	0.050	0.001	0.050	0.035	0.000	0.026	0.011	0.000	0.040	0.015
		300	0.000	0.055	0.046	0.000	0.034	0.027	0.000	0.024	0.019	0.000	0.021	0.014	0.000	0.034	0.017
	0.5	50	0.044	0.184	0.141	0.024	0.164	0.130	0.020	0.156	0.114	0.004	0.109	0.079	0.001	0.088	0.055
		150	0.029	0.088	0.078	0.013	0.053	0.042	0.001	0.038	0.034	0.000	0.028	0.017	0.000	0.044	0.018
		300	0.041	0.060	0.059	0.000	0.025	0.016	0.000	0.023	0.020	0.000	0.021	0.014	0.000	0.034	0.018
	0.8	50	0.020	0.142	0.114	0.012	0.147	0.119	0.013	0.143	0.099	0.003	0.103	0.079	0.000	0.091	0.060
		150	0.038	0.068	0.071	0.015	0.046	0.037	0.002	0.036	0.030	0.000	0.028	0.019	0.000	0.044	0.018
		300	0.053	0.050	0.056	0.001	0.025	0.015	0.000	0.023	0.020	0.000	0.021	0.014	0.000	0.034	0.018
	1.0	50	0.004	0.139	0.109	0.005	0.136	0.103	0.008	0.137	0.096	0.000	0.104	0.078	0.000	0.091	0.061
		150	0.042	0.054	0.059	0.012	0.046	0.038	0.001	0.036	0.030	0.000	0.028	0.019	0.000	0.044	0.018
		300	0.073	0.041	0.068	0.001	0.025	0.017	0.000	0.023	0.020	0.000	0.021	0.014	0.000	0.034	0.018
	1.5	50	0.002	0.130	0.102	0.001	0.131	0.098	0.001	0.137	0.096	0.000	0.104	0.079	0.000	0.091	0.061
		150	0.011	0.039	0.033	0.001	0.046	0.034	0.000	0.036	0.030	0.000	0.028	0.019	0.000	0.044	0.018
		300	0.022	0.034	0.036	0.000	0.025	0.016	0.000	0.023	0.020	0.000	0.021	0.015	0.000	0.034	0.018
5	0.3	50	0.014	0.181	0.137	0.012	0.168	0.139	0.011	0.174	0.135	0.004	0.127	0.088	0.002	0.085	0.057
		150	0.003	0.086	0.066	0.000	0.070	0.055	0.001	0.048	0.035	0.000	0.033	0.014	0.000	0.041	0.018
		300	0.000	0.057	0.046	0.000	0.035	0.026	0.000	0.023	0.019	0.000	0.021	0.015	0.000	0.036	0.020
	0.5	50	0.050	0.190	0.158	0.028	0.179	0.131	0.020	0.162	0.125	0.007	0.116	0.085	0.001	0.092	0.063
		150	0.028	0.092	0.079	0.012	0.057	0.046	0.002	0.038	0.035	0.000	0.032	0.014	0.000	0.044	0.019
		300	0.041	0.062	0.057	0.000	0.023	0.016	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
	0.8	50	0.022	0.149	0.122	0.017	0.152	0.127	0.012	0.137	0.098	0.001	0.112	0.083	0.000	0.093	0.065
		150	0.038	0.072	0.073	0.013	0.048	0.037	0.003	0.037	0.033	0.000	0.032	0.015	0.000	0.044	0.020
		300	0.053	0.051	0.059	0.001	0.022	0.015	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
	1.0	50	0.007	0.139	0.110	0.006	0.137	0.112	0.007	0.126	0.091	0.000	0.112	0.084	0.000	0.093	0.066
		150	0.042	0.054	0.058	0.012	0.048	0.038	0.001	0.037	0.032	0.000	0.032	0.015	0.000	0.044	0.022
		300	0.074	0.042	0.068	0.001	0.022	0.016	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
	1.5	50	0.002	0.140	0.114	0.001	0.134	0.108	0.001	0.127	0.093	0.000	0.112	0.086	0.000	0.093	0.066
		150	0.010	0.041	0.037	0.001	0.048	0.036	0.000	0.037	0.032	0.000	0.032	0.015	0.000	0.044	0.022
		300	0.021	0.035	0.035	0.000	0.022	0.016	0.000	0.022	0.020	0.000	0.021	0.015	0.000	0.036	0.019
10	0.3	50	0.033	0.192	0.155	0.033	0.205	0.161	0.022	0.205	0.158	0.009	0.142	0.103	0.005	0.114	0.089
		150	0.003	0.089	0.067	0.000	0.074	0.053	0.001	0.047	0.032	0.000	0.035	0.017	0.000	0.052	0.028
		300	0.000	0.057	0.042	0.000	0.034	0.024	0.000	0.023	0.019	0.000	0.022	0.016	0.000	0.040	0.021
	0.5	50	0.060	0.211	0.177	0.046	0.196	0.153	0.030	0.181	0.136	0.011	0.132	0.096	0.003	0.112	0.080
		150	0.024	0.090	0.075	0.010	0.054	0.048	0.002	0.042	0.031	0.000	0.037	0.018	0.000	0.055	0.028
		300	0.041	0.063	0.058	0.000	0.023	0.014	0.000	0.021	0.018	0.000	0.022	0.016	0.000	0.040	0.022
	0.8	50	0.038	0.160	0.130	0.024	0.170	0.130	0.018	0.151	0.111	0.004	0.128	0.090	0.000	0.113	0.083
		150	0.039	0.075	0.076	0.012	0.046	0.035	0.002	0.040	0.030	0.000	0.037	0.019	0.000	0.055	0.028
		300	0.056	0.052	0.060	0.001	0.022	0.013	0.000	0.021	0.018	0.000	0.022	0.017	0.000	0.040	0.023
	1.0	50	0.014	0.158	0.130	0.013	0.161	0.125	0.011	0.148	0.107	0.002	0.129	0.091	0.000	0.113	0.083
		150	0.042	0.055	0.060	0.012	0.047	0.037	0.002	0.040	0.029	0.000	0.037	0.019	0.000	0.055	0.029
		300	0.076	0.042	0.066	0.001	0.022	0.014	0.000	0.021	0.018	0.000	0.022	0.017	0.000	0.040	0.023
	1.5	50	0.002	0.159	0.133	0.003	0.159	0.127	0.002	0.150	0.107	0.000	0.129	0.092	0.000	0.114	0.084
		150	0.009	0.045	0.037	0.001	0.047	0.033	0.000	0.040	0.029	0.000	0.037	0.019	0.000	0.055	0.029
		300	0.022	0.038	0.035	0.000	0.022	0.013	0.000	0.021	0.018	0.000	0.022	0.018	0.000	0.040	0.023

Table B.11: Case A. Empirical power with two structural breaks, $\gamma = \gamma^*$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.028	0.195	0.154	0.022	0.196	0.157	0.027	0.193	0.146	0.014	0.166	0.123	0.009	0.119	0.082
		150	0.001	0.076	0.062	0.002	0.082	0.058	0.001	0.056	0.040	0.000	0.041	0.027	0.000	0.088	0.054
		300	0.004	0.056	0.044	0.000	0.040	0.031	0.000	0.023	0.017	0.000	0.035	0.017	0.000	0.117	0.067
	0.5	50	0.050	0.199	0.164	0.032	0.183	0.144	0.021	0.182	0.133	0.008	0.134	0.108	0.004	0.117	0.084
		150	0.030	0.093	0.078	0.014	0.063	0.045	0.000	0.046	0.034	0.000	0.041	0.028	0.000	0.090	0.052
		300	0.042	0.062	0.061	0.001	0.026	0.019	0.000	0.020	0.015	0.000	0.035	0.017	0.000	0.117	0.069
	0.8	50	0.023	0.160	0.124	0.021	0.157	0.121	0.018	0.154	0.130	0.002	0.134	0.108	0.000	0.122	0.088
		150	0.049	0.070	0.076	0.014	0.052	0.036	0.000	0.040	0.029	0.000	0.041	0.028	0.000	0.090	0.056
		300	0.051	0.046	0.058	0.001	0.026	0.019	0.000	0.020	0.014	0.000	0.035	0.017	0.000	0.117	0.071
	1.0	50	0.010	0.156	0.120	0.007	0.155	0.118	0.007	0.147	0.124	0.000	0.134	0.108	0.000	0.122	0.089
		150	0.053	0.063	0.064	0.011	0.050	0.039	0.001	0.040	0.029	0.000	0.041	0.029	0.000	0.090	0.056
		300	0.075	0.041	0.065	0.001	0.026	0.020	0.000	0.020	0.014	0.000	0.035	0.017	0.000	0.117	0.073
	1.5	50	0.004	0.145	0.110	0.001	0.151	0.115	0.001	0.147	0.128	0.000	0.134	0.109	0.000	0.123	0.089
		150	0.015	0.050	0.041	0.001	0.050	0.036	0.000	0.040	0.030	0.000	0.041	0.029	0.000	0.090	0.056
		300	0.025	0.033	0.037	0.000	0.026	0.019	0.000	0.020	0.014	0.000	0.035	0.017	0.000	0.117	0.072
5	0.3	50	0.767	0.760	0.827	0.784	0.792	0.844	0.788	0.820	0.848	0.738	0.828	0.829	0.718	0.909	0.874
		150	0.237	0.324	0.315	0.199	0.382	0.368	0.196	0.392	0.351	0.141	0.827	0.777	0.156	0.973	0.960
		300	0.108	0.139	0.133	0.100	0.170	0.156	0.050	0.259	0.210	0.003	0.955	0.939	0.027	0.996	0.992
	0.5	50	0.582	0.561	0.639	0.586	0.590	0.653	0.554	0.591	0.645	0.516	0.666	0.657	0.532	0.826	0.792
		150	0.168	0.205	0.225	0.117	0.191	0.193	0.092	0.242	0.207	0.053	0.813	0.766	0.091	0.972	0.969
		300	0.095	0.109	0.131	0.030	0.108	0.092	0.002	0.221	0.177	0.001	0.955	0.942	0.000	0.996	0.993
	0.8	50	0.432	0.453	0.513	0.403	0.466	0.501	0.400	0.492	0.502	0.403	0.648	0.621	0.415	0.822	0.781
		150	0.175	0.143	0.191	0.112	0.165	0.166	0.080	0.238	0.200	0.032	0.813	0.767	0.008	0.972	0.969
		300	0.110	0.077	0.113	0.020	0.106	0.088	0.004	0.221	0.177	0.000	0.955	0.942	0.000	0.996	0.993
	1.0	50	0.297	0.417	0.437	0.292	0.448	0.449	0.278	0.488	0.480	0.262	0.648	0.597	0.257	0.822	0.775
		150	0.170	0.124	0.170	0.107	0.165	0.163	0.079	0.238	0.199	0.012	0.813	0.767	0.002	0.972	0.970
		300	0.135	0.069	0.116	0.025	0.106	0.088	0.002	0.221	0.177	0.000	0.955	0.942	0.000	0.996	0.993
	1.5	50	0.081	0.356	0.331	0.081	0.434	0.395	0.076	0.489	0.439	0.043	0.649	0.574	0.031	0.823	0.771
		150	0.060	0.101	0.101	0.034	0.165	0.141	0.016	0.238	0.188	0.001	0.813	0.768	0.000	0.972	0.970
		300	0.046	0.055	0.060	0.003	0.106	0.084	0.000	0.221	0.178	0.000	0.955	0.943	0.000	0.996	0.994
10	0.3	50	0.997	0.993	1.000	0.993	0.996	0.999	0.995	0.998	1.000	0.998	1.000	1.000	0.999	1.000	0.999
		150	0.975	0.894	0.970	0.980	0.915	0.981	0.987	0.945	0.983	0.999	1.000	1.000	1.000	1.000	1.000
		300	0.748	0.536	0.714	0.767	0.690	0.765	0.803	0.889	0.883	0.974	1.000	1.000	0.997	1.000	1.000
	0.5	50	0.988	0.938	0.998	0.990	0.954	1.000	0.995	0.953	0.997	0.992	0.982	0.999	0.983	1.000	1.000
		150	0.863	0.624	0.868	0.863	0.654	0.858	0.871	0.794	0.895	0.970	1.000	1.000	0.969	1.000	1.000
		300	0.460	0.328	0.460	0.450	0.477	0.550	0.410	0.850	0.813	0.588	1.000	1.000	0.791	1.000	1.000
	0.8	50	0.932	0.884	0.989	0.936	0.885	0.987	0.948	0.895	0.991	0.929	0.980	0.997	0.910	1.000	1.000
		150	0.807	0.472	0.773	0.787	0.567	0.779	0.819	0.780	0.863	0.904	1.000	1.000	0.792	1.000	1.000
		300	0.435	0.226	0.396	0.369	0.465	0.503	0.336	0.850	0.810	0.435	1.000	1.000	0.285	1.000	1.000
	1.0	50	0.854	0.854	0.974	0.868	0.858	0.974	0.868	0.896	0.980	0.835	0.980	0.994	0.777	1.000	1.000
		150	0.768	0.415	0.742	0.785	0.563	0.769	0.809	0.780	0.865	0.826	1.000	1.000	0.595	1.000	1.000
		300	0.462	0.212	0.394	0.395	0.465	0.503	0.364	0.850	0.811	0.295	1.000	1.000	0.070	1.000	1.000
	1.5	50	0.520	0.815	0.888	0.493	0.852	0.912	0.471	0.895	0.931	0.401	0.980	0.983	0.345	1.000	0.999
		150	0.398	0.360	0.485	0.393	0.563	0.615	0.361	0.780	0.783	0.218	1.000	1.000	0.078	1.000	1.000
		300	0.215	0.170	0.241	0.141	0.465	0.442	0.075	0.850	0.808	0.001	1.000	1.000	0.000	1.000	1.000

Table B.12: Case A. Empirical power with two structural breaks, $\gamma = \gamma^* T^{1/2}$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.997	0.993	1.000	0.996	0.999	0.999	0.993	0.999	0.999	0.996	1.000	1.000	0.991	1.000	1.000
		150	1.000	0.995	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.997	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
	0.5	50	0.963	0.854	0.966	0.962	0.877	0.971	0.958	0.900	0.973	0.955	0.983	0.990	0.958	1.000	1.000
		150	0.997	0.802	0.986	0.995	0.912	0.990	0.999	0.989	0.999	0.995	1.000	1.000	0.945	1.000	1.000
		300	1.000	0.795	0.988	1.000	0.981	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.815	1.000	1.000
	0.8	50	0.846	0.720	0.855	0.836	0.756	0.864	0.841	0.807	0.876	0.825	0.979	0.979	0.784	1.000	1.000
		150	0.956	0.673	0.929	0.964	0.871	0.958	0.977	0.981	0.994	0.961	1.000	1.000	0.699	1.000	1.000
		300	0.965	0.660	0.941	0.992	0.978	0.990	0.999	1.000	1.000	0.910	1.000	1.000	0.368	1.000	1.000
	1.0	50	0.755	0.679	0.808	0.758	0.726	0.836	0.749	0.797	0.859	0.714	0.978	0.980	0.642	1.000	1.000
		150	0.930	0.633	0.895	0.955	0.869	0.942	0.963	0.981	0.993	0.888	1.000	1.000	0.503	1.000	1.000
		300	0.952	0.615	0.904	0.982	0.978	0.989	0.995	1.000	1.000	0.798	1.000	1.000	0.169	1.000	1.000
	1.5	50	0.415	0.620	0.674	0.418	0.710	0.755	0.383	0.797	0.814	0.275	0.978	0.975	0.183	1.000	1.000
		150	0.712	0.542	0.746	0.728	0.869	0.906	0.682	0.981	0.989	0.404	1.000	1.000	0.109	1.000	1.000
		300	0.831	0.516	0.783	0.856	0.978	0.981	0.806	1.000	1.000	0.295	1.000	1.000	0.013	1.000	1.000
5	0.3	50	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.997	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.997	0.999	1.000	0.999	0.999	1.000	0.997	1.000	1.000	0.998	1.000	1.000	0.993	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
	0.8	50	0.998	0.995	1.000	0.997	0.999	1.000	0.996	0.998	1.000	0.987	1.000	1.000	0.956	1.000	1.000
		150	1.000	0.989	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.990	1.000	1.000	0.921	1.000	1.000
		300	1.000	0.988	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.751	1.000	1.000
	1.0	50	0.990	0.992	1.000	0.990	0.996	1.000	0.987	1.000	1.000	0.969	1.000	1.000	0.899	1.000	1.000
		150	1.000	0.993	1.000	1.000	0.999	1.000	0.999	1.000	1.000	0.981	1.000	1.000	0.828	1.000	1.000
		300	1.000	0.982	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.963	1.000	1.000	0.551	1.000	1.000
	1.5	50	0.938	0.985	1.000	0.927	0.995	1.000	0.909	1.000	1.000	0.836	1.000	1.000	0.723	1.000	1.000
		150	1.000	0.985	1.000	0.997	0.999	1.000	0.987	1.000	1.000	0.901	1.000	1.000	0.536	1.000	1.000
		300	1.000	0.972	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.813	1.000	1.000	0.200	1.000	1.000
10	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.999	1.000	1.000	0.999	1.000	1.000	0.997	1.000	1.000	0.998	1.000	1.000	0.995	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	1.000	0.999	1.000	0.998	1.000	1.000	0.997	1.000	1.000	0.989	1.000	1.000	0.974	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.979	1.000	1.000
		300	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.909	1.000	1.000
	1.0	50	0.995	0.996	1.000	0.996	1.000	1.000	0.994	1.000	1.000	0.984	1.000	1.000	0.942	1.000	1.000
		150	1.000	0.997	1.000	1.000	0.999	1.000	1.000	1.000	1.000	0.990	1.000	1.000	0.903	1.000	1.000
		300	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.992	1.000	1.000	0.704	1.000	1.000
	1.5	50	0.977	0.993	1.000	0.968	1.000	1.000	0.956	1.000	1.000	0.904	1.000	1.000	0.819	1.000	1.000
		150	1.000	0.995	1.000	1.000	0.999	1.000	0.998	1.000	1.000	0.951	1.000	1.000	0.678	1.000	1.000
		300	1.000	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.891	1.000	1.000	0.331	1.000	1.000

Table B.13: Case A. Empirical power with two structural breaks, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.3$

γ^*	$\bar{c}$	T	$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
			S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.001	0.140	0.105	0.000	0.146	0.103	0.000	0.155	0.119	0.000	0.107	0.076	0.000	0.067	0.045
		150	0.000	0.093	0.075	0.000	0.093	0.072	0.001	0.054	0.043	0.000	0.021	0.008	0.000	0.038	0.020
		300	0.000	0.073	0.056	0.000	0.075	0.053	0.000	0.033	0.021	0.000	0.023	0.009	0.000	0.031	0.012
	0.5	50	0.025	0.235	0.200	0.011	0.192	0.151	0.010	0.200	0.162	0.000	0.091	0.061	0.000	0.073	0.049
		150	0.010	0.138	0.111	0.003	0.076	0.055	0.000	0.044	0.031	0.000	0.018	0.008	0.000	0.037	0.019
		300	0.021	0.115	0.089	0.000	0.041	0.028	0.000	0.029	0.015	0.000	0.023	0.008	0.000	0.031	0.012
	0.8	50	0.028	0.178	0.159	0.019	0.170	0.137	0.009	0.142	0.117	0.000	0.089	0.059	0.000	0.074	0.050
		150	0.040	0.124	0.113	0.005	0.064	0.042	0.000	0.039	0.027	0.000	0.018	0.008	0.000	0.037	0.019
		300	0.034	0.086	0.078	0.000	0.042	0.029	0.000	0.029	0.016	0.000	0.023	0.008	0.000	0.031	0.012
	1.0	50	0.019	0.202	0.170	0.008	0.164	0.136	0.003	0.135	0.112	0.000	0.089	0.060	0.000	0.074	0.050
		150	0.041	0.120	0.119	0.002	0.064	0.043	0.000	0.039	0.027	0.000	0.018	0.008	0.000	0.037	0.019
		300	0.052	0.087	0.090	0.000	0.042	0.029	0.000	0.029	0.016	0.000	0.023	0.008	0.000	0.031	0.012
	1.5	50	0.006	0.169	0.137	0.002	0.152	0.122	0.001	0.136	0.110	0.000	0.089	0.062	0.000	0.074	0.051
		150	0.024	0.103	0.093	0.000	0.064	0.042	0.000	0.039	0.027	0.000	0.018	0.008	0.000	0.037	0.019
		300	0.028	0.079	0.078	0.000	0.042	0.029	0.000	0.029	0.016	0.000	0.023	0.008	0.000	0.031	0.012
5	0.3	50	0.003	0.160	0.132	0.002	0.148	0.118	0.001	0.180	0.135	0.002	0.128	0.096	0.000	0.117	0.086
		150	0.000	0.093	0.074	0.000	0.089	0.067	0.001	0.061	0.048	0.000	0.028	0.015	0.000	0.064	0.031
		300	0.000	0.073	0.057	0.000	0.071	0.054	0.000	0.032	0.023	0.000	0.026	0.009	0.000	0.043	0.021
	0.5	50	0.021	0.219	0.180	0.013	0.190	0.163	0.009	0.197	0.156	0.000	0.115	0.082	0.000	0.099	0.071
		150	0.017	0.134	0.105	0.004	0.074	0.055	0.000	0.048	0.033	0.000	0.026	0.014	0.000	0.065	0.032
		300	0.024	0.117	0.094	0.000	0.042	0.029	0.000	0.028	0.016	0.000	0.026	0.009	0.000	0.043	0.021
	0.8	50	0.029	0.174	0.155	0.017	0.173	0.134	0.012	0.150	0.122	0.000	0.112	0.077	0.000	0.103	0.076
		150	0.041	0.125	0.105	0.002	0.064	0.047	0.000	0.043	0.028	0.000	0.026	0.015	0.000	0.065	0.032
		300	0.036	0.087	0.082	0.000	0.045	0.030	0.000	0.028	0.015	0.000	0.026	0.009	0.000	0.043	0.021
	1.0	50	0.023	0.198	0.164	0.015	0.162	0.123	0.003	0.149	0.117	0.000	0.112	0.077	0.000	0.103	0.076
		150	0.043	0.122	0.115	0.001	0.065	0.047	0.000	0.043	0.028	0.000	0.026	0.015	0.000	0.065	0.032
		300	0.054	0.087	0.089	0.000	0.045	0.030	0.000	0.028	0.015	0.000	0.026	0.009	0.000	0.043	0.022
	1.5	50	0.006	0.176	0.144	0.002	0.157	0.114	0.001	0.149	0.115	0.000	0.112	0.080	0.000	0.103	0.076
		150	0.023	0.103	0.086	0.000	0.065	0.046	0.000	0.043	0.028	0.000	0.026	0.015	0.000	0.065	0.032
		300	0.028	0.080	0.076	0.000	0.045	0.030	0.000	0.028	0.015	0.000	0.026	0.009	0.000	0.043	0.022
10	0.3	50	0.017	0.225	0.167	0.012	0.233	0.190	0.010	0.230	0.178	0.003	0.228	0.179	0.001	0.230	0.176
		150	0.000	0.098	0.083	0.000	0.089	0.069	0.001	0.074	0.054	0.000	0.068	0.046	0.000	0.130	0.087
		300	0.000	0.075	0.058	0.000	0.070	0.054	0.000	0.036	0.024	0.000	0.042	0.022	0.000	0.128	0.070
	0.5	50	0.041	0.217	0.184	0.020	0.218	0.172	0.022	0.204	0.168	0.004	0.162	0.126	0.000	0.188	0.150
		150	0.021	0.125	0.113	0.008	0.071	0.058	0.000	0.056	0.043	0.000	0.071	0.045	0.000	0.130	0.087
		300	0.027	0.115	0.098	0.000	0.044	0.028	0.000	0.030	0.019	0.000	0.041	0.022	0.000	0.128	0.070
	0.8	50	0.035	0.183	0.156	0.025	0.181	0.150	0.024	0.156	0.132	0.000	0.161	0.128	0.000	0.189	0.151
		150	0.043	0.113	0.108	0.002	0.065	0.053	0.000	0.051	0.037	0.000	0.071	0.045	0.000	0.130	0.090
		300	0.038	0.088	0.080	0.000	0.044	0.029	0.000	0.029	0.018	0.000	0.041	0.023	0.000	0.128	0.071
	1.0	50	0.035	0.192	0.162	0.025	0.168	0.129	0.010	0.153	0.120	0.000	0.161	0.128	0.000	0.189	0.152
		150	0.042	0.117	0.110	0.002	0.065	0.052	0.000	0.051	0.037	0.000	0.071	0.046	0.000	0.130	0.091
		300	0.057	0.086	0.089	0.000	0.044	0.029	0.000	0.029	0.018	0.000	0.041	0.023	0.000	0.128	0.071
	1.5	50	0.009	0.185	0.158	0.005	0.158	0.119	0.001	0.150	0.117	0.000	0.161	0.130	0.000	0.188	0.153
		150	0.024	0.100	0.085	0.001	0.065	0.051	0.000	0.051	0.037	0.000	0.071	0.046	0.000	0.130	0.091
		300	0.027	0.078	0.073	0.000	0.044	0.030	0.000	0.029	0.019	0.000	0.041	0.023	0.000	0.128	0.073

Table B.14: Case A. Empirical power with two structural breaks, $\gamma = \gamma^*$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.005	0.171	0.135	0.008	0.180	0.134	0.005	0.189	0.148	0.004	0.162	0.115	0.000	0.149	0.103
		150	0.000	0.102	0.087	0.000	0.093	0.073	0.001	0.071	0.055	0.000	0.077	0.054	0.000	0.177	0.113
		300	0.000	0.070	0.059	0.000	0.069	0.057	0.000	0.043	0.032	0.000	0.100	0.059	0.000	0.351	0.246
	0.5	50	0.030	0.218	0.179	0.017	0.201	0.162	0.013	0.202	0.153	0.002	0.134	0.091	0.000	0.133	0.094
		150	0.023	0.119	0.108	0.009	0.076	0.061	0.000	0.062	0.043	0.000	0.079	0.054	0.000	0.177	0.113
		300	0.032	0.112	0.095	0.000	0.044	0.030	0.000	0.035	0.027	0.000	0.100	0.058	0.000	0.351	0.246
	0.8	50	0.033	0.183	0.161	0.019	0.184	0.144	0.016	0.153	0.122	0.000	0.131	0.086	0.000	0.137	0.099
		150	0.045	0.115	0.106	0.003	0.067	0.053	0.000	0.056	0.038	0.000	0.079	0.054	0.000	0.177	0.114
		300	0.041	0.084	0.081	0.000	0.043	0.031	0.000	0.035	0.027	0.000	0.100	0.058	0.000	0.351	0.249
	1.0	50	0.027	0.190	0.161	0.018	0.176	0.128	0.004	0.153	0.119	0.000	0.131	0.087	0.000	0.137	0.099
		150	0.046	0.119	0.110	0.004	0.067	0.052	0.000	0.056	0.039	0.000	0.079	0.055	0.000	0.177	0.114
		300	0.055	0.086	0.091	0.000	0.043	0.031	0.000	0.035	0.027	0.000	0.100	0.059	0.000	0.351	0.249
	1.5	50	0.006	0.181	0.151	0.003	0.161	0.120	0.001	0.152	0.119	0.000	0.131	0.088	0.000	0.138	0.100
		150	0.025	0.099	0.092	0.001	0.067	0.051	0.000	0.056	0.039	0.000	0.079	0.055	0.000	0.177	0.114
		300	0.027	0.076	0.073	0.000	0.043	0.031	0.000	0.035	0.027	0.000	0.100	0.058	0.000	0.351	0.250
5	0.3	50	0.874	0.942	0.946	0.878	0.954	0.951	0.882	0.960	0.951	0.861	0.979	0.974	0.873	0.989	0.988
		150	0.370	0.568	0.547	0.357	0.669	0.646	0.327	0.747	0.700	0.079	0.983	0.978	0.104	0.976	0.974
		300	0.208	0.301	0.287	0.140	0.443	0.376	0.045	0.711	0.635	0.000	0.978	0.975	0.000	0.965	0.961
	0.5	50	0.626	0.684	0.717	0.615	0.716	0.730	0.606	0.732	0.739	0.530	0.886	0.864	0.482	0.967	0.965
		150	0.230	0.316	0.336	0.201	0.362	0.353	0.091	0.548	0.488	0.011	0.981	0.976	0.002	0.977	0.974
		300	0.132	0.181	0.186	0.024	0.297	0.227	0.000	0.650	0.559	0.000	0.977	0.974	0.000	0.965	0.961
	0.8	50	0.475	0.544	0.568	0.443	0.581	0.608	0.443	0.625	0.646	0.351	0.879	0.841	0.286	0.966	0.966
		150	0.200	0.241	0.258	0.099	0.320	0.295	0.029	0.541	0.475	0.000	0.981	0.975	0.000	0.977	0.975
		300	0.099	0.123	0.135	0.001	0.292	0.218	0.000	0.649	0.553	0.000	0.977	0.974	0.000	0.965	0.961
	1.0	50	0.393	0.494	0.512	0.370	0.552	0.565	0.347	0.615	0.622	0.224	0.877	0.840	0.137	0.966	0.966
		150	0.187	0.202	0.218	0.085	0.318	0.287	0.024	0.541	0.477	0.000	0.981	0.976	0.000	0.977	0.975
		300	0.107	0.105	0.119	0.000	0.292	0.218	0.000	0.649	0.552	0.000	0.977	0.974	0.000	0.965	0.961
	1.5	50	0.193	0.437	0.421	0.154	0.544	0.524	0.128	0.614	0.591	0.024	0.877	0.841	0.003	0.966	0.965
		150	0.099	0.167	0.165	0.026	0.318	0.278	0.002	0.541	0.477	0.000	0.981	0.976	0.000	0.977	0.975
		300	0.047	0.080	0.077	0.000	0.292	0.219	0.000	0.649	0.557	0.000	0.977	0.974	0.000	0.965	0.961
10	0.3	50	1.000	1.000	1.000	0.998	1.000	1.000	0.996	1.000	1.000	0.996	1.000	1.000	0.993	1.000	1.000
		150	0.998	0.977	0.996	1.000	0.989	0.998	1.000	0.997	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		300	0.936	0.836	0.897	0.939	0.924	0.937	0.959	0.996	0.995	0.995	1.000	1.000	0.995	1.000	1.000
	0.5	50	0.991	0.950	1.000	0.992	0.962	1.000	0.994	0.973	1.000	0.988	0.999	1.000	0.973	1.000	1.000
		150	0.936	0.698	0.897	0.934	0.826	0.934	0.941	0.950	0.968	0.988	1.000	1.000	0.924	1.000	1.000
		300	0.600	0.473	0.587	0.491	0.767	0.757	0.305	0.992	0.982	0.258	1.000	1.000	0.465	1.000	1.000
	0.8	50	0.976	0.880	0.993	0.978	0.898	0.995	0.974	0.939	0.997	0.952	0.999	1.000	0.873	1.000	1.000
		150	0.803	0.564	0.771	0.771	0.773	0.846	0.774	0.946	0.949	0.806	1.000	1.000	0.612	1.000	1.000
		300	0.408	0.359	0.428	0.180	0.756	0.716	0.061	0.991	0.981	0.023	1.000	1.000	0.001	1.000	1.000
	1.0	50	0.942	0.843	0.985	0.939	0.875	0.984	0.932	0.931	0.992	0.885	0.999	1.000	0.779	1.000	1.000
		150	0.766	0.528	0.728	0.755	0.769	0.828	0.718	0.946	0.947	0.685	1.000	1.000	0.395	1.000	1.000
		300	0.399	0.306	0.365	0.155	0.756	0.714	0.042	0.991	0.981	0.000	1.000	1.000	0.000	1.000	1.000
	1.5	50	0.704	0.787	0.929	0.688	0.869	0.956	0.665	0.928	0.977	0.540	0.999	1.000	0.388	1.000	1.000
		150	0.502	0.437	0.550	0.437	0.769	0.771	0.318	0.946	0.934	0.050	1.000	1.000	0.000	1.000	1.000
		300	0.243	0.245	0.271	0.043	0.756	0.710	0.002	0.991	0.981	0.000	1.000	1.000	0.000	1.000	1.000

Table B.15: Case A. Empirical power with three structural breaks, $\gamma = \gamma^* T^{1/2}$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.995	0.952	0.975	0.994	0.974	0.992	0.997	0.980	0.990	0.995	0.986	0.996	0.993	1.000	0.997
		150	1.000	0.977	0.997	1.000	0.990	0.998	1.000	0.996	0.998	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.978	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
	0.5	50	0.988	0.777	0.931	0.991	0.819	0.957	0.986	0.816	0.946	0.981	0.922	0.962	0.968	0.993	0.990
		150	1.000	0.769	0.978	1.000	0.850	0.992	1.000	0.961	0.995	0.995	1.000	1.000	0.944	1.000	1.000
		300	1.000	0.734	0.985	1.000	0.937	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.828	1.000	1.000
	0.8	50	0.946	0.629	0.852	0.936	0.676	0.865	0.927	0.699	0.881	0.900	0.913	0.937	0.842	0.991	0.990
		150	0.999	0.624	0.924	1.000	0.790	0.966	0.999	0.958	0.992	0.955	1.000	1.000	0.720	1.000	1.000
		300	1.000	0.601	0.937	1.000	0.929	0.990	1.000	1.000	1.000	0.919	1.000	1.000	0.372	1.000	1.000
	1.0	50	0.882	0.576	0.793	0.867	0.643	0.822	0.860	0.696	0.847	0.809	0.912	0.933	0.717	0.991	0.990
		150	0.996	0.556	0.868	0.998	0.787	0.939	0.990	0.958	0.989	0.898	1.000	1.000	0.540	1.000	1.000
		300	1.000	0.547	0.877	1.000	0.929	0.981	1.000	1.000	1.000	0.815	1.000	1.000	0.189	1.000	1.000
	1.5	50	0.591	0.530	0.660	0.558	0.632	0.716	0.546	0.694	0.764	0.430	0.912	0.909	0.296	0.991	0.988
		150	0.883	0.503	0.771	0.862	0.787	0.886	0.823	0.958	0.977	0.540	1.000	1.000	0.164	1.000	1.000
		300	0.973	0.475	0.772	0.953	0.929	0.965	0.908	1.000	1.000	0.392	1.000	1.000	0.025	1.000	1.000
5	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	0.999	0.998	1.000	0.999	0.999	1.000	0.998
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.998	1.000	0.999	0.999	1.000	0.998	0.999	1.000	0.999	0.996	1.000	1.000	0.995	1.000	1.000
		150	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
		300	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000
	0.8	50	0.994	0.998	0.999	0.996	0.999	1.000	0.996	0.999	1.000	0.985	1.000	1.000	0.961	1.000	1.000
		150	1.000	0.995	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.992	1.000	1.000	0.915	1.000	1.000
		300	1.000	0.994	0.994	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	0.747	1.000	1.000
	1.0	50	0.986	0.995	0.999	0.991	0.996	0.999	0.992	0.999	1.000	0.966	1.000	1.000	0.912	1.000	1.000
		150	1.000	0.995	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.978	1.000	1.000	0.818	1.000	1.000
		300	1.000	0.987	0.992	1.000	1.000	1.000	1.000	1.000	1.000	0.962	1.000	1.000	0.549	1.000	1.000
	1.5	50	0.929	0.988	0.997	0.920	0.997	0.999	0.903	0.999	1.000	0.839	1.000	1.000	0.724	1.000	1.000
		150	0.998	0.987	0.993	0.997	1.000	1.000	0.991	1.000	1.000	0.891	1.000	1.000	0.532	1.000	1.000
		300	1.000	0.978	0.990	1.000	1.000	1.000	0.999	1.000	1.000	0.811	1.000	1.000	0.178	1.000	1.000
10	0.3	50	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	0.999	0.998	1.000	0.999	0.999	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	1.000	1.000	0.999	1.000	1.000	0.998	0.999	1.000	0.999	0.997	1.000	1.000	0.996	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	0.998	1.000	1.000	0.999	0.999	1.000	0.999	1.000	1.000	0.992	1.000	1.000	0.978	1.000	1.000
		150	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000	0.966	1.000	1.000
		300	1.000	0.999	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.895	1.000	1.000
	1.0	50	0.992	0.998	0.999	0.996	1.000	1.000	0.996	1.000	1.000	0.982	1.000	1.000	0.949	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.990	1.000	1.000	0.902	1.000	1.000
		300	1.000	0.996	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	0.700	1.000	1.000
	1.5	50	0.970	0.997	1.000	0.964	1.000	1.000	0.962	1.000	1.000	0.917	1.000	1.000	0.826	1.000	1.000
		150	1.000	0.997	0.999	1.000	1.000	1.000	0.997	1.000	1.000	0.944	1.000	1.000	0.670	1.000	1.000
		300	1.000	0.995	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.893	1.000	1.000	0.317	1.000	1.000

Table B.16: Case A. Empirical power with three structural breaks, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.018	0.165	0.120	0.011	0.162	0.130	0.013	0.168	0.125	0.005	0.126	0.089	0.001	0.097	0.062
		150	0.003	0.084	0.067	0.000	0.074	0.053	0.001	0.051	0.035	0.000	0.026	0.015	0.000	0.037	0.019
		300	0.000	0.057	0.046	0.000	0.035	0.027	0.000	0.025	0.019	0.000	0.020	0.013	0.000	0.033	0.016
	0.5	50	0.040	0.186	0.144	0.026	0.172	0.127	0.022	0.166	0.126	0.004	0.110	0.080	0.001	0.090	0.063
		150	0.026	0.092	0.083	0.014	0.057	0.043	0.001	0.038	0.034	0.000	0.028	0.018	0.000	0.039	0.020
		300	0.040	0.061	0.058	0.000	0.023	0.015	0.000	0.023	0.019	0.000	0.020	0.013	0.000	0.033	0.015
	0.8	50	0.021	0.138	0.115	0.015	0.150	0.118	0.015	0.141	0.103	0.002	0.104	0.073	0.000	0.095	0.066
		150	0.038	0.068	0.074	0.015	0.047	0.038	0.003	0.036	0.032	0.000	0.028	0.018	0.000	0.039	0.020
		300	0.053	0.050	0.057	0.001	0.022	0.015	0.000	0.023	0.020	0.000	0.020	0.014	0.000	0.033	0.017
	1.0	50	0.006	0.142	0.112	0.007	0.135	0.105	0.007	0.134	0.100	0.000	0.104	0.073	0.000	0.095	0.066
		150	0.040	0.055	0.058	0.013	0.047	0.039	0.001	0.036	0.031	0.000	0.028	0.018	0.000	0.039	0.020
		300	0.073	0.041	0.069	0.001	0.022	0.016	0.000	0.023	0.020	0.000	0.020	0.014	0.000	0.033	0.017
	1.5	50	0.002	0.137	0.107	0.001	0.129	0.099	0.001	0.134	0.101	0.000	0.104	0.074	0.000	0.096	0.066
		150	0.012	0.042	0.036	0.001	0.047	0.035	0.000	0.036	0.031	0.000	0.028	0.018	0.000	0.039	0.020
		300	0.022	0.034	0.035	0.000	0.022	0.015	0.000	0.023	0.020	0.000	0.020	0.014	0.000	0.033	0.017
5	0.3	50	0.026	0.158	0.123	0.015	0.174	0.131	0.018	0.184	0.142	0.008	0.135	0.097	0.005	0.085	0.064
		150	0.003	0.084	0.066	0.000	0.073	0.056	0.001	0.055	0.038	0.000	0.031	0.019	0.000	0.045	0.026
		300	0.000	0.061	0.043	0.000	0.035	0.026	0.000	0.024	0.020	0.000	0.021	0.011	0.000	0.036	0.020
	0.5	50	0.044	0.193	0.153	0.032	0.178	0.127	0.024	0.144	0.117	0.008	0.115	0.082	0.002	0.097	0.076
		150	0.028	0.090	0.080	0.011	0.059	0.046	0.002	0.041	0.034	0.000	0.034	0.018	0.000	0.047	0.031
		300	0.042	0.062	0.059	0.000	0.022	0.014	0.000	0.021	0.018	0.000	0.021	0.011	0.000	0.036	0.020
	0.8	50	0.022	0.135	0.105	0.015	0.140	0.111	0.015	0.127	0.096	0.003	0.115	0.090	0.000	0.101	0.076
		150	0.042	0.068	0.072	0.014	0.048	0.039	0.002	0.038	0.032	0.000	0.034	0.019	0.000	0.047	0.030
		300	0.050	0.049	0.062	0.001	0.021	0.014	0.000	0.021	0.018	0.000	0.021	0.011	0.000	0.036	0.020
	1.0	50	0.010	0.128	0.106	0.009	0.134	0.109	0.011	0.119	0.090	0.000	0.115	0.090	0.000	0.101	0.077
		150	0.050	0.052	0.057	0.013	0.048	0.040	0.001	0.038	0.031	0.000	0.034	0.019	0.000	0.047	0.030
		300	0.075	0.040	0.069	0.001	0.021	0.015	0.000	0.021	0.018	0.000	0.021	0.011	0.000	0.036	0.020
	1.5	50	0.002	0.133	0.106	0.001	0.127	0.095	0.001	0.119	0.089	0.000	0.115	0.090	0.000	0.101	0.078
		150	0.010	0.042	0.033	0.001	0.048	0.037	0.000	0.038	0.031	0.000	0.034	0.019	0.000	0.047	0.030
		300	0.021	0.033	0.035	0.000	0.021	0.014	0.000	0.021	0.018	0.000	0.021	0.011	0.000	0.036	0.020
10	0.3	50	0.041	0.196	0.147	0.036	0.198	0.154	0.050	0.209	0.161	0.022	0.167	0.115	0.014	0.142	0.104
		150	0.006	0.086	0.067	0.000	0.080	0.055	0.001	0.052	0.036	0.000	0.035	0.027	0.000	0.074	0.042
		300	0.001	0.057	0.045	0.000	0.031	0.027	0.000	0.026	0.019	0.000	0.027	0.015	0.000	0.059	0.038
	0.5	50	0.067	0.199	0.166	0.052	0.177	0.137	0.043	0.157	0.134	0.020	0.122	0.089	0.008	0.129	0.093
		150	0.026	0.082	0.069	0.009	0.055	0.040	0.002	0.044	0.030	0.000	0.039	0.026	0.000	0.076	0.045
		300	0.043	0.061	0.063	0.000	0.023	0.015	0.000	0.025	0.017	0.000	0.026	0.015	0.000	0.059	0.038
	0.8	50	0.041	0.141	0.112	0.042	0.147	0.122	0.028	0.136	0.103	0.007	0.121	0.087	0.002	0.129	0.094
		150	0.046	0.071	0.077	0.013	0.045	0.033	0.001	0.039	0.022	0.000	0.039	0.026	0.000	0.076	0.045
		300	0.055	0.051	0.061	0.001	0.021	0.015	0.000	0.025	0.017	0.000	0.026	0.015	0.000	0.059	0.038
	1.0	50	0.023	0.132	0.104	0.015	0.146	0.106	0.012	0.133	0.097	0.004	0.121	0.088	0.000	0.129	0.093
		150	0.052	0.055	0.056	0.015	0.047	0.036	0.002	0.039	0.022	0.000	0.039	0.027	0.000	0.076	0.045
		300	0.079	0.044	0.064	0.001	0.021	0.016	0.000	0.025	0.017	0.000	0.026	0.015	0.000	0.059	0.038
	1.5	50	0.004	0.138	0.119	0.003	0.138	0.102	0.001	0.133	0.097	0.000	0.121	0.087	0.000	0.130	0.093
		150	0.014	0.046	0.036	0.001	0.047	0.033	0.000	0.039	0.022	0.000	0.039	0.028	0.000	0.076	0.045
		300	0.022	0.033	0.033	0.000	0.021	0.015	0.000	0.025	0.017	0.000	0.026	0.015	0.000	0.058	0.038

Table B.17: Case A. Empirical power with three structural breaks, $\gamma = \gamma^*$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.035	0.162	0.119	0.024	0.188	0.135	0.036	0.188	0.142	0.008	0.135	0.096	0.004	0.110	0.063
		150	0.006	0.088	0.068	0.000	0.082	0.061	0.001	0.058	0.039	0.000	0.044	0.030	0.000	0.090	0.058
		300	0.004	0.058	0.044	0.001	0.030	0.026	0.000	0.026	0.020	0.000	0.042	0.025	0.000	0.145	0.090
	0.5	50	0.047	0.186	0.152	0.041	0.168	0.119	0.031	0.137	0.116	0.012	0.116	0.085	0.004	0.097	0.073
		150	0.030	0.084	0.068	0.010	0.053	0.040	0.002	0.044	0.027	0.000	0.046	0.032	0.000	0.092	0.062
		300	0.043	0.055	0.059	0.000	0.024	0.018	0.000	0.025	0.018	0.000	0.042	0.025	0.000	0.145	0.089
	0.8	50	0.028	0.136	0.110	0.024	0.147	0.111	0.014	0.120	0.089	0.006	0.119	0.088	0.000	0.098	0.072
		150	0.051	0.070	0.078	0.017	0.045	0.036	0.002	0.037	0.022	0.000	0.046	0.032	0.000	0.092	0.063
		300	0.054	0.046	0.063	0.001	0.022	0.018	0.000	0.025	0.018	0.000	0.042	0.025	0.000	0.145	0.092
	1.0	50	0.009	0.132	0.105	0.009	0.133	0.102	0.011	0.118	0.090	0.002	0.118	0.088	0.000	0.098	0.072
		150	0.055	0.056	0.059	0.016	0.047	0.037	0.002	0.037	0.024	0.000	0.046	0.033	0.000	0.092	0.063
		300	0.080	0.045	0.065	0.001	0.022	0.019	0.000	0.025	0.018	0.000	0.042	0.025	0.000	0.145	0.092
	1.5	50	0.002	0.130	0.110	0.001	0.128	0.090	0.001	0.118	0.087	0.000	0.118	0.087	0.000	0.098	0.073
		150	0.015	0.050	0.041	0.001	0.047	0.035	0.000	0.037	0.024	0.000	0.046	0.033	0.000	0.092	0.063
		300	0.022	0.035	0.032	0.000	0.022	0.018	0.000	0.025	0.018	0.000	0.042	0.026	0.000	0.145	0.093
5	0.3	50	0.953	0.782	0.880	0.951	0.857	0.911	0.957	0.870	0.925	0.934	0.906	0.919	0.928	0.959	0.945
		150	0.546	0.403	0.462	0.483	0.479	0.503	0.491	0.504	0.506	0.344	0.962	0.947	0.381	0.990	0.985
		300	0.316	0.178	0.182	0.251	0.222	0.212	0.131	0.415	0.350	0.020	0.990	0.981	0.138	0.994	0.993
	0.5	50	0.819	0.559	0.711	0.815	0.590	0.728	0.775	0.586	0.696	0.726	0.736	0.763	0.712	0.914	0.888
		150	0.328	0.230	0.302	0.286	0.234	0.293	0.222	0.350	0.340	0.111	0.958	0.936	0.231	0.990	0.986
		300	0.187	0.089	0.148	0.075	0.133	0.113	0.007	0.366	0.295	0.002	0.990	0.981	0.000	0.994	0.994
	0.8	50	0.658	0.411	0.546	0.656	0.445	0.584	0.666	0.476	0.600	0.622	0.721	0.727	0.577	0.911	0.878
		150	0.307	0.132	0.210	0.236	0.199	0.241	0.185	0.342	0.325	0.070	0.958	0.937	0.019	0.990	0.987
		300	0.167	0.057	0.117	0.042	0.127	0.110	0.004	0.366	0.293	0.000	0.990	0.981	0.000	0.994	0.994
	1.0	50	0.591	0.361	0.477	0.578	0.412	0.512	0.591	0.472	0.568	0.529	0.721	0.713	0.430	0.909	0.879
		150	0.306	0.117	0.191	0.258	0.198	0.228	0.187	0.342	0.317	0.035	0.958	0.938	0.003	0.990	0.987
		300	0.199	0.046	0.092	0.059	0.127	0.108	0.006	0.366	0.294	0.000	0.990	0.981	0.000	0.994	0.994
	1.5	50	0.274	0.320	0.328	0.271	0.403	0.404	0.246	0.472	0.450	0.159	0.724	0.670	0.089	0.910	0.866
		150	0.147	0.095	0.107	0.096	0.198	0.175	0.046	0.342	0.297	0.000	0.958	0.938	0.000	0.990	0.987
		300	0.101	0.037	0.036	0.015	0.127	0.098	0.001	0.366	0.300	0.000	0.990	0.981	0.000	0.994	0.994
10	0.3	50	0.997	0.997	0.995	0.998	0.998	0.998	0.997	0.998	0.998	0.996	1.000	0.999	0.995	1.000	0.997
		150	1.000	0.922	0.993	1.000	0.952	0.998	1.000	0.982	0.997	1.000	1.000	1.000	1.000	1.000	1.000
		300	0.990	0.659	0.898	0.993	0.791	0.925	0.995	0.971	0.984	1.000	1.000	1.000	0.996	1.000	1.000
	0.5	50	0.994	0.917	0.971	0.993	0.939	0.979	0.995	0.941	0.984	0.991	0.992	0.995	0.979	1.000	1.000
		150	1.000	0.654	0.960	0.999	0.718	0.975	0.999	0.882	0.982	0.995	1.000	1.000	0.925	1.000	1.000
		300	0.847	0.367	0.735	0.820	0.599	0.758	0.772	0.944	0.934	0.823	1.000	1.000	0.676	1.000	1.000
	0.8	50	0.980	0.846	0.941	0.979	0.850	0.950	0.985	0.870	0.959	0.953	0.989	0.994	0.886	1.000	1.000
		150	0.983	0.473	0.862	0.979	0.645	0.910	0.976	0.875	0.948	0.948	1.000	1.000	0.678	1.000	1.000
		300	0.754	0.241	0.573	0.664	0.585	0.688	0.592	0.943	0.929	0.586	1.000	1.000	0.216	1.000	1.000
	1.0	50	0.943	0.802	0.910	0.947	0.819	0.925	0.947	0.865	0.954	0.896	0.989	0.993	0.787	1.000	1.000
		150	0.964	0.422	0.787	0.967	0.642	0.868	0.962	0.875	0.934	0.867	1.000	1.000	0.480	1.000	1.000
		300	0.770	0.208	0.489	0.701	0.585	0.667	0.649	0.943	0.931	0.469	1.000	1.000	0.067	1.000	1.000
	1.5	50	0.733	0.747	0.867	0.713	0.810	0.885	0.680	0.863	0.923	0.571	0.989	0.992	0.421	1.000	1.000
		150	0.771	0.353	0.602	0.748	0.642	0.745	0.686	0.875	0.883	0.396	1.000	1.000	0.097	1.000	1.000
		300	0.532	0.157	0.264	0.391	0.585	0.568	0.234	0.943	0.921	0.005	1.000	1.000	0.000	1.000	1.000

Table B.18: Case A. Empirical power with three structural breaks, $\gamma = \gamma^* T^{1/2}$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	1.000	0.048	0.589	1.000	0.062	0.607	1.000	0.062	0.615	1.000	0.046	0.563	1.000	0.043	0.568
		150	1.000	0.023	0.800	1.000	0.025	0.788	1.000	0.012	0.798	1.000	0.004	0.729	1.000	0.007	0.654
		300	1.000	0.018	0.876	1.000	0.012	0.862	1.000	0.002	0.846	1.000	0.000	0.759	1.000	0.001	0.680
	0.5	50	0.994	0.124	0.465	0.994	0.131	0.470	0.994	0.123	0.482	0.998	0.079	0.425	1.000	0.048	0.410
		150	1.000	0.106	0.620	1.000	0.072	0.546	1.000	0.036	0.480	1.000	0.007	0.365	1.000	0.007	0.348
		300	1.000	0.077	0.644	1.000	0.019	0.536	1.000	0.003	0.435	1.000	0.001	0.365	1.000	0.001	0.365
	0.8	50	0.739	0.145	0.333	0.742	0.156	0.332	0.733	0.149	0.323	0.603	0.079	0.172	0.457	0.048	0.095
		150	0.987	0.105	0.580	0.995	0.088	0.476	0.996	0.038	0.399	1.000	0.007	0.329	1.000	0.007	0.319
		300	0.998	0.094	0.598	1.000	0.019	0.440	1.000	0.003	0.371	1.000	0.000	0.349	1.000	0.001	0.347
	1.0	50	0.538	0.146	0.282	0.540	0.165	0.280	0.496	0.150	0.256	0.301	0.079	0.113	0.183	0.047	0.062
		150	0.898	0.112	0.517	0.932	0.088	0.407	0.926	0.038	0.329	0.885	0.007	0.156	0.881	0.007	0.093
		300	0.961	0.105	0.559	0.983	0.019	0.397	0.988	0.003	0.323	0.998	0.000	0.283	1.000	0.001	0.254
	1.5	50	0.348	0.176	0.257	0.322	0.172	0.225	0.301	0.150	0.192	0.181	0.079	0.093	0.161	0.046	0.061
		150	0.645	0.137	0.380	0.559	0.088	0.234	0.410	0.038	0.135	0.177	0.007	0.034	0.171	0.007	0.032
		300	0.706	0.094	0.390	0.566	0.019	0.153	0.336	0.003	0.045	0.110	0.000	0.009	0.163	0.001	0.024
5	0.3	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.620	1.000	0.000	0.634	1.000	0.000	0.628	1.000	0.000	0.669	1.000	0.000	0.664
		300	1.000	0.000	0.489	1.000	0.000	0.516	1.000	0.000	0.505	1.000	0.000	0.515	1.000	0.000	0.511
	0.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.228	1.000	0.000	0.192	1.000	0.000	0.197	1.000	0.000	0.206	1.000	0.000	0.217
		300	1.000	0.000	0.487	1.000	0.000	0.515	1.000	0.000	0.507	1.000	0.000	0.515	1.000	0.000	0.511
	0.8	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.045	1.000	0.000	0.051	1.000	0.000	0.053	1.000	0.000	0.055	1.000	0.000	0.060
		300	1.000	0.000	0.520	1.000	0.000	0.517	1.000	0.000	0.507	1.000	0.000	0.515	1.000	0.000	0.511
	1.0	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		300	1.000	0.000	0.426	1.000	0.000	0.398	1.000	0.000	0.392	1.000	0.000	0.387	1.000	0.000	0.381
	1.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		300	1.000	0.000	0.192	1.000	0.000	0.184	1.000	0.000	0.187	1.000	0.000	0.185	1.000	0.000	0.183
10	0.3	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.668	1.000	0.000	0.687	1.000	0.000	0.666	1.000	0.000	0.671	1.000	0.000	0.664
		300	1.000	0.000	0.489	1.000	0.000	0.516	1.000	0.000	0.505	1.000	0.000	0.515	1.000	0.000	0.511
	0.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.656	1.000	0.000	0.663	1.000	0.000	0.676	1.000	0.000	0.669	1.000	0.000	0.664
		300	1.000	0.000	0.487	1.000	0.000	0.515	1.000	0.000	0.507	1.000	0.000	0.515	1.000	0.000	0.511
	0.8	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.655	1.000	0.000	0.661	1.000	0.000	0.673	1.000	0.000	0.669	1.000	0.000	0.664
		300	1.000	0.000	0.520	1.000	0.000	0.517	1.000	0.000	0.507	1.000	0.000	0.515	1.000	0.000	0.511
	1.0	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.653	1.000	0.000	0.661	1.000	0.000	0.673	1.000	0.000	0.669	1.000	0.000	0.664
		300	1.000	0.000	0.528	1.000	0.000	0.517	1.000	0.000	0.507	1.000	0.000	0.515	1.000	0.000	0.511
	1.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.634	1.000	0.000	0.638	1.000	0.000	0.650	1.000	0.000	0.647	1.000	0.000	0.642
		300	1.000	0.000	0.540	1.000	0.000	0.517	1.000	0.000	0.507	1.000	0.000	0.515	1.000	0.000	0.511

Table B.19: Case A. Empirical power with three structural breaks, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.001	0.140	0.106	0.000	0.149	0.107	0.000	0.155	0.121	0.000	0.102	0.074	0.000	0.070	0.044
		150	0.000	0.093	0.076	0.000	0.092	0.071	0.001	0.056	0.042	0.000	0.020	0.009	0.000	0.038	0.019
		300	0.000	0.074	0.055	0.000	0.076	0.051	0.000	0.033	0.021	0.000	0.023	0.009	0.000	0.031	0.012
	0.5	50	0.026	0.236	0.199	0.012	0.197	0.151	0.009	0.197	0.155	0.000	0.085	0.061	0.000	0.074	0.052
		150	0.011	0.138	0.110	0.003	0.076	0.053	0.000	0.045	0.031	0.000	0.017	0.009	0.000	0.037	0.018
		300	0.021	0.115	0.088	0.000	0.041	0.028	0.000	0.028	0.016	0.000	0.023	0.009	0.000	0.031	0.012
	0.8	50	0.031	0.180	0.161	0.019	0.163	0.133	0.011	0.143	0.110	0.000	0.083	0.059	0.000	0.075	0.054
		150	0.040	0.129	0.112	0.004	0.065	0.045	0.000	0.039	0.027	0.000	0.017	0.009	0.000	0.037	0.018
		300	0.037	0.087	0.081	0.000	0.042	0.029	0.000	0.028	0.016	0.000	0.023	0.009	0.000	0.031	0.012
	1.0	50	0.019	0.199	0.165	0.011	0.160	0.132	0.004	0.137	0.106	0.000	0.083	0.061	0.000	0.075	0.054
		150	0.042	0.121	0.117	0.002	0.065	0.044	0.000	0.039	0.027	0.000	0.017	0.009	0.000	0.037	0.018
		300	0.052	0.087	0.090	0.000	0.042	0.029	0.000	0.028	0.016	0.000	0.023	0.009	0.000	0.031	0.012
	1.5	50	0.006	0.178	0.145	0.002	0.149	0.121	0.001	0.138	0.103	0.000	0.083	0.061	0.000	0.076	0.055
		150	0.024	0.107	0.092	0.000	0.065	0.043	0.000	0.039	0.027	0.000	0.017	0.009	0.000	0.037	0.018
		300	0.029	0.080	0.079	0.000	0.042	0.029	0.000	0.028	0.016	0.000	0.023	0.009	0.000	0.031	0.012
5	0.3	50	0.004	0.170	0.140	0.002	0.189	0.134	0.004	0.205	0.161	0.001	0.154	0.121	0.001	0.133	0.102
		150	0.000	0.096	0.077	0.000	0.092	0.076	0.001	0.060	0.045	0.000	0.044	0.022	0.000	0.071	0.042
		300	0.000	0.073	0.057	0.000	0.069	0.055	0.000	0.034	0.022	0.000	0.031	0.013	0.000	0.065	0.029
	0.5	50	0.031	0.222	0.184	0.012	0.204	0.170	0.010	0.203	0.172	0.002	0.124	0.103	0.000	0.115	0.091
		150	0.017	0.131	0.112	0.005	0.073	0.059	0.000	0.052	0.035	0.000	0.044	0.018	0.000	0.070	0.040
		300	0.025	0.112	0.096	0.000	0.044	0.029	0.000	0.032	0.016	0.000	0.031	0.014	0.000	0.065	0.031
	0.8	50	0.033	0.201	0.159	0.027	0.185	0.147	0.015	0.152	0.127	0.000	0.127	0.106	0.000	0.117	0.096
		150	0.042	0.125	0.110	0.004	0.065	0.050	0.000	0.042	0.029	0.000	0.044	0.018	0.000	0.070	0.040
		300	0.038	0.089	0.081	0.000	0.044	0.029	0.000	0.032	0.015	0.000	0.031	0.014	0.000	0.065	0.032
	1.0	50	0.023	0.206	0.175	0.019	0.180	0.147	0.006	0.149	0.123	0.000	0.127	0.106	0.000	0.117	0.096
		150	0.044	0.125	0.115	0.002	0.065	0.048	0.000	0.042	0.029	0.000	0.044	0.019	0.000	0.070	0.040
		300	0.054	0.088	0.088	0.000	0.044	0.029	0.000	0.032	0.015	0.000	0.031	0.014	0.000	0.065	0.032
	1.5	50	0.006	0.195	0.158	0.004	0.168	0.132	0.001	0.149	0.121	0.000	0.127	0.107	0.000	0.117	0.097
		150	0.024	0.104	0.093	0.000	0.065	0.047	0.000	0.042	0.029	0.000	0.044	0.020	0.000	0.070	0.041
		300	0.028	0.080	0.077	0.000	0.044	0.029	0.000	0.032	0.015	0.000	0.031	0.014	0.000	0.065	0.032
10	0.3	50	0.019	0.252	0.214	0.013	0.251	0.207	0.015	0.275	0.230	0.002	0.255	0.197	0.002	0.215	0.162
		150	0.000	0.102	0.082	0.000	0.117	0.086	0.001	0.075	0.060	0.000	0.078	0.052	0.000	0.158	0.107
		300	0.000	0.070	0.059	0.000	0.071	0.055	0.000	0.035	0.029	0.000	0.062	0.035	0.000	0.169	0.104
	0.5	50	0.043	0.238	0.201	0.033	0.236	0.196	0.029	0.240	0.190	0.001	0.197	0.157	0.000	0.192	0.151
		150	0.025	0.136	0.117	0.006	0.084	0.065	0.000	0.068	0.053	0.000	0.082	0.047	0.000	0.159	0.107
		300	0.028	0.110	0.091	0.000	0.040	0.027	0.000	0.030	0.024	0.000	0.062	0.034	0.000	0.169	0.106
	0.8	50	0.054	0.213	0.182	0.042	0.204	0.171	0.034	0.186	0.155	0.001	0.189	0.153	0.000	0.192	0.153
		150	0.047	0.118	0.121	0.002	0.070	0.055	0.000	0.058	0.044	0.000	0.082	0.047	0.000	0.159	0.107
		300	0.040	0.088	0.080	0.000	0.041	0.027	0.000	0.029	0.024	0.000	0.062	0.034	0.000	0.169	0.108
	1.0	50	0.044	0.213	0.180	0.032	0.204	0.173	0.021	0.190	0.150	0.000	0.189	0.155	0.000	0.192	0.154
		150	0.050	0.126	0.126	0.001	0.070	0.055	0.000	0.058	0.044	0.000	0.082	0.048	0.000	0.159	0.107
		300	0.053	0.086	0.084	0.000	0.041	0.027	0.000	0.029	0.024	0.000	0.062	0.034	0.000	0.169	0.108
	1.5	50	0.012	0.210	0.181	0.008	0.192	0.158	0.003	0.189	0.151	0.000	0.189	0.156	0.000	0.192	0.156
		150	0.026	0.104	0.100	0.001	0.071	0.054	0.000	0.058	0.045	0.000	0.082	0.049	0.000	0.159	0.108
		300	0.029	0.082	0.074	0.000	0.041	0.027	0.000	0.029	0.024	0.000	0.062	0.035	0.000	0.169	0.107

Table B.20: Case A. Empirical power with three structural breaks, $\gamma = \gamma^*$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.009	0.204	0.163	0.004	0.208	0.159	0.006	0.235	0.183	0.003	0.194	0.148	0.001	0.172	0.129
		150	0.000	0.112	0.092	0.000	0.122	0.097	0.001	0.089	0.062	0.000	0.094	0.066	0.000	0.200	0.134
		300	0.000	0.084	0.065	0.000	0.077	0.058	0.000	0.051	0.039	0.000	0.133	0.075	0.000	0.411	0.273
	0.5	50	0.038	0.238	0.190	0.023	0.219	0.183	0.018	0.212	0.179	0.001	0.137	0.116	0.000	0.153	0.115
		150	0.028	0.136	0.119	0.007	0.091	0.071	0.000	0.073	0.062	0.000	0.099	0.067	0.000	0.203	0.133
		300	0.030	0.105	0.090	0.000	0.042	0.033	0.000	0.038	0.028	0.000	0.132	0.076	0.000	0.411	0.274
	0.8	50	0.038	0.203	0.161	0.030	0.189	0.158	0.023	0.171	0.139	0.000	0.138	0.116	0.000	0.156	0.116
		150	0.048	0.121	0.120	0.002	0.074	0.058	0.000	0.062	0.051	0.000	0.099	0.068	0.000	0.203	0.133
		300	0.043	0.092	0.083	0.000	0.043	0.032	0.000	0.038	0.028	0.000	0.132	0.076	0.000	0.411	0.276
	1.0	50	0.033	0.213	0.170	0.022	0.185	0.154	0.011	0.169	0.134	0.000	0.138	0.116	0.000	0.157	0.117
		150	0.051	0.125	0.123	0.001	0.074	0.058	0.000	0.062	0.051	0.000	0.099	0.068	0.000	0.203	0.133
		300	0.054	0.086	0.085	0.000	0.043	0.032	0.000	0.038	0.028	0.000	0.132	0.076	0.000	0.411	0.277
	1.5	50	0.009	0.203	0.154	0.006	0.177	0.141	0.002	0.168	0.132	0.000	0.138	0.116	0.000	0.157	0.117
		150	0.026	0.106	0.105	0.001	0.074	0.057	0.000	0.062	0.051	0.000	0.099	0.068	0.000	0.203	0.134
		300	0.031	0.078	0.074	0.000	0.043	0.032	0.000	0.038	0.028	0.000	0.132	0.077	0.000	0.411	0.278
5	0.3	50	0.955	0.111	0.366	0.960	0.125	0.367	0.965	0.133	0.376	0.961	0.106	0.374	0.962	0.089	0.347
		150	0.610	0.200	0.267	0.534	0.204	0.267	0.540	0.196	0.254	0.211	0.169	0.140	0.039	0.169	0.116
		300	0.370	0.212	0.227	0.289	0.211	0.222	0.097	0.191	0.169	0.000	0.378	0.277	0.000	0.365	0.270
	0.5	50	0.592	0.206	0.286	0.545	0.211	0.278	0.537	0.210	0.285	0.394	0.146	0.167	0.221	0.106	0.098
		150	0.346	0.210	0.254	0.258	0.197	0.208	0.132	0.193	0.164	0.003	0.172	0.121	0.000	0.168	0.116
		300	0.214	0.160	0.177	0.029	0.177	0.149	0.000	0.186	0.155	0.000	0.379	0.275	0.000	0.365	0.268
	0.8	50	0.298	0.213	0.237	0.279	0.217	0.242	0.255	0.218	0.224	0.109	0.146	0.126	0.069	0.105	0.086
		150	0.194	0.161	0.189	0.091	0.187	0.169	0.017	0.192	0.153	0.000	0.172	0.121	0.000	0.168	0.116
		300	0.115	0.130	0.143	0.001	0.170	0.140	0.000	0.186	0.151	0.000	0.379	0.273	0.000	0.365	0.268
	1.0	50	0.252	0.206	0.230	0.227	0.230	0.236	0.195	0.217	0.217	0.094	0.146	0.126	0.057	0.104	0.081
		150	0.178	0.150	0.173	0.073	0.185	0.163	0.014	0.192	0.154	0.000	0.172	0.121	0.000	0.168	0.116
		300	0.108	0.127	0.135	0.000	0.170	0.140	0.000	0.186	0.151	0.000	0.379	0.275	0.000	0.365	0.268
	1.5	50	0.199	0.217	0.231	0.170	0.234	0.224	0.141	0.217	0.200	0.037	0.145	0.117	0.005	0.104	0.076
		150	0.132	0.138	0.145	0.031	0.185	0.159	0.003	0.192	0.154	0.000	0.172	0.121	0.000	0.168	0.117
		300	0.086	0.114	0.109	0.000	0.170	0.141	0.000	0.186	0.153	0.000	0.379	0.276	0.000	0.365	0.269
10	0.3	50	1.000	0.012	0.552	1.000	0.010	0.529	1.000	0.012	0.513	1.000	0.010	0.526	1.000	0.009	0.508
		150	1.000	0.049	0.570	1.000	0.052	0.546	1.000	0.046	0.533	1.000	0.012	0.478	1.000	0.014	0.439
		300	1.000	0.134	0.397	1.000	0.113	0.408	1.000	0.051	0.364	1.000	0.027	0.364	1.000	0.029	0.351
	0.5	50	1.000	0.056	0.716	1.000	0.055	0.704	1.000	0.057	0.716	1.000	0.031	0.657	1.000	0.025	0.611
		150	0.999	0.143	0.516	0.997	0.108	0.446	0.999	0.064	0.386	1.000	0.013	0.329	1.000	0.014	0.320
		300	0.812	0.167	0.390	0.777	0.113	0.288	0.615	0.053	0.156	0.108	0.027	0.025	0.039	0.029	0.016
	0.8	50	1.000	0.084	0.559	1.000	0.082	0.569	1.000	0.074	0.555	1.000	0.033	0.496	1.000	0.023	0.449
		150	0.807	0.137	0.425	0.804	0.123	0.332	0.727	0.063	0.231	0.385	0.013	0.047	0.208	0.014	0.024
		300	0.433	0.154	0.262	0.153	0.111	0.113	0.008	0.053	0.037	0.000	0.027	0.016	0.001	0.029	0.014
	1.0	50	0.991	0.083	0.511	0.994	0.085	0.512	0.996	0.076	0.492	0.999	0.033	0.435	1.000	0.023	0.399
		150	0.646	0.147	0.351	0.533	0.123	0.243	0.400	0.063	0.145	0.081	0.013	0.020	0.082	0.014	0.021
		300	0.355	0.141	0.207	0.066	0.111	0.098	0.003	0.053	0.037	0.000	0.027	0.015	0.000	0.029	0.013
	1.5	50	0.768	0.103	0.377	0.774	0.091	0.361	0.767	0.073	0.336	0.650	0.032	0.174	0.534	0.023	0.114
		150	0.455	0.162	0.280	0.294	0.123	0.176	0.183	0.063	0.084	0.036	0.013	0.011	0.011	0.014	0.008
		300	0.299	0.129	0.171	0.038	0.111	0.093	0.001	0.053	0.037	0.000	0.027	0.013	0.000	0.029	0.014

Table B.21: Case A. Empirical power with four structural breaks, $\gamma = \gamma^* T^{1/2}$ and $w = 0.15$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	1.000	0.993	0.987	0.996	0.993	0.983	0.993	0.998	0.991	0.997	0.999	0.994	0.994	1.000	0.992
		150	1.000	0.992	0.998	1.000	0.997	0.997	1.000	0.999	0.998	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	0.992	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.987	0.903	0.939	0.995	0.926	0.951	0.996	0.940	0.966	0.990	0.982	0.985	0.970	1.000	0.996
		150	1.000	0.883	0.945	1.000	0.923	0.972	1.000	0.980	0.992	0.994	1.000	1.000	0.955	1.000	1.000
		300	1.000	0.860	0.943	1.000	0.981	0.994	1.000	1.000	1.000	0.999	1.000	1.000	0.842	1.000	1.000
	0.8	50	0.960	0.831	0.877	0.957	0.849	0.897	0.971	0.868	0.913	0.928	0.979	0.984	0.867	1.000	0.998
		150	1.000	0.779	0.872	1.000	0.882	0.937	0.999	0.975	0.984	0.960	1.000	1.000	0.755	1.000	1.000
		300	1.000	0.783	0.891	1.000	0.974	0.987	1.000	1.000	1.000	0.926	1.000	1.000	0.404	1.000	1.000
	1.0	50	0.898	0.809	0.844	0.899	0.824	0.879	0.897	0.860	0.892	0.833	0.979	0.980	0.737	1.000	0.998
		150	1.000	0.745	0.836	0.996	0.879	0.922	0.990	0.975	0.978	0.904	1.000	1.000	0.569	1.000	1.000
		300	1.000	0.726	0.840	1.000	0.974	0.979	1.000	1.000	1.000	0.823	1.000	1.000	0.216	1.000	1.000
	1.5	50	0.592	0.771	0.783	0.581	0.812	0.827	0.550	0.858	0.859	0.450	0.980	0.973	0.314	1.000	0.998
		150	0.889	0.709	0.755	0.862	0.879	0.890	0.828	0.975	0.974	0.519	1.000	1.000	0.191	1.000	1.000
		300	0.977	0.672	0.739	0.954	0.974	0.973	0.919	1.000	1.000	0.406	1.000	1.000	0.026	1.000	1.000
5	0.3	50	1.000	1.000	0.992	1.000	1.000	0.994	1.000	1.000	0.992	1.000	1.000	0.998	1.000	1.000	0.993
		150	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	0.998	1.000	0.996	1.000	1.000	0.999	1.000	1.000	0.999	0.997	1.000	0.998	0.994	1.000	0.997
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000
	0.8	50	0.995	1.000	1.000	0.997	1.000	1.000	0.996	1.000	1.000	0.985	1.000	1.000	0.959	1.000	1.000
		150	1.000	0.997	0.999	1.000	0.999	1.000	1.000	1.000	1.000	0.989	1.000	1.000	0.935	1.000	1.000
		300	1.000	0.997	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	0.764	1.000	1.000
	1.0	50	0.987	0.999	0.999	0.991	1.000	1.000	0.992	1.000	0.999	0.964	1.000	1.000	0.911	1.000	1.000
		150	1.000	0.997	0.999	1.000	0.999	1.000	1.000	1.000	1.000	0.976	1.000	1.000	0.834	1.000	1.000
		300	1.000	0.996	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.968	1.000	1.000	0.564	1.000	1.000
	1.5	50	0.924	1.000	1.000	0.916	0.999	1.000	0.905	0.999	1.000	0.827	1.000	1.000	0.723	1.000	1.000
		150	0.999	0.996	0.999	0.997	0.999	1.000	0.991	1.000	1.000	0.891	1.000	1.000	0.558	1.000	1.000
		300	1.000	0.996	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.818	1.000	1.000	0.202	1.000	1.000
10	0.3	50	1.000	1.000	0.995	1.000	1.000	0.994	1.000	1.000	0.993	1.000	1.000	0.998	1.000	1.000	0.996
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0.5	50	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.998	0.998	1.000	0.997
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.8	50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.992	1.000	1.000	0.979	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000	1.000	0.981	1.000	1.000
		300	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.907	1.000	1.000
	1.0	50	0.993	1.000	1.000	0.995	1.000	1.000	0.995	1.000	1.000	0.983	1.000	1.000	0.951	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	1.000	1.000	0.913	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.992	1.000	1.000	0.716	1.000	1.000
	1.5	50	0.978	1.000	1.000	0.973	1.000	1.000	0.967	1.000	1.000	0.915	1.000	1.000	0.834	1.000	1.000
		150	1.000	0.999	1.000	0.999	1.000	1.000	0.997	1.000	1.000	0.946	1.000	1.000	0.701	1.000	1.000
		300	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.894	1.000	1.000	0.345	1.000	1.000

Table B.22: Case A. Empirical power with four structural breaks, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.15$

			$\alpha=1$			$\alpha=0.95$			$\alpha=0.9$			$\alpha=0.7$			$\alpha=0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.016	0.172	0.131	0.012	0.164	0.123	0.010	0.170	0.127	0.004	0.132	0.094	0.001	0.100	0.062
		150	0.003	0.085	0.069	0.000	0.073	0.050	0.001	0.050	0.035	0.000	0.029	0.016	0.000	0.038	0.018
		300	0.000	0.057	0.046	0.000	0.035	0.027	0.000	0.025	0.019	0.000	0.022	0.013	0.000	0.034	0.017
	0.5	50	0.042	0.193	0.146	0.027	0.164	0.131	0.024	0.173	0.122	0.006	0.115	0.085	0.001	0.085	0.059
		150	0.027	0.092	0.077	0.011	0.056	0.041	0.001	0.037	0.032	0.000	0.030	0.019	0.000	0.040	0.022
		300	0.042	0.061	0.061	0.000	0.023	0.015	0.000	0.024	0.020	0.000	0.022	0.013	0.000	0.034	0.016
	0.8	50	0.022	0.154	0.118	0.015	0.158	0.123	0.013	0.148	0.103	0.002	0.112	0.080	0.000	0.092	0.064
		150	0.039	0.066	0.073	0.012	0.043	0.037	0.003	0.035	0.028	0.000	0.030	0.019	0.000	0.040	0.023
		300	0.053	0.049	0.058	0.001	0.022	0.014	0.000	0.024	0.020	0.000	0.022	0.013	0.000	0.034	0.017
	1.0	50	0.006	0.150	0.114	0.004	0.146	0.114	0.007	0.140	0.100	0.000	0.112	0.083	0.000	0.093	0.064
		150	0.041	0.054	0.057	0.012	0.043	0.038	0.001	0.035	0.027	0.000	0.030	0.019	0.000	0.040	0.023
		300	0.073	0.042	0.071	0.001	0.022	0.015	0.000	0.024	0.020	0.000	0.022	0.014	0.000	0.034	0.017
	1.5	50	0.001	0.146	0.104	0.001	0.140	0.106	0.001	0.140	0.100	0.000	0.112	0.082	0.000	0.094	0.065
		150	0.012	0.039	0.033	0.001	0.043	0.034	0.000	0.035	0.027	0.000	0.030	0.019	0.000	0.040	0.023
		300	0.022	0.034	0.036	0.000	0.022	0.015	0.000	0.024	0.020	0.000	0.022	0.014	0.000	0.034	0.017
5	0.3	50	0.026	0.180	0.136	0.022	0.184	0.132	0.024	0.202	0.144	0.013	0.162	0.118	0.003	0.124	0.092
		150	0.003	0.080	0.063	0.001	0.079	0.055	0.001	0.055	0.040	0.000	0.034	0.018	0.000	0.060	0.034
		300	0.001	0.061	0.049	0.000	0.036	0.029	0.000	0.025	0.019	0.000	0.023	0.014	0.000	0.039	0.025
	0.5	50	0.050	0.196	0.150	0.039	0.185	0.150	0.029	0.179	0.129	0.012	0.141	0.112	0.006	0.127	0.096
		150	0.027	0.090	0.082	0.010	0.058	0.044	0.001	0.038	0.030	0.000	0.036	0.021	0.000	0.059	0.036
		300	0.043	0.062	0.064	0.001	0.022	0.015	0.000	0.023	0.018	0.000	0.023	0.015	0.000	0.039	0.025
	0.8	50	0.032	0.156	0.131	0.018	0.180	0.137	0.017	0.167	0.133	0.002	0.133	0.108	0.000	0.129	0.098
		150	0.045	0.068	0.073	0.015	0.048	0.039	0.003	0.036	0.028	0.000	0.036	0.023	0.000	0.059	0.037
		300	0.054	0.049	0.060	0.001	0.021	0.013	0.000	0.023	0.018	0.000	0.023	0.015	0.000	0.039	0.025
	1.0	50	0.014	0.155	0.113	0.005	0.163	0.123	0.005	0.156	0.122	0.000	0.133	0.108	0.000	0.129	0.098
		150	0.048	0.052	0.058	0.013	0.047	0.039	0.001	0.036	0.027	0.000	0.036	0.023	0.000	0.059	0.038
		300	0.073	0.042	0.068	0.001	0.021	0.015	0.000	0.023	0.018	0.000	0.023	0.015	0.000	0.039	0.025
	1.5	50	0.003	0.143	0.107	0.001	0.155	0.120	0.001	0.159	0.123	0.000	0.133	0.108	0.000	0.130	0.099
		150	0.012	0.049	0.036	0.000	0.047	0.037	0.000	0.036	0.027	0.000	0.036	0.023	0.000	0.059	0.038
		300	0.021	0.034	0.036	0.000	0.021	0.014	0.000	0.023	0.018	0.000	0.023	0.015	0.000	0.039	0.025
10	0.3	50	0.059	0.255	0.208	0.060	0.253	0.197	0.056	0.298	0.226	0.036	0.268	0.216	0.019	0.230	0.179
		150	0.003	0.084	0.064	0.002	0.090	0.070	0.002	0.062	0.042	0.000	0.053	0.036	0.000	0.087	0.059
		300	0.002	0.062	0.046	0.000	0.037	0.026	0.000	0.026	0.018	0.000	0.035	0.021	0.000	0.064	0.042
	0.5	50	0.092	0.207	0.172	0.062	0.216	0.181	0.055	0.200	0.168	0.029	0.208	0.159	0.016	0.204	0.158
		150	0.034	0.094	0.086	0.015	0.064	0.044	0.001	0.039	0.029	0.000	0.054	0.034	0.000	0.089	0.057
		300	0.041	0.061	0.063	0.003	0.024	0.016	0.000	0.026	0.016	0.000	0.035	0.019	0.000	0.064	0.042
	0.8	50	0.067	0.174	0.138	0.049	0.191	0.160	0.041	0.178	0.149	0.013	0.199	0.155	0.004	0.203	0.163
		150	0.051	0.069	0.077	0.015	0.055	0.044	0.002	0.035	0.026	0.000	0.055	0.034	0.000	0.089	0.057
		300	0.054	0.048	0.058	0.001	0.023	0.016	0.000	0.026	0.015	0.000	0.035	0.021	0.000	0.064	0.042
	1.0	50	0.039	0.176	0.141	0.019	0.184	0.149	0.019	0.174	0.140	0.005	0.199	0.155	0.000	0.203	0.162
		150	0.050	0.057	0.063	0.013	0.053	0.040	0.002	0.035	0.026	0.000	0.055	0.035	0.000	0.089	0.057
		300	0.075	0.041	0.063	0.001	0.023	0.016	0.000	0.026	0.015	0.000	0.035	0.021	0.000	0.064	0.042
	1.5	50	0.008	0.155	0.128	0.002	0.177	0.143	0.003	0.175	0.141	0.000	0.199	0.156	0.000	0.203	0.165
		150	0.015	0.058	0.045	0.000	0.053	0.039	0.000	0.035	0.026	0.000	0.055	0.036	0.000	0.089	0.057
		300	0.024	0.035	0.035	0.000	0.023	0.015	0.000	0.026	0.015	0.000	0.035	0.021	0.000	0.064	0.042

Table B.23: Case A. Empirical power with four structural breaks, $\gamma = \gamma^*$ and $w = 0.15$

γ^*	$\bar{c}$	T	$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
			S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.034	0.197	0.150	0.033	0.208	0.155	0.035	0.232	0.168	0.022	0.196	0.151	0.009	0.154	0.116
		150	0.004	0.095	0.072	0.002	0.098	0.069	0.002	0.069	0.045	0.000	0.062	0.041	0.000	0.111	0.075
		300	0.004	0.057	0.044	0.000	0.039	0.029	0.000	0.030	0.020	0.000	0.060	0.039	0.000	0.160	0.094
	0.5	50	0.067	0.195	0.153	0.051	0.197	0.162	0.038	0.179	0.146	0.014	0.161	0.127	0.006	0.153	0.117
		150	0.037	0.095	0.082	0.014	0.062	0.045	0.000	0.039	0.026	0.000	0.060	0.039	0.000	0.113	0.073
		300	0.041	0.058	0.061	0.003	0.025	0.016	0.000	0.024	0.015	0.000	0.060	0.037	0.000	0.160	0.098
	0.8	50	0.042	0.166	0.128	0.028	0.175	0.145	0.023	0.170	0.136	0.003	0.157	0.125	0.002	0.156	0.120
		150	0.048	0.069	0.075	0.014	0.056	0.045	0.002	0.035	0.024	0.000	0.060	0.041	0.000	0.113	0.074
		300	0.059	0.044	0.063	0.001	0.024	0.016	0.000	0.024	0.015	0.000	0.060	0.037	0.000	0.160	0.100
	1.0	50	0.025	0.172	0.123	0.010	0.168	0.137	0.007	0.164	0.124	0.000	0.157	0.126	0.000	0.156	0.120
		150	0.052	0.057	0.062	0.013	0.054	0.043	0.003	0.035	0.024	0.000	0.060	0.041	0.000	0.113	0.074
		300	0.073	0.040	0.059	0.002	0.024	0.016	0.000	0.024	0.015	0.000	0.060	0.037	0.000	0.160	0.100
	1.5	50	0.005	0.152	0.116	0.002	0.159	0.127	0.001	0.165	0.124	0.000	0.157	0.126	0.000	0.156	0.121
		150	0.015	0.059	0.049	0.000	0.054	0.040	0.000	0.035	0.024	0.000	0.060	0.041	0.000	0.113	0.075
		300	0.026	0.033	0.034	0.000	0.024	0.015	0.000	0.024	0.015	0.000	0.060	0.038	0.000	0.160	0.101
5	0.3	50	0.990	0.934	0.932	0.981	0.929	0.926	0.976	0.948	0.947	0.975	0.969	0.966	0.970	0.994	0.979
		150	0.638	0.569	0.590	0.624	0.666	0.647	0.572	0.698	0.654	0.376	0.983	0.970	0.402	0.996	0.990
		300	0.392	0.313	0.282	0.320	0.385	0.339	0.171	0.558	0.493	0.021	0.993	0.992	0.146	0.996	0.995
	0.5	50	0.858	0.756	0.795	0.860	0.792	0.809	0.841	0.799	0.808	0.826	0.890	0.875	0.806	0.976	0.963
		150	0.436	0.386	0.428	0.367	0.413	0.405	0.275	0.513	0.475	0.126	0.981	0.968	0.246	0.996	0.993
		300	0.213	0.179	0.194	0.094	0.240	0.202	0.018	0.502	0.427	0.003	0.993	0.992	0.000	0.996	0.995
	0.8	50	0.772	0.656	0.675	0.742	0.670	0.702	0.731	0.710	0.708	0.719	0.881	0.859	0.668	0.974	0.964
		150	0.403	0.257	0.300	0.315	0.351	0.340	0.234	0.510	0.463	0.069	0.981	0.969	0.029	0.996	0.993
		300	0.188	0.124	0.151	0.061	0.241	0.197	0.012	0.502	0.427	0.000	0.993	0.992	0.000	0.996	0.995
	1.0	50	0.692	0.602	0.621	0.664	0.646	0.665	0.661	0.702	0.696	0.600	0.881	0.850	0.519	0.974	0.963
		150	0.412	0.213	0.245	0.326	0.350	0.328	0.235	0.510	0.457	0.032	0.981	0.970	0.003	0.996	0.993
		300	0.230	0.101	0.118	0.074	0.241	0.192	0.012	0.502	0.429	0.000	0.993	0.992	0.000	0.996	0.995
	1.5	50	0.340	0.555	0.536	0.333	0.634	0.602	0.299	0.701	0.657	0.201	0.882	0.843	0.112	0.974	0.963
		150	0.198	0.189	0.174	0.128	0.350	0.305	0.062	0.510	0.441	0.000	0.981	0.970	0.000	0.996	0.993
		300	0.120	0.086	0.078	0.009	0.241	0.187	0.000	0.502	0.432	0.000	0.993	0.992	0.000	0.996	0.995
10	0.3	50	1.000	1.000	0.990	0.998	1.000	0.991	0.996	1.000	0.992	0.999	1.000	0.997	0.997	1.000	0.989
		150	1.000	0.967	0.988	1.000	0.990	0.994	1.000	0.994	0.992	1.000	1.000	1.000	1.000	1.000	1.000
		300	0.998	0.800	0.886	0.998	0.908	0.931	0.997	0.988	0.990	1.000	1.000	1.000	0.998	1.000	1.000
	0.5	50	0.993	0.978	0.974	0.996	0.978	0.981	0.997	0.981	0.990	0.992	0.999	0.995	0.981	1.000	0.998
		150	1.000	0.796	0.900	0.999	0.833	0.943	1.000	0.945	0.971	0.991	1.000	1.000	0.944	1.000	1.000
		300	0.921	0.539	0.733	0.896	0.740	0.823	0.854	0.975	0.967	0.851	1.000	1.000	0.694	1.000	1.000
	0.8	50	0.982	0.942	0.958	0.984	0.945	0.971	0.988	0.957	0.977	0.954	0.998	0.995	0.895	1.000	1.000
		150	0.998	0.673	0.825	0.993	0.775	0.880	0.992	0.935	0.953	0.949	1.000	1.000	0.709	1.000	1.000
		300	0.856	0.395	0.569	0.773	0.721	0.761	0.668	0.974	0.963	0.627	1.000	1.000	0.230	1.000	1.000
	1.0	50	0.948	0.928	0.943	0.947	0.932	0.958	0.948	0.953	0.971	0.899	0.998	0.995	0.808	1.000	1.000
		150	0.992	0.626	0.769	0.985	0.773	0.857	0.983	0.935	0.939	0.877	1.000	1.000	0.526	1.000	1.000
		300	0.856	0.355	0.501	0.796	0.721	0.738	0.722	0.974	0.963	0.500	1.000	1.000	0.078	1.000	1.000
	1.5	50	0.730	0.907	0.913	0.711	0.922	0.943	0.677	0.952	0.966	0.562	0.998	0.995	0.425	1.000	1.000
		150	0.805	0.569	0.634	0.791	0.773	0.802	0.702	0.935	0.923	0.402	1.000	1.000	0.110	1.000	1.000
		300	0.615	0.302	0.332	0.474	0.721	0.699	0.276	0.974	0.964	0.003	1.000	1.000	0.000	1.000	1.000

Table B.24: Case A. Empirical power with four structural breaks, $\gamma = \gamma^* T^{1/2}$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	1.000	0.003	0.712	1.000	0.007	0.731	1.000	0.007	0.758	1.000	0.003	0.698	1.000	0.000	0.685
		150	1.000	0.002	0.858	1.000	0.001	0.836	1.000	0.001	0.805	1.000	0.000	0.779	1.000	0.000	0.843
		300	1.000	0.001	0.864	1.000	0.001	0.847	1.000	0.000	0.791	1.000	0.000	0.826	1.000	0.000	0.906
	0.5	50	1.000	0.022	0.450	1.000	0.021	0.450	1.000	0.021	0.447	1.000	0.010	0.471	1.000	0.001	0.435
		150	1.000	0.019	0.609	1.000	0.009	0.580	1.000	0.004	0.537	1.000	0.000	0.471	1.000	0.000	0.432
		300	1.000	0.011	0.648	1.000	0.006	0.601	1.000	0.000	0.498	1.000	0.000	0.404	1.000	0.000	0.377
	0.8	50	0.846	0.030	0.265	0.838	0.038	0.265	0.849	0.032	0.254	0.722	0.010	0.130	0.538	0.001	0.057
		150	0.998	0.032	0.509	1.000	0.016	0.451	1.000	0.004	0.409	1.000	0.000	0.413	1.000	0.000	0.407
		300	1.000	0.022	0.528	1.000	0.006	0.424	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	1.0	50	0.600	0.040	0.176	0.572	0.042	0.161	0.541	0.031	0.120	0.255	0.010	0.041	0.092	0.001	0.010
		150	0.962	0.036	0.469	0.988	0.016	0.416	0.985	0.004	0.376	0.970	0.000	0.275	0.942	0.000	0.145
		300	0.996	0.018	0.528	1.000	0.006	0.413	1.000	0.000	0.379	1.000	0.000	0.371	1.000	0.000	0.363
	1.5	50	0.252	0.048	0.097	0.185	0.042	0.070	0.149	0.031	0.053	0.037	0.010	0.014	0.018	0.001	0.006
		150	0.640	0.049	0.308	0.503	0.016	0.136	0.265	0.004	0.044	0.002	0.000	0.000	0.000	0.000	0.000
		300	0.751	0.036	0.379	0.520	0.006	0.095	0.171	0.000	0.012	0.000	0.000	0.000	0.000	0.000	0.000
5	0.3	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.357	1.000	0.000	0.401	1.000	0.000	0.397	1.000	0.000	0.414	1.000	0.000	0.410
		300	1.000	0.000	0.387	1.000	0.000	0.405	1.000	0.000	0.379	1.000	0.000	0.374	1.000	0.000	0.368
	0.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.398	1.000	0.000	0.400	1.000	0.000	0.400	1.000	0.000	0.413	1.000	0.000	0.407
		300	1.000	0.000	0.382	1.000	0.000	0.398	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	0.8	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.105	1.000	0.000	0.109	1.000	0.000	0.114	1.000	0.000	0.106	1.000	0.000	0.099
		300	1.000	0.000	0.384	1.000	0.000	0.397	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	1.0	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		300	1.000	0.000	0.396	1.000	0.000	0.397	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	1.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		300	1.000	0.000	0.396	1.000	0.000	0.397	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
10	0.3	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.357	1.000	0.000	0.401	1.000	0.000	0.397	1.000	0.000	0.414	1.000	0.000	0.410
		300	1.000	0.000	0.387	1.000	0.000	0.405	1.000	0.000	0.379	1.000	0.000	0.374	1.000	0.000	0.368
	0.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.398	1.000	0.000	0.400	1.000	0.000	0.400	1.000	0.000	0.413	1.000	0.000	0.407
		300	1.000	0.000	0.382	1.000	0.000	0.398	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	0.8	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.388	1.000	0.000	0.402	1.000	0.000	0.396	1.000	0.000	0.413	1.000	0.000	0.407
		300	1.000	0.000	0.384	1.000	0.000	0.397	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	1.0	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.372	1.000	0.000	0.402	1.000	0.000	0.396	1.000	0.000	0.413	1.000	0.000	0.407
		300	1.000	0.000	0.396	1.000	0.000	0.397	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368
	1.5	50	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
		150	1.000	0.000	0.373	1.000	0.000	0.402	1.000	0.000	0.396	1.000	0.000	0.413	1.000	0.000	0.407
		300	1.000	0.000	0.396	1.000	0.000	0.397	1.000	0.000	0.380	1.000	0.000	0.374	1.000	0.000	0.368

Table B.25: Case A. Empirical power with four structural breaks, $\gamma = \gamma^* T^{-1/2}$ and $w = 0.3$

			$\alpha = 1$			$\alpha = 0.95$			$\alpha = 0.9$			$\alpha = 0.7$			$\alpha = 0.5$		
γ^*	$\bar{c}$	T	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
1	0.3	50	0.001	0.141	0.108	0.000	0.148	0.106	0.000	0.152	0.123	0.000	0.108	0.075	0.000	0.073	0.047
		150	0.000	0.092	0.074	0.000	0.092	0.070	0.001	0.060	0.041	0.000	0.020	0.011	0.000	0.037	0.022
		300	0.000	0.075	0.055	0.000	0.076	0.052	0.000	0.031	0.021	0.000	0.024	0.009	0.000	0.032	0.014
	0.5	50	0.028	0.238	0.198	0.010	0.206	0.160	0.008	0.198	0.158	0.000	0.088	0.063	0.000	0.077	0.049
		150	0.013	0.136	0.105	0.003	0.076	0.055	0.000	0.044	0.031	0.000	0.017	0.010	0.000	0.036	0.021
		300	0.022	0.116	0.090	0.000	0.042	0.028	0.000	0.027	0.016	0.000	0.024	0.009	0.000	0.032	0.015
	0.8	50	0.032	0.182	0.167	0.018	0.165	0.136	0.009	0.155	0.116	0.000	0.087	0.064	0.000	0.080	0.051
		150	0.040	0.123	0.111	0.004	0.063	0.045	0.000	0.039	0.027	0.000	0.017	0.011	0.000	0.036	0.021
		300	0.035	0.086	0.081	0.000	0.043	0.029	0.000	0.027	0.016	0.000	0.024	0.009	0.000	0.032	0.015
	1.0	50	0.019	0.201	0.165	0.010	0.165	0.135	0.003	0.148	0.112	0.000	0.087	0.065	0.000	0.080	0.052
		150	0.041	0.119	0.114	0.002	0.063	0.044	0.000	0.039	0.027	0.000	0.017	0.011	0.000	0.036	0.021
		300	0.052	0.088	0.092	0.000	0.043	0.029	0.000	0.027	0.016	0.000	0.024	0.010	0.000	0.032	0.015
	1.5	50	0.006	0.177	0.148	0.002	0.153	0.121	0.001	0.149	0.110	0.000	0.087	0.066	0.000	0.081	0.053
		150	0.024	0.104	0.091	0.000	0.063	0.043	0.000	0.039	0.027	0.000	0.017	0.011	0.000	0.036	0.021
		300	0.029	0.080	0.079	0.000	0.043	0.029	0.000	0.027	0.016	0.000	0.024	0.010	0.000	0.032	0.015
5	0.3	50	0.006	0.189	0.152	0.005	0.198	0.155	0.001	0.221	0.175	0.001	0.178	0.134	0.001	0.161	0.125
		150	0.000	0.102	0.080	0.000	0.090	0.077	0.001	0.071	0.047	0.000	0.048	0.027	0.000	0.088	0.054
		300	0.000	0.074	0.058	0.000	0.072	0.056	0.000	0.033	0.024	0.000	0.040	0.015	0.000	0.076	0.037
	0.5	50	0.035	0.216	0.182	0.015	0.210	0.168	0.014	0.207	0.163	0.001	0.151	0.111	0.000	0.143	0.107
		150	0.018	0.128	0.111	0.004	0.081	0.061	0.000	0.055	0.040	0.000	0.046	0.023	0.000	0.086	0.055
		300	0.025	0.119	0.094	0.000	0.045	0.027	0.000	0.030	0.019	0.000	0.040	0.014	0.000	0.076	0.037
	0.8	50	0.038	0.197	0.166	0.024	0.183	0.147	0.014	0.170	0.138	0.000	0.144	0.113	0.000	0.146	0.112
		150	0.040	0.114	0.110	0.003	0.067	0.057	0.000	0.045	0.034	0.000	0.046	0.023	0.000	0.086	0.055
		300	0.038	0.088	0.084	0.000	0.046	0.028	0.000	0.029	0.018	0.000	0.040	0.015	0.000	0.076	0.038
	1.0	50	0.030	0.196	0.162	0.021	0.182	0.142	0.009	0.164	0.132	0.000	0.145	0.115	0.000	0.146	0.112
		150	0.045	0.115	0.116	0.001	0.067	0.057	0.000	0.045	0.034	0.000	0.046	0.024	0.000	0.086	0.055
		300	0.054	0.095	0.087	0.000	0.046	0.029	0.000	0.029	0.018	0.000	0.040	0.015	0.000	0.076	0.038
	1.5	50	0.009	0.193	0.160	0.003	0.169	0.135	0.002	0.164	0.132	0.000	0.145	0.115	0.000	0.147	0.112
		150	0.022	0.101	0.094	0.000	0.067	0.056	0.000	0.045	0.034	0.000	0.046	0.024	0.000	0.086	0.055
		300	0.028	0.083	0.078	0.000	0.046	0.029	0.000	0.029	0.020	0.000	0.040	0.016	0.000	0.077	0.038
10	0.3	50	0.022	0.218	0.180	0.022	0.230	0.178	0.016	0.265	0.215	0.008	0.230	0.183	0.004	0.200	0.153
		150	0.000	0.107	0.088	0.000	0.116	0.083	0.001	0.095	0.069	0.000	0.092	0.064	0.000	0.154	0.100
		300	0.000	0.080	0.053	0.000	0.070	0.054	0.000	0.044	0.032	0.000	0.077	0.041	0.000	0.209	0.126
	0.5	50	0.054	0.227	0.186	0.038	0.224	0.193	0.028	0.230	0.172	0.008	0.211	0.162	0.000	0.186	0.140
		150	0.034	0.130	0.114	0.008	0.088	0.066	0.000	0.071	0.049	0.000	0.094	0.062	0.000	0.151	0.098
		300	0.028	0.117	0.096	0.000	0.043	0.031	0.000	0.040	0.028	0.000	0.076	0.040	0.000	0.209	0.127
	0.8	50	0.053	0.194	0.160	0.044	0.197	0.162	0.032	0.210	0.175	0.001	0.202	0.153	0.000	0.185	0.141
		150	0.048	0.115	0.113	0.002	0.070	0.056	0.000	0.064	0.041	0.000	0.094	0.063	0.000	0.151	0.101
		300	0.039	0.090	0.080	0.000	0.045	0.032	0.000	0.040	0.027	0.000	0.076	0.042	0.000	0.209	0.127
	1.0	50	0.049	0.192	0.152	0.041	0.191	0.155	0.020	0.208	0.169	0.000	0.202	0.154	0.000	0.185	0.142
		150	0.052	0.115	0.110	0.002	0.070	0.056	0.000	0.064	0.041	0.000	0.094	0.064	0.000	0.151	0.101
		300	0.051	0.089	0.090	0.000	0.045	0.032	0.000	0.040	0.028	0.000	0.076	0.042	0.000	0.209	0.127
	1.5	50	0.016	0.183	0.144	0.008	0.184	0.146	0.003	0.207	0.166	0.000	0.203	0.154	0.000	0.185	0.142
		150	0.024	0.105	0.092	0.001	0.070	0.055	0.000	0.064	0.041	0.000	0.094	0.064	0.000	0.152	0.101
		300	0.027	0.080	0.077	0.000	0.045	0.032	0.000	0.040	0.028	0.000	0.076	0.042	0.000	0.209	0.129

Table B.26: Case A. Empirical power with four structural breaks, $\gamma = \gamma^*$ and $w = 0.3$

γ^*	$\bar{c}$	T	$\alpha=1$ S_1	S_0	S_U	$\alpha=0.95$ S_1	S_0	S_U	$\alpha=0.9$ S_1	S_0	S_U	$\alpha=0.7$ S_1	S_0	S_U	$\alpha=0.5$ S_1	S_0	S_U
1	0.3	50	0.015	0.221	0.173	0.011	0.224	0.178	0.005	0.251	0.203	0.005	0.217	0.173	0.002	0.190	0.144
		150	0.000	0.112	0.093	0.000	0.119	0.100	0.001	0.106	0.077	0.000	0.099	0.070	0.000	0.181	0.120
		300	0.000	0.079	0.063	0.000	0.082	0.064	0.000	0.056	0.040	0.000	0.144	0.085	0.000	0.375	0.241
	0.5	50	0.046	0.223	0.184	0.016	0.230	0.188	0.016	0.220	0.174	0.003	0.196	0.153	0.000	0.172	0.132
		150	0.036	0.128	0.116	0.009	0.095	0.070	0.000	0.074	0.059	0.000	0.104	0.074	0.000	0.180	0.122
		300	0.030	0.106	0.095	0.000	0.050	0.037	0.000	0.049	0.033	0.000	0.145	0.081	0.000	0.375	0.242
	0.8	50	0.047	0.192	0.164	0.036	0.202	0.160	0.024	0.199	0.155	0.001	0.183	0.143	0.000	0.176	0.138
		150	0.052	0.120	0.119	0.002	0.079	0.062	0.000	0.069	0.053	0.000	0.104	0.074	0.000	0.180	0.122
		300	0.043	0.084	0.079	0.000	0.049	0.036	0.000	0.049	0.032	0.000	0.145	0.081	0.000	0.375	0.245
	1.0	50	0.044	0.186	0.163	0.030	0.192	0.158	0.015	0.193	0.147	0.000	0.183	0.147	0.000	0.175	0.138
		150	0.056	0.113	0.115	0.002	0.079	0.062	0.000	0.069	0.053	0.000	0.104	0.075	0.000	0.180	0.122
		300	0.051	0.089	0.089	0.000	0.049	0.036	0.000	0.049	0.033	0.000	0.145	0.081	0.000	0.375	0.246
	1.5	50	0.011	0.191	0.158	0.004	0.186	0.150	0.002	0.192	0.146	0.000	0.183	0.147	0.000	0.175	0.139
		150	0.025	0.105	0.093	0.001	0.079	0.061	0.000	0.069	0.053	0.000	0.104	0.075	0.000	0.180	0.123
		300	0.028	0.081	0.071	0.000	0.049	0.036	0.000	0.049	0.033	0.000	0.145	0.084	0.000	0.375	0.247
5	0.3	50	0.999	0.027	0.436	0.996	0.026	0.428	0.996	0.029	0.403	0.997	0.019	0.435	0.997	0.009	0.419
		150	0.809	0.103	0.316	0.753	0.107	0.335	0.749	0.087	0.295	0.322	0.043	0.075	0.050	0.033	0.019
		300	0.531	0.143	0.241	0.437	0.172	0.214	0.179	0.123	0.117	0.000	0.120	0.078	0.000	0.093	0.066
	0.5	50	0.702	0.071	0.239	0.656	0.078	0.205	0.683	0.085	0.214	0.535	0.048	0.120	0.287	0.020	0.053
		150	0.434	0.147	0.227	0.324	0.126	0.182	0.155	0.108	0.115	0.000	0.045	0.025	0.000	0.033	0.014
		300	0.278	0.138	0.181	0.045	0.164	0.129	0.000	0.120	0.093	0.000	0.120	0.076	0.000	0.093	0.066
	0.8	50	0.302	0.093	0.149	0.269	0.109	0.126	0.232	0.106	0.107	0.074	0.051	0.035	0.024	0.021	0.020
		150	0.216	0.116	0.162	0.100	0.136	0.120	0.009	0.107	0.085	0.000	0.045	0.023	0.000	0.033	0.014
		300	0.141	0.101	0.118	0.001	0.161	0.121	0.000	0.120	0.090	0.000	0.120	0.074	0.000	0.093	0.065
	1.0	50	0.222	0.101	0.135	0.177	0.114	0.119	0.160	0.110	0.098	0.051	0.051	0.033	0.020	0.021	0.017
		150	0.184	0.121	0.147	0.068	0.136	0.111	0.005	0.107	0.085	0.000	0.045	0.024	0.000	0.033	0.015
		300	0.123	0.093	0.112	0.000	0.161	0.120	0.000	0.120	0.090	0.000	0.120	0.075	0.000	0.093	0.065
	1.5	50	0.178	0.107	0.115	0.138	0.117	0.113	0.107	0.109	0.084	0.029	0.051	0.029	0.003	0.021	0.012
		150	0.167	0.114	0.128	0.041	0.136	0.110	0.002	0.107	0.089	0.000	0.045	0.025	0.000	0.033	0.015
		300	0.096	0.079	0.078	0.000	0.161	0.121	0.000	0.120	0.091	0.000	0.120	0.076	0.000	0.093	0.066
10	0.3	50	1.000	0.000	0.010	1.000	0.000	0.009	1.000	0.000	0.007	1.000	0.000	0.012	1.000	0.000	0.016
		150	1.000	0.009	0.755	1.000	0.008	0.773	1.000	0.002	0.788	1.000	0.001	0.770	1.000	0.000	0.682
		300	1.000	0.038	0.417	1.000	0.036	0.436	1.000	0.016	0.407	1.000	0.000	0.375	1.000	0.000	0.368
	0.5	50	1.000	0.003	0.687	1.000	0.001	0.671	1.000	0.001	0.657	1.000	0.001	0.604	1.000	0.000	0.614
		150	1.000	0.045	0.482	1.000	0.024	0.443	1.000	0.013	0.420	1.000	0.001	0.414	1.000	0.000	0.407
		300	0.938	0.067	0.413	0.922	0.051	0.368	0.836	0.020	0.250	0.295	0.000	0.021	0.038	0.000	0.000
	0.8	50	1.000	0.004	0.525	1.000	0.003	0.535	1.000	0.004	0.557	1.000	0.001	0.520	1.000	0.000	0.494
		150	0.903	0.052	0.423	0.926	0.036	0.363	0.885	0.011	0.292	0.496	0.001	0.054	0.164	0.000	0.003
		300	0.540	0.068	0.254	0.189	0.051	0.061	0.014	0.020	0.011	0.000	0.000	0.000	0.000	0.000	0.000
	1.0	50	1.000	0.005	0.395	0.999	0.005	0.396	1.000	0.004	0.412	1.000	0.001	0.402	1.000	0.000	0.394
		150	0.723	0.064	0.347	0.643	0.035	0.214	0.421	0.011	0.092	0.012	0.001	0.001	0.000	0.000	0.000
		300	0.369	0.077	0.188	0.046	0.051	0.040	0.000	0.020	0.009	0.000	0.000	0.000	0.000	0.000	0.000
	1.5	50	0.797	0.010	0.211	0.831	0.006	0.218	0.824	0.004	0.180	0.682	0.001	0.068	0.477	0.000	0.017
		150	0.388	0.077	0.210	0.196	0.035	0.065	0.051	0.011	0.011	0.000	0.001	0.001	0.000	0.000	0.000
		300	0.272	0.072	0.151	0.010	0.051	0.037	0.000	0.020	0.010	0.000	0.000	0.000	0.000	0.000	0.000

Table B.27: Case B. Empirical size, $\pi = 0.5$, known $\overline{\boldsymbol{c}}$

			$w = 0.10$			$w = 0.15$			$w = 0.20$			$w = 0.25$			$w = 0.30$		
$(\bar{c}_1, \bar{c}_2)$	T	α	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
$(0.3, 0.5)$	50	1	0.049	0.224	0.187	0.045	0.225	0.181	0.044	0.209	0.173	0.051	0.205	0.168	0.058	0.245	0.186
		0.95	0.038	0.233	0.184	0.045	0.198	0.160	0.040	0.180	0.141	0.039	0.176	0.134	0.040	0.208	0.166
		0.9	0.034	0.222	0.179	0.028	0.188	0.144	0.026	0.184	0.124	0.026	0.173	0.119	0.032	0.193	0.159
		0.7	0.012	0.172	0.135	0.009	0.129	0.093	0.007	0.102	0.075	0.010	0.098	0.063	0.010	0.102	0.069
		0.5	0.004	0.155	0.104	0.003	0.099	0.067	0.004	0.077	0.053	0.003	0.070	0.041	0.003	0.070	0.045
	150	1	0.060	0.101	0.093	0.053	0.092	0.092	0.049	0.109	0.097	0.051	0.142	0.122	0.052	0.160	0.152
		0.95	0.013	0.074	0.058	0.019	0.055	0.049	0.025	0.067	0.055	0.024	0.068	0.065	0.021	0.086	0.069
		0.9	0.000	0.046	0.032	0.002	0.030	0.021	0.003	0.038	0.028	0.004	0.059	0.041	0.002	0.057	0.037
		0.7	0.000	0.028	0.017	0.000	0.017	0.007	0.000	0.020	0.012	0.000	0.028	0.016	0.000	0.021	0.007
		0.5	0.000	0.036	0.019	0.000	0.025	0.011	0.000	0.027	0.010	0.000	0.036	0.017	0.000	0.039	0.018
	300	1	0.048	0.057	0.064	0.052	0.082	0.091	0.060	0.087	0.101	0.054	0.131	0.129	0.050	0.155	0.150
		0.95	0.000	0.023	0.011	0.001	0.029	0.023	0.001	0.039	0.028	0.001	0.048	0.032	0.001	0.051	0.035
		0.9	0.000	0.017	0.007	0.000	0.022	0.017	0.000	0.026	0.021	0.000	0.035	0.021	0.000	0.029	0.017
		0.7	0.000	0.015	0.007	0.000	0.021	0.015	0.000	0.023	0.009	0.000	0.019	0.009	0.000	0.020	0.008
		0.5	0.000	0.020	0.007	0.000	0.033	0.018	0.000	0.032	0.013	0.000	0.031	0.010	0.000	0.028	0.010
$(0.5, 1)$	50	1	0.035	0.281	0.224	0.041	0.262	0.219	0.042	0.230	0.190	0.046	0.233	0.187	0.052	0.281	0.232
		0.95	0.012	0.201	0.149	0.011	0.194	0.140	0.017	0.180	0.132	0.015	0.161	0.118	0.016	0.214	0.154
		0.9	0.001	0.178	0.134	0.001	0.151	0.098	0.001	0.142	0.101	0.003	0.120	0.087	0.005	0.168	0.120
		0.7	0.000	0.183	0.137	0.000	0.118	0.079	0.000	0.102	0.071	0.000	0.086	0.060	0.000	0.093	0.057
		0.5	0.000	0.153	0.111	0.000	0.089	0.055	0.000	0.083	0.050	0.000	0.064	0.041	0.000	0.071	0.045
	150	1	0.049	0.176	0.159	0.047	0.148	0.132	0.042	0.162	0.134	0.043	0.201	0.176	0.042	0.232	0.206
		0.95	0.000	0.049	0.031	0.000	0.034	0.022	0.000	0.062	0.038	0.000	0.057	0.045	0.000	0.076	0.049
		0.9	0.000	0.046	0.029	0.000	0.034	0.025	0.000	0.035	0.027	0.000	0.041	0.029	0.000	0.044	0.033
		0.7	0.000	0.025	0.014	0.000	0.022	0.009	0.000	0.025	0.014	0.000	0.029	0.016	0.000	0.020	0.005
		0.5	0.000	0.037	0.019	0.000	0.026	0.010	0.000	0.030	0.012	0.000	0.035	0.017	0.000	0.039	0.016
	300	1	0.053	0.129	0.116	0.051	0.167	0.140	0.050	0.137	0.126	0.052	0.184	0.171	0.047	0.197	0.172
		0.95	0.000	0.013	0.009	0.000	0.025	0.015	0.000	0.027	0.016	0.000	0.049	0.028	0.000	0.039	0.025
		0.9	0.000	0.016	0.009	0.000	0.023	0.017	0.000	0.025	0.018	0.000	0.033	0.019	0.000	0.029	0.014
		0.7	0.000	0.015	0.006	0.000	0.021	0.012	0.000	0.023	0.008	0.000	0.019	0.009	0.000	0.020	0.006
		0.5	0.000	0.021	0.007	0.000	0.033	0.014	0.000	0.032	0.011	0.000	0.031	0.010	0.000	0.028	0.009

Table B.28: Case B. Empirical size of the modified statistics, $\pi = 0.5$, known $\overline{\boldsymbol{c}}$

			$w = 0.10$			$w = 0.15$			$w = 0.20$			$w = 0.25$			$w = 0.30$		
$(\overline{c}_1, \overline{c}_2)$	T	α	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U	S_1	S_0	S_U
$(0.3, 0.5)$	50	1	0.058	0.163	0.124	0.051	0.172	0.131	0.046	0.158	0.130	0.059	0.161	0.136	0.061	0.176	0.140
		0.95	0.038	0.154	0.121	0.048	0.152	0.121	0.045	0.142	0.110	0.046	0.123	0.105	0.047	0.140	0.116
		0.9	0.030	0.153	0.116	0.031	0.156	0.115	0.033	0.143	0.093	0.032	0.125	0.092	0.036	0.140	0.118
		0.7	0.015	0.118	0.082	0.010	0.105	0.068	0.009	0.088	0.068	0.015	0.082	0.055	0.014	0.087	0.060
		0.5	0.005	0.092	0.072	0.005	0.074	0.050	0.006	0.064	0.044	0.003	0.061	0.035	0.003	0.058	0.041
	150	1	0.056	0.085	0.081	0.057	0.069	0.074	0.056	0.079	0.072	0.057	0.087	0.087	0.052	0.098	0.098
		0.95	0.017	0.073	0.056	0.025	0.050	0.048	0.030	0.065	0.061	0.024	0.065	0.062	0.021	0.078	0.064
		0.9	0.002	0.041	0.025	0.005	0.028	0.021	0.002	0.036	0.027	0.003	0.053	0.037	0.002	0.050	0.033
		0.7	0.000	0.027	0.012	0.000	0.016	0.006	0.000	0.020	0.011	0.000	0.028	0.016	0.000	0.022	0.008
		0.5	0.000	0.032	0.018	0.000	0.024	0.011	0.000	0.022	0.013	0.000	0.034	0.019	0.000	0.039	0.016
	300	1	0.044	0.042	0.048	0.047	0.061	0.071	0.056	0.059	0.074	0.053	0.091	0.093	0.047	0.099	0.104
		0.95	0.000	0.020	0.010	0.001	0.024	0.020	0.002	0.037	0.026	0.001	0.046	0.031	0.001	0.045	0.031
		0.9	0.000	0.013	0.003	0.000	0.020	0.014	0.000	0.025	0.019	0.000	0.029	0.019	0.000	0.029	0.015
		0.7	0.000	0.014	0.005	0.000	0.021	0.012	0.000	0.022	0.010	0.000	0.024	0.009	0.000	0.022	0.008
		0.5	0.000	0.017	0.005	0.000	0.033	0.016	0.000	0.031	0.015	0.000	0.032	0.011	0.000	0.022	0.006
$(0.5, 1)$	50	1	0.041	0.133	0.105	0.050	0.147	0.115	0.046	0.123	0.101	0.048	0.118	0.100	0.041	0.145	0.135
		0.95	0.019	0.135	0.102	0.023	0.144	0.113	0.031	0.132	0.106	0.034	0.121	0.090	0.034	0.141	0.111
		0.9	0.011	0.125	0.088	0.014	0.127	0.077	0.023	0.127	0.090	0.023	0.103	0.076	0.023	0.133	0.098
		0.7	0.007	0.122	0.078	0.008	0.098	0.065	0.006	0.088	0.063	0.007	0.073	0.059	0.004	0.079	0.052
		0.5	0.000	0.095	0.073	0.001	0.066	0.042	0.003	0.070	0.046	0.002	0.062	0.043	0.002	0.061	0.046
	150	1	0.050	0.103	0.086	0.054	0.077	0.083	0.058	0.077	0.084	0.051	0.090	0.098	0.050	0.104	0.105
		0.95	0.000	0.049	0.021	0.000	0.037	0.024	0.009	0.055	0.036	0.008	0.052	0.041	0.006	0.066	0.045
		0.9	0.000	0.043	0.023	0.001	0.028	0.025	0.000	0.034	0.026	0.000	0.036	0.027	0.000	0.040	0.026
		0.7	0.000	0.024	0.010	0.000	0.020	0.009	0.000	0.022	0.013	0.000	0.027	0.013	0.000	0.022	0.007
		0.5	0.000	0.033	0.017	0.000	0.025	0.010	0.000	0.024	0.013	0.000	0.035	0.018	0.000	0.039	0.015
	300	1	0.032	0.043	0.043	0.035	0.068	0.059	0.035	0.062	0.075	0.040	0.081	0.082	0.048	0.088	0.089
		0.95	0.000	0.013	0.009	0.000	0.023	0.013	0.000	0.025	0.017	0.000	0.043	0.030	0.000	0.039	0.023
		0.9	0.000	0.013	0.007	0.000	0.020	0.013	0.000	0.024	0.017	0.000	0.028	0.018	0.000	0.029	0.012
		0.7	0.000	0.014	0.005	0.000	0.021	0.010	0.000	0.022	0.009	0.000	0.024	0.007	0.000	0.022	0.007
		0.5	0.000	0.018	0.005	0.000	0.033	0.014	0.000	0.031	0.014	0.000	0.032	0.009	0.000	0.022	0.006

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