PHYSICAL REVIEW D **85,** 086007 (2012)

Diagnostics of plasma photoemission at strong coupling

Babiker Hassanain*

The Rudolf Peierls Centre for Theoretical Physics, Department of Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, United Kingdom

Martin Schvellinger†

IFLP-CCT-La Plata, CONICET, and Departamento de Física, Universidad Nacional de La Plata, Calle 49 y 115, C.C. 67, (1900) La Plata, Buenos Aires, Argentina

(Received 7 October 2011; published 23 April 2012)

We compute the spectrum of photons emitted by the finite-temperature large-N $SU(N)$ $\mathcal{N}=4$ supersymmetric Yang-Mills plasma coupled to electromagnetism, at strong yet finite 't Hooft coupling. We work in the holographic dual description, extended by the inclusion of the full set of $\mathcal{O}(\alpha'^3)$ type IIB string theory operators that correct the minimal supergravity action. We find that, as the t' Hooft coupling decreases, the peak of the spectrum increases, and the momentum of maximal emission shifts toward the infrared, as expected from weak-coupling computations. The total number of emitted photons also increases as the 't Hooft coupling weakens.

DOI: 10.1103/PhysRevD.85.086007 PACS numbers: 11.25.Tq, 12.38.Mh

I. INTRODUCTION

The analysis of data from heavy ion collision experiments at RHIC and LHC indicates that the quark-gluon plasmas (QGPs) produced are in the strongly coupled regime [1], where the 't Hooft coupling (λ) governing the interactions of the microscopic constituents of the plasma is larger than 1. Being electrically charged, these microscopic constituents will emit photons. The number of photons emitted with a given momentum, i.e. the photoemission spectrum, yields valuable information about the structure of the plasma. A theoretical study of this spectrum at strong coupling is therefore an essential step for investigating the QGP. Gauge/string duality lends itself perfectly to such a computation, because it allows the investigation of strongly coupled gauge theories in terms of their weakly coupled supergravity dual description [2]. Although there is no complete string theory dual model which accounts for all the relevant properties of QCD, the microscopic theory governing the behavior of plasma produced at RHIC and LHC, one can attempt to approach the real world using the holographic dual of the large-N $SU(N)$ $\mathcal{N}=4$ super Yang-Mills (SYM) plasma. Moreover, holography stipulates that λ in the gauge theory maps to α'^{-2} in the string dual, so that the minimal supergravity (i.e. the zeroth-order string theory description in a small-curvature expansion), obtained for $\alpha' \to 0$, corresponds to the gauge theory at $\lambda \to \infty$. With these two caveats in mind, the work of [3] considered two limits: photoemission from infinitely strongly coupled SYM plasma, tackled via minimal type IIB supergravity on the anti de Sitter (AdS)-Schwarzschild black hole (BH) $\mathrm{AdS_{BH}} \times S^5$, and photoemission from weakly coupled SYM plasma, computed using the traditional tools of perturbative quantum field theory. The real-world QGP lies somewhere in between these two illuminating yet unrealistic regimes. Our aim in this letter is to compute the photoemission rate of $\mathcal{N}=4$ SYM plasma at large finite λ. We work in the holographic dual extended by the inclusion of the full $\mathcal{O}(\alpha'^3)$ type IIB string theory corrections to the supergravity action. The relation $\lambda \sim \alpha'^{-2}$ immediately dictates that the corrections to the $\lambda \to \infty$ result start at $\mathcal{O}(\lambda^{-3/2})$. We compute characteristic properties of the photoemission spectrum at large finite λ, such as the evolution of the height and position of the photoemission peak as a function of λ. We thereby quantify the interpolation between the photoemission spectrum from strongly coupled plasma and that from weakly coupled plasma.

II. PHOTOEMISSION RATE AND SPECTRAL FUNCTION

$\mathcal{N}=4$ SYM theory is a supersymmetric gauge theory with gluons, fermions, and scalars all in the adjoint representation of $SU(N)$, and a (global) R-symmetry group $SU(4)$. To model electromagnetism in this theory, one of the $U(1)$ subgroups of the R-symmetry group is gauged with coupling e. The Lagrangian can then be written as [3]

$$L = L_{\mathrm{SYM}} + eJ^{\mathrm{em}}_{\mu}\mathcal{A}^{\mu} - \tfrac{1}{4}\mathcal{F}^2, \tag{1}$$

where L_{SYM} is the Lagrangian of $\mathcal{N}=4$ SYM theory, and the interactions internal to this Lagrangian are governed by the 't Hooft coupling $\lambda = g^2_{\mathrm{YM}}N$. The field $\mathcal{A}^{\mu}$ is the photon (introduced by hand), with kinetic term $\mathcal{F}^2$, and J^{em}_{μ} is the electromagnetic current. The number of photons produced by a thermally equilibrated plasma per unit time per unit volume is given by

*babiker@thphys.ox.ac.uk
†martin@fisica.unlp.edu.ar

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{\rm em}}{\pi} k n_b(k) \eta^{\mu\nu} \chi_{\mu\nu}(k), \tag{2}$$

where k is the 3-momentum of the (on-shell) photon, $n_b(k) = 1/(e^{k/T} - 1)$, T is the temperature, and $\alpha_{\rm em} \equiv e^2/4\pi$. This holds to all orders in λ, and to leading order in e. The quantity $\chi_{\mu\nu}(k)$ is the lightlike spectral density of the plasma defined via the retarded electromagnetic current correlator $R_{\mu\nu}$ as $\chi_{\mu\nu} = -2\,{\rm Im} R_{\mu\nu}$, where

$$R_{\mu\nu}(k) = -i\int d^4x e^{iK.X}\Theta(t) < [J^{\rm em}_\mu(x), J^{\rm em}_\nu(0)] >, \tag{3}$$

$\Theta(t)$ is the step function, and $K = (k, \vec{k})$, with $k = |\vec{k}|$.

We wish to compute $R_{\mu\nu}$ using holography, but we do not have the holographic dual of the theory defined by Eq. (1); we do, however, have the holographic dual of pure $\mathcal{N} = 4$ SYM. The crucial point is that, to leading order in e, the electromagnetic current $J^{\rm em}_\nu = J_\nu$, where J_ν is purely the R-symmetry current associated with the $U(1)$ subgroup. Therefore, the two-point function of the electromagnetic current can be replaced by the two-point function of the R-symmetry current, computed entirely within the $\mathcal{N} = 4$ SYM itself [3]. Our aim in this paper is to compute the retarded correlator of the R-symmetry currents of $\mathcal{N} = 4$ SYM at strong 't Hooft coupling, retrieve the spectral function $\chi_{\mu\nu}$, and insert the latter into Eq. (2) to obtain the photoemission rate. Given that we wish to keep λ strong yet finite, we must therefore work in the supergravity dual to $\mathcal{N} = 4$ SYM plasma extended by the addition of finite α' corrections.

III. TYPE IIB STRING THEORY SETUP AT $\mathcal{O}(\alpha'^3)$

The type IIB supergravity action corrected to $\mathcal{O}(\alpha'^3)$ is $S_{\rm IIB} = S^0_{\rm IIB} + S^{\alpha'}_{\rm IIB}$, where

$$S^0_{\rm IIB} = \frac{1}{2\kappa^2_{10}}\int d^{10}x\sqrt{-G}\left[R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4.5!}(F_5)^2\right], \tag{4}$$

where ϕ is the dilaton, F_5 the five-form, and R_{10} is the curvature. The leading 't Hooft coupling corrections are contained in $S^{\alpha'}_{\rm IIB}$, and given schematically by [4,5]

$$S^{\alpha'}_{\rm IIB} = \frac{\gamma R^6}{2\kappa^2_{10}}\int d^{10}x\sqrt{-G}e^{-\frac{3}{2}\phi}(C + \mathcal{T})^4, \tag{5}$$

where $\gamma = \frac{1}{8}\zeta(3)(\alpha'/R^2)^3 = \frac{1}{8}\zeta(3)\lambda^{-3/2}$, with $R^4 = g^2_{YM} N\alpha'^2$ and ζ is the Riemann Zeta function. C is the ten-dimensional (10D) Weyl tensor, and $\mathcal{T}_{abcdef}$ is defined as

$$i\nabla_a F^+_{bcdef} + \tfrac{1}{16}(F^+_{abcmn}F^{+mn}_{def} - 3F^+_{abfmn}F^{+mn}_{dec}),$$

where the right-hand side is antisymmetrized in $[a, b, c]$ and $[d, e, f]$ and symmetrized with respect to interchange of $abc \leftrightarrow def$ [5]. Also, $F^+ = \frac{1}{2}(1 + *)F_5$. The operators in Eq. (5) are dimension-eight operators obtained by various independent contractions of C and $\mathcal{T}$ that can be found in [5]. The background solution to this action is the corrected metric G_{MN} [6], with $f(u) = 1 - u^2$,

$$ds^2 = \left(\frac{r_0}{R}\right)^2\frac{1}{u}(-f(u)K^2(u)dt^2 + d\vec{x}^2) + \frac{R^2}{4u^2 f(u)}P^2(u)du^2 + R^2L^2(u)d\Omega^2_5, \tag{6}$$

where $d\Omega^2_5$ is the line element on the S^5, and

$$K(u) = e^{\gamma[a(u)+4b(u)]}, \quad P(u) = e^{\gamma b(u)}, \quad L(u) = e^{\gamma c(u)},$$

where the exponents are given by the expressions

$$\begin{aligned} a(u) &= -\tfrac{1625}{8}u^2 - 175u^4 + \tfrac{10\,005}{16}u^6, \\ b(u) &= \tfrac{325}{8}u^2 + \tfrac{1075}{32}u^4 - \tfrac{4835}{32}u^6, \\ c(u) &= \tfrac{15}{32}(1 + u^2)u^4. \end{aligned} \tag{7}$$

The extremality parameter is $r_0 = \pi T R^2/(1 + 265\gamma/16)$, where T is the physical equilibrium temperature of the plasma. The boundary of the AdS space is at $u = 0$, and the horizon of the black hole at $u = 1$. In addition, both F_5 and the dilaton have nontrivial background solutions, but their explicit forms are of no consequence in this work, as we explain shortly. The crucial point is that the tensor $\mathcal{T}$ is zero for the background solution [4].

IV. THE VECTOR PERTURBATION

We now pursue the usual recipe involved in all holographic computations: firstly, perturb the supergravity background along the directions ψ which are dual to the field theory operators $\mathcal{J}$ whose correlation functions we are interested in, and plug the perturbed background into $S_{\rm IIB}$. This yields the action $\mathcal{S}(\psi)$ of the perturbation ψ. Then, solve the equations of motion (EOM) of $\mathcal{S}(\psi)$ subject to $\psi = \psi_0$ on the boundary of the space $u = 0$, and evaluate the on-shell action for these solutions, giving $\mathcal{Z}(\psi_0)$, the generating functional of correlation functions of the operators $\mathcal{J}$. Differentiating $\mathcal{Z}(\psi_0)$ twice with respect to ψ_0 yields $<\mathcal{J}.\mathcal{J}>$, and we are done. The details of this prescription in real time are described in [7]. For the present case, the perturbation field ψ dual to the R-symmetry currents J_μ of the four-dimensional theory is the vector perturbation A_μ of the gravitational background obtained as a solution of the EOM of $S_{\rm IIB}$. The vector perturbation A_μ perturbs the metric *and* the F_5 solution, yielding

$$ds^2 = g_{mn}dx^m dx^n + R^2L(u)^2\sum_{i=1}^{3}\left[d\mu_i^2 + \mu_i^2\left(d\phi_i + \frac{2}{\sqrt{3}}A_\mu dx^\mu\right)^2\right], \tag{8}$$

where $g_{mn} = G_{mn}$ for $m, n \in [0, 4]$ and μ_i are the direction cosines for the sphere, and

$$F_5 = -\frac{4}{R}\bar{\epsilon} + \frac{R^3 L(u)^3}{\sqrt{3}}\left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i\right) \wedge \bar{*}F_2, \quad (9)$$

where $F_2 = dA$ is the Abelian field strength of A_μ and $\bar{\epsilon}$ is a deformation of the volume form of the metric of the $\mathrm{AdS_{BH}}$. The Hodge duals $*$ and $\bar{*}$ are taken with respect to the 10D metric and five-dimensional-$\mathrm{AdS_{BH}}$ metric, respectively. Notice that the $\gamma \to 0$ limits of Eqs. (8) and (9) are known exactly, see for instance [8]. We assume that Eqs. (8) and (9) are correct to linear order in the gauge fields.

We will insert these *Ansätze* of Eqs. (8) and (9) into $S_{\mathrm{IIB}} = S^0_{\mathrm{IIB}} + S^{\alpha'}_{\mathrm{IIB}}$ below, obtaining an effective Lagrangian for A_μ which is at most *quadratic* in A_μ. Two important points must be stated to this end, dictated by the fact that we work strictly to linear order in γ. For insertion into S^0_{IIB}, we require the *Ansätze* to linear order in γ. However, S^0_{IIB} only contains quadratic powers of F_5 and, therefore, the $\bar{\epsilon}$ part of the F_5 *Ansatz cannot* contribute to the quadratic effective Lagrangian of A_μ. On the other hand, $S^{\alpha'}_{\mathrm{IIB}}$ contains operators which are higher than quadratic in F_5, so here $\bar{\epsilon}$ can contribute to the quadratic Lagrangian for A_μ, but the crucial point is that $S^{\alpha'}_{\mathrm{IIB}}$ contains an explicit factor of γ already, and so we only require $\bar{\epsilon}$ to zero order in γ. This is of course nothing but the volume form on the AdS space [8]. The incredibly simplifying upshot of these observations is that *we do not require the $\mathcal{O}(\gamma)$ terms in $\bar{\epsilon}$ for our computation*. This is why we do not care to state the explicit form of $\bar{\epsilon}$. All we need to know is $\lim_{\gamma\to 0}\bar{\epsilon}$. The same observations can be made for the contribution to the effective Lagrangian of A_μ of operators containing the dilaton (and in fact any other field which is trivial in the zero-order background supergravity solution). These statements make the following work possible.

V. THE EFFECTIVE LAGRANGIAN OF THE VECTOR PERTURBATION

Without loss of generality, we may set the photon momenta to $(k, 0, 0, k)$. In order to study the photoemission rate, we only need to consider the transverse fluctuation $A_x(t, x, u)$. Inserting the *Ansätze* of Eqs. (8) and (9) into S_{IIB}, and integrating out the S^5, we obtain a complicated Lagrangian for $A_x(t, x, u)$. This action can be coverted into a more useful form by use of the following field redefinitions: write $\Psi(u) = A_k(u)/(\sqrt{f(u)}[1 + \gamma p(u)])$, where $p(u)$ is a polynomial in u beginning at $\mathcal{O}(u^2)$ and $A_k(u)$ is the Fourier transform of $A_x(t, x, u)$. This takes us into the Schrödinger basis, such that the action is given by

$$S = -\frac{N^2 r_0^2}{16\pi^2 R^4}\int \frac{d^4k}{(2\pi)^4}\int_0^1 du\left[\frac{1}{2}\Psi\mathcal{L}\Psi + \partial_u\Phi\right], \quad (10)$$

where $\mathcal{L}\Psi(u) = 0$ is the EOM $\Psi''(u) = V(u)\Psi(u)$, where the Schrödinger-like potential is given by

$$V(u) = -\frac{1}{f^2(u)}\Big(1 + q^2u - \frac{\gamma}{144}f(u)[-11700 + 2098482u^2 - 4752055u^4 + 1838319u^6 + q^2u(-16470 + 245442u^2 + 1011173u^4)]\Big), \quad (11)$$

and $q = k/(2\pi T)$. The boundary term can be simplified to $\Phi = \Psi'(u)\Psi(u)$. We solve the Schrödinger equation analytically for the region $k \ll T$, the so-called hydrodynamic regime of the plasma, and the high-energy regime $k \gg T$. In the intermediate momentum region, we resort to a numerical solution of the EOM. Once we have the solution of the Schrödinger equation, the trace of the spectral function is given by

$$\chi^\mu_\mu(k) = \frac{N^2T^2}{2}\left(1 - \frac{265}{8}\gamma\right)\mathrm{Im}\left[\frac{\Psi'(u)}{\Psi(u)}\right]\Big|_{u=0}, \quad (12)$$

with $(1 - \frac{265}{8}\gamma)$ coming from the factors of r_0 in Eq. (10).

A. The EOM of the vector perturbation

We solve the Schrödinger equation using perturbation theory. Write $\Psi(u) = \Psi_0(u) + \gamma\Psi_1(u)$, and insert into the Schrödinger equation, separating the powers of γ. The equation for $\Psi_0(u)$ is given by $\Psi_0''(u) = (-f^{-2}(u) \times (1 + q^2u))\Psi_0(u)$, and solves to give

$$\Psi_0(u) = (1-u)^{-\frac{1}{2}(1+iq)}(1+u)^{-\frac{1}{2}(1+q)}{}_2F_1\left(1 - \frac{(1+i)q}{2}, -\frac{(1+i)q}{2}, 1 - iq, \frac{1-u}{2}\right). \quad (13)$$

The equation for $\Psi_1(u)$ is solved numerically (if necessary). The trace of the spectral function $\chi^\mu_\mu(k)$ is

$$\frac{N^2T^2}{2}\,\mathrm{Im}\left[\left(1 - \frac{265}{8}\gamma\right)\frac{\Psi_0'}{\Psi_0} + \gamma\left[-\frac{\Psi_0'}{\Psi_0}\frac{\Psi_1}{\Psi_0} + \frac{\Psi_1'}{\Psi_0}\right]\right]\Big|_{u=0}$$

which is exact to linear order in γ. We note that $\chi^\mu_\mu(k)$ at $\lambda \to \infty$ is known [3], so our task here is to compute the 't Hooft coupling corrections to that result.

VI. ASYMPTOTIC LIMIT OF THE SPECTRAL FUNCTION

The trace of the spectral function $\chi^\mu_\mu(k)$ can be evaluated analytically for low and high momentum, and numerically for the remaining momentum domain. We do not discuss the details of the computations, referring the reader to [3], and we simply display the results:

$$\frac{\chi^\mu_\mu(q)}{\frac{1}{2}N^2T^2} = \begin{cases} \left(1 + \frac{14\,993}{9}\gamma\right)q + \mathcal{O}(q^3) & q \ll 1 \\ \frac{3^{5/6}}{2}\frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})}(1 + 5\gamma)q^{2/3} + \mathcal{O}(1) & q \gg 1. \end{cases} \quad (14)$$

The coefficient of q in the low-q regime of Eq. (14) means that the electrical conductivity of the strongly coupled plasma is enhanced by a factor $(1 + \frac{14\,993}{9}\gamma)$ due to the finite λ corrections [9]. This is as expected from the perturbative computations in [3]: the weakly coupled plasma has a larger mean-free-path per collision, allowing more efficient diffusion of electric charge, and hence a higher electrical conductivity. The high-q region however, poses a question that we (as of yet) cannot answer: the authors of [3] claim that the spectral function at weak coupling should go like $q^{1/2}$ in the UV. Given that the spectral function at $\lambda \to \infty$ goes like $q^{2/3}$, in that regime one would have expected our result in Eq. (14) to display some smooth interpolation between $q^{1/2}$ and $q^{2/3}$. We do not obtain such an interpolation, finding instead that the finite coupling corrections *do not* change the q dependence in the UV. Moreover, we find an enhancement by a factor $(1 + 5\gamma)$ in that regime (see also [10]). The fact that the leading q behavior is unchanged by the corrections could have been seen from the Schrödinger-like potential in Eq. (11): the only q dependence is q^2, identically to the $\lambda \to \infty$ case. Terms like q^4, which could have changed the high-q functional dependence of $\chi^{\mu}_{\mu}(q)$, vanish. We shall revisit this point below.

VII. THE PHOTOEMISSION SPECTRUM

We plug the obtained $\chi^{\mu}_{\mu}(k)$ into Eq. (2) to give the photoemission spectrum. We display the results in Fig. 1. Clearly, the corrected result at strong coupling approaches the weakly coupled result (taken from Ref. [3]). Firstly, the corrected curves exhibit a steeper tangent at the origin, due to the enhancement of the electric conductivity by the factor $(1 + \frac{14\,993}{9}\gamma)$ in Eq. (14). Secondly, the peak of the photoemission is enhanced by the corrections, and the momentum of maximal emission shifts toward the IR,

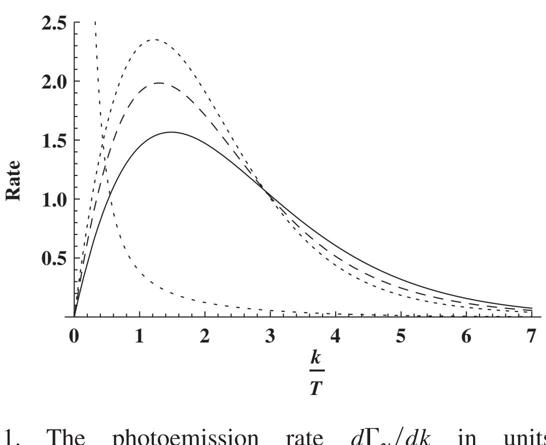

FIG. 1. The photoemission rate $d\Gamma_\gamma/dk$ in units of $0.01\alpha_{\rm em}N^2T^3$ for different values of λ, as a function of k/T. Solid, dashed, and small-dashed lines correspond to decreasing values of $\lambda \to \infty$, 75, and 50, respectively. The dotted line to the extreme left is the weak-coupling result at $\lambda = 0.5$ taken from [3].

taking the corrected curves closer to the weakly coupled result. Simple numerical analysis on the lightlike spectral function yields that the maximal rate is given by

$$\frac{d\Gamma_\gamma}{dk}\bigg|_{\max} \simeq 0.0156695\left(1+\left[1115.3-\frac{265}{8}\right]\gamma\right)+\mathcal{O}(\gamma^2), \tag{15}$$

in units of $\alpha_{\rm em}N^2T^3$, where we have made explicit the factor $-265/8\gamma$ coming from the overall normalization of the action. The position of the peak $k_{\max}$ is

$$k_{\max} \simeq 1.484\,69(1 - 842.425\gamma)T + \mathcal{O}(\gamma^2). \tag{16}$$

This quantity can analytically be shown to be independent of the overall normalization of the action, making it an excellent candidate for comparing disparate gauge theories. One more quantity which is of interest is the total number of photons emitted, given by the area under the curves in Fig. 1. This is enhanced by a factor

$$\frac{N_{\rm total}(\gamma)}{N_{\rm total}(0)} \simeq 1 + \left[461.941 - \frac{265}{8}\right]\gamma + \mathcal{O}(\gamma^2), \tag{17}$$

due to the fact that the weakly coupled theory dominates in the IR, where Bose suppression [due to $n_b(k)$] is small.

It is interesting to notice that the corrections we find to the spectral function and also for the electrical conductivity, both $14\,993/9\gamma$, and to the photoemission rate, $(1115.3 - 265/8)\gamma$, are significantly large compared with corrections found for observables which do not depend on correlation functions of the electromagnetic currents, and thus are not influenced by terms depending on F_5. For instance, we may compare the correction to the spectral function which is about 100% and the one to the photoemission rate about 64% for $\lambda = 40$, with the small correction received by quantities such as the shear viscosity which was found to be about 100% for values of λ as small as 7 [4]. Also, for example, the free energy receives a similar order correction (about 100%) when $\lambda \sim 2$. This suggests that the strong coupling expansion converges especially slowly for the spectral function, the electrical conductivity, and the photoemission rate, all obtained from the correlation functions of electromagnetic currents. This is an important finding of our work which shows the effects of the terms arising from the supersymmetric completion to the C^4 term (only depending on the 10D Weyl tensor).

We finally make two comments about the behavior of the photoemission rate for high q. Firstly, there is a (λ-independent) crossover point around $k/T \sim 2.92$, where the corrected curves dip below the $\lambda \to \infty$ result. This is expected from the weak-coupling computations of [3]. What is surprising, as we mentioned above, is that the asymptotic values of the λ-corrected curves for large k/T are given by $(1 + 5\gamma)$ times the infinite coupling result, as in Eq. (14) (note that the domain of Fig. 1 does not extend to cover this asymptotic behavior). This means that the finite-λ corrections *enhance* the photoemission rate in the deep UV regime, contrary to the expectations of [3]. Obviously, we are not guaranteed that the weakly coupled result should be approached by strongly coupled corrections computed in perturbation theory, especially not for a situation where the functional dependence on momenta is expected to be different, so we are not unduly concerned by this apparent discrepancy. It would be very revealing to understand these crossover points, as well as their scaling with λ. An important extension of our work would be to determine if the universality found in [11] for the energy-momentum spectral functions operates for the R-symmetry current spectral function computed here.

ACKNOWLEDGMENTS

We thank Gert Aarts, Alex Buchel, Carlos Núñez, Miguel Paulos, and Jorge Russo for valuable comments. The work of M. S. has been partially supported by the CONICET, the ANPCyT-FONCyT Grant No. PICT-2007-00849, and the CONICET Grant No. PIP-2010-0396.

[1] E. Shuryak, Prog. Part. Nucl. Phys. **53**, 273 (2004); J. Schukraft (f. t. A. Collaboration), arXiv:1103.3474; N. Armesto *et al.*, J. Phys. G **35**, 054001 (2008).
[2] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999).
[3] S. Caron-Huot, P. Kovtun, G. Moore, A. Starinets, and L. Yaffe, J. High Energy Phys. 12 (2006) 015.
[4] R. C. Myers, M. F. Paulos, and A. Sinha, Phys. Rev. D **79**, 041901 (2009).
[5] M. F. Paulos, J. High Energy Phys. 10 (2008) 047.
[6] S. S. Gubser, I. R. Klebanov, and A. A. Tseytlin, Nucl. Phys. **B534**, 202 (1998); J. Pawelczyk and S. Theisen, J. High Energy Phys. 09 (1998) 010; A. Buchel, J. Liu, and A. Starinets, Nucl. Phys. **B707** (2005) 56.
[7] D. T. Son and A. O. Starinets, J. High Energy Phys. 09 (2002) 042.G. Policastro, D. T. Son, and A. O. Starinets, J. High Energy Phys. 09 (2002) 043.
[8] A. Chamblin, R. Emparan, C. Johnson, and R. Myers, Phys. Rev. D **60**, 064018 (1999); M. Cvetic, M. Duff, P. Hoxha, J. Liu, H. L, J. Lu, R. Martinez-Acosta, C. Pope, H. Sati, and T. Tran, Nucl. Phys. **B558**, 96 (1999).
[9] B. Hassanain and M. Schvellinger, J. High Energy Phys. 10 (2010) 068; B. Hassanain and M. Schvellinger, J. High Energy Phys. 01 (2012) 114.
[10] B. Hassanain and M. Schvellinger, J. High Energy Phys. 04 (2010) 012.
[11] A. Buchel, R. Myers, M. Paulos, and A. Sinha, Phys. Lett. B **669**, 364 (2008).