Gravitational memory of boson stars

Diego F. Torres[♣]
Departamento de Física, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina

Andrew R. Liddle[◇] and Franz E. Schunck[♠]
Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QJ, United Kingdom
(April 16, 2018)

'Gravitational memory' refers to the possibility that, in cosmologies with a time-varying gravitational 'constant', objects such as black holes may retain a memory of conditions at the time of their birth. We consider this phenomenon in a different physical scenario, where the objects under consideration are boson stars. We construct boson star solutions in scalar–tensor gravity theories, and consider their dependence on the asymptotic value of the gravitational strength. We then discuss several possible physical interpretations, including the concept of pure gravitational stellar evolution.

PACS numbers: 04.40.Dg, 04.50.+h   Sussex preprint SUSSEX-AST 97/10-2, gr-qc/9710048

I. INTRODUCTION

A few years ago, Barrow introduced the concept of gravitational memory [1] by posing the following problem: what happens to black holes during the subsequent evolution of the universe if the gravitational coupling G evolves with time? He envisaged two possible scenarios. In the first, dubbed scenario A, the black hole evolves quasi-statically, in order to adjust its size with the changing G. If true, this means that there are no static black holes, even classically, during any period in which G changes. In the alternative possibility, scenario B, the local value of G within the black hole is preserved, while the asymptotic value evolves with a cosmological rate. This would mean that the black hole keeps a *memory* of the strength of gravity at the moment of its formation. Further analysis of the striking phenomena which arise in both of these scenarios was made in Ref. [2]. Black holes are of particular interest in this context, because primordial black holes may have formed in the very early stages of the Universe. At those times no direct evidence of the strength of gravity is available; nucleosynthesis is the earliest epoch at which significant constraints apply. Gravitational memory may provide a unique probe of these early epochs. However, in this paper our aim is to consider the gravitational memory phenomenon in a completely stellar arena. The motivation is twofold. On one side, since there are no singularities or event horizons as in the black hole case, there are many calculational advantages which may provide a more direct route towards shedding light on which of the scenarios stated above will happen in practice. On the other, a comprehensive understanding of the effects of a varying-G cosmology upon astrophysical objects is far from complete.

To make a proper study of a varying gravitational strength, it is imperative to operate within a self-consistent framework rather than simply writing in a variation 'by hand'. The most useful such framework is scalar–tensor theories [3], which have an action

$$S=\int \frac{\sqrt{-g}}{16\pi}\,dx^4\left[\phi R-\frac{\omega(\phi)}{\phi}\,\partial_\mu\phi\,\partial^\mu\phi+16\pi\mathcal{L}_{\rm m}\right]. \quad (1)$$

Here $g_{\mu\nu}$ is the metric, R is the scalar curvature, ϕ is the Brans–Dicke field, $\mathcal{L}_{\rm m}$ is the Lagrangian of the matter content of the system, and $\omega(\phi)$ gives the strength of the coupling between the scalar ϕ and the metric. The special case of a constant ω is the Jordan–Brans–Dicke (JBD) theory, and general relativity is regained in the limit of large ω. The gravitational constant G of the Einstein–Hilbert action is replaced by a dynamical field, ϕ^{-1}. In the usual applications, it is either spatially-constant but time-varying (cosmological solutions) or spatially-varying but time-independent (astrophysical solutions). Our situation will be unusual in including both types of variation.

Our choice of stellar object is motivated by two concerns. Firstly, it should be simple enough as to allow us to isolate the effects caused by a varying G. Secondly, in order to show the changes due to the particular gravitational theory, it should be fully relativistic, in the sense that its equilibrium configuration must be obtained from Einstein-like field equations. There is one type of stellar object which meets these criteria, though so far it is known to exist only as a theoretical construct. It is the boson star, the analogue of a neutron star formed when a large collection of bosonic particles (normally taken to be scalars) becomes gravitationally bound. Such configurations were introduced by Kaup [4] and by Ruffini and Bonazzola [5] in the nineteen sixties, but current interest was sparked by Colpi et al. [6] who showed that, provided the scalar field has a self-interaction, the boson star masses could be of the same order of magnitude as, or even much greater than, the Chandrasekhar mass. This led to the study of many properties, which are summarized in two reviews [7]. It seems possible for them to form through gravitational collapse of a scalar field [8], though little is known as yet. The observational status of boson stars was analyzed very recently in Ref. [9], where it was asked whether radiating baryonic matter moving around a boson star could be converted into an observational signal. Unfortunately, any direct detection looks a long way off.

As far as we are aware, only three papers have studied boson stars in scalar–tensor theories. Gunderson and Jensen [10] studied the JBD scenario, concluding that typically the mass of equilibrium solutions would be reduced by a few percent. This work was generalized by Torres [11], who looked at several coupling functions $\omega(\phi)$, chosen to be compatible with known weak-field limit and nucleosynthesis constraints. Similar scenarios were further studied in Ref. [12]. In this paper we shall study boson stars formed at different times of cosmic history and in addition consider the possible physical evolution of such objects. One might further hope that results found for boson stars might be representative of other stellar candidates, especially neutron stars but perhaps even indicative of what might happen with long-lived hydrogen-burning stars such as our own Sun.

II. EQUILIBRIUM CONFIGURATIONS

We begin with a review of the formalism, which can be found in more detail in Ref. [11]. The material from which the boson star is made is a complex, massive, self-interacting scalar field ψ, which is unrelated to the Brans–Dicke scalar already described. Its Lagrangian is

$$\mathcal{L}_{\rm m} = -\frac{1}{2} g^{\mu\nu}\, \partial_\mu \psi^* \partial_\nu \psi - \frac{1}{2} m^2 |\psi|^2 - \frac{1}{4}\lambda |\psi|^4 \,. \qquad (2)$$

The $U(1)$ symmetry leads to conservation of boson number. Varying the action with respect to $g^{\mu\nu}$ and ϕ we obtain the field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega(\phi)}{\phi}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi^{,\alpha}\phi_{,\alpha}\right) + \frac{1}{\phi}\left(\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi\right) , \qquad (3)$$

$$\Box\phi = \frac{1}{2\omega+3}\left[8\pi T - \frac{d\omega}{d\phi}\phi^{,\alpha}\phi_{,\alpha}\right] , \qquad (4)$$

where $T_{\mu\nu}$ is the energy–momentum tensor for the matter fields (Eq. (5) of Ref. [11]) and T, its trace. Commas and semicolons are derivatives and covariant derivatives, respectively.

At first, we seek static equilibrium solutions. The metric of a spherically-symmetric system can be written as

$$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 d\Omega^2 \,. \qquad (5)$$

We also demand a spherically-symmetric form for the ψ field. Crucially, the most general ansatz consistent with the static metric permits a time-dependent ψ of the form

$$\psi(r,t) = \chi(r) \exp\left[-i\varpi t\right] . \qquad (6)$$

To write the equations of structure of the star, we use a rescaled radial coordinate, given by

$$x = mr \,. \qquad (7)$$

From now on, a prime will denote a derivative with respect to the variable x. We also define dimensionless quantities by

$$\Omega = \frac{\varpi}{m} \,,\quad \Phi = \frac{\phi}{m_{\rm Pl}^2} \,,\quad \sigma = \sqrt{4\pi}\frac{\chi(r)}{m_{\rm Pl}} \,,\quad \Lambda = \frac{\lambda}{4\pi}\left(\frac{m_{\rm Pl}}{m}\right)^2 , \qquad (8)$$

where $m_{\rm Pl} \equiv G_0^{-1/2}$ is the present Planck mass. Our observed gravitational coupling implies $\Phi = 1$. In order to consider the total amount of mass of the star within a radius x we change the function A in the metric to its Schwarzschild form,

$$A(x) = \left(1 - \frac{2M(x)}{x\,\Phi(\infty)}\right)^{-1} . \qquad (9)$$

The issue of the definition of mass in JBD theory is quite a subtle one [13]. The above gives the Schwarzschild mass; there are arguments that the tensor mass is more appropriate [13], but we have verified numerically that for the large couplings we use the difference is negligible.

Note that a factor $\Phi(\infty)$ appears in Eq. (9). This is crucial to obtain the correct value of the mass, which is given by

$$M_{\rm star} = M(\infty)\,\Phi(\infty)\,\frac{m_{\rm Pl}^2}{m} \,, \qquad (10)$$

for a given value of m. The $\Phi(\infty)$ factor allows for the asymptotic gravitational coupling to be different from that presently observed.* With all these definitions, the non-trivial equations of structure are [11]

$$\sigma'' + \sigma'\left(\frac{B'}{2B} - \frac{A'}{2A} + \frac{2}{x}\right) + A\left[\left(\frac{\Omega^2}{B} - 1\right)\sigma - \Lambda\sigma^3\right] = 0 \,, \qquad (11)$$

$$\Phi'' + \Phi'\left(\frac{B'}{2B} - \frac{A'}{2A} + \frac{2}{x}\right) + \frac{1}{2\omega+3}\frac{d\omega}{d\Phi}\Phi'^2 - \frac{2A}{2\omega+3}\left[\left(\frac{\Omega^2}{B} - 2\right)\sigma^2 - \frac{\sigma'^2}{A} - \Lambda\sigma^4\right] = 0 \,, \qquad (12)$$

---

*Note that the $\Phi(\infty)$ factor was mistakenly forgotten in Ref. [11]. This affects Table III of that work, where $M(\infty)$ is actually $M(\infty)/\Phi(\infty)$. The $\Phi(0)$ values and the conclusions extracted from it are unaffected.

$$\frac{B'}{xB} - \frac{A}{x^2}\left(1-\frac{1}{A}\right) = \frac{A}{\Phi}\left[\left(\frac{\Omega^2}{B}-1\right)\sigma^2 + \frac{\sigma'^2}{A} - \frac{\Lambda}{2}\sigma^4\right] + \frac{\omega}{2}\left(\frac{\Phi'}{\Phi}\right)^2 + \left(\frac{\Phi''}{\Phi} - \frac{1}{2}\frac{\Phi'}{\Phi}\frac{A'}{A}\right) + \frac{1}{2\omega+3}\frac{d\omega}{d\Phi}\frac{\Phi'^2}{\Phi} - \frac{A}{\Phi}\frac{2}{2\omega+3}\left[\left(\frac{\Omega^2}{B}-2\right)\sigma^2 - \frac{\sigma'^2}{A} - \Lambda\sigma^4\right] , \quad (13)$$

$$\frac{2BM'}{x^2\Phi(\infty)} = \frac{B}{\Phi}\left[\left(\frac{\Omega^2}{B}+1\right)\sigma^2 + \frac{\sigma'^2}{A} + \frac{\Lambda}{2}\sigma^4\right] + \frac{\omega}{2}\frac{B}{A}\left(\frac{\Phi'}{\Phi}\right)^2 + \frac{B}{\Phi}\frac{2}{2\omega+3}\left[\left(\frac{\Omega^2}{B}-2\right)\sigma^2 - \frac{\sigma'^2}{A} - \Lambda\sigma^4\right] - \frac{B}{A(2\omega+3)}\frac{d\omega}{d\Phi}\frac{\Phi'^2}{\Phi} - \frac{1}{2}\frac{\Phi'}{\Phi}\frac{B'}{A} . \quad (14)$$

To solve these equations numerically, we use a fourth-order Runge–Kutta method, for which details may be found in Ref. [11]. Here we shall study two kinds of theories.

1. JBD theory with $\omega = 400$. This is comparable to current observational limits [14,15].
2. A scalar–tensor theory with a coupling function of the form
$$2\omega + 3 = 2B_1|1-\Phi|^{-\alpha} ,$$
where we choose $\alpha = 0.5$ and $B_1 = 5$. The cosmological setting of this model has been studied by Barrow and Parsons [16].

As explained in Ref. [11], the results for these couplings may be thought of as the general behavior of any scalar–tensor theory, by suitable expanding a general coupling in a Taylor or Laurent series. The positive exponent of this theory must be obtained from power-law couplings, but the behavior of these was found to be similar to the pure Brans–Dicke ones.

In addition to the freedom to choose the fundamental parameters, there are two free boundary conditions. One is the value of the boson field at the centre of the star, $\sigma(0)$, which we call the central density. The second is the asymptotic value of the Brans–Dicke field, which determines the asymptotic strength of gravity. The other boundary conditions are fixed by demanding non-singularity and finite mass [7,11]. This still leaves an infinite discrete set of solutions with different ϖ, corresponding to a different number of nodes in $\sigma(x)$. We choose the nodeless solution, which is the only stable one. Higher node solutions are generated in Ref. [12].

III. QUASI-STATIC EVOLUTION

We first assess the likely cosmological variation of ϕ, concentrating on JBD theory. During radiation domination, the attractor behavior is actually exactly that of general relativity, namely a constant ϕ and $a \propto t^{1/2}$. This changes with the onset of matter domination, when the attractor solution becomes [17,18]

$$a(t) \propto t^{(2-n)/3} \quad ; \quad G(t) \propto t^{-n} , \quad (15)$$

where $n = 2/(4+3\omega)$, so G exhibits a slow decrease. Assuming that matter–radiation equality took place near the general relativity value, at $z_{\rm eq} = 24\,000\,\Omega_0 h^2$, then, for critical density and $h = 0.5$, the fractional change in G since equality is

$$\frac{G(t_0)}{G(t_{\rm eq})} = 6000^{-1/(1+\omega)} . \quad (16)$$

For $\omega = 400$, the ratio is 0.98, so G will have changed value by about two percent since equality.

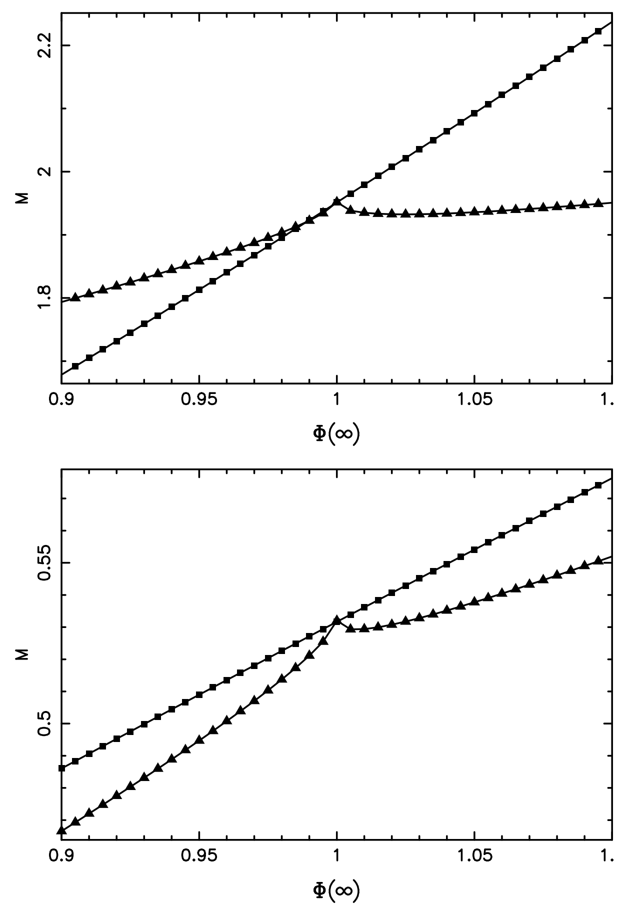

FIG. 1. Boson star masses as a function of $\Phi(\infty)$; squares are for the JBD theory and circles the scalar–tensor theory. The upper panel shows models with $\Lambda = 100$ and $\sigma(0) = 0.06$. The lower one has $\Lambda = 0$ and $\sigma(0) = 0.1$.

Informed by the likely range of variation, we compute static configurations with different values of the asymptotic gravitational strength, denoted $\Phi(\infty) = 1/G$. In Fig. 1, we see that if Λ and the central density $\sigma(0)$ are kept fixed, then the mass changes with a different $\Phi(\infty)$. In the JBD case, the mass is a growing function

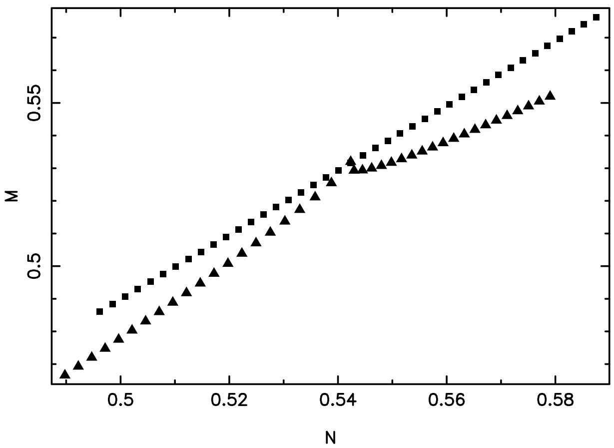

FIG. 2. This shows the boson mass as a function of particle number in the two models, with squares for the JBD theory and triangles for the scalar–tensor one. Both have $\Lambda = 0$ and $\sigma(0) = 0.1$. The leftmost point in each case corresponds to $\Phi(\infty) = 0.9$ and the rightmost to $\Phi(\infty) = 1.1$. As $M(N)$ is single valued, there can be no change of stability.

of $\Phi(\infty)$, while in the more complicated second model there is actually a small peak in the mass at our present gravitational strength. That peak is due to the special form of the coupling function, which selects out our own observed coupling strength.

Let us now suppose that evolution does drive the system through a series of quasi-static states, and naïvely estimate the sorts of energies involved. Fig. 1 shows that variation of $\Phi(\infty)$ by a few percent can change the masses of configurations by a few percent. As it happens, this is very similar to, or even greater than, the fraction of the mass–energy which can be liberated by nuclear reactions over the lifetime of a star, a process which for most stars occurs on similar timescales to the cosmological ones of the time-varying G.

Before analyzing this in further detail, we shall first show the computation for the number density of the configurations, defined by

$$\frac{m^2}{m_{\rm Pl}^2} N = \Omega \int_0^\infty \sigma^2 \sqrt{\frac{A}{B}}\, x^2\, dx\,, \tag{17}$$

We found that, for both models under consideration, the number of bosons is a growing function of $\Phi(\infty)$. Fig. 2 shows the bifurcation diagram, number density against mass. There is no cusp structure in the sense of catastrophe theory, which implies that there is no change in the stability criterion for values of G close to the present one [19]. In fact, we found this result for a wider range of $\Phi(\infty)$ than shown.

IV. SCENARIOS

Now we discuss possible interpretations of the computational results described above, in the same context which led Barrow to propose the gravitational memory hypothesis. We describe the possible subsequent histories a boson star might have, after forming at a time $t_{\rm f}$ with a mass $M(t_{\rm f})$.

A. The gravitational memory hypothesis

The star remains completely static, without change in the values of its mass or central density. Such a situation would be reminiscent say of a virialized galaxy after gravitational collapse, which has become decoupled from the cosmological expansion of the Universe around it and in particular does not participate in further expansion. In such a scenario the mass $M(t_{\rm f})$ is a function of the formation time, and hence of $\Phi(\infty\,,\, t_{\rm f})$, as well as the central density; the star keeps memory of the value of G at formation. This situation clearly cannot be precisely correct since the asymptotic gravitational constant does evolve, but it may well be that the effect on the star is much slower even than the cosmological evolution of G, so that in practice the star can be viewed as static.

In this scenario, there is the interesting feature that stars of the same mass may differ in other physical properties (their radius, for example) depending on the formation time.

B. Quasi-static evolution hypothesis

The opposite regime has the adjustment time for the star being much shorter than the asymptotic evolution of G. If true, the star should quasi-statically evolve, either changing its mass or its central density or both. In this scheme, when intervals of time are short enough compared with the scale of cosmological evolution, the star can be taken as static and the solutions computed hold in this limit.

An interesting subcase is *pure gravitational evolution.* This assumes that the mass of the star is preserved all along the quasi-static evolution, through the central density evolving in the appropriate way. This leads to the remarkable conclusion that stars may be able to evolve even without absorption or emission of energy, and without nuclear burning. Such quiescent evolution would be entirely gravitational in its nature.

Purely gravitational evolution would doubtless have significant consequences for stellar evolution, and indeed may well be quite strongly constrained. Yet within the quasi-static evolution hypothesis, it is actually rather conservative, because the other possibility is that mass is lost during the evolution, if dissipative processes are required to keep tracking the evolutionary sequence. We have seen that the variation in G can easily lead to changes in the mass of up to a few percent, which can be of the same order as the energy liberated in a conventional star by nuclear burning.

The reduction in central density seems reasonable when we recall the force balance in polytropic stars. Since gravity is reducing in strength as time goes by, the equilibrium configurations can become more diffuse and hence drop in central density. In an extreme situation, one might wonder whether the continuation of this process can in fact lead to the complete destruction of the star, though this may be prevented by the initial negative binding energy of a stable boson star, since boson number is conserved.

The quasi-static evolution hypothesis has an important difference from the gravitational memory hypothesis; in it, stars of the same mass are identical in all their other physical properties too.

C. Feedback on the asymptotic gravitational constant?

Finally, one can ask whether the formation of boson stars can significantly influence the 'asymptotic' gravitational constant; when the stars form gravity becomes weaker in their interior as the ϕ value increases. Following the results of Ref. [11] (Table III), this change can be about 1% between the internal and the external G value. In a static configuration, the radius at which ϕ finally approaches its asymptotic value is quite a bit larger than the region in which the ψ field is localized. If a very high density of boson stars formed, might they be able to reduce the gravitational interaction strength in quite a significant region around themselves? It is an interesting possibility, though unlikely in practice as the density of material available to make the stars is so small; boson stars would be expected to be separated by similar distances to conventional stars (which could also contribute to the effect).

V. DISCUSSION

In this paper, we have not directly tried to answer the question as to whether or not the gravitational memory phenomenon exists. Rather, we have introduced a new framework in which it can be studied, that of stellar systems. As a spinoff, it allows us to introduce the concept of pure gravitational stellar evolution. As well as their possible physical relevance, boson stars have the considerable advantage of being much simpler than their black hole analogues. They are also much simpler than neutron stars, as they are based directly on a field theory description. In that light, we have studied static configurations in two different scalar–tensor theories, in particular emphasizing the dependence of the mass on the asymptotic value of the gravitational coupling. We have also been able to make some preliminary statements on the dynamical stability of the configurations, an issue we leave for further study in a future publication. The next step would be numerical simulation including dynamical evolution, starting from a static solution and varying the asymptotic gravitational coupling. This looks a promising avenue for determining which of the proposed scenarios is the correct one, and that knowledge may allow a test of general relativity not just at the present epoch, but in the distant past, through astrophysical systems.

ACKNOWLEDGMENTS

D.F.T. was supported by a British Council Fellowship and CONICET, A.R.L. by the Royal Society and F.E.S. by an European Union Marie Curie TMR fellowship. We thank John Barrow, Eckehard W. Mielke, Hisa-aki Shinkai and Andrew Whinnett for useful discussions.

♣ E-mail address: dtorres@venus.fisica.unlp.edu.ar
◇ E-mail address: a.liddle@sussex.ac.uk
♠ E-mail address: fs@astr.cpes.susx.ac.uk

[1] J. D. Barrow, Phys. Rev. D **46**, 3227 (1992), Gen. Rel. Grav. **26**, 1 (1994).
[2] J. D. Barrow and B. J. Carr, Phys. Rev. D **54**, 3920 (1996).
[3] C. Brans and R. H. Dicke, Phys. Rev. **124** , 925 (1961); P. G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968); K. Nortvedt, Astrophys. J. **161**, 1059 (1970); R. V. Wagoner, Phys. Rev. D **1**, 3209 (1970).
[4] D. J. Kaup, Phys. Rev. **172**, 1331 (1968).
[5] R. Ruffini and S. Bonazzola, Phys. Rev. **187**, 1767 (1969).
[6] M. Colpi, S. L. Shapiro and I. Wasserman, Phys. Rev. D **57**, 2485 (1986).
[7] P. Jetzer, Phys. Rep. **220**, 163 (1992); A. R. Liddle and M. S. Madsen, Int. J. Mod. Phys. **D1**, 101 (1992).
[8] M. S. Madsen and A. R. Liddle, Phys. Lett. B **251**, 507 (1990); I. I. Tkachev, Phys. Lett. B **261**, 289 (1991).
[9] F. E. Schunck and A. Liddle, Phys. Lett. B **404**, 25 (1997).
[10] M. A. Gunderson and L. G. Jensen, Phys. Rev. D **48**, 5628 (1993).
[11] D. F. Torres, Phys. Rev. **D56**, 3478 (1997).
[12] G. L. Comer and H. Shinkai, Report No. gr-qc/9708071 (1997).
[13] A. W. Whinnett, Report No. gr-qc/9711080 (1997).
[14] R. D. Reasenberg et al., Astrophys. J. **234**, L219 (1979).
[15] F. S. Accetta, L. M. Krauss and P. Romanelli, Phys. Lett. B **248**, 146 (1990); J. A. Casas, J. García-Bellido and M. Quirós, Phys. Lett. B **278**, 94 (1992).
[16] J. D. Barrow and P. Parsons, Phys. Rev. D **55**, 3906 (1997).
[17] H. Nariai, Prog. Theor. Phys. **42**, 544 (1969).
[18] L. E. Gurevich, A. M. Finkelstein and V. A. Ruban, Astrophys. Space Sci. **98**, 101 (1973).
[19] F. V. Kusmartsev, E. W. Mielke and F. E. Schunck, Phys. Rev. D **43**, 3895 (1991); Phys. Lett. A **157**, 465 (1991).