

Non(anti)commutative superspace with coordinate-dependent deformation

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We consider non(anti)commutative superspace with coordinate-dependent deformation parameters $C^{\alpha\beta}$. We show that a chiral $\mathcal{N} = 1/2$ supersymmetry can be defined and that chiral and antichiral superfields are still closed under the Moyal-Weyl associative product implementing the deformation. A consistent $\mathcal{N} = 1/2$ super Yang-Mills deformed theory can be constructed provided $C^{\alpha\beta}$ satisfies a suitable condition which can be connected with the graviphoton background at the origin of the deformation. After adding matter, we also discuss the Konishi anomaly and the gluino condensation.

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I. INTRODUCTION

Recently, a deformation of the supersymmetry algebra attracted much attention due to its connection with string dynamics in nontrivial Ramond-Ramond backgrounds [1–11], the deformation parameter $C^{\alpha\beta}$ being related to a constant graviphoton field strength. Moreover, a relation between C -deformed super Yang-Mills theory [3] and conventional $\mathcal{N} = 1$ super Yang-Mills theory [supersymmetry (SUSY) gluodynamics] has been signaled in Ref. [12]. According to this last work, such a C deformation turns out to be related to a spectral degeneracy in SUSY gluodynamics (which, due to the planar equivalence, can be related to one flavor QCD).

It was conjectured in Ref. [12] that, in fact, $\mathcal{N} = 1/2$ supersymmetry should remain valid for a coordinate-dependent $C^{\alpha\beta}$ deformation. In this paper, we investigate such a possibility by analyzing a deformed algebra for the fermionic coordinates θ with $C^{\alpha\beta}$ depending on the chiral variable y . As it happens for constant $C^{\alpha\beta}$, we shall see that the subalgebra satisfied by Q_α is preserved when $C^{\alpha\beta} = C^{\alpha\beta}(y)$ so that a chiral $\mathcal{N} = 1/2$ supersymmetry can be defined. We also show that chiral and antichiral superfields are still closed under the Moyal-Weyl associative product implementing the deformation, but the case of antichiral superfields should be handled with care due to the fact that the chiral covariant derivative D_α violates the Leibnitz rule. Moreover, chiral and antichiral superfield strengths do not in general transform covariantly under general supergauge transformations. However, one can still consistently define a super Yang-Mills deformed theory by adopting, from the start, the Wess-Zumino gauge and appropriately restricting supergauge transformations. We show that demanding gauge invariance of the resulting deformed theory imposes a remarkable condition on the coordinate-dependent $C^{\alpha\beta}$. Finally, we discuss the Konishi anomaly and the gluino condensation for coordinate-dependent deformation.

The paper is organized as follows. We introduce in Sec. II the coordinate-dependent deformation, discuss how a Moyal-Weyl associative product of superfields can be implemented, and present the supercharge algebra.

Then in Sec. III we introduce chiral and vector superfields, carefully analyzing the condition under which a gauge invariant $\mathcal{N} = 1/2$ supersymmetric Lagrangian can be defined. Coupling the super Yang-Mills multiplet to matter in the fundamental, we present in Sec. IV the supersymmetric Lagrangian in a component, analyzing the essential features of the deformed terms. We also discuss the Konishi anomaly in the commutator leading to the gluino condensation, showing that it remains unchanged by the coordinate-dependent deformation. Finally, we summarize and discuss our results in Sec. V.

II. DEFORMED SUPERSPACE

We consider the deformation of four-dimensional Euclidean $\mathcal{N} = 1$ superspace parametrized by chiral bosonic coordinates $y^\mu = x^\mu + i\hat{\theta}^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}}$ and chiral and antichiral fermionic coordinates $\hat{\theta}^\alpha, \bar{\theta}^{\dot{\alpha}}$ satisfying the Clifford algebra

$$\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = C^{\alpha\beta}(y), \quad \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0, \quad \{\hat{\theta}^\alpha, \bar{\theta}^{\dot{\beta}}\} = 0, \quad (1)$$

where $C^{\alpha\beta}$ is some chiral coordinate-dependent symmetric matrix. (We follow the conventions of Ref. [13] for lowering and raising spinor indices.) We indicate with a hat that the θ subalgebra is deformed. Following Ref. [3], we also define

$$[y^\mu, y^\nu] = [y^\mu, \hat{\theta}^\alpha] = [y^\mu, \bar{\theta}^{\dot{\alpha}}] = 0. \quad (2)$$

Because of the nonanticommutativity of the $\hat{\theta}$'s coordinates, functions in this deformed superspace have to be ordered. One can show that, as it is the case of constant $C^{\alpha\beta}$, a Weyl ordering can be easily implemented [14]. Indeed, consider the Fourier transform \tilde{f} of a function f in ordinary superspace given by

$$f(y; \theta, \bar{\theta}) = \int d^2\pi e^{-\pi\theta} \tilde{f}(y; \pi, \bar{\theta}) \quad (3)$$

and define a one to one map between Weyl symbols \hat{f} in deformed superspace $(y, \hat{\theta}, \bar{\theta})$ and functions f in ordinary

superspace $(y, \theta, \bar{\theta})$ via the formula

$$\hat{f}(y; \hat{\theta}, \bar{\theta}) \equiv \int d^2\pi e^{-\pi\hat{\theta}} \tilde{f}(y; \pi, \bar{\theta}). \quad (4)$$

The product \cdot of symbols, $\hat{f}_1(y, \hat{\theta}, \bar{\theta}) \cdot \hat{f}_2(y, \hat{\theta}, \bar{\theta})$, can then be written as:

$$\begin{aligned} \hat{f}_1(y; \hat{\theta}, \bar{\theta}) \cdot \hat{f}_2(y; \hat{\theta}, \bar{\theta}) &= \int d^2\pi_1 d^2\pi_2 e^{-\pi_1\hat{\theta}} \cdot e^{-\pi_2\hat{\theta}} \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) \\ &= \int d^2\pi_1 d^2\pi_2 e^{-(\pi_1+\pi_2)\hat{\theta}} e^{-1/2\pi_{1\alpha}C^{\alpha\beta}(y)\pi_{2\beta}} \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}), \end{aligned} \quad (5)$$

where the Baker-Campbell-Hausdorff formula has been used,

$$e^{-\pi_1\hat{\theta}} \cdot e^{-\pi_2\hat{\theta}} = e^{-(\pi_1+\pi_2)\hat{\theta}} e^{-1/2\pi_{1\alpha}C^{\alpha\beta}(y)\pi_{2\beta}}.$$

After a change of integration variables, $\pi = \pi_1 + \pi_2$, $\pi' = \pi_1 - \pi_2$, Eq. (5) becomes

$$\hat{f}_1 \cdot \hat{f}_2(y; \hat{\theta}, \bar{\theta}) = \int d^2\pi d^2\pi' e^{-\pi\hat{\theta}} e^{-1/8(\pi+\pi')_{\alpha}C^{\alpha\beta}(y)(\pi-\pi')_{\beta}} \tilde{f}_1\left(y; \frac{\pi+\pi'}{2}, \bar{\theta}\right) \tilde{f}_2\left(y; \frac{\pi-\pi'}{2}, \bar{\theta}\right) \quad (6)$$

so that a product of symbols gives

$$\widetilde{f_1 \cdot f_2}(y; \pi, \bar{\theta}) = \int d^2\pi' e^{-1/8(\pi+\pi')_{\alpha}C^{\alpha\beta}(y)(\pi-\pi')_{\beta}} \tilde{f}_1\left(y; \frac{\pi+\pi'}{2}, \bar{\theta}\right) \tilde{f}_2\left(y; \frac{\pi-\pi'}{2}, \bar{\theta}\right). \quad (7)$$

Using (5), it is easy to see that the product is associative,

$$\begin{aligned} \hat{f}_1 \cdot (\hat{f}_2 \cdot \hat{f}_3) &= \int d^2\pi_1 d^2\pi_2 d^2\pi_3 e^{-\pi_1\hat{\theta}} \cdot e^{-(\pi_2+\pi_3)\hat{\theta}} e^{-1/2\pi_2 C \pi_3} \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) \tilde{f}_3(y; \pi_3, \bar{\theta}) \\ &= \int d^2\pi_1 d^2\pi_2 d^2\pi_3 e^{-(\pi_1+\pi_2+\pi_3)\hat{\theta}} e^{-1/2(\pi_1 C \pi_2 + \pi_1 C \pi_3 + \pi_2 C \pi_3)} \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) \tilde{f}_3(y; \pi_3, \bar{\theta}) \\ &= \int d^2\pi_1 d^2\pi_2 d^2\pi_3 e^{-(\pi_1+\pi_2)\hat{\theta}} \cdot e^{-\pi_3\hat{\theta}} e^{-1/2\pi_1 C \pi_2} \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) \tilde{f}_3(y; \pi_3, \bar{\theta}) = (\hat{f}_1 \cdot \hat{f}_2) \cdot \hat{f}_3. \end{aligned} \quad (8)$$

Moreover, a mapping between the product $\hat{f}_1 \cdot \hat{f}_2$ in deformed superspace and a star product of the corresponding functions in ordinary space can be established,

$$\hat{f}_1 \cdot \hat{f}_2 = \int d^2\pi \exp(-\pi\hat{\theta}) (\widetilde{f_1 * g})(y; \pi, \bar{\theta}), \quad (9)$$

where the Moyal-Weyl star product is defined by

$$\begin{aligned} f_1(y, \theta, \bar{\theta}) * f_2(y, \theta, \bar{\theta}) &\equiv f_1(y, \theta, \bar{\theta}) \exp\left(-\frac{1}{2}C^{\alpha\beta}(y) \frac{\vec{\partial}}{\partial\theta^{\alpha}} \frac{\vec{\partial}}{\partial\theta^{\beta}}\right) f_2(y, \theta, \bar{\theta}) \\ &= f_1(y, \theta, \bar{\theta}) \left(1 - \frac{1}{2}C^{\alpha\beta}(y) \frac{\vec{\partial}}{\partial\theta^{\alpha}} \frac{\vec{\partial}}{\partial\theta^{\beta}} - \det C(y) \frac{\vec{\partial}}{\partial\theta\theta} \frac{\vec{\partial}}{\partial\theta\theta}\right) f_2(y, \theta, \bar{\theta}), \end{aligned} \quad (10)$$

with

$$\frac{\vec{\partial}}{\partial\theta^{\alpha}} \theta^{\beta} \equiv \frac{\partial}{\partial\theta^{\alpha}} \theta^{\beta} = \delta_{\alpha}^{\beta}, \quad \theta^{\alpha} \frac{\vec{\partial}}{\partial\theta^{\beta}} \equiv -\delta_{\alpha}^{\beta}, \quad \frac{\partial}{\partial\theta\theta} \equiv \frac{1}{4} \epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^{\alpha}} \frac{\partial}{\partial\theta^{\beta}}. \quad (11)$$

Using the inverse Fourier transformation, we have

$$\begin{aligned}
f_1 \widetilde{*} f_2(y; \pi, \bar{\theta}) &= \int d^2\theta e^{\pi\theta} (f_1(y; \theta, \bar{\theta}) * f_2(y; \theta, \bar{\theta})) \\
&= \int d^2\theta e^{\pi\theta} \left(\int d^2\pi_1 e^{-\pi_1\theta} \tilde{f}_1(y; \pi_1, \bar{\theta}) \right) * \left(\int d^2\pi_2 e^{-\pi_2\theta} \tilde{f}_2(y; \pi_2, \bar{\theta}) \right) \\
&= \int d^2\pi_1 d^2\pi_2 d^2\theta \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) e^{\pi\theta} (e^{-\pi_1\theta} * e^{-\pi_2\theta}) \\
&= \int d^2\pi_1 d^2\pi_2 d^2\theta \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) e^{\pi\theta} e^{-(\pi_1+\pi_2)\theta - 1/2\pi_1 C \pi_2} \\
&= \frac{1}{2} \int d^2\pi_1 d^2\pi_2 \tilde{f}_1(y; \pi_1, \bar{\theta}) \tilde{f}_2(y; \pi_2, \bar{\theta}) \delta(\pi - \pi_1 - \pi_2) e^{-1/2\pi_1 C \pi_2} \\
&= \int d^2\pi' \tilde{f}_1\left(y; \frac{\pi + \pi'}{2}, \bar{\theta}\right) \tilde{f}_2\left(y; \frac{\pi - \pi'}{2}, \bar{\theta}\right) e^{-1/8(\pi + \pi') C (\pi - \pi')}. \tag{12}
\end{aligned}$$

Thus, we see that $f_1 \widetilde{*} f_2$ in (12) coincides with $f_1 \widetilde{\cdot} f_2$ in (7), and then $(f_1 \cdot f_2)(\bar{\theta})$ in deformed superspace is mapped to $(f_1 * f_2)(\bar{\theta})$ in the ordinary superspace.

We can then formulate a field theory in the $C(y)$ -deformed superspace as defined above, by working in ordinary four-dimensional Euclidean superspace but multiplying superfields with the Moyal-Weyl product (10). Let us first discuss how the algebra of the supercharges and covariant derivatives are modified when a y -dependent C deformation is introduced. Supercharges and covariant derivatives in chiral coordinates take the form

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha}, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + 2i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \frac{\partial}{\partial y^\mu}, \tag{13}$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial y^\mu}, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}. \tag{14}$$

The covariant derivative algebra is not modified by (1) and the same happens for the supercharge-covariant derivative algebra. Concerning the supercharge algebra, it gets modified according to

$$\{Q_\alpha, Q_\beta\}_* = 0,$$

$$\{\bar{Q}_{\dot{\alpha}}, Q_\alpha\}_* = 2i\sigma_{\alpha\dot{\alpha}}^\mu \frac{\partial}{\partial y^\mu},$$

$$\begin{aligned}
\{\bar{Q}_{\dot{\beta}}, \bar{Q}_{\dot{\alpha}}\}_* &= -2\sigma_{\alpha\dot{\alpha}}^\mu \sigma_{\beta\dot{\beta}}^\nu \left(\partial_\mu C^{\alpha\beta} \frac{\partial}{\partial y^\nu} + \partial_\nu C^{\alpha\beta} \frac{\partial}{\partial y^\mu} \right. \\
&\quad \left. + 2C^{\alpha\beta} \frac{\partial^2}{\partial y^\mu \partial y^\nu} \right). \tag{15}
\end{aligned}$$

Only the chiral subalgebra generated by Q_α is still pre-

served, and this defines the chiral $\mathcal{N} = 1/2$ supersymmetry algebra. By a similar analysis, it can be seen that, as it happens for constant $C^{\alpha\beta}$, the complete $\mathcal{N} = 1$ superconformal group is broken for the y -dependent deformation to the subgroup generated by $\tilde{M}_{\mu\nu}, D_{\text{new}} \equiv D - \frac{1}{2}R, P_\mu, Q_\alpha, \bar{S}^{\dot{\alpha}}$, which is known as the $\mathcal{N} = 1/2$ superconformal group.

III. SCALAR AND VECTOR SUPERFIELDS

A chiral superfield Φ satisfies the condition $\bar{D}_{\dot{\alpha}} * \Phi = 0$. As usual, it can be written in the form

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{16}$$

The Moyal-Weyl product multiplication of two chiral superfields $\Phi_1(y, \theta)$ and $\Phi_2(y, \theta)$ takes the form

$$\begin{aligned}
\Phi_1(y, \theta) * \Phi_2(y, \theta) &= \Phi_1(y, \theta)\Phi_2(y, \theta) \\
&\quad - C^{\alpha\beta}(y)\psi_{1\alpha}(y)\psi_{2\beta}(y) \\
&\quad + \sqrt{2}C^{\alpha\beta}(y)\theta_\beta(\psi_{1\alpha}(y)F_2(y) \\
&\quad - \psi_{2\alpha}(y)F_1(y)) \\
&\quad - \det C(y)F_1(y)F_2(y). \tag{17}
\end{aligned}$$

It is easy to see that the right-hand side is a function of y and θ solely, so that the product of two chiral superfields is still a chiral superfield. This result could have been advanced, provided the covariant derivative $\bar{D}_{\dot{\alpha}}$ satisfies the Leibnitz rule. That this is the case can be seen by considering $\bar{D}_{\dot{\alpha}}$ acting on a product of two superfields,

$$\begin{aligned}
\bar{D}_{\dot{\alpha}}(\Phi(y, \theta, \bar{\theta}) * \Psi(y, \theta, \bar{\theta})) &= \bar{D}_{\dot{\alpha}} \left[\Phi \exp\left(-\frac{1}{2}C^{\alpha\beta}(y) \frac{\bar{\partial}}{\partial\theta^\alpha} \frac{\bar{\partial}}{\partial\theta^\beta}\right) \Psi \right] \\
&= \bar{D}_{\dot{\alpha}}(\Phi) \exp\left(-\frac{1}{2}C^{\alpha\beta}(y) \frac{\bar{\partial}}{\partial\theta^\alpha} \frac{\bar{\partial}}{\partial\theta^\beta}\right) \Psi + (-1)^{F[\Phi]} \Phi \exp\left(-\frac{1}{2}C^{\alpha\beta}(y) \frac{\bar{\partial}}{\partial\theta^\alpha} \frac{\bar{\partial}}{\partial\theta^\beta}\right) \bar{D}_{\dot{\alpha}} \Psi \\
&= \bar{D}_{\dot{\alpha}}(\Phi) * \Psi + (-1)^{F[\Phi]} \Phi * \bar{D}_{\dot{\alpha}} \Psi. \tag{18}
\end{aligned}$$

The case of antichiral superfields is more involved. An antichiral superfield is defined by $D_\alpha * \bar{\Phi} = 0$. As usual, $\bar{\Phi}$ depends only on $\bar{\theta}$ and the antichiral coordinates $\bar{y}^\mu = y^\mu - 2i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}}$. Written in terms of the chiral variable y^μ , $\bar{\Phi}$ takes the form

$$\begin{aligned} \bar{\Phi}(y - 2i\theta\sigma\bar{\theta}, \bar{\theta}) &= \bar{\phi}(y - 2i\theta\sigma\bar{\theta}) + \sqrt{2}\bar{\theta}\bar{\psi}(y - 2i\theta\sigma\bar{\theta}) + \bar{\theta}\bar{\theta}\bar{F}(y - 2i\theta\sigma\bar{\theta}) \\ &= \bar{\phi}(y) + \sqrt{2}\bar{\theta}\bar{\psi}(y) - 2i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}(y) + \bar{\theta}\bar{\theta}(\bar{F}(y) + i\sqrt{2}\theta\sigma^\mu\partial_\mu\bar{\psi}(y) + \theta\theta\partial^\mu\partial_\mu\bar{\phi}). \end{aligned} \quad (19)$$

The product of two antichiral superfields takes the form

$$\bar{\Phi}_1(y - 2i\theta\sigma\bar{\theta}, \bar{\theta}) * \bar{\Phi}_2(y - 2i\theta\sigma\bar{\theta}, \bar{\theta}) = \bar{\Phi}_1(y - 2i\theta\sigma\bar{\theta}, \bar{\theta})\bar{\Phi}_2(y - 2i\theta\sigma\bar{\theta}, \bar{\theta}) + 2\bar{\theta}\bar{\theta}C^{\mu\nu}(y)\partial_\mu\bar{\phi}_1(y)\partial_\nu\bar{\phi}_2(y). \quad (20)$$

Since the term $C^{\mu\nu}(y)\partial_\mu\bar{\phi}_1(y)\partial_\nu\bar{\phi}_2(y)$ appears multiplied by $\bar{\theta}\bar{\theta}$, its arguments can alternatively be taken as the antichiral coordinate \bar{y}^μ . It is then clear that the product of two antichiral superfields is another antichiral superfield. This result, however, turns out to be unexpected if one notes that, due to the coordinate dependence of the deformation $C^{\alpha\beta}$, the covariant derivative D_α violates the Leibnitz rule. One can see this from the product of two generic superfields

$$\begin{aligned} D_\alpha(\Phi(y, \theta, \bar{\theta}) * \Psi(y, \theta, \bar{\theta})) &= D_\alpha\left[\Phi \exp\left(-\frac{1}{2}C^{\alpha\beta}(y)\frac{\tilde{\partial}}{\partial\theta^\alpha}\frac{\tilde{\partial}}{\partial\theta^\beta}\right)\Psi\right] \\ &= D_\alpha(\Phi) \exp\left(-\frac{1}{2}C^{\alpha\beta}(y)\frac{\tilde{\partial}}{\partial\theta^\alpha}\frac{\tilde{\partial}}{\partial\theta^\beta}\right)\Psi + (-1)^{F[\Phi]}\Phi \exp\left(-\frac{1}{2}C^{\alpha\beta}(y)\frac{\tilde{\partial}}{\partial\theta^\alpha}\frac{\tilde{\partial}}{\partial\theta^\beta}\right)D_\alpha\Psi \\ &\quad + \Phi D_\alpha\left[\exp\left(-\frac{1}{2}C^{\alpha\beta}(y)\frac{\tilde{\partial}}{\partial\theta^\alpha}\frac{\tilde{\partial}}{\partial\theta^\beta}\right)\right]\Psi \\ &= D_\alpha(\Phi) * \Psi + (-1)^{F[\Phi]}\Phi * D_\alpha\Psi + \Phi D_\alpha(*)\Psi, \end{aligned} \quad (21)$$

where we have introduced the notation $D_\alpha(*)$ to denote

$$\Phi D_\alpha(*)\Psi = -2i(\sigma^\mu\bar{\theta})_\alpha\left(\frac{1}{2}\partial_\mu(C^{\beta\gamma})\Phi\frac{\tilde{\partial}}{\partial\theta^\beta}\frac{\tilde{\partial}}{\partial\theta^\gamma}\Psi + \partial_\mu(\det C)\Phi\frac{\tilde{\partial}}{\partial\theta^\theta}\frac{\tilde{\partial}}{\partial\theta^\theta}\Psi\right). \quad (22)$$

Let us now discuss vector superfields. We shall consider a $U(N_c)$ gauge group with Lie algebra Hermitian generators T^a satisfying $[T^a, T^b] = if^{abc}T^c$ and $\text{tr}T^aT^b = \frac{1}{2}\delta^{ab}$. In four-dimensional $\mathcal{N} = 1$ Euclidean superspace, no reality condition analogous to the Minkowski case can be imposed on superfields. Hence, a vector superfield V containing the gauge field can be defined as the one which changes under supergauge transformations according to

$$e^V \rightarrow e^{V'} = e^{-i\bar{\Lambda}} * e^V * e^{i\Lambda}, \quad (23)$$

where $V = V^aT^a$, and $\Lambda = \Lambda^aT^a$ and $\bar{\Lambda} = \bar{\Lambda}^aT^a$ are chiral and antichiral superfields taking values in the Lie algebra of $U(N_c)$. In all the expressions above, exponentials are defined through their *-product expansion,

$$e^{i\Omega} \equiv 1 + i\Omega + \frac{i^2}{2}\Omega * \Omega + \dots \quad (24)$$

Taking the standard expressions for the chiral and antichiral superfield strengths,

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar{D} * \bar{D} * e^{-V} * D_\alpha * e^V, \\ \bar{W}_{\dot{\alpha}} &= \frac{1}{4}D * D * e^V * \bar{D}_{\dot{\alpha}} * e^{-V}, \end{aligned} \quad (25)$$

and using Eqs. (18) and (21), one verifies that these superfields transform under an infinitesimal supergauge transformation according to

$$\begin{aligned} \delta W_\alpha &= i(W_\alpha * \Lambda - \Lambda * W_\alpha) - \frac{i}{4}\bar{D}\bar{D}[e^{-V} * (e^V D_\alpha(*)\Lambda - \bar{\Lambda}D(*)e^V)], \\ \delta \bar{W}_{\dot{\alpha}} &= i(\bar{W}_{\dot{\alpha}} * \bar{\Lambda} - \bar{\Lambda} * \bar{W}_{\dot{\alpha}}) + \frac{i}{2}D_\alpha(e^V * \bar{D}_{\dot{\alpha}}e^{-V}) \\ &\quad \times D^\alpha(*)\bar{\Lambda} - \frac{i}{2}\bar{\Lambda}D^\alpha(*)D_\alpha(e^V * \bar{D}_{\dot{\alpha}}e^{-V}). \end{aligned} \quad (26)$$

It is then clear that neither chiral nor antichiral superfield strengths transform covariantly under a general supergauge transformation unless the deformation parameter $C^{\alpha\beta}$ is constant. Gauge covariance cannot be invoked to transform an arbitrary vector superfield to the Wess-Zumino gauge. One can, however, still handle a $C = C(y)$ deformation if one starts with a vector superfield V already satisfying the Wess-Zumino condition. As in the case of constant deformation, it is convenient to identify the component fields in V according to

$$\begin{aligned}
V(y, \theta, \bar{\theta}) &= -\theta \sigma^\mu \bar{\theta} A_\mu(y) - i \bar{\theta} \bar{\theta} \theta^\alpha (\lambda_\alpha(y) \\
&\quad + \frac{1}{4} \varepsilon_{\alpha\beta} C^{\beta\gamma}(y) \sigma_{\gamma\dot{\gamma}}^\mu \{\bar{\lambda}^{\dot{\gamma}}(y), A_\mu(y)\}) \\
&\quad + i \theta \bar{\theta} \bar{\lambda}(y) + \frac{1}{2} \theta \bar{\theta} \bar{\theta} (D(y) - i \partial_\mu A^\mu(y)).
\end{aligned} \tag{27}$$

In this gauge

$$\begin{aligned}
V_*^2 &\equiv V * V \\
&= -\frac{1}{2} \bar{\theta} \bar{\theta} \left[\theta \theta A_\mu A^\mu + C^{\mu\nu} A_\mu A_\nu \right. \\
&\quad \left. - \frac{i}{2} \theta_\alpha C^{\alpha\beta} \sigma_{\beta\dot{\beta}}^\mu [A_\mu, \bar{\lambda}^{\dot{\beta}}] + \frac{1}{4} |C|^2 \bar{\lambda} \bar{\lambda} \right], \\
V_*^3 &= 0,
\end{aligned} \tag{28}$$

where $C^{\mu\nu} = C^{\alpha\beta} \varepsilon_{\beta\gamma} (\sigma^{\mu\nu})_\alpha^\gamma$ is self-dual, and $|C|^2 \equiv C^{\mu\nu} C_{\mu\nu} = 4 \det C$.

Chiral and antichiral superfield strengths written in components take the form

$$\begin{aligned}
W_\alpha &= W_\alpha(C=0) + \varepsilon_{\alpha\beta} C^{\beta\gamma} \theta_\gamma \bar{\lambda} \bar{\lambda}, \\
\bar{W}_{\dot{\alpha}} &= \bar{W}_{\dot{\alpha}}(C=0) - \bar{\theta} \bar{\theta} \left[\frac{C^{\mu\nu}}{2} \{F_{\mu\nu}, \lambda_{\dot{\alpha}}\} \right. \\
&\quad \left. + C^{\mu\nu} \left\{ A_\nu, \mathcal{D}_\mu \bar{\lambda}_{\dot{\alpha}} - \frac{i}{4} [A_\mu, \bar{\lambda}_{\dot{\alpha}}] \right\} \right. \\
&\quad \left. + \frac{i}{16} |C|^2 \{ \bar{\lambda} \bar{\lambda}, \bar{\lambda}_{\dot{\alpha}} \} + \partial_\mu C^{\mu\nu} \{ \bar{\lambda}_{\dot{\alpha}}, A_\nu \} \right],
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{i}{2} [A_\mu, A_\nu], \\
\mathcal{D}_\mu \bar{\lambda}_{\dot{\alpha}} &= \partial_\mu \bar{\lambda}_{\dot{\alpha}} + \frac{i}{2} [A_\mu, \bar{\lambda}_{\dot{\alpha}}].
\end{aligned} \tag{30}$$

One can still perform infinitesimal supergauge transformations preserving the Wess-Zumino gauge (27) through

$$\begin{aligned}
\Lambda &= -\varphi(y), \\
\bar{\Lambda} &= -\varphi(y) + 2i\theta \sigma^\mu \bar{\theta} \partial_\mu \varphi(y) - \theta \bar{\theta} \bar{\theta} \partial^\mu \partial_\mu \varphi(y) \\
&\quad - \frac{i}{2} \bar{\theta} \bar{\theta} C^{\mu\nu} \{ \partial_\mu \varphi, A_\nu \}.
\end{aligned} \tag{31}$$

As in Ref. [3], a particular parametrization of the coefficient of $\bar{\theta} \bar{\theta} \theta$ in V is adopted to ensure that the gauge transformation above acts on the component fields in the standard way,

$$\begin{aligned}
\delta A_\mu &= -2\partial_\mu \varphi + i[\varphi, A_\mu], & \delta \lambda_\alpha &= i[\varphi, \lambda_\alpha], \\
\delta \bar{\lambda}_{\dot{\alpha}} &= i[\varphi, \bar{\lambda}_{\dot{\alpha}}], & \delta D &= i[\varphi, D].
\end{aligned} \tag{32}$$

For the case of the supergauge transformation (31), the transformation of the superfield strengths reduces to

$$\begin{aligned}
\delta W_\alpha &= i(W_\alpha * \Lambda - \Lambda * W_\alpha), \\
\delta \bar{W}_{\dot{\alpha}} &= i(\bar{W}_{\dot{\alpha}} * \bar{\Lambda} - \bar{\Lambda} * \bar{W}_{\dot{\alpha}}) + 2\bar{\theta} \bar{\theta} \partial_\mu C^{\mu\nu} \{ \bar{\lambda}_{\dot{\alpha}}, \partial_\nu \varphi \}.
\end{aligned} \tag{33}$$

Finally, gauge covariance under the set of transformations (31) is achieved, provided the condition

$$\partial_\mu C^{\mu\nu} = 0 \tag{34}$$

holds.

As discussed for the case of constant deformations in Refs. [1–4], $C^{\mu\nu}$ is related to the graviphoton field-strength background through the formula $(\alpha')^2 F^{\mu\nu} = C^{\mu\nu}$, and $F^{\mu\nu}$ is taken as self-dual in order to avoid backreaction of the metric. It is then natural to interpret condition (34), that in our approach follows from gauge covariance arguments, as the graviphoton equation of motion, provided the relation between the coordinate-dependent deformation and a self-dual graviphoton field remains valid. Concerning supersymmetry, one easily checks, following the analysis in Ref. [4], that a coordinate-dependent self-dual graviphoton background does not affect the 4 chiral supercharges Q_α . In principle, the antichiral supercharges will be broken for a nonconstant background (but this should be investigated more thoroughly). As we shall see below from the Lagrangian written in components, condition (34) guarantees the gauge invariance of the theory.

Let us end this section with a comment on supersymmetry transformations. Infinitesimal SUSY transformations are generated by the operator $Q_\alpha = \partial/\partial\alpha_\alpha$ acting on superfields. As it is well known, this operation takes the vector superfield away from the Wess-Zumino (W-Z) gauge. For consistency, we have to be able to restore the W-Z gauge in V after the SUSY transformation, while maintaining supergauge invariance of $\text{tr} W^2$ and $\text{tr} \bar{W}^2$. The supergauge transformation restoring the W-Z gauge is in the present case generated by

$$\Lambda = 0, \quad \bar{\Lambda} = i\xi \sigma^\mu \bar{\theta} A_\mu - \bar{\theta} \bar{\theta} (\xi \lambda - \frac{1}{2} \xi \sigma_\mu \bar{\lambda} C^{\mu\nu} A_\nu). \tag{35}$$

It can be seen from (26) that both W and \bar{W} transform covariantly under the supergauge transformation (35). Such composition of SUSY and gauge transformations gives

$$\begin{aligned}
\delta A_\mu &= -i\bar{\lambda} \bar{\sigma}_\mu \xi \rightarrow \delta F_{\mu\nu} = i\xi (\sigma_\nu \mathcal{D}_\mu - \sigma_\mu \mathcal{D}_\nu) \bar{\lambda}, \\
\delta \lambda &= iD\xi + \sigma^{\mu\nu} \xi \left(F_{\mu\nu} + \frac{i}{2} C_{\mu\nu} \bar{\lambda} \bar{\lambda} \right), \\
\delta \bar{\lambda} &= 0, & \delta D &= -\xi \sigma^\mu \mathcal{D}_\mu \bar{\lambda}.
\end{aligned} \tag{36}$$

IV. EUCLIDEAN $\mathcal{N} = 1/2$ SQCD

From the results above, we see that once condition (34) is imposed, it should be possible to consistently construct a Lagrangian invariant under both generic supersymmetric

and particular supergauge transformations which correspond to the standard transformations of the component fields. The super Yang-Mills Lagrangian in $C(y)$ -deformed superspace takes the form

$$\mathcal{L}^{\text{SYM}} = \frac{1}{8g^2} \left(\text{tr} \int d^2\theta W^\alpha * W_\alpha + \text{tr} \int d^2\bar{\theta} \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\alpha}} \right). \quad (37)$$

Using the expressions (29) for the superfield strengths, the F terms are

$$\begin{aligned} \text{tr} W^\alpha * W_\alpha |_{\theta\theta} &= \text{tr} W^\alpha W_\alpha (C=0) |_{\theta\theta} - iC^{\mu\nu} \text{tr} F_{\mu\nu} \bar{\lambda} \bar{\lambda} \\ &\quad + \frac{|C|^2}{4} \text{tr} (\bar{\lambda} \bar{\lambda})^2, \\ \text{tr} \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\alpha}} |_{\bar{\theta}\bar{\theta}} &= \text{tr} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} (C=0) |_{\bar{\theta}\bar{\theta}} \\ &\quad - iC^{\mu\nu} \text{tr} F_{\mu\nu} \bar{\lambda} \bar{\lambda} + \frac{|C|^2}{4} \text{tr} (\bar{\lambda} \bar{\lambda})^2 \\ &\quad - 2i \text{tr} \partial_\mu (C^{\mu\nu} A_\nu \bar{\lambda} \bar{\lambda}). \end{aligned} \quad (38)$$

Then, disregarding the surface term, the $\mathcal{N} = 1/2$ super Yang-Mills Lagrangian in terms of the component fields is

$$\begin{aligned} \mathcal{L}_C^{\text{SYM}} &= \mathcal{L}_{C=0}^{\text{SYM}} - \frac{i}{4g^2} C^{\mu\nu}(y) \text{tr} F_{\mu\nu} \bar{\lambda} \bar{\lambda} \\ &\quad + \frac{1}{16g^2} |C(y)|^2 \text{tr} (\bar{\lambda} \bar{\lambda})^2, \end{aligned} \quad (39)$$

which coincides with the usual expression for $\mathcal{N} = 1/2$ deformed Super Yang-Mills theory for constant $C^{\alpha\beta}$ [3].

Let us now add matter fields in order to consider a supersymmetric version of QCD with $U(N_c)$ gauge group and N_f flavors defined in the deformed superspace. The matter fields are pairs of chiral superfields $\{\Phi, \Psi\}$ transforming as $\{\mathbf{N}_c, \bar{\mathbf{N}}_c\}$ multiplets of the color group. The super QCD (SQCD) Lagrangian is defined as

$$\mathcal{L}^{\text{SQCD}} = \mathcal{L}_C^{\text{SYM}} + \mathcal{L}_C^{\text{matter}}, \quad (40)$$

where we have redefined the super Yang-Mills (SYM) Lagrangian in order to incorporate a θ term,

$$\mathcal{L}_C^{\text{SYM}} = \frac{-i\tau}{32\pi} \int d^2\theta \text{tr} W^\alpha * W_\alpha + \frac{i\bar{\tau}}{32\pi} \int d^2\bar{\theta} \text{tr} \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\alpha}}, \quad (41)$$

with

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \quad (42)$$

and $\bar{\tau}$ its complex conjugate. Concerning the matter Lagrangian,

$$\begin{aligned} \mathcal{L}_C^{\text{matter}} &= \int d^4\theta (\bar{\Phi} * e^V * \Phi + \Psi * e^{-V} * \bar{\Psi}) \\ &\quad + \int d^2\theta m \Psi * \Phi + \int d^2\bar{\theta} \bar{m} \bar{\Phi} * \bar{\Psi}. \end{aligned} \quad (43)$$

The matter Lagrangian (43) is invariant under local $U(N_c)$ supergauge transformations,

$$\begin{aligned} (\Phi, \bar{\Phi}) &\rightarrow (e^{-i\Lambda} * \Phi, \bar{\Phi} * e^{i\bar{\Lambda}}), \\ (\Psi, \bar{\Psi}) &\rightarrow (\Psi * e^{i\Lambda}, e^{-i\bar{\Lambda}} * \bar{\Psi}), \\ e^V &\rightarrow e^{-i\bar{\Lambda}} * e^V * e^{i\Lambda}. \end{aligned} \quad (44)$$

In order to have the ordinary gauge transformation laws for the component fields under the supergauge transformation generated by (31), we parametrize the matter superfields as in Ref. [15],

$$\begin{aligned} \Phi(y, \theta) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F_\phi(y), \\ \bar{\Phi}(\bar{y}, \bar{\theta}) &= \bar{\phi}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) \\ &\quad + \bar{\theta}\bar{\theta}(\bar{F}_{\bar{\phi}} + iC^{\mu\nu}\partial_\mu(\bar{\phi}A_\nu) - \frac{1}{4}C^{\mu\nu}\bar{\phi}A_\mu A_\nu)(\bar{y}), \\ \Psi(y, \theta) &= \eta(y) + \sqrt{2}\theta\chi(y) + \theta\theta F_\eta(y), \\ \bar{\Psi}(\bar{y}, \bar{\theta}) &= \bar{\eta}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\chi}(\bar{y}) \\ &\quad + \bar{\theta}\bar{\theta}(\bar{F}_{\bar{\eta}} + iC^{\mu\nu}\partial_\mu(A_\nu\bar{\eta}) - \frac{1}{4}C^{\mu\nu}A_\mu A_\nu\bar{\eta})(\bar{y}). \end{aligned} \quad (45)$$

Written in components, infinitesimal transformations read

$$\begin{aligned} \delta\phi &= i\varphi\phi, & \delta\bar{\phi} &= -i\bar{\phi}\varphi, & \delta\psi &= i\varphi\psi, \\ \delta\bar{\psi} &= -i\bar{\psi}\varphi, & \delta F_\phi &= i\varphi F_\phi, & \delta\bar{F}_{\bar{\phi}} &= -i\bar{F}_{\bar{\phi}}\varphi, \\ \delta\bar{\eta} &= i\varphi\bar{\eta}, & \delta\eta &= -i\eta\varphi, & \delta\bar{\chi} &= i\varphi\bar{\chi}, \\ \delta\chi &= -i\chi\varphi, & \delta\bar{F}_{\bar{\eta}} &= i\varphi\bar{F}_{\bar{\eta}}, & \delta F_\eta &= -iF_\eta\varphi. \end{aligned} \quad (46)$$

The different terms in the matter Lagrangian written in components fields read

$$m\Psi * \Phi |_{\theta\theta} = m\eta F_\phi + mF_\eta\phi - m\chi\psi, \quad (47)$$

$$\bar{m}\bar{\Phi} * \bar{\Psi} |_{\bar{\theta}\bar{\theta}} = \bar{m}\bar{\phi}\bar{F}_{\bar{\eta}} + \bar{m}\bar{F}_{\bar{\phi}}\bar{\eta} - \bar{m}\bar{\psi}\bar{\chi} + \frac{i}{2}\bar{m}C^{\mu\nu}\bar{\phi}F_{\mu\nu}\bar{\eta} + i\bar{m}C^{\mu\nu}\partial_\mu(\bar{\phi}A_\nu\bar{\eta}) + 2\bar{m}C^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\eta}, \quad (48)$$

$$\begin{aligned}
\bar{\Phi} * e^V * \Phi|_{\theta\theta\bar{\theta}\bar{\theta}} + \Psi * e^{-V} * \bar{\Psi}|_{\theta\theta\bar{\theta}\bar{\theta}} &= \bar{\Phi} e^V \Phi(C=0)|_{\theta\theta\bar{\theta}\bar{\theta}} + \Psi e^{-V} \bar{\Psi}(C \\
&= 0)|_{\theta\theta\bar{\theta}\bar{\theta}} + \frac{i}{2} C^{\mu\nu} \bar{\phi} F_{\mu\nu} F_\phi - \frac{1}{16} |C|^2 \bar{\phi} \bar{\lambda} \bar{\lambda} F_\phi - \frac{\sqrt{2}}{2} \overline{D_\mu \phi} (\sigma^\mu \bar{\lambda})_\alpha C^{\alpha\beta} \psi_\beta \\
&\quad + \frac{i}{2} C^{\mu\nu} F_\eta F_{\mu\nu} \bar{\eta} - \frac{1}{16} |C|^2 F_\eta \bar{\lambda} \bar{\lambda} \bar{\eta} - \frac{\sqrt{2}}{2} \chi_\alpha C^{\alpha\beta} (\sigma^\mu \bar{\lambda})_\beta D_\mu \bar{\eta},
\end{aligned} \tag{49}$$

where

$$\bar{\Phi} e^V \Phi(C=0)|_{\theta\theta\bar{\theta}\bar{\theta}} = \bar{F}_{\bar{\phi}} F_\phi - i \bar{\psi} \bar{\sigma}^\mu D_\mu \psi - \overline{D_\mu \phi} D^\mu \phi + \frac{1}{2} \bar{\phi} D \phi + \frac{i}{\sqrt{2}} (\bar{\phi} \lambda \psi - \bar{\psi} \bar{\lambda} \phi), \tag{50}$$

$$\Psi e^{-V} \bar{\Psi}(C=0)|_{\theta\theta\bar{\theta}\bar{\theta}} = F_\eta \bar{F}_{\bar{\eta}} - i \chi \sigma^\mu D_\mu \bar{\chi} - \bar{D}_\mu \eta D^\mu \bar{\eta} - \frac{1}{2} \eta D \bar{\eta} + \frac{i}{\sqrt{2}} (\eta \bar{\lambda} \bar{\chi} - \chi \lambda \bar{\eta}), \tag{51}$$

and

$$\begin{aligned}
D_\mu \phi &= \partial_\mu \phi + \frac{i}{2} A_\mu \phi, & \overline{D_\mu \phi} &= \partial_\mu \bar{\phi} - \frac{i}{2} \bar{\phi} A_\mu, & D_\mu \psi &= \partial_\mu \psi + \frac{i}{2} A_\mu \psi, & D_\mu \bar{\eta} &= \partial_\mu \bar{\eta} + \frac{i}{2} A_\mu \bar{\eta}, \\
\bar{D}_\mu \eta &= \partial_\mu \eta - \frac{i}{2} \eta A_\mu, & D_\mu \bar{\chi} &= \partial_\mu \bar{\chi} + \frac{i}{2} A_\mu \bar{\chi}.
\end{aligned} \tag{52}$$

Putting all this together, the Euclidean $\mathcal{N} = 1/2$ SQCD Lagrangian in components takes the form

$$\mathcal{L}^{\text{SQCD}} = \mathcal{L}_{C=0}^{\text{SQCD}} + \sum_{i=1}^6 \mathcal{L}_i, \tag{53}$$

$$\begin{aligned}
\mathcal{L}_1 &= -\frac{i}{8g^2} C^{\mu\nu}(y) \text{tr} F_{\mu\nu} \bar{\lambda} \bar{\lambda} + \frac{1}{32g^2} |C(y)|^2 \text{tr} (\bar{\lambda} \bar{\lambda})^2, & \mathcal{L}_2 &= \frac{i}{2} C^{\mu\nu}(y) \bar{\phi} F_{\mu\nu} F_\phi - \frac{1}{16} |C(y)|^2 \bar{\phi} \bar{\lambda} \bar{\lambda} F_\phi, \\
\mathcal{L}_3 &= -\frac{\sqrt{2}}{2} \overline{D_\mu \phi} (\sigma^\mu \bar{\lambda})_\alpha C^{\alpha\beta}(y) \psi_\beta, & \mathcal{L}_4 &= \frac{i}{2} C^{\mu\nu}(y) F_\eta F_{\mu\nu} \bar{\eta} - \frac{1}{16} |C(y)|^2 F_\eta \bar{\lambda} \bar{\lambda} \bar{\eta}, \\
\mathcal{L}_5 &= -\frac{\sqrt{2}}{2} C^{\alpha\beta}(y) \chi_\alpha (\sigma^\mu \bar{\lambda})_\beta D_\mu \bar{\eta}, & \mathcal{L}_6 &= \frac{i\bar{m}}{2} C^{\mu\nu}(y) \bar{\phi} F_{\mu\nu} \bar{\eta}.
\end{aligned} \tag{54}$$

In obtaining (54), we have used Eq. (34) and discarded surface terms. Auxiliary fields can be eliminated using their equations of motion,

$$\begin{aligned}
F_\phi &= -\bar{m} \bar{\eta}, & \bar{F}_{\bar{\phi}} &= -m \eta - \frac{i}{2} C^{\mu\nu} \bar{\phi} F_{\mu\nu} + \frac{1}{16} |C|^2 \bar{\phi} \bar{\lambda} \bar{\lambda}, & F_\eta &= -\bar{m} \bar{\phi}, \\
\bar{F}_{\bar{\eta}} &= -m \phi - \frac{i}{2} C^{\mu\nu} F_{\mu\nu} \bar{\eta} + \frac{1}{16} |C|^2 \bar{\lambda} \bar{\lambda} \bar{\eta}.
\end{aligned} \tag{55}$$

The $\mathcal{N} = 1/2$ supersymmetric variations of the matter component fields under which this Lagrangian is invariant are [15]

$$\begin{aligned}
\delta \phi &= \sqrt{2} \xi \psi, & \delta \bar{\phi} &= 0, & \delta \psi &= \sqrt{2} \xi F_\phi, & \delta \bar{\psi}_{\dot{\alpha}} &= -i \sqrt{2} \overline{D_\mu \phi} (\xi \sigma^\mu)_{\dot{\alpha}}, & \delta F_\phi &= 0, \\
\delta \bar{F}_{\bar{\phi}} &= -i \sqrt{2} \overline{D_\mu \psi} \bar{\sigma}^\mu \xi - i \bar{\phi} \xi \lambda + C^{\mu\nu} \bar{D}_\mu (\bar{\phi} \xi \sigma_\nu \bar{\lambda}), & \delta \eta &= \sqrt{2} \xi \chi, & \delta \bar{\eta} &= 0, & \delta \chi &= \sqrt{2} \xi F_\eta, \\
\delta \bar{\chi}_{\dot{\alpha}} &= -i \sqrt{2} D_\mu \bar{\eta} (\xi \sigma^\mu)_{\dot{\alpha}}, & \delta F_\eta &= 0, & \delta \bar{F}_{\bar{\eta}} &= -i \sqrt{2} D_\mu \bar{\chi} \bar{\sigma}^\mu \xi - i \xi \lambda \bar{\eta} + C^{\mu\nu} D_\mu (\xi \sigma_\nu \bar{\lambda} \bar{\eta}).
\end{aligned} \tag{56}$$

It was pointed out in Ref. [16] that the variation of the C -deformed super Yang-Mills Lagrangian (with constant $C_{\alpha\beta}$) can be written as a Q commutator. Then, if supersymmetry is not spontaneously broken, the partition function and, in general, correlation functions of Q invariant operators will not depend on C . The extension to the case in which massless matter fields are present was considered in Ref. [17], also for the constant C case. One can see that such a formal analysis can be done in the case of a $\mathcal{N} = 1/2$ SQCD Lagrangian (54) with $C = C(y)$. Indeed, after some straightforward calculations, one finds

$$\begin{aligned}
\frac{\delta \mathcal{L}_1}{\delta C^{\mu\nu}} &= \frac{i}{16g^2} \text{tr}\{Q^\alpha, (\sigma_{\mu\nu})_{\alpha\beta} \lambda^\beta \bar{\lambda}\}, & \frac{\delta \mathcal{L}_2}{\delta C^{\mu\nu}} &= \frac{i}{4} \left\{ Q^\alpha, \bar{\phi} (\sigma_{\mu\nu})^\beta_{\alpha} \lambda_\beta F_\phi + i \frac{\sqrt{2}}{8} C_{\mu\nu} \bar{\phi} \bar{\lambda} \bar{\lambda} \psi_\alpha \right\}, \\
\frac{\delta \mathcal{L}_3}{\delta C^{\mu\nu}} &= \frac{i}{4} \{Q^\alpha, \bar{\psi} \bar{\lambda} (\sigma_{\mu\nu})^\beta_{\alpha} \psi_\beta\}, & \frac{\delta \mathcal{L}_4}{\delta C^{\mu\nu}} &= \frac{i}{4} \left\{ Q^\alpha, F_\eta (\sigma_{\mu\nu})^\beta_{\alpha} \lambda_\beta \bar{\eta} + i \frac{\sqrt{2}}{8} C_{\mu\nu} \chi_\alpha \bar{\lambda} \bar{\eta} \right\}, \\
\frac{\delta \mathcal{L}_5}{\delta C^{\mu\nu}} &= -\frac{i}{4} \{Q^\alpha, (\sigma_{\mu\nu})^\beta_{\alpha} \chi_\beta \bar{\chi}\}, & \frac{\delta \mathcal{L}_6}{\delta C^{\mu\nu}} &= \frac{\bar{m}}{2} \left\{ Q^\alpha, \bar{\phi} (\sigma_{\mu\nu})^\beta_{\alpha} \lambda_\beta - \frac{1}{4} C_{\mu\nu} \bar{\phi} A_\rho (\sigma^\rho \bar{\lambda})_\alpha \bar{\eta} \right\}.
\end{aligned} \tag{57}$$

It should be stressed that SUSY transformations (56) were used in order to write the different C -dependent terms in the Lagrangian as Q commutators. To confirm that the connection still holds at the quantum level, one should analyze whether no anomalous contributions modify the classical identities. To this end, one should proceed to a calculation similar to that presented at the end of this section in the analysis of Konishi anomaly and gluino condensation. We leave the details of the complete analysis confirming these identities for a forthcoming work.

At the formal level, if we assume that supersymmetry is not spontaneously broken, and, hence, $Q|0\rangle = 0$, we can then write

$$\frac{1}{Z} \frac{\delta Z}{\delta C^{\mu\nu}(y)} = 0 \tag{58}$$

with

$$Z = \int D \text{ fields} \exp\left(-\int d^4x \mathcal{L}^{\text{SQCD}}\right). \tag{59}$$

A. Konishi anomaly and gluino condensation

Given the Lagrangian (53), it is easy to verify that the anomalous commutator leading to the gluino condensation in the $\mathcal{N} = 1/2$ supersymmetric theory with coordinate-dependent deformation gives the same answer as in the undeformed case. That is, we shall show that the following relation holds:

$$\frac{1}{2\sqrt{2}} \{Q^\alpha, \chi_\alpha(y) \bar{\eta}(y)\} = -\bar{m} \bar{\phi} \bar{\eta}(y) + \frac{g^2}{32\pi^2} \bar{\lambda} \bar{\lambda}(y), \tag{60}$$

where the last term, corresponding to the Konishi anomaly [18,19], results from a consistent regularization of the ill-defined product in the commutator [20]. Consider, for instance, the point splitting method where one defines

$$\begin{aligned}
\frac{1}{2\sqrt{2}} \{Q^\alpha, \chi_\alpha(y) \bar{\eta}(y)\}_{\text{reg}} &\equiv \lim_{\epsilon \rightarrow 0} \frac{1}{2\sqrt{2}} \{Q^\alpha, \chi_\alpha(y + \epsilon) \\
&\quad \times \exp(-i\epsilon^\mu A_\mu) \bar{\eta}(y - \epsilon)\} \\
&= \{Q^\alpha, \chi_\alpha(y) \bar{\eta}(y)\}_{\text{naive}} \\
&\quad + \lim_{\epsilon \rightarrow 0} \epsilon^\mu \chi_\alpha(y + \epsilon) \sigma_\mu^{\alpha\dot{\alpha}} \bar{\lambda}_{\dot{\alpha}}(y) \\
&\quad \times \bar{\eta}(y - \epsilon).
\end{aligned} \tag{61}$$

When inserted in a correlation function containing a product of local operators, the second term in the right-hand side of (61) gives a finite contribution in the $\epsilon \rightarrow 0$ limit. This is due to a contribution from a linear ultraviolet divergent term that results from the integration of a loop containing propagators that arise from contractions with the Yukawa interaction term $\eta \bar{\lambda} \bar{\chi}$ present in $\mathcal{L}^{\text{SQCD}}_{C=0}$. The final answer coincides with Eq. (60). One can also check that the new $C(y)$ -dependent vertices in Lagrangian (54) and (55) do not give rise to new finite contributions so that Eq. (60) holds for $C(y) \neq 0$ as it does in undeformed superspace.

V. SUMMARY AND DISCUSSION

Our work was motivated by the observation in Ref. [12] relating the spectral degeneracy in conventional $\mathcal{N} = 1$ SUSY gluodynamics with a C deformation of the anticommuting superspace coordinates, suggesting that $\mathcal{N} = 1/2$ supersymmetry might be defined for a coordinate-dependent C parameter.

In contrast with the case of ordinary noncommutative geometry, where implementation of an associative $*$ product becomes rather complicated for space-time dependent $\theta_{\mu\nu}(x)$ [21] (see also [22], and references therein), the case in which $C_{\alpha\beta}$ depends on the chiral variable y can be rather simply handled and a Moyal-Weyl star product can be defined [according to Eq. (10)] so that associativity and other basic properties remain valid (see also [23]). One can then see that the subalgebra generated by Q_α is still preserved, and, hence, as in the constant C case, a chiral $\mathcal{N} = 1/2$ supersymmetry can be defined with the superconformal group broken to the so-called $\mathcal{N} = 1/2$ superconformal group. Multiplication of chiral superfields proceeds as in the constant C case, while that of antichiral ones is more involved because the Leibnitz rule for the derivative of a product ceases to be valid. Concerning vector superfields, a remarkable condition arises when studying the covariance of superfield strengths, namely, $\partial_\mu C^{\mu\nu} = 0$, which is consistent with the requirement of self-duality of the graviphoton field background present in the supergravity model at the origin of (constant) C deformations.

With all these ingredients, a $\mathcal{N} = 1/2$ SQCD Lagrangian was constructed and, from its expression in components, the effects of the deformation were discussed. In particular, studying the Konishi anomaly, we confirmed that, as suggested in Ref. [12], the anomalous commutator

contribution leading to gluino condensation has the same form as in the ordinary case.

Various interesting issues related to our work can be envisaged. In particular, one should analyze whether anomalous terms arise when computing at the quantum level commutators such as those in Eq. (57) as it happens in (61) [18]. Another line to pursue concerns the corrections introduced by the coordinate-dependent deformation on Bogomol'nyi-Prasad-Sommerfield equations, as was already discussed for constant C in Refs. [24–28]. We hope to come back to these issues and those related to the connection with string theory dynamics in nontrivial

graviphoton background elsewhere.

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