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Research Article

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Linking Photography and Mathematics with the Use of Technology

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Abstract: In this paper, we describe an inclusive, collaborative learning approach in which both able and less able students engaged together in a contest combining Photography, Mathematics and Technology, in our case GeoGebra. Following three cycles of the contest, we analysed a selection of entries and the related post-participation survey. Based on the analysis, we concluded that the combination of these approaches could help to engage students with mathematical content by relating art, their surrounding environment with mathematics. This approach enabled us to offer different ways of mathematics learning through providing contexts for exploration and students' creation of their own contents. Results also showed that school teaching is sometimes insufficient to solve emerging problems and this challenged students to find new kinds of solutions supported by new tools and concepts. It also allowed students to think creatively by inviting them to work together to develop their own questions, problems, and facilitating learning by research.

Keywords: Photography, GeoGebra, Creativity, ICTs in Education, STEAM Education .

1 Introduction

It is common in most countries that many young students believe that mathematics is boring and useless. This is why teachers and researchers from all over the world are concerned about how to engage children in mathematical tasks. There is a new trend making explicit connections between mathematics and other science, technology, engineering (STEM) subjects and more recently with the arts. According to Lavicza et al. (2018), "Creative activities may support students to recognize that doing 'real' mathematics is creative thinking; and creative thinking in mathematics means, that you do your own mathematics" (p. 110). In this paper, we outline approaches connecting mathematics, arts and other STEM subjects through photography.

There are several initiatives integrating photography in mathematics teaching and learning. For instance, Muñoz Casado (2018) invited his students to walk around their city observing the implicit geometry in architecture. Then, students used photography and GeoGebra (2009) to model and study geometry of buildings. Similarly, Sharp, Garofalo and Thompson (2004) proposed using photography to analyse the concepts of slope (studying different kind of rooftops) and vanishing points (in paintings). These researchers used photography to provide context for problem solving and proportional thinking. According to the authors, "digital imagery can help students develop a strong sense of authenticity in their mathematics learning" (Sharp et al., 2004, p. 32).

In our study, we used GeoGebra, because it is free software that provides users with an environment to explore, conjecture, discover, learn by trial and error in a dynamic way. Also, it is a user-friendly and versatile tool, widely used in Argentina, allowing easy incorporation of pictures and connecting different kinds of mathematical objects with inserted pictures. This feature helped us to mathematize an image and perform further analyses. Furner and Marinas (2014) stated that "by importing photography into the GeoGebra software, teachers can explain math concepts and make the learning of math more real-world and relevant" (p. 134). Later, we will share some reflexions about a Mathematics, Photography & GeoGebra contest that we organize yearly, called Fotogebra (Rizzo, 2009, 2016). We invite students to

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"catch" a mathematical concept in a picture, set up a motto, develop a problem involving the selected concept to be solved by analysing the picture, and, finally, solve the problem. This didactic proposal is strongly related to the authors' ideas for teaching and learning: students create their own mathematics, by creating their own questions and strategies to answer them; they are encouraged to observe the world; they use digital imagery to establish relations between art, their environment, STEM subjects and mathematics.

As we will point out throughout this work, our photo contest enables students to work in groups, to create their own questions and problems, and develop solutions by themselves. Also, this scenario encourages students to learn mathematical topics beyond the school curriculum, and to relate scholarly content with arts and their own surroundings.

2 Contest Organization

The photography and GeoGebra contest is organized in four student categories: (1) Lower middle school students, (2) Higher middle school students, (3) Teacher trainees (first and second year), (4) Teacher trainees (third and fourth year). In the first and third category, students had to choose a geometrical concept, and for second and fourth category, they had to develop a problem involving functions.

Students participated in the contest in small groups (2-3) in order to encourage collaborative learning among students with different profiles, in particular relating to aptitude and perception of both Maths and Art. The objective is to both awaken mathematical interests of less able maths students who often do not like the subject, and also provide a skill learning opportunity for more able students. They were supported by their school teachers and one of them is assigned the role of a mentor. Teachers, for necessary professional reasons, had the freedom to adapt and integrate the contest activities into their own annual planning. Thus, this contest can be considered as providing both formal and informal teaching and learning. Teachers accompanied students, serving as tutors and helped them to develop their projects. The mentor preselected some works to present in the contest (only five works by category and by institution are accepted), so they performed a preliminary screening. Organisers offered workshops for teachers and tutorials to assist them with this task.

An expert jury evaluated the works presented by students. They took into account: from the artistic point of view the quality of photography, its aesthetics, originality, compositional balance, dynamism and framing; and from the mathematics point of view the creativity in the production of the problem situation, the applied specific knowledge and the use of GeoGebra. Mathematics aspects had a larger weight compared to the artistic elements as was outlined in the contest description. Winners were selected and awarded by the jury.

Also, two photo exhibitions were prepared by organizers (one of them is physical and the other one is virtual – it took place in social media). Visitors of the exhibition could also vote for their favourite works. In these cases, the public gave the votes based on their own criteria, they didn't have the obligation to take into account the mathematical value of the works and the winners in this instance receive a special "public vote award".

3 Photography Analysis and Learning Outcomes

In this section, we share some of the works presented by students from the second category (high school students, looking for functions in their environment). These selected works will serve as examples of the value of the contest for teaching and learning. We analysed each submission based on what mathematical and GeoGebra tools they used, how it was related to school content, and what they had to learn *ad hoc* to solve the problem they worked on.

The first one, shown in Figure 1, *Cumpleaños exponencialmente feliz* (in English, *Exponentially Happy Birthday*) used a picture of one of the authors' dog to find out how long should a fake beard be that would fit the dog for a birthday party. In GeoGerba they placed some points on the dog's face and they used GeoGebra *FitExp* command to fit an exponential function to them. Then, they used the *Length* command to solve their problem. These students had studied exponential functions at school, but they didn't know about fitting of curves and about calculating their length. These topics are usually taught in Argentina at the university level. Students had to find out by themselves what mathematical tools were available to solve their problem, using the user-friendly GeoGebra, allowing them to focus on the mathematics rather than how to use the tool *per se*.

The second example is *Dulce o Miau* (in English, *Sweet or Meow*), which is shown in Figure 2. The author wanted to estimate how much fabric was needed to create a Halloween-costume for his cat. So, he took a picture of the animal and used the *Polynomial* command to fit the contour of its body. Then, he used the *Integral* command to find out the area, which helped him to estimate the area of the fabric needed. Integral is a concept usually taught at university level in Argentina, but it did not stop him from solving this problem. He investigated the concept, its use and how to find the desired result with GeoGebra.

The same student presented another work in the next year: it was entitled *Cuy Sediento* (in English, *Thirsty Guinea Pig*), and it is shown in Figure 3. This time he worked with a partner. He took several pictures during his winter holydays, thinking about possibilities for participating in the contest. Back in school, he discussed with his partner and teacher which of the pictures would be the most appropriate. They chose the picture in Figure 3, which made them ask themselves how long the spiral paths are, to estimate how much time would a Guinea Pig need to get to the water inside the spiral. They used GeoGebra sliders to fit a spiral, then they used a formula in order to calculate the length. They researched the average speed of this animal, and used this value to estimate the time to get the water, which required, besides photography, mathematics, GeoGebra, and the application of a basic Physics concept. In his second contest entry, this student showed more enthusiasm, engaged a partner, they changed the way to fit the curve (the first time he had used a static command, but this time he used a dynamic form). Besides all this, they used an animation to simulate the movement of the Guinea Pig[1].

The fourth example we want to share is *Una foto en el momento justo* (in English, *A just-in-time picture*), shown in Figure 4. The authors wanted to know how high the ball reaches when they play Voleyball. They took several pictures of the launch, and then they overlapped them using a specific software for digital image editing. They used *FitPoly* command to fit the parabola and then they used previously known formulas to get the maximum.

In many of the presented works, as it is shown by the previous examples, mathematical content emerged from the picture analysis exceeded the content studied at school, because it corresponds to a higher level of education. Students chose an image they could link with scholarly content, but the question triggered by it exceeded

Figure 1: One of the works presented by students, Cumpleaños exponencialmente feliz (Exponentially happy Birthday). By: Milagros Candia.

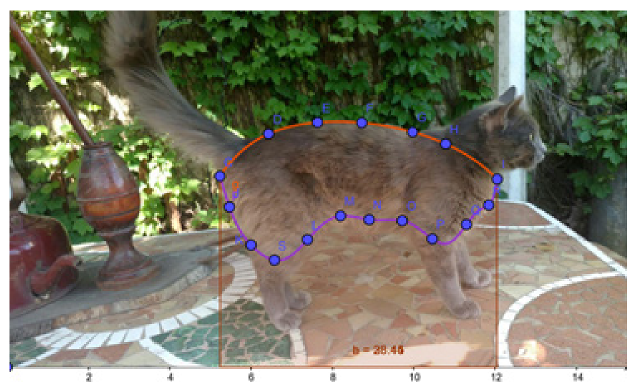

Figure 2: Dulce o miau (Sweet or Meaw). By: Danilo Fuentes.

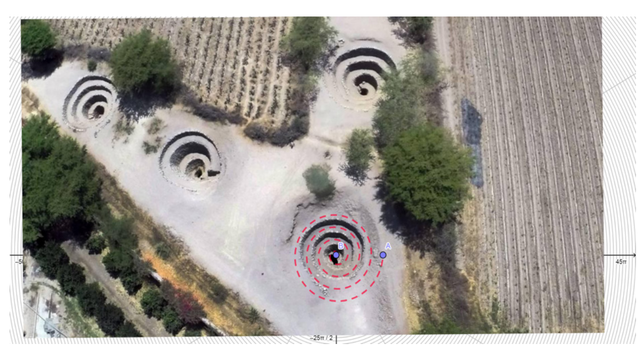

Figure 3: Cuy Sediento (Thirsty Guinea Pig). By Abril Oviedo and Danilo Fuentes.

1 The description of the mathematical problem, as well as background information about the picture can be accessed at: https://www.geogebra.org/m/vhxnnfez

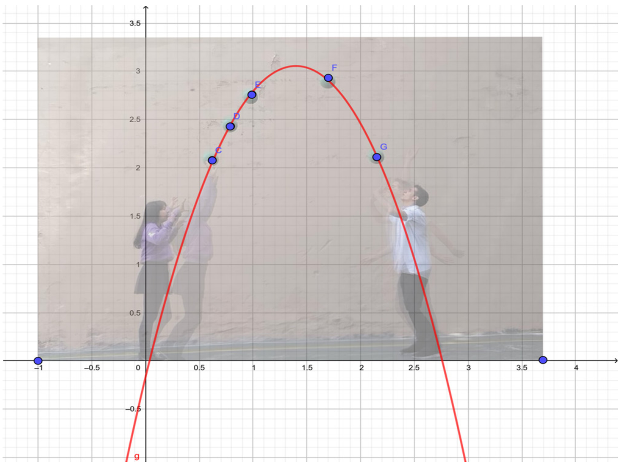

Figure 4: Una foto en el momento justo (A just-in-time picture). By Ignacio Derudi and Vera Aymará.

what they previously studied, and this challenged them to learn new mathematics and conduct research (on the web or consulting the teacher) and GeoGebra helped with calculations that was beyond their actual capabilities. It is important to say that, even though the solutions are not strictly correct from a mathematical point of view (because they don't take into account the projection problem) we found them valuable from a didactic point of view, because they led students to ask their own questions, link them with mathematical concepts and get an estimation of the result.

4 Survey Analysis

After the contest, we asked participants (students and tutors) questions about their experiences through an anonymous survey. For students, the survey contained had five questions, whereas for tutors, it had six. As shown in this section, their answers correspond to the analysis we made in the previous section about learning outcomes and new ways of learning mathematics that were triggered by this didactic proposal.

One of the questions of the students' survey was: Do you think that participating in the contest helped you learn new mathematics? If you do, which areas?

We share below some of the answers:

– "Yes, of course, I learned lots of things. For example, I saw the world surrounding us characterized by mathematics and physical objects. By modelling photography you can learn or revise mathematical content in an interesting and catchy way. Working within a team, looking for a motto, finding photographical art and enjoying mathematics was awesome".
– "It was useful for me because it made me think that mathematics is not always heavy, there are more approachable ways of learning it. Also, I learned how to use GeoGebra, which I had thought was a difficult app".
– "Using GeoGebra and the fact of looking for new concepts for developing my own projects".

These answers show us that students value this approach because they used to think that mathematics was boring and useless, but they found that it is not necessarily the case. Non-routine activities in the classroom certainly empower students with this subject. They also found integration of technological artefacts to help them solve a problem valuable. In this case, GeoGebra, which is easy to use and user friendly software.

Another question of the students' survey was: What did you like the most about taking part in the contest? What did you like the least?

Some of the answers were:

– "I liked thinking for two months about what to photograph, what would be the sense of doing it to study with GeoGebra. I didn't like the waiting". By waiting, this participant meant that he had to wait for the results of the contest more than he had expected.
– "Making new friends and having fun learning new things that otherwise would never have caught my eye".
– "What I liked the most was interacting with my partner while doing the project. Also, being able to relate Mathematics to everyday life".
– "What I liked the most was working with pictures and graphics. There is nothing I didn't like".

Reading these answers made us note enthusiasm in participants, emotion, and collaborative learning. We think that the collaborative approach helped to spread the passion, mitigating the fear of the subject, and to show students another face of STEM or STEAM.

Regarding the responses of the tutors, they invited their students to engage in new ways with mathematical concepts, integrating digital resources in order to get more significant learning outcomes. They thought that the contest allowed students to relate their everyday life with scholarly content. They liked the most the response of their students and the enthusiasm they showed while learning.

The very positive responses of both teachers and students suggested inclusiveness (engaging both able and less able students) and sustainability (it is more likely to be continued if both students and teachers support the activity).

Next steps would be to explore differences in responses between able and less able students to see how/which students become more positive comparing pre and post attitudes towards learning involving maths. Similarly, it would be valuable to learn how able and less able students work together in the collaborative process in, for example, defining the problem, agreeing on respective roles and tasks, recovering from mistakes and using this as a vehicle for learning and supporting each other emotionally.

5 Summary and Conclusions

In this paper we presented a didactical approach integrating Photography and GeoGebra for enhancing mathematical learning and students' enthusiasm towards the subject. The approach consisted of a contest in which students had to "catch" a mathematical concept in a picture, set up a motto, make up a problem that can be solved using it, insert the picture in GeoGebra software and analyse it using GeoGebra tools to solve the problem.

We noticed that, by participating in this contest students were enabled to integrate different ways of learning, many of which they had been using in other contexts, but not in school. They took pictures that they can relate some mathematical school concepts, but the questions they created were beyond what they had studied. This, however, did not deter them. Instead of quitting, they investigated new mathematical concepts, tools and procedures that helped them reach the solution. They also used GeoGebra, a user-friendly, easy-to-learn, exploration tool, to fit a model to the situation and make further calculations. By this didactical approach learning through research was also encouraged.

Several participants, motivated by their own enthusiasm and willingness to take on new challenges, chose to participate in the contest more than once, even when they stopped working with teachers who encouraged them to do that. These students improved their performances year after year: they deepened their knowledge of new mathematical concepts; they improved their skills of using the GeoGebra software, explored new tools and features; they challenged themselves with new questions, and were less apprehensive about searching for and learning about new mathematical objects, had less fear of making mistakes. They learned to learn from mistakes as GeoGebra helped to achieve this, because it easily allows to undo, erase and try again different constructions.

All this supports the goals of the authors mentioned in the introduction: students improving their own mathematics skills by developing their own questions and strategies to answer them; working together; encouraged to observe the world; using digital imagery to establish relations between art, their environment and mathematics.

The same conclusion emerges from both the analysis of students' work and of survey results. For this reason we believe that this didactical approach could be valuable and deserves to be further improved and investigated in future projects.

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