Remarks on Goldstone bosons and hard thermal loops

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May 6, 2021

Abstract

The hard thermal loop effective action for Goldstone bosons is deduced by symmetry arguments from the corresponding result for gauge bosons. Pseudoscalar mesons in Chromodynamics and magnons in an antiferromagnet are discussed as special cases, including the hard thermal loop contribution to their scattering.

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The importance of hard thermal loops (HTL's) in a thermal gauge theory was recognized a few years ago [1]. The proper identification of the HTL-contributions and the resummation of Feynman diagrams to take into account their effects are a crucial first step towards a thermal perturbation theory for gauge fields, which is free of infrared singularities. The HTL-contributions in a gauge theory can be summarized by an effective action, different versions of which have been analyzed in detail by various groups [1, 2, 3, 4]. More recently, it has been pointed out that there are HTL-contributions in the chiral model for pions or more generally in a theory of Goldstone bosons [5, 6]. Since Goldstone bosons behave in a way similar to the longitudinal polarizations of massive gauge bosons, we can expect that the HTL's for Goldstone bosons should be related to the HTL's for gauge bosons via symmetry arguments. Some elements of this connection are evident in references [5, 6]. Nevertheless the arguments presented there are not entirely symmetry-based. It should be possible to deduce the HTL effective action for Goldstone bosons purely by symmetry arguments starting from the HTL action for gauge bosons. In this note, we present the relevant arguments, for Goldstone bosons corresponding to a global symmetry group G being spontaneously broken to $H \subset G$. The basic strategy is to rewrite the dynamics of the Goldstone bosons as a gauge theory with gauge group H and then to use this gauge theory result with appropriate minor changes. As special cases, we consider $G = SU_L(N_f) \times SU_R(N_f)\,, H = SU_{L+R}(N_f)$ corresponding to the pseudo scalar mesons and $G = SU(2)\,, H = U(1)$ corresponding to magnons or spin waves in an antiferromagnet.

The Goldstone boson fields corresponding to the symmetry breaking $G \to H$ take values in the coset G/H and their dynamics can be described by a nonlinear sigma model with target space G/H. We begin with a brief description of this theory as a theory with H-gauge symmetry [7]. Let $T^\alpha,\ \ \alpha = 1, \cdots, dim\ G$ denote the generators of $\mathcal{G}$ and $t^a,\ \ a = 1, \cdots, dim\ H$ denote the generators of $\mathcal{H}$. We assume the standard normalization $Tr(T^\alpha T^\beta) = 1/2\ \delta^{\alpha\beta}$, for the fundamental representation of the generators. The generators in the orthogonal complement of $\mathcal{H}$ in $\mathcal{G}$ will be denoted by $S^i,\ \ i = 1, \cdots, dim\ G - dim\ H$. The commutation rules are of the form

$$\begin{aligned} &[t^a, t^b] = if^{abc} t^c\ , \qquad [t^a, S^i] = i\,(D^a)^{ij}\, S^j \\ &[S^i, S^j] = i f^{aij} t^a. \end{aligned} \tag{1}$$

The structure of these commutation rules, with $[S, S] \approx t$ implies that we are considering the case when G/H is a symmetric space. Let $g(x)$ be a G-

valued field. Define

$$V_\mu^a = 2\,Tr\left(t^a \partial_\mu g g^{-1}\right)\ , \qquad E_\mu^i = 2\,Tr\left(S^i \partial_\mu g g^{-1}\right). \tag{2}$$

This corresponds to the decomposition $\partial_\mu g g^{-1} = V_\mu + E_\mu,\ \ V_\mu = t^a V_\mu^a,\ \ E_\mu = S^i E_\mu^i$. Under H- transformations of G on the left, *i.e.,* under $g \to g' = hg$, V_μ transforms as a gauge potential, namely,

$$V_\mu(hg) = hV_\mu h^{-1} + \partial_\mu h h^{-1}. \tag{3}$$

The field strength associated with this gauge potential is given by

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - [V_\mu, V_\nu] \\ &= (-it^a)\, f^{aij} E_\mu^i E_\nu^j. \end{aligned} \tag{4}$$

The gauge potential V_μ also allows us to the define the covariant derivative

$$D_\mu g = \partial_\mu g - V_\mu g. \tag{5}$$

The Lagrangian for the G/H- sigma model may be written as

$$L = -\alpha Tr\left(D_\mu g g^{-1}\ D^\mu g g^{-1}\right). \tag{6}$$

This Lagrangian has invariance under the global G transformations $g \to g\ U\ ,\ \ U \in G$, as expected for a theory for which the symmetry breaking $G \to H$ is only spontaneous. Further, it has invariance under the local H-gauge transformations $g(x) \to h(x)g(x)$. The field $g(x)$ has $dim\ G$ degrees of freedom. The H-gauge invariant shows that it is possible to "gauge away" the degrees of freedom corresponding to H, leaving only G/H-degrees of freedom. (This can general be done only locally in some parametrizations of g and H, since, in general, $G \neq G/H \times H$.) With the splitting $\partial_\mu g g^{-1} = V_\mu + E_\mu$, we find $L = -\alpha/2\ E_\mu^i E^{i\mu}$, which is proportional to the Cartan-Killing metric on the coset space G/H. Thus (6) is indeed equivalent to the standard sigma model for G/H.

The Lagrangian (6) describes the G/H-model as a theory of "matter fields" minimally coupled to an H-gauge potential V_μ. At finite temperature, therefore we expect a hard thermal loop mass term for the gauge field V_μ, due to the electrical screening effects of the matter fields in G/H. Now, the

HTL-effective action for a pure gauge theory with no matter fields, is given in terms of the gauge potential A_μ as [3]

$$\Gamma[A]_{gauge} = \frac{C_G T^2}{6} \int d\Omega \; d^2x^T \; S_{WZW}(N^{-1}M) \tag{7}$$

where C_G is the quadratic Casimir for the adjoint representation of the group and S_{WZW} is the Wess-Zumino-Witten action defined on the two-dimensional space of $x^\pm = 1/2\;(x^0 \mp \vec{Q}\cdot\vec{x})$. i.e.,

$$\begin{aligned} S_{WZW}(U) &= \frac{1}{2\pi}\int_M dx^+ dx^- \;\; \mathrm{tr}(\partial_+ U \partial_- U^{-1}) \\ & \qquad - \frac{i}{12\pi}\int_{M^3} d^3x \; \epsilon^{\mu\nu\alpha}\, \mathrm{tr}(U^{-1}\partial_\mu U U^{-1}\partial_\nu U U^{-1}\partial_\alpha U) \end{aligned} \tag{8}$$

M, N are defined by $A_+ = 1/2\;(A_0 + \vec{Q}\cdot\vec{A}) = -\partial_+ M M^{-1}$, $A_- = 1/2\;(A_0 - \vec{Q}\cdot\vec{A}) = -\partial_- N N^{-1}$. $d\Omega$ denotes integration over the orientation of the unit vector $\vec{Q}$; integration over coordinates transverse to $\vec{Q}$, *viz.*,x^T, is explicitly shown in (7) while integration over $x^\pm$ is included in the definition of S_{WZW}.

For the G/H-model, the result should be similar to (7) with A_μ replaced by $V_\mu = t^a \; 2Tr\,(t^a \partial_\mu g g^{-1})$. The overall coefficient will be different. In the case of gauge bosons, there are two polarization states which contribute to the screening; for Goldstone bosons we have only one polarization state. This should give an additional factor of 1/2. Further, for the G/H model, the V_μ-fields couple only to the G/H-degrees of freedom, the coupling charge matrices being f^{aij} from (1). Since $f^{aij}f^{bij} = f^{a\alpha\beta}f^{b\alpha\beta} - f^{acd}f^{bcd} = (C_G - C_H)\;\delta^{ab}$, we see that C_G in eq.(1 should be replaced by $C_G - C_H$. The HTL-effective action for the Goldstone modes in G/H can thus be written as

$$\begin{aligned} \Gamma[V] &= \frac{T^2}{12}\,(C_G - C_H)\int d\Omega d^2x^T \; S_{WZW}(N^{-1}M) \\ &= \frac{1}{2}\,\frac{C_G - C_H}{C_G}\;\Gamma[A]\Big|_{A_\mu \to V_\mu} . \end{aligned} \tag{9}$$

This result has been obtained purely by symmetry arguments. It can be checked by explicit calculations or by comparison to previous calculations as we shall do shortly.

Notice that Γ as given by (9), is at least quartic in the Goldstone fields. Since Γ is gauge-invariant, the H-degrees of freedom can be removed; by orthogonality of t^a and S^i and the commutation rules (1), up to an H-gauge transformation, V_μ is at least quadratic in the Goldstone fields:

$$\begin{aligned} V^a_\mu &= 2Tr\left(t^a\, \partial_\mu e^{i\pi^i S^i}\, e^{-i\pi^i S^i}\right) \\ &\approx 2Tr\; t^a(i\partial_\mu \pi^i S^i + \partial_\mu \pi^i \pi^j [S^i, S^j] + \cdots) \approx i f^{aij} \partial_\mu \pi^i \pi^j + \cdots \end{aligned} \tag{10}$$

Γ being quadratic in V_μ's, the lowest order term in (9) is quartic in the Goldstone fields.

A comment regarding the direct evaluation of the result in terms of the Goldstone fields is in order. In terms of the gauge field V_μ, the leading term in (9) is quadratic and this can be evaluated by the two-point vacuum polarization diagram with V_μ on the external lines. A comparison of the overall coefficient in (9) can thus be done with the explicit evaluation of the vacuum polarization diagram. However, for the term with four external Goldstone particles, higher diagrams with upto four external lines can in principle contribute. Directly in terms of Goldstone fields, the orders of various terms can get mixed up, since V_μ is itself made of the Goldstone fields and obeys identities like (4) (where the curl of V_μ is related to a term quadratic in the fields). In seeking a covariant generalization of the result of the vacuum polarization diagram, this point must be taken care of. One must keep V_μ as an arbitrary external field and compare the coefficient of (9) with the evaluation of the vacuum polarization diagram. This seems to account for the discrepancy of a factor of 4 between references [5] and [6].

The result for pions given in references [5, 6] also include the leading T^2-correction to the coefficient α in the chiral Lagrangian (6). Such a correction, which can contribute at the quadratic order in the Goldstone fields, is not, from our point of view, a hard thermal loop contribution. To see how this arises, consider a background field expansion of (6). Writing $g = U\ B$, where B denotes the background field, and $U = exp\,(i\varphi^j S^j)$ we find

$$L = \frac{1}{2}\left(D_\mu \varphi\right)^2 + 2\mathcal{A}^i_\mu \mathcal{A}^i_\mu + 2\varphi^j \varphi^k f^{jml} f^{knl} \mathcal{A}^m_\mu \mathcal{A}^n_\mu + \cdots \tag{11}$$

where $\mathcal{A}^i_\mu = 1/2\ \left(\partial_\mu B B^{-1}\right)^i$, $D^{ij}_\mu = \partial_\mu \delta^{ij} + f^{ija} V^a_\mu$, $V^a_\mu = 1/2\ \left(\partial_\mu B B^{-1}\right)^a$. The first term shows the H-gauge invariant structure and leads to the result (9) as we have argued. The last term gives, upon Wick contraction of φ's with a thermal propagator,

$$\delta\Gamma = -2\left[\frac{T^2}{24}(C_G - C_H)\right] \int\ \mathcal{A}^2 \tag{12}$$

which corresponds to the modification $\alpha \to \alpha(T)$,

$$\alpha(T) = \alpha - \frac{T^2}{24}(C_G - C_H). \tag{13}$$

To leading order in T^2 and in HTL-approximation, (9) and (12) are the only corrections.

We now consider the specialization of the results (9), (12) to the case of pions or pseudoscalar mesons. In this case $G = SU_L(N_f) \times SU_R(N_f)\,, H = SU_{L+R}(N_f)$. G may be parametrized by $(g_1, g_2)\ ,\ \ g_i(x) \in SU(N_f)$. The gauge potential is given by$V_\mu = 1/2\ \left(\partial_\mu g_1 g_1^{-1} - g_2^{-1}\partial_\mu g_2\right)$ with H transformations acting as $g_1 \to h(x) g_1\ ,\ \ g_2 \to g_2 h^{-1}(x)\ ,\ \ h(x) \in SU(N_f)$. Global G-transformations act as $g_1 \to g_1 U_L\ ,\ \ g_2 \to U_R g_2\ ,\ \ U_L, U_R \in G$. The Lagrangian (6) becomes

$$\begin{aligned} L &= -\alpha\, Tr\left[\left(g_1^{-1} D_\mu g_1\right)^2 + \left(g_2 D_\mu g_2^{-1}\right)^2\right] \\ &= -2\alpha Tr\left(\mathcal{A}_\mu^2\right) \end{aligned} \tag{14}$$

where $D_\mu = \partial_\mu - V_\mu$ and $\mathcal{A}_\mu = 1/2\ \left(\partial_\mu g_1 g_1^{-1} + g_2^{-1}\partial_\mu g_2\right)$. The H-symmetry allows us to chose a gauge where $g_2 = 1$ or equivalently we can consider $g_2 g_1 = U(x) \in SU(N_f)$ as the residual degrees of freedom. In this gauge $V_\mu = \mathcal{A}_\mu = 1/2\ (\partial_\mu U U^{-1})$ and $L = -\alpha/2\ Tr\,(\partial_\mu U U^{-1})^2$ which is the usual chiral Lagrangian with $\alpha = 2 f_\pi^2$. In this case, by expansion of (9) in powers of V_μ, we can check by direct comparison that (9) agrees with the result of references [5, 6]. Furthermore, from (13),

$$f_\pi^2(T) = f_\pi^2 - \frac{N_f T^2}{48} \tag{15}$$

which also agrees with the result in references [5, 6], noting that with our normalization for the generators, our f_π^2 is 1/4 of the f_π^2 used in [5, 6].

Using equation (9) we can evaluate the pion-pion scattering amplitude for the process $(E_1, \vec{k}_1, e^1), (E_2, \vec{k}_2, e^2) \to (E_3, \vec{k}_3, e^3), (E_4, \vec{k}_4, e^4)$, ($e^1, e^2, e^3$, and e^4 are polarization vectors), where the pion fields are related to the field U through the identity $U = exp\ (i\pi^i t^i / f_\pi)$ (we are considering here the case $N_f = 2$).The result can be computed to be:

$$\mathcal{A} \ = \ \frac{i\ \delta^4(k_1 + k_2 - k_3 - k_4)}{(2\pi)^2\ \prod_i \sqrt{2\ E_i}}\ \mathcal{M},$$

$$
\begin{aligned}
\mathcal{M} &= A\,(e^1\cdot e^2)\,(e^3\cdot e^4)\ +\ B\,(e^1\cdot e^3)\,(e^2\cdot e^4)\ +\\
&\qquad C\,(e^1\cdot e^4)\,(e^2\cdot e^3)\ ,\\
A &= \frac{1}{4f_\pi^2(T)}\,(k_1\cdot k_2+k_3\cdot k_4)-\frac{T^2}{192f_\pi^4}\Big[(k_1+k_3)_\mu\,M_{\mu\nu}(k_1-k_3)\times\\
&\quad (k_2+k_4)_\nu+(k_1+k_4)_\mu\,M_{\mu\nu}(k_1-k_4)\,(k_2+k_3)_\nu\Big]\ ,\\
B &= -\frac{1}{4f_\pi^2(T)}\,(k_1\cdot k_3+k_2\cdot k_4)+\frac{T^2}{192f_\pi^4}\Big[(k_1-k_2)_\mu\,M_{\mu\nu}(k_1+k_2)\times\\
&\quad (k_3-k_4)_\nu+(k_1+k_4)_\mu\,M_{\mu\nu}(k_1-k_4)\,(k_2+k_3)_\nu\Big]\ ,\\
C &= -\frac{1}{4f_\pi^2(T)}\,(k_1\cdot k_4+k_2\cdot k_3)-\frac{T^2}{192f_\pi^4}\Big[(k_1-k_2)_\mu\,M_{\mu\nu}(k_1+k_2)\times\\
&\quad (k_3-k_4)_\nu-(k_1+k_3)_\mu\,M_{\mu\nu}(k_1-k_3)\,(k_2+k_4)_\nu\Big]\ .
\end{aligned}
\tag{16}
$$

The bilinear kernel $M_{\mu\nu}(p)$ is given by

$$
M_{\mu\nu}(p)=g_{\mu0}\;g_{\nu0}-p^0\int\frac{d\Omega_Q}{4\pi}\;\frac{Q_\mu Q_\nu}{p\cdot Q}
\tag{17}
$$

(here Q is the null vector $(1,\vec{q})$, $\vec{q}^2=1$).

The expression (16) takes a particularly simple form if the total (spatial) momentum is zero: $\vec{k}_1+\vec{k}_2=0$, $E_i\equiv E=|\vec{k}_1|$ and the scattering angle is defined by $\vec{k}_1\cdot\vec{k}_3=|\vec{k}_1||\vec{k}_3|\cos\theta$. Then

$$
\begin{aligned}
A &= \frac{E^2}{f_\pi^2(T)}\left(1-\frac{T^2}{24f_\pi^2(T)}\right)\approx\frac{E^2}{f_\pi^2},\\
B &= B_1-B_2\;cos(\theta)\ ,\qquad C=B_1+B_2\;cos(\theta)\ ,\\
B_1 &= -\frac{E^2}{2\;f_\pi^2(T)}\left(1-\frac{T^2}{24\;f_\pi^2(T)}\right)\approx-\frac{E^2}{2\;f_\pi^2},\\
B_2 &= -\frac{E^2}{2\;f_\pi^2(T)}\left(1-\frac{T^2}{72\;f_\pi^2(T)}\right)\approx-\frac{E^2}{2\;f_\pi^2}\left(1+\frac{T^2}{36f_\pi^2}\right).
\end{aligned}
\tag{18}
$$

Notice that the contribution of the hard thermal loops is comparable, and with opposite sign, to the other leading T-dependent corrections. Moreover, for a scattering angle of $\theta=\pm\pi/2$ the scattering amplitude is independent of the temperature.

We now consider the case of spin waves or magnons in an antiferromagnet [8]. Since the dispersion relation is linear for antiferromagnetic magnons (as opposed to the ferromagnetic case), it is for this case that it is possible to adapt equations (9) and (12) in a simple way. The groups involved are $G = SU(2)$ and $H = U(1)$. A convenient parametrization for $g \in SU(2)$ is

$$g = \lambda \begin{pmatrix} 1 & z \\ -\bar{z} & 1 \end{pmatrix} \frac{1}{\sqrt{1 + z\bar{z}}} \tag{19}$$

where $\lambda = \exp(i\sigma^3\theta/2) \in U(1)$. $(z, \bar{z})$ parametrize the coset $SU(2)/U(1)$. From $\partial_\mu g\ g^{-1}$, we identify

$$V_\mu = i\frac{(\bar{z}\partial_\mu z - \partial_\mu \bar{z} z)}{(1 + z\bar{z})} \tag{20}$$

Specialization of (9) to the magnon case is obtained by taking $G = SU(2), H = U(1)$ and $A_\mu^{1,2} = 0, A_\mu^3 = V_\mu$. In addition, we have to incorporate the fact that magnons have a propagation speed v which is not 1. The dispersion relation $\omega = v|\vec{k}|$ shows that every spatial derivative should carry a factor of v. In other words, we need $\partial_\mu \to \tilde{\partial}_\mu = (\partial_0, v\partial_i)$. Further there must be a factor of $(1/v^3)$ in Γ for dimensional reasons. This can also be seen diagrammatically as arising from $d^3k = k^2 dk d\Omega = (1/v^3)\omega^2 d\omega d\Omega$. Putting all this together

$$\Gamma = -\frac{T^2}{24\pi v^3} \int \frac{d^4k}{(2\pi)^4} \left(\frac{\bar{z}\tilde{\partial}_\mu z - z\tilde{\partial}_\mu \bar{z}}{1 + z\bar{z}} \right)(-k) M_{\mu\nu}(\tilde{k}) \left(\frac{\bar{z}\tilde{\partial}_\mu z - z\tilde{\partial}_\mu \bar{z}}{1 + z\bar{z}} \right)(k) \tag{21}$$

where $M_{\mu\nu}$ is given in equation (17).

The kinetic energy term or the sigma model part of the action is given by (6) with appropriate changes as

$$S_0 = 2\alpha \frac{\tilde{\partial}_\mu z \tilde{\partial}_\mu \bar{z}}{(1 + z\bar{z})^2} = \frac{1}{2} \frac{\tilde{\partial}_\mu \varphi_i \tilde{\partial}_\mu \varphi_i}{(1 + \frac{\varphi_i \varphi_i}{4\alpha})^2} \tag{22}$$

where $2\sqrt{\alpha}\ z = (\varphi_1 - i\varphi_2)$ and $\alpha(T) = \alpha(0) - (T^2/12v^3)$.

The hard thermal loop contribution is at least quartic in the magnon fields and so can contribute to a T-dependent term to magnon-magnon scattering. The quartic term in (22) also contributes to such a process. The magnon wave function can be taken to be

$$\varphi_i = e_i^{(\lambda)} \frac{\exp(-i(\omega t - \vec{k} \cdot \vec{x}))}{\sqrt{2\omega V}} \tag{23}$$

where $e_i^{(\lambda)}$ is the polarization and we choose normalization in a volume V. Consider the scattering process $(k_1, e^1), (k_2, e^2) \rightarrow (k_3, e^3), (k_4, e^4)$. The amplitude for this process can be calculated to be

$$\begin{aligned}
\mathcal{A} &= \frac{i(2\pi)^4 \delta(k_1 + k_2 - k_3 - k_4)}{\prod_i \sqrt{2\omega_i V}}\, \mathcal{M}, \\
\mathcal{M} &= A\,(e^1 \cdot e^2)\,(e^3 \cdot e^4) \;+\; B\,(e^1 \cdot e^3)\,(e^2 \cdot e^4) \;+ \\
&\qquad C\,(e^1 \cdot e^4)\,(e^2 \cdot e^3) \\
A &= \frac{1}{\alpha(T)}\,(k_1 \cdot k_2 + k_3 \cdot k_4) - \frac{T^2}{48\pi v^3 \alpha(T)^2} \Big[(k_1 + k_3)_\mu\, M_{\mu\nu}(k_1 - k_3) \times \\
&\quad (k_2 + k_4)_\nu + (k_1 + k_4)_\mu\, M_{\mu\nu}(k_1 - k_4)\,(k_2 + k_3)_\nu \Big] \\
B &= -\frac{1}{\alpha(T)}\,(k_1 \cdot k_3 + k_2 \cdot k_4) + \frac{T^2}{48\pi v^3 \alpha(T)^2} \Big[(k_1 - k_2)_\mu\, M_{\mu\nu}(k_1 + k_2) \times \\
&\quad (k_3 - k_4)_\nu + (k_1 + k_4)_\mu\, M_{\mu\nu}(k_1 - k_4)\,(k_2 + k_3)_\nu \Big] \\
C &= -\frac{1}{\alpha(T)}\,(k_1 \cdot k_4 + k_2 \cdot k_3) - \frac{T^2}{48\pi v^3 \alpha(T)^2} \Big[(k_1 - k_2)_\mu\, M_{\mu\nu}(k_1 + k_2) \times \\
&\quad (k_3 - k_4)_\nu - (k_1 + k_3)_\mu\, M_{\mu\nu}(k_1 - k_3)\,(k_2 + k_4)_\nu \Big] \;. \qquad (24)
\end{aligned}$$

Again, this expression is enormously reduced if the combined momentum of the incoming magnons vanishes. In this case we have

$$\begin{aligned}
A &= \frac{4\omega^2}{\alpha(T)} \left(1 - \frac{T^2}{6v^3 \alpha(T)} \right) \approx \frac{4\omega^2}{\alpha(0)} \left(1 - \frac{T^2}{12 v^3 \alpha(0)} \right), \\
B &= B_1 - B_2\, cos(\theta)\;, \qquad C = B_1 + B_2\, cos(\theta)\;, \\
B_1 &= -\frac{2\omega^2}{\alpha(T)} \left(1 - \frac{T^2}{6v^3 \alpha(T)} \right) \approx -\frac{2\omega^2}{\alpha(0)} \left(1 - \frac{T^2}{12 v^3 \alpha(0)} \right), \\
B_2 &= \frac{2\omega^2}{\alpha(T)} \left(1 - \frac{T^2}{18 v^3 \alpha(T)} \right) \approx \frac{2\omega^2}{\alpha(0)} \left(1 + \frac{T^2}{36 v^3 \alpha(0)} \right) \;. \qquad (25)
\end{aligned}$$

As in the case of pion scattering, the contribution of the hard thermal loops is of the same order of magnitude as the other leading T-dependent corrections. The temperature at which $\alpha(T)$ vanishes, and thereby restores disorder, gives an estimate of the Néel temperature T_N as $T_N^2 = 12 v^3 \alpha(0)$. This is of course rather crude, the calculation of $\alpha(T)$ cannot be trusted very near the transition point; nevertheless it gives a rough estimate. The corrections to scattering are thus seen to be proportional to (T^2/T_N^2).

To recapitulate, we have shown in this article that the hard thermal loop effective action for Goldstone bosons corresponding to a symmetry breaking pattern $G \to H$ can be deduced entirely by symmetry arguments. In particular we discuss two examples: pseudoscalar mesons and magnons in an antiferromagnet. In both of these cases we see that the Goldstone boson scattering amplitude is modified significantly by the contribution from the hard thermal loop term.

Acknowledgements

We wish to thank C. Manuel for a critical reading of the manuscript. G.A. and R.R. were supported in part by a PSC-CUNY grant. E.F.M. was supported by CONICET. V.P.N. was supported in part by the National Science Foundation grant number PHY-9605216.

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