

Early universe constraints on time variation of fundamental constants

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We study the time variation of fundamental constants in the early Universe. Using data from primordial light nuclei abundances, cosmic microwave background, and the 2dFGRS power spectrum, we put constraints on the time variation of the fine structure constant α and the Higgs vacuum expectation value $\langle v \rangle$ without assuming any theoretical framework. A variation in $\langle v \rangle$ leads to a variation in the electron mass, among other effects. Along the same line, we study the variation of α and the electron mass m_e . In a purely phenomenological fashion, we derive a relationship between both variations.

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I. INTRODUCTION

Unification theories, such as super-string [1–6], brane-world [7–10] and Kaluza-Klein theories [11–15], allow fundamental constants, such as the fine structure constant α and the Higgs vacuum expectation value $\langle v \rangle$, to vary over cosmological time scales. A variation in $\langle v \rangle$ leads to a variation in the electron mass, among other effects. On the other hand, theoretical frameworks based in first principles were developed by different authors [16–18] in order to study the variation of certain fundamental constants. Since each theory predicts a specific time behavior, by setting limits on the time variation of fundamental constants some of these theories could be set aside.

Limits on the present rate of variation of α and $\mu = \frac{m_e}{m_p}$ (where m_e is the electron mass and m_p the proton mass) are provided by atomic clocks [19–24]. Data from the Oklo natural fission reactor [25,26] and half-lives of long lived β decayers [27] allow one to constrain the variation of fundamental constants at $z \simeq 1$. Recent astronomical data based on the analysis of spectra from high-redshift quasar absorption systems suggest a possible variation of α and μ [28–34]. However, another analysis of similar data gives null variation of α [35–38]. Big bang nucleosynthesis (BBN) and cosmic microwave background (CMB) also provide constraints on the variation of fundamental constants. Although the limits imposed by BBN and CMB are less stringent than the previous ones, they are still important since they refer to the earliest cosmological times.

In previous works, we have studied the time variation of the fine structure constant in the early Universe to test the Bekenstein model [39] and the time variation of the elec-

tron mass to test the Barrow-Magueijo model [40]. However, unifying theories predict relationships among the variation of gauge coupling constants which depend on the theoretical framework. In this work, we perform a phenomenological analysis of the joint time variation of α and $\langle v \rangle$ in the early Universe without assuming a theoretical framework.

The model developed by Barrow and Magueijo [18] predicts the variation of m_e over cosmological time scales. This model could be regarded as the low energy limit of a more sophisticated unified theory. In such a case, the unifying theory would also predict variation of gauge coupling constants and in consequence the variation of α . Thus, in order to provide bounds to test such a kind of models, we also analyze in this paper the joint variation of α and m_e without assuming a theoretical framework.

The dependence of the primordial abundances on α has been analyzed by Bergstrom *et al.* [41] and improved by Nollett and Lopez [42], while the dependence on $\langle v \rangle$ has been analyzed by Yoo and Scherrer [43]. Semianalytical analyses have been performed by some of us in earlier works [44,45]. Several authors [46–48] studied the effects of the variation of fundamental constants on BBN in the context of a dilaton superstring model. Müller *et al.* [49] calculated the primordial abundances as a function of the Planck mass, fine structure constant, Higgs vacuum expectation value, electron mass, nucleon decay time, deuterium binding energy, and neutron-proton mass difference and studied the dependence of the last three quantities as functions of the fundamental coupling and masses. Coc *et al.* [50] set constraints on the variation in the neutron lifetime and neutron-proton mass difference using the pri-

mordial abundance of ${}^4\text{He}$. Cyburt *et al.* [51] studied the number of relativistic species at the time of BBN and the variations in fundamental constants α and G_N . Dent *et al.* [52] studied the dependence of the primordial abundances with nuclear physics parameters such as G_N , nucleon decay time, α , m_e , the average nucleon mass, the neutron-proton mass difference, and binding energies. Finally, limits on cosmological variations of α , Λ_{QCD} , and quark mass (m^q) from optical quasar absorption spectra, laboratory atomic clocks, and from BBN have been established by Flambaum *et al.* [53,54].

In this paper, we study the effects of a possible variation of α and $\langle v \rangle$ on the primordial abundances, including the dependence of the masses of the light elements on the cross sections, and using the dependence on $\langle v \rangle$ of the deuterium binding energy calculated in the context of usual quantum theory with a phenomenological potential. We use all available observational data of D, ${}^4\text{He}$, and ${}^7\text{Li}$ to set constraints on the joint variation of fundamental constants at the time of BBN.

However, we do not consider a possible variation of Λ_{QCD} . Indeed, the dependence of the physical quantities involved in the calculation of the primordial abundances with a varying Λ_{QCD} is highly dependent on the model. The analyses of Refs. [46,47], for example, are done in the context of a string dilaton model. Therefore, we will not consider such dependencies even though it has been analyzed in the literature [46,47,53–55]. Our analysis, instead, is a model independent one.

Previous analysis of CMB data (earlier than the WMAP three-year release) including a possible variation of α have been performed by Refs. [56–58] and including a possible variation of m_e have been performed by Refs. [43,58]. The work of Ichikawa *et al.* [58] is the only one that assumes that both variations are related in the context of string dilaton models. In this work, we follow a completely different approach, by assuming that the fundamental constants vary independently.

The paper is organized as follows. In Sec. II, we present bounds on the variation of the fine structure constant and the Higgs vacuum expectation value during big bang nucleosynthesis. We also discuss the difference between considering $\langle v \rangle$ variation and m_e variation during this epoch. In Sec. III, we use data from the CMB and from the 2dFGRS power spectrum to put bounds on the variation α and $\langle v \rangle$ (or m_e) during recombination, allowing also other cosmological parameters to vary. In Sec. IV, we use the $\alpha - m_e$ and $\alpha - \langle v \rangle$ confidence contours to obtain a phenomenological relationship between both variations and then discuss our results. Conclusions are presented in Sec. V.

II. BOUNDS FROM BBN

Big bang nucleosynthesis (BBN) is one of the most important tools to study the early universe. The standard

model has a single free parameter, the baryon to photon ratio η_B , which can be determined by comparison between theoretical calculations and observations of the abundances of light elements. Independently, the value of the baryonic density $\Omega_B h^2$ (related to η_B) can be obtained with great accuracy from the analysis of the cosmic microwave background data [59–61]. Provided this value, the theoretical abundances are highly consistent with the observed D but not with all ${}^4\text{He}$ and ${}^7\text{Li}$ data. If the fundamental constants vary with time, this discrepancy might be solved and we may have insight into new physics beyond the minimal BBN model.

In this section, we use available data of D, ${}^4\text{He}$, and ${}^7\text{Li}$ to put bounds on the joint variation of α and $\langle v \rangle$ and on the joint variation of α and m_e at the time of primordial nucleosynthesis. The observational data for D have been taken from Refs. [62–69]. For ${}^7\text{Li}$ we consider the data reported by Refs. [70–76]. For ${}^4\text{He}$, we use the data from Refs. [77,78] (see Ref. [39] for details).

We checked the consistency of each group of data following Ref. [79] and found that the ideogram method plots are not Gaussian-like, suggesting the existence of unmodeled systematic errors. We take them into account by increasing the errors by a fixed factor, 2.10, 1.40, and 1.90 for D, ${}^4\text{He}$, and ${}^7\text{Li}$, respectively. A scaling of errors was also suggested by Ref. [80].

The main effects of the variation of the fine structure constant during BBN are the variation of the neutron-to-proton ratio in thermal equilibrium produced by a variation in the neutron-proton mass difference, the weak decay rates, and the cross sections of the reactions involved during the first three minutes of the Universe. The main effects of the variation of the Higgs vacuum expectation value during BBN are the variation of the electron mass, the Fermi constant, the neutron-proton mass difference, and the deuterium binding energy, affecting mostly the neutron-to-proton ratio, the weak decay rates, and the initial abundance of deuterium. In Appendix A we give more details about how the physics at BBN is modified by a possible change in α , $\langle v \rangle$, and m_e . We modify the Kawano code [81] in order to consider time variation of α and $\langle v \rangle$ and time variation of α and m_e during BBN. The coulomb, radiative, and finite temperature corrections were included following Ref. [82]. We follow the analysis of Refs. [41,42] to introduce the variation in α on the reaction rates. The main effects of a change in α in nuclear reaction rates are variations in the Coulomb barrier for charged-induced reactions and radiative captures. We introduce the dependence of the light nuclei masses on α , correction that affects the reaction rates, their inverse coefficients, and their Q values [44]. We also update the value of the reaction rates following Ref. [41].

To illustrate the effect of the variation in the fine structure constant on the reactions rates, we present the nuclear

reaction rate of $d + d \rightarrow p + t$ ($R[dd; pt] = 0.93 \times 10^{-3} \Omega_B h^2 T_9^3 N_A \langle \sigma v \rangle$) as a function of $\frac{\Delta\alpha}{\alpha_0}$ ($\Delta\alpha = \alpha - \alpha_0$ and α_0 is the current value of the fine structure constant):

$$R[dd; pt] = 2.369 \times 10^{-3} \Omega_B h^2 T_9^{7/3} \alpha_0^{1/3} \left(1 + \frac{\Delta\alpha}{\alpha_0}\right)^{4/3} \mu^{-1/3} e^{-9.545 \times 10^{10} ((\mu\alpha_0^2/T_9)(1+(\Delta\alpha/\alpha_0))^2)^{1/3}} \left[1 + 0.16 \left(1 + \frac{\Delta\alpha}{\alpha_0}\right)\right] \\ \times \left(1 + 4.365 \times 10^{-12} \mu^{-1/3} \alpha_0^{-2/3} \left(1 + \frac{\Delta\alpha}{\alpha_0}\right)^{-2/3} T_9^{1/3} + 1.161 \times 10^{10} \mu^{1/3} \alpha_0^{2/3} \left(1 + \frac{\Delta\alpha}{\alpha_0}\right)^{2/3} T_9^{2/3} \right. \\ \left. + 0.355 T_9 - 5.104 \times 10^{18} \mu^{2/3} \alpha_0^{4/3} \left(1 + \frac{\Delta\alpha}{\alpha_0}\right)^{4/3} T_9^{4/3} - 3.966 \times 10^8 \mu^{1/3} \alpha_0^{2/3} \left(1 + \frac{\Delta\alpha}{\alpha_0}\right)^{2/3} T_9^{5/3}\right), \quad (1)$$

where T_9 is the temperature in units of 10^9 K and μ is the reduced mass. The reduced mass also changes if the fine structure constant varies with time (see Appendix A). This nuclear reaction is important for calculating the final deuterium abundance since this reaction destroys deuterium and produces tritium which is crucial to form ${}^4\text{He}$. In Fig. 1 we present the value of $N_A \langle \sigma v \rangle$ for this reaction as a function of the temperature, for different values of $\frac{\Delta\alpha}{\alpha_0}$. If the fine structure constant is greater than its present value, the reaction rate is lower than in the case of no α variation. A decrease in the value of this reaction rate results in an increase in the deuterium abundance.

The ${}^4\text{He}$ abundance is less sensitive to changes in the nuclear reaction rates than the other abundances (deuterium and ${}^7\text{Li}$) [41] and very sensitive to variations in the parameters that fixed the neutron-to-proton ratio. In thermal equilibrium, this ratio is

$$\frac{Y_n}{Y_p} = e^{-\Delta m_{np}/T}, \quad (2)$$

where Y_n (Y_p) is the neutron (proton) abundance, Δm_{np} is the neutron-proton mass difference, and T is the temperature in MeV. When the weak interaction rates become

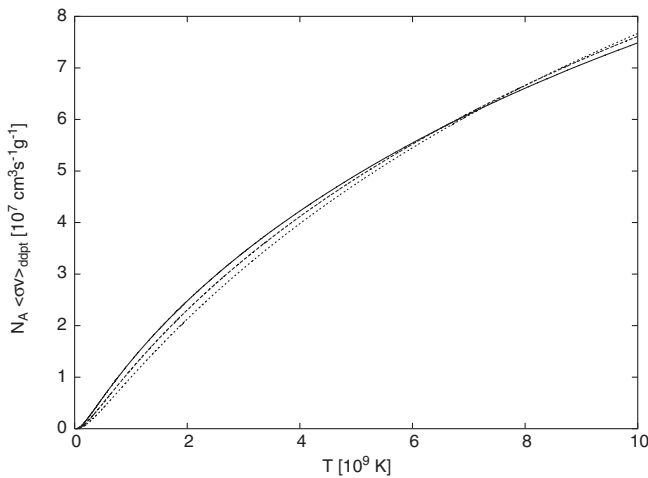


FIG. 1. $N_A \langle \sigma v \rangle$ (in units of $10^7 \text{ cm}^3 \text{ s}^{-1} \text{ g}^{-1}$) for the reaction $d + d \rightarrow p + t$, as a function of the temperature (in units of 10^9 K), when $\frac{\Delta\alpha}{\alpha_0} = -0.1$ (solid line), $\frac{\Delta\alpha}{\alpha_0} = 0.0$ (dashed line), and $\frac{\Delta\alpha}{\alpha_0} = 0.1$ (dotted line).

slower than the Universe expansion rate the neutron-to-proton ratio freezes out at temperature T_f . Afterwards, nearly all the available neutrons are captured in ${}^4\text{He}$ [41], this abundance can be estimated by

$$Y_4 \sim 2 \left(\frac{Y_n}{Y_p}\right)_f \left[1 + \left(\frac{Y_n}{Y_p}\right)_f\right]^{-1}, \quad (3)$$

where $\left(\frac{Y_n}{Y_p}\right)_f = e^{-\Delta m_{np}/T_f}$. The neutron-proton mass difference is affected by a change in the fine structure constant and in the Higgs vacuum expectation value:

$$\frac{\delta \Delta m_{np}}{\Delta m_{np}} = -0.587 \frac{\Delta\alpha}{\alpha_0} + 1.587 \frac{\Delta \langle v \rangle}{\langle v \rangle_0}. \quad (4)$$

An increase in the fine structure constant results in a decrease in Δm_{np} , this produces a larger equilibrium neutron-to-proton ratio and a larger abundance of ${}^4\text{He}$. However, an increase in $\langle v \rangle$ leads to an increase in Δm_{np} . This produces a smaller equilibrium neutron-to-proton equilibrium ratio and a smaller abundance of ${}^4\text{He}$ [43].

The freeze-out temperature of weak interactions is crucial to determinate the amount of available neutrons and therefore the primordial abundance of ${}^4\text{He}$. This temperature is modified if the Higgs vacuum expectation value is changed during BBN due to changes in the weak reaction rates (see Appendix A). A larger Higgs vacuum expectation value during BBN results in: (i) a smaller G_F leading to earlier freeze-out of the weak reactions ($n \leftrightarrow p$), producing more ${}^4\text{He}$; (ii) an increase in m_e , a decreasing of $n \leftrightarrow p$ reaction rates and also producing more ${}^4\text{He}$ [43].

The dependence of the deuterium binding energy on the Higgs vacuum expectation value is extremely model dependent. Beane and Savage [83] studied this dependence using chiral perturbation theory and their results were applied by several authors [43,49,55]. We performed another estimation in the context of usual quantum theory, using the effective Reid potential [84] for the nucleon-nucleon interaction (paper in preparation). Even though Yoo and Scherrer had shown that very different values for $\frac{\partial \epsilon_D}{\partial m_\pi}$ lead to similar constraints on the change of $\langle v \rangle$, we perform our calculation using two different relationships (see Table I): (i) that obtained by Ref. [43] using the results of Ref. [83]; (ii) that obtained using the effective Reid

TABLE I. Values used in this work for $\frac{\partial \epsilon_D}{\partial m_\pi}$ and the coefficient κ in the relationship $\frac{\Delta \epsilon_D}{(\epsilon_D)_0} = \kappa \frac{\Delta \langle v \rangle}{\langle v \rangle_0}$.

	$\frac{\partial \epsilon_D}{\partial m_\pi}$	κ
Yoo and Scherrer	-0.159	-5.000
Reid potential	-0.198	-6.230

potential. From Table I, it follows that the value obtained using the effective Reid potential lies in the range allowed by the estimation of Beane and Savage [83]. The variation of the deuterium binding energy due to a time variation of the Higgs vacuum expectation value is related to $\frac{\partial \epsilon_D}{\partial m_\pi}$ as

$$\frac{\Delta \epsilon_D}{(\epsilon_D)_0} = \frac{\partial \epsilon_D}{\partial m_\pi} \frac{m_\pi}{2(\epsilon_D)_0} \frac{\Delta \langle v \rangle}{\langle v \rangle_0}. \quad (5)$$

We call $\kappa = \frac{m_\pi}{2(\epsilon_D)_0} \frac{\partial \epsilon_D}{\partial m_\pi}$ hereafter.

An increase in the Higgs vacuum expectation value results in a decrease in the deuterium binding energy, leading to a smaller initial deuterium abundance:

$$Y_d = \frac{Y_n Y_p e^{11.605 \epsilon_D / T_9}}{0.471 \times 10^{-10} T_9^{3/2}}, \quad (6)$$

where ϵ_D is in MeV. The production of ${}^4\text{He}$ begins later, leading to a smaller helium abundance but also to an increase in the final deuterium abundance [43].

To assume time variation of the electron mass during BBN is not exactly the same as to assume time variation of the Higgs vacuum expectation value since the weak interactions are important during this epoch. There exists some well tested theoretical model that predicts time variation of the electron mass [18]. For this reason we compute the light nuclei abundances and perform a statistical analysis using the observational data mentioned above to obtain the best fit values for the parameters for the following cases:

- (i) variation of α and $\langle v \rangle$ allowing η_B to vary,
- (ii) variation of α and $\langle v \rangle$ keeping η_B fixed,
- (iii) variation of α and m_e allowing η_B to vary,
- (iv) variation of α and m_e keeping η_B fixed.

Even though the WMAP data are able to constrain the baryon density with great accuracy, there is still some degeneracy between the parameters involved in the statistical analysis. For this reason, we allow the joint variation of baryon density and the other two constants to obtain an independent estimation for η_B . In the cases where η_B is fixed, we assume the value reported by the WMAP team [60] ($\eta_B^{\text{WMAP}} = (6.108 \pm 0.219) \times 10^{-10}$). We also present in Table II, as an independent estimation, the best fit for the baryon density when all the constants are fixed at their present value. It is shown that the only reasonable fit is found when the lithium data is removed from the data set, and the value for η_B is consistent with the value reported by Spergel *et al.* [60]. The fits considering the

 TABLE II. Best fit parameter values, 1σ errors for the BBN constraints on η_B (in units of 10^{-10}) keeping all the fundamental constants fixed at their present value.

	$\eta_B \pm \sigma [10^{-10}]$	$\frac{\chi^2_{\text{min}}}{N-1}$
D + ${}^4\text{He}$ + ${}^7\text{Li}$	4.310 ± 0.050	10.00
${}^4\text{He}$ + ${}^7\text{Li}$	3.920 ± 0.080	6.53
D + ${}^7\text{Li}$	4.270 ± 0.060	9.27
D + ${}^4\text{He}$	$6.710^{+0.400}_{-0.360}$	1.61

data of ${}^7\text{Li}$ are not reasonable and the best fit for η_B is not consistent with the WMAP value.

Table III shows the results for the analysis of the variation of α and $\langle v \rangle$ when η_B is allowed to vary. These results correspond to the relationship between ϵ_D and $\langle v \rangle$ obtained using the Reid potential. This fit is consistent within 1σ with that obtained considering the value of κ calculated by Yoo and Scherrer. We will not perform the statistical analysis again excluding one group of data for the case where η_B is allowed to vary since we would have two groups of data and three unknown variables. A reasonable fit can be found when α , $\langle v \rangle$, and η_B are allowed to vary. This fit is consistent within 6σ with non-null values for the variations of α and $\langle v \rangle$ while the value for η_B is not consistent with the estimation of WMAP within 3σ .

The left part of Fig. 2 shows a strong degeneracy between $\frac{\Delta \alpha}{\alpha_0}$ and η_B , $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$ and η_B , and $\frac{\Delta \alpha}{\alpha_0}$ and $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$, ($\Delta \langle v \rangle = \langle v \rangle - \langle v \rangle_0$ and $\langle v \rangle_0$ is the present value of the Higgs vacuum expectation value). From this phenomenological approach, the variation of the fundamental constants can be used to reconcile the observed primordial abundances.

When α , m_e , and η_B are considered as free parameters, we also obtain a reasonable fit (see Table III). Once again, we will not perform the statistical analysis again excluding one group of data since we would have two groups of data and three unknown variables. There is consistency within 6σ with variation of α and m_e but the value for η_B is not consistent with the estimation of WMAP within 3σ . The right part of Fig. 2 shows a strong degeneracy between $\frac{\Delta \alpha}{\alpha_0}$ and η_B , $\frac{\Delta m_e}{(m_e)_0}$ and η_B , and $\frac{\Delta \alpha}{\alpha_0}$ and $\frac{\Delta m_e}{(m_e)_0}$ [$\Delta m_e = m_e - (m_e)_0$, $(m_e)_0$ is the present value of the electron mass]. From this result it might be possible to reconcile the observed primordial abundances and the WMAP estimation for the baryon density.

 TABLE III. Best fit parameter values and 1σ errors for the BBN constraints on $\frac{\Delta \alpha}{\alpha_0}$, $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$, $\frac{\Delta m_e}{(m_e)_0}$, allowing η_B (in units of 10^{-10}) to vary and considering D + ${}^4\text{He}$ + ${}^7\text{Li}$. We use the estimation $\frac{\Delta \epsilon_D}{(\epsilon_D)_0} = -6.230 \frac{\Delta \langle v \rangle}{\langle v \rangle_0}$ to obtain the fit on $\frac{\Delta \alpha}{\alpha_0}$, $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$, and η_B .

$\frac{\Delta \alpha}{\alpha_0} \pm \sigma$	$\frac{\Delta \langle v \rangle}{\langle v \rangle_0} \pm \sigma$	$\frac{\Delta m_e}{(m_e)_0} \pm \sigma$	$\eta_B \pm \sigma [10^{-10}]$	$\frac{\chi^2_{\text{min}}}{N-3}$
$0.198^{+0.013}_{-0.014}$	$0.043^{+0.003}_{-0.004}$...	$8.005^{+0.552}_{-0.553}$	1.16
$0.210^{+0.015}_{-0.013}$...	$-0.250^{+0.015}_{-0.018}$	$7.533^{+0.447}_{-0.502}$	1.11

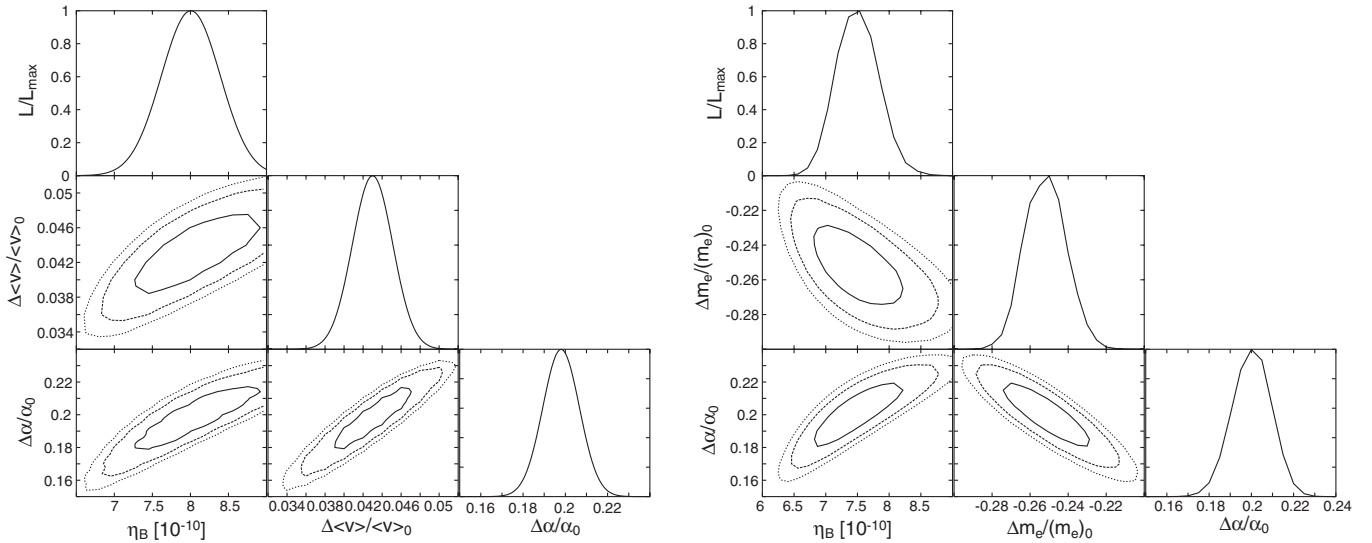


FIG. 2. Left figure: Likelihood contours for $\frac{\Delta\alpha}{\alpha_0}$, $\frac{\Delta\langle v \rangle}{\langle v \rangle_0}$, and η_B (in units of 10^{-10}) and one-dimensional likelihood (using $\frac{\Delta\epsilon_D}{(\epsilon_D)_0} = -6.230 \frac{\Delta\langle v \rangle}{\langle v \rangle_0}$). Right figure: Likelihood contours for $\frac{\Delta\alpha}{\alpha_0}$, $\frac{\Delta m_e}{(m_e)_0}$, and η_B (in units of 10^{-10}) and one-dimensional likelihood.

By comparing the results presented in Tables II and III, it can be noticed that the variation of the fundamental constants improves the statistical analysis providing a $\chi^2_{\min}/(N-3)$ value that is closer to one.

We now consider the joint variation of the fine structure constant and the Higgs vacuum expectation value with η_B fixed at the WMAP estimation. In this case, it is reasonable to repeat the analysis excluding one group of data at the time. The results are presented in Table IV and were obtained using $\frac{\Delta\epsilon_D}{(\epsilon_D)_0} = -6.230 \frac{\Delta\langle v \rangle}{\langle v \rangle_0}$. The fits obtained using $\frac{\Delta\epsilon_D}{(\epsilon_D)_0} = -5.000 \frac{\Delta\langle v \rangle}{\langle v \rangle_0}$ are consistent, within 1σ , with the ones presented. There is good fit for the whole data set and also excluding one group of data at each time. In any case, there is a strong degeneracy between $\frac{\Delta\alpha}{\alpha_0}$ and $\frac{\Delta\langle v \rangle}{\langle v \rangle_0}$ (see Fig. 3). Considering all data or ${}^4\text{He} + {}^7\text{Li}$ we find variation of both fundamental constants, α and $\langle v \rangle$, even at 6σ . However, if the statistical analysis is performed with D + ${}^4\text{He}$ we find null variation for both constants within 1σ . For completeness we also modified Kawano's code in order to calculate the different primordial abundances for two values inside

TABLE IV. Best fit parameter values, 1σ errors for the BBN constraints on $\frac{\Delta\alpha}{\alpha_0}$ and $\frac{\Delta\langle v \rangle}{\langle v \rangle_0}$, with η_B fixed at the WMAP estimation. The results correspond to the estimation $\frac{\Delta\epsilon_D}{(\epsilon_D)_0} = -6.230 \frac{\Delta\langle v \rangle}{\langle v \rangle_0}$.

Data	$\frac{\Delta\alpha}{\alpha_0} \pm \sigma$	$\frac{\Delta\langle v \rangle}{\langle v \rangle_0} \pm \sigma$	$\frac{\chi^2_{\min}}{N-2}$
D + ${}^4\text{He} + {}^7\text{Li}$	0.140 ± 0.006	0.032 ± 0.002	2.52
${}^4\text{He} + {}^7\text{Li}$	$0.148^{+0.004}_{-0.008}$	$0.033^{+0.002}_{-0.003}$	1.23
D + ${}^7\text{Li}$	$0.090^{+0.017}_{-0.022}$	$-0.070^{+0.024}_{-0.026}$	1.15
D + ${}^4\text{He}$	$-0.030^{+0.035}_{-0.030}$	$-0.002^{+0.007}_{-0.008}$	1.03

the range of $\frac{\partial\epsilon_D}{\partial m_\pi}$ calculated by Refs. [43,85] ($-0.15 < \frac{\partial\epsilon_D}{\partial m_\pi} < -0.05$) and performed the statistical test in order to obtain the constraints on the variation of the fundamental constants α and $\langle v \rangle$. All the results are consistent within 2σ with the ones presented above.

Table V and Fig. 4 show the results obtained when only α and m_e are allowed to vary. There is a strong degeneracy between the variations of α and the variations of m_e in all the cases considered. We find reasonable fits for the whole data set and also excluding one group of data at each time. Considering all data or ${}^4\text{He} + {}^7\text{Li}$, we find variation of α and m_e , even at 6σ . On the other hand, if the statistical analysis is performed with D + ${}^4\text{He}$ we find null variation for both constants within 1σ .

Richard *et al.* [86] have pointed out that a better understanding of turbulent transport in the radiative zones of the stars is needed in order to get a reliable estimation of the ${}^7\text{Li}$ abundance, while Meléndez and Ramírez [87] have reanalyzed the ${}^7\text{Li}$ data and obtained results that are marginally consistent with the WMAP estimate. On the other hand, Prodanović and Fields [88] put forward that the discrepancy with the WMAP data can worsen if contamination with ${}^6\text{Li}$ is considered. Therefore, we adopt the conservative criterion that the bounds on the variation of fundamental constants obtained in this paper are those where only the data of D and ${}^4\text{He}$ are fitted to the theoretical predictions of the abundances. This paper shows evidence for variation of fundamental constants in the early Universe if the reported values for the ${}^7\text{Li}$ abundance are confirmed by future observations and/or improvement of the theoretical analyses.

In Table VI we summarize our results for the variation of α , $\langle v \rangle$, and m_e , using D + ${}^4\text{He}$ data in the statistical analysis. The sign of α variation is the same for all the

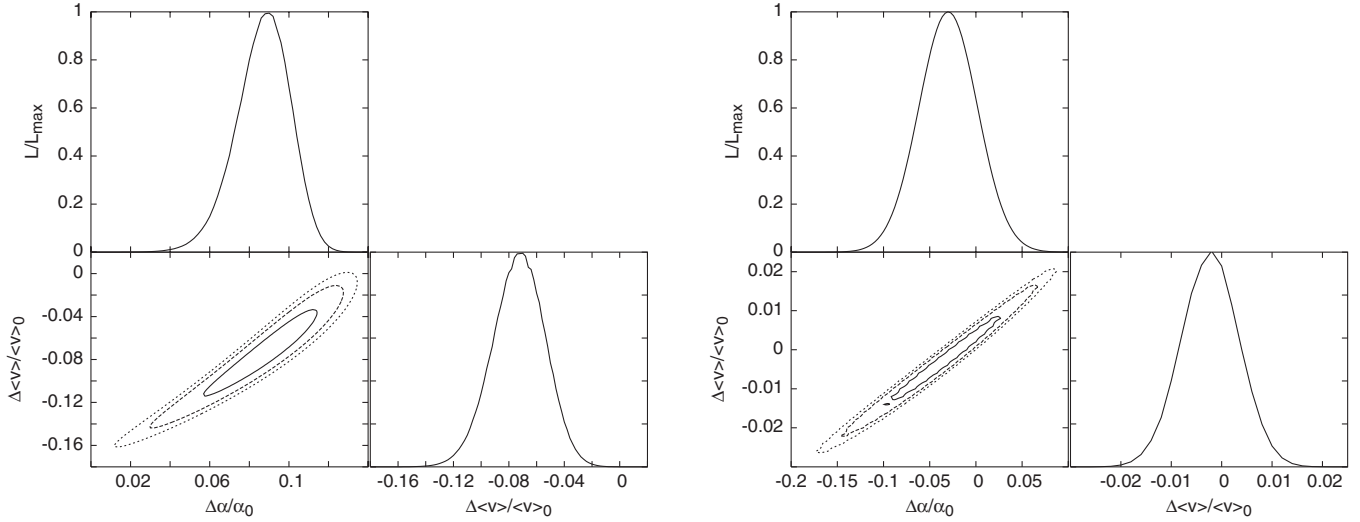


FIG. 3. Likelihood contours for $\frac{\Delta\alpha}{\alpha_0}$ vs $\frac{\Delta\langle v \rangle}{\langle v \rangle_0}$ and one-dimensional likelihood. Left figure: D + ${}^7\text{Li}$ data; right figure: D + ${}^4\text{He}$. The figures were obtained using $\frac{\Delta\epsilon_D}{(\epsilon_D)_0} = -6.230 \frac{\Delta\langle v \rangle}{\langle v \rangle_0}$.

TABLE V. Best fit parameter values, 1σ errors for the BBN constraints on $\frac{\Delta\alpha}{\alpha_0}$ and $\frac{\Delta m_e}{(m_e)_0}$, with η_B fixed at the WMAP estimation.

Data	$\frac{\Delta\alpha}{\alpha_0} \pm \sigma$	$\frac{\Delta m_e}{(m_e)_0} \pm \sigma$	$\frac{\chi^2_{\min}}{N-2}$
D + ${}^4\text{He}$ + ${}^7\text{Li}$	0.159 ± 0.008	-0.213 ± 0.012	1.85
${}^4\text{He}$ + ${}^7\text{Li}$	0.163 ± 0.008	-0.218 ± 0.013	1.00
D + ${}^7\text{Li}$	$0.067^{+0.022}_{-0.015}$	0.447 ± 0.134	1.00
D + ${}^4\text{He}$	$-0.036^{+0.052}_{-0.053}$	$0.020^{+0.066}_{-0.064}$	1.00

cases. However, the sign of the variation of $\langle v \rangle$ or m_e changes depending on whether the joint variation with α is considered or not. In all cases, we have found null variation of the fundamental constants within 3σ and the values of $\frac{\chi^2_{\min}}{N-g}$ (where $g = 2$ for the case where two constants are allowed to vary, and $g = 1$ when only one

fundamental constant is allowed to vary) are closer to one, resulting in reasonable fits for all the cases.

III. BOUNDS FROM CMB

The cosmological parameters can be estimated by an analysis of the cosmic microwave background (CMB)

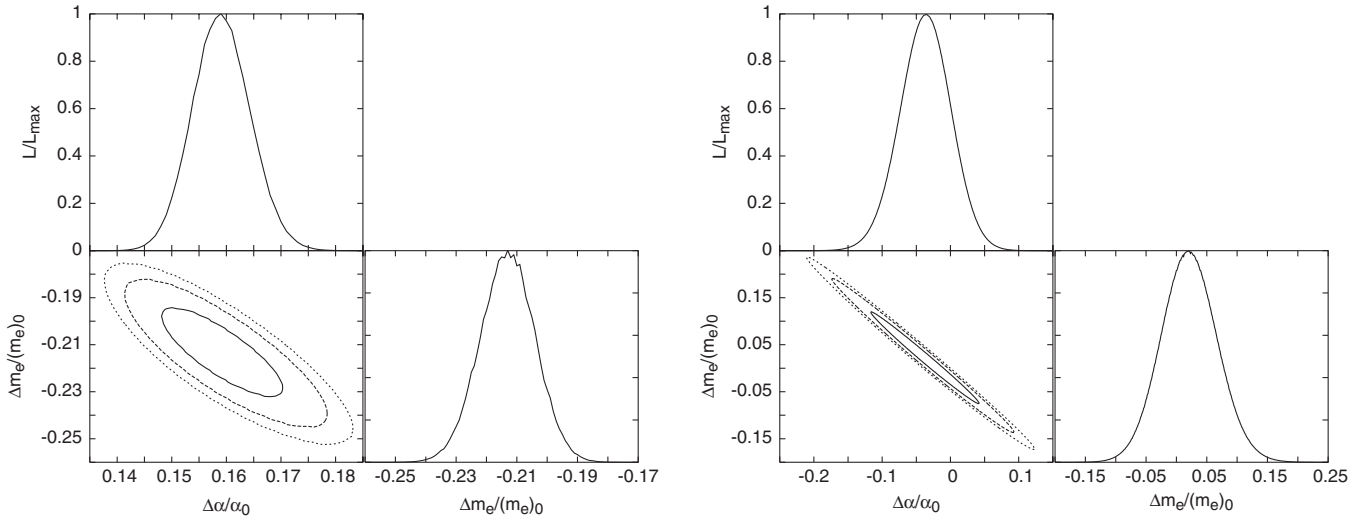


FIG. 4. Likelihood contours for $\frac{\Delta\alpha}{\alpha_0}$ vs $\frac{\Delta m_e}{(m_e)_0}$ and one-dimensional likelihood. Left figure: D + ${}^4\text{He}$ + ${}^7\text{Li}$ data; right figure: D + ${}^4\text{He}$.

TABLE VI. Comparison between the best fit parameter values and 1σ errors for the BBN constraints on $\frac{\Delta\alpha}{\alpha_0}$, $\frac{\Delta\langle v \rangle}{\langle v \rangle_0}$, and $\frac{\Delta m_e}{(m_e)_0}$, when one or two constants are allowed to vary [39,40]. We also present the values of $\frac{\chi^2_{\min}}{N-g}$ with $g = 2$ for the case where two constants are allowed to vary, whereas $g = 1$ when only one fundamental constant is allowed to vary. We consider D + ^4He data and η_B fixed at the WMAP estimation.

Parameter	Time variation of				
	α and $\langle v \rangle$	α and m_e	α	$\langle v \rangle$	m_e
$\Delta\alpha/\alpha_0 \pm \sigma$	$-0.030^{+0.035}_{-0.030}$	$-0.036^{+0.052}_{-0.053}$	-0.020 ± 0.007
$\Delta\langle v \rangle/\langle v \rangle_0 \pm \sigma$	$-0.002^{+0.007}_{-0.008}$	0.004 ± 0.002	...
$\Delta m_e/(m_e)_0 \pm \sigma$...	$0.020^{+0.066}_{-0.064}$	-0.024 ± 0.008
$\chi^2_{\min}/(N-g)$	1.03	1.00	0.90	0.97	0.95

radiation, which gives information about the physical conditions in the Universe just before decoupling of matter and radiation.

The variation of fundamental constants affects the physics during recombination (see Appendix B for details). At this stage of the Universe history, the only consequence of the time variation of $\langle v \rangle$ is a variation in m_e . The main effect of α and m_e variations is the shift of the epoch of recombination to higher z as α or m_e increases. This is easy to understand since the binding energy B_n scales as $\alpha^2 m_e$, so photons should have higher energy to ionize hydrogen atoms. In Fig. 5 we show how the ionization history is affected by changes in α and in m_e , in a flat universe with cosmological parameters $(\Omega_b h^2, \Omega_{\text{CDM}} h^2, h, \tau) = (0.0223, 0.1047, 0.73, 0.09)$. When α and/or m_e have higher values than the present ones, recombination occurs earlier (higher redshifts). The ionization history is more sensitive to α than to m_e because of the B_n dependence on these constants.

The most efficient thermalizing mechanism for the photon gas in the early universe is Thomson scattering on free electrons. Therefore, another important effect produced by the variation of fundamental constants is a shift in the Thomson scattering cross section σ_T , which is proportional to $m_e^{-2} \alpha^2$.

The visibility function, which measures the differential probability that a photon last scattered at conformal time η , depends on α and m_e . This function is defined as

$$g(\eta) = e^{-\kappa} \frac{d\kappa}{d\eta}, \quad \text{where} \quad \frac{d\kappa}{d\eta} = x_e n_p a \sigma_T \quad (7)$$

is the differential optical depth of photons due to Thomson scattering, n_p is the total number density of protons (both free and bound), x_e is the fraction of free electrons, and a is the scale factor. The strongest effect of variations of α and m_e on the visibility function occurs due to the alteration of the ionization history $x_e(\eta)$. In Fig. 6 we show that if α and/or m_e were smaller (larger) at recombination than their present values, the peak in the visibility function would shift towards smaller (larger) redshifts, and its width would slightly increase (decrease).

The signatures on the CMB angular power spectrum due to varying fundamental constants are similar to those produced by changes in the cosmological parameters, i.e. changes in the relative amplitudes of the Doppler peaks and a shift in their positions. Indeed, an increase in α or m_e leads to a higher redshift of the last-scattering surface, which corresponds to a smaller sound horizon. The position of the first peak (ℓ_1) is inversely proportional to the latter, so a larger ℓ_1 results. Also a larger early integrated

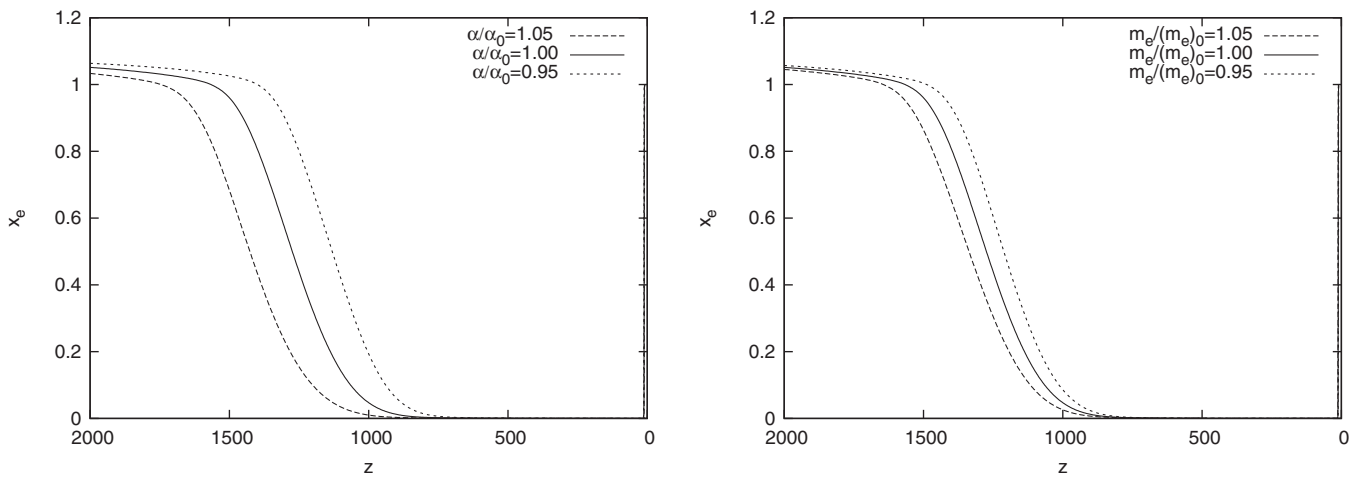


FIG. 5. Ionization history as a function of redshift, for different values of α (left panel) and m_e (right panel) at recombination time.

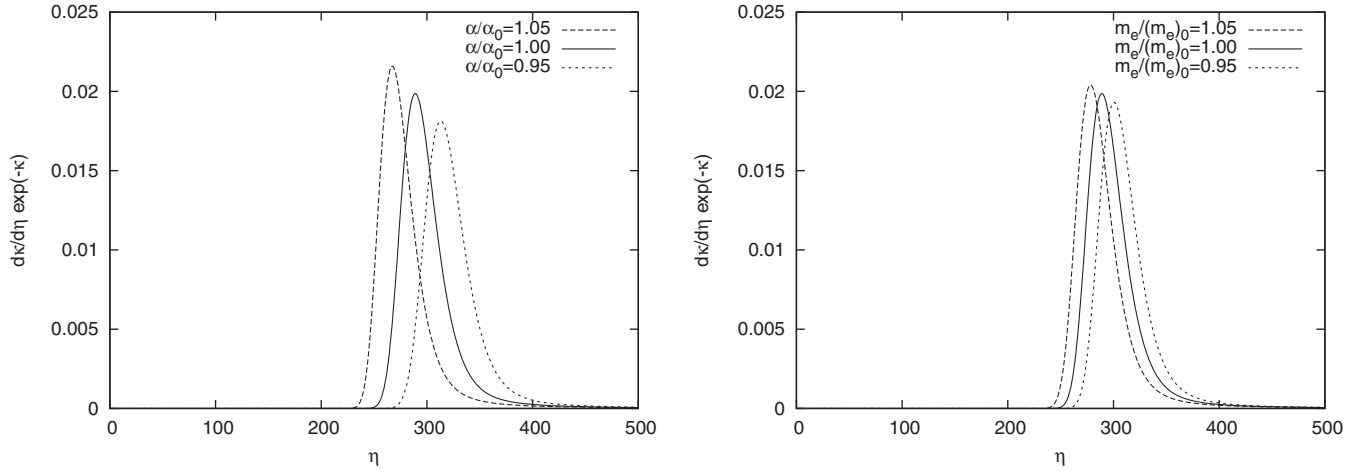


FIG. 6. Visibility function as a function of conformal time in Mpc, for different values of α (left panel) and m_e (right panel).

Sach-Wolfe effect is produced, making the first Doppler peak higher. Moreover, an increment in α or m_e decreases the high- ℓ diffusion damping, which is due to the finite thickness of the last-scattering surface, and thus, increases the power on very small scales [89–91]. All these effects are illustrated in Fig. 7.

To put constraints on the variation of α and $\langle v \rangle$ during recombination time, we performed a statistical analysis using data from the WMAP 3-year temperature and temperature-polarization power spectrum [60], and other CMB experiments such as CBI [92], ACBAR [93], and BOOMERANG [94,95], and the power spectrum of the 2dFGRS [96]. We consider a spatially flat cosmological model with adiabatic density fluctuations, and the following parameters:

$$P = \left(\Omega_B h^2, \Omega_{\text{CDM}} h^2, \Theta, \tau, \frac{\Delta \alpha}{\alpha_0}, \frac{\Delta \langle v \rangle}{\langle v \rangle_0}, n_s, A_s \right), \quad (8)$$

where $\Omega_{\text{CDM}} h^2$ is the dark matter density in units of the

critical density, Θ gives the ratio of the comoving sound horizon at decoupling to the angular diameter distance to the surface of last scattering, τ is the reionization optical depth, n_s the scalar spectral index, and A_s is the amplitude of the density fluctuations.

The parameter space was explored using the Markov chain Monte Carlo method implemented in the COSMOMC code of Ref. [97] which uses CAMB [98] to compute the CMB power spectra and RECFAST [99] to solve the recombination equations. We modified these numerical codes in order to include the possible variation of α and $\langle v \rangle$ (or m_e) at recombination. We ran eight Markov chains and followed the convergence criterion of Ref. [100] to stop them when $R - 1 < 0.0149$. Results are shown in Table VII and Fig. 8.

It is noticeable the strong degeneracies that exist between $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$ and $\Omega_{\text{CDM}} h^2$, $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$ and Θ , $\frac{\Delta \alpha}{\alpha_0}$ and n_s , and also between $\frac{\Delta \alpha}{\alpha_0}$ and $\frac{\Delta \langle v \rangle}{\langle v \rangle_0}$. The values obtained for $\Omega_B h^2$, $\Omega_{\text{CDM}} h^2$, τ , and n_s agree, within 1σ , with those of the

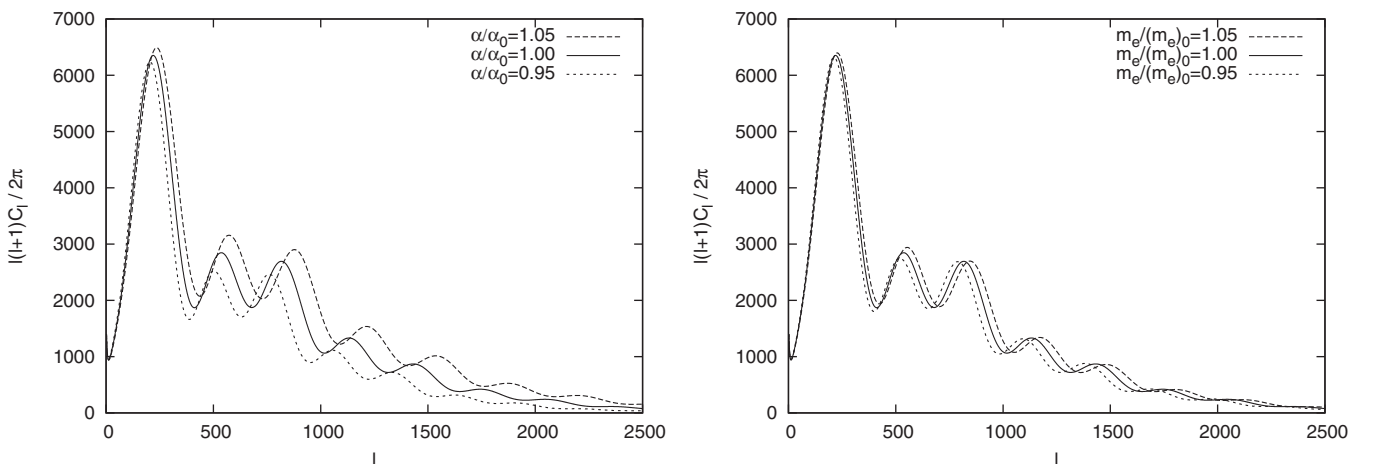


FIG. 7. The spectrum of CMB fluctuations for different values of α (left panel) and m_e (right panel).

TABLE VII. Mean values and 1σ errors for the parameters including α and $\langle v \rangle$ variation. For comparison, results where only one fundamental constant is allowed to vary are also shown [39,40]. H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$.

Parameter	α and $\langle v \rangle$ variation	α variation	$\langle v \rangle$ variation
$\Omega_B h^2$	0.0218 ± 0.0010	0.0216 ± 0.0009	0.0217 ± 0.0010
$\Omega_{\text{CDM}} h^2$	0.106 ± 0.011	0.102 ± 0.006	0.101 ± 0.009
Θ	$1.033^{+0.028}_{-0.029}$	1.021 ± 0.017	1.020 ± 0.025
τ	0.090 ± 0.014	0.092 ± 0.014	$0.091^{+0.013}_{-0.014}$
$\Delta\alpha/\alpha_0$	-0.023 ± 0.025	-0.015 ± 0.012	...
$\Delta\langle v \rangle / \langle v \rangle_0$	0.036 ± 0.078	...	-0.029 ± 0.034
n_s	0.970 ± 0.019	0.965 ± 0.016	0.960 ± 0.015
A_s	3.054 ± 0.073	$3.039^{+0.064}_{-0.065}$	3.020 ± 0.064
H_0	$70.4^{+6.6}_{-6.8}$	$67.7^{+4.7}_{-4.6}$	$68.1^{+5.9}_{-6.0}$

WMAP team [60], where no variation of α nor $\langle v \rangle$ is considered. It is interesting to note that our results for the cosmological parameters are similar to those obtained considering the variation of one constant at each time [39,40]. Our results are consistent within 1σ with no variation of α and $\langle v \rangle$ at recombination.

In Figs. 9 and 10 we compare the degeneracies that exist between different cosmological parameters and the fundamental constants when one or both constants are allowed to vary. In any case, the allowable region in the parameter space is larger when both fundamental constants are allowed to vary. This is to be expected since when the

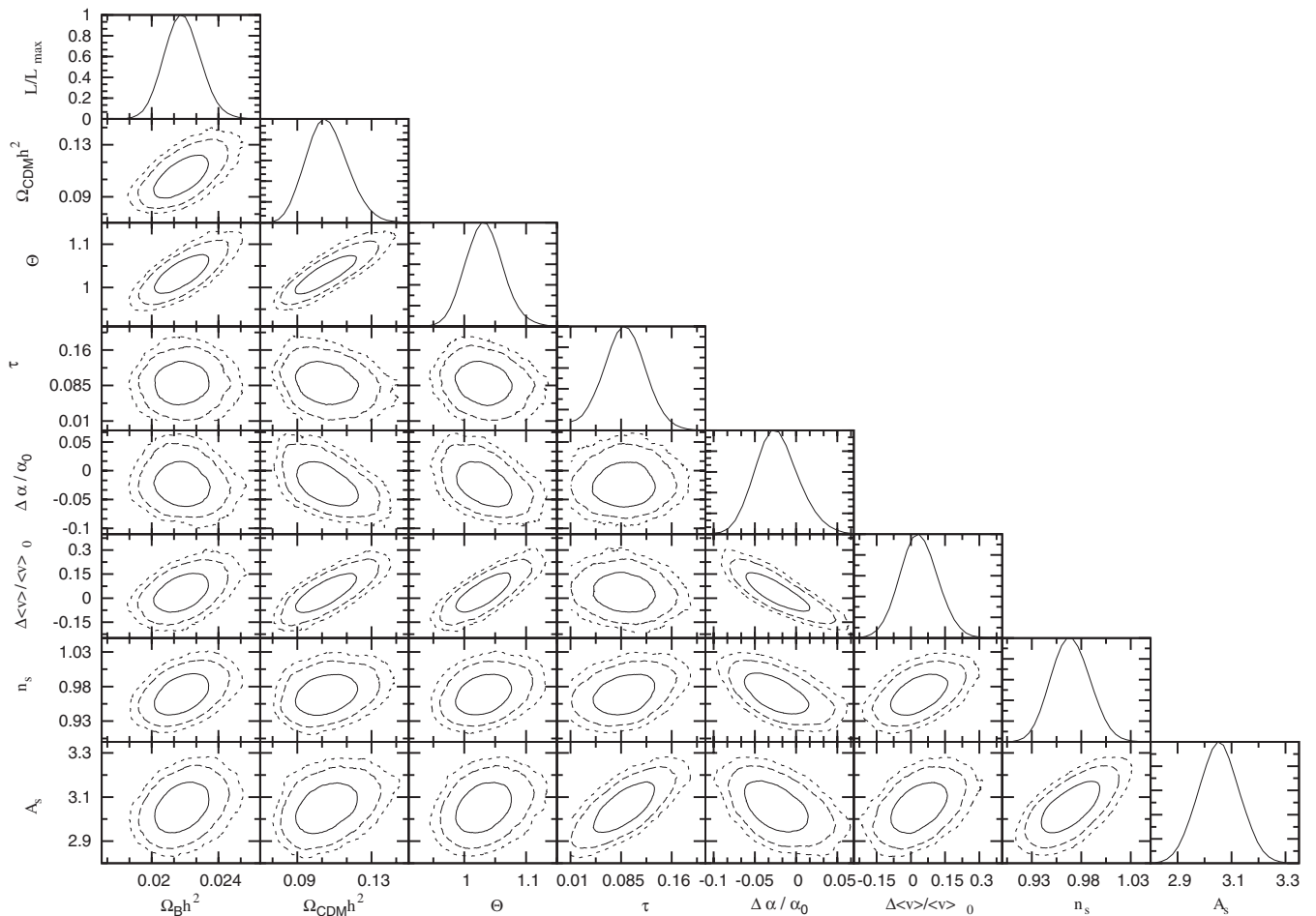


FIG. 8. Marginalized posterior distributions obtained with CMB data, including the WMAP 3-year data release plus 2dFGRS power spectrum. The diagonal shows the posterior distributions for individual parameters, the other panels show the 2D contours for pairs of parameters, marginalizing over the others.

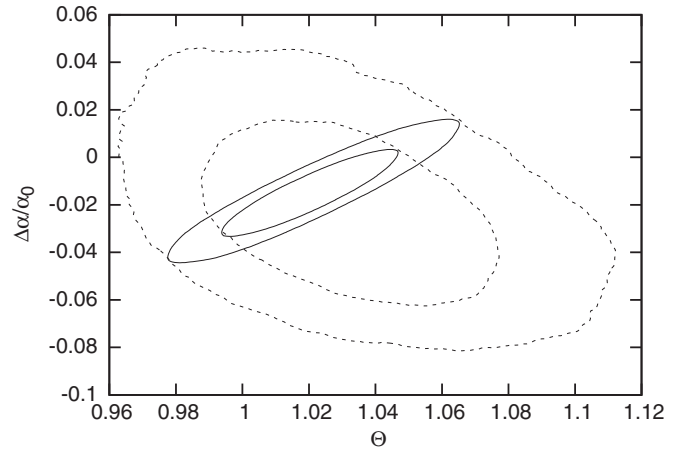
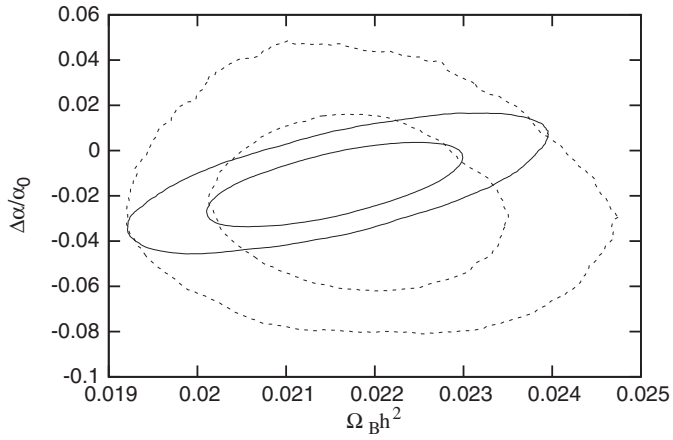


FIG. 9. 1σ and 2σ contour levels. Dotted line: variation of α and $\langle\nu\rangle$; solid line: only α variation. The cosmological parameters are free to vary in both cases.

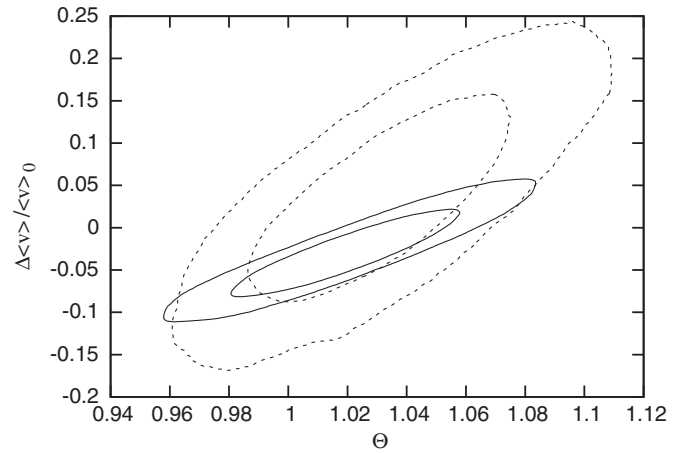
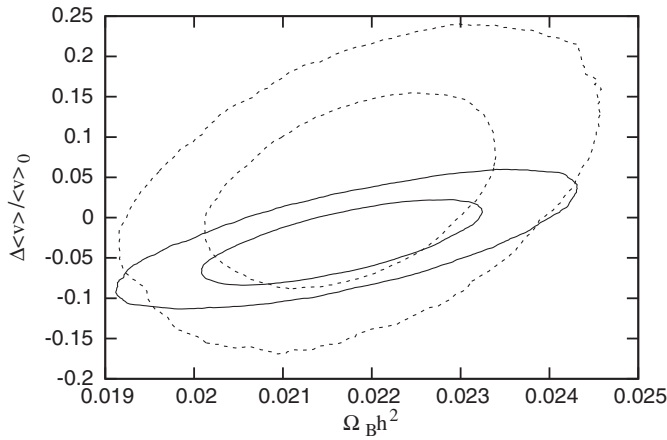


FIG. 10. 1σ and 2σ contour levels. Dotted line: variation of α and $\langle\nu\rangle$; solid line: only $\langle\nu\rangle$ variation. The cosmological parameters are free to vary in both cases.

parameter space has a higher dimension the uncertainties in the parameters are larger. The correlations of $\frac{\Delta\alpha}{\alpha_0}$ with the other cosmological parameters change sign when $\langle\nu\rangle$ is also allowed to vary. On the contrary, the correlations of $\frac{\Delta\langle\nu\rangle}{\langle\nu\rangle_0}$ with cosmological parameters do not change sign when α is also allowed to vary.

When only one fundamental constant is allowed to vary, the correlation between this constant and any particular cosmological parameter has the same sign, no matter whether the fundamental constant is α or $\langle\nu\rangle$. This is because both constants enter the same physical quantities. However, since the functional forms of the dependence on α and $\langle\nu\rangle$ are different, the best fit mean values for the time variations of these fundamental constants are different and the probability distribution is more extended in one case than in the other. Nevertheless, in the cases when only one constant is allowed to vary, it prefers a lower value than the present one.

IV. DISCUSSION

In Sec. II, we obtained bounds on the variation of α and $\langle\nu\rangle$ using the observational abundances of D, ${}^4\text{He}$, and ${}^7\text{Li}$. We performed different analyses: (i) we allow η_B to vary and (ii) we keep η_B fixed. We also performed the same analyses for two different estimations of the dependence of the deuterium binding energy on the pion mass or the Higgs vacuum expectation value: (i) that obtained by Yoo and Scherrer who considered the coefficient for the linear dependence obtained by Beane and Savage [83]; (ii) that obtained using the Reid potential for the description of the nucleon-nucleon interaction and without using chiral perturbation theory. The best fits for these two different cases are consistent within 1σ . We found reasonable fits for the variation of α , $\langle\nu\rangle$, and η_B for the whole data set and for the variation of α , $\langle\nu\rangle$, keeping η_B fixed, for the whole data set and also when we exclude one group of data. We only found variation of the fundamental constants when the ${}^7\text{Li}$

abundance is included in the statistical analysis. We also calculated the light abundances, keeping η_B fixed at the WMAP estimation, for different values of the dependence of ϵ_D on the Higgs vacuum expectation value, inside the range proposed in Ref. [85], and performed the statistical analysis. These results are consistent within 1σ with the ones presented in Sec. II.

We also considered the joint variation of α and m_e with η_B variable and fixed at the WMAP estimation. In this case, we also obtained reasonable fits for the whole data set. When the ${}^7\text{Li}$ abundance was included in the fit, we obtained results consistent with variation of fundamental constants (and η_B consistent with the WMAP value). From a phenomenological point of view, to vary α and m_e solves the discrepancy between the ${}^7\text{Li}$ data, the other abundances, and the WMAP estimate. However, it is important to mention that the theoretical motivations for m_e being the varying fundamental constant are weak.

We have discussed in Sec. II that there is still no agreement within the astronomical community in the value of the ${}^7\text{Li}$ abundance. We think that more observations of ${}^7\text{Li}$ are needed in order to arrive to stronger conclusions. However, if the present values of ${}^7\text{Li}$ abundances are correct, we may have insight into new physics and varying fundamental constants would be a good candidate for solving the discrepancy between the light elements abundances and the WMAP estimates.

In Sec. III, we calculated the time variation of α and $\langle\nu\rangle$ (or m_e) with data from CMB observations and the final 2dFGRS power spectrum. In this analysis, we also allowed other cosmological parameters to vary. We found no variation of α and $\langle\nu\rangle$ within 1σ , and the values for the cosmological parameters agree with those obtained by Ref. [60] within 1σ .

In Fig. 11 we show the 2D contours for $\frac{\Delta\alpha}{\alpha_0}$ and $\frac{\Delta\langle\nu\rangle}{\langle\nu\rangle_0}$ obtained from BBN and CMB data. The correlation coefficients are -0.82 for CMB and 0.77 for BBN. There is a

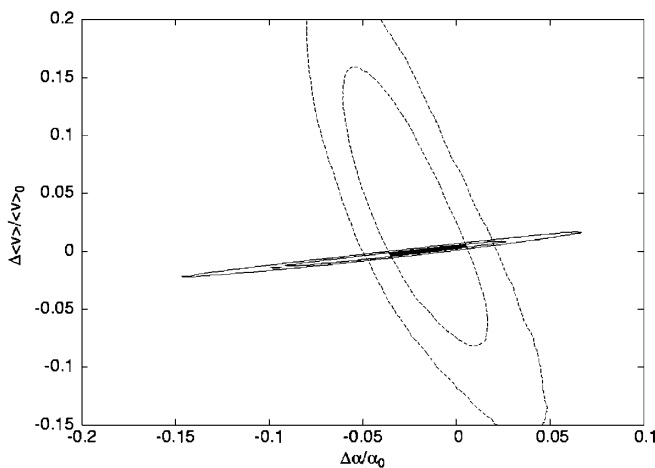


FIG. 11. 2D contour levels for variation of α and $\langle\nu\rangle$ from BBN (solid line) and CMB (dotted line) data.

small region where the two contours superpose which is consistent with null variation of both constants. However, the results do not exclude the possibility that the fundamental constants have values different from their present ones but constant in the early universe. It is possible to obtain a linear relationship (between $\frac{\Delta\alpha}{\alpha_0}$ and $\frac{\Delta\langle\nu\rangle}{\langle\nu\rangle_0}$) from the BBN and CMB contours:

$$\frac{\Delta\langle\nu\rangle}{\langle\nu\rangle_0} = a_{\text{BBN}} \frac{\Delta\alpha}{\alpha_0} + b_{\text{BBN}} \quad \text{for BBN,} \quad (9)$$

$$\frac{\Delta\langle\nu\rangle}{\langle\nu\rangle_0} = a_{\text{CMB}} \frac{\Delta\alpha}{\alpha_0} + b_{\text{CMB}} \quad \text{for CMB,} \quad (10)$$

where $a_{\text{BBN}} = 0.181 \pm 0.003$, $b_{\text{BBN}} = 0.0046 \pm 0.0002$, $a_{\text{CMB}} = -3.7_{-0.5}^{+0.1}$, and $b_{\text{CMB}} = -0.053_{-0.027}^{+0.009}$.

Figure 12 shows the 2D contours for $\frac{\Delta\alpha}{\alpha_0}$ and $\frac{\Delta m_e}{(m_e)_0}$ obtained from BBN and CMB data. In this case, the correlation coefficients are -0.82 for CMB and -0.91 for BBN. A phenomenological relationship between the variation of the fundamental constants α and m_e can be obtained by adjusting a linear function. These two linear fits are different for both cases:

$$\frac{\Delta m_e}{(m_e)_0} = c_{\text{BBN}} \frac{\Delta\alpha}{\alpha_0} + d_{\text{BBN}} \quad \text{for BBN,} \quad (11)$$

$$\frac{\Delta m_e}{(m_e)_0} = c_{\text{CMB}} \frac{\Delta\alpha}{\alpha_0} + d_{\text{CMB}} \quad \text{for CMB,} \quad (12)$$

where $c_{\text{BBN}} = -1.229 \pm 0.008$, $d_{\text{BBN}} = -0.0234 \pm 0.005$, $c_{\text{CMB}} = -3.7_{-0.5}^{+0.1}$, and $d_{\text{CMB}} = -0.053_{-0.027}^{+0.009}$ (the time variation of $\langle\nu\rangle$ during CMB has the same effects as the variation of the electron mass).

It is important to point out that BBN degeneration suggests phenomenological relationships between the var-

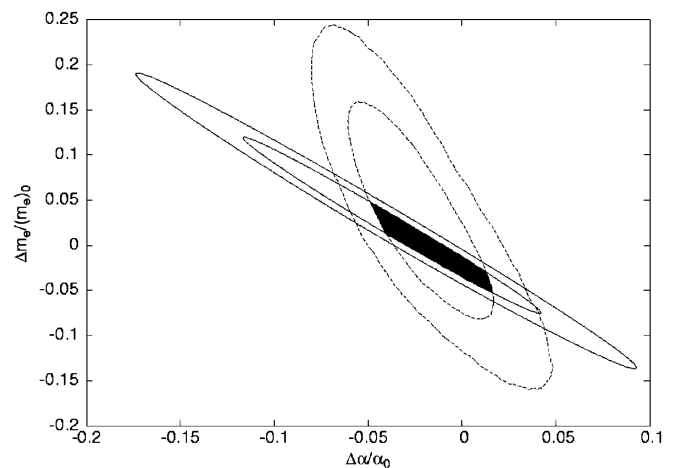


FIG. 12. 2D contour levels for variation of α and m_e from BBN (solid line) and CMB (dotted line) data.

iations of both constants, while the CMB contours are not thin enough to assure any conclusion.

Our results suggest that the model used by Ichikawa *et al.* [58] where the variation of fundamental constants is driven by the time evolution of a dilaton field can be discarded, since these models predict $m_e \simeq \alpha^{1/2}$.

V. SUMMARY AND CONCLUSION

In this work we have studied the joint time variation of the fine structure constant and the Higgs expectation value and the joint variation of α and m_e in the early Universe. We used the observational abundances of D, ${}^4\text{He}$, and ${}^7\text{Li}$ to put bounds on the joint variation of α and $\langle v \rangle$ and on the joint variation of α and m_e during primordial nucleosynthesis. We used the 3 yr WMAP data together with other CMB experiments and the 2dfGRS power spectrum to put bounds on the variation of α and $\langle v \rangle$ (or m_e) at the time of neutral hydrogen formation.

From our analysis we arrive at the following conclusions:

- (1) The consideration of different values of $\frac{\partial \epsilon_D}{\partial m_\pi}$ leads to similar constraints on the time variation of the fundamental constants.
- (2) We obtain non-null results for the joint variation of α and $\langle v \rangle$ at 6σ in two cases: (i) when η_B is allowed to vary and all abundances are included in the data set used to perform the fit and (ii) when only ${}^4\text{He}$ and ${}^7\text{Li}$ are included in the data set used to perform the fit and η_B is fixed to the WMAP estimation. In the first case, the best fit value of η_B is inconsistent with the WMAP estimation.
- (3) We obtain non-null results for the joint variation of α and m_e at 6σ in two cases: (i) when η_B is allowed to vary and all abundances are included in the data set used to perform the fit and (ii) when only ${}^4\text{He}$ and ${}^7\text{Li}$ are included in the data set used to perform the fit and η_B is fixed to the WMAP estimation. In the first case, the best fit value of η_B is inconsistent with the WMAP estimation.
- (4) We also obtain non-null results for the joint variation of α and $\langle v \rangle$ and α and m_e when all abundances are included in the data set and η_B is fixed to the WMAP estimation. The statistical significance of these results is too low to claim a variation of fundamental constants.
- (5) Excluding ${}^7\text{Li}$ abundance from the data set used to perform the fit, and keeping η_B fixed, we find results that are consistent with no variation of fundamental constants within 1σ .
- (6) The bounds obtained using data from CMB and 2dfGRS are consistent with null variation of α and $\langle v \rangle$ (or m_e) at recombination within 1σ .
- (7) We find phenomenological relationships for the variations of α and $\langle v \rangle$, and for the variations of α and m_e , at the time of primordial nucleosynthesis and at

the time of recombination. All the phenomenological relationships correspond to linear fits.

- (8) From our phenomenological approach, it follows that the relationship between the variations of the two pairs of constants considered in this paper is different at the time of nucleosynthesis than at the time of neutral hydrogen formation.
- (9) The dilaton model proposed by Ichikawa *et al.* [58] can be discarded.

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APPENDIX A: PHYSICS AT BBN

We discuss the dependencies on α , $\langle v \rangle$, and m_e of the physical quantities involved in the calculation of the abundances of the light elements. We also argue how these quantities are modified in the Kawano code.

1. Variation of the fine structure constant

The variation of the fine structure constant affects several physical quantities relevant during BBN. These quantities are the cross sections, the Q values of reaction rates, the light nuclei masses, and the neutron-proton mass difference (along with the neutrons and protons initial abundances and the $n \leftrightarrow p$ reaction rates).

The cross sections were modified following Refs. [41,42,44,58] and replacing α by $\alpha_0 (1 + \frac{\Delta\alpha}{\alpha_0})$ in the numerical code. The Q values of reaction rates were modified following Ref. [44].

To consider the effect of the variation of the fine structure constant upon the light nuclei masses, we adopted

$$\frac{\Delta m_x}{(m_x)_0} = P \frac{\Delta\alpha}{\alpha_0}, \quad (\text{A1})$$

where P is a constant of the order of 10^{-4} (see Ref. [44] for details) and m_x is the mass of the nuclei x . These changes affect all of the reaction rates, their Q values and their inverse coefficients.

If the fine structure constant varies with time, the neutron-proton mass difference also changes. Following Ref. [101],

$$\frac{\delta \Delta m_{np}}{\Delta m_{np}} = -0.587 \frac{\Delta\alpha}{\alpha_0}. \quad (\text{A2})$$

This modifies the $n \leftrightarrow p$ and the neutrons and protons initial abundances. The $n \rightarrow p$ reaction rate is calculated by

$$\begin{aligned} \lambda_{n \rightarrow p} = & K \int_{m_e}^{\infty} dE_e \frac{E_e p_e}{1 + e^{E_e/T_\gamma}} \frac{(E_e + \Delta m_{np})^2}{1 + e^{-(E_e + \Delta m_{np})/T_\nu - \xi_l}} \\ & + K \int_{m_e}^{\infty} dE_e \frac{E_e p_e}{1 + e^{-E_e/T_\gamma}} \frac{(E_e - \Delta m_{np})^2}{1 + e^{(E_e - \Delta m_{np})/T_\nu + \xi_l}}, \end{aligned} \quad (\text{A3})$$

where K is a normalization constant proportional to G_F^2 , E_e and p_e are the electron energy and momentum, respectively, T_γ and T_ν are the photon and neutrino temperature, and ξ_l is the ratio between the neutrino chemical potential and the neutrino temperature. In order to include the variation of Δm_{mp} , we replace this quantity by $\Delta m_{mp} (1 + \frac{\delta \Delta m_{mp}}{\Delta m_{mp}})$ in Kawano code. The neutrons and protons initial abundances are calculated by

$$Y_n = \frac{1}{1 + e^{\Delta m_{np}/T + \xi}}, \quad (\text{A4})$$

$$Y_p = \frac{1}{1 + e^{-\Delta m_{np}/T - \xi}}. \quad (\text{A5})$$

2. Variation of the electron mass

If the electron mass can have a different value than the present one during primordial nucleosynthesis, the sum of the electron and positron energy densities, the sum of the electron and positron pressures, and the difference of the electron and positron number densities must be modified in order to include this change. These quantities are calculated in Kawano code as

$$\rho_{e^-} + \rho_{e^+} = \frac{2}{\pi^2} \frac{(m_e c^2)^4}{(\hbar c)^3} \sum_n (-1)^{n+1} \cosh(n\phi_e) M(nz), \quad (\text{A6})$$

$$\frac{p_{e^-} + p_{e^+}}{c^2} = \frac{2}{\pi^2} \frac{(m_e c^2)^4}{(\hbar c)^3} \sum_n \frac{(-1)^{n+1}}{nz} \cosh(n\phi_e) N(nz), \quad (\text{A7})$$

$$\begin{aligned} \frac{\pi^2}{2} \left[\frac{\hbar c^3}{m_e c^2} \right]^3 z^3 (n_{e^-} - n_{e^+}) &= z^3 \sum_n (-1)^{n+1} \\ &\times \sinh(n\phi_e) L(nz), \end{aligned} \quad (\text{A8})$$

where $z = \frac{m_e c^2}{kT_\gamma}$, ϕ_e is the electron chemical potential, and $L(z)$, $M(z)$, and $N(z)$ are combinations of the modified Bessel function $K_i(z)$ [81,102]. The change in these quantities affects their derivatives and the expansion rate through the Friedmann equation:

$$H^2 = \frac{8\pi}{3} G \left(\rho_T + \frac{\Lambda}{3} \right), \quad (\text{A9})$$

where G is the Newton constant, Λ is the cosmological constant, and

$$\rho_T = \rho_\gamma + \rho_{e^-} + \rho_{e^+} + \rho_\nu + \rho_b. \quad (\text{A10})$$

The $n \leftrightarrow p$ reaction rates [see Eq. (A3)] and the weak decay rates of heavy nuclei are also modified if the electron mass varies with time.

It is worthwhile mentioning that the most important changes in the primordial abundances (due to a change in m_e) arrive from the change in the weak rates rather than from the change in the expansion rate [43].

3. Variation of the Higgs vacuum expectation value

If the value of $\langle v \rangle$ during BBN is different than the present value, the electron mass, the Fermi constant, the neutron-proton mass difference, and the deuterium binding energy take different values than the current ones. The electron mass is proportional to the Higgs vacuum expectation value, then

$$\frac{\Delta m_e}{(m_e)_0} = \frac{\Delta \langle v \rangle}{\langle v \rangle_0}. \quad (\text{A11})$$

The Fermi constant is proportional to $\langle v \rangle^{-2}$ [103]; this dependence affects the $n \leftrightarrow p$ reaction rates. The neutron-proton mass difference changes by [101]

$$\frac{\delta \Delta m_{np}}{\Delta m_{np}} = 1.587 \frac{\Delta \langle v \rangle}{\langle v \rangle_0}, \quad (\text{A12})$$

affecting $n \leftrightarrow p$ reaction rates [see Eq. (A3)] and the initial neutron and proton abundances [see Eq. (A4)].

The deuterium binding energy must be corrected by

$$\frac{\Delta \epsilon_D}{(\epsilon_D)_0} = \kappa \frac{\Delta \langle v \rangle}{\langle v \rangle_0}, \quad (\text{A13})$$

where κ is a model dependent constant. This constant can be found: (i) using chiral perturbation theory, as was done by Beane and Savage [104]; (ii) using effective potentials to describe the nucleon-nucleon interaction. This correction affects the initial value of the deuterium abundance

$$Y_d = \frac{Y_n Y_p e^{11.605 \epsilon_D / T_9}}{0.471 \times 10^{-10} T_9^{3/2}}, \quad (\text{A14})$$

where T_9 is the temperature in units of 10^9 K, and ϵ_D is in MeV.

APPENDIX B: PHYSICS AT RECOMBINATION

During the recombination epoch, the ionization fraction, $x_e = n_e/n$ (where n_e and n are the number density of free electrons and of neutral hydrogen, respectively), is determined by the balance between photoionization and recombination.

In this paper, we solved the recombination equations using RECAST [99], taking into account all of the dependencies on α and m_e [43,56–58]. To get a feeling of the dependencies of the physical quantities relevant during recombination, we consider here the Peebles recombination scenario [105]. The recombination equation is

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = C\left(\frac{\alpha_c n_e^2}{n} - \beta_c \frac{n_{1s}}{n} e^{-(B_1 - B_2)/kT}\right), \quad (\text{B1})$$

where

$$C = \frac{(1 + K\Lambda_{2s,1s}n_{1s})}{(1 + K(\beta_c + \Lambda_{2s,1s})n_{1s})} \quad (\text{B2})$$

is the Peebles factor, which inhibits the recombination rate due to the presence of Lyman- α photons, n_{1s} is the number density of hydrogen atoms in the ground state, and B_n is the binding energy of hydrogen in the n th principal quantum

number. The redshift of the Lyman- α photons is $K = \frac{\lambda_a^3 a}{8\pi a}$, with $\lambda_a = \frac{8\pi\hbar c}{3B_1}$, and $\Lambda_{2s,1s}$ is the rate of decay of the $2s$ excited state to the ground state via two-photon emission, and scales as $\alpha^8 m_e$. Recombination directly to the ground state is strongly inhibited, so the case B recombination takes place. The case B recombination coefficient α_c is proportional to $\alpha^3 m_e^{-3/2}$. The photoionization coefficient depends on α_c , but it also has an additional dependence on m_e ,

$$\beta_c = \alpha_c \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-B_2/kT}. \quad (\text{B3})$$

The most important effects of changes in α and m_e during recombination are due to their influence upon Thomson scattering cross section $\sigma_T = \frac{8\pi\hbar^2}{3m_e^2 c^2} \alpha^2$, and the binding energy of hydrogen $B_1 = \frac{1}{2} \alpha^2 m_e c^2$.

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- [1] Y. Wu and Z. Wang, Phys. Rev. Lett. **57**, 1978 (1986).
 - [2] K. Maeda, Mod. Phys. Lett. A **3**, 243 (1988).
 - [3] S.M. Barr and P.K. Mohapatra, Phys. Rev. D **38**, 3011 (1988).
 - [4] T. Damour and A.M. Polyakov, Nucl. Phys. **B423**, 532 (1994).
 - [5] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. Lett. **89**, 081601 (2002).
 - [6] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. D **66**, 046007 (2002).
 - [7] D. Youm, Phys. Rev. D **63**, 125011 (2001).
 - [8] D. Youm, Phys. Rev. D **64**, 085011 (2001).
 - [9] G.A. Palma, P. Brax, A.C. Davis, and C. van de Bruck, Phys. Rev. D **68**, 123519 (2003).
 - [10] P. Brax, C. van de Bruck, A.-C. Davis, and C.S. Rhodes, Astrophys. Space Sci. **283**, 627 (2003).
 - [11] T. Kaluza, Sitzungber. Preuss. Akad. Wiss. K **1**, 966 (1921).
 - [12] O. Klein, Z. Phys. **37**, 895 (1926).
 - [13] S. Weinberg, Phys. Lett. **125B**, 265 (1983).
 - [14] M. Gleiser and J.G. Taylor, Phys. Rev. D **31**, 1904 (1985).
 - [15] J.M. Overduin and P.S. Wesson, Phys. Rep. **283**, 303 (1997).
 - [16] J.D. Bekenstein, Phys. Rev. D **25**, 1527 (1982).
 - [17] J.D. Barrow, H.B. Sandvik, and J. Magueijo, Phys. Rev. D **65**, 063504 (2002).
 - [18] J.D. Barrow and J. Magueijo, Phys. Rev. D **72**, 043521 (2005).
 - [19] S. Bize *et al.*, Phys. Rev. Lett. **90**, 150802 (2003).
 - [20] M. Fischer *et al.*, Phys. Rev. Lett. **92**, 230802 (2004).
 - [21] E. Peik, B. Lipphardt, H. Schnatz, T. Schneider, C. Tamm, and S.G. Karshenboim, Phys. Rev. Lett. **93**, 170801 (2004).
 - [22] J.D. Prestage, R.L. Tjoelker, and L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995).
 - [23] Y. Sortais *et al.*, Phys. Scr. T **95**, 50 (2001).
 - [24] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
 - [25] Y. Fujii, A. Iwamoto, T. Fukahori, T. Ohnuki, M. Nakagawa, H. Hidaka, Y. Oura, and P. Möller, Nucl. Phys. **B573**, 377 (2000).
 - [26] T. Damour and F. Dyson, Nucl. Phys. **B480**, 37 (1996).
 - [27] K.A. Olive, M. Pospelov, Y.Z. Qian, G. Manhès, E. Vangioni-Flam, A. Coc, and M. Cassé, Phys. Rev. D **69**, 027701 (2004).
 - [28] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
 - [29] J.K. Webb, M.T. Murphy, V.V. Flambaum, V.A. Dzuba, J.D. Barrow, C.W. Churchill, J.X. Prochaska, and A.M. Wolfe, Phys. Rev. Lett. **87**, 091301 (2001).
 - [30] M.T. Murphy, J.K. Webb, V.V. Flambaum, V.A. Dzuba, C.W. Churchill, J.X. Prochaska, J.D. Barrow, and A.M. Wolfe, Mon. Not. R. Astron. Soc. **327**, 1208 (2001).
 - [31] M.T. Murphy, J.K. Webb, V.V. Flambaum, J.X. Prochaska, and A.M. Wolfe, Mon. Not. R. Astron. Soc. **327**, 1237 (2001).
 - [32] M.T. Murphy, J.K. Webb, and V.V. Flambaum, Mon. Not. R. Astron. Soc. **345**, 609 (2003).
 - [33] A. Ivanchik, P. Petitjean, D. Varshalovich, B. Aracil, R. Srianand, H. Chand, C. Ledoux, and P. Boissé, Astron. Astrophys. **440**, 45 (2005).
 - [34] P. Tzanavaris, M.T. Murphy, J.K. Webb, V.V. Flambaum, and S.J. Curran, Mon. Not. R. Astron. Soc. **374**, 634 (2007).
 - [35] A.F. Martínez Fiorenzano, G. Vladilo, and P. Bonifacio, Soc. Astronom. Ital. Mem. Suppl. **3**, 252 (2003).
 - [36] R. Quast, D. Reimers, and S.A. Levshakov, Astron. Astrophys. **415**, L7 (2004).
 - [37] J.N. Bahcall, C.L. Steinhardt, and D. Schlegel, Astrophys. J. **600**, 520 (2004).
 - [38] R. Srianand, H. Chand, P. Petitjean, and B. Aracil, Phys. Rev. Lett. **92**, 121302 (2004).

- [39] M.E. Mosquera, C.G. Scóccola, S.J. Landau, and H. Vucetich, *Astron. Astrophys.* **478**, 675 (2008).
- [40] C.G. Scóccola, M.E. Mosquera, S.J. Landau, and H. Vucetich, *Astrophys. J.* **681**, 737 (2008).
- [41] L. Bergström, S. Iguri, and H. Rubinstein, *Phys. Rev. D* **60**, 045005 (1999).
- [42] K.M. Nollett and R.E. Lopez, *Phys. Rev. D* **66**, 063507 (2002).
- [43] J.J. Yoo and R.J. Scherrer, *Phys. Rev. D* **67**, 043517 (2003).
- [44] S.J. Landau, M.E. Mosquera, and H. Vucetich, *Astrophys. J.* **637**, 38 (2006).
- [45] N. Chamoun, S.J. Landau, M.E. Mosquera, and H. Vucetich, *J. Phys. G* **34**, 163 (2007).
- [46] B.A. Campbell and K.A. Olive, *Phys. Lett. B* **345**, 429 (1995).
- [47] K. Ichikawa and M. Kawasaki, *Phys. Rev. D* **65**, 123511 (2002).
- [48] K. Ichikawa and M. Kawasaki, *Phys. Rev. D* **69**, 123506 (2004).
- [49] C.M. Müller, G. Schäfer, and C. Wetterich, *Phys. Rev. D* **70**, 083504 (2004).
- [50] A. Coc, N.J. Nunes, K.A. Olive, J.-P. Uzan, and E. Vangioni, *Phys. Rev. D* **76**, 023511 (2007).
- [51] R.H. Cyburt, B.D. Fields, K.A. Olive, and E. Skillman, *Astropart. Phys.* **23**, 313 (2005).
- [52] T. Dent, S. Stern, and C. Wetterich, *Phys. Rev. D* **76**, 063513 (2007).
- [53] V.V. Flambaum and E.V. Shuryak, *Phys. Rev. D* **65**, 103503 (2002).
- [54] V.V. Flambaum, D.B. Leinweber, A.W. Thomas, and R.D. Young, *Phys. Rev. D* **69**, 115006 (2004).
- [55] J.P. Kneller and G.C. McLaughlin, *Phys. Rev. D* **68**, 103508 (2003).
- [56] C.J.A.P. Martins, A. Melchiorri, R. Trotta, R. Bean, G. Rocha, P.P. Avelino, and P.T.P. Viana, *Phys. Rev. D* **66**, 023505 (2002).
- [57] G. Rocha, R. Trotta, C.J.A.P. Martins, A. Melchiorri, P.P. Avelino, and P.T.P. Viana, *New Astron. Rev.* **47**, 863 (2003).
- [58] K. Ichikawa, T. Kanzaki, and M. Kawasaki, *Phys. Rev. D* **74**, 023515 (2006).
- [59] D.N. Spergel *et al.*, *Astrophys. J. Suppl. Ser.* **148**, 175 (2003).
- [60] D.N. Spergel *et al.*, *Astrophys. J. Suppl. Ser.* **170**, 377 (2007).
- [61] A.G. Sanchez, C.M. Baugh, W.J. Percival, J.A. Peacock, N.D. Padilla, S. Cole, C.S. Frenk, and P. Norberg, *Mon. Not. R. Astron. Soc.* **366**, 189 (2006).
- [62] M. Pettini and D.V. Bowen, *Astrophys. J.* **560**, 41 (2001).
- [63] J.M. O’Meara, D. Tytler, D. Kirkman, N. Suzuki, J.X. Prochaska, D. Lubin, and A.M. Wolfe, *Astrophys. J.* **552**, 718 (2001).
- [64] D. Kirkman, D. Tytler, N. Suzuki, J.M. O’Meara, and D. Lubin, *Astrophys. J. Suppl. Ser.* **149**, 1 (2003).
- [65] S. Burles and D. Tytler, *Astrophys. J.* **499**, 699 (1998).
- [66] S. Burles and D. Tytler, *Astrophys. J.* **507**, 732 (1998).
- [67] N.H.M. Crighton, J.K. Webb, A. Ortiz-Gil, and A. Fernández-Soto, *Mon. Not. R. Astron. Soc.* **355**, 1042 (2004).
- [68] J.M. O’Meara, S. Burles, J.X. Prochaska, G.E. Prochter, R.A. Bernstein, and K.M. Burgess, *Astrophys. J. Lett.* **649**, L61 (2006).
- [69] C.M. Oliveira, H.W. Moos, P. Chayer, and J.W. Kruk, *Astrophys. J.* **642**, 283 (2006).
- [70] S. Ryan, T. Beers, K. Olive, B.D. Fields, and J.E. Norris, *Astrophys. J.* **530**, L57 (2000).
- [71] P. Bonifacio, P. Molaro, and L. Pasquini, *Mon. Not. R. Astron. Soc.* **292**, L1 (1997).
- [72] P. Bonifacio and P. Molaro, *Mon. Not. R. Astron. Soc.* **285**, 847 (1997).
- [73] Bonifacio *et al.*, *Astron. Astrophys.* **390**, 91 (2002).
- [74] M. Asplund, D.L. Lambert, P.E. Nissen, F. Primas, and V.V. Smith, *Astrophys. J.* **644**, 229 (2006).
- [75] A.M. Boesgaard, M.C. Novicki, and A. Stephens, in *Proceedings of IAU Symposium No. 228: “From Lithium to Uranium: Elemental Tracers of Early Cosmic Evolution”*, edited by V. Hill, P. Francois, and F. Primas (Cambridge University Press, Cambridge, England, 2005), p. 29.
- [76] P. Bonifacio *et al.*, *Astron. Astrophys.* **462**, 851 (2007).
- [77] M. Peimbert, V. Luridiana, and A. Peimbert, *Astrophys. J.* **666**, 636 (2007).
- [78] Y.I. Izotov, T.X. Thuan, and G. Stasińska, *Astrophys. J.* **662**, 15 (2007).
- [79] Yao *et al.*, *J. Phys. G* **33**, 1 (2006), <http://pdg.lbl.gov>.
- [80] K.A. Olive and E.D. Skillman, *Astrophys. J.* **617**, 29 (2004).
- [81] L. Kawano, Report No. FERMILAB-PUB-92-004-A, 1992.
- [82] D.A. Dicus, E.W. Kolb, A.M. Gleeson, E.C.G. Sudarshan, V.L. Teplitz, and M.S. Turner, *Phys. Rev. D* **26**, 2694 (1982).
- [83] S.R. Beane and M.J. Savage, *Nucl. Phys.* **A717**, 91 (2003).
- [84] R.V. Reid, Jr., *Ann. Phys. (N.Y.)* **50**, 411 (1968).
- [85] E. Epelbaum, U. Meißner, and W. Glöckle, *Nucl. Phys.* **A714**, 535 (2003).
- [86] O. Richard, G. Michaud, and J. Richer, *Astrophys. J.* **619**, 538 (2005).
- [87] J. Meléndez and I. Ramírez, *Astrophys. J. Lett.* **615**, L33 (2004).
- [88] T. Prodanović and B.D. Fields, *Phys. Rev. D* **76**, 083003 (2007).
- [89] R.A. Battye, R. Crittenden, and J. Weller, *Phys. Rev. D* **63**, 043505 (2001).
- [90] S.J. Landau, D.D. Harari, and M. Zaldarriaga, *Phys. Rev. D* **63**, 083505 (2001).
- [91] P.P. Avelino, C.J.A.P. Martins, G. Rocha, and P. Viana, *Phys. Rev. D* **62**, 123508 (2000).
- [92] A.C.S. Readhead *et al.*, *Astrophys. J.* **609**, 498 (2004).
- [93] C. Kuo *et al.* (ACBAR Collaboration), *Astrophys. J.* **600**, 32 (2004).
- [94] F. Piacentini *et al.*, *Astrophys. J.* **647**, 833 (2006).
- [95] W.C. Jones *et al.*, *Astrophys. J.* **647**, 823 (2006).
- [96] S. Cole *et al.* (2dFGRS Collaboration), *Mon. Not. R. Astron. Soc.* **362**, 505 (2005).
- [97] A. Lewis and S. Bridle, *Phys. Rev. D* **66**, 103511 (2002).
- [98] A. Lewis, A. Challinor, and A. Lasenby, *Astrophys. J.* **538**, 473 (2000).
- [99] S. Seager, D.D. Sasselov, and D. Scott, *Astrophys. J. Lett.* **523**, L1 (1999).

- [100] A.E. Raftery and S.M. Lewis, in *Bayesian Statistics*, edited by J.M. Bernardo (Oxford University Press, New York, 1992), p. 765.
- [101] H.R. Christiansen, L.N. Epele, H. Fanchiotti, and C.A. García Canal, *Phys. Lett. B* **267**, 164 (1991).
- [102] L. Kawano, Report No. FERMILAB-PUB-88-034-A, 1988.
- [103] V.V. Dixit and M. Sher, *Phys. Rev. D* **37**, 1097 (1988).
- [104] S.R. Beane and M.J. Savage, *Nucl. Phys.* **A713**, 148 (2003).
- [105] P.J.E. Peebles, *Astrophys. J.* **153**, 1 (1968).