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Path-integral fermion-boson decoupling at finite temperature

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Abstract

We show how to extend the standard functional approach to bosonisation, based on a decoupling change of path-integral variables, to the case in which a finite temperature is considered. As examples, in order to both illustrate and check the procedure, we derive the thermodynamical partition functions for the Thirring and Schwinger models.

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1　Introduction

Some time ago a path-integral approach to bosonisation was developed [1], which has been shown to be very useful in the study of a variety of (1+1) problems such as the Kondo effect [2], fermion fields in topological backgrounds [3], 1D many-body systems [4], etc. This method is based on a decoupling change of path-integral variables, whose corresponding Fujikawa [5] Jacobian J_F yields a non-trivial contribution to the kinetic piece of the bosonised Lagrangian density. In a relevant contribution, Reuter and Dittrich [6] have computed J_F at non-zero temperature, paving thus the road for the application of the decoupling technique in the context of finite temperature Quantum Field Theory (QFT) [7] [8] [9] [10]. However, only very recently this path-integral approach was employed to compute the fermion condensate at finite density [11], whereas a systematic and detailed computation of thermodynamical quantities within the functional bosonisation scheme is still lacking. The principal aim of this work is to help filling this gap. Apart from its academic interest, the formulation we present here is a necessary first step towards the implementation of a finite-temperature functional treatment of models that try to describe some features of the recently developed 1D semiconductors [12].

In order to illustrate the procedure we shall consider the two most popular (1+1) QFT's, namely, the Thirring [13] and Schwinger (QED_2) [14] models. These theories have been extensively studied at $T \neq 0$, following operational and functional approaches different from ours. In particular, the thermodynamical properties of the Thirring model were examined by Ruiz Ruiz and Alvarez-Estrada [15] and Yokota [16], who found contradictory answers for the corresponding partition function. This issue has been recently reconsidered by Sachs and Wipf [17], whose result agrees with that of ref.[15]. In the next Section we rederived the partition function for the Thirring model by using the above mentioned path-integral route to bosonization. Our result provides an independent confirmation of the validity of the expression first obtained in [15].

For completeness, in Section 3 we show how to get the thermodynamical partition funtion of the Schwinger model [6] [18] within our framework. There is no controversy in this case, and thus our result coincides with that of ref.[15], as expected. We end this paper by briefly summarizing our results and commenting on possible applications of this formulation.

2 The Thirring model

In this section we consider the two-dimensional Thirring model at finite temperature using the imaginary time formalism developed by Bernard [7] and Matsubara [8]. We start from the Euclidean partition function

$$Z = NN_F(\beta) \int_{antiper} D\bar{\Psi} \; D\Psi \exp\{-\int_{\beta} d^2x \; [\bar{\Psi} i\partial\!\!\!/\Psi - \frac{g^2}{2}(\bar{\Psi}\gamma_\mu\Psi)^2]\} \quad (2.1)$$

where $\int_{\beta} d^2x$ means $\int_0^{\beta} dx^0 \int dx^1$ and $\beta = \frac{1}{k_B T}$ with k_B the Boltzman's constant and T the temperature. Here N is an infinite constant which does not depend on temperature. On the other hand, $N_F(\beta)$ is given by (please see the paragraph following eq.(2.18))

$$lnN_F(\beta) = 2ln\beta \sum_n \int \frac{dk}{2\pi} \quad (2.2)$$

As is well-known, the functional integral in (2.1) must be extended over the paths with antiperiodicity conditions in the Euclidean time variable x^0:

$$\begin{aligned} \Psi(x^0 + \beta, x^1) &= -\Psi(x^0, x^1) \\ \bar{\Psi}(x^0 + \beta, x^1) &= -\bar{\Psi}(x^0, x^1) \end{aligned} \quad (2.3)$$

Exactly as one does in the zero-temperature case, we can eliminate the quartic fermionic interaction introducing a vector field A_μ through the identity

$$\exp\{-\frac{g^2}{2}\int_{\beta} d^2x(\bar{\Psi}\gamma_\mu\Psi)^2\} \propto \int_{per} DA_\mu \exp\{\int_{\beta} d^2x[\frac{A_\mu^2}{2} + g\bar{\Psi}A\!\!\!/\Psi]\} \quad (2.4)$$

We have to impose periodicity conditions for the bosonic A_μ field over the range $[0, \beta]$. The partition function then results

$$Z = NN_F(\beta) \int_{antiper} D\bar\Psi \ D\Psi \int_{per} DA_\mu \exp\{-\int_\beta d^2x[\bar\Psi i\partial\!\!\!/\Psi + \frac{A_\mu^2}{2} + g\bar\Psi A\!\!\!/\Psi]\}$$

(2.5)

Of course, the above equation can be expressed in terms of a β-dependent fermionic determinant (satisfying the corresponding antiperiodic conditions) as

$$Z = NN_F(\beta) \int_{periodic} DA_\mu \det(i\partial\!\!\!/ + gA\!\!\!/) \exp\{-\int_\beta d^2x \frac{A_\mu^2}{2}\}$$

(2.6)

Now, following the path-integral bosonization scheme [1] we write the vector field A_μ in terms of two scalar fields in the form

$$A_\mu = -\epsilon_{\mu\nu}\partial_\nu\phi + \partial_\mu\eta$$

(2.7)

and make a chiral transformation in the fermionic variables

$$\begin{aligned}
\Psi &= e^{\gamma_5\phi+i\eta}\chi \\
\bar\Psi &= \bar\chi e^{\gamma_5\phi-i\eta}
\end{aligned}$$

(2.8)

so that we can decouple A_μ from the fermion fields in the determinant. The Jacobian associated to this change in the fermionic path-integral measure has been first computed, for the $T \neq 0$ case, by Reuter and Dittrich [6]. Their result allows one to write

$$\det(i\partial\!\!\!/ + gA\!\!\!/) = \det(i\partial\!\!\!/) \ e^{-\frac{g^2}{2\pi}\int_\beta d^2x\partial_\mu\phi\partial_\mu\phi}$$

(2.9)

Let us stress that in writing this equation we are restricting the present analysis to the gauge-invariant sector of the Thirring model. Indeed, the evaluation of the fermionic Jacobian requires a regularization prescription, which, in turn, involves an arbitrary parameter to be fixed on symmetry grounds (In passing we note that the implementation of a regularization scheme is essentially temperature independent [19]). On the other hand the Thirring model is not a gauge theory and consequently one should have a partition function depending on the regularization parameter, reflecting the well-known existence of a family of solutions for this model [13]. Among all

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these solutions we shall keep only the gauge-invariant ones for two reasons: firstly to facilitate comparison with previous results [15] [16], and secondly because we have in mind the application of our procedure in the context of many-body systems with charge conservation [4]. However, it is interesting to point out that the present formulation seems to be specially appropriate to examine the non-gauge-invariant sectors of the Thirring theory at $T \neq 0$. Such a model could be useful to describe an open (without charge conservation), finite-length many-body ensemble.

At this stage we must also emphasize that, in contrast to the $T = 0$ case, in which the Jacobian associated to the change of bosonic variables (2.7) plays no relevant role due to the fact that it is field-independent, in the present case its contribution is crucial since it depends on temperature through a bosonic determinant. To be specific let us now write down the corresponding change in the bosonic measure,

$$DA_\mu = \det(-\Box)D\phi D\eta \qquad (2.10)$$

where $\Box = \partial_\mu \partial_\mu$.

Inserting (2.9) and (2.10) in (2.6) one readily obtains

$$
\begin{aligned}
Z \;=\; & NN_F(\beta)Z_0 \det(-\Box) \int_{per} D\phi \exp\{-\frac{1}{2}(1+\frac{g^2}{\pi}) \int_\beta d^2x(\partial_\mu\phi)^2\} \\
& \int_{per} D\eta \exp\{-\frac{1}{2}\int_\beta d^2x(\partial_\mu\eta)^2\} \qquad (2.11)
\end{aligned}
$$

where
$$Z_0 = \int_{antiper} D\bar{\Psi}D\Psi e^{-\int_\beta d^2x \bar{\Psi}i\slashed{\partial}\Psi} = det(i\slashed{\partial}) \qquad (2.12)$$

From now on we shall disregard in (2.11) the prefactors that are independent of both temperature and coupling constant. Expressing the bosonic path-integrals in terms of determinants and using the property $ln\,detA = tr\,lnA$, with A the corresponding operator, we have

$$
\begin{aligned}
lnZ \;=\; & ln\,N_F(\beta) + tr\,ln(i\slashed{\partial}) + tr\,ln(-\Box) - \frac{1}{2}tr\,ln(1+\frac{g^2}{\pi}) \\
& - \frac{1}{2}tr\,ln(-\Box) - \frac{1}{2}tr\,ln(-\Box) \qquad (2.13)
\end{aligned}
$$

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Note that the only contribution which survives from the bosonic part of the complete partition function is β-independent, because the pieces containing $ln(-\square)$, which depend on temperature, cancel each other.

In order to evaluate lnZ_0 we follow the pioneering work of Bernard [7] and expand the fermionic fields $\Psi(x^0, x^1)$, which are antiperiodic in the interval $0 \leq x \leq \beta$, in a Fourier series:

$$\Psi(x^0, x^1) = \frac{1}{\beta} \sum_n \int \frac{dk}{2\pi} e^{ikx^1} e^{-i\omega_n x^0} \Psi_n(k) \qquad (2.14)$$

where

$$\Psi_n(k) = \int dx^1 \int_0^\beta dx^0 e^{-ikx^1} e^{-i\omega_n x^0} \Psi(x^0, x^1) \qquad (2.15)$$

and

$$\omega_n = \frac{(2n+1)\pi}{\beta} \qquad (2.16)$$

Taking into account that in eq.(2.13) trace means $\sum_n \int \frac{dk}{2\pi}$, it is straight-forward to obtain

$$lnZ = lnN_F(\beta) + \frac{1}{2}\sum_n \int \frac{dk}{2\pi}(k^2 + \omega_n^2) - \frac{1}{2}\sum_n \int \frac{dk}{2\pi}ln(1 + \frac{g^2}{\pi}) \qquad (2.17)$$

At this point some algebraic manipulations similar to those performed in ref.[9] allows to write

$$\begin{aligned} lnZ &= lnN_F(\beta) + 2\int_0^\infty \frac{dk}{2\pi}[\frac{\beta k}{2} + ln(1 + e^{-\beta k})] \\ &- 2ln\beta \sum_n \int \frac{dk}{2\pi} - \frac{1}{2}\sum_n \int \frac{dk}{2\pi}ln(1 + \frac{g^2}{\pi}) \end{aligned} \qquad (2.18)$$

We see that if we identify $lnN_F = 2ln\beta \sum_n \int \frac{dk}{2\pi}$, as it stays in (2.2), we get

$$Z = Z_{FD}\exp\{-\frac{1}{2}ln(1 + \frac{g^2}{\pi})\sum_n \int \frac{dk}{2\pi}\} \qquad (2.19)$$

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where Z_{FD} is the Fermi-Dirac distribution for massless electrons:

$$lnZ_{FD} = 2 \int_0^\infty \frac{dk}{2\pi} [\frac{\beta k}{2} + ln(1 + e^{-\beta k})] \qquad (2.20)$$

Therefore the partition function for the Thirring model at finite-temperature differs from the one corresponding to free massless fermions in an infinite constant which depends on the coupling parameter g but not on temperature. This result exactly coincides with the expression first obtained in ref.[15], in contrast to the identification between Thirring and free fermions partition functions claimed in ref.[16]. As stressed in [17], this is a relevant difference, since it affects the value of the zero-point pressure of the system.

3   The Schwinger model

Let us now study the thermodynamical partition function of the Schwinger model. As it is a gauge theory its quantization requires a gauge fixing [7], which gives rise to the appearence of the Faddev-Popov determinant. We choose to work in the Lorentz gauge $\partial_\mu A_\mu = 0$. We start from the expression

$$
\begin{aligned}
Z_S &= NN(\beta) \int_{antiper} D\bar\Psi \, D\Psi \int_{per} DA_\mu \Delta_{FP} \delta(\partial_\mu A_\mu) \\
&\quad \exp\{-\int_\beta d^2x[\bar\Psi \,(i\slashed\partial + e\slashed A)\,\Psi - \frac{1}{4}F_{\mu\nu}^2]\}
\end{aligned}
\tag{3.21}
$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $N(\beta)$ is the same constant defined in (2.2). Taking into account that in the Lorentz gauge one has $\Delta_{FP} = det(-\Box)$, the above equation can be rewritten as

$$
\begin{aligned}
Z_S &= NN(\beta)det(-\Box) \int_{antiper} D\bar\Psi \, D\Psi \int_{per} DA_\mu \delta \,(\partial_\mu A_\mu) \\
&\quad \exp\ \{-\int_\beta d^2x[\bar\Psi \,(i\slashed\partial + e\slashed A)\,\Psi - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu A_\mu)^2]\}
\end{aligned}
\tag{3.22}
$$

In order to decouple the fermionic from the bosonic fields we perform the transformation (2.8) and write the gauge field as in (2.7). It is evident that the gauge condition $\partial_\mu A_\mu = 0$ becomes $\Box \eta = 0$. Thus, using the identity $\delta(\Box \eta) = \frac{1}{det(\Box)}\delta(\eta)$, and taking into account the Jacobian generated by these changes of variables we arrive at:

$$
\begin{aligned}
Z_S &= N \, N(\beta) \, det(i\slashed\partial) \, det(-\Box) \int D\eta D\phi \\
&\quad \exp\ \{-\frac{1}{2} \int_\beta d^2x\phi[-\frac{e^2}{\pi}\Box + \Box\Box]\phi\}
\end{aligned}
\tag{3.23}
$$

In this expression we see that the η field is completely decoupled from the ϕ field and has no dynamics. This means that its corresponding functional integral is an infinite β independent constant that we will absorb in the normalization.

Taking logarithm in (3.23), and keeping the terms that depend on the temperature and the coupling constant, we get

$$
\begin{aligned}
lnZ_S &= ln\ N(\beta) + ln\ det(i\not{\partial}) + tr\ ln(-\Box) - \\
&- \frac{1}{2}ln[(\frac{e^2}{\pi} - \Box)(-\Box)]
\end{aligned}
\tag{3.24}
$$

From now on we shall work in momentum space, where we have

$$
\begin{aligned}
tr\ ln(-\Box) &= \sum_n \int \frac{dk}{2\pi} ln(\omega_n^2 + k^2) \\
tr\ ln(\frac{e^2}{\pi} - \Box) &= \sum_n \int \frac{dk}{2\pi} ln(\omega_n^2 + k'^2)
\end{aligned}
$$

$$\tag{3.25}$$

with $k'^2 = \frac{e^2}{\pi} + k^2$ and $\omega_n = \frac{2n\pi}{\beta}$.

Hence we get

$$
\begin{aligned}
lnZ_S &= lnN(\beta) + \frac{1}{2}\sum_n \int \frac{dk}{2\pi}(k^2 + \omega_n^2) + \\
&+ \frac{1}{2}\{\int \frac{dk}{2\pi}2ln\ senh\frac{k\beta}{2} - 2ln\beta \sum_n \int \frac{dk}{2\pi}\} \\
&- \frac{1}{2}\{\int \frac{dk}{2\pi}2ln\ senh\frac{k'\beta}{2} - 2ln\beta \sum_n \int \frac{dk}{2\pi}\}
\end{aligned}
\tag{3.26}
$$

where one sees that the infinite β-dependent terms coming from the evaluation of $tr\ ln(-\Box)$ and $tr\ ln(-\frac{e^2}{\pi}\Box + \Box\Box)$ cancel each other. After some more algebra one finally obtains

$$
\begin{aligned}
lnZ_S &= 2\int \frac{dk}{2\pi}\{\frac{k\beta}{2} + ln(1 + e^{-k\beta})\} + \\
&+ \int \frac{dk}{2\pi}\{\frac{\beta}{2}(k - k') + ln(\frac{1 - e^{-k\beta}}{1 - e^{-k'\beta}})\}
\end{aligned}
\tag{3.27}
$$

which is in agreement with previous results [15]. Notice that this partition function is not equal to that corresponding to the massive boson times the

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one of free massless fermions, as one could have naively expected. Indeed, there is also a factor associated to the zero-mass gauge excitation which appears in the Lowenstein-Swieca solution for the massless Schwinger model [14]. Due to this contribution, in the case $e = 0$, one recovers the partition function for free fermions.

4 Summary and looking ahead

In this work we have shown how to implement, at finite temperature, a by now standard approach to the study of (1+1) QFT's, originally developed in the context of systems at $T = 0$ [1]. As examples, in Sections 2 and 3, we have evaluated the partition functions for the Thirring and Schwinger models, respectively. In both cases we were able to reobtain the results previously presented in the literature [15]. Concerning our expression for the Thirring thermodynamical partition function, it is of interest in itself since there was some controversy between the first computations of Ruiz Ruiz and Alvarez-Estrada [15] and Yokota [16]. Our result provides an independent confirmation, through a different method, of the answer given in [15] and [17].

Besides its simplicity, one specific advantage of our technique is the possibility of examining the general (non gauge-invariant) sectors of the Thirring model at $T \neq 0$ in a natural way. A physical realization of this model could be found if one examines the thermodynamical behaviour of an open, non charge-preserving system of many particles in a finite volume. Within this formulation the lack of gauge invariance manifests itself through the appearence of an additional dimensionless parameter associated to the regularization of the fermionic determinant which is at the root of the approach. This work could be followed in several directions, ranging from the investigation of fermionic models in topological backgrounds [3] to the analysis of the Kondo effect at low but finite temperatures, according to the lines of ref.[2]. However, we think that the application of the present formalism will be particularly fruitful when considering the non-local Thirring model [4], which has been recently proposed to describe 1D many-body systems. Work on this last subject is currently in progress and will be reported elsewhere.

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