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On charge quantization and abelian gauge horizontal symmetries

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Abstract

Under the assumption that there exists a local gauge horizontal symmetry G_H wich allows only for a top quark mass at tree level, we look for the constraints that charge quatization and the family structure of the standard model imposes on that symmetry. © 1997 Elsevier Science B.V.

1. Introduction

Among the many questions that the successful Standard Model (SM) leaves unanswered, one of the most basic is why the gauge group is $G_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. In fact, given N_F Weyl fermion fields, the kinetic energy term, with the convention of taking all fields to be left-handed, possesses a global $U(N_F)$ symmetry; then why does Nature gauges only $G_{\rm SM}$ which is a tiny subgroup of this enormous global group? Certainly there is a reason why the entire $U(N_F) = SU(N_F) \otimes U(1)$ group cannot be gauged: the resulting theory would be anomalous [1]. Nevertheless, groups larger than $G_{\rm SM}$ should in principle be allowed.

Looking the problem upside down, one notices that the SM $U(1)_Y$ hypercharge assignments, for the standard fermions, are least to say, peculiar [2]. This observation leads us to ask a simpler question: given $SU(3)_c \otimes SU(2)_L$ as a local gauge group with the standard matter spectrum (supported by experimental facts)

$$Q_L \sim (3,2); \quad U_L^c \sim (\bar{3},1); \quad D_L^c \sim (\bar{3},1); \quad F_L \sim (1,2); \quad E_L \sim (1,1) \tag{1}$$

(where (k,l) refers to $(SU(3)_c, SU(2)_L)$ labels), which is the most general $U(1)$ group associated with hypercharge that can be gauged together with $SU(3)_c \otimes SU(2)_L$? Is the answer the standard $U(1)_Y$ with the mentioned ''peculiar'' assignment of hypercharge quantum numbers?

The purpose of this note is precisely to specify the minimal set of conditions that selects the standard model hypercharges. Our analysis is mainly based on anomaly cancellation considerations and includes horizontal symmetries that when broken, give rise to a mass hierarchy where only the top quark acquire mass at the tree level. In going forward with our analysis we will recall well known results relevant for our purposes.

Notice that $SU(3)_c \otimes SU(2)_L$ gauge theory with the spectrum referred in Eq. (1) is free of the Adler-Bell-Jackiw anomaly [3] because $SU(2)$ is real (pseudoreal), and the standard matter spectrum is vector-like with respect to $SU(3)_c$. Also it is free of the global $SU(2)_L$ anomaly [4] because the number of $SU(2)_L$ doublets is even. Consequently, anomalies could come only from extra $U(1)$ gauge groups.

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The first step is to look for all the $U(1)$'s that could be included under general conditions. For 3 families, the most general symmetry of the fermion kinetic energy which commutes with $SU(3)_c \otimes SU(2)_L$ is:

$$G = U(3)_Q \otimes U(3)_U \otimes U(3)_D \otimes U(3)_F \otimes U(3)_E \equiv [U(3)]^5,$$

where $U(3)_\eta = SU(3)_\eta \otimes U(1)_\eta (\eta = Q,U,D,F,E)$ and $U(1)_\eta$ is a global (family independent) abelian factor. Again, the entire group G can not be gauged because it is anomalous, but which are the abelian subgroups of G that can be gauged so that the resulting theory will be anomaly free? If they are family independent, in the sense that the same hypercharge value is assigned to related multiplets of each one of the three families, they must be subgroups of

$$U(1)_Q \otimes U(1)_U \otimes U(1)_D \otimes U(1)_F \otimes U(1)_E \equiv [U(1)]^5.$$

If they assign different hypercharge values to related multiplets in each family, they must be subgroups of

$$SU(3)_Q \otimes SU(3)_U \otimes SU(3)_D \otimes SU(3)_F \otimes SU(3)_E \equiv [SU(3)]^5.$$

Let us present the analysis of a family independent $U(1)$ abelian factor. In this case the anomaly cancellation constraint equations are [5]:

$$[SU(2)]^2 U(1): \quad 3Y_Q + Y_F = 0, \tag{2}$$

$$[SU(3)]^2 U(1): \quad 2Y_Q + Y_U + Y_D = 0, \tag{3}$$

$$[\mathrm{grav}]^2 U(1): \quad 6Y_Q + 3Y_U + 3Y_D + 2Y_F + Y_E = 0, \tag{4}$$

$$[U(1)]^3: \quad 6Y_Q^3 + 3Y_U^3 + 3Y_D^3 + 2Y_F^3 + Y_E^3 = 0, \tag{5}$$

where $Y_\eta (\eta = Q,U,D,F,D)$ are the $U(1)$ hypercharges for the corresponding multiplet. Here we have included the constraint comming from the mixed gauge-gravitational anomaly [6] resulting from triangle diagrams involving two energy momentum tensors (gravitons) vertices and a $U(1)$ gauge vertex.

Eqs. (2)–(5) have the following three different solutions:

Sol.A $\quad Y_F = -3x, \quad Y_E = 6x, \quad Y_Q = x, \quad Y_U = -4x, \quad Y_D = 2x$

Sol.B $\quad Y_F = -3y, \quad Y_E = 6y, \quad Y_Q = y, \quad Y_U = 2y, \quad Y_D = -4y$

Sol.C $\quad Y_F = Y_E = Y_Q = 0, Y_U = -Y_D = z,$

where x, y and z are arbitrary real parameters, fixed by normalization. Solution **A** with $x = 1/3$ yields the usual $U(1)_Y$ hypercharge of the SM ($x = -1/3$ is related to mirror families), while solutions **B** and **C** represent two different $U(1)$ quantum number assignments, not identified with conventional physics.

Solution **B** appears as a consequence of the symmetry $Y_U \leftrightarrow Y_D$ in the anomaly constraint equations, which means that from the anomaly cancellation alone one can not distinguishes between U and D quark fields. So, solution **A** and solution **B** became equivalent when Y_U and Y_D are interchanged, but this would be not the case if an extra-interaction, which distinguishes between U and D, is introduced [7]. Since we wish to analize horizontal symmetries which could perform this distinction, we treat the three solutions independently. Then, anomaly cancellation by itself is not enough to fix completely the standard gauge group [5]. In fact, this ambiguity is absent in the SM standard formulation because the Higgs sector of the theory plays a central role. The introduction of a Higgs field ϕ_{SM} with $U(1)$ hypercharge Y_ϕ which couples to Up and Down quarks and to charged leptons in each family, reduces the number of free hypercharge parameters in the analysis via the relationships:

$$Y_Q + Y_U = -Y_Q - Y_D = -Y_F - Y_E = Y_\phi. \tag{6}$$

and the solution **A** is unambigously singled out [5]. Notice that the SM Higgs provides mass to every fermion field. This means that the phenomenological quark and lepton mass spectrum hierarchy should appear from a fine tunning of the Yukawa couplings, turning the scheme quite unnatural.

There is also an alternative view of that problem, used in the last two papers of Ref. [5]. There one defines an electric charge operator $Q_{EM} = T_{3L} + Y/2$, with the extra assumption that the $U(1)_{EM}$ associated with Q_{EM} is vectorlike.

Before exploring the possibilities to single out solution **A**, we can pursue the analysis in an almost general case, taking into account all three solutions, namely $U(1)_{Y_A}$,$U(1)_{Y_B}$ and $U(1)_{Y_C}$ respectively. In this context, one can ask oneself whether $U(1)_{Y_A} \otimes U(1)_{Y_B} \otimes U(1)_{Y_C}$ can be gauged simultaneusly, or at least two of them at the same time. The answer is **no** because the triangle anomalies $[U(1)_{Y_\alpha}]^2 U(1)_{Y_\beta}$ with $\alpha, \beta = A, B$ or C and $\alpha \neq \beta$, do not cancel. So, the hypercharges Y_B and Y_C should be automatically excluded if $U(1)_{Y_A}$ is gauged as the correct hypercharge. However, the cancellation of the triangle anomalies allows that once one gauges solution **A** for a particular value of x, one can gauge as many $U(1)_{Y_A}$ as one wishes, each one for a different value of x. In other words, the only family independent hypercharge that one can gauge simultaneously with $G_{\rm SM}$ is a $U(1)_{Y'}$ of hypercharge $Y' \sim Y_A$ [8,9].

In summary, from anomaly cancellation considerations alone, one can specify as a sensible gauge goup: $SU(3)_c \otimes SU(2)_L \otimes U(1)$, but with an ambiguity in the assignement of the abelian charge, namely hypercharges Y_A, Y_B or Y_C. In what follow we analyse the uselfulness of the inclusion of a further symmetry related to families, namely a local gauge horizontal symmetry G_H in selecting the SM ''peculiar'' hypercharges. In so doing we ask, as a further requirement, that the spontaneous breaking of the symmetry provides mass, at the tree level, only to the top quark. This last condition sounds clearly reasonable whenever one recalls that $m_b/m_t \sim 3\%$, i.e., of the same order as the amount of quantum (radiative) corrections.

2. Charge quantization from horizontal symmetries

In this section we introduce an Abelian gauge horizontal symmetry for three families designed to allow only for a tree level top quark mass. The generation of masses for the remaining known fermion fields via radiative corrections demands for a "Diophantine solution" [10] to the new anomaly contraint equations. We then show that the implementation of this program is compatible only with solution **A** of the previous section, ruling out solutions **B** and **C**, implying thus discrete values for the electric charge.

2.1. Horizontal symmetries

Since there is no evidence for an specific local gauge horizontal symmetry, we work within the following frame which is at least consistent with experimental facts: There exist only three complete chiral families of ordinary matter (without right-handed neutrinos) togheter with a local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_H$, where $G_H \subset [U(3)]^5$, the most general symmetry allowed. This scheme is implemented with the following symmetry breaking chain

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_H \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_{EM}.$$

We also expect that an effective Lagrangean of the type [11]

$$\mathscr{L}^{eff} = \sum_{i,j} \left[\left(y^U_{ij} \phi^\dagger_{\rm SM} U^c_{iL} \left(\frac{\Lambda}{M} \right)^{n^U_{ij}} \left(\frac{\Lambda^\star}{M} \right)^{n'^U_{ij}} + y^D_{ij} \tilde{\phi}^\dagger_{\rm SM} D^c_{iL} \left(\frac{\Lambda}{M} \right)^{n^D_{ij}} \left(\frac{\Lambda^\star}{M} \right)^{n'^D_{ij}} \right) Q_{jL} \right.$$
$$\left. + y^E_{ij} \tilde{\phi}^\dagger_{\rm SM} E^c_{iL} F_{jL} \left(\frac{\Lambda}{M} \right)^{n^E_{ij}} \left(\frac{\Lambda^\star}{M} \right)^{n'^E_{ij}} \right] + \text{h.c.} \tag{7}$$

will provide, after the symmetry breaking and diagonalization, the appropriate masses and mixing angles for all the ordinary fields. In $\mathscr{L}^{eff}$, Λ is the breaking scale of G_H, M is an undetermined mass scale, n^{κ}_{ij} and n'^{κ}_{ij}, ($\kappa = U,D,E$) are integer numbers for $i,j = 1,2,3$ and y^{κ}_{ij} are Yukawa coupling constants of order one. To properly fulfil all the requirements in $\mathscr{L}^{eff}$, one needs a G_H able to distinguish Up from Down, and integer G_H quantum numbers to relate them to the loop exponentials n^{κ}_{ij} and n'^{κ}_{ij}.

The simplest local gauge horizontal symmetries contained in $[U(3)]^5$ that one may consider are $U(1)_H$ and $U(1)_{H_1} \otimes U(1)_{H_2}$. That they are reasonable horizontal symmetries for understanding the mass spectrum of the elementary fermions and their mixings has been disscused in Refs. [9,12]. In what follows we are going to restrict ourselves to those two cases [13]. Notice that these symmetries could be realized both in a family independent and in a family dependent way. As we have stated above, this means that the same horizontal charge is assigned to related multiplets of each family or not.

2.2. Family independent

We consider first a family independent $U(1)_H$ as our horizontal symmetry group. For a Higgs field with $U(1)_H$ charge Y_ϕ, a Yukawa coupling for the top quark is allowed if $Y_{Q_3} + Y_{U_3} = Y_\phi$, whereas a bottom quark coupling is forbidden if $Y_{Q_3} + Y_{D_3} \neq -Y_\phi$ which is in contradition with equation anomaly constraint equivalent to Eq. (3). Therefore for a $U(1)_H$ family independent, if a top quark mass arises at tree level, a bottom mass arises as well at the same level [9]. So, only a $U(1)_H$ family dependent symmetry could be consistent with our working hypothesis. A similar analysis and conclusions follow for a family independent $U(1)_{H_1} \otimes U(1)_{H_2}$.

2.3. Family dependent

For three families, an $U(1)_H$ family dependent symmetry group must be a subgroup of $[SU(3)]^5$. Consequently, for each horizontal η multiplet ($\eta = Q,U,D,F,E$), the hypercharges must be traceless, and even further they must be of the form $(\delta_\eta, 0, -\delta_\eta)$ or $(\delta_\eta, \delta_\eta, -2\delta_\eta)$. These are the forms of the corresponding $U(1)$ subgroups of $SU(3)$. Then, for the most general $U(1)_H$ extracted from $[SU(3)]^5$, we must have

$$\mathrm{tr}[U(1)_H] = \sum_{i=1}^{3} H_{\eta_i} = 0$$

for $\eta = Q,U,D,F,E$ and where H_{η_i} stand for the horizontal charges.

Now, if the anomalies are going to be cancelled by an interplay among the three families, the linear cancellation constraints $[SU(2)_L]^2 U(1)_H$, $[SU(3)_c]^2 U(1)_H$, $[\mathrm{grav}]^2 U(1)_H$, and $[U(1)]^2 U(1)_H$ are automatically satisfied, and we have to worried only about the following two new constraints:

$$[U(1)_H]^2 U(1): \quad \sum_i \left(6Y_Q H^2_{Q_i} + 3Y_U H^2_{U_i} + 3Y_D H^2_{D_i} + 2Y_F H^2_{F_i} + Y_E H^2_{E_i}\right) = 0, \tag{8}$$

$$[U(1)_H]^3: \quad \sum_i \left(6H^3_{Q_i} + 3H^3_{U_i} + 3H^3_{D_i} + 2H^3_{F_i} + H^3_{E_i}\right) = 0, \tag{9}$$

where $i = 1,2,3$ sums over the multiplets for the three families, and the Y_η hypercharge for $\eta = Q,U,D,F,E$, appearing in Eq. (8), are in principle any one of the three possible solutions **A**, **B**, or **C**.

For an $U(1)_H$ of the form $(\delta_\eta, 0, -\delta_\eta)$, the quadratic Eq. (8) becomes:

$$6Y_Q\delta_Q^2 + 3Y_U\delta_U^2 + 3Y_D\delta_D^2 + 2Y_F\delta_F^2 + Y_E\delta_E^2 = 0, \tag{10}$$

and the cubic equation is trivially satisfied.

For an $U(1)_H$ of the form $(\delta_\eta, \delta_\eta, -2\delta_\eta)$, the quadratic equation is again given by Eq. (10) and the cubic equation reads:

$$6\delta_Q^3 + 3\delta_U^3 + 3\delta_D^3 + 2\delta_F^3 + \delta_E^3 = 0, \tag{11}$$

which do not depend upon the $U(1)$ hypercharge Y_η, so it does not play any role on the solution of the ambiguity in the $U(1)$ hypercharge (eventhough it may play an important role on the form of the mass matrices).

Now, since at tree level the mass matrix for the Up quark sector must be a rank one matrix, we may, without lost of generality, relabel the weak eigenstates in such a way that only the (3,3) entry in the Up quark mass matrix is different from zero. Then it is just natural to demand for a symmetric mass matrix for the entire Up quark sector [12], which is achieved only if $\delta_Q = \delta_U (\equiv \delta)$.

On the other hand, since Eq. (10) is also $U \leftrightarrow D$ symmetric, in order to distinguishes Up from Down let us look for a $G_H = U(1)_{H_U} \otimes U(1)_{H_D}$ where $U(1)_{H_U}$ is inert (zero charge value) in the Down sector and $U(1)_{H_D}$ is inert in the Up sector. Then for a $G_H = U(1)_{H_U} \otimes U(1)_{H_D}$ and a symmetric Up quark mass matrix we have the following anomaly constraint quadratic equations:

$$[U(1)_{H_U}]^2 U(1): \quad 3\delta^2(2Y_Q + Y_U) + 2Y_F\delta_F^2 + Y_E\delta_E^2 = 0, \tag{12}$$

$$[U(1)_{H_D}]^2 U(1): \quad 3Y_D\delta_D'^2 + 2Y_F\delta_F'^2 + Y_E\delta_E'^2 = 0, \tag{13}$$

$$[U(1)_{H_U}]^2 U(1)_{H_D}: \quad 2\delta_F^2\delta_F' + \delta_E^2\delta_E' = 0, \tag{14}$$

$$[U(1)_{H_D}]^2 U(1)_{H_U}: \quad \delta_F\delta_F'^2 + \delta_E\delta_E'^2 = 0, \tag{15}$$

where the unprimed charges are related to the $U(1)_{H_U}$ factor and the primed charge to the $U(1)_{H_D}$ factor. A Diophantine analysis [10] of the last two equations shows that it is impossible to find rational value for $\delta_F, \delta_F', \delta_E, \delta_E'$ which satisfies them. Since irrational charges are not allowed in our approach, we must ask for non-overlapping $U(1)_{H_U}$ and $U(1)_{H_D}$ factors, which in turn implies that the last two equations must be trivially satisfied (0 = 0).

If Y_η are given by solution **C**, Eqs. (12) and (13) are satisfied only for $\delta = \delta_D' = 0$. This solution will produce tree level masses in the Up and Down sectors (which can be seen from the matrix quantum numbers for the Up and Down quark sectors presented below) and must be ruled out.

On the other hand, if Y_η is given by solution **B**, Eqs. (12) and (13) become:

$$2\delta^2 - \delta_F^2 + \delta_E^2 = 0, \tag{16}$$

$$2\delta_D'^2 + \delta_F'^2 - \delta_E'^2 = 0. \tag{17}$$

Non-overlapping real solutions are produced only for $\delta_E = \delta_F' = 0$. But for those values there is no rational solution. Then, solution **B** is also ruled out.

If Y_η is given by solution **A**, Eqs. (12) and (13) become:

$$\delta^2 + \delta_F^2 - \delta_E^2 = 0, \tag{18}$$

$$\delta_D'^2 - \delta_F'^2 + \delta_E'^2 = 0, \tag{19}$$

where non-overlaping real solutions exist for $\delta_F = \delta_E' = 0$. Those solutions are $\delta_E = \pm\delta$ and $\delta_F' = \pm\delta_D'$.

Consequently, only an $U(1)_Y$ with the hypercharge given by solution **A** is consistent with our working hypothesis, toghether with a $U(1)_{H_U} \otimes U(1)_{H_D}$ local gauge horizontal symmetry which assigns charge values to ordinary fields according to the values in Table 1. Since the two factors $U(1)_{H_U}$ and $U(1)_{H_D}$ do not overlap, $U(1)_{H_U} \otimes U(1)_{H_D}$ is equivalent to a single $U(1)_H$ with hypercharge values as in Table 1.

3. Mass matrices

Let us briefly comment on the mass matrices produced by the simplest solution of the anomaly constraint equations, presented in Table 1.

3.1. Case $U(1)_H$ of the form $(\delta_\eta, 0, -\delta_\eta)$

For an $U(1)_H$ of this form, Eq. (10) with the hypercharge values Y_η given by solution **A** reads:

$$6x\left(\delta_Q^2 - 2\delta_u^2 + \delta_D^2 - \delta_F^2 + \delta_E^2\right) = 0, \tag{20}$$

which is satisfied by the $U(1)_H$ charge values of Table 1 as it should be (although a more general Diophantine analysis of this equation may be performed). Since the cubic equation is trivially satisfied by a $U(1)_H$ of the form $(\delta_\eta, 0, -\delta_\eta)$, there are no further constraints for the $U(1)_H$ charge values in Table 1. For those values the $U(1)_H$ matrix quantum numbers, for the Up quark sector, is given by

$$\begin{pmatrix} 2\delta & \delta & 0 \\ \delta & 0 & -\delta \\ 0 & -\delta & -2\delta \end{pmatrix}.$$

Similarly, for the Down quark sector we obtain

$$\begin{pmatrix} \delta+\delta' & \delta & \delta-\delta' \\ \delta' & 0 & -\delta' \\ -\delta+\delta' & -\delta & -\delta-\delta' \end{pmatrix}.$$

Then, a ϕ_{SM} Higgs field with a $U(1)_H$ charge value -2δ will produce, at tree level, a rank one mass matrix for the Up quark sector ($M_{33}^{up} \sim m_t$ as wished) and a zero mass matrix for the Down quark sector, as far as $\delta' \neq \pm\delta, \pm 2\delta, \pm 3\delta$.

Now, under the (crude) assumption that $y_{ij}^\eta = 1$ for $\eta = U,D,E; ij = 1,2,3$, the effective Lagrangean produces, for $\delta' = -5\delta$ and the appropriate $U(1)_H$ charge value for the necessary Higgs, the following quark mass matrices [12]:

$$M_U \sim \begin{pmatrix} \theta^2 & 0 & \theta \\ 0 & \theta & 0 \\ \theta & 0 & 1 \end{pmatrix},$$

and

$$M_D \sim \begin{pmatrix} \theta^3 & 0 & \theta^2 \\ 0 & \theta & 0 \\ \theta^4 & 0 & \theta \end{pmatrix},$$

where $\theta = \Lambda/M$. The diagonalization of M_U and M_D produces $m_t \sim 1 + \theta^2, m_c \sim \theta, m_u \sim 0, m_b \sim \theta + \theta^5, m_s \sim \theta$ and $m_d \sim \theta^3 - \theta^5$. These values may be brought closer to the experimental results by the appropriate selection of θ and $y_{ij}(\sim 1)$ values.

Table 1
Hypercharge values for $U(1)_{H_U}$, $U(1)_{H_D}$ and $U(1)_H$ local gauge horizontal symmetries

	δ_Q	δ_U	δ_D	δ_F	δ_E
$U(1)_{H_U}$	δ	δ	0	0	$\pm\delta$
$U(1)_{H_D}$	0	0	δ'	$\pm\delta'$	0
$U(1)_H$	δ	δ	δ'	$\pm\delta'$	$\pm\delta$

Notice by the way that the matrix quantum numbers for the charged lepton sector is equivalent to the matrix quantum numbers for the Down quark sector, which in turn implies a relationship between charged leptons and Down quark masses as it is usually expected.

3.2. Case $U(1)_H$ of the form $(\delta_\eta, \delta_\eta, -2\delta_\eta)$

For $U(1)_H$ of this form, both Eqs. (11) and (20) must be be satisfied. As mentioned above, Eq. (20) is satisfied by the $U(1)_H$ charge values in Table 1, and a Diophantine solution to Eq. (11) brings the further constraints

$$\delta_E = -\delta, \quad \delta_D = -\delta_F = -2\delta,$$

then the $U(1)_H$ matrix quantum numbers for the Up quark sector for this case reads

$$\begin{pmatrix} 2\delta & 2\delta & -\delta \\ 2\delta & 2\delta & -\delta \\ -\delta & -\delta & -4\delta \end{pmatrix},$$

and for the Down quark sector it reads

$$\begin{pmatrix} -\delta & -\delta & 5\delta \\ -\delta & -\delta & 5\delta \\ -4\delta & -4\delta & 2\delta \end{pmatrix};$$

then a $\phi_{\rm SM}$ Higgs field with a $U(1)_H$ charge value -4δ will produce also a rank one mass matrix at tree level for the Up quark sector (again $M_{33}^{up} \sim m_t$) and a zero mass matrix for the Down quark sector. The analysis of the mass matrices for the simplest solutions presented in this case does not improve the results obtained for the previous case.

4. Conclusions

The previous analysis allows us to conclude that: - In order to provide a tree level mass only for the top quark, at least an Abelian horizontal family dependent symmetry $U(1)_H$ is needed. - The Abelian horizontal family dependent symmetry can be only of the form $(\delta_\eta, 0 - \delta_\eta)$ or $(\delta_\eta, \delta_\eta, -2\delta_\eta)$ or a combination of both (for $\eta = Q,U,D,E,F$). - The presence of a horizontal gauge symmetry, allowing only for a top quark mass at the tree level, selects the Standard Model hypercharges among the three different weak hypercharge assignments compatible with anomaly cancellation. As a matter of fact, the very simple election of $G_H = U(1)_{H_U} \otimes U(1)_{H_D}$ (equivalent to $U(1)_H$), with the horizontal charges summarized in Table 1, does the job, breaking simultaneously the $Y_U \leftrightarrow Y_D$ symmetry.

In our approach the tree level "top" mass, generated by the Yukawa coupling with an appropriate Higgs boson, is expected to be the seed mass that gives rise to the three fermion mass matrices via quantum effects.

In summary, a realistic Standard Model in the sense of observing the experimental mass hierarchy, could emerge from the inclusion of a horizontal local gauge symmetry. Moreover, the proposed mechanism based on anomaly cancellation, is sufficently restrictive as to uniquely fix the standard weak hypercharges.

A further analysis of the mass matrices for more general Diophantine solutions of the anomaly constraint Eq. (10), and for a general $U(1)_H$ of the form $(\delta_\eta, 0 - \delta_\eta) \oplus (\delta_{\eta'}, \delta_{\eta'}, -2\delta_{\eta'})$, $\eta \neq \eta'$, is under way.

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