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Essays On Firm Dynamics, Innovation And Growth

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Abstract

This dissertation studies various aspects of firm dynamics, and its relation to innovation and economic growth. Chapter 1 studies how incorporation, which provides limited liability protection to firm owners, affects firm behavior and economic growth. I propose an endogenous growth model of heterogeneous firms, where firms spend resources to improve their productivity and choose whether to incorporate or not. The model underlines incentive and selection effects of incorporation that generate the observed differences between incorporated and unincorporated firms. I calibrate the model to the Danish firm-level data to study the importance of these two effects quantitatively and draw several policy conclusions. Chapter 2 (joint with Ufuk Akcigit and Michael Peters) studies managerial delegation and its importance on the process of firm growth and aggregate productivity. We construct a model of firm dynamics where entrepreneurs have a fixed time endowment to run daily operations and need to hire outside managers as they grow. We calibrate the model to plant-level data from the US and India and quantify the importance of frictions in the managerial delegation to explain the differences in the firm dynamics and aggregate productivity between two countries. Chapter 3 (joint with Daron Acemoglu, Ufuk Akcigit, Nicholas Bloom and William Kerr) develops a model of endogenous reallocation and innovation with heterogeneous firms. Our main focus is on the reallocation (and misallocation) of R&D inputs, which emphasizes that misallocation may affect equilibrium growth as well. We estimate the model by using US Census Bureau micro data to study the effects of various counterfactual policies and gain insights about whether substantial improvements in economic growth and welfare are possible. Our results highlight the potential pitfalls of industrial policies supporting incumbents.

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ABSTRACT

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Harun Alp

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This dissertation studies various aspects of firm dynamics, and its relation to innovation and economic growth. Chapter 1 studies how incorporation, which provides limited liability protection to firm owners, affects firm behavior and economic growth. I propose an endogenous growth model of heterogeneous firms, where firms spend resources to improve their productivity and choose whether to incorporate or not. The model underlines incentive and selection effects of incorporation that generate the observed differences between incorporated and unincorporated firms. I calibrate the model to the Danish firm-level data to study the importance of these two effects quantitatively and draw several policy conclusions. Chapter 2 (joint with Ufuk Akcigit and Michael Peters) studies managerial delegation and its importance on the process of firm growth and aggregate productivity. We construct a model of firm dynamics where entrepreneurs have a fixed time endowment to run daily operations and need to hire outside managers as they grow. We calibrate the model to plant-level data from the US and India and quantify the importance of frictions in the managerial delegation to explain the differences in the firm dynamics and aggregate productivity between two countries. Chapter 3 (joint with Daron Acemoglu, Ufuk Akcigit, Nicholas Bloom and William Kerr) develops a model of endogenous reallocation and innovation with heterogeneous firms. Our main focus is on the reallocation (and misallocation) of R&D inputs, which emphasizes that misallocation may affect equilibrium growth as well. We estimate the model by using US Census Bureau micro data to study the effects of various counterfactual policies and gain insights about whether substantial improvements in economic growth and welfare are possible. Our results highlight the potential pitfalls of industrial policies supporting incumbents.

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Chapter 1

Incorporation, Selection and Firm Dynamics: A Quantitative Exploration

Abstract

This paper studies how incorporation, which provides limited liability to firm owners, affects firm dynamics and macroeconomy. I propose an endogenous growth model of firm dynamics with endogenous entry and exit, where firms spend resources to improve their productivity and choose whether to incorporate or not. Incorporation provides liability protection which ensures that firm value is bounded from below, at the expense of high set-up and maintaining cost. An important model feature is that firms have heterogeneous (high and low) types which differ in their capacity to improve productivity. This heterogeneity allows for the possibility of selection as high-type firms, who have higher growth potential, benefit more from incorporation. I calibrate the model by using Danish firm-level data, specifically exploiting the heterogeneity in exit rates by age conditional on size to identify firm types in growth potential and therefore selection. The quantitative results suggest that both treatment and selection effects of incorporation are important and accounting for firm heterogeneity is quantitatively relevant in explaining the observed better performance of incorporated firms. Conditional on the firm type, incorporated firms choose an expansion rate, the rate at which firms improve their productivity, 50% higher than unincorporated firms do on average. Upon entry, 90% (15%) of the incorporated (unincorporated) firms are high-types, which are estimated twice as efficient as low-types in improving their productivity. This underlines a significant selection effect which is more pronounced among incumbents as the exit rate of high-type firms is lower. I find significant welfare gains from subsidizing incorporated firms and large welfare losses from removing incorporation choice. These welfare results are largely driven by the change in the degree of selection, i.e. the change in the composition of firm types.

1.1 Introduction

An extensive empirical literature has documented striking differences across firms. While many firms fail in their early years of existence and most of those that survive do not grow, others grow rapidly and significantly contribute to job creation and aggregate productivity growth. This reflects substantial firm heterogeneity in many aspects, one of which is the legal form they choose to operate in. For example in the U.S. roughly half of all business owners prefer to shield themselves against the downside risks by attaining limited liability through incorporating their businesses. How does limited liability affect firm behavior and the macroeconomy? How does the choice of legal form interact with ex-ante and ex-post firm heterogeneity?

To answer these questions, this paper proposes a macroeconomic model of firm dynamics with endogenous entry and exit, where entrepreneurs choose whether or not to incorporate their firms. In the model, firms invest in resources to improve their productivities which determine their profitability and contribute to economic growth. Firms have heterogeneous (high and low) types that differ in their efficiency to improve productivity. In other words, firms are heterogeneous in terms of their growth potential. Successful entrepreneurs increase their firm productivity and stay in the economy, whereas unsuccessful ones end up exiting the economy, either endogenously due to the deterioration in their profitability or due to exogenous shocks that render the firm unproductive. Firms are subject to an exit cost which is proportional to their size. Due to this cost, the firm value falls below zero in the case of exit. By paying a sunk cost, entrepreneurs can incorporate their firms to ensure that their losses are limited to the initial cost of setting up the firm. In other words, incorporation provides insurance to the owner by bounding the firm value from below.

This environment underlines two main effects that generate differences in firm dynamics between incorporated and unincorporated firms. The first one is a treatment effect of incorporation: since incorporation protects firms from downside risks, it incentivizes

them to invest more in improving their productivity, subsequently grow large and exit less often. The second one is a selection effect due to the presence of firm heterogeneity: entrepreneurs with higher growth potential (i.e. more efficient in improving productivity) are more likely to choose incorporation as it is more valuable to large firms. The strength of this selection effect is determined by the interplay between endogenous entry, investment, and exit decisions.

To quantify the importance of these effects and study their macroeconomic implications, I calibrate the model to firm-level micro data from Denmark. Calibration targets several key empirical moments of firm dynamics for incorporated and unincorporated firms. Specifically, the calibrated model is able to quantitatively match the observed differences between incorporated and unincorporated firms: incorporated firms have higher employment upon entry, grow faster, and exit less often conditional on their size and age, compared to unincorporated firms. Furthermore, to validate the model, I show that a variety of moments that are not targeted in the estimation are in line with the data.

My calibration strategy exploits the heterogeneity in firms exit rates by age conditional on size and legal form to identify firm heterogeneity in growth potential. The model implies that without this firm type heterogeneity, the likelihood of exit would be independent of age conditional on size. In data, however, such conditional exit rates are decreasing in firm age for both incorporated and unincorporated firms. My framework rationalizes this pattern through the interaction between firm heterogeneity and endogenous selection in that the share of firms with high growth potential, which have lower exit rates conditional on size, increases within a given cohort as the cohort ages.

The quantitative results suggest that both treatment and selection effects are important and accounting for firm heterogeneity is quantitatively relevant in explaining the observed better performance of incorporated firms. Conditional on the firm type, incorporated firms choose an expansion rate, the rate at which firms improve their productivity, 50% higher than unincorporated firms do on average. This indicates a significant

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positive treatment effect of incorporation on firm-level productivity growth. Among entrants, 90% (15%) of the firms that choose (not) to incorporate are high-types, highlighting a significant selection effect upon entry. Among incumbents, the selection effect becomes more pronounced where the share of high-types rises to 99% within incorporated firms. To further explore the importance of selection effect, I consider a counterfactual economy where the incorporation decision is randomized within firm types, while keeping the distribution of firm types upon entry constant. In this counterfactual economy, the difference in the average size of incorporated and unincorporated firms decreases by 32% compared to the baseline economy and the aggregate productivity growth decreases from 3% to 2.7%. Aggregate productivity growth declines mainly because the randomization of legal form decisions deteriorates the equilibrium composition of firm types.

Finally, I use the model to conduct two experiments to assess the value of incorporation. First, I consider a case where the option of incorporation is not available to the firms. The absence of incorporation not only eliminates the positive treatment effect on firms expansion rates but also mitigates the selection of high-growth potential firms in the economy. Consequently, the growth rate decreases to 2.49% and welfare decreases by 4.6% (in consumption equivalent terms). On the other hand, subsidizing the incorporated firms provides significant welfare gains. This last result is largely driven by the change in the degree of selection, i.e. the change in the composition of firm types.

Related Literature   This paper is linked to a number of different literatures. Recently, the macroeconomic implications of firms legal forms have attracted some attention. For example, Dyrda et al. (2019) and Barro and Wheaton (2019) have investigated the recent trend of pass-through entities and C-corporations among the U.S. businesses. Specifically, Dyrda et al. (2019) focus on the trade-off entrepreneurs face between running the C-corporation versus pass-through entity in manufacturing and services sector, while Barro and Wheaton (2019) assess the effects of business taxation on choices of legal form and subsequently productivity in an empirical framework. Unlike their work, my pa-

per focuses on the presence of limited liability and how it affects firm dynamics and the macroeconomy.

To the best of my knowledge, the papers that come closest to mine are Herranz et al. (2015) and Short and Glover (2011). Herranz et al. (2015) point out that less risk averse entrepreneurs, because they operate larger more risky projects and therefore would gain the most from limited liability, are those who would most likely incorporate if they are given the option. Short and Glover (2011) propose a model where they study the bankruptcy and incorporation decisions of entrepreneurs in order to understand the types of risks faced by entrepreneurs. My paper is different from theirs in several aspects. First, their papers consider the choice of incorporation as an individual decision in the tradition of Quadrini (2000) and Cagetti and De Nardi (2006) where their main focus is entrepreneurs. By contrast, my paper mainly focuses on the firm behavior and proposes a theory that is able to capture the stylized facts of firm dynamics and growth in the economy. Second, while they consider firm productivity as exogenous, the productivity process takes the center stage for firm growth in my paper where firms choose the level of investment to improve their productivity. In other words, the productivity process is endogenous in my framework. Therefore, my paper allows the legal environment to affect this investment decision and hence aggregate productivity growth, which is absent in their framework.

One distinct feature of my model is that it explicitly allows for heterogeneity in firms growth potential, which is essential in capturing the observed pattern of firm dynamics in data and allowing for selection effect. There is ample empirical evidence for the importance of such heterogeneity. Schoar (2010) and Decker et al. (2014) argue that some entrepreneurs are "transformative" and have the necessary skills to expand, while subsistence entrepreneurs may simply never grow independently of the environment they operate in. Hurst and Pugsley (2012) provide evidence that many firms in the U.S. intentionally choose to remain small. On the theoretical side, Luttmer (2011), Lentz and

Mortensen (2016) and Jones and Kim (2018) argue that models without heterogeneity in growth potential are unable to explain the very rapid growth of a subset of firms. Gabaix et al. (2016) argue that theories which build on a random growth mechanism generate transition dynamics that are too slow and allowing the presence of "high-growth types" can explain the observed fast rise in income inequality. Acemoglu et al. (2018) emphasize the importance of heterogeneity in innovative capacity for designing optimal R&D policies.

The rest of the paper is organized as follows: In Section 1.2, I describe the theoretical model. Section 1.3 summarizes the data that I use in the quantitative analysis and discusses the identification of the model. In Section 1.4, I present the calibration results, and assess the model fit based on various out-of-sample moments. In Section 1.5, I provide the main analysis to quantify the importance of treatment and selection effects on firm dynamics and the aggregate economy. Section 1.6 concludes. All proofs and additional details are contained in the Appendix A.

1.2 Model

1.2.1 Preferences, Technology, and Static Allocations

The economy is in continuous time and admits a representative household with per-period log utility function

$$U_0 = \int e^{-\rho t} \ln C(t) dt \tag{1.1}$$

where $C(t)$ is consumption at time t and $\rho > 0$ is the discount rate. The household is populated by a continuum of individuals with measure one. Each member is endowed with one unit of labor that is supplied inelastically and earns the wage rate $w(t)$ determined endogenously. Households own all the firms in the economy and are subject the

following budget constraint:

$$\dot{A}(t) + C(t) = r(t)A(t) + w(t) \tag{1.2}$$

where $A(t)$ is the asset position of the representative household (equal to the value of firm assets in equilibrium) and $r(t)$ is the equilibrium interest rate on assets.

The individuals consume a unique final good $Y(t)$, which is also used for other purposes as will be discussed below. The final good is produced competitively by labor $L(t)$ and a continuum of intermediate goods over the set $\mathcal{N}(t)$, with measure Φ_t, following the production technology given below

$$Y(t) = \frac{L(t)^{\beta}}{1-\beta} \int_{\mathcal{N}(t)} q_j(t)^{\beta} y_j(t)^{1-\beta} dj \tag{1.3}$$

where $q_j(t)$ and $y_j(t)$ are the quality and quantity of intermediate good j, respectively. The measure of intermediate goods produced in the economy is determined endogenously through entry and exit decisions. The price of the final good is normalized to be one in every period without loss of generality. In what follows, I will drop the time subscript t whenever it does not cause any confusion.

Each intermediate good $j \in \mathcal{N}$ is produced by a single firm which monopolistically competes against other firms active in the economy. Therefore index j also refers to the firm that produces intermediate good j. These firms have access to a linear technology of the form

$$y_j = \bar{q} l_j \tag{1.4}$$

where l_j is the amount of labor that firm j hires for the production, and $\bar{q} \equiv \frac{\int_{\mathcal{N}} q_j dj}{\Phi}$ is the average quality in the economy. In addition to the labor cost, production requires also a fixed cost of operation $\psi \bar{q}$ at every period in terms of the final good. As will be discussed later, this fixed cost is allowed to be different for different legal forms chosen by the firm.

The maximization problem of the final goods producer generates the inverse demand $p_j = L^\beta q_j^\beta y_j^{-\beta}$. Given the production technology, each firm is faced with a constant marginal cost of production given by $w/\bar{q}$, where w is the wage rate in the economy. Therefore, for a given level of quality q_j, we can write firm j's static profit maximization problem as

$$\pi(q_j) = \max_{y_j \geq 0} \left\{ L^\beta q_j^\beta y_j^{1-\beta} - \frac{w}{\bar{q}} y_j \right\}.$$

where $\pi(q_j)$ is the per-period profit of firm j (before paying the fixed cost of operation) with quality q_j. The price and output level of firm follow from this maximization as

$$p_j = \frac{1}{(1-\beta)} \frac{w}{\bar{q}} \quad \text{and} \quad y_j = \left[(1-\beta) \frac{\bar{q}}{w} \right]^{\frac{1}{\beta}} L q_j, \tag{1.5}$$

implying that the price is a constant markup over the marginal cost, and firm's optimal output is proportional to quality. As shown in Section A.1 in Appendix, the maximization in the final goods sector, together with (1.5), implies that the wage rate is proportional to average quality in the economy. Given the production technology in (1.4), optimal output choice of the firm implies that labor hiring of the firm is proportional to the quality of the firm *relative to* average quality in the economy, $q/\bar{q}$. Therefore relative quality can be considered as a summary statistics for firm size.

Finally, the resulting equilibrium profits can then be written as

$$\pi(q_j) = \Pi q_j, \tag{1.6}$$

where $\Pi = \beta \left[(1-\beta) \right]^{\frac{1-\beta}{\beta}} \left(\frac{\bar{q}}{w} \right)^{\frac{1-\beta}{\beta}}$, i.e., profits are increasing in quality q_j. Therefore firms have profit incentives to improve their product quality, which is the source of firm growth and will be discussed in the next subsection.

1.2.2 Evolution of Firm Quality

Quality at the firm level evolves over time depending on the firm's investments in improving its quality. This process is modeled as a controlled stochastic process as in Akcigit and Kerr (2018) and Atkeson and Burstein (2010). In particular, I assume that by investing R in terms of final good, an incumbent firm with quality q improves its quality at the Poisson flow rate x such that

$$x = \theta \left(\frac{R}{q} \right)^{\eta} \tag{1.7}$$

where $\eta \in (0,1)$ and θ is the *efficiency* of the investment technology. This particular investment technology assumes that the cost required to increase the quality scales with the size of the firm. This implies that, consistent with Gibrat's law, the growth rate of sufficiently large firms (large quality) is independent of their size.

When the investment is successful, the current quality of the firm improves from q to $q + J(q, \bar{q})$ where

$$J(q, \bar{q}) = \lambda \left[\omega \bar{q} + (1 - \omega)q \right], \quad \omega \in [0,1]. \tag{1.8}$$

That is, improvement in the quality is a convex combination of current quality of the firm q and the average quality in the economy $\bar{q}$. This formulation is a generalization of Acemoglu et al. (2018), where quality improvements depend only on average quality in the economy, and Akcigit and Kerr (2018), where quality improvements are proportional to current quality of the firm.[1]

Firm Heterogeneity Firms are heterogeneous in how *efficient* they are at improving their quality. It is this heterogeneity across firms that gives rise to the possibility of selection: entrepreneurs with higher growth potential (i.e. more efficient in improving productivity)

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[1]Having average quality in (1.8) introduces spillovers between firms: each firm's improvement in its quality adds to the average quality, which in turn provides bigger quality improvement for all the firms in the economy. Therefore the parameter ω controls the extend of this spillover.

are more likely to choose incorporation as it is more valuable to large firms. Formally, I assume that firms differ in their efficiency of the investment technology θ and can be either low-type (θ_L) or high-type (θ_H). A firm's type is persistent and determined upon entry. New entrant draws its type from a Bernoulli distribution

$$\theta = \begin{cases} \theta_L & \text{with probability } \alpha \\ \theta_H & \text{with probability } 1 - \alpha \end{cases}. \tag{1.9}$$

where $\alpha \in (0,1)$ and $\theta_H > \theta_L > 0$. As will be discussed later in detail, allowing this heterogeneity is not only important in quantifying the scope of firm selection into different legal forms, but is also quantitatively relevant in accounting for the firm growth and exit heterogeneity within legal forms.[2]

1.2.3 Entry and Exit

A unit mass of potential entrants attempts to enter the economy at any point in time. They use a similar investment technology as the incumbent firms, where the flow rate of entry x_e is related to the spending on entry efforts R_e according to $x_e = \theta_e \left(\frac{R_e}{\bar{q}} \right)^{\eta}$. Following a successful entry, the entrant first draws its initial quality from a distribution $\Psi(q)$ and its type, $\theta \in \{\theta_L, \theta_H\}$, then decides whether to incorporate its firm or not. This description implies the following optimization problem for entrants:

$$\max_{x_e} \left\{ x_e \mathbb{E} \left(v(q,\theta) \right) - c_e(x_e, \theta_E) \right\} \tag{1.10}$$

where $\mathbb{E} \left(v(q,\theta) \right)$ is the expected value of entry (and the expectation is over the quality the successful entrants will obtain and firm type θ) and $c_e(x_e, \theta_E)$ is the cost of entry implied by the investment technology. Given that there is a unit measure of potential entrants, x_e is also equal to the total entry flow rate.

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[2]For the relevance of this type of heterogeneity, see Acemoglu et al. (2018) in the context of optimal industrial policies and Jones and Kim (2018) in the context of top income inequality.

A firm's exit happens either due to (i) an exogenous death shock at Poisson rate $\kappa > 0$ or (ii) firms' choosing to exit *endogenously*: firms will voluntarily shut down when their quality is low enough such that they are no longer sufficiently profitable relative to the fixed cost of operation. When firms exit, they stop producing and their flow profits drop to zero. Importantly, I assume that firms are subject to an *exit cost* that is proportional to the quality of the firm at the time of exit, $c_E \times q$ where c_E is a parameter. This cost can be considered as a liquidation or firing cost as in Hopenhayn and Rogerson (1993a) and Poschke (2009). Recall that firm's optimal output and the amount of labor are proportional to its quality as in equation (1.5), motivating the exit cost assumption being proportional to the quality. Importantly, the presence of exit cost drives the value of the firm to the owner below zero in the case of exit. As will be discussed in the next subsection, the presence of this exit cost creates the main motivation for a firm owner to incorporate her business.

1.2.4 Legal Form Choice

New entrants choose their legal form (incorporate or not) after they learn their initial quality q and their type $\theta \in \{\theta_L, \theta_H\}$. Incumbent firms have the option to switch between legal forms at arrival rate μ. Incorporation entails a sunk setup cost in terms of final good $C_I \bar{q}$.[3] In this setting, a firm with quality q and type θ chooses to incorporate if and only if

$$V_I(q;\theta) - C_I \bar{q} > V_U(q;\theta) \tag{1.11}$$

where V_I and V_U denote the value of an incorporated and unincorporated firm, respectively. Incorporation provides liability protection which ensures that firm owner does not suffer any losses beyond the value of the firm.[4] In other words, in the case of exit, incorpo-

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[3]Like fixed cost of operation, setup cost of incorporation is assumed to grow with the average quality in the economy to ensure stationarity.

[4]Initial costs of starting a firm such as the cost of entry given by (1.10) and the setup cost of incorporation C_I are considered as sunk.

rated firms do not suffer loses due to liquidation or firing costs that derive the firm value below zero. This benefit comes at the expense of set-up and maintaining cost. In short, incorporation trade-offs exit cost, which is proportional to the firm size, with higher fixed cost of operation and setup cost and it provides insurance to the firm owner by bounding the firm value from below. This trade-off and its implications on firm behavior will be more clear in the next subsection.

1.2.5 Firm Decision and Value Functions

I normalize all the growing variables by average quality in the economy, $\bar{q}(t)$, to keep the stationary equilibrium values constant and denote the relative quality $q/\bar{q}$ as $\hat{q}$. Moreover, let g denote the growth rate of average quality, which is also the aggregate growth rate in the economy endogenously determined in equilibrium. Then the stationary equilibrium value function for incorporated firms with relative quality $\hat{q}$ and type θ can be written as

$$
\rho v_I(\hat{q};\theta) = \max \left\{ 0, \max_{x_I \geq 0} \left[\begin{array}{c} \Pi\hat{q} - c(x_I, \hat{q}, \theta) - \psi_I \\ -g\hat{q}\frac{\partial v_I(\hat{q};\theta)}{\partial \hat{q}} \\ +x_I \left[v_I\left(\hat{q}_+;\theta\right) - v_I(\hat{q};\theta) \right] \\ +\kappa \left[0 - v_I(\hat{q};\theta) \right] \\ +\mu \left[\max\{v_I(\hat{q};\theta), v_U(\hat{q};\theta)\} - v_I(\hat{q};\theta) \right] \end{array} \right] \right\} \tag{1.12}
$$

where $v_I(\hat{q};\theta)$ is the value of the firm and $\hat{q}_+$ denotes the new level of relative quality after a successful investment.[5] Notice that the type of the firm affects the firm value through the investment cost function for quality improvements $c(x,\hat{q},\theta) = \hat{q}\left(\frac{x}{\theta}\right)^{\frac{1}{\eta}}$ implied by equation (1.7). This value function implicitly defines (i) a threshold level of relative quality at which firms choose to exit $\hat{q}_{min}$ and (ii) firms' optimal rate of expansion x_I

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[5]Full derivation of the value function is provided in Section **??** in Appendix.

which determines the rate of quality growth at the firm level.

The value function above can be interpreted as follows. Given discounting at the rate ρ, the left-hand side is the flow value of firm with relative productivity $\hat{q}$. The right-hand side includes the components that make up this flow value. The outer maximization problem determines the endogenous exit decision of the firm. Since owner of an incorporated firm is not liable losses of her business beyond the value of the firm, the value of choosing exit is zero. The first line includes the instantaneous profits, minus the cost of quality enhancing investment and the fixed costs of operation. The second line reflects the change in firm value due to the increase in the average quality in the economy, which happens at the rate g. This term accounts for the fact that as the average quality increases, the relative quality at which the firm operates declines, leading to the erosion of profits. The third line expresses the change in firm value when the firm is successful with its investment in improving quality at the rate x_I. The fourth line shows the change in value when the firm has to exit due to an exogenous death shock at the rate κ. The last line includes the change in firm value if firm decides to switch to being unincorporated.

The value function for unincorporated firms v_U is given by

$$
\rho v_U(\hat{q};\theta) = \max \left\{ -c_E \hat{q}, \max_{x_U \geq 0} \left[\begin{array}{c} \Pi \hat{q} - c(x_U, \hat{q}, \theta) - \psi_U \\ -g\hat{q}\frac{\partial v_U(\hat{q};\theta)}{\partial \hat{q}} \\ +x_U \left[v_U\left(\hat{q}_+;\theta\right) - v_U(\hat{q};\theta) \right] \\ +\kappa(-c_E \hat{q} - v_U(\hat{q};\theta)) \\ +\mu \left[\max\left\{ v_I(\hat{q};\theta) - C_I, v_U(\hat{q};\theta) \right\} - v_U(\hat{q};\theta) \right] \end{array} \right] \right\} . \quad (1.13)
$$

The interpretation of the value function is same as above. The main difference between incorporated and unincorporated firms is that the value of the unincorporated firm falls below zero in the case of exit due to presence of exit cost: the owner has full liability

and needs to pay the exit cost $c_E \hat{q}$. This happens either when the relative quality is too low to be profitable so that firm chooses exit endogenously or due to firm experiencing exogenous death shock at the rate κ.

The optimal expansion decision of a firm with legal form $l \in \{I, U\}$ and firm type $\theta \in \{\theta_L, \theta_H\}$ is given by

$$x_l(\hat{q}; \theta) = \theta^{\frac{1}{1-\eta}} \eta^{\frac{\eta}{1-\eta}} \left[\frac{v_l(\hat{q}_+; \theta) - v_l(\hat{q}; \theta)}{\hat{q}} \right]^{\frac{\eta}{1-\eta}}, \tag{1.14}$$

i.e. incentives to invest on quality depend on its marginal return of the investment $\frac{v_l(\hat{q}_+; \theta) - v_l(\hat{q}; \theta)}{\hat{q}}$ as well as how efficient the investment technology is, θ. To provide further intuition regarding firm decision, Figures 1 and 2 depict a visual comparison of the value functions for different legal forms and firm types.

Panel A of Figure 1 shows the value of an incorporated and unincorporated firm by its size (quality) for a given firm type. First notice that firm value is constant below a certain quality threshold which determines the region of qualities where firms choose to exit. Since incorporated firm owners are protected by limited liability, their exit value is higher. This higher exit value, combined with a higher fixed cost of operation, typically results in a higher exit quality threshold for incorporated firms ($\hat{q}_{I,min} > \hat{q}_{U,min}$). Moreover, the value function of incorporated firms is steeper than that of unincorporated firms when firm size is above a certain level. This means that marginal returns from improving quality under incorporation is higher. As a result, incorporated firms choose a higher expansion rate conditional on firm type, given by (1.14).

Panel B of Figure 1 makes the same comparison, but this time between a high-type and a low-type firm, given a legal form. The value function of high-type firms is steeper and the quality threshold at which they exit is lower. The former implies that they choose a higher expansion rate.[6] The latter indicates that high-types exit less often conditional on

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[6]In addition to the higher marginal returns from expansion, high-types also directly benefit from having a more efficient investment technology. That is, keeping the marginal returns constant, a higher θ implies a

firm size. Therefore the presence of type heterogeneity generates exit rate heterogeneity conditional on firm size. This implication of the model is crucial in identifying the type heterogeneity, which will be discussed in Section 1.3.2.

Panel A: Incorporated v.s. Unincorporated   Panel B: High- v.s. Low- type
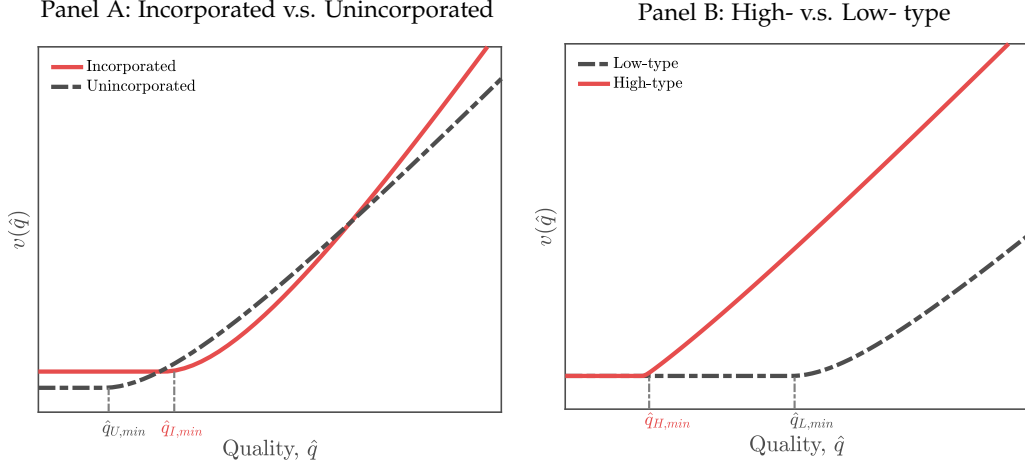
Notes: Panel A depicts the value of incorporated and unincorporated firms by size *condition on* firm type. Panel B shows the value of high- and low-type firms *condition on* legal form. In both figures, $\hat{q}_{min}$ refers to the relative quality threshold at which firms choose to exit endogenously.

Figure 1: Firm Value

So far, I have discussed the expansion and exit decisions of firms based on Figure 1. Figure 2 focuses on legal form decision by showing how value differences between incorporated and unincorporated firms change by firm size for both high- and low- types. Although it shows some non-monotonicity for low-types, this difference typically increases in firm size. This implies that for a type θ firm, there exists a relative quality level $\hat{q}_\theta$ such that firms above this level of quality choose to incorporate:

$$v_I(\hat{q};\theta) - C_I \geq v_U(\hat{q},\theta), \;\; for \; \hat{q} \geq \hat{q}_\theta. \tag{1.15}$$

Importantly, this threshold size is smaller for high-types $(\hat{q}_{\theta_H} < \hat{q}_{\theta_L})$ since the value difference between incorporated and unincorporated firms $(v_I(\hat{q};\theta) - v_U(\hat{q},\theta))$ is higher for high-types at any quality level. These results indicate that the likelihood of a firm

more efficient (cheaper) investment technology and implies a higher optimal expansion rate.

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being incorporated is increasing in firm size and higher for high-type firms conditional on firm size, highlighting the selection effects due to firm size and firm type heterogeneity.
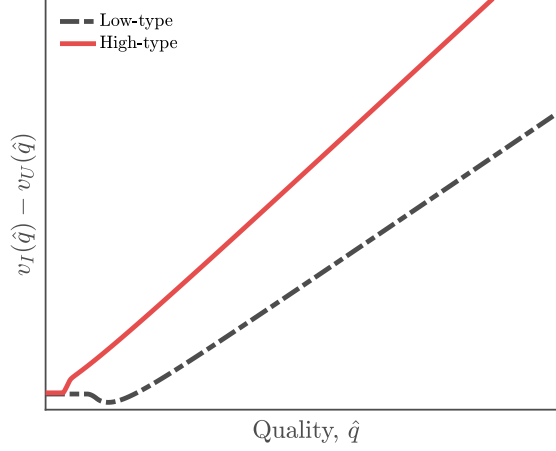
Notes: This figure the differences between value of incorporated and unincorporated firms by size for high- and low-type firms.

Figure 2: Choice of Legal Form

1.2.6 Firm Size Distribution and Aggregate Growth

As the quality improvements are stochastic in nature, firms (within each legal form and firm type category) are heterogeneous in terms of quality. Along balance growth path, the stationary distribution of relative qualities, which determines firm size, emerges as the result of the expansion and exit decisions of all firms, and characterizes the long-run state of the economy. For a given firm type θ, the distribution of relative qualities for incorporated firms in stationary equilibrium satisfies [7]

$$
g\hat{q}f'_{I,\theta}(\hat{q}) = (x_{I,\theta}(\hat{q}) + \kappa - g)f_{I,\theta}(\hat{q}) - x_{I,\theta}(\hat{q}_-)f_{I,\theta}(\hat{q}_-)\frac{1}{1 + \lambda(1 - \omega)}
$$
$$
- x_e\Psi(\hat{q}) - \mathbb{1}\left(\hat{q} \in \mathbb{Q}_{U\to I}\right)\mu f_{U,\theta}(\hat{q}) \tag{1.16}
$$

---

[7]Details of the derivations are provided in Section A.2 in Appendix.

where $x_{I,\theta}(\hat{q})$ is the expansion rate, $f_{I,\theta}(\cdot)$ is the unnormalized density function with boundary conditions $f_{I,\theta}(\hat{q}) = 0$ for $\hat{q} < \hat{q}_{min}^{I,\theta}$ where $\hat{q}_{min}^{I,\theta}$ is the exit quality threshold solved from (1.12) and $\lim_{\hat{q}\to\infty} f_{I,\theta}(\hat{q}) = 0$. $Q_{U\to I}$ denotes the quality region at which $v_I(\hat{q};\theta) - C_I \geq v_U(\hat{q},\theta)$ is satisfied, i.e. an unincorporated firm switches to incorporation. The distribution of qualities for unincorporated firms is analogous to above expression.

Given the distribution of firm qualities $f_{l,\theta}(.)$, the growth rate of average quality in the economy g is given by

$$g = \frac{\sum_{l\in\{I,U\}}\sum_{\theta\in\{\theta_L,\theta_H\}}\left[\int J(\hat{q},1)x_{l,\theta}(q)f_{l,\theta}(\hat{q})d\hat{q}\right] - \kappa + \frac{x_e}{\Phi}\mathbb{E}(\hat{q}_{entry})}{1 + \sum_{l\in\{I,U\}}\sum_{\theta\in\{\theta_L,\theta_H\}} f_{l,\theta}(\hat{q}_{min}^{l,\theta})\left(\hat{q}_{min}^{l,\theta}\right)^2} \tag{1.17}$$

where $J(\hat{q},1)$ is the amount of quality improvement defined in (1.8). The intuition for the growth rate in is as follows. The numerator has the contribution of entrants and different types and legal forms of incumbent firms to the quality distribution. This contribution happens at the rate $x_{l,\theta}(\hat{q})$, which underlines the connection between firm-level quality improvements and aggregate growth. The denominator on the other hand adjusts for the improvements in quality distribution due to the firms exiting the economy endogenously.

1.2.7 Dynamic Equilibrium

Given the above description of the environment, I can now formally define the full dynamic equilibrium for this economy.

Definition 1 Consider the environment described above. A stationary equilibrium of this economy is a tuple

$$\{y_j, p_j, l_j, v_I(\hat{q};\theta), v_U(\hat{q};\theta), x_I(\hat{q};\theta), x_U(\hat{q};\theta), x_e, f_{I,\theta}(\hat{q}), f_{U,\theta}(\hat{q}), g\}$$

such that (i) representative household maximize utility; (ii) y_j and p_j maximize profits as in (1.5) and the labor demand l_j satisfies (1.4); (iii) $v_I(\hat{q}; \theta)$ and $v_U(\hat{q}; \theta)$ are given by the incorporated and unincorporated firm value functions in (1.12) and (1.13); (iv) $x_I(\hat{q}; \theta)$ and $x_I(\hat{q}; \theta)$ are given by the optimal expansion rate decision in (1.14) and x_e solves the entrants problem in (1.10); (v) the stationary equilibrium relative quality distributions $f_{I,\theta}(\hat{q})$ and $f_{U,\theta}(\hat{q})$ satisfy (1.16); (vi) the growth rate of average quality g is given by (1.17); (vii) labor market clears as in (A.4).

1.3 Data and Calibration Strategy

1.3.1 Data

The quantitative analysis of the model uses both firm-level and individual-level data for the years between 1999 and 2014. To measure the properties of the firm dynamics process, I rely on micro data for the population of non-farm and non-financial businesses from the Danish Business Statistics Register. The variables used in each year include the two-digit industry identifier, employment level, firm age, and legal form of the business. As the focus of the paper is on how limited liability affects the incorporation decision and firm dynamics, I restrict the sample to firms with a single owner. This allows me to mitigate the importance of other incorporation benefits, such as issuing equity. To identify the owners of the businesses, I use the Danish Entrepreneurship Database which provides information on the primary founder of all privately owned firms in Denmark. I further restrict the sample to those firms that are active. I define active firms as firms with minimum employment of one full-time equivalent in addition to the founder. Following Gjerlv-Juel and Dahl (2012), I consider the firm exited after two successive years without activity.

The central moments in the calibration are firm entry rate and employment share of

entrants in the economy, employment level by age, exit rate by age and size, the share of incorporated firms by age, transitions between the legal forms over time, and aggregate productivity growth. The moments related to entry/exit rates and legal form transitions are in per annum terms.

1.3.2 Calibration

I fix three of the parameters exogenously and calibrate the remaining parameters by minimizing the distance between several empirical moments and their model counterparts. Discount rate ρ, is set to 0.02, which roughly corresponds to an annual discount factor of 97%. The share of quality in final good β determines the price markup for firms through equation (1.5). Therefore I choose $\beta = 0.33$ to get a markup of 1.5 reported in De Loecker and Eeckhout (2018). The curvature of expansion production function η is set to 0.5, implying a quadratic expansion cost function, following Akcigit and Kerr (2018) and Acemoglu et al. (2018).

The remaining parameters, which are listed in Table 1, are calibrated by minimizing the distance between several empirical moments and their model counterparts. In particular, let Ω denote the set of parameters to be calibrated, M^E denote the vector of S empirical moments and $M(\Omega)$ denote the vector of model-simulated moments. I then chose Ω to minimize the absolute relative deviation between the model and data:

$$\min_{\Omega} \sum_{m=1}^{S} \frac{|M_m^E - M_m(\Omega)|}{|M_m^E|}.$$

Even though the parameters are calibrated jointly, below I provide a heuristic description of the relationship between the parameters and the specific moments that are informative.

The expansion efficiencies for low-type and high-type firms and exit costs are mostly identified from the firms' employment growth and the growth differences between incorporated and unincorporated firms. Therefore, I use average employment growth from

age 0 to age 20 for incorporated and unincorporated firms to discipline these parameters. I assume that entrants draw their initial quality from an exponential distribution, the rate parameter of which is identified from the employment share of entrants. On the other hand, the entry efficiency parameter θ_E is mainly determined by the aggregate entry rate.

Since the fixed operation cost affects the quality threshold at which firms choose to exit endogenously, I use firm exit rates for incorporated and unincorporated firms to inform this parameter. The setup cost of incorporation is mainly identified by the share of entrepreneurs that choose to incorporate their business upon entry. Exit rates for large firms inform the exogenous death shock κ as it is the only cause for large firms to exit. As shown in Section A.2 in Appendix, the model endogenously generates a Pareto-tailed distribution of firm size and the shape parameter of the distribution depends on ω which controls the extent of the spillovers in firm-level quality improvements. Therefore I target Pareto shape parameter implied by the empirical firm size distribution to pin down ω.[8] Aggregate growth rate is informative about the step size of quality improvements, λ.

To identify the share of low-type firms upon entry α, which determines the distribution of firm types among entrants, I focus on the age-profile of exit rates conditional on firm size. The model implies that without firm type heterogeneity in growth potential, the likelihood of exit would be independent of age *conditional on size* within a legal form. In data, however, such conditional exit rates are decreasing in firm age for both incorporated and unincorporated firms. Through the lens of the model, this pattern is rationalized by the interaction between firm heterogeneity and endogenous selection process: the share of high-type firms, which have lower exit rates conditional on size than low-type firms, increases within a given cohort as the cohort ages. This is shown in Figure 3 for incorporated firms, where I display the exit rate of small firms by age for different values of the share of low-types upon entry, α. Without any type heterogeneity, i.e. $\alpha = 0$, the conditional exit rates by age would be flat. Moreover, the lower the value of α, the less

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[8]I use firms with more than 50 employees for the tail parameter estimation.

steep the decline in exit rate by age since the scope of selection is lower. Therefore, to inform α, I use the exit rate by age profile of small firms, the firms with less than or equal to 2 employees.
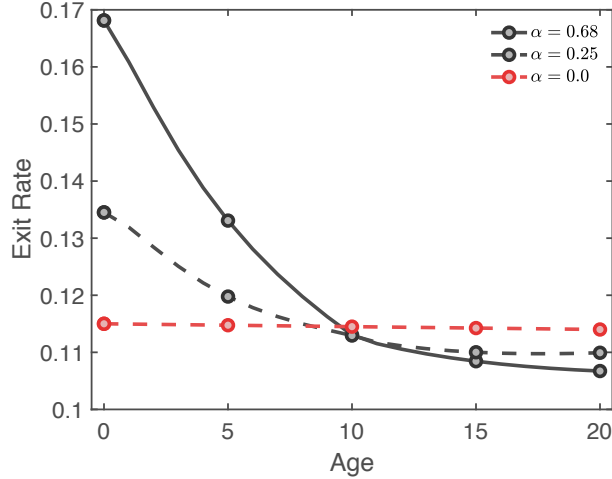
Notes: This figure shows the exit rate of small incorporated firms (firms from bottom 1% of the firm size distribution) by age for different values of share of low-type firms among entrants, α, while keeping the rest of the parameters constant.

Figure 3: Exit rate by age, conditional on size (Incorporated Firms)

1.4 Calibration Results

In this section I present the calibration results. Section 1.4.1 contains the structural parameters and targeted moments. In Section 1.4.2, I show that the calibrated model is also consistent with a variety of non-targeted moments.

1.4.1 Calibrated Parameters and Targeted Moments

Tables 1 and 2 contain the jointly calibrated parameters and the targeted moments, respectively. The estimates of the fixed cost of operation indicate that maintaining incorporated firms costs five times more than maintaining unincorporated firms. My estimates also show that high-type firms are about 2.5 times as efficient as the low-type firms in terms of improving their quality ($\theta_H/\theta_L \approx 2.5$) and entrants have a 68% chance of being

a low-type firm ($\alpha = 0.678$). The parameter ω, which controls the weight of average quality in quality improvements, is estimated as 0.349, implying significant spillover effects between firms. The rate at which incumbent firms switch their legal form μ is estimated at 2.2%, reflecting the fact that legal form transitions among incumbent firms are rare.

Parameter		Value
Fixed cost (Incorporated)	ψ_I	0.250
Fixed cost (Unincorporated)	ψ_U	0.043
Exogenous death rate	κ	0.036
Exit cost	c_E	2.061
Step size	λ	0.156
Share of low types upon entry	α	0.678
Expansion efficiency (high type)	θ_H	0.932
Expansion efficiency (low type)	θ_L	0.392
Entry efficiency	θ_E	0.388
Incorporation setup cost	C_I	4.014
Entry dist. (rate)	χ	11.24
Legal form switching rate	μ	0.022
Share of average quality in step size	ω	0.349

Table 1: Parameter Estimates

Table 2 reports the targeted empirical moments and the predicted values from the model. It shows a good fit between model-implied moments and data. Overall, the model is able to replicate important characteristics of the data and the observed differences between incorporated and unincorporated firms. In particular, the model matches the better performance of incorporated firms in terms of employment growth: while incorporated firms grow by a factor of 4, compared to their entry size, by the time they are 20 years old; unincorporated firms reach only around 1.5 of their entry size. Moreover, the calibrated model also matches exit rate heterogeneity between incorporated and unincorporated firms in terms of both levels and changes by age: unincorporated firms have higher exit rates and the exit rates of small unincorporated firms show a steeper decline by age.

	Model	Data
Aggregate productivity growth	0.030	0.029
Entry rate	0.078	0.085
Employment share of entrants	0.031	0.022
Employment at age 20 (Incorporated)	4.14	4.28
Employment at age 20 (Unincorporated)	1.51	1.54
Share of incorporated firms at age 0	0.21	0.23
Share of incorporated firms at age 10	0.39	0.38
Exit rate of small firms (Incorporated)	0.08	0.06
Exit rate of small firms (Unincorporated)	0.17	0.12
Exit ratio of small firms, age 0 to 20 (I)	1.42	1.53
Exit ratio of small firms, age 0 to 20 (U)	2.46	2.67
Exit rate of large firms	0.036	0.036
Tail of firm size dist.	2.04	2.04
Share of incumbents switching legal form	0.046	0.043

Notes: Table reports both the data moments and the corresponding moments in the model. "Employment at age 20" moment refers to the average employment at age 20 relative to the entry employment level. I define small firms as firms with 1-2 employees in the data (including the firm owner).

Table 2: Targeted Moments

1.4.2 Non-targeted Moments

In this section, I assess the performance of the calibrated model in how well it matches a variety of non-targeted moments. This strategy thus provides an out-of-sample test of the structure imposed by the model. Table 3 and Figure 4 summarize the results, which suggest that the model performs fairly well. In particular, the model is able to capture the average firm size differences between incorporated and unincorporated firms, the direction of legal form transition among incumbents, and the heterogeneity in firm size by age for both incorporated and unincorporated firms. Moreover, Figure 4 shows that the calibrated model performs well in replicating the share of incorporated firms in the overall economy as well as by firm size. This last result is especially reassuring as it suggests that the quantified model is able to capture the value of incorporation by firm size, which is reflected in the choice of incorporation by firms at different sizes.

	Model	Data
Average firm size (I/U)	3.97	4.92
Share of switchers from U to I (cond. on switching)	0.93	0.99
Standard dev. of log employment age 10 (I)	1.49	1.32
Standard dev. of log employment age 10 (U)	1.12	0.95

Table 3: Non-targeted Moments
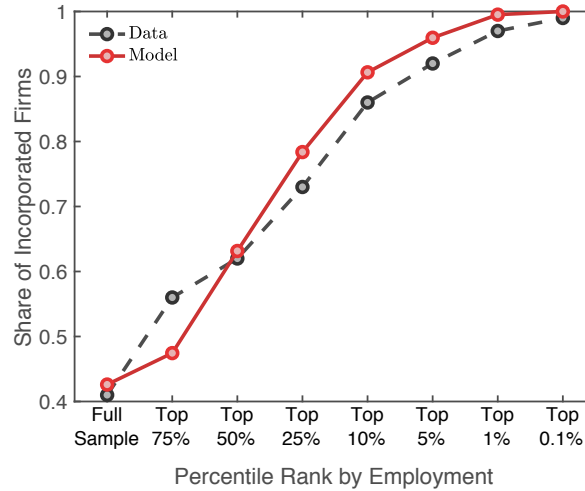
Notes: This figure shows the share of incorporated firms in the top x% of the firm-size distribution for x = 0.1%, 1%, 5%, ... I report the data using a black dashed line and the model using a red solid line.

Figure 4: Share of Incorporation by Firm Size

1.5 Quantitative Results

In this section, I study the equilibrium properties of the calibrated model and its implications. I start by focusing on how the availability of incorporation choice affects firm incentives, equilibrium firm heterogeneity and the selection pattern in the economy. Then, to study the importance of the selection effect, I consider a counterfactual economy where the incorporation decision is randomized within firm types. Finally, I use the model to conduct two policy experiments to assess the value of incorporation.

1.5.1 Equilibrium Allocation: Firm Growth and Selection

Table 4 presents the key equilibrium objects for each legal form (incorporated or unincorporated) and firm type (low-type or high-type) category and it summarizes the heterogeneity in firms' growth incentives and in the composition of firm types within legal forms.

The first row reports the average expansion rates[9], the rate at which firms choose to improve their quality (see equation (1.14)). The expansion rate is a good summary statistics for firm growth as firms grow through quality improvements in the model. First, note that the ex-ante firm heterogeneity generates substantial firm growth rate differentials: conditional on legal form, the average expansion rate of high-type firms is around 7 times as high as that of low-type firms. Second, the choice of legal form also has direct effect on expansion rates and therefore firm growth: for both low- and high- types, incorporation increases average expansion rates by 50% (from 0.03 to 0.047 for low-types, from 0.20 to 0.31 for high-types), which can be considered as the *treatment effect* of incorporation on firm growth.[10] This treatment effect arises because, in the model, incorporation protects firm owners from downside risks, which incentivizes them to invest more in improving their product quality, subsequently grow large.

	Unincorporated		Incorporated	
	Low-type	High-type	Low-type	High-type
Average expansion rates	0.03	0.20	0.047	0.31
Shares among entrants	0.66	0.13	0.02	0.19
Shares among incumbents	0.36	0.22	0.005	0.42

Notes: The table contains various equilibrium objects for each legal form (incorporated or unincorporated) and firm type (low-type or high-type). The second and third rows refer to the share of each category such that they sum up to one. The model is parametrized according to Table 1.

Table 4: Firm growth and selection

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[9]Average expansion rate is calculated based on the firm size distribution within a legal form-firm type category.

[10]Note that this reflects the effect of incorporation on firm growth in partial equilibrium.

The second and third row of Table 4 shows the distribution of firm types by legal form among entrants and incumbents such that the values in each row sum up to one. The former provides a measure for the selection of types into different legal forms upon entry, whereas the latter emphasizes the selection through competition, growth and the exit behavior of incumbent firms. These results reflect two important features of the entrants and incumbents.

First, consider the selection among entrants. Note that the *unconditional* probability of choosing incorporation upon entry is 21% (= 2% + 19%), however the probabilities *conditional* on firm type are drastically different: conditional on being a high-type, the probability of choosing incorporation upon entry is 59% $\left(= \frac{0.19}{0.13+0.19}\right)$. This result implies that high-type entrants disproportionately choose to become incorporated as high-types benefit more from incorporation compared to the low-types. This is because incorporation is valuable especially for large firms and high-type firms expect to grow larger than low-type firms. This pattern of firm types selecting into legal forms results in a significant firm type heterogeneity across legal forms: while the share of high-types is 90% among the incorporated entrants, this share among unincorporated entrants is only around 15%.

Second, the selection process becomes more pronounced among incumbents as is shown in the third row of Table 4. Note that the share of firms in a given legal form and firm type category would remain the same between entrants (row 2 in Table 4) and incumbents (row 3 in Table 4) if the average exit rates were uniform across these categories. In other words, the selection is driven by the resulting heterogeneity in the exit rates: the higher the exit rate differences, the stronger the selection effect. The results show that, within both incorporated and unincorporated incumbents, the share of high-types increases relative to entrants, showing a significant positive selection of high-types across the board and resulting in a 64% of high-type firm share in the economy. Importantly, the share of high-type incorporated firms shows the most significant increase compared to their entry share: the share of incorporated firms among incumbents reaches 43%, al-

most all of which are high-types.[11] Figure 5 also depicts the extent of selection among incumbents for a given cohort.
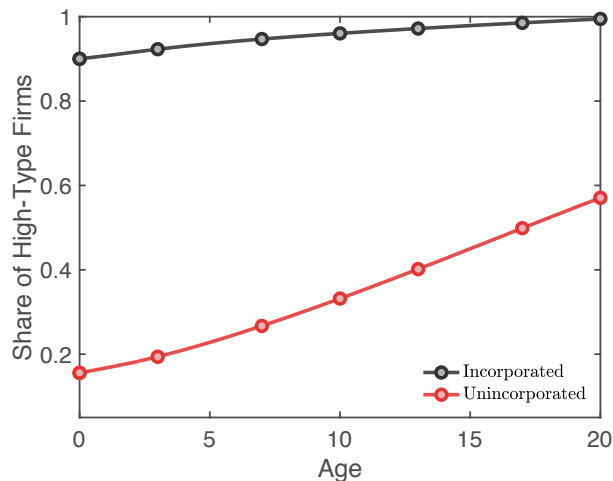
Notes: The figure shows the share of high-type firms for a given cohort by ages and legal form for the baseline economy.

Figure 5: Share of High-Types by Age

1.5.2 Counterfactual Exercise

To study the importance of the selection effect, I consider a counterfactual economy where the incorporation decision upon entry is randomized such that (i) the probabilities of choosing incorporation among low- and high- type entrants are same and equal to the unconditional probability of incorporation in the baseline economy and (ii) the distribution of firm types upon entry are kept same as the baseline economy. This exercise effectively shuts down the selection effect among entrants.[12] Figure 6 illustrates the resulting effect of this counterfactual on selection pattern and firm growth. In Panel A, I depict the share of high-type firms of a given cohort by age. In both baseline and counterfactual economy, initial entrants have the same type heterogeneity by design. However as the cohort gets older, the share of high-types grows slower under the counterfactual

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[11]Recall that share of incorporated firms among incumbents was not targeted in the calibration. Despite that, the model matches this moment very well as shown in Figure 4 (first data point from the left).

[12]Note that selection process among incumbents is still in place due to the heterogeneity in growth rates and exit decisions.

economy, implying a weaker selection process. Overall, the share of high-types among incumbents decline from the baseline value of 64% to 57%. This is because, due to the randomization of legal form choice, a lower share of high-type firms benefits from incorporation (21% as opposed to 59% in the baseline). Therefore, on average, high-type firms grow slower and exit more often, compared to the baseline economy.

Panel B shows the effect of randomizing the legal form choice on employment growth. As seen from the figure, the average employment by age differences between legal forms decrease significantly: for 20-year-old firms, the average employment difference between incorporated and unincorporated firms decreases by 44%. This pattern of the resulting change in employment growth leads to a decrease in the average size differences between legal forms by 32%. Overall, the aggregate productivity growth decreases from 3% to 2.7%.
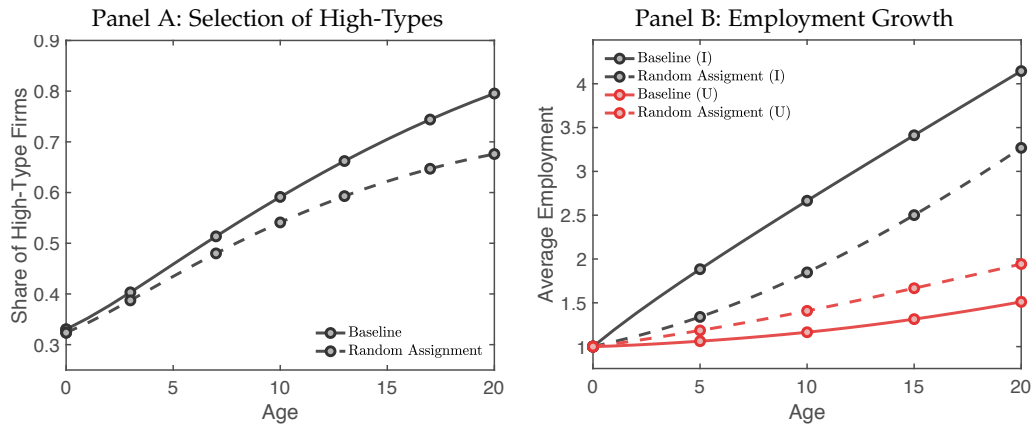
Notes: Panel A depicts the share of high-type firms by age. Panel B shows the average employment by age for incorporated and unincorporated firms. I show the baseline model results using solid lines and the counterfactual model results using dashed lines.

Figure 6: Counterfactual: Random Assignment of Legal Status

1.5.3 Policy Experiments

I use the model to conduct two experiments to assess the value of incorporation. First, I consider a case where the incorporation option is eliminated, i.e. all firms are un-

incorporated. The effects of this experiment are summarized in Table 5. The absence of incorporation choice hurts the high-type firms the most as they are the ones that benefit most from the positive treatment effect of incorporation. The average expansion rate of high-type firms decreases significantly compared to the baseline economy. Low-types' expansion rate increases slightly, mainly because they experience less competitive pressure from high-types firms, incentivizing them to invest more in their quality improvements. High-type firms' lower incentive to grow in turn creates a weaker selection process in the economy. The share of high-type firms among incumbent is now lower (53% as oppose to 64% in the baseline economy).[13] The combination of lower expansion rates and weaker selection leads to a decline in aggregate productivity growth to 2.5% and welfare decreases by 4.6% (in consumption equivalent terms).

	Low-type	High-type
Average expansion rates	0.04 (0.03)	0.22 (0.28)
Shares among entrants	0.68 (0.68)	0.32 (0.32)
Shares among incumbent	0.47 (0.36)	0.53 (0.64)

Notes: The table contains various equilibrium objects for an economy where incorporation choice is not available. The second and third rows refer to the share of each category such that they sum up to one. Numbers from the baseline model are given in parenthesis for comparison.

Table 5: Eliminating Incorporation Choice

Next, I consider a policy that incentivizes incorporation by subsidizing the incorporated incumbent firms. In particular, I introduce a 5% subsidy to the incorporated incumbent firms' profit, which corresponds to 0.3% of the final output.[14] This policy not only encourages firms to choose incorporation but also incentivizes incorporated firms to invest more in quality improvements. Table 6 summarizes the impact of this policy on the equilibrium of the economy. As seen from Panel A, the subsidy policy increases the expansion rate of both low- and high- type incorporated firms but the increases are relatively small. The policy has a more significant impact on the selection pattern in the

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[13]Notice that share of firm types among entrance is the same as baseline economy by design.

[14]In order to focus on the implication of this policy on firm incentives and selection, I abstract from the costs of financing the subsidy.

economy. The share of high-type firms that choose incorporation upon entry increases significantly to 75% (from 59% in the baseline) and half of the firms are high-type incorporated firms among incumbents. This subsidy policy would increase the aggregate productivity growth to 3.2%, mostly thanks to the change in the composition of firm types in favor of high-types who has higher expansion rates.

	Unincorporated		Incorporated	
	Low-type	High-type	Low-type	High-type
Panel A: Average Expansion Rates				
Baseline	0.03	0.20	0.047	0.31
Subsidy	0.02	0.18	0.049	0.33
Panel B: Shares among Entrants				
Baseline	0.66	0.13	0.02	0.19
Subsidy	0.63	0.09	0.01	0.27
Panel C: Shares among Incumbents				
Baseline	0.36	0.22	0.005	0.42
Subsidy	0.32	0.16	0.011	0.51

Notes: The table contains various equilibrium objects for an economy where incorporated firm profit is subsidized by 5%, together with the baseline results for comparison. Panel B and C refer to the share of each category such that they sum up to one.

Table 6: Subsidizing Incorporated Firms

1.6   Conclusion

This paper develops an equilibrium model of firm dynamics with endogenous entry and exit, where firms spend resources to improve their productivity and choose whether to incorporate or not. I use the model to study how incorporation, which provides limited liability to firm owners, affects firm dynamics and macroeconomy. An important model feature is that firms have heterogeneous (high and low) types which differ in their capacity to improve productivity.

The model underlines two main effects that generate the differences in firm dynamics

between incorporated and unincorporated firms. The first one is a treatment effect of incorporation: since incorporation protects firms from downside risks, it incentivizes them to invest more in improving their productivity, subsequently grow large and exit less often. The second one is a selection effect due to the presence of firm heterogeneity: entrepreneurs with higher growth potential (i.e. more efficient in proving productivity) are more likely to choose incorporation as it is more valuable to large firms. The strength of this selection effect is determined by the interplay between endogenous entry, investment, and exit decisions.

To quantify the importance of these effects, I estimate the model with firm-level micro data from Denmark, specifically exploiting the heterogeneity in exit rates by age conditional on size to identify firm types in growth potential and therefore selection. My model fits the key moments from micro-data reasonably well, and also performs well on non-targeted moments in the data.

The calibration results suggest that accounting for firm heterogeneity in growth potential is quantitatively important in explaining the observed better performance of incorporated firms. In a counterfactual economy where the incorporation decision is randomized within firm types, both the productivity growth and the difference in the average size of incorporated and unincorporated firms would decline. To assess the value of incorporation, I also use the model to conduct two experiments. First, I consider a case where the option of incorporation is not available to the firms. The absence of incorporation not only eliminates the positive treatment effect on firms expansion rates but also mitigates the selection of high-growth potential firms in the economy, resulting in lower growth rates and welfare. Second, subsidizing the incorporated firms provides significant welfare gains. This is largely driven by the change in the degree of selection, i.e., the change in the composition of firm types.

Chapter 2

Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries

This chapter is co-authored with Ufuk Akcigit and Michael Peters.

Abstract

Delegating managerial tasks is essential for firm growth. Most firms in developing countries, however, do not hire outside managers but instead rely on family members. In this paper, we ask if this lack of managerial delegation can explain why firms in poor countries are small and whether it has important aggregate consequences. We construct a model of firm growth where entrepreneurs have a fixed time endowment to run their daily operations. As firms grow large, the need to hire outside managers increases. Firms' willingness to expand therefore depends on the ease at which delegation can take place. We calibrate the model to plant-level data from the US and India. We identify the key parameters of our theory by targeting the experimental evidence on the effect of managerial practices on firm performance from Bloom et al. (2013). The inefficient delegation environment in India reduces income per capita by 11%. It also contributes to the small size of Indian producers, but would cause substantially more harm for US firms. The reason is that US firms are larger on average and managerial delegation is especially valuable for large firms. This makes delegation efficiency and other factors affecting firm growth complements.

2.1 Introduction

Managerial delegation is essential for firm growth. In the developed world, many family-owned industrial giants, such as Walmart, The Lego Group, or Ford Motor Co., have managed to expand to hundreds of thousands of employees by relying on professional managers to run their daily operations. In contrast, firms in developing economies often shun outside managers and recruit managers exclusively among family members. Are such cross-country differences in the ease of managerial delegation important determinants of the process of firm growth? Might such limits to delegation allow small and unproductive firms in poor countries to survive because they limit the competitive pressure from more productive producers? And do they have important macroeconomic implications by reducing aggregate productivity and income per capita? In this paper, we answer these questions both theoretically and quantitatively.

To do so, we propose a macroeconomic model of firm dynamics where the need for managerial delegation takes center stage. Firms are run by entrepreneurs, who have the opportunity to increase their productivity in order to expand. As the entrepreneur's own managerial time is a fixed factor, production features decreasing returns and marginal profits decrease in firm size. This reduces firms' incentives to grow large. Entrepreneurs can endogenously overcome such limits to their span of control by hiring outside managers. If delegating managerial responsibilities to outside managers is riddled with problems, entrepreneurs have no incentive to invest in productivity growth as they anticipate not being able to efficiently delegate as they grow. Improvements in the efficiency of delegation therefore raise the returns to growing large and increase aggregate productivity.

Our theory highlights an inherent complementarity between managerial delegation and firm size. Small firms do not consider the fixed managerial human capital of their entrepreneurs a drag on profitability. It is only once firms reach a certain size that the entrepreneur's span of control becomes binding and outside managers valuable. This

non-homotheticity, whereby the demand for outside managers is increasing in firm size, implies that frictions in the process of delegation affect the equilibrium distribution of firm size and the process of reallocation in a specific way. Firms with growth potential are hurt if outside managers cannot be employed efficiently and hence reduce their expansion efforts. In contrast, stagnant firms, which never grow beyond a certain size, benefit from such imperfections: not only do they not hire any managers themselves, but they are more likely to survive as they are shielded against the competition from their dynamic counterparts.

To quantify the importance of this mechanism, we calibrate our model to plant-level micro data from India and the US Our quantitative methodology has two main features. First, we allow the structural parameters of our model to be country-specific and calibrate them to the Indian and US data independently. This approach is important to address the identification problem implied by the non-homotheticity of managerial demand: are firms in India small and managerial delegation rare because it is difficult to delegate? Or do other frictions in India keep firms small and hence reduce the demand for outside managers in equilibrium? Our calibration strategy explicitly recognizes that firms in India might face higher barriers to growth (for example, due to capital market inefficiencies or distortionary regulation), that entry costs might be higher (for example, due to frictions in the access to start-up capital), or that many firms in India might be "subsistence entrepreneurs", who may simply lack the ability to grow their firms beyond a certain size. By allowing these features of the environment to be arbitrarily correlated with the efficiency of delegation, we refrain from attributing all differences between the US and India to our mechanism of interest.

Secondly, we use well-identified micro-estimates as "identified moments" to calibrate our structural model (Nakamura and Steinsson, 2018). Specifically, we exploit variation in managerial practices based on the randomized experiment by Bloom et al. (2013) to

estimate the production function for managerial inputs via indirect inference.[15] Bloom et al. (2013) provided a randomly selected group of Indian textile companies with management consulting to introduce American-style frontier management practices. They show that this intervention increased the profitability among treatment firms: after two years, the firms that benefited from the intervention produced 9% more than firms in the control group. By explicitly using this estimated treatment effect as a moment for our structural model, we ensure that our model generates the right microeconomic response to the experimental "management" intervention.

Our estimated model reveals stark differences between the US and India. First, we estimate that the efficiency of delegation is indeed substantially smaller in India: a given manager is only half as efficient in an Indian firm, relative to a firm in the US Second, we find that share of subsistence firms with little growth potential is substantially higher in India. Finally, the few Indian firms with the potential to expand are substantially less efficient in doing so relative to the US. Such differences could, for example, reflect credit market imperfections or distortions to market entry, which prevent firms from expanding or keep innovative firms out of the market entirely.

Taken together, this implies that the Indian economy suffers from a significant lack of selection, where subsistence producers survive because firms with growth potential have low incentives to expand. Hence, the glut of small firms in India is not merely a reflection of frictions that those small firms face but rather an indication of a lack of competition stemming from larger firms. Policies aimed at supporting small firms, e.g., micro-finance programs, while potentially desirable for their redistributive properties, could be harmful by reducing the reallocation of resources from small stagnant firms to firms with growth potential.

We then use our calibrated model to quantify the importance of frictions in the delegation process to explain such differences in the process of firm dynamics between the

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[15]We are very grateful to Nick Bloom and his coauthors that they were willing to share their data with us.

US and India. This analysis yields two main conclusions. First, we show that frictions to delegate managerial tasks in India are partly responsible for this lack of selection. If Indian firms could use outside managers as efficiently as firms in the U.S, their incentives to expand would be higher. This would increase aggregate productivity and income per capita in India. Our estimates imply that the low efficiency of delegation reduces income per capita in India by about 11%.

Second, the complementarity of firm size and delegation implies an important interaction between the ease of delegation and other differences between India and the US While the process of firm dynamics in India does depend on the delegation environment, the implications are modest. We find that an increase in the efficiency of delegation to US standards would increase average firm size by around 4% and reduce the employment share of small firms by a similar amount. If, in contrast, US firms could use outside managers only as inefficiently as firms in India, the consequences would be much more severe: average firm size would decline by around 13%, and the employment share of small firms would increase by 19%. The reason is that managerial delegation and other non-managerial factors that determine firm expansion naturally interact.

Related Literature That managerial delegation might be a key determinant of firm dynamics and macroeconomic performance goes back to the early work of Alfred Chandler (Chandler, 1990) and Edith Penrose, who argue that managerial resources are essential for firms to expand and that a scarcity of managerial inputs prevents the weeding out of small firms as "bigger firms have not got around to mopping them up" (Penrose, 1959, p. 221). Recently, more systematic evidence for the importance of managerial inputs has accumulated. In particular, managerial practices differ systematically across countries, and firms in developed economies are larger and delegate more managerial tasks to outside managers (Bloom and Van Reenen, 2007, 2010; McKenzie and Woodruff, 2017).

We formalize and quantify the macroeconomic importance of such managerial con-

siderations by providing a new theory of firm dynamics and the resulting firm size distribution in developing countries.[16] Our theory incorporates limits to firms' span of control, as in Lucas (1978), into a micro-founded model of Schumpeterian growth following Klette and Kortum (2004), which has been shown to provide a tractable and empirically successful theory of firm dynamics (see for instance (Acemoglu et al., 2018; Akcigit and Kerr, 2018; Garcia-Macia et al., 2016; Lentz and Mortensen, 2008)).[17] By explicitly allowing firms to hire outside managers, our model makes firms' span of control an endogenous variable which is jointly determined with the process of firm dynamics and the equilibrium distribution of firm size.

Frictions in the market for managerial inputs are also highlighted in Caselli and Gennaioli (2013), Powell (2012), Grobvosek (2015) and Bloom et al. (2016). In contrast to our theory, all of these papers assume that firm productivity is exogenous, so that there is no interaction between the delegation environment and firm growth. Guner et al. (2015), Roys and Seshadri (2014) and Xi (2016) present dynamic models of (managerial) human capital accumulation but do not focus on the implications for firm dynamics. Finally, there is a large literature on the internal organization of the firm - see, for example, Garicano and Rossi-Hansberg (2015) for a survey. This literature has a much richer micro structure of firms' delegation environment and the substitutability of managerial skills, but does not focus on the resulting properties of firm dynamics.[18]

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[16]An overview of some regularities of the firm size distributions in India, Indonesia, and Mexico is contained in Hsieh and Olken (2014). There is a large literature to explain cross-country difference in allocative efficiency across firms as diagnosed in (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). This literature highlights credit market frictions (Buera et al., 2011; Moll, 2014; Midrigan and Xu, 2014), size-dependent policies (Guner et al., 2008; Garicano et al., 2016; Gourio and Roys, 2014), monopolistic market power (Peters, 2016) and adjustment costs (Collard-Wexler et al., 2011). See Hopenhayn (2014) for a synthesis of this literature.

[17]As in Aghion and Howitt (1992), firm dynamics are determined through creative destruction, whereby successful firms expand through replacing other producers. See Aghion et al. (2014) for a survey of the Schumpeterian growth literature and Akcigit (2017) for the importance of firm dynamics for the process of economic growth.

[18]There is also a large empirical literature on family firms - see e.g. Bertrand and Schoar (2006). La Porta et al. (1999) document that family members are regularly controlling shareholders in most countries. Bennedsen et al. (2007) use variation in the gender of the CEO's firstborn child to present causal evidence that family successions have a negative impact on performance. In contrast, Mueller and Philippon (2011) argue that family ownership has distinct benefits in environments of hostile labor relations.

Our model explicitly allows for heterogeneity in firms' innate growth potential. This heterogeneity is important to formalize the idea that limits to delegation affect the extent to which firms with growth potential replace stagnant, subsistence producers. There is ample empirical evidence for the importance of such heterogeneity. Schoar (2010) and Decker et al. (2014) argue that some entrepreneurs are "transformative" and have the necessary skills to expand, while subsistence entrepreneurs may simply never grow independently of the environment they operate in. Hurst and Pugsley (2012) provide evidence that many firms in the US intentionally choose to remain small. In the context of developing countries, Banerjee et al. (2015) and De Mel et al. (2008) stress the importance of persistent differences in growth potential. On the theoretical side, Luttmer (2011) and Lentz and Mortensen (2016) argue that models without heterogeneity in growth potential are unable to explain the very rapid growth of a subset of the US firms.

Finally, on the methodological front, our paper adds to the recent literature in macroeconomics that uses well-identified microeconomic estimates to identify structural models (Nakamura and Steinsson, 2018). Recent examples in the literature on growth and development are Lagakos et al. (2018), Kaboski and Townsend (2011) and Brooks and Donovan (2017). To the best of our knowledge, our paper is the first to use this methodology to estimate a model of firm dynamics.

The remainder of the paper is organized as follows. In Section 2.2, we describe the theoretical model. Section 2.3 summarizes the data that we use in our quantitative analysis and discusses the identification of the model. Section 2.4 contains the calibration results and discusses a variety of non-targeted moments. In Section 2.5, we provide our main analysis to quantify the role of the delegation environment for firm dynamics and the aggregate economy. Section 2.6 provides various robustness checks of the main quantitative results. Section 2.7 concludes. All proofs and additional details are contained in the Appendix B.

2.2 Theory

2.2.1 Technology, Preferences and Static Allocations

We consider a continuous time economy, where a representative household values the consumption of a unique final good, maximizes the stream of per-period utilities $U(C_t) = \ln(C_t)$ and discounts the future at rate ρ. Labor is supplied inelastically and the members of the household can work as either managers or production workers. The final good Y is a Cobb-Douglas composite of a unit continuum of varieties,

$$\ln Y_t = \int_0^1 \ln y_{jt} dj, \tag{2.1}$$

and is used for consumption (C_t) and for productivity enhancing investments by incumbents (R_t) and entrants ($R_{E,t}$). The aggregate resource constraint is therefore given by

$$Y_t = C_t + R_t + R_{E,t}. \tag{2.2}$$

To save on notation we will drop the time subscript t whenever it does not cause any confusion.

Producing the variety y_j requires both production workers and managerial inputs. In particular, we assume that managers increase the efficiency of production workers so that firm f can produce good j according to

$$y_{jf} = q_{jf} \phi\left(e_{jf}\right) l_{jf}, \tag{2.3}$$

where q_{jf} is the firm-product specific efficiency, l_{jf} is the number of production workers employed in producing intermediate good j, e_{jf} denotes the amount of managerial services firm f allocates toward the production of good j, and $\phi\left(e_{jf}\right) \geq 1$ is an increasing function translating managerial services into physical productivity units. Letting w_P de-

note the equilibrium wage for production workers, the production labor cost of producing one unit of y is therefore given by $MC = \frac{w_P}{q\phi(e)}$.

Firms can produce multiple products $j \in [0,1]$. In equilibrium, product j will be produced by the firm with the highest productivity q_{jf}. Firm f will therefore produce n_f products if it has the highest productivity in n_f product markets. We denote the producer's (i.e., the highest) productivity of variety j by q_j.

In order to focus on the interaction between managerial delegation and the resulting equilibrium process of firm dynamics, we keep the static market structure as tractable as possible. To do so, we assume that in each market j, the producing firm competes against a competitive fringe of potential producers that can produce variety j at marginal costs w_P/q_j.[19] Because the demand function stemming from (2.1) has a unitary elasticity, the producing firm engages in limit pricing and sets its price equal to the marginal costs of the competitive fringe. The gross profits after paying for production workers l_j (but before paying any managers the firm might decide to hire) are therefore given by[20]

$$\pi_j(e_j) = p_j y_j - w_P l_j = \left(\frac{\phi(e_j) - 1}{\phi(e_j)} \right) Y. \tag{2.4}$$

Hence, profits on variety j are increasing in the amount of managerial services e_j because managerial inputs increase physical productivity and hence profitability. For analytical convenience, we assume that $\phi(e) = \frac{1}{1 - e^\sigma}$, where $e \in [0, 1)$ and $\sigma < 1$. This implies that

$$\pi(e_j) = e_j^\sigma Y, \tag{2.5}$$

i.e., profits are a simple power function of managerial effort parameterized by the elastic-

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[19]This assumption allows us to abstract from strategic pricing decisions of firms who compete with firms of different productivity. A model with strategic pricing behavior is analyzed in Peters (2016). In terms of primitives, the fringe firms have access to the same technology as the leading firm and to a level of managerial services ϕ^{fringe}, which we normalize to unity.

[20]To see this, note that $p_j y_j - w_P l_j = \left(1 - \frac{w_P l_j}{p_j y_j}\right) p_j y_j = \left(1 - \frac{1}{\phi(e_j)}\right) Y$ as $p_j = w_P/q_j$ and $p_j y_j = Y$.

ity σ.

Managerial resources not only affect firm profitability but also the aggregate alloca-
tions. In particular (see Section B.1.4 in the Appendix), aggregate output Y is given by:

$$Y = Q\mathcal{M}L^P, \tag{2.6}$$

where $L^P = \int_0^1 l_j dj$ denotes the mass of production workers, $\ln Q = \int_0^1 \ln q_j dj$ is an
index of aggregate physical productivity, and $\mathcal{M} = \left(1 - \int_0^1 e_j^\sigma dj\right)^{-1}$ summarizes the static
effect of managerial services on aggregate productivity. Note that $\mathcal{M}$ is increasing in e_j,
reflecting the positive effect of managerial inputs on labor productivity at the firm-level.

2.2.2 Delegation, Span of Control and Firms' Incentives to Grow Large

At the heart of our theory is the link between managerial delegation and firms' in-
centives to grow large. As in Klette and Kortum (2004), firms produce multiple products
and grow by expanding into new product markets. In particular, by replacing the current
producer of variety j, the firm adds new products to its portfolio and grows in sales,
employment and profits.

Because profits of each product depend directly on the amount of managerial ser-
vices e, their availability is a key determinant of firms' incentives to expand. We assume
that firms are run by entrepreneurs, who have a fixed endowment $T < 1$ of managerial
efficiency units they provide inelastically to their firms.[21] If an entrepreneur is the current
producer in n markets, she will have $e_j = T/n$ units of managerial services per product.
That she will want to spread her managerial time equally across all product lines follows

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[21]Recall that $e < 1$ for $\phi(e) = (1 - e^\sigma)^{-1}$ to be well-defined. It can be shown that $T < 1$ is sufficient to
ensure that this condition is satisfied.

directly from the concavity of π. The total profits of a firm of size n are hence,

$$\Pi(n) = \sum_{j=1}^{n} \pi(e_j) = n \times \pi\left(\frac{T}{n}\right) = T^\sigma n^{1-\sigma} Y.$$

This expression has a simple but important implication: while profits are increasing in the number of products n, they do so at a decreasing rate. The reason is that the owner's fixed endowment T limits her span of control, as in Lucas (1978). As the existing supply of managerial resources is spread over more and more production units, the marginal profitability declines. This implies that firm size n and the entrepreneur's managerial endowment T are complements in that the marginal return to a unit of additional managerial resources is increasing in firm size

$$\frac{\partial^2 \Pi(n)}{\partial n \partial T} > 0.$$

Hence, entrepreneurs with larger firms consider their fixed time endowment more of a bottleneck.

Delegation To counteract these decreasing returns, the entrepreneur can hire *outside* managers to augment her own endowment of managerial resources. It is this distinction between entrepreneurs and outside managers that makes firms' span of control endogenous in our theory: while entrepreneurial human capital T is in fixed supply at the firm level, outside managers can be hired on the market. We assume that the entrepreneur's and the managers' human capital are perfect substitutes and that the relative efficiency of outside managers within the firm is given by α. More specifically, if an owner of a firm of size n hires m units of managerial human capital for the production of product j, the total amount of managerial services e is given by

$$e(m) = T/n + \alpha \times m. \tag{2.7}$$

The parameter α is the key parameter for our analysis. It governs the efficiency of delegating tasks to outside managers and we therefore refer to it as the *delegation efficiency*. The higher α, the more managerial services a given outside manager generates within the firm.

We want to highlight that α is a parameter of the firms' production structure. Consider, for example, an entrepreneur in India looking to expand. One reason why the entrepreneur might decide to stay small is because the supply of sufficiently talented managers might be low. Another reason might be that the pool of managers might be fine, but that he could not prevent them from shirking on the job. The former is about managerial human capital embedded in m. The latter is summarized in the delegation efficiency α.

One can think of many reasons why delegation might be less efficient in a developing economy like India. First of all, there is large empirical literature that argues that the prevalence of efficient management practices, such as quality standards, monitoring, or meritocratic promotions, varies systematically with the level of development (see e.g., Bloom et al. (2012) or Bloom and Van Reenen (2010)). Secondly, the efficiency of delegation could depend on the level of technology. For example, if delegation is complementary to IT equipment, technological differences across countries will be a source of variation in α (see, for example, Bloom et al. (2009)). Finally, α can be interpreted as a reduced form specification of the prevailing institutional or cultural environment. If, for example, contractual imperfections are severe or the level of trust is low, entrepreneurs might need to spend substantial amounts of their own time monitoring their managerial personnel.[22]

We assume that outside managers are hired on a spot market at a given wage w_M. This implies that the firm's delegation decision is static. Using (2.5) and (2.7), total profits

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[22]In Section (B.1.5) in the Appendix, we provide a simple micro-founded example, where a contractual game between the owner and outside managers leads to equation (2.7) and α is a combination of explicit structural parameters.

net of managerial payments of a firm of size n are given by:

$$\Pi(n) \equiv \sum_{j=1}^{n} \max_{m_j \geq 0} \left\{ \left(\frac{T}{n} + \alpha m_j \right)^{\sigma} Y - w_M m_j \right\}. \tag{2.8}$$

The maximization problem in (2.8) defines both firms' demand for managerial inputs and their final profit function. Two properties are noteworthy. First of all, the entrepreneur's own managerial input T generates a well-defined extensive margin for managerial hiring. In particular, the firm only hires outside managers if the size of the firm exceeds the endogenous delegation cutoff $n^*(\alpha)$, which is given by

$$n^*(\alpha) = T \times \left(\frac{\omega_M}{\sigma \alpha} \right)^{\frac{1}{1-\sigma}}, \tag{2.9}$$

where $\omega_M = w_M / Y$. Hence, small firms rely purely on the time of the owner and only start delegating once they reach a size $n > n^*(\alpha)$. Note that the cutoff $n^*(\alpha)$ is decreasing in α, as even small firms utilize outside managers if it is easy to delegate. Second, it is easy to verify that the optimal managerial demand per product $m(n)$, conditional on hiring, i.e., if $n > n^*(\alpha)$, is given by

$$m(n) = \left(\frac{\sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}} \alpha^{\frac{\sigma}{1-\sigma}} - \frac{1}{\alpha} \frac{T}{n}. \tag{2.10}$$

Note first that $m(n)$ is increasing in n, i.e. larger firms hire more managers per product to make up for the fact that their own managerial resources are spread thinner and thinner as the firm gets larger. Hence, the demand for outside managerial resource is non-homothetic as larger firms hire managers more intensely. Moreover, the demand for outside managers is increasing in the delegation efficiency α, holding ω_M constant.

Substituting firms' optimal delegation policies into (2.8) implies that firm profits are given by

$$\Pi(n; \alpha) = \tilde{\pi}(n; \alpha) \times Y \tag{2.11}$$

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where

$$
\tilde{\pi}(n;\alpha) = \begin{cases} T^{\sigma} n^{1-\sigma} & \text{if} \quad n < n^*(\alpha) \\ T\frac{\omega_M}{\alpha} + (1-\sigma)\left(\frac{\sigma\alpha}{\omega_M}\right)^{\frac{\sigma}{1-\sigma}} n & \text{if} \quad n \geq n^*(\alpha) \end{cases} . \tag{2.12}
$$

This profit function is a crucial object in our analysis, because it reflects the firm's span of control, i.e., the marginal return to increasing the number of markets in which it is active. Importantly, the possibility of delegation endogenizes the firm's span of control and makes it directly dependent on α.

To see this, consider the left panel in Figure 7, where we depict the profit function $\tilde{\pi}(n;\alpha)$ for two different levels of $\alpha^L < \alpha^H$. Small firms are run only by their owner and are subject to diminishing returns: as long as they do not delegate, the marginal profit from producing an additional product is declining, i.e., $\tilde{\pi}(n;\alpha)$ is concave in n. Once firms reach the delegation cutoff n^* and start hiring outside managers, however, the profit function becomes linear in n. Hence, entrepreneurs overcome their limited span of control by delegating managerial tasks to outside managers.

Now consider an increase in the efficiency of delegation. This reduces the delegation cutoff, and smaller firms start to rely on outside managers. Importantly, an increase in α also increases the *slope* of the profit function. It is this channel that links the delegation environment and the process of firm dynamics: a higher α increases firms' span of control and raises the returns to growing large.

Our model nests two workhorse models in the literature as special cases. When $\alpha = 0$, there is no scope for outside delegation. In that case, $n^* = \infty$, and all firms are subject to diminishing returns, as in Lucas (1978). In contrast, when α is sufficiently large so that $n^* < 1$, every firm delegates, the limited span of control of the owner's own time T is not a bottleneck, and firms' profit functions are linear as in the baseline version of Klette and Kortum (2004). Hence, our model offers a simple framework where the firm's span

of control is endogenous and determined in equilibrium.

Panel A: Delegation and Span of Control   Panel B: Optimal Expansion Rate
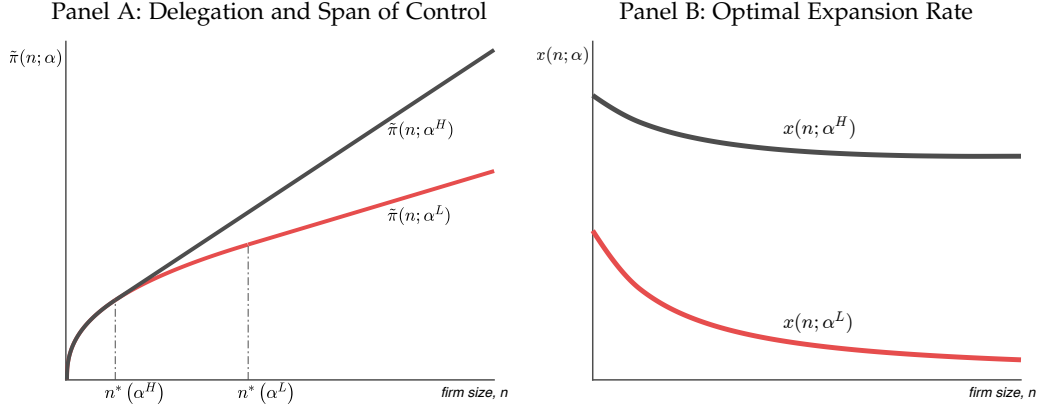
Notes: In Panel A we depict the profit function $\tilde{\pi}(n; \alpha)$ characterized in (2.12) for α^L and α^H, $\alpha^L < \alpha^H$. In Panel B we depict the optimal expansion schedule $x(n; \alpha)$ in (2.15).

Figure 7: Delegation, Span of Control and Expansion Incentives

Firm Expansion   The efficiency of delegation is a crucial determinant of firms' incentives to expand. For now, we consider the behavior of an individual firm. In Section 2.2.3, we embed this structure into a general equilibrium model.

We model firm growth as a stochastic process where the firm can choose the rate at which it improves the productivity q of a randomly selected product by $\gamma_t > 1$ and thereby replace the existing firm. In particular, if a firm with n varieties invests R units of the final good, it expands into a new product line at rate

$$X(R; \theta, n) = \theta[R/Q]^{\zeta} n^{1-\zeta}, \tag{2.13}$$

where θ, which we refer to as firms' growth potential, determines the efficiency of innovation, $\zeta < 1$ parametrizes the convexity of the expansion cost function and Q_t is the productivity index defined in (2.6).[23] At the same time, each product the firm currently

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[23]Because we denote innovation costs in terms of the final good, the scaling variable Q is required to keep the model stationary. We also assume that firms' innovation costs depend on the number of varieties n to generate deviations from Gibrat's law solely through incomplete delegation. In particular, if the profit

produces gets improved upon by other firms at rate τ_t. This rate of creative destruction is of course endogenous and determined in equilibrium, but firms take it as given.

To characterize the firm's optimal expansion policy, we need to solve for its value function. The value of a firm with n products, $V_t(n)$, solves the Hamilton-Jacobi-Bellman equation

$$r_t V_t(n) - \dot{V}_t(n) = \Pi_t(n;\alpha) - n\tau_t \left[V_t(n) - V_t(n-1)\right]$$
$$+ \max_X \left\{ X\left[V_t(n+1) - V_t(n)\right] - Q_t n^{\frac{\zeta-1}{\zeta}} \left[\frac{X}{\theta}\right]^{\frac{1}{\zeta}} \right\}, \qquad (2.14)$$

where $\dot{V}_t \equiv \partial V_t / \partial t$. The right-hand side of (2.14) consists of three parts. First, the firm earns the flow profits $\Pi_t(n;\alpha)$ given in (2.12). Second, the firm might lose one of its products to other firms. This occurs at the endogenous rate of creative destruction $n\tau_t$ (because each product gets replaced at rate τ_t). Finally, the value function incorporates the option value of expansion: with flow rate X, the firm expands into a new market and experiences a capital gain of $V_t(n+1) - V_t(n)$. The associated costs of expanding into a new market stem from (2.13). Note that the function V_t directly depends on the delegation efficiency α via the profit function.

This value function implicitly defines firms' optimal rate of expansion and productivity growth. Letting $x \equiv X/n$ denote the expansion intensity, optimality requires that

$$x_t(n;\alpha) = \theta^{\frac{1}{1-\zeta}} \zeta^{\frac{\zeta}{1-\zeta}} \times \left(\frac{V_t(n+1) - V_t(n)}{Q_t} \right)^{\frac{\zeta}{1-\zeta}}. \qquad (2.15)$$

Naturally, the incentives to expand depend on the *marginal* return to doing so, $V_t(n+1) - V_t(n)$. It is this marginal return that links firms' innovation incentives to the ease of delegation. In equation (2.12) and the left panel of Figure 7, we showed that α determines the concavity of the profit function and hence the marginal flow profit of expansion. Because the value function inherits the properties of the profit function, α also determines

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function in (2.12) were linear, the specification in (2.13) would imply that firm growth is independent of size.

the slope of the value function and hence the optimal innovation rate for firms of different sizes.

In the right panel of Figure 7, we depict the optimal innovation rate $x(n, \alpha)$. The concavity of the profit and value function implies that firms' expansion incentives are declining in size. An increase in α affects this schedule in two ways. First, an increase in delegation efficiency shifts the whole expansion schedule upwards. Intuitively, if firms anticipate being able to hire outside managers more efficiently once they reach the delegation cutoff n^*, their incentives to expand will already be higher today. Similarly, firms that are already delegating also increase their expansion efforts as their profitability increases. Secondly, innovation incentives increase more for larger firms, so that the schedule $x(n; \alpha)$ becomes flatter. Hence, improvements in the delegation environment are particularly important for large firms, which rely heavily on outside managers.

2.2.3 Firm Dynamics and Delegation in General Equilibrium

To determine the aggregate effects of the delegation environment, we now embed this model of firm growth into a general equilibrium model of firm-dynamics. At each point in time there is a set of existing firms whose innovation rates are given by (2.15), and a set of potential entrants that enter the economy by improving upon existing producers.

Firm Heterogeneity We explicitly allow firms to be heterogeneous in their growth potential. Formally, we assume that firms differ in their innovation efficiency θ and can be either *transformative* (high, θ_H) or *subsistence* (low, θ_L) types. A firm's type is persistent and determined upon entry. Each new entrant draws a firm type $\theta \in \{\theta_H, \theta_L\}$ from a Bernoulli distribution, where

$$\theta = \begin{cases} \theta_H & \text{with probability } \delta \\ \theta_L & \text{with probability } 1 - \delta \end{cases}. \tag{2.16}$$

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To capture the existence of subsistence entrepreneurs, we assume that $\theta_L = 0$, so that low-type firms are entirely stagnant. This polar case is conceptually useful because the sole difference in firm dynamics across countries then stems from the innovation incentives for high types – it is the high types' appetite for expansion that determines the degree of selection, i.e., the time it takes for low-type firms to be replaced.

In addition, we also allow firms to potentially differ in the rate at which they *lose* products due to differences in their reputation, customer loyalty, or organizational capital. Letting τ_H and τ_L be the rates at which high and low-type firms lose a given product to other firms (both of which will be determined in equilibrium), we assume that $\tau_L = \beta \tau_H$. If $\beta > 1$, low-type firms are easier to replace (or are targeted by expanding firms more intensely), if $\beta < 1$, the opposite is the case. The parameter β is one of our structural parameters which we will calibrate from the data. Allowing for $\beta \neq 1$ is not conceptually important; we introduce it mostly for quantitative reasons.

To summarize, the behavior of high types is described by the optimal expansion rate in (2.15) and the value function in (2.14) (which from now on we denote by $V_t^H(n)$). Subsistence entrepreneurs, in contrast, never innovate and hence never grow beyond a single product; they exit at rate $\tau_{L,t}$. Their value function is therefore simply given by

$$r_t V_t^L - \dot{V}_t^L = \Pi_t (1; \alpha) - \tau_{L,t} V_t^L. \tag{2.17}$$

Entry   A unit mass of potential entrants attempts to enter the economy at any point in time. They use a similar innovation technology as incumbent firms, where the flow rate of entry z is related to the spending on entry efforts R_E according to $z = \theta_E [R_E/Q]^\zeta$. Entrants enter the economy with a single, randomly selected product. Given that an
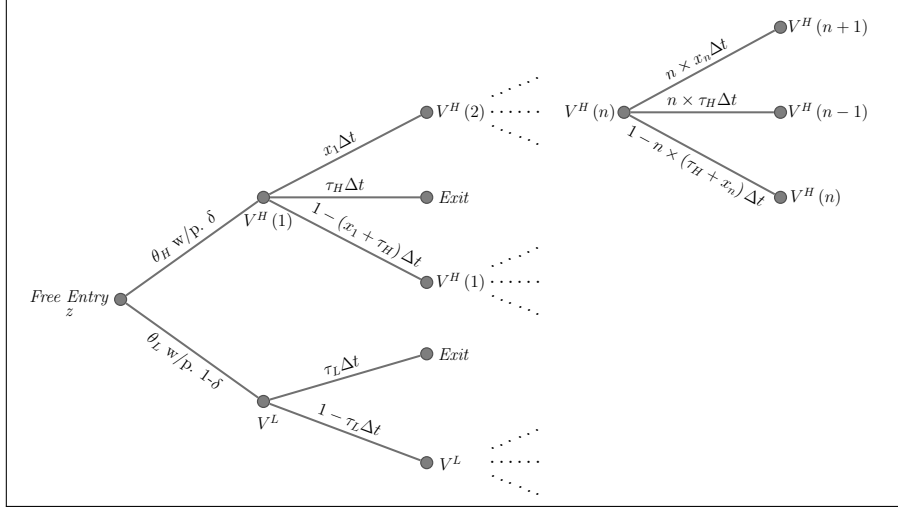
Figure 8: Overview of the Life-Cycle Dynamics in the Model

entrant becomes a high-type with probability δ, the equilibrium entry flow is given by:

$$
z_t(\alpha) = \underset{z}{\mathrm{argmax}} \left\{ z \left[\delta V_t^H(1) + (1-\delta) V_t^L \right] - Q_t \theta_E^{-\frac{1}{\zeta}} z^{\frac{1}{\zeta}} \right\}
$$

$$
= \theta_E^{\frac{1}{1-\zeta}} \zeta^{\frac{\zeta}{1-\zeta}} \left[\frac{\delta V_t^H(1) + (1-\delta) V_t^L}{Q_t} \right]^{\frac{\zeta}{1-\zeta}}. \tag{2.18}
$$

Note that the equilibrium entry flow depends on the delegation environment α through firms' value function.

Figure 8 provides an overview of the life cycle dynamics in our model. Firms enter the economy with a single product and are either transformative, high-type entrepreneurs (with probability δ) or subsistence, low-type entrepreneurs (with probability $1 - \delta$). The corresponding value functions are $V^H(1)$ and V^L. Within the next time interval Δt, high-type firms either expand (with probability $x_1 \Delta t$), lose their only product and exit (with probability $\tau_H \Delta t$), or remain a one-product firm (with probability $1 - x_1 \Delta t - \tau_H \Delta t$). In contrast, low-type firms never expand but instead either exit (at rate τ_L) or remain in the economy by serving their initial market.

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Delegation Efficiency and the Firm Size Distribution   The equilibrium firm size distribution is endogenously determined from firms' expansion and entry incentives and hence depends on the delegation environment α. Let F_{nt}^H be the mass of high-type producers with n products and F_t^L be the mass of low-type producers (all of which have a single product). In a stationary equilibrium, these are constant over time and have a simple expressions. In particular, as we show in Section B.1.1 in the Appendix, they are given by

$$F_n^H(\alpha) = \frac{\delta z(\alpha)}{nx(n;\alpha)} \prod_{j=1}^{n} \left(\frac{x(j;\alpha)}{\tau_H(\alpha)} \right) \text{ and } F^L(\alpha) = \frac{(1-\delta)z(\alpha)}{\tau_L(\alpha)}. \tag{2.19}$$

These expressions follow directly from the flow equations of the firm size distribution. Consider, for example, the case of F^L. Because subsistence firms exit the economy at rate τ_L and $z(1-\delta)$ subsistence entrepreneurs enter each instant, the equilibrium mass of low type firms is given by $(1-\delta)z/\tau_L$ as in (2.19). Furthermore, the aggregate rate of creative destruction is given by

$$\tau(\alpha) = \sum_{n=1}^{\infty} nx(n;\alpha)F_n^H(\alpha) + z(\alpha), \tag{2.20}$$

because existing producers get replaced both by other incumbent firms and new entrants. Note that (2.19) and (2.20) fully determine the equilibrium firm size distribution as a function of $x_t(n;\alpha)$ and $z_t(\alpha)$ because $\tau_L = \beta\tau_H$ and consistency requires that $\tau = \tau_H(1-F^L) + \tau_L F^L$, as F_t^L is the share of products which are produced by subsistence entrepreneurs.

The expressions in (2.19) are useful to build intuition how managerial delegation shapes the distribution of firm size. Recall that firm sales are proportional to the number of products n. The aggregate share of sales of firms with $n+1$ products relative to firms with n products is given by

$$\frac{(n+1)F_{n+1}^H}{nF_n^H} = \frac{x(n;\alpha)}{\tau_H(\alpha)}. \tag{2.21}$$

This expression shows that the relative importance of large producers is directly determined by the size-dependent innovation schedule $x(n;\alpha)$: the faster $x(n;\alpha)$ is declining

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in n, the smaller the aggregate importance of large firms. The right panel of Figure 7 therefore already suggests the link between delegation and the endogenous firm size distribution. If α is low, firms' span of control is a bottleneck for large firms, the optimal innovation rate $x(n; \alpha)$ declines steeply in size n and so does the aggregate importance of large firms. Improvements in the efficiency of delegation therefore induce reallocation towards large producers. Similarly, the expression for the equilibrium mass of subsistence firms F^L shows why inefficiencies in the process of delegation reduce selection and keep low-type firms alive: by harming large firms more than small firms, they reduce creative destruction more than the entry rate. Environments where delegation is difficult therefore make it easy for low-type firms to survive. In our quantitative analysis, we show that these intuitions carry through once general equilibrium effects are taken into account.

Creative Destruction and Aggregate Growth The rate of creative destruction is also the driver of aggregate growth in our economy. Recall that each successful innovation increases productivity by the step size γ_t. And because the rate of creative destruction is exactly the rate at which such innovations take place, the aggregate growth rate of the productivity index Q_t is given by (see Appendix B.1.2)

$$g_t(\alpha) \equiv \frac{\dot{Q}_t}{Q_t} = \ln(\gamma_t) \times \tau_t(\alpha). \tag{2.22}$$

This expression highlights the relationship between delegation and aggregate growth. In our model, more efficient delegation increases aggregate growth through its effect on expansion and entry and hence creative destruction. Whether this leads to persistent differences in growth rates across countries depends on the step size γ_t. As far as the process of firm dynamics is concerned, we do not have to take a stand on γ_t, because our model permits a stationary firm-size distribution even if the step size γ_t varies over time; see Section B.1.3 of the Appendix where we prove this property formally. However, in order to quantify the effect of delegation on long-run productivity differences, we

consider a model where γ_t is endogenous and the long-run distribution of income across countries is stationary. Hence, differences in α between the US and India will result in level differences, not growth differences. See Section 2.5.2 below.

2.2.4   The Labor Market Equilibrium for Outside Managers

To complete the characterization of the equilibrium, we need to specify the supply and demand of managerial inputs. The demand for outside managers results from firms' optimal hiring decisions. Because of the non-homotheticity of managerial demand, larger firms delegate more intensely, and the aggregate demand for managerial inputs depends on the endogenous firm size distribution. Using the optimal hiring rule in (2.10), a firm with $n \geq n^*$ products hires a total of $nm(n)$ managerial efficiency units. The demand for outside managers, H^{OM}, is therefore given by

$$
\begin{aligned}
H^{OM} &= \sum_{n=1}^{\infty} \mathbb{1}(n \geq n^*) m\,(n)\, n F_n\,(\alpha) \\
&= \sum_{n=1}^{\infty} \mathbb{1}(n \geq n^*) \underbrace{\left(\left(\frac{\sigma}{w_M/Y} \right)^{\frac{1}{1-\sigma}} \alpha^{\frac{\sigma}{1-\sigma}} n - \frac{T}{\alpha} \right)}_{\text{Managerial demand given firm size}} \underbrace{F_n\,(\alpha)}_{\text{Effect of FSD}} \;,
\end{aligned}
\tag{2.23}
$$

where $F_n = F_n^H + \mathbb{1}(n = 1)F^L$. This expression highlights two important determinants of managerial demand. Holding the firm size distribution constant, aggregate demand is increasing in α. In addition, because managerial demand is non-homothetic, the firm size distribution $F_n\,(\alpha)$ itself also affects managerial demand directly: if firms are small, outside managers are in low demand because small firms can be run by their owners. This highlights an important identification challenge which our empirical strategy has to address: do we see few outside managers in India because delegation is difficult? Or do other frictions keep Indian firms small and hence no managers are required?

To model the supply of managerial workers, we assume that each individual is endowed with a single efficiency unit of production labor and h units of managerial human

capital, distributed according to $G(h)$. Individuals make their occupational choice to maximize total earnings, i.e., individual i works as an outside manager if $h_i w_M > w_P$. Labor market clearing therefore requires that

$$H^{OM} = \int_{h \geq \frac{w_P}{w_M}} h g(h) dh, \tag{2.24}$$

where $g(h)$ is the density associated with $G(h)$.

In our application, we assume that h is drawn from a Pareto distribution, i.e., $G(h) = 1 - \left(\frac{\vartheta-1}{\vartheta}\mu_M\right)^{\vartheta} \times h^{-\vartheta}$. Here μ_M parametrizes the average level of managerial skills and $\vartheta > 1$ governs the heterogeneity in managerial talent. Using this functional form, the labor market clearing condition in (2.24) is given by

$$H^{OM} = \left(\frac{\vartheta-1}{\vartheta}\mu_M\right)^{\vartheta} \left(\frac{w_M}{w_P}\right)^{\vartheta-1} \frac{\vartheta}{\vartheta-1}. \tag{2.25}$$

Note that the supply of outside managers is increasing in the relative wage with an elasticity of $\vartheta - 1$. Moreover, holding relative wages fixed, the managerial skill supply is increasing in the average level of managerial human capital μ_M.

An equilibrium in our economy is then defined in the following way:

Definition 1. *Consider the environment described above. A dynamic equilibrium path is characterized by a time path of*

$$\left[p_{jt}, y_{jt}, \{V_t^H(n)\}_{n=1}^{\infty}, V_t^L, \{x_t(n)\}_{n=1}^{\infty}, z_t, w_{t,M}, w_{t,P}, \{F_{nt}^H\}_{n=1}^{\infty}, F_t^L, r_t, g_t \right]_{t=0}^{\infty},$$

such that (i) p_{jt} and y_{jt} maximize monopoly profits in (2.4), (ii) the value functions $V_t^H(n)$ and V_t^L are given by (2.14) and (2.17) (iii) the innovation rates $x_t(n)$ are optimal and given in (2.15), (iv) the entry rate z_t satisfies (2.18), (v) $w_{t,P}$ and $w_{t,M}$ clear the labor market for production and managerial labor, (vi) the numbers of firms of each size $[F_{nt}^H, F_t^L]$ are consistent with the flow equations in Section B.1.1 in the Appendix, (vii) the interest rate r_t satisfies the household's Euler

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equation, and (viii) the aggregate productivity growth rate is consistent with (2.22)*.*

2.2.5   Taking Stock

We have developed a theory to link the efficiency of delegation to firms' growth incentives and hence the process of firm dynamics and the equilibrium firm size distribution. At the heart of our model is the insight that a higher efficiency of delegation endogenously increases firms' span of control and hence their incentives to grow large.

To summarize the effects of an increase in delegation efficiency α, consider Figure 9, where we depict the qualitative relationships between α and various equilibrium outcomes.[24] In Panel A, we show that there is a positive relationship between delegation efficiency and firms' life-cycle growth. This follows directly from the resulting increase in firms' expansion incentives, in particular for large firms. This faster growth at the firm level shifts the firm-size distribution to the right so that the employment share of small firms declines (Panel B). These changes at the firm level are accompanied by changes in the labor market. In particular, the employment share of outside managers is increasing in α both because firms' demand for managers increases and because the firm size distribution shifts to the right, which further increases managerial demand as large firms are manager intensive (Panel C). Finally, because firms are heterogeneous in their growth potential, an increase in α will also be accompanied by selection. As subsistence entrepreneurs are small in equilibrium, they do not benefit from the opportunity to hire managers. In contrast, they *lose* from improvements in delegation efficiency because they are less likely to survive (Panel D).

These patterns are qualitatively consistent with stylized facts on firm dynamics in poor countries where firms are small and do not grow, subsistence producers are abun-

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[24]While these relationships stem from our quantitative model and we currently do not have an analytical proof, we have yet to find a counterexample. Hence, we suspect that these comparative static results hold true regardless of the particular parametrization of the model.

dant, and outside managers are rare. Importantly, the glut of small, stagnant firms in poor countries might not solely reflect frictions these firms face, but also result from more productive firms not being able to overcome limits to their span of control. Improvements in the efficiency of delegation enable firms with growth potential to overcome these decreasing returns and speed up the aggregate selection process. In the remainder of this paper, we analyze whether this mechanism can quantitatively account for the observed differences in the firm size distribution between the US and India and whether it has important implications for differences in income per capita.

Panel A: Life cycle growth

Panel B: Employment share of small firms

Panel C: Managerial employment share

Panel D: Share of subsistence firms
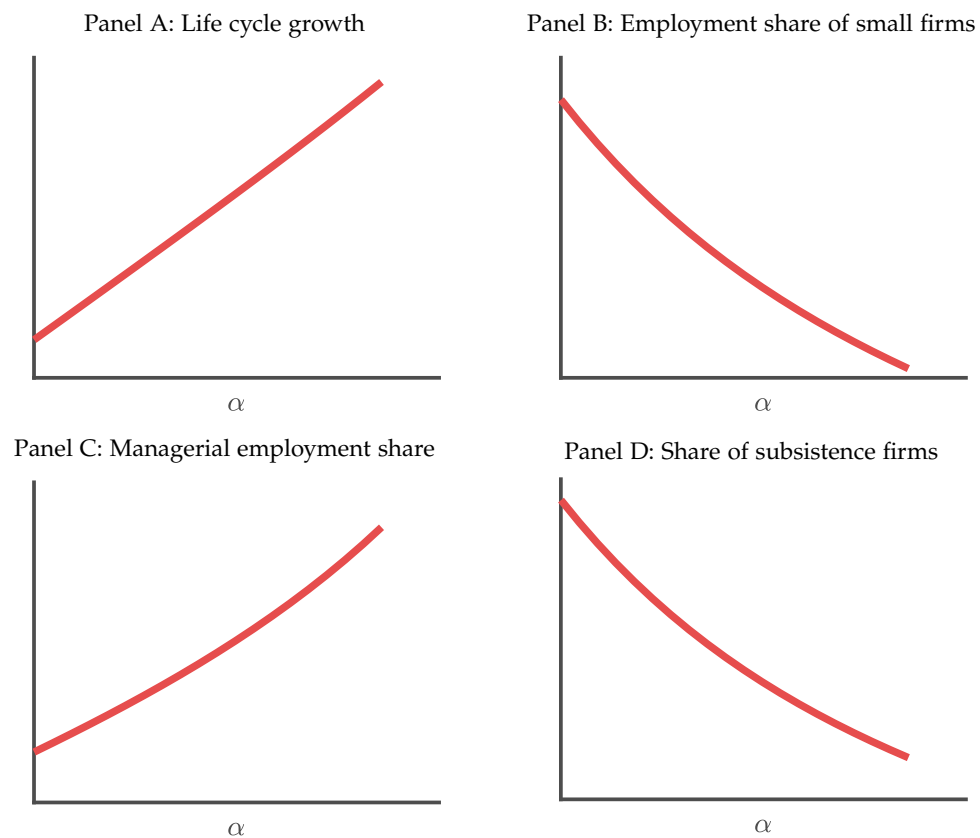
Notes: The figure summarizes the qualitative implications of changes in the delegation efficiency α for firms' life cycle growth (Panel A), the employment share of small firms (Panel B), the managerial employment share (Panel C) and the equilibrium share of low-type firms (Panel D).

Figure 9: Taking Stock: Delegation, Selection and Firm Dynamics

2.3 Data and Calibration Strategy

2.3.1 Data

Here we briefly describe the main data sources. A detailed description is contained in Section B.2.1 of the Appendix.

Establishment level data for the US and India: We calibrate our model to data for the manufacturing sector of the US and India. For the US we rely on publicly available data for the population of manufacturing plants from the Business Dynamics Statistics (BDS). The BDS is provided by the US Census Bureau and compiled from the Longitudinal Business Database (LBD), which provides data on employment and age for each establishment with paid employees. We focus on the data from 2012.

Analyzing data for the manufacturing sector in India is less straightforward as there does not exist a single database that provides this information. To capture the entirety of the manufacturing sector, we follow Hsieh and Klenow (2014a) and Hsieh and Olken (2014) and combine the Annual Survey of Industries (ASI) and the survey of the unorganized manufacturing sector from the National Sample Survey (NSS). The ASI focuses on the formal sector and covers all establishments employing ten or more workers using electric power or employing twenty or more workers without electric power. The NSS, every five years, surveys a random sample of the population of manufacturing establishments outside the sampling frame of the ASI. Hence, the firms in the NSS are mostly informal firms, which are decidedly smaller - more than 80% of plants have at most two employees and less than 1% have more than 15 employees (see Table 35 in the Appendix, where we report the firm size distribution in the NSS). We merge these two datasets using the sampling weights provided in the data and focus on the year 2010, which is the latest year for which both datasets are available.

For our analysis, we treat this union of the ASI and NSS data as representing the population of manufacturing firms in India. To provide direct evidence for representativeness of this data, we compared it to the Indian Economic Census, which is a complete count of all economic units in India. As we show in Section B.2.1 in the Appendix, the cross-sectional firm size distributions of the ASI/NSS sample and the Economic Census are very similar. We cannot rely on the Economic Census for our main analysis, because it does not contain information on firm age and hence cannot be used to estimate the employment life cycle or to measure firm entry.

Table 7 contains some basic descriptive statistics about the distribution of establishment size in the US and India.[25] Expectedly, the importance of large firms differs enormously. In the US, two-thirds of manufacturing employment is concentrated in establishments with at least 100 employees, and only one-third of the establishments have fewer than four employees. In India, more than nine out of ten establishments have fewer than four employees, and they account for more than half of aggregate employment. Because the Indian data is collected at the level of the establishment, our benchmark analysis will focus on individual establishments. We will conduct robustness checks using firm-level data for the US in Section 2.6.

Data on Managerial Employment: To measure managerial employment we rely on national census data provided by the IPUMS project. We focus on male workers in the manufacturing industry working in private-sector jobs. We always use the most recent data available, which is 2004 in the case of India and 2010 in the case of the US Our theory stresses the importance of *outside* managers. We therefore classify employees as managers if they are assigned the occupational code "Legislator, Senior official, and manager" *and*

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[25]Recently, Rotemberg and White (2017) argued that the data in the US and India differ in terms of their data cleaning strategies. These concerns are less relevant for our study because we only rely on sample averages of the reported employment data and do not utilize information on any higher moments, which are important for the measurement of misallocation. However, we did recalculate all estimation moments after dropping firms in the top and bottom 2% of the employment distribution (both in the population of firms and conditional on age) and found that this did not affect our analysis much.

| | | Establishment Size | | | | Empl. share |
| | | 1 - 4 employees | | $\geq$ 100 employees | | of outside |
	Average empl.	Share	Empl. share	Share	Empl. share	managers
US	42.7	32.8%	1.8%	8.8%	65.6%	12.4%
India	2.7	93.0%	54.8%	0.1%	18.6%	1.7%

Notes: The table contains summary statistics from the firm size distribution in the US and India. The US data come from the BDS in 2012, the data for India from the NSS and ASI in 2010. In the last column, we report the share of outside managers, i.e. all workers who are classified as managers according to the occupation classification ISCO and who are hired as wage workers. This data stems from IPUMS.

Table 7: Establishment Size and Managerial Employment in the US and India

they are hired as wage workers instead of being, for example, unpaid family members or the owner themselves. As shown in the last column of Table 7, in the US roughly 12.4% of employees satisfy this criterion. In India, less than 2% are employed as outside managers.

Insisting on outside managers is important. For the case of the US, roughly 14% of the labor force is classified as managers according to their occupational code. The majority, namely 90%, are wage workers and hence outside managers in the sense of our theory. This is very different in the case of India, where, conditional on working in a managerial occupation, only 12% of individuals are wage workers, and the remainder of individuals working in managerial occupations are either entrepreneurs themselves or unpaid family members. Hence, Indian firms acquire managerial services mostly from their owners or close family members. This pattern is very much the exception in the US

An important implication of our model is that firms' demand for outside managers is non-homothetic: larger firms have higher managerial employment shares. In Table 8, we show that such non-homotheticities are the norm in the Indian firm-level data.[26] While firms with 1-4 employees have essentially no managerial personnel, firms with more

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[26]The definition of outside managers is similar between the firm-level data and the data on IPUMS. The firm-level data has an employment category "supervisory and managerial staff". This category contains everyone who holds positions of supervision and management and who are working proprietors and managers when paid a regular salary. This is distinct from the category "working proprietors", which comprises all owners who are actively engaged in the work of the enterprise and all unpaid working proprietors. We use the managerial employment share from IPUMS as our main calibration target to ensure that the classification is consistent between the US and India.

than 100 employees have managerial employment shares exceeding 10%. The aggregate managerial share as measured from the firm-level data is 2.8%, which is reasonably close to the 1.7% reported in IPUMS. Below we show that the predictions of our model are also quantitatively in line with Table 8.

| | Number of employees | | | | | | | Full |
	1-4	5-9	10-19	20-49	50-99	100-999	+1000	Sample
Share of managers	0.002	0.017	0.043	0.077	0.079	0.101	0.147	0.028

Notes: The table reports the share of managerial employment among firms of a given size and for the aggregate economy. The data combines the NSS data from 1995 and the ASI data from 1999. 1995 is the only year where we observe managerial hiring in the NSS data and 1999 is the closest year for which we have access to the ASI data.

Table 8: Non-homothetic managerial demand in India

While it is natural to measure such non-homotheticities from the firm-level data, doing so has the disadvantage that we cannot report Table 8 for the US (because the BDS data does not have information on managerial employment). In Section B.2.1 in the Appendix (see in particular Figure 19), we use data from the Current Population Survey (CPS), which shows that managerial hiring is also non-homothetic in the US. In addition, because the IPUMS data for India (but not for the US) contains information on the size of establishment individuals work in, we also corroborate the results reported in Table 8 using the data from IPUMS.

2.3.2 Identification and Calibration

Our model has 12 parameters:

$$\Omega \equiv \{\underbrace{\alpha, \sigma, T, \mu_M, \vartheta}_{\text{Management}}, \underbrace{\theta, \theta_E, \zeta, \delta, \beta}_{\text{Firm dynamics}}, \underbrace{\gamma_t, \rho}_{\text{Macro}}\},$$

Five parameters are directly related to the demand for and supply of managerial services: the delegation efficiency (α), the managerial output elasticity (σ), the owners' own human capital (T), and the distribution of managerial skills (μ_M and ϑ). The process of firm

dynamics is captured by the expansion and entry efficiencies (θ and θ_E), the convexity of the cost function ζ, the share of high-type entrants (δ), and the difference in type-specific creative destruction rates (β). Finally, the remaining "macro" parameters include the innovation step size (γ_t) and the discount rate (ρ).

As highlighted above, we estimate most of these parameters separately for the US and India. We restrict three parameters to be the same across countries: ρ, ζ, and ϑ. We fix ρ and ζ exogenously and calibrate the remaining parameters by minimizing the distance between several empirical moments and their model counterparts.[27] In particular, let M^E denote the vector of S empirical moments and $M(\Omega)$ denote the vector of model-simulated moments. We then chose Ω to minimize the absolute relative deviation between the model and data, i.e., we solve

$$\min_{\Omega} \sum_{m=1}^{S} \frac{|M_m^E - M_m(\Omega)|}{|M_m^E|}.$$

Even though our parameters are calibrated jointly, below we provide a heuristic description of the relationship between the parameters and specific moments. In Appendix B.2.2, we give a more formal identification discussion and verify these relationships numerically using a sensitivity matrix, where we report the elasticity of each moment used in the internal calibration with respect to the parameters of the model (see Table 39 in Section B.2.5 in the Appendix).

Note that we allow the innovation step size γ_t to be country-specific and time-varying. In particular, we allow for the Indian economy to be along a transition path, i.e., catching-up with the US As far as the firm size distributions are concerned, we estimate the parameters under the assumption that the distributions are stationary. As we show formally in Section B.1.3 of the Appendix, our model implies that the firm-size distribution will

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[27]As we do not have data on spending on innovation, we do not attempt to estimate the curvature of the expansion cost function, ζ. Instead we follow the microeconomic literature, whose estimates imply a quadratic cost function, i.e., $\zeta = 0.5$. See Akcigit and Kerr (2018) and Acemoglu et al. (2018), who discuss this evidence in more detail. In Section 2.6 we provide a battery of robustness checks. We set the discount rate ρ equal to 5%.

remain stationary during the transition, i.e., despite the fact that the aggregate economy has not reached a BGP yet.[28] This allows us to calibrate all parameters independently of γ_t. In Section 2.5.2, we describe in detail how we discipline the evolution of γ_t.

Firm Dynamics: Identifying θ, δ, β and θ_E. The expansion efficiency θ is mostly identified from the profile of firms' life-cycle growth. This is seen in Panel A of Figure 10, where we depict average employment by age for different values of θ, holding all other parameters fixed. The higher θ, the faster firms grow conditional on survival. To identify the share of high-type producers δ, we focus on the age-profile of exit rates *conditional* on firm size. Without type heterogeneity, the likelihood of exit would be independent of age conditional on size. In the data, however, such conditional exit rates are strongly decreasing in firm age (see e.g., Haltiwanger et al. (2013a)). Through the lens of our model, this pattern is rationalized through endogenous selection, whereby the share of low-type firms within a given cohort declines as the cohort ages. This is shown in Panel B of Figure 10, where we display the exit rate of small firms by age for different values of δ. Without any heterogeneity, i.e., $\delta = 1$, the conditional exit hazard is flat. The parameter β, which determines how quickly low-type firms lose market share, is identified from the aggregate employment share of old firms. Intuitively, as high-type firms are older on average, the aggregate size of old cohorts is informative about this parameter. Finally, the entry efficiency θ_E is identified from the aggregate entry rate.

Identifying the delegation efficiency α. The delegation efficiency α is a crucial parameter of our analysis. As α directly affects firms' managerial demand, we aim to identify it from the aggregate employment share of outside managers. Doing so, however, requires us to address an important identification problem. Because the share of managers is increasing in firm size, the firm size distribution directly affects the aggregate managerial employment share. To see this, consider Figure 11, where we plot the managerial

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[28] Empirically, the firm size distribution in India is relatively stable over time, despite the fast convergence in income per capita (see Section B.2.8 in the Appendix).

| | Panel A: The Life-Cycle | | | | | Panel B: Exit Rate of Small Firms by Age |
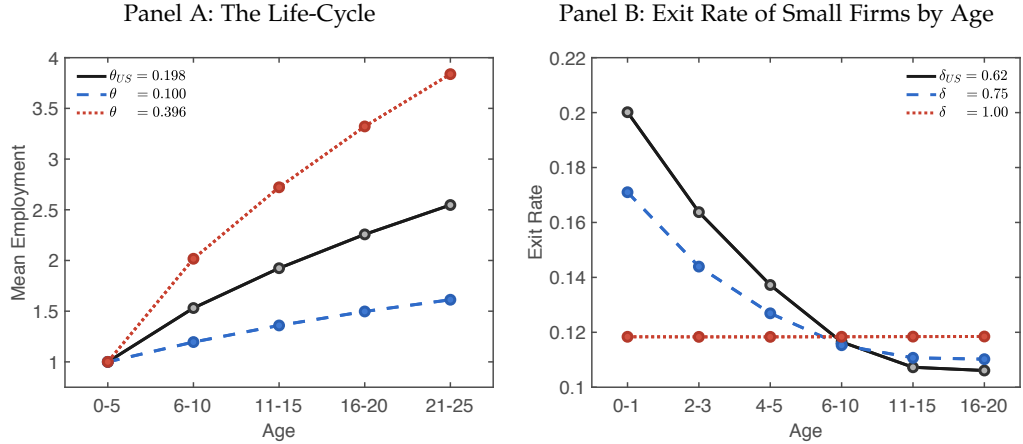
Notes: The left panel shows the employment life-cycle, i.e. average employment by age, for different values of θ. The right panel shows the exit rate of one-product firms by age for different values of δ. The black line depicts the US calibration (i.e., $\theta_{US} = 0.198$ in the left panel and $\delta_{US} = 0.62$ in the right panel). The other lines are obtained by varying θ (left panel) or δ (right panel) while keeping the rest of the parameters constant.

Figure 10: Identification of δ and θ

employment share by firm size and the employment distribution in India from our calibrated model. Holding α constant, the managerial share is higher for larger firms. More importantly, holding firm size fixed, the equilibrium managerial share is increasing in α. Because the aggregate managerial share is simply the integral of the firm level managerial shares with respect to the employment distribution, this raises the question whether managerial delegation in India is rare because it is difficult to delegate or whether other frictions keep Indian firms small and hence reduce the share of outside managers in the aggregate.

To credibly identify the efficiency of delegation α, we therefore need to simultaneously match the aggregate managerial employment share and the firm size distribution. Our model and calibration strategy allow us to do so. In particular, recall that the equilibrium firm size distribution is determined from firms' expansion schedules x_n and the entry rate z (see (2.19) and (2.20)). And by allowing the fundamental determinants of x_n and z, namely the firm-dynamics parameters θ, δ, β, and θ_E, to vary between the US and India in an unrestricted way, our calibration can match the firm size distribution using

these parameters and identify α from the residual variation in managerial employment shares between the US and India.[29]
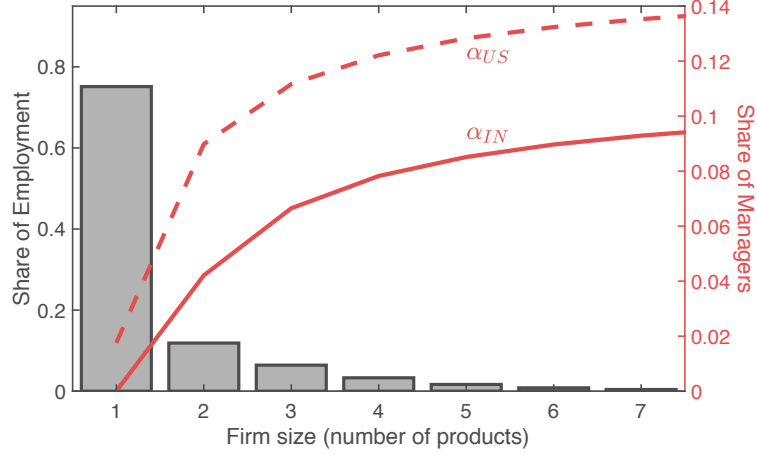
Notes: This figure shows the share of managers by firm size for two values of α and the calibrated Indian firm size distribution.

Figure 11: Identification of α

Identifying the "management elasticity" σ. We identify the parameter σ from the relationship between firm profits and managerial efficiency e. Using the profit function in (2.12) and the optimal amount of managerial efficiency $e = (\alpha\sigma/\omega_M)^{\frac{1}{1-\sigma}}$, profits can be written as

$$\tilde{\pi}(n) = (1 - \sigma)\, e^{\sigma} n + \sigma T e^{-(1-\sigma)}. \tag{2.26}$$

Equation (2.26) highlights that σ governs the relationship between managerial services e and firm profits. In fact, if firms' managerial demand was homothetic, i.e., if T was equal to zero, σ would exactly be the elasticity of profits with respect to e holding firm size n constant.

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[29]Differences in high types' growth potential θ could - in a reduced form way - capture differences in capital market efficiency which might prevent Indian firms from investing (see, for instance, Cole et al., 2016) or size-dependent policies, whereby Indian firms might be subject to steeper (implicit) tax rates (see, for example, Hsieh and Klenow, 2014a, Guner et al., 2008, Ulyssea, 2016, and Bento and Restuccia, 2017). Similarly, inefficiencies in the allocation of start-up capital, bureaucratic red tape or frictions in the labor market might induce more subsistence firms to enter in India ($\delta_{IND} < \delta_{US}$) or entry costs to be higher ($\theta^E_{IND} < \theta^E_{US}$).

An ideal way to estimate σ is to exploit exogenous variation in managerial inputs at the firm-level and subsequent changes in firm profitability. We therefore estimate σ via indirect inference and target the experimental evidence on the relationship between management practices and firm performance from Bloom et al. (2013).[30] The authors provided free consulting on the efficacy of 38 management practices to a set of randomly chosen textile establishments in India. These practices, which are standard in US firms, centered on factory operations, formalized quality control and inventory practices, and changes in human resource management like performance-based incentive pay. Using the random assignment of this managerial intervention, Bloom et al. (2013) estimate the treatment effect of managerial practices on subsequent output growth using the specification

$$\ln Output_{i,t} = \beta \times TREAT_{i,t} + f_i + \epsilon_{i,t}, \tag{2.27}$$

where $TREAT_{i,t}$ takes the value of one for the treatment plants starting one month after the end of the intervention period and f_i are a full set of plant dummies. They estimate (2.27) at the weekly level and find a treatment effect of 9% for a horizon of 100 weeks.

It is this treatment effect which we use as an "identified moment" to identify σ (Nakamura and Steinsson, 2018). To implement this experiment in our model, we need to take a stand on what the treatment means in our theory, i.e., how we translate the ordinal nature of the treatment into a cardinal increase in managerial services e among treated firms. Our strategy is as follows. In our model, firms' managerial environment is fully summarized by their managerial services e. We therefore relate firm f's optimally chosen managerial services e_f to the share of practices which firm f chooses to adopt and which we denote by MP_f. Note that like e in our theory, the adoption decision of the managerial practices in the experiment was also endogenous. In particular, the experimental intervention provided management consulting but left the eventual choice of which practices to adopt to the firms. Bloom et al. (2013, p. 22) explicitly report that the adoption deci-

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[30]See also Bruhn et al. (2018) for a related management intervention for small and medium enterprises in Mexico.

sion "was endogenous and it presumably varied with the cost-benefit calculation for each practice".

To link the unobservable e_f to the observable MP_f, we consider the measurement equation $e_f = v MP_f^\varrho$, where v and ϱ are positive parameters. Letting MP_{IND}^{Treat} be the share of managerial practices adopted by Indian firms after the treatment and MP_{IND} be the share among control plants, this implies that

$$\frac{e_{IND}^{Treat}}{e_{IND}} = \left(\frac{MP_{IND}^{Treat}}{MP_{IND}} \right)^{\varrho}.$$

For a given parameter ϱ, we can therefore infer the change in managerial service e due to the treatment from the change in managerial practices MP. To determine ϱ, we use data on differences in managerial practices between the US and India and the model-implied differences in managerial services, e_{IND} and e_{US}. In particular, letting MP_{US} denote the share of practices adopted by US firms, our measurement equation implies that $\frac{e_{IND}}{e_{US}} = \left(\frac{MP_{IND}}{MP_{US}} \right)^{\varrho}$. Hence, we can map the observed change in managerial practices among treatment firms to the change in e as

$$\ln \left(\frac{e_{IND}^{Treat}}{e_{IND}} \right) = \varrho \times \ln \left(\frac{MP_{IND}^{Treat}}{MP_{IND}} \right) = \frac{\ln \left(e_{IND} / e_{US} \right)}{\ln \left(MP_{IND} / MP_{US} \right)} \times \ln \left(\frac{MP_{IND}^{Treat}}{MP_{IND}} \right). \qquad (2.28)$$

In the microdata of the experiment, we find that $MP_{IND} = 0.25$, i.e. prior to the treatment, Indian firms adopt roughly 1/4 of the managerial practices. The treatment increases the adoption rate to $MP_{IND}^{Treat} = 0.63$. Given that all of these practices "have been standard for decades in the developed world" (Bloom et al., 2013, p. 43), we assume that firms in the US adopt all these practices, i.e. $MP_{US} = 1$.[31] Furthermore, for a given calibration of our model, we can calculate e_{IND} and e_{US}. We can then use (2.28) to calculate e_{IND}^{Treat}.[32]

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[31] In the Appendix B, we also provide additional corroborating evidence using the reported management scores from Bloom and Van Reenen (2007) (which are available both for firms in the US and for firms in India pre-treatment) for our assumption that $MP_{US} = 1$.

[32] To give a concrete example, our baseline calibration implies that Indian firms utilize only 71% as many managerial services as firms in the US, i.e. $e_{IND} / e_{US} = 0.71$. Together with $MP_{US} = 1$, $MP_{IND} = 0.25$ and $MP_{IND}^{Treat} = 0.63$, (2.28) implies that $e_{IND}^{Treat} / e_{IND} = 1.26$, i.e. we infer that the endogenous adoption of managerial practices from 0.25 to 0.63 corresponds to an increase in managerial efficiency in treatment firms

As we describe in detail in Section B.2.3 in the Appendix, our implementation takes the endogeneity of e_{IND}^{Treat} explicitly into account. In particular, we have to take a stand on *how* the experiment induced treatment firms to increase their e. Because the intervention provided information on how to use such managerial practices optimally, we model the treatment as a proportional increase in the productivity of treated firms' endogenous managerial services. Specifically, we assume that treated firms' total managerial resources are given by ξe and we choose ξ such that ξe^{Treat} coincides with the value implied by (2.28), where e^{Treat} denotes the optimal choice of e given ξ. In Section B.2.3 in the Appendix we show that ξ is given by $\xi = \left(e_{IND}^{Treat}/e_{IND}\right)^{1-\sigma}$. Importantly, we keep all general equilibrium variables constant in order to implement a partial equilibrium analysis consistent with the experiment.

We then relate this increase in managerial services to the resulting profits to estimate σ. Specifically, we take 50 firms from the very top of the firm size distribution of our calibrated Indian economy (consistent with the sample selection in Bloom et al. (2013)), treat them with the management intervention as described above, simulate their evolution for 100 weeks, and then estimate the treatment effect according to (2.27) in the model-generated data. While Bloom et al. (2013) estimate (2.27) using physical output as a measure of firm performance, we focus on total profits as the dependent variable in our model counterpart. We do so because profits are at the heart of our theory to link managerial services to firm performance.

Because the experiment was only conducted for firms in India, this strategy forces us to assume that σ is common across countries.[33] Because of the importance of this parameter, we also implement a complementary identification strategy which does not rely on the experimental evidence at all but only uses standard accounting data. The standard intuition from a constant elasticity production function suggests that the output

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by 26%.

[33]Bloom et al. (2016) use managerial scores to estimate production functions for managerial inputs across countries. They find that the coefficients on the managerial scores are very similar across countries.

elasticity should be related to relative cost shares. The same intuition is true in our model: the higher σ, the larger the share of managerial compensation relative to profits. More specifically, our model implies that

$$\frac{w_M nm(n)}{\Pi(n)} = \frac{\sigma}{1-\sigma}\left(1 - \frac{Tw_M}{\sigma\alpha\Pi(n)}\right), \tag{2.29}$$

where $w_M nm$ and $\Pi(n)$ denote total managerial payments and profits, respectively. Note that if firms had to rely only on outside managers, i.e., if $T = 0$, the demand for outside managers would be homothetic and σ would exactly reflect the relative compensation share. In our model, this mapping is slightly more complicated, but (2.29) shows that the managerial compensation share is directly affected by σ. Because we can measure this moment both for the US and India, this approach allows us estimate σ separately for both countries. As we discuss in Section 2.6, both of these approaches lead to similar results. In particular, the estimates for σ are almost identical between the US and India and only slightly lower than the estimates implied by our indirect inference strategy.

Identifying the remaining management parameters μ_M, ϑ and T. As we discuss in Section B.2.2 in the Appendix, all allocations in the model only depend on $\mu_M \times \alpha$. To separately identify the efficiency of managers within firms α from the supply of managerial skills μ_M, we require variation in the demand for managerial skills holding managerial human capital fixed. Intuitively, we would want to observe the same manager working with both the US and the Indian α. We mimic this experiment by using data from the New Immigrant Survey (NIS), which contains information about the pre- and post-migration occupations of recent immigrants to the U.S and has recently been used by Hendricks and Schoellman (2016). In Section B.2.4 in the Appendix, we show in detail how we can use this information to identify μ_M and α separately. Intuitively, Indian immigrants to the US are almost as likely to work in managerial occupations as US residents. However, they are much more likely to have worked in managerial jobs *prior* to emigrating. This

implies that the average managerial human capital of the non-selected, non-migrant Indian population is lower than in the US These two moments separately identify α and μ_M and allow us to perform our counterfactual, where we change the delegation efficiency α while holding the supply of managerial skills μ_M constant.

To identify the dispersion of the managerial skill distribution, ϑ, we note that it can be directly calibrated to match the dispersion in managerial earnings. In particular, the model implies that the variance of log managerial earnings is given by ϑ^{-2}. Finally, the owner's time endowment T is a fixed factor and firm profits are a renumeration for the provision of these services. We therefore calibrate T by targeting the entrepreneurial profit share, which is given by

$$\frac{\text{Aggregate Profits}}{\text{Total Sales}} = \frac{\sum_n \Pi(n)F_n}{Y} = \sum_n \tilde{\pi}(n)F_n, \tag{2.30}$$

where $F_n = F_n^H + \mathbb{1}(n=1)F^L$ is the number of firms with n products and $\tilde{\pi}(n)$ is increasing in T, holding aggregate prices fixed (see (2.12)).

2.4   Estimation Results

In this section we discuss our estimation results. Section 2.4.1 contains the structural parameters and targeted moments. In Section 2.4.2 we show that our model is also consistent with a variety of non-targeted moments. Finally, in Section 2.4.3 we use our estimated model to assess why firms in India are small.

2.4.1   Calibrated Parameters and Targeted Moments

Tables 9 and 10 contain the calibrated parameters and the targeted moments. For convenience, Table 9 also reports the main target for the respective parameter even though the parameters are calibrated jointly. For the US, we estimate 7 parameters and for India,

we estimate 8 parameters.

Panel A. Internal Calibration				
Parameter	Interpretation	Target	US	India
Firm Dynamics				
θ	Expansion efficiency	Employment life-cycle	0.198	0.059
δ	Share of high types	Exit profile by age (cond. on size)	0.620	0.107
β	Relative creative destruction	Empl. share of old firms	4.365	2.827
θ_E	Entry efficiency	Entry rate	0.100	0.099
Managerial Environment				
α	Delegation efficiency	Managerial employment share	0.429	0.203
μ_M	Average managerial human capital	Occupational sorting by immigrants	1.000[†]	0.420
ϑ	Dispersion of managerial human capital	Var of ln managerial earnings	1.429	1.429[*]
σ	Managerial output elasticity	Treatment effect of Bloom et al. (2013)	0.464[*]	0.464
T	Entrepreneurial time endowment	Average entrepreneurial profit share	0.156	0.261
Panel B. External Calibration				
ζ	Convexity of expansion costs		0.50	0.50
ρ	Discount rate		0.05	0.05

Notes: The table reports the parameter values that yield the model moments reported in Table 10. We denote normalized parameters by "[†]" and parameters which we do not estimate by "[*]".

Table 9: Estimated Parameters for the US and India

Consider first Table 9. The top panel shows that 90% of entering firms in India are subsistence entrepreneurs. This is very different in the US, where entrants are about six times as likely to be high types ($\delta_{US} \approx 6 \times \delta_{IND}$). In addition, such firms in the US are around 3.5 times as efficient in expanding into new markets as their Indian counterparts ($\theta_{US} \approx 3.5 \times \theta_{IND}$). At the same time, the costs of creating such superior firms are almost the same between the US and India ($\theta_{E,US} \approx \theta_{E,IND}$). Economically, we find these estimates plausible in that they capture the myriad reasons why firms in India might not expand (e.g., due to the presence of credit constraints or size-dependent policies) or why unproductive firms are abundant upon entry (e.g., because of low opportunity costs of entrepreneurship in India).

The next panel contains our estimates of the delegation environment. Our estimation implies that delegation in the US is about twice as efficient as in India ($\alpha_{US} \approx 2 \times \alpha_{IND}$). As highlighted above, this low estimate of α_{IND} is conditional on the other determinants of the firm size distribution, i.e. θ, δ, and θ_E. In fact, if we only calibrated our model to

the Indian firm-dynamic moments in Panel A, but kept the delegation efficiency at the US level, the managerial employment share would be around 5%, i.e., exceeding the level observed in India. Hence, while the fact that firms in India are small accounts for a sizable part of the lower share of managerial inputs, a less efficient delegation environment α is also required to explain the data.

We also estimate that managers in the US have more human capital, i.e., $\mu_{M,US} > \mu_{M,IND}$. This is inferred from the fact that the share of managers among Indian immigrants in the US is 12.9% (hence very similar to the overall manager share in the US), but they are much *more* likely to work as managers prior to migrating compared to the Indian population. Therefore, the unselected population in India has a comparative disadvantage in managerial occupations.

	US		India	
	Data	Model	Data	Model
Firm Dynamics				
Entry rate (%)	7.35	7.35	5.60	5.60
Exit profile by age (cond. on size)	1.55	1.55	1.10	1.09
Employment life-cycle	2.55	2.55	1.12	1.12
Employment share of old firms (%)	8.10	6.30	7.70	6.00
Managerial Environment				
Managerial employment share (%)	12.4	12.4	1.70	1.70
Treatment effect from Bloom et al. (2013) (%)	n/a	n/a	9.00	9.00
Relative managerial share of Indian migrants	n/a	n/a	2.11	2.11
Average entrepreneurial profit share (%)	21.0	21.0	48.3	45.8
Variance of ln manager earnings	0.49	0.49	0.45*	0.49

Notes: The table reports both the data moments and the corresponding moments in the model for the US and India. We define "old" and "young" firms as firms of age 21 - 25 years and 1-5 years respectively. We define small firms as firms with 1-4 employees in the data and with a single product in the model. See Section B.2.1 in the Appendix for details. "*" denotes that the moment is not targeted in the calibration.

Table 10: Moments for the US and India

In Table 10, we report the targeted moments. The first two columns contain the US calibration. Our model is able to rationalize most moments well. In particular, it matches the observed employment life cycle (whereby firms of age 21-25 years are about 2.5 times

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as large as firms younger than 5 years), the aggregate entry rate, and the differences in exit rates (whereby small young firms, which exit at a rate of 21% per year, are around 1.5 times as likely to exit as small old firms, which have an exit rate of 14%). The model slightly underestimates the aggregate employment share of old firms.[34]

The model also matches the aggregate share of managerial workers of 12.4% reported in Table 7, an entrepreneurial profit share of about 20%, and the dispersion of log managerial earnings.[35] Although we assume ϑ to be identical across countries for simplicity, the dispersion of log managerial earnings in India is essentially the same as in the US[36]

The model is similarly successful to match the moments of the Indian economy reported in columns 3 and 4. In particular, it replicates the essentially flat life-cycle of Indian establishments, the low share of aggregate managerial employment, and that young establishments exit almost at the same rate as old establishments. As is the case for the US calibration, the model slightly underestimates the share of old firms in the economy.[37] Also note that firms in India have a much higher share of entrepreneurial profits compared to firms in the US This is due to the fact that most firms in India are small so that most of their sales are attributed as entrepreneurial compensation for the provision of the fixed factor T.

Finally, the model is able to replicate the treatment effect of Bloom et al. (2013). This

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[34]One reason is that in our model growth is only driven by the extensive margin of adding products. Hence, the process of growth and the resulting exit hazard are tightly linked. If we allowed for growth on the intensive margin (e.g., through quality innovations within existing product lines as in Akcigit and Kerr, 2018, or Garcia-Macia et al., 2016), we could break this link.

[35]Empirically, we target the variance of residual log managerial earnings in the manufacturing sector after controlling for a quadratic in age and industry fixed effects within the manufacturing sector.

[36]Note also that our distributional assumption of managerial human capital implies that the average wage of managers relative to production workers within a country is given by $\vartheta/(\vartheta - 1)$. When we look at this implication in the micro-data, we find that managers in the US (India) earn a premium of 0.54 log points (0.78 log points). Both of these are lower than the model-implied premium given the estimate of ϑ, which is 1.19 log points. Because ϑ plays the role of a labor supply elasticity, we prefer to target the dispersion in wages, which is more directly related to the scope of selection. In Section 2.6 we discuss how different assumptions about this supply elasticity affect our results.

[37]At first glance it might be surprising that old firms, i.e., firms of ages 21-25, have roughly the same aggregate employment share in the US and India. The reason is that the aggregate employment share of *very* old firms is much higher in the US In the US (India) the share of firms older than 25 years is 55% (20%). See Sections B.2.7 and B.2.8 in the Appendix for details.

is important, because in order to credibly quantify the aggregate effects of changes in the efficiency of delegation, it is reassuring that our model is quantitatively consistent with well-identified microeconomic evidence on the dynamic effects of changes in managerial efficiency at the firm-level. Matching the estimated treatment effect requires an estimate of σ around 0.46. As discussed in detail above, for our baseline analysis we restrict σ to be the same across countries. In Section 2.6, we discuss an alternative strategy where we estimate σ from accounting data and allow it to be country-specific.

2.4.2 Non-targeted Moments

Our model also performs well in matching a variety of non-targeted moments. In particular, we focus on the non-homotheticity of managerial demand, firms' survival hazards and the number of products firms sell. Additionally, we also discuss some qualitative patterns in the delegation decisions of Indian firms based on a regression analysis and compare them to the predictions of our theory.

Non-homothetic Managerial Demand A key mechanism of our model is that large firms endogenously increase their span of control by hiring outside managers. In particular, larger firms are more likely to hire any outside managers and they hire more per product, conditional on hiring. Because the Indian data reports managerial hiring at the firm-level, we can look for these implications in the data.

Our model predicts both the extensive and intensive margin of managerial hiring well. Regarding the extensive margin our model implies that 73% of all Indian firms run their operations without outside managers. Empirically, we find that 77.5% of firms in India do not hire any managers. In Figure 12, we show that our model is also quantitatively consistent with the relationship between managerial employment shares and firm size conditional on hiring. To compare the model and the data (which we reported below in Table 8), we focus on quantiles of the firm size distribution. In particular, going from

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right to left, we plot the share of managerial employment among the largest 0.1%, the largest 1%, the largest 5% of firms, and so on. Hence, by going from right to left, we trace out the average managerial share as a function of the firm size distribution. At the far left, we report the share among the 100% largest firms, which is simply the entire sample of firms. Hence, in the data, the managerial share is the sample average of 2.8% (see Table 8), and in the model, it is 1.7%, our calibration target from the IPUMS data. Figure 12 shows that our model replicates the "delegation-firm size" relationship observed in India very well even though we do not target it explicitly.
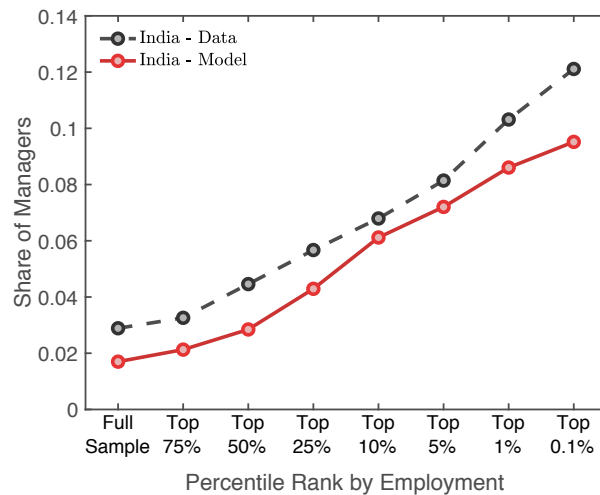
Notes: This figures shows the employment share of managers among firms in the top *x*% of the firm-size distribution for $x = 0.1\%, 1\%, 5\%, ...$ We report the data using a black dashed line and the model using a red solid line. See also Table 8 for a summary of the data.

Figure 12: Managerial Demand By Firm Size

Survival Hazards In Figure 13, we compare our model to two measures of the degree of selection. In Panel A, we depict the survival rate, i.e., the size of a given age cohort relative to the entering cohort. The rate of firm survival is reasonably similar in the US and India – both in the data and in the model.[38] In Panel B, we show the share of small

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[38]As for the category of 26+ firms: Note that this is the *accumulated* stock of surviving firms, who are older than 26 years. Hence, even though the US exit rates are only slightly lower than those in India, the small differences in the flow of exit add up to a sizable difference in the stock of old firms. See also Figures

firms by age (relative to their share among young firms). While the share of small firms in the US declines to 40% by the age of 25, the vast majority of old firms in India are still small. Our model again replicates these patterns reasonably well.

Panel A: The Survival of a Cohort   Panel B: The Share of Small Firms By Age
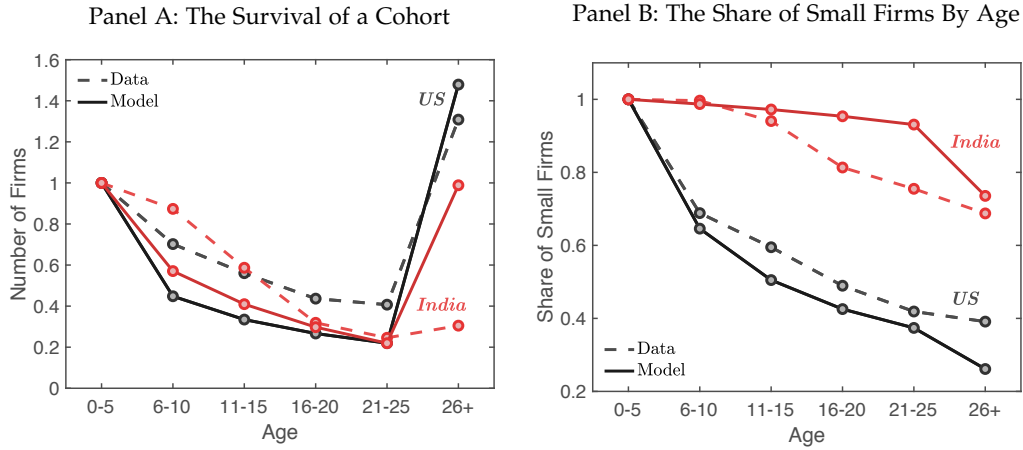
Notes: Panel A depicts the share of firms by age relative to the share of firms in the youngest age category. Panel B shows the share of small firms by age. We show the data using solid lines and the model using dashed lines. In the US small firms are firms with 1 - 4 employees. In India small firms are firms with one employee.

Figure 13: Firm Selection in the US and India

The Product Line Distribution   In our model, a firm is a collection of product lines. Our calibration focuses only on employment data to measure firm size and does not use data at the product level. Both the US and the Indian data, however, contain information on the number of 5-digit product codes in which individual firms are operating.[39] In Figure 14, we plot the distribution of firm-level product counts in the data and the model. Our model matches this aspect of the data remarkably well, despite the fact that this moment is not targeted. In particular, the vast number of Indian firms indeed produce only a single product.

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2 and 3 in Hsieh and Klenow (2014a), who show that exit rates are only slightly lower in the US but that the aggregate employment share of old firms is vastly larger in the US

[39]The data for the US firms come from Acemoglu et al. (2018)
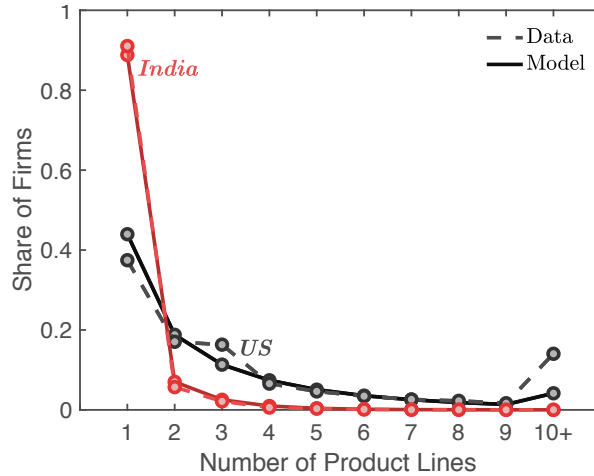
Figure 14: The Product Line Distribution

Qualitative Predictions on Delegation in the Indian Micro Data   Finally, we can look at some qualitative predictions of our theory.[40]   Our theory implies that firms do not hire outside managers if their size falls short of the delegation cutoff, i.e., if $n < n^* = T \left(\frac{\omega_M}{\sigma \alpha} \right)^{\frac{1}{1-\sigma}}$. Hence, firms are more likely to delegate if (i) firm size n increases, (ii) delegation becomes more efficient, i.e. α increases, and (iii) the owner's inelastically provided managerial human capital T is smaller.

To take these predictions to the data, we follow Bloom et al. (2013, p. 4), who argue that for Indian textile firms "managerial time was constrained by the number of male family members. Non-family members were not trusted by firm owners with any decision-making power, and as a result firms did not expand beyond the size that could be managed by close (almost always male) family members." Hence, we take the size of the entrepreneur's family as a proxy for T. Moreover, we use regional variation in trust within India to proxy for variation in α. The latter is calculated from the World Values Survey as the share of people providing the answer "Most people can be trusted" within the Indian state where the firm is located. This is the most common measure of trust used in the literature (see, for instance, La Porta et al. (1997)).

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[40]See Section B.2.6 for the details of the empirical analysis. There we also provide an explicit derivation of the regression equations based on our theory.

We then regress firms' managerial hiring decision on firm size, household size and regional trust in 22 Indian states. We always control for the market of a firm, i.e. whether or not the firm is urban or rural, firm age, state-level GDP per capita, and 2-digit sector fixed effects. Due to space constraints, we only report the estimated equation; the full analysis can be found in Appendix B.2.6. We find that:

$$\mathbb{1}(\textbf{\textit{Firm hires managers}}) = \underset{(0.003)^{***}}{0.039} \times \textbf{\textit{Firm Size}} \underset{(0.001)^{**}}{-0.003} \times \textbf{\textit{Family Size}} \underset{(0.006)^{***}}{+0.013} \times \textbf{\textit{Trust}},$$

where *"Firm Size"* and *"Family Size"* are the logarithms of the number of employees and household members, respectively. Hence, as predicted by our theory, firm size and regional trust correlate positively, whereas family size correlates negatively, with the probability of hiring an outside manager. These results are consistent with Bloom et al. (2012) who, using data from a survey on managerial practices, show that high trust areas delegate more decision power to managers.

Our model also has implications for the relationship between family size and firm size. In our model, managerial resources within the family, T, are the constraining factor for firm size. This constraint, however, is less important the higher the delegation efficiency α becomes. Hence, while family size should be a predictor of firm size, the effect should be particularly strong in regions where trust, and hence the possibility of delegation, is less developed. We can test this prediction from the interaction between trust and family size. This also allows us to include a full set of state-fixed effects in the regression to control for all characteristics (including the level of trust) which are constant within Indian states. As before, we also control for the location of the firm (rural vs. urban), firm age, and 2-digit sector fixed effects. We find that

$$\textbf{\textit{Firm Size}} = \underset{(0.278)^{***}}{0.812} \times \textbf{\textit{Family Size}} \underset{(0.758)^{*}}{- 1.329} \times \textbf{\textit{Family Size}} \times \textbf{\textit{Trust}},$$

i.e., there is a positive correlation between family size and firm size which is particularly strong in low-trust regions. Through the lens of our model, this occurs due to the imperfections in delegation in those regions.

2.4.3 Why are Indian Firms Small? Role of Selection and Creative Destruction

The estimated model allows us to give a structural interpretation of the observed differences in firm dynamics between the US and India. Our theory stresses that two key determinants are the extent of selection and the rate of creative destruction. Although neither of these mechanisms is directly observable, we can measure them through the lens of the model.

In Table 11, we report a set of statistics from the stationary distribution. First of all, note that our calibration implies that creative destruction in the US is twice as large as in India. At first glance, it seems surprising that we infer large differences in creative destruction despite the fact that both aggregate entry and exit rates and firms' survival probabilities by age are quite similar (see Figure 13). The key to reconciling these facts is to realize that the underlying distributions of firm size are vastly different in the US and India. Recall that the number of exiting firms is the *product* of the mass of firms operating in a single market and the rate of creative destruction. The fact that exit rates are quite similar despite the fact that many firms in India are small and hence close to the exit threshold implies that creative destruction in India has to be substantially smaller. Conversely, most creative destruction in the US takes place in infra-marginal markets where firms lose market share without exiting.

In the remaining rows of Table 11, we report different aspects of the degree of selection. In the stationary distribution of the US, around 95% of firms are high-type firms (compared to 62% at the time of entry), and they have a combined employment share of 98%, as they are bigger on average. In India, even in the long-run, high-type firms

	India	US
Rate of creative destruction, τ	0.054	0.126
Share of high-type firms upon entry (δ)	0.107	0.620
Long-run share of high-type firms	0.337	0.946
Long-run employment share of high-type firms	0.466	0.985
Long-run share of high-type firms among firms of age 21-25	0.291	0.999

Notes: The table contains various equilibrium objects from the stationary distribution of the calibrated models. The models are parametrized according to Table 9.

Table 11: Creative Destruction and Selection in India and the US

account for only 34% of firms and 47% of aggregate employment. This slower weeding out process of low-type firms in India is also highlighted by the fact that even among old firms, more than two-thirds of them are subsistence entrepreneurs. This is in stark contrast to the US, where the population of old firms is only comprised of high types.

In Figure 15, we display the dynamics of this "shake-out" process by tracing out the share of high-type firms within a cohort at different ages. Not only is the share of high-type firms in the US significantly greater among the entering cohort, they also grow much faster, creating a much stronger selection force. This selection process is dampened in India: even among 30-year-old plants, more than half are low-type firms. Importantly, this lack of selection in India is not only due to fact that there are few high-type firms to begin with. To illustrate this distinction, we simulate a counterfactual cohort of US firms which starts with the initial type distribution of India, i.e., where the initial share of high-type firms is δ_{IND}. Figure 15 shows that differences in growth incentives of high-type firms in the US and India are a key aspect of the selection dynamics: by the age of 15, this counterfactual cohort in the US would again be populated by mostly high-type firms.

2.5   The Aggregate Importance of Delegation Efficiency

To what extent are differences in the efficiency of delegation responsible for the observed differences in firm dynamics and aggregate economic performance between the
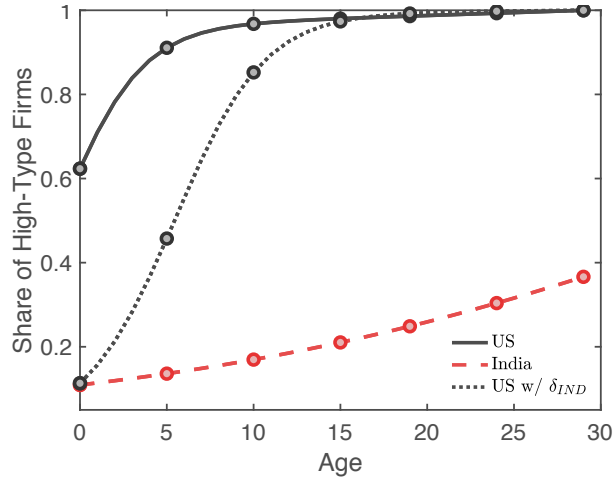
Notes: The figure shows the share of high-type firms by age both for the India calibration (red line) and for the US calibration (black line). It also shows the counterfactual share of high-type firms by age if the initial share of high-type firms in a cohort in the US is given by its Indian counterpart δ_{IND}. All calibrated parameters are taken from Table 9.

Figure 15: Endogenous Selection

US and India? To answer these questions, we study a counterfactual Indian economy where we increase α from α_{IND} to α_{US} while keeping the rest of the parameters at their calibrated levels. We first quantify the effects on firm-level outcomes. We then turn to the aggregate effects and study the link between α and aggregate income differences.

2.5.1 Delegation Efficiency and Firm Dynamics

The firm-level implications are summarized in Table 12. In Panel A, we focus on the changes in firm expansion, entry, and creative destruction. Incumbents' expansion incentives are much more responsive than the entry margin. While firms' expansion rates increase by 24% on average, the entry intensity increases only by 1.5%. These differences are due to the fact that outside managers are complementary to firm size and therefore not very important for subsistence firms, which never grow. This complementarity also implies that the expansion rate of large firms is particularly responsive.

At the aggregate level, however, the increase in creative destruction is much closer to the change in the entry intensity. The reason is that the market share of high-type firms

	Panel A: Equilibrium outcomes					
	Average	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
Expansion rate $x(n;\alpha)$	+24.42%	+14.62%	+19.11%	+20.80%	+21.54%	+21.89%
Entry intensity $z(\alpha)$			+1.51%			
Creative destruction τ			+4.32%			
Share of outside managers			+141%			
	Panel B: Implications for firm dynamics					
Average firm size			+3.79%			
Share of high type firms			+3.40%			
Empl. share of small firms			−3.33%			
	Effects by age					
	<=5	6-10	11-15	16-20	21-25	+26
Average firm size	2.08%	2.12%	2.27%	2.54%	2.95%	5.92%
Share of small firms	-0.08%	-0.30%	-0.58%	-0.99%	-1.53%	-5.90%

Notes: The table reports the changes in various equilibrium outcomes after increasing the delegation efficiency in India from α_{IND} to α_{US}. "Small firms" are those with a single product. All changes refer to changes in the stationary distribution.

Table 12: Increasing the Delegation Efficiency in India: Firm-level implications

in India is relatively small, so that the majority of creative destruction is accounted for by new entrants. Finally, the equilibrium employment share of outside managers would more than double to 4.1%. Note that this is still way below the level in the U.S, because Indian firms are still substantially smaller than their US counterparts.

In Panel B, we report the implications for the resulting process of firm-dynamics. If Indian firms could employ outside managers as efficiently as firms in the US, average firm size would increase by 3.8%, the share of high-type firms would increase by 3.4%, and the importance of small producers would decline by 3.3%.[41] The last two rows of Panel B show that these changes stem mostly from older firms, which are on average larger and hence more likely to rely on outside managers. Quantitatively, firms between 21 and 25 years old see their average employment rise by 3% and their share of single-product firms decline by 1.5%. The reason why these effects are small compared to the increase

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[41]Our calibrated model predicts that firms in the US are on average roughly 2.5 times as large as firms in India. Note that this number is not comparable to the empirical size difference of 15.8 as reported in Table 7. The reason is that in our model entrants in the US start at the same size as entrants in India. Empirically, entrants in the US have on average 13.7 employees, while entrants in India have 2.5. Entrants in the US are therefore 5.5 times as large as entrants in India. Hence, relative to the initial size difference, US firms are 15.8/5.5 = 2.8 times as large as firms in India.

in high types' expansion rate, $x(n; \alpha)$, is again due to the lack of selection as even among old firms, the majority of firms in India are subsistence producers. The effect of α on the process of firm dynamics in India is therefore modest.

The Importance of Complementarities   The results in Table 12 highlight the interaction between the ease of delegation and other aspects of the economy. In particular, improvements in the efficiency of delegation are more potent if high-type firms are plentiful and those firms can expand easily. To see that this intuition is indeed correct, Table 13 presents the US analogue of Table 12.[42] Compared to the results for the Indian economy, we find that a decrease in the efficiency of delegation in the US to the Indian level would affect firm growth substantially. In particular, the rate of creative destruction decreases by 25%, average firm size declines by 13%, and the employment share of small firms increases by 19%. Similarly, the effects on managerial hiring are also larger in the US If outside managers were as inefficient as their Indian counterparts, the equilibrium managerial share would decline from 12.4% to 5.3%. The reason for such stark differences is that high-type firms are abundant in the US and their expansion costs are low. Preventing these dynamic entrepreneurs from growing affects the process of firm dynamics substantially.

Panel A: Equilibrium outcomes			Panel B: Implications for firm dynamics			
Average Expansion rate	Entry intensity	Creative Destruction	Average Firm Size	Share of high type firms	Empl. Share small firms	Share of managers
-28.5%	-10.0%	-24.8%	-13.3%	-0.3%	+18.7%	-57.1%

Notes: The table reports the changes in various equilibrium outcomes after decreasing the efficiency of delegation in the US from α_{US} to α_{IND}. "Small firms" are those with a single product. All changes refer to changes in the stationary distribution.

Table 13: Decreasing Delegation Efficiency in the US

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[42]For brevity we only report the aggregate outcomes. The results by firm size and firm age are available upon request.

2.5.2 Delegation Efficiency and Aggregate Income Differences

How important are frictions to delegate decision power in Indian firms for the gap in income per capita gap between India and the US? To answer this question we need to specify the evolution of the step size γ_t. Because we can estimate all other parameters of the model independently, our earlier results do not depend on these assumptions in any way.

We consider a parametrization of our model where the distribution of income between the US and India is stationary in the long-run. To achieve this, we assume that the Indian economy (by being technologically backward relative to the US) benefits from "catch-up" growth and a higher step-size γ. To capture this intuition in a parsimonious way, we assume that the Indian step-size $\gamma_{IND,t}$ is related to the technological gap $Q_{US,t}/Q_{IND,t}$ and given by

$$\gamma_{IND,t} = \gamma_{US} \times \left(\frac{Q_{US,t}}{Q_{IND,t}} \right)^{\lambda}, \tag{2.31}$$

where $\lambda \geq 0$ and γ_{US} is the step size for the US, which we assume to be constant.[43] Equation (2.31) captures – in a reduced form way – the presence of knowledge spillovers. If $\lambda > 0$, the lower the relative technology in India, the higher the innovation step size. If $\lambda = 0$, there are no "advantages from backwardness" (Gerschenkron, 1962). Importantly, the formulation in (2.31) implies that income differences between the US and India will be constant in the long-run. To see this, note that along a BGP where $g = \ln(\gamma_{US})\tau_{US} = \ln(\gamma_{IND})\tau_{IND}$, equation (2.31) implies that

$$\ln \left(\frac{Q_{IND,t}}{Q_{US,t}} \right) = \frac{\ln \gamma_{US} - \ln \gamma_{IND}}{\lambda} = \frac{\ln \gamma^{US}}{\lambda} \times \left(\frac{\tau_{IND} - \tau_{US}}{\tau_{IND}} \right). \tag{2.32}$$

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[43]Taking the US as the frontier economy is purely for simplicity. Suppose there is an exogenous technological frontier $Q_{F,t}$, which grows at rate g. Suppose that the step size in country c is given by (2.31) *relative* to this frontier, i.e. $\gamma_{c,t} = \gamma \times (Q_{F,t}/Q_{c,t})^{\lambda}$. If the US economy has already reached its BGP, (2.31) holds with $\gamma_{US} = g/\tau_{US}$.

This expression highlights that the long-run distribution of technology Q across countries is stationary and determined by differences in creative destruction. Differences in delegation efficiency α, by affecting the rate of creative destruction, therefore manifest themselves in level differences, not in growth differences in the long run. During the transition, an increase in α increases the growth rate of $Q_{IND,t}$. In addition, a change in α has static consequences as it increases the amount of managerial efficiency units, $\mathcal{M}_t$, and hence raises income per capita, holding the level of Q_t fixed (see (2.6)).

To quantify the strength of these forces, we consider an experiment where in 2010 th delegation efficiency α in India increases unexpectedly and permanently from α_{IND} to α_{US}. We then trace out the dynamic evolution of the Indian economy. To do so, we need to calibrate γ_{US}, λ, and the initial productivity differences between the US and India. We assume that the US economy is on a BGP and choose γ^{US} to match a growth rate of 2%, given the rate of creative destruction reported in Table 11. India, in contrast, is still catching up to the US economy. Empirically, relative productivity in the US, vis-à-vis India, decreased substantially from about 4 in 1985 to 3.2 in 2005 (see Section B.2.2 in the Appendix, in particular Figure 20). We therefore calibrate λ and the relative productivity between the US and India in 1985, $Q_{IND,1985}/Q_{US,1985}$, to match these time-series dynamics. This exercise implies that $\lambda = 0.3$.[44]

In Table 14, we summarize the aggregate implications of this experiment. In Panel A, we report the implications for the growth rate of the technology index Q_t. On impact, the growth rate increases by about 0.16 percentage points in 2010. Over time, this growth rate differential between the baseline and the counterfactual Indian economy declines, and in the long run, both countries grow at the same rate. In Panel B, we calculate the cumulative effect of this higher growth rate on the (relative) level of Q_t. In 2000, the

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[44]While we use plant level data from the manufacturing sector for the firm-related moments, here we rely on data about aggregate TFP. As long as relative TFP in the manufacturing sector, $TFP_{IND}^{Manu}/TFP_{US}^{Manu}$ shows the same rate of catch-up, our analysis will be valid. If aggregate TFP in India were to show faster catch-up (e.g., due to the reallocation of workers out of agriculture), our estimate of λ would be upward biased and we would underestimate the aggregate consequences of changes in α - see equation (2.32).

technology in India is about 26.8% of the US level. Our baseline estimates imply that long-run technological differences between the US and India would be about 49%. If delegation in India were as seamless as in the US, relative technology in India would be equal to 52%. Hence, limits to delegation can account for $\frac{51.9-49.3}{100-49.3} \approx 5.1\%$ of the long-run technological gap between the US and India.

The effects on income per capita, shown in Panel C, are larger. In the long run, an increase in the efficiency of delegating managerial tasks would raise relative income per capita in India from 51.3% to around 56.8%. This accounts for $\frac{56.8-51.3}{100-51.3} \approx 11\%$ of the aggregate gap in income per capita. The effects are larger because of the static effects captured by $\mathcal{M}$. In particular, the magnitudes of the static effects of better delegation and the dynamic effects operating though higher creative destruction are roughly equal.[45] For completeness, we also report the long run change in consumption per capita in Panel D, which - in contrast to the comparison of income per capita - also takes the resources spent on entry and expansion efforts into account.

2.6   Robustness

In this section, we discuss the robustness of our results. For each specification, we recalibrate both the US and the Indian economy and redo our analysis. Overall, we find that our main conclusions are fairly robust. All results are reported in Table 15. We report the implied levels of creative destruction in both countries (columns 1 and 2) as a summary statistic of the respective calibrations and the changes in creative destruction, relative technology and income, average firm size, and the share of small firms among 21 to 25 year-old firms in India due to an increase in α to the US level. In Panel A of Table 15, we report our baseline results for comparison.

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[45]Additionally, the increase in α also reduces the number of production workers as individuals sort into managerial occupations. Quantitatively, the number of production workers declines by about 2.4% along the BGP.

Year:	2000	2010	2020	2030		∞
Panel A: Productivity Growth g_Q						
Baseline	3.00%	2.85%	2.72%	2.61%	...	2.00%
$\alpha = \alpha_{US}$	3.00%	3.01%	2.84%	2.70%	...	2.00%
Panel B: Relative productivity Q_{IND}/Q_{US}						
Baseline	26.8%	29.4%	31.7%	33.9%	...	49.3%
$\alpha = \alpha_{US}$	26.8%	29.4%	32.2%	34.8%	...	51.9%
Panel C: Relative income pc y_{IND}/y_{US}						
Baseline	27.8%	30.5%	33.0%	35.3%	...	51.3%
$\alpha = \alpha_{US}$	27.8%	32.6%	35.6%	38.4%	...	56.8%
Panel D: Relative Consumtion c_{IND}/c_{US}						
Baseline	29.2%	32.0%	34.6%	37.0%	...	53.9%
$\alpha = \alpha_{US}$	29.2%	34.0%	37.2%	40.1%	...	59.5%

Notes: The table reports the aggregate implications of an increase of the efficiency of delegation in India from α_{IND} to α_{US} in the year 2010. We report the rate of growth of the productivity index Q_t (Panel A), the differences in Q_t between the US and India (Panel B), the differences in income per capita (Panel C), and the differences in consumption per capita (Panel D). This results are based on an estimate for λ of 0.300 (see Section B.2.2 in the Appendix).

Table 14: Increasing Delegation Efficiency in India: Macroeconomic Implications

To summarize: our baseline calibration is qualitatively robust across the different alternatives we consider. The most important parameters are the "management elasticity" σ, the elasticity of labor supply, and the dispersion of managerial human capital ϑ. If anything, we find that the aggregate results of our baseline calibration are likely to be conservative.

Alternative estimates of σ: Our baseline estimates of σ are identified from the estimated treatment effect of the managerial intervention of Bloom et al. (2013). A concern with this strategy is that we had to restrict σ to be constant across countries. In Panels B and C, we report the results from an alternative strategy that addresses these limitations. In Panel B, we consider a calibration, which does not rely on the experimental results of Bloom et al. (2013), but instead uses the share of managerial compensation in total

| | τ_{IND} | τ_{US} | Change in ... due to the increase from α_{IND} to α_{US} | | | | |
			τ_{IND}	Q_{IND}/Q_{US}	y_{IND}/y_{US}	Avg. firm size	Share of small firms
Panel A. Baseline Calibration							
	0.054	0.126	4.32%	5.25%	10.72%	3.79%	-1.53%
Panel B. Estimating country-specific σ from accounting information							
	0.057	0.119	4.66%	6.23%	12.99%	0.76%	-1. 02%
Panel C. Estimating country-specific σ from Bloom et al. (2013) and accounting information							
	0.055	0.129	5.77%	8.19%	15.80%	3.58%	-1.64%
Panel D. Entry elasticity ζ_e							
$\zeta_e^L = 0.4$	0.054	0.126	3.96%	4.82%	10.21%	3.91%	-1.52%
$\zeta_e^H = 0.6$	0.054	0.126	4.79%	5.81%	11.39%	3.63%	-1.56%
Panel E. Convexity of expansion technology ζ							
$\zeta^L = 0.4$	0.054	0.124	4.42%	5.42%	11.50%	2.30%	-1.13%
$\zeta^L = 0.6$	0.054	0.129	4.07%	4.86%	9.60%	5.57%	-2.07%
Panel F. Estimation with firm level data							
	0.054	0.115	4.35%	5.41%	11.04%	3.23%	-1.53%
Panel G. Strength of knowledge diffusion λ							
$\lambda^L = 0.220$	0.054	0.126	4.32%	7.23%	12.81%	3.79%	-1.53%
$\lambda^H = 0.429$	0.054	0.126	4.32%	3.65%	9.04%	3.79%	-1.53%
Panel H. Elastic labour supply in the manufacturing sector							
$\Delta L/L = 2\%$	0.054	0.126	5.85%	7.06%	15.11%	4.00%	-1.82%
$\Delta L/L = 5\%$	0.054	0.126	8.13%	9.72%	21.80%	4.31%	-2.26%
Panel I. Dispersion in managerial human capital ϑ							
	0.052	0.121	1.61%	2.18%	3.27%	6.63%	-0.73%

Table 15: Robustness

profits to identify σ (see (2.29)).[46] Because we observe this moment in both countries, this strategy allows us to let σ vary across countries. Our calibrated model is able to match this moment precisely in both countries. We estimate that $\sigma_{IND} = 0.57$ and $\sigma_{US} = 0.59$. While these are higher than our baseline estimate of $\sigma = 0.46$, it is reassuring to see that they are very similar in the US and India. In Panel C, we use both the estimated treatment effect and the managerial compensation shares as moments and we find that $\sigma_{IND} = 0.45$ and $\sigma_{US} = 0.63$. These estimates for σ would amplify the aggregate consequences of an increase in α as managerial services are a more important factor in production.

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[46]In Section B.2.1 in the Appendix we discuss in detail how we measure this moment.

Entry: In our benchmark specification, we assume that entrants have access to the same innovation technology as incumbent firms, i.e., the cost function has an elasticity governed by $\zeta_e = \zeta = 0.5$. To assess the importance of this parameter, we recalibrate our model, both for the US and India, while setting ζ_e to alternative values. The higher the value of ζ_e, the more responsive are entrants to changes in the value of entry. As shown in Panel D, if we set ζ_e to 0.4 (0.6), the effects of improving the efficiency of outside managers are smaller (larger). In terms of income per capita, our baseline results decrease (increase) by 0.5 percentage point. As expected, a higher entry elasticity reduces the effect on average firm size.

Convexity of incumbents' expansion technology: Similarly, we studied how the convexity of the expansion cost function for incumbent firms changes our results. Interestingly, the results are exactly the opposite of the ones found in Panel D: the higher (lower) the elasticity of incumbent innovation, the stronger (weaker) the response of aggregate income and creative destruction to changes in α. The reason is that, in India, entrants account for most creative destruction. The higher the incumbent expansion elasticity, the more entrants are crowded out. While this increases average firm size, it actually reduces the aggregate impact of changes in α.

Firm-Level Analysis: For our baseline analysis, we have focused solely on establishment-level data. We did so to ensure comparability between the US and India since we cannot link individual establishments to specific firms in the Indian data. Panel F shows that this choice has no substantial implications for our conclusions - the counterfactual implications of an increase in α are quantitatively similar when we calibrate the US parameters to firm-level moments.[47]

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[47]The model is able to match the firm-level moments quite well. The main difference between establishments and firms at the horizon of age 21-25 is the life-cycle, the aggregate employment share, and the relative exit rate. The life-cycle is slightly steeper, the employment share is lower (because very old firms are much bigger than very old establishments), and the relative exit rate of young firms is higher than that of older establishments, because old firms exit less frequently than older establishments. Moreover, the aggregate entry rate is slightly lower at the firm level. In Section B.2.7 in the Appendix, we provide more details on establishment-firm comparison for the US

Strength of Knowledge Diffusion: Our benchmark analysis estimates the diffusion parameter λ from the time series of TFP differences between India and the US. Our estimate implies a half-life of around 50 years. We considered two alternative values for λ which increase (reduce) the speed of convergence by 25%. Recall that this parameter only affects aggregate income differences and not the firm size distribution. Panel G of Table 15 shows that a faster transition speed (i.e., a high level of λ) decreases the impact of α on productivity and income differences. This follows directly from (2.32), which shows that Q_{IND}/Q_{US} is less sensitive to changes in τ if λ is large. The quantitative results are, however, in the ballpark of our baseline estimates.

Elastic Labor Supply: In our main analysis we treated aggregate labor supply as exogenous and hence non-responsive to an increase in α. If an increase in delegation efficiency in the manufacturing sector raises productivity, we might expect the manufacturing sector to draw in workers from the rest of the economy. In Panel H, we report the results when we assume that the total workforce in the manufacturing sector increases by 2% or 5% when α is increased to the US level. This amplifies our results because an increase in the workforce increases creative destruction and hence reduces income differences.

Dispersion in managerial human capital ϑ: For our baseline estimates, we use the dispersion in log managerial earnings to calibrate the dispersion in managerial human capital ϑ. Our assumption regarding the managerial skill distribution implies that average managerial earnings relative to those of production workers are given by $\vartheta/(\vartheta - 1)$ (see also footnote 36). The managerial earnings premium of 0.54 log points in the US implies a higher ϑ value of 2.4. Panel I shows the results based on this higher value. This parameter is quite important in that the change in relative income per capita due to the increase in α declines from 11% to 3.3%. The main reason is that a higher ϑ makes the labor supply of managers more elastic. This implies that a given change in α induces a sharper decline in the number workers. This in turn tends to lower profits and hence weakens the effect on expansion, entry, and creative destruction.

2.7 Conclusion

Are inefficiencies in delegating managerial tasks to outside managers an important determinant of the process of firm dynamics and aggregate income in poor countries? To answer this question, we proposed a novel model of firm growth that highlights the interaction between managerial delegation, firms' incentives to expand, and aggregate productivity. Our theory predicts an inherent complementarity between the efficiency of delegation and firm size, as delegation only becomes necessary once firms reach a certain scale. If firms anticipate that they will not be able delegate efficiently once they grow large, their incentives to expand are throttled. At the micro-level, this implies that most firms stay small. At the macro-level, this reduces the extent of reallocation, allows stagnant, subsistence producers to survive, and lowers aggregate productivity.

To quantify the strength of this mechanism, we calibrate our model to plant-level data from India and the US To credibly identify the link between managerial inputs and firms' incentives to expand, we estimate our structural model to the experimental evidence on the relationship between management practices and firm performance reported in Bloom et al. (2013).

We draw three lessons from our quantitative analysis. First, we find that the Indian economy suffers from a lack of selection, which allows subsistence firms to survive. The glut of small firms in poor countries may therefore not result from frictions these firms face, but rather a sign that other, more dynamic firms do not grow sufficiently. Policies targeted at small firms could therefore end up supporting stagnant producers and have unintended consequences.

Second, we find that inefficiencies in delegating managerial tasks have non-trivial macroeconomic implications. Our estimates imply that a given manager is only half as efficient in an Indian firm, relative to a firm in the US If Indian firms could use managers

as efficiently as US firms, income per capita in the long-run would increase by 11%. This increase is due to both static and dynamic effects which are of roughly equal size.

Finally, we find a strong complementarity between delegation efficiency and other factors affecting firm growth. While an increase to US standards would raise average firm size in India only modestly, firms in the US would shrink substantially if they had to operate with the delegation environment common in India. Hence, for improvements in the efficiency of delegation to have sizable effects in India, other determinants of firm growth also need to be addressed: even if one of its tires is fixed, a car cannot run when the rest of its tires remain broken.

Chapter 3

Innovation, Reallocation and Growth

This chapter is co-authored with Daron Acemoglu, Ufuk Akcigit, Nicolas Bloom and William R. Kerr.

Abstract

We build a model of firm-level innovation, productivity growth and reallocation featuring endogenous entry and exit. A new and central economic force is the selection between high- and low-type firms, which differ in terms of their innovative capacity. We estimate the parameters of the model using US Census micro data on firm-level output, R&D and patenting. The model provides a good fit to the dynamics of firm entry and exit, output and R&D. Taxing the continued operation of incumbents can lead to sizable gains (of the order of 1.4% improvement in welfare) by encouraging exit of less productive firms and freeing up skilled labor to be used for R&D by high-type incumbents. Subsidies to the R&D of incumbents do not achieve this objective because they encourage the survival and expansion of low-type firms.

3.1 Introduction

Industrial policies that subsidize (often large) incumbent firms, either permanently or when they face distress, are pervasive. They have been the mainstay of government policies in China over the last two and a half decades as well as widely used in Europe (e.g., Owen, 1999; Lerner, 2009).[48] The majority of regional aid in Europe also ends up going to larger firms because they tend to be more effective at obtaining subsidies (Criscuolo et al., 2012). Despite the ubiquity of such policies, their effects are poorly understood. They may encourage incumbents to undertake greater investments, increase productivity and protect employment (e.g., Aghion et al., 2015). But they may also reduce economic growth by slowing down reallocation and even discouraging innovation by both continuing firms and new entrants.[49]

In this paper, we develop a model of endogenous reallocation and innovation with heterogeneous firms to investigate the implications of different types of industrial policies. Our model builds on the endogenous technological change literature (e.g., Romer, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991) and in particular, on Klette and Kortum (2004)'s and Lentz and Mortensen (2008)'s analyses of firm-level innovation, but extends these models by incorporating endogenous exit and reallocation. These margins are critical for our investigation of different types of industrial policies as we explain below.

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[48]The amount spent on bailouts and industrial policy by the European Union in 2010 was about 1.18 trillion euros, which amounts to 9.6% of EU GDP (European Commission, 2011, page 8).

[49]The impact of these policies on the reallocation of resources may be particularly important to take into account. Foster et al. (2001, 2006) report that reallocation, broadly defined to include entry and exit, accounts for around 50% of manufacturing and 90% of US retail productivity growth. These figures probably underestimate the full contribution of reallocation since entrants' prices tend to be below industry average leading to a downward bias in their estimated TFP (Foster, Haltiwanger, and Syverson, 2008). As a result the contribution of reallocation to aggregate productivity growth in the US across all sectors is probably substantially higher. Numerous papers looking at productivity growth in other countries also find a similarly important role for differences in reallocation in accounting for differences in aggregate productivity growth. For example, Hsieh and Klenow (2009, 2014b), Bartelsman et al. (2013) and Syverson (2011) discuss how variations in reallocation across countries play a major role in explaining differences in productivity levels.

In our model, incumbents and entrants hire skilled labor to perform R&D. Successful innovation enables a firm to take over a leading-edge technology from its current holder, adding to the number of product lines the firm is operating. Because operating a product line entails a fixed cost (which is also in terms of skilled labor), firms may decide to exit some of the product lines in which they have the leading-edge technology when this technology has sufficiently low productivity relative to the equilibrium wage. Finally, firms have heterogeneous (high and low) types, which determine their "innovative capacity". We assume that firm type changes over time, and in particular, high-type firms can become low-type, which is important for accommodating the possibility that firms that have grown large over time may have ceased to be innovative.

The interplay of endogenous exit and innovation and exogenous transitions from high to low type introduces a *selection effect*, determining the composition of active product lines operated by high-type firms. There is positive selection as the fraction of active product lines operated by high-type firms expands over time because low-type firms innovate less and are more likely to exit endogenously. Countering this there is also negative selection resulting from the fact that high-type firms transition to low type. The balance of these two forces will determine whether young (and small) firms are more innovative and contribute more to growth.

The key market failure in our model is related to skilled labor. Because of the quality ladder structure (whereby firms build on the quality level of existing leaders), R&D creates positive spillovers on other firms. This implies there will be underinvestment in R&D, and thus lower than socially optimal demand for the factor of production used in R&D, skilled labor. This implies that too high a fraction of skilled workers will be employed in operation activities, and thus all else equal, a welfare-maximizing social planner would like to reallocate skilled labor back to R&D, and especially away from the operations of low-type firms. However, our quantitative analysis will show that, despite the underinvestment in R&D and the emphasis on R&D subsidies in the previous literature,

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this objective cannot be successfully achieved by R&D subsidies to either incumbents or entrants, because such subsidies would go to both high- and low-type firms. Rather, taxing the continued operation of the incumbents (or alternatively subsidizing exit) is much more powerful in freeing up skilled labor, because such taxes fall disproportionately on low-type firms, which are more likely to be near the exit margin.

Our focus on the reallocation (and misallocation) of R&D inputs, which are critical for productivity growth, is different from that of much of the literature, which emphasizes the reallocation of production inputs. Though in practice there is not a hard line demarcating R&D and production inputs, our separation of these two sets of inputs enables us to highlight our main contribution in a more transparent manner, and emphasizes that misallocation may affect equilibrium growth as well.

Despite the various dimensions of firm-level decisions, heterogeneity, and selection effects, which will prove important in our estimation and quantitative exercises, we show that the model is tractable and that much of the equilibrium can be characterized in closed form (conditional on the wage rate, which does not admit a closed-form solution). This equilibrium characterization then enables the estimation of the model's parameters using simulated method of moments.

The data we use for estimation come from the Census Bureau's Longitudinal Business Data-base and Census of Manufacturers, the National Science Foundation's Survey of Industrial Research and Development, and the NBER Patent Database. We design our sample around innovative firms that are in operation during the 1987-1997 period. As discussed in greater detail below, the combination of these data sources and our sample design permits us to study the full distribution of innovative firms, which is important when considering reallocation of resources for innovation, and to match the model's focus on R&D-based firms. Our model closely links the growth dynamics of firms to their underlying innovation efforts and outcomes, and we quantify the reallocation of resources necessary for innovation. Our sample contains over 98% of the industrial R&D conducted

in the US during this period.

We compute 18 moments capturing key features of firm-level R&D behavior, shipments growth, employment growth and exit, and how these moments vary by firm size and age. We use these moments to estimate the 8 parameters of our model and 5 parameters are calibrated using conventional values. The model performs well and matches these 18 moments quite closely. In addition, we show that a variety of correlations implied by the model (not targeted in the estimation) are similar to the same correlations computed from the data, bolstering our confidence in the model and our subsequent policy analysis.

We then use our model to study the effects of various counterfactual policies and gain insights about whether substantial improvements in economic growth and welfare are possible. In addition to illustrating the aforementioned effects of different types of policies, our quantitative analysis enables us to compute the socially optimal allocation chosen by a planner who controls R&D investments, and entry and exit decisions of different types of firms. We find that such an allocation would achieve a 2.94% growth rate per annum (relative to 2.26% in equilibrium) and a 4.47% increase in welfare. The social planner achieves this by forcing low-type incumbents to exit at a substantial rate, reducing their R&D, and increasing the R&D of high-type incumbents. These policies induce a strong selection away from low-type firms where the productivity of skilled labor is less than in high-type firms. The socially optimal allocation is not achievable without type-specific taxes, however. Instead, with just (uniform) taxes on operations and subsidies to incumbent R&D, growth can be increased to about 2.54% and welfare can be increased by 1.4%. Optimal policies in this case involve a sizable tax, of about 70%, on the continued operation of incumbents alone, which leverages the selection effect (just like what the social planner was able to achieve directly).

Our baseline empirical analysis uses unweighted moments and focuses on continuously-innovative firms. We show that both our estimation results and quantitative policy conclusions are robust if we instead use employment-weighted moments or also include non-

innovative firms in our sample (which however forces us to drop the R&D moments). The results are also not sensitive to excluding mergers and acquisitions related activities. We further document that our results are robust to various variations of the model, including modifying the technology of fixed costs so that it depends on both skilled and unskilled labor; including costs of factor reallocation; generalizing the model to more than two types of firms; and incorporating endogenous supply of skills.

Our paper is linked to a number of different literatures. First, it is most closely related to models of firm innovation and dynamics in general equilibrium pioneered by Klette and Kortum (2004) and Lentz and Mortensen (2008). As already mentioned, we extend these papers in a number of noteworthy dimensions. Most importantly, both papers assume unit elastic demands and no fixed costs of operations, and thus do not feature endogenous exit (obsolescence) of low-productivity products, which removes the issues related to our main focus in this paper—the impact of different types of policies on equilibrium reallocation and selection of firms. In addition, though Lentz and Mortensen allow for firm heterogeneity, this does not affect innovative capacity in their model, ruling out any misallocation of R&D inputs, which is central for our focus and policy analysis. Second, our paper is related to the growing literature on firm dynamics, reallocation and misallocation[50], but is distinguished by our framework which marries the issue of reallocation to innovation, and by our focus on the reallocation and misallocation of R&D inputs (skilled labor). We are also not aware of any papers in these two literatures that investigate the equilibrium implications of different types of industrial policies, including R&D subsidies. On this last point, some of our emphasis on the distortions that are caused by R&D subsidies are related to Goolsbee (1998), Romer (2001) and Wilson (2009) who point out that R&D subsidies may primarily increase the wages of inelastic inputs (such as R&D workers) rather than spurring additional innovation, and to Akcigit et al. (2016a) who suggest that R&D subsidies may be ineffective when other complementary

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[50]For example, Jovanovic (1982); Hopenhayn (1992, 2012); Hopenhayn and Rogerson (1993b); Ericson and Pakes (1995); Davis et al. (2006); Restuccia and Rogerson (2008); Guner et al. (2008); Hsieh and Klenow (2009, 2014b); Jones (2011); Peters (2016); Garcia-Macia et al. (2016); and Hsieh et al. (2013).

investments in basic science are not subsidized as well. None of these papers develop a comprehensive framework for studying the effects of different types of policies on selection, reallocation and innovation, nor do they obtain our main substantive conclusions on the ineffectiveness of R&D subsidies and the critical role of taxing incumbents for generating positive selection across firms and productivity growth.

The rest of the paper is organized as follows. Section 3.2 presents the model. Section 3.3 describes our data and quantitative framework. Section 3.4 presents our quantified parameter estimates, assesses the model's fit with the data, and provides validation tests. Section 3.5 examines the impact of counterfactual policy experiments on the economy's innovation and growth. Section 3.6 reports the results from a number of robustness exercises. The last section concludes, while Appendix C contains some of the proofs omitted from the text.

3.2   Model

In this section, we introduce our theoretical framework and characterize the stationary balanced growth equilibrium.

3.2.1   Preferences and Final Good Technology

Our economy is in continuous time and admits a representative household with the following CRRA preferences

$$U_0 = \int_0^\infty \exp\left(-\rho t\right) \frac{C\left(t\right)^{1-\vartheta} - 1}{1 - \vartheta} dt, \tag{3.1}$$

where $\rho > 0$ is the discount factor and $C\left(t\right)$ is a consumption aggregate given by

$$C\left(t\right) = \left(\int_{\mathcal{N}(t)} c_j\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.2}$$

where $c_j(t)$ is the consumption of product j at time t, $\varepsilon > 1$ is the elasticity of substitution between products, and $\mathcal{N}(t) \subset [0,1]$ is the set of active product lines at time t. The reason why not all products are active at each point in time will be made clear below. Throughout we choose this consumption aggregate as the numeraire.

We assume that the economy is closed, and because R&D and production costs are in terms of labor, we have $c_j(t) = y_j(t)$, where $y_j(t)$ is the amount of product j produced at time t. This also implies that aggregate output (GDP) is equal to aggregate consumption,

$$Y(t) = C(t). \tag{3.3}$$

There are two types of labor in the economy, skilled and unskilled. Unskilled workers are used in the production of the active products (total labor demand denoted by L^P), while skilled workers perform R&D functions (total labor demand L^{RD}) and are also employed to cover the (fixed) costs of operations, such as management, back-office functions and other non-production work (total labor demand L^F). We assume that the operation of each product requires $\phi > 0$ units of skilled labor.

The representative household has a fixed skilled labor supply of measure L^S and an unskilled labor supply of measure 1, both supplied inelastically. The labor market-clearing condition then equates total labor demand to labor supply for each type of labor:

$$L^P = 1 \quad \text{and} \quad L^F + L^{RD} = L^S. \tag{3.4}$$

With this specification, the representative household maximizes its utility (3.1) subject to the flow budget constraint

$$\dot{A}(t) + C(t) \le r(t)A(t) + w^u(t) + L^S w^s(t), \tag{3.5}$$

and the usual no-Ponzi condition, $\int_0^\infty \exp(-r(t)t)A(t)dt \ge 0$, where $A(t) = \int_{\mathcal{N}(t)} V_j(t)\,dj$

is the asset position of the representative household, $r(t)$ is the equilibrium interest rate on assets, and $w^s(t)$ and $w^u(t)$ denote skilled and unskilled wages, respectively. In what follows, we focus on stationary equilibria and drop the time subscripts when this causes no confusion.

For future reference, we also note that the representative household utility maximization problem delivers the standard Euler equation,

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\vartheta}. \tag{3.6}$$

3.2.2 Intermediate Good Production

Intermediate good (product) j is produced by the monopolist who has the best (leading-edge) technology in that product line, though a single monopolist can own multiple product lines and can produce multiple intermediate goods simultaneously.

At any given point in time, there are two different sets of firms: (i) a set of active firms $\mathcal{F}$ that own at least one product line; and (ii) a set of potential entrants of measure 1 that do not currently own any product line but invest in R&D for innovation.

Consider firm $f \in \mathcal{F}$ that has the leading-edge technology in product j. We assume that, once it hires ϕ units of skilled labor for operation, this firm has access to a linear technology in product line j of the form

$$y_{f,j} = q_{f,j} l_{f,j}, \tag{3.7}$$

where $q_{f,j}$ is the leading-edge technology of firm f in intermediate good j (which means that firm f has the best technology for this intermediate good), and $l_{f,j}$ is the number of workers it employs for producing this good.

Let us denote by $\mathcal{J}_f$ the set of *active* product lines where firm f has the leading-edge

technology and chooses to produce, and by n_f the cardinality of this set, and also define

$$\mathcal{Q}_f \equiv \{q_{f,j_1}, q_{f,j_2}, ..., q_{f,j_n}\}$$

as the set of productivities of firm f in product lines in the set $\mathcal{J}_f$. In what follows, we also drop the f subscript when this causes no confusion; for example, we refer to $q_{f,j}$ as q_j.

With this notation, equation (3.7) implies that the marginal cost of production in line j is simply w^u / q_j. Since all allocations will depend on productivity relative to the unskilled wage, we define the *relative productivity* of a product with productivity q as

$$\hat{q} \equiv \frac{q}{w^u}. \tag{3.8}$$

We also define the productivity index of the economy as

$$Q \equiv \left(\int_{\mathcal{N}} q_j^{\varepsilon-1} dj \right)^{\frac{1}{\varepsilon-1}}. \tag{3.9}$$

3.2.3 Firm Heterogeneity and Dynamics

Firms differ in terms of their innovative capacities. Upon successful entry into the economy, each firm draws its type $\theta \in \{\theta^H, \theta^L\}$, corresponding to one of two possible types high (H) and low (L). We assume:

$$\Pr\left(\theta = \theta^H\right) = \alpha \text{ and } \Pr\left(\theta = \theta^L\right) = 1 - \alpha,$$

where $\alpha \in (0,1)$ and $\theta^H > \theta^L > 0$. Firm type impacts innovation as described below. We assume that while low-type is an absorbing state, high-type firms transition to low-type at the exogenous flow rate $\nu > 0$.

In addition to the transition from high to low type, each firm is also subject to an exogenous destructive shock at the rate φ. Once a firm is hit by this shock, its value declines to zero and it exits the economy.

Innovation by incumbents is modeled as follows. When firm f with type θ_f hires h_f workers for developing a new product, it adds one more product into its portfolio at the flow rate

$$X_f = \theta_f^{\gamma} n_f^{\gamma} h_f^{1-\gamma}, \tag{3.10}$$

where $\gamma \in (0,1)$ and n_f is the number of product lines that firm f owns in total. Suppressing the f subscripts again, this implies the following cost function for R&D

$$C(x,n,\theta) = w^s n x^{\frac{1}{1-\gamma}} \theta^{-\frac{\gamma}{1-\gamma}} \equiv w^s n G(x,\theta), \tag{3.11}$$

where $x \equiv X/n$ is the "innovation intensity" (innovation effort per product) and $G(x,\theta) \equiv x^{\frac{1}{1-\gamma}} \theta^{-\frac{\gamma}{1-\gamma}}$, defined in (3.11), denotes the skilled labor requirement for a firm with innovative capacity θ to generate a per product innovation rate of x.

We assume that research is *undirected* across all product lines, meaning that firms do not know ex ante upon which particular product line they will innovate. This implies that their expected return to R&D is the expected value across all product lines $j \in [0,1]$.

When a firm innovates over a product line j, it increases the productivity of this product line j by $\lambda \bar{q}$, where $\lambda > 0$ and

$$\bar{q} \equiv \int_0^1 q_j dj$$

is the *average* quality over all product lines. That is,

$$q_j(t+) = q_j + \lambda \bar{q}, \tag{3.12}$$

where $t+$ refers to the instant after time t. Note also that equation (3.12) applies even if

product line j is not currently active so that the dynamics of productivity at the product line level are independent of whether the product line in question is currently active or not.

What happens following innovation? The firm with the improved technology in product line j takes over this product line, but in principle, the firm that previously had the leading-edge technology might still compete if the current owner tried to set a very high price. To prevent this possibility, we follow Acemoglu et al. (2012) and assume that there is a two-stage pricing game between any firm that wishes to supply a product $j \in [0, 1]$, whereby each firm first has to enter and pay a small cost $\epsilon > 0$, and then all firms that have entered simultaneously set prices. We take $\epsilon \to 0$ for simplicity. Since the price setting after entry forces Bertrand competition, the more productive firm will be able to make any sales and profits, and thus only this firm will pay the cost ϵ and enter. But then in equilibrium, the firm with the leading-edge technology can charge the monopoly price, regardless of the productivity gap between itself and the next best technology. This enables us to characterize prices in a simple fashion in the next subsection.

3.2.4 Equilibrium Prices and Profits

First note that from the utility function in (3.2), the inverse demand function for active product line $j \in \mathcal{N}$ is

$$p_j = C^{\frac{1}{\epsilon}} c_j^{-\frac{1}{\epsilon}}.$$

Given the market structure described in the previous subsection, the firm with the leading-edge technology can act as a monopolist and thus solves the following maximization problem,

$$\pi\left(\hat{q}_j\right) = \max_{c_j \geq 0} \left\{ \left(C^{\frac{1}{\epsilon}} c_j^{-\frac{1}{\epsilon}} - \hat{q}_j^{-1} \right) c_j \right\},$$

where we use $\pi\left(\hat{q}_j\right)$ to designate the firm's profit as a function of only its relative quality $\hat{q}_j$ after substituting for the unskilled wage, w^u, from (3.8). The price and consumption

level of intermediate good j follow from this maximization as

$$p_j = \frac{\varepsilon}{\varepsilon - 1} \hat{q}_j^{-1} \text{ and } c_j = \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon} C \hat{q}_j^{\varepsilon}, \tag{3.13}$$

and equilibrium profits can then be computed as

$$\pi \left(\hat{q}_j \right) = \frac{1}{\varepsilon - 1} \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon} C \hat{q}_j^{\varepsilon - 1}.$$

Since the final good is the numeraire, (3.2) also implies

$$\left(\int_{\mathcal{N}} p_j^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} = 1.$$

Substituting c_j from (3.13) into the production function (3.2) and integrating over $\mathcal{N}$, we obtain the unskilled wage rate as

$$w^u = \frac{\varepsilon - 1}{\varepsilon} Q, \tag{3.14}$$

where Q is given in (3.9).

3.2.5 Entry and Exit

There is a unit measure of potential entrants. Each entrant has access to an R&D technology $G\left(x^{entry}, \theta^E\right)$, where the function G was defined in (3.11) above and specifies the number of skilled workers necessary for generating an innovation rate of $x^{entry} > 0$. Thus an entrant wishing to achieve an innovation rate of x^{entry} would need to hire

$$h^{entry} = G\left(x^{entry}, \theta^E\right) \tag{3.15}$$

skilled workers. This specification implies that a potential entrant has access to the same R&D technology that an incumbent with innovative capacity θ^E and a single active product would have had.

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Following a successful innovation, the entrant improves the productivity of a randomly chosen product line by $\lambda \bar{q}$, and at this point, the initial type of a firm, $\theta \in \{\theta^H, \theta^L\}$ is also realized. This description implies the following optimization problem for entrants:

$$\max_{x^{entry} \geq 0} \left\{ x^{entry} \mathbb{E} V^{entry} \left(\hat{q} + \lambda \bar{\hat{q}}, \theta \right) - w^s G \left(x^{entry}, \theta^E \right) \right\}, \qquad (3.16)$$

where $\mathbb{E} V^{entry} \left(\cdot \right)$ is the expected value of entry (and the expectation is over the relative productivity $\hat{q}$ of the single product the successful entrants will obtain and firm type $\theta \in \{\theta^H, \theta^L\}$). The maximization in (3.16) determines the R&D intensity of an entrant. Given that there is a unit measure of potential entrants, x^{entry} is also equal to the total entry flow rate.

Exit (of products and firms) has three causes:

1. There is an exogenous *destructive shock* at the rate $\varphi > 0$, which causes the firm to exit and shut down all its product lines.

2. There will be *creative destruction*, because of innovation by other firms replacing the leading-edge technology in a particular product line.

3. There will be *endogenous obsolescence*, meaning that firms will voluntarily shut down some product lines because they are no longer sufficiently profitable relative to the fixed cost of operation.

Due to the first and third factors, the measure of inactive product lines will be positive.

3.2.6 Value Functions

We normalize all the growing variables by $Q(t)$ to keep the stationary equilibrium values constant. Let us denote the normalized value of a generic variable X by $\tilde{X}$.

Let τ denote the *average creative destruction rate* which is endogenously determined in equilibrium. Then the stationary equilibrium value function for a low-type firm can be written as

$$
r\tilde{V}_l\left(\hat{\mathcal{Q}}\right) = \max\left\{0, \max_{x\geq 0}\left[\begin{array}{c}\displaystyle\sum_{\hat{q}\in\hat{\mathcal{Q}}}\left[\begin{array}{c}\tilde{\pi}\left(\hat{q}\right)-\tilde{w}^s\phi+\\ \tau\left[\tilde{V}_l\left(\hat{\mathcal{Q}}\setminus\{\hat{q}\}\right)-\tilde{V}_l\left(\hat{\mathcal{Q}}\right)\right]+\frac{\partial\tilde{V}_l(\mathcal{Q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}\end{array}\right]\\ -n\tilde{w}^s G\left(x,\theta^L\right)\\ +nx\left[\mathbb{E}\tilde{V}_l\left(\hat{\mathcal{Q}}\cup\{\hat{q}+\lambda\bar{q}\}\right)-\tilde{V}_l\left(\hat{\mathcal{Q}}\right)\right]\\ +\varphi\left[0-\tilde{V}_l\left(\hat{\mathcal{Q}}\right)\right]\end{array}\right]\right\},
$$
(3.17)

where $\hat{\mathcal{Q}}\cup\{\hat{q}_{j'}\}$ denotes the new portfolio of the firm after successfully innovating in product line j'. Similarly $\hat{\mathcal{Q}}\setminus\{\hat{q}_j\}$ denotes the loss of a product with technology $\hat{q}_j$ from firm f's portfolio $\hat{\mathcal{Q}}$ due to creative destruction.[51]

The value function (3.17) can be interpreted as follows. Given discounting at the rate r, the left-hand side is the flow value of a low-type firm with a set of product lines given by $\hat{\mathcal{Q}}$. The right-hand side includes the components that make up this flow value. The first line (inside the summation) includes the instantaneous operating profits, minus the fixed costs of operation, plus the change in firm value if any of its products gets replaced by another firm through creative destruction at the rate τ, plus the change in firm value due to the the increase in the economy-wide wage. This last term accounts for the fact that as the wage rate increases, the relative productivity of each of the products that the firm operates declines. The second line subtracts the R&D expenditure by firm f. The third line expresses the change in firm value when the low-type firm is successful with its R&D investment at the rate x. The last line shows the change in value when the firm has to exit due to an exogenous destructive shock at the rate φ.

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[51]Note that in writing this expression, we have made use of the fact that there is a continuum of products, and thus even for a firm with a large number of product lines, the probability that it will innovate on one of its own products is zero. Consequently, τ is both the average creative destruction rate and the average innovation rate in the economy.

Similarly, we can write the value function of a high-type firm as

$$
r\tilde{V}_h\left(\hat{\mathcal{Q}}\right) = \max\left\{0, \max_{x\geq 0}\left[\begin{array}{c}\sum_{\hat{q}\in\hat{\mathcal{Q}}}\left[\begin{array}{c}\tilde{\pi}\left(\hat{q}\right) - \tilde{w}^s\phi + \\ \tau\left[\tilde{V}_h\left(\hat{\mathcal{Q}}\setminus\{\hat{q}\}\right) - \tilde{V}_h\left(\hat{\mathcal{Q}}\right)\right] + \frac{\partial\tilde{V}_h\left(\hat{\mathcal{Q}}\right)}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}\end{array}\right] \\ -n\tilde{w}^s G\left(x,\theta^H\right) \\ +nx\left[\mathbb{E}\tilde{V}_h\left(\hat{\mathcal{Q}}\cup\{\hat{q}+\lambda\bar{\hat{q}}\}\right) - \tilde{V}_h\left(\hat{\mathcal{Q}}\right)\right] \\ +\varphi\left[0 - \tilde{V}_h\left(\hat{\mathcal{Q}}\right)\right] \\ +\nu\left[\tilde{V}_l\left(\hat{\mathcal{Q}}\right) - \tilde{V}_h\left(\hat{\mathcal{Q}}\right)\right]\end{array}\right]\right\}.
$$

(3.18)

The major difference from (3.17) is in the last line, where we incorporate the possibility of a transition to a low-type status at the rate ν. The remaining terms have the same interpretation as (3.17).

The next lemma shows that the value of each firm can be expressed as the sum of the *franchise values* of each of their product lines, defined as the net present discounted value of profits from a product line (as we will see these franchise values depend on the type of the firm).

Lemma 1. *The value function of a $k\in\{h,l\}$ type firm takes an additive form*

$$
\tilde{V}_k\left(\hat{\mathcal{Q}}\right) = \sum_{\hat{q}\in\hat{\mathcal{Q}}}\Upsilon_k\left(\hat{q}\right),
$$

where $\Upsilon_k\left(\hat{q}\right)$ is the franchise value of a product line of relative quality $\hat{q}$ to a firm of type k, and $\Upsilon_k\left(\hat{q}\right)$ is nondecreasing and increasing when it is greater than zero. Moreover, there exist thresholds $\hat{q}_{k,\min}$ such that a firm of type k shuts down a product line with relative quality $\hat{q} < \hat{q}_{k,\min}$ (and $\Upsilon_k\left(\hat{q}\right) > 0$ when $\hat{q} > \hat{q}_{k,\min}$).

Proof. See the Appendix C. □

The next lemma characterizes the franchise value of a single product line as the solution to a simple differential equation and the type of the firm with the leading-edge best

technology in this product line.

Lemma 2. *The franchise values of a product line of relative productivity $\hat{q}$ to low-type and high-type firms, respectively, are given by the following differential equations*

$$(r + \tau + \varphi) \, Y^l \, (\hat{q}) - \frac{\partial Y^l \, (\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t} = \Pi \hat{q}^{\varepsilon - 1} - \tilde{w}^s \phi + \Omega^l \; \text{if } \hat{q} > \hat{q}_{l,\min} \quad (3.19)$$

$$Y^l \, (\hat{q}) = 0 \; \text{otherwise}$$

and

$$(r + \tau + \varphi) \, Y^h \, (\hat{q}) - \frac{\partial Y^h \, (\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t} = \left\{ \begin{array}{c} \Pi \hat{q}^{\varepsilon - 1} - \tilde{w}^s \phi + \Omega^h + \\ \\ \nu \left[Y^l \, (\hat{q}) - Y^h \, (\hat{q}) \right] \end{array} \right\} \; \text{if } \hat{q} > \hat{q}_{h,\min}$$

$$Y^h \, (\hat{q}) = 0 \; \text{otherwise}$$

where $\Pi \equiv \frac{1}{\varepsilon - 1} \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon}$, and

$$\Omega^k \equiv \max_{x \geq 0} \left\{ \, -\tilde{w}^s G \left(x, \theta^k \right) + x \mathbb{E} Y^k \, (\hat{q} + \lambda \bar{\hat{q}}) \, \right\}, \; \text{for } k \in \{L, H\}$$

is the R&D value of a k-type firm. Moreover, the R&D policy function of a k-type firm is

$$x^k = \theta^k \left[\frac{(1 - \gamma) \, \mathbb{E} Y^k \, (\hat{q} + \lambda \bar{\hat{q}})}{\tilde{w}^s} \right]^{\frac{1 - \gamma}{\gamma}} \; \text{for } k \in \{L, H\}. \quad (3.20)$$

Finally, $\hat{q}_{k,\min}$ is given by

$$\hat{q}_{k,\min} = \left(\frac{\tilde{w}^s \phi - \Omega^k}{\Pi} \right)^{\frac{1}{\varepsilon - 1}} \; \text{for } k \in \{L, H\}. \quad (3.21)$$

Proof. This follows from the proof of Lemma 1. □

The expressions in this lemma are intuitive. So long as this product line remains active, the firm receives two returns: a flow of profits depending on $\hat{q}$, $\Pi \hat{q}^{\varepsilon - 1}$, and an R&D value, denoted by Ω^k for a firm of type k. The R&D value accounts for the fact that

the firm can undertake R&D building on the knowledge embedded in this active product line. While operating this product line, the firm also incurs the fixed cost of operation $\tilde{w}^s\phi$. The differential equation also takes into account that the relative productivity of this product line is declining proportionately at the growth rate of the economy, g, reducing profits at the rate $(\varepsilon - 1)g$, and that this product line is replaced by a higher productivity one at the rate τ and the firm exits for exogenous reasons at the rate φ, making the effective discount rate $r + \tau + \varphi$. If this product line is not replaced or the firm does not exit by the time its relative productivity reaches $\hat{q}_{k,\min}$ (for a firm of type k), at $\hat{q}_{k,\min}$ it will become "obsolete", providing a boundary condition for the differential equation. Finally, for high-type firms there is an additional term incorporating the possibility of switching to low-type.

The differential equations in Lemma 2 can be solved explicitly, and in the next proposition, we provide the solution for low-type firms, which is simpler. We present the solution for high-type firms in Appendix C.

Proposition 1. *Let g and $\tilde{w}^s$ be the stationary equilibrium growth rate of the economy and the normalized skilled wage rate, respectively. Moreover, let*

$$F_k(x) \equiv 1 - \left(\frac{\hat{q}_{k,\min}}{\hat{q}}\right)^x.$$

Then, the franchise value of a product line with relative productivity $\hat{q}$ for a low-type firm is

$$Y^l(\hat{q}) = \frac{\Pi\hat{q}^{\varepsilon-1}}{r + \tau + \varphi + (\varepsilon - 1)g}F_l\left(\frac{r + \tau + \varphi + (\varepsilon - 1)g}{g}\right) + \frac{\Omega^l - \tilde{w}^s\phi}{r + \tau + \varphi}F_l\left(\frac{r + \tau + \varphi}{g}\right),$$

where $\Pi \equiv \frac{1}{\varepsilon-1}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}$.

Proof. See Appendix C. □

Intuitively, the franchise value of a product line can be obtained in closed-form be-

cause it is given by a combination of two forces: a proportional decline in the value of a product line as the unskilled wage rate increases (and the relative quality of the product line declines), accounting for the term $(\varepsilon - 1)g$, and effective discounting coming from the interest rate, creative destruction and exogenous firm exit, accounting for the term $r + \tau + \varphi$.

3.2.7 Labor Market and Stationary Equilibrium Distributions

The relative productivity distribution for type-k firms has a stationary equilibrium distribution function, $F_k(\hat{q})$ on $[\hat{q}_{k,\min}, \infty)$. Let the shares of product lines that belong to two different types of firms and inactive product lines be denoted by Φ^h, Φ^l and Φ^{np}, respectively. Naturally,

$$\Phi^h + \Phi^l + \Phi^{np} = 1.$$

Then the labor market-clearing condition for unskilled workers is

$$\int_{\mathcal{N}} l(\hat{q}_j) \, dj = \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\varepsilon} (w^u)^{-\varepsilon} C \int_{\mathcal{N}} q_j^{\varepsilon - 1} dj = 1. \tag{3.22}$$

Using (3.9), (3.13) and (3.14), the previous labor market condition gives

$$Y = C = Q. \tag{3.23}$$

The labor market-clearing for skilled workers, on the other hand, sets the total demand, made up of demand from entrants (first term) and demand from incumbents (second term), equal to the total supply, L^S:

$$G\left(x^{entry}, \theta^E\right) + \sum_{k \in \{h,l\}} \Phi^k \left[h_k(w^s) + \phi\right] = L^S. \tag{3.24}$$

To solve for the labor market-clearing condition, we need to characterize the measures

of active product lines Φ^k and the stationary equilibrium productivity distributions conditional on firm type k. These detailed derivations are provided in Lemma 4 in Appendix C.

3.2.8 Aggregate Growth

Equation (3.23) shows that aggregate output is equal to the productivity index, Q. Thus the growth rate of aggregate output is given by $g = \dot{Q}/Q$. The following proposition characterizes the growth rate.

Proposition 2. *The growth rate of the economy is equal to*

$$g = \lambda \tau. \tag{3.25}$$

Proof. See Appendix C. $\qquad\qquad\square$

The intuition for the growth rate in (3.25) is as in standard quality ladder models, linking growth to the frequency and size of innovations.

Finally we summarize the equilibrium of this economy.

Definition 2 (Stationary Equilibrium). *A stationary equilibrium of this economy is a tuple*

$$\{y_j, p_j, l_j, \tilde{V}_l, \tilde{V}_h, \hat{q}_{h,\min}, \hat{q}_{l,\min}, x^h, x^l, x^{entry}, h^h, h^l, h^{entry}, \Phi^h, \Phi^l, \Phi^{np}, F_l(\hat{q}), F_h(\hat{q}), w^s, w^u, g, r\}$$

such that [**i**] y_j *and* p_j *maximize profits as in* (3.13) *and the labor demand* l_j *satisfies* (3.7); [**ii**] $\tilde{V}_l$ *and* $\tilde{V}_h$ *are given by the low-type and high-type value functions in* (3.17) *and* (3.18); [**iii**] $(\hat{q}_{h,\min}, \hat{q}_{l,\min})$ *satisfy the threshold rule in* (3.21); [**iv**] x^h *and* x^l *are given by the R&D policy functions in* (3.20) *and* x^{entry} *solves the entrants' problem in* (3.16); [**v**] *the skilled worker demands* h^h, h^l *and* h^{entry} *satisfy* (3.10) *and* (3.15); [**vi**] *the stationary equilibrium productivity distributions* $(\tilde{F}_l(\hat{q}), \tilde{F}_h(\hat{q}))$ *and the product line shares* $(\Phi^h, \Phi^l, \Phi^{np})$ *satisfy Lemma 4;* [**vii**] *the growth rate is given by* (3.25); [**viii**] *the interest rate satisfies the Euler equation* (3.6); *and* [**ix**]

w^s and w^u are consistent with labor market-clearing for unskilled and skilled workers as given by (3.22) and (3.24).

Though the stationary equilibrium in this model is a relatively complex object, the values for different types of firms can be computed in closed form given the equilibrium wage as shown in Proposition 1. There are no closed-form solutions for the equilibrium wage rate and stationary distributions, but these can be computed numerically. We will also use this computation for the simulated method of moments estimation as outlined in Section 3.3.2.

3.2.9   Welfare and Distortions

Recall that output and consumption are equal to the productivity index Q, so that the initial level of consumption satisfies $C_0 = Q_0$, where

$$Q_0 = \left(\int_{\mathcal{N}_0} q_{j0}^{\varepsilon-1} dj \right)^{\frac{1}{\varepsilon-1}}.$$

We normalize the initial productivity level of all active product lines to 1, i.e., $q_{j0} = 1$ for all $j \in \mathcal{N}_0$, which implies, $C_0 = Q_0 = \Phi^{\frac{1}{\varepsilon-1}}$, where $\Phi_0 = \Phi_0^h + \Phi_0^l$ is the endogenous measure of active product lines at date $t = 0$. Then welfare can be obtained as

$$U_0\left(C_0, g\right) = \int_0^\infty \exp\left(-\rho t\right) \frac{C_t^{1-\vartheta} - 1}{1 - \vartheta} dt = \frac{1}{1 - \vartheta} \left[\frac{\Phi_0^{\frac{1-\vartheta}{\varepsilon-1}}}{\rho - (1-\vartheta)g} - \frac{1}{\rho} \right], \qquad (3.26)$$

where the first equality simply repeats the definition of discounted utility from (3.1), the second equality imposes the assumption that we are in stationary equilibrium (thus implying that we are not evaluating welfare implications of transitioning from one stationary equilibrium to another), and solves the integral using $C_t = C_0 e^{gt}$ and $C_0 = \Phi_0^{\frac{1}{\varepsilon-1}}$.

In comparing welfare in two economies, say with subsidy policies s_1 and s_2, and resulting growth rates $g\left(s_1\right)$ and $g\left(s_2\right)$ and initial consumption levels $C_0\left(s_1\right)$ and $C_0\left(s_2\right)$,

we compute consumption-equivalent changes in welfare by considering the fraction of initial consumption ζ that will ensure the same discounted utility with the new growth rate as with the initial allocation. More formally, the consumption-equivalent change ζ is given such that

$$U_0\left(\zeta C_0\left(s_2\right), g\left(s_2\right)\right) = U_0\left(C_0\left(s_1\right), g\left(s_1\right)\right).$$

It is also useful at this point to note that the decentralized equilibrium is typically inefficient. As in models of endogenous technological change, there is insufficient R&D because firms do not appropriate the full value of new innovations (see, e.g., Acemoglu, 2008, for a discussion). In our model, this lack of appropriation results because future innovations build on the current knowledge stock, as captured by equation (3.12), and thus current innovations create a positive spillover to future innovators. The resulting underinvestment takes the form of too little employment of skilled workers in R&D, and thus too much employment in operations (covering the fixed costs of active firms).[52] However, this underinvestment does not apply to the two types of firms equally. The social value of one more active product is greater in the hands of a high-type firm, because such a firm is more productive in R&D, and thus is more likely to undertake a socially valuable (and under-provided) innovation. Consequently, the social planner would like to allocate more skilled labor to R&D, and to be able to do this, she would need to free up this labor from operations, especially from the operations of low-type firms. We will see below how different policies achieve this objective.

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[52]Counteracting this lack of full appropriation are two other effects. First, as in other quality ladder models such as Aghion and Howitt (1992), there is a business stealing effect, encouraging firms to undertake R&D in order to capture monopoly profits. Second, the love-for-variety resulting from the imperfect substitution of different varieties means that consumers benefit from having more active products. Nevertheless, these two effects are typically dominated by the lack of full appropriation, which leads to underinvestment in R&D.

We should also note that even though there are monopoly markups in this model, these do not directly distort the allocation, since there is no elastic supply of production inputs.

3.3 Estimation and Quantitative Analysis

To perform the policy experiments described in the Introduction, we first estimate the parameters of our model using simulated method of moments (SMM). In this section, we describe our data set and estimation procedures, and the next two sections provide our results and policy counterfactual experiments.

3.3.1 Data

We employ the Longitudinal Business Database (LBD), the Census of Manufacturers (CMF), the NSF Survey of Industrial Research and Development (RAD), and the NBER Patent Database (PAT). The LBD and CMF are the backbone for our study. The LBD is a business registry that contains annual employment levels for every private-sector establishment in the United States with payroll from 1976 onward. The CMF is conducted every five years and provides detailed records on manufacturing plant and firm operations (e.g., output). Sourced from US tax records and Census Bureau surveys, these micro-records document the universe of establishments and firms, enabling us to study reallocation, entry/exit, and related firm dynamics.

The Survey of Industrial Research and Development (RAD) is the US government's primary instrument for surveying the R&D expenditures and innovative efforts of US firms. This is an annual or biannual survey conducted jointly by the Census Bureau and NSF. The survey includes with certainty all public and private firms, as well as foreign-owned firms, undertaking over one million dollars of R&D within the US. The survey frame also subsamples firms conducting less than the certainty expenditure threshold. The certainty threshold was raised after 1996 to five million dollars of R&D for future years (before subsequently being lowered after our sample frame). RAD surveys are linked to the LBD's and CMF's operating data through Census Bureau identifiers. These

micro-records begin in 1972 and provide the most detailed statistics available on firm-level R&D efforts. In 1997, 3,741 firms reported positive R&D expenditures that sum to $158 billion (Foster and Grim (2010) provide additional details on the data). To complement the RAD, we also match patent data into the Census Bureau data. We employ the individual records of all patents granted by the United States Patent and Trademark Office (USPTO) from January 1975 to May 2009. Each patent record provides information about the invention and the inventors submitting the application. Hall et al. (2001) provide extensive details about these data, and Griliches (1990) surveys the use of patents as economic indicators of technology advancement. We only employ patents (i) filed by inventors living in the US at the time of the patent application; and (ii) assigned to industrial firms. In 1997, this group comprised about 77 thousand patents. We match these patent data to the LBD using firm name and location matching algorithms.[53]

Our main sample focuses on "continuously-innovative" firms (though we later consider the broader manufacturing sample). We define a firm as "innovative" if it is conducting R&D or patenting within the US. Our operating data come from the years 1987, 1992, and 1997 when the CMF is conducted, and the data are specific to those years. We develop our measures of innovation using five-year windows surrounding these CMF years (e.g., 1985-1989 for the 1987 CMF). These local averages assist with RAD's biannual reporting when it occurs, and they ensure that we include two RAD surveys with the lower certainty threshold for the 1997 CMF group. The local averages also provide a more consistent measure of patent filings, which can be lumpy for firms with few patents. We describe the use of patents in further detail shortly.

The "continuous" part of our sample selection is important and is structured as follows. We only include a firm in our sample if it conducts R&D or patents during the five-year window surrounding each CMF year in which it is operating (i.e., has positive

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[53]Akcigit and Kerr (2018) discuss the R&D and patent data in greater detail. The patent matching builds upon the prior work of Balasubramanian and Sivadasan (2011) and Kerr and Fu (2008). See also Kogan et al. (2017).

employment and sales in the CMF). Thus, a firm that is in operation in 1987 and 1992 is included in our sample if it is also conducting R&D or patents during 1985-1989 and 1990-1994. Similarly, a firm that is in operation in 1992 and 1997 is included in our sample if it is also conducting R&D or patents during 1990-1994 and 1995-1999. The firm does not need to conduct R&D or patent in every year of the five-year window, but must do one of the two activities at least once.

This selection process has several features to point out. First, the entrants in our sample (i.e., firms first appearing in the 1992 or 1997 CMF) will be innovative throughout their lifecycle until the 1995-1999 period. Second, we do not consider switches into innovation among already existing firms. For example, we exclude firms that are present in the 1987 and 1992 CMF, patent or conduct R&D in the 1990-1994 period, but do not patent or conduct R&D during 1985-1989 (the probability that an existing, non-innovative firm commences R&D or patenting over the ensuing five years, conditional on survival, is only about 1%). Third, and on a similar note, we do not include in our sample firms that cease to be innovative but continue in operation. Exits in our economy are thus defined over firms that patent or conduct R&D until they cease to operate.

Finally, our sample does not condition on innovative activity before 1985-1989. Thus, the incumbents in our sample who were in operation prior to the 1987 CMF may have had some point in their past when they did not conduct R&D or patent. We only require that incumbents be innovative in every period when they are in operation during our sample. This choice allows us to construct a full distribution of innovative firms in the economy, which is important when considering the reallocation of resources for innovation. Of course, this choice is also partly due to necessity as we do not observe the full history of older incumbents. We discuss further below the aggregate implications for reallocation and growth measurement of this design.[54]

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[54]It would have been impossible to build a consistent sample for "ever innovative" firms rather than for continuously innovative firms. To see why, consider keeping all of the past records for firms that conduct R&D in 1997. In both 1987 and 1992, this approach would induce a mismeasurement of exit propensities and growth dynamics because a portion of the sample will include firms conditioned on survival until 1997.

We now describe the use of the patenting data. In accordance with our model, the moments below focus on R&D intensities (i.e., inputs into the innovation production function) as well as employment, sales and exit dynamics. We face the challenge that the RAD subsamples firms conducting less than one million dollars in R&D. By contrast, the patent data are universally observed. To provide a more complete distribution, we use patents to impute R&D values for firms that are less than the certainty threshold and not sub-sampled. Thus, our moments combine the R&D and patent data into a single measure that accords with the model. As the R&D expenditures in these sub-sampled cases are very low (by definition), this imputation choice versus treating unsurveyed R&D expenditures as zero expenditures conditional on patenting is not very important.

Overall, our compiled dataset includes innovative manufacturing firms from the years 1987, 1992, and 1997 when the CMF is conducted. A record in our dataset is a firm-year observation that aggregates over the firm's manufacturing establishments. We have 17,055 observations from 9,835 firms. By abstracting from the extensive margin of entry or exit into innovation for continuing firms, all of our moments are consistently defined and well measured in the data. At the same time, our selection procedures provide as complete a distribution of innovative firms as possible, which is important when considering real-location. Our sample accounts for 98% of industrial R&D conducted during the period. When compared to a single cross-section of data, our sample is slightly more skewed towards larger firms. Specifically, in the average year during our sample period, 22% of the firms conducting R&D or patenting have more than 500 employees. In our sample, 32% of observations have more than 500 employees.

Our main sample thus focuses on the reallocation of resources for innovation and thus excludes firms that do not report R&D or patents, which we define as "non-innovative firms". It is important to place our sample within the overall distribution of economic activity. Our sample of continuously-innovative firms accounts for 2% of firms, 50% of employment, and 64% of sales within manufacturing. The greater share of employment

and sales activity than firm counts is because the great majority of small firms are non-innovative. In a similar manner and due to the link of innovation to growth, our sample accounts for a substantial portion of reallocation occurring. Many small firms are not oriented for growth (e.g., Haltiwanger et al., 2013b) and thus play a limited role in reallocation. As one statistic, our sample includes 58% and 65% of employment and sales reallocation, respectively, among continuing manufacturing firms between 1987 and 1997. As a second statistic, among firms that were either very small (fewer than 20 employees) or did not exist in 1987, we capture 94% of those that then grew to 10,000+ employees by 1997. We likewise capture 80% of small firms or new entrants that grow to one billion dollars in sales by 1997.

Our central moments are firm entry/exit rates, the age and size distribution of firms, transitions across the firm size distribution over time, firm growth rates by age and size, firm innovation intensity by age and size, and entrants' share of employment in the economy. Large firms are defined to be those with more than 200 employees, which is roughly the median firm size in our sample. The age distribution is similarly separated into whether a firm is 0-9 years or 10+ years old. We calculate firm age as the count of years since the firm was first observed in the LBD with positive employment, and we later consider robustness checks that exclude inorganic entry and exit (e.g., spinouts and acquisitions). We define moments related to entry/exit, growth, and age-size distribution transitions as changes between CMF years expressed in per annum terms.[55] Shipments are deflated using the 2009 NBER Productivity Database.[56]

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[55]We measure growth rates relative to base years over the five-year period to allow a direct decomposition to per annum terms. These growth rates are winsorized at their 0.5% and 99.5% values. The patterns are similar when expressing growth relative to the average of base and end years. We then calculate geometric averages over these firm-level growth rates. We similarly winsorize R&D intensities to be conservative.

[56]Though prices in industries related to computers and semiconductors behave differently from those in other parts of manufacturing, we find very similar moments when excluding these industries from our moment calculations.

3.3.2 Computational Algorithm

The model can be solved computationally as a fixed point of the following vector of six aggregate equilibrium variables

$$\left\{ \tilde{w}^s, \Phi^h, \Phi^l, \bar{\hat{q}}, \mathbb{E}[Y^h(\hat{q} + \lambda\bar{\hat{q}})], \mathbb{E}[Y^l(\hat{q} + \lambda\bar{\hat{q}})] \right\}. \tag{3.27}$$

Our characterization above shows that equilibrium innovation decisions can be determined given these aggregate variables. While the skilled wage $\tilde{w}^s$ directly gives the cost of innovation, the rest of the variables in (3.27) determine the expected return to innovation. We can solve for the stationary equilibrium by first posing a conjecture for (3.27), then solving for the individual innovation decisions and then verifying the initial conjecture. Specifically, using the guess for these variables:

1. we compute the innovation rates (x^h, x^l, x^{entry}), R&D values (Ω^h, Ω^l), and growth rate g;

2. using the innovation intensities, we calculate the stationary equilibrium distribution over active/inactive product lines and over values of $\hat{q}$ by using Lemma 4;

3. we check the labor market-clearing conditions using the innovation intensities and the above distributions and compute the equilibrium wage rates from (3.22) and (3.24), updating $\tilde{w}^s$;

4. we update the values for $\bar{\hat{q}}$, $\mathbb{E}[Y^h(\hat{q} + \lambda\bar{\hat{q}})]$ and $\mathbb{E}[Y^l(\hat{q} + \lambda\bar{\hat{q}})]$ by using the productivity distribution and Lemma 2.

This procedure gives us (3.27) as a fixed point and also generates the stationary equilibrium distributions of relative productivities. Note that all these variables are determined at the product-line level. We compute firm-level moments by simulating the evolution of a panel of 2^{17} firms until they reach approximate stationary equilibrium after

10,000 periods. Each period corresponds to 0.02 of a year, and hence the total simulation time comes out to 200 years. At each iteration, firms gain and lose products according to the flow probabilities specified in the model.

3.3.3 Estimation

We set the discount rate equal to $\rho = 2\%$, which roughly corresponds to an annual discount factor of 97%, and the inverse of the intertemporal elasticity of substitution to $\vartheta = 2$. We choose $L^S = 0.166$ to match the share of managers, scientists and engineers in the workforce in 1990, which is 14.2% $(= 0.166/1.166)$. Following Broda and Weinstein (2006), we take the elasticity of substitution between different products to be $\varepsilon = 2.9$.

#	Parameter	Description	Value
1.	ε	CES	2.900
2.	L^S	Measure of high-skilled workers	0.166
3.	γ	Innovation elasticity	0.500
4.	ϑ	Inverse of the intertemporal elasticity of substitution	2.000
5.	ρ	Discount rate	0.020

Table 16: Calibrated Parameters

Following the microeconometric innovation literature, we choose the elasticity of successful innovation with respect to R&D γ as 0.5. In particular, using count data models, Blundell et al. (2002) report an elasticity of $\gamma = 0.5$, while Hall and Ziedonis (2001) find similar results in a study of the semiconductor industry. Estimates exploiting variations in tax credits also yield similar elasticities. Both studies exploiting over-time variation in the US tax code (e.g., Hall, 1993) and those relying on cross-state variation in R&D tax credits (e.g., Bloom et al., 2002; Wilson, 2009) typically estimate a tax elasticity of R&D around unity. These tax elasticities are equivalent to the R&D elasticity with respect to the scientist wage, w^s, since this is the only cost of R&D in our model. Because $\frac{\%\Delta R\&D}{\%\Delta w^s} = \frac{\gamma-1}{\gamma}$,

a unit tax elasticity also implies $\gamma = 0.5$ in our setup.[57]

The remaining 8 parameters, which are listed in Table 17, are estimated with SMM.[58] We compute the model-implied moments from the simulation strategy described above and compare them to the data-generated moments to minimize

$$\min \sum_{i=1}^{18} \frac{|\text{model}\,(i) - \text{data}\,(i)|}{\frac{1}{2}\,|\text{model}\,(i)| + \frac{1}{2}\,|\text{data}\,(i)|},$$

where we index each moment by i. SMM iteratively searches repeatedly across sets of parameter values in the model until the model's moments are as close as possible to the empirical moments.

Our SMM procedure targets the 18 moments outlined in Table 18. These moments center on firm entry (measured through employment shares), exit rates, size transition rates, employment and sales growth rates, and innovation intensities, selected in each case because of their economic importance for the mechanisms of the model. We have a single aggregate moment, the growth of output per worker in our sample of firms, and we give this moment 5 times the weight of the micro-moments to ensure that we are in the ballpark of matching the aggregate growth.

We compute the standard errors of the data moments by bootstrap. Specifically, we draw samples of equal size to our original sample from either the Census Bureau data or from the Census of Populations. We use 1000 iterations in each case. For the firm data, we stratify the sample draws by firm age, size, year and industry. The sample draws are conducted at the firm-year level and retain the firm-specific information like whether the firm is an entrant in that year and its forward growth rates for sales and employment. We recalculate our aggregate moments like entrant shares of employment and overall growth rate for each bootstrap sample. The resulting standard errors are quite similar across a

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[57]To see this, substitute the equilibrium innovation choice (3.20) into R&D cost function (3.11) to obtain $R\&D = n\theta^k\,(w^s)^{\frac{\gamma-1}{\gamma}}\left[(1-\gamma)\,\mathbb{E}Y^k\,(\hat{q}+\lambda\bar{q})\right]^{\frac{1}{\gamma}}.$

[58]See McFadden (1989) and Pakes and Pollard (1989) for the statistical properties of the SMM estimator.

range of techniques, such as removing the firm selection stratification or sampling whole firm histories (i.e., retaining all years of a sampled firm).

Standard errors of the parameter estimates are also computed by bootstrap. We estimate the model parameters 1000 times by targeting the empirical moments that are randomly generated based on the bootstrapped distribution of the data moments, and then derive their standard errors from their distribution across these 1000 estimations.[59]

3.4 Results

In this section, we present our estimation results and evaluate the fit of our model to various targeted and non-targeted moments in the data.

3.4.1 Parameter Estimates

Table 17 reports the parameter estimates from our SMM procedure and the bootstrapped standard errors, which are uniformly very small reflecting the size of our micro data.

#	Parameter	Description	Value	St Errors
1.	ϕ	Fixed cost of operation	0.216	(0.012)
2.	θ^H	Innovative capacity of high-type firms	1.751	(0.020)
3.	θ^L	Innovative capacity of low-type firms	1.391	(0.017)
4.	θ^E	Innovative capacity of entrants	0.024	(0.001)
5.	α	Probability of being high-type entrant	0.926	(0.023)
6.	ν	Transition rate from high-type to low-type	0.206	(0.005)
7.	λ	Innovation step size	0.132	(0.010)
8.	φ	Exogenous destruction rate	0.037	(0.001)

Table 17: Parameter Estimates

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[59]Due to disclosure restrictions, we cannot use the bootstrapped distribution of the data moments directly. Instead, we generate the data moments from a multivariate normal distribution with mean and covariance matrix that are calculated from bootstrapped data moments.

The estimate of the fixed cost of operation indicates that the ratio of fixed workers to variable production workers is around 13.3%. Our estimates also show that high-type firms are about 26% more innovative than low-type firms ($\theta^H / \theta^L \approx 1.26$). Entrants have a 93% chance of being a high-type firm ($\alpha = 0.93$), and high-type firms face an annual 21% probability of transitioning to low-type ($\nu = 0.206$). This pattern implies a very high degree of negative selection—firms are much more likely to be high-type when young than later in their life cycle. The parameter λ is estimated as 0.132, which implies that an innovation leads to 13.2% increase in quality for an average product line. We also estimate a small exogenous destruction rate, $\varphi = 0.037$. Recall, however, that the overall rate of firm exit will be higher than this because of endogenous exit due to creative destruction and obsolescence, as we show below.

3.4.2 Goodness of Fit

Table 18 reports the empirical moments that we target (together with their standard errors) and the predicted values from our model. The solid bars in Figures 16(a)-(e) for the model-implied moments provides a graphical depiction.

# Moments	Model	Data	St Error	# Moments	Model	Data	St Error
1. Firm exit (small-young)	0.097	0.107	(0.002)	10. Sales growth (small-young)	0.101	0.107	(0.006)
2. Firm exit (small-old)	0.092	0.077	(0.002)	11. Sales growth (small-old)	0.040	0.024	(0.004)
3. Firm exit (large-old)	0.036	0.036	(0.001)	12. Sales growth (large-old)	-0.005	-0.003	(0.002)
4. Trans. from large to small	0.021	0.010	(0.001)	13. R&D to sales (small-young)	0.086	0.064	(0.004)
5. Trans. from small to large	0.038	0.014	(0.001)	14. R&D to sales (small-old)	0.066	0.059	(0.004)
6. Prob. of small (cond on entry)	0.848	0.753	(0.005)	15. R&D to sales (large-old)	0.059	0.037	(0.001)
7. Emp. growth (small-young)	0.101	0.106	(0.004)	16. 5-year entrant share	0.336	0.393	(0.003)
8. Emp. growth (small-old)	0.040	0.035	(0.003)	17. Fixed cost-R&D labor ratio	4.175	5.035	(0.015)
9. Emp. growth (large-old)	-0.005	-0.005	(0.002)	18. Aggregate growth	0.023	0.022	(0.007)

Table 18: Model and Data Moments

Both Table 18 and Figure 16 show a relatively good fit between our model-implied

moments and data. Our model replicates salient characteristics of data, including lower exit rates for larger and older firms, similar transition rates across firm sizes and entry/exit, quite similar growth rates for sales and employment by firm size and age bins, and similar R&D/sales intensities by type of firm. The last three economy-wide moments are also well aligned. On the whole, despite the overidentification of matching 18 moments with 8 parameters, the fit is quite good.[60]

Table 19 shows the important equilibrium implications in our baseline economy (all numbers in this and subsequent tables, except welfare, are in percentage points). These moments will be used extensively for comparison in our policy analysis in the next section. The equilibrium growth rate is $g = 2.26\%$. This is driven by entry as well as R&D investments by high- and low-type firms. The table shows that the per product innovation rate of high-type firms is about 50% greater than that of low-type firms, which reflects their greater innovative capacity ($x^h = 38.1\%$ vs. $x^l = 25.9\%$). As explained above, the distribution of product lines across high- and low-type firms is determined by different rates of innovation for these two types of firms, different obsolescence rates, and negative selection due to transitions to low-type. Our model finds 6.3% of product lines are held by high-type firms (Φ^h), 55% by low-types firms (Φ^l), and 38.7% are inactive. Together with the 0.51% flow rate of innovations by entrants, these innovation efforts lead to the employment of about 19.9% of all skilled workers in R&D ($L^{R\&D}/L^S$) and an average creative destruction rate of $\tau = 17.2\%$. We also normalize baseline welfare to 100 for ease of comparison in our subsequent policy analysis.

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[60]We do not report tests of the overidentifying restrictions for the usual reason that, given our sample size, standard errors are tiny, and even the most minor deviation from these 18 moments would constitute a rejection of the overidentifying restrictions. At the bottom of this, of course, is the fact that standard errors are computed without allowing for "model misspecification".
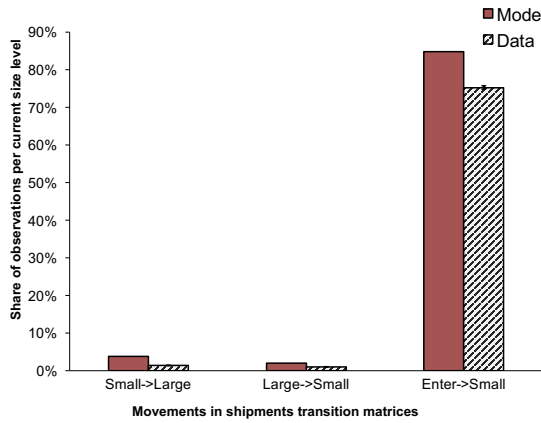
(a) Transition Rates
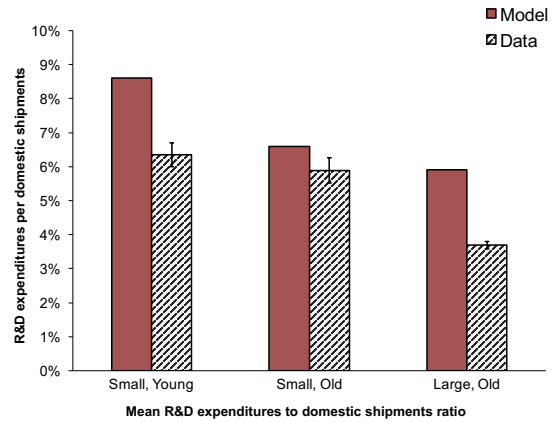
(b) R&D Intensity
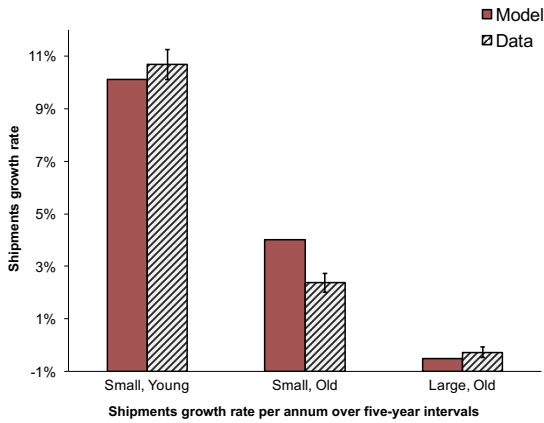
(c) Sales Growth
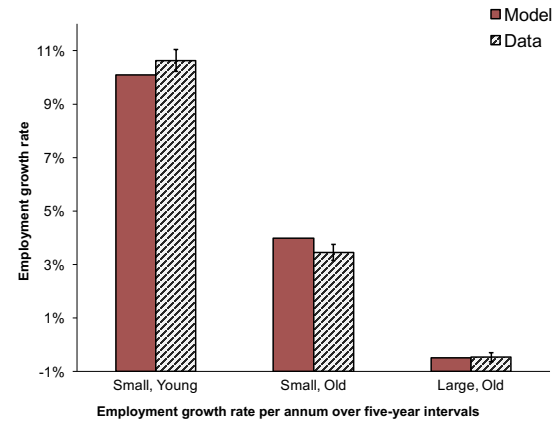
(d) Employment Growth
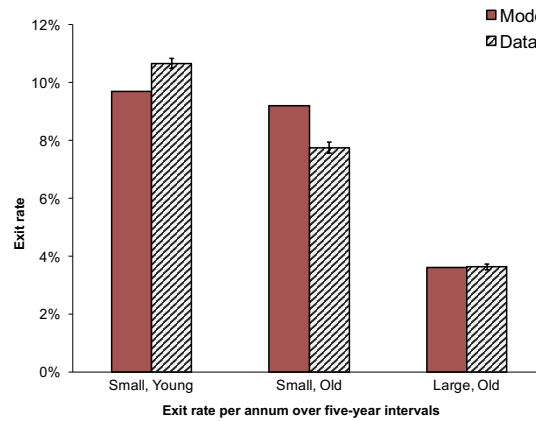
(e) Exit Rates

Figure 16: Data and Simulated Moments

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
0.51	25.90	38.13	55.04	6.28	147.26	130.33	19.86	17.16	2.26	100

Table 19: Baseline Economy

Figure 17 shows the productivity distribution across product lines among high- and low-type firms. An important point to note is that the threshold productivity for high-type firms is lower because of their greater R&D value of operating a product line ($\hat{q}_{h,\min} = 1.30$ vs. $\hat{q}_{l,\min} = 1.47$).

(a)   Productivity Distribution and Obsolescence

(b) Impact of Incumbent Tax
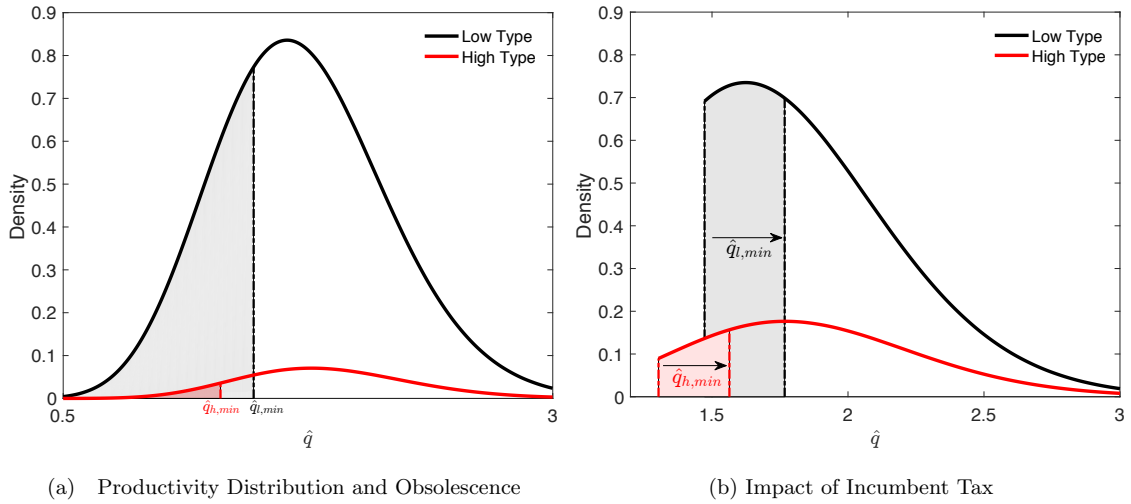
Figure 17: Productivity Distribution and Selection

3.4.3   Non-Targeted Moments

We assess the performance of our model by comparing its implications for a range of non-targeted moments, which capture important economic quantities, but have played no role in our estimation. This strategy thus provides an out-of-sample test of the structure imposed by our model and the values of the parameters we have estimated. Reassuringly, we will see that our model performs fairly well, raising our confidence in the model's ability to provide a good approximation to data and the conclusions that will follow from

the policy experiments.

First, Panel A of Table 20 considers persistence in growth rates among firms that survive over the whole sample period. Table 19 shows that about 10% of active product lines are operated by high-type firms in our model, and so we look at persistence in the second period of differences between the top 10% of firms and the remaining 90% in terms of first-period employment growth. For both employment growth and the R&D-to-sales ratio, the model generates patterns consistent with the data, though the differences in the data are somewhat larger in our model. For example, the future employment growth of bottom 90% and top 10% of firms in the data are 0.011 and 0.016, respectively, while they are 0.011 and 0.037 in our model.

Moments	Model	Data Mean	St Error
Panel A			
Employment growth of bottom 90%	0.011	0.011	(0.004)
Employment growth of top 10%	0.016	0.037	(0.016)
R&D to sales of bottom 90%	0.061	0.038	(0.004)
R&D to sales of top 10%	0.071	0.052	(0.018)
Panel B			
R&D per employee ratio (high to low)	1.737	1.461	(*n/a*)
Patent per employee (high to low)	1.578	1.838	(*n/a*)
Panel C			
Productivity distribution - 75/25: small/young	1.328	1.344	(0.006)
Productivity distribution - 75/25: small/old	1.231	1.311	(0.006)
Productivity distribution - 75/25: large/young	1.150	1.388	(0.017)
Productivity distribution - 75/25: large/old	1.087	1.294	(0.004)
Panel D			
Add a product (or more)	2.9%	8.0%	(0.003)
Drop a product (or more)	2.3%	8.3%	(0.003)
Do both	91.7%	76.8%	(0.005)
Do neither	3.1%	7.0%	(0.003)

Table 20: Non-targeted Moments

Panel B uses the recent Management and Organizational Practices Survey (MOPS)

conducted by the U.S. Census Bureau. Bloom et al. (2017) summarize some initial findings from MOPS 2010 survey, which we compare with the implications of our model. These authors group firms into deciles of management scores. Since high-type firms contain 10% of the active product lines in our simulations, we compare the innovation performances our high-type firms to the top-decile of the Bloom et al. (2017) sample. The ratio of the R&D per employee of the top 10% of firms to the bottom 90% is 1.5 in the data, and our model predicts a similar rate, 1.7. Likewise, the patent per employee ratio is 1.8 in the data versus 1.6 in our model. All in all, the model performs fairly well with respect to these non-targeted comparisons, which is reassuring.

Panel C presents the ratio of productivities at the $75th$ and $25th$ percentiles by size and age group. In the data, these are calculated with 5% fuzzy bands around each percentile point to allow for disclosure. Both the model and data exhibit similar productivity distributions within each size and age category, even though these distributions were not targeted in our estimation.

Panel D reports the fractions of firms that gained at least one product without losing any in a year, lost at least one product without gaining any, both gained and lost at least one product and neither gained nor lost any product. The model and data exhibit similar patterns for these numbers as well.

Finally we compare the product line distribution that is generated from our model to its empirical counterpart in Figure 18.
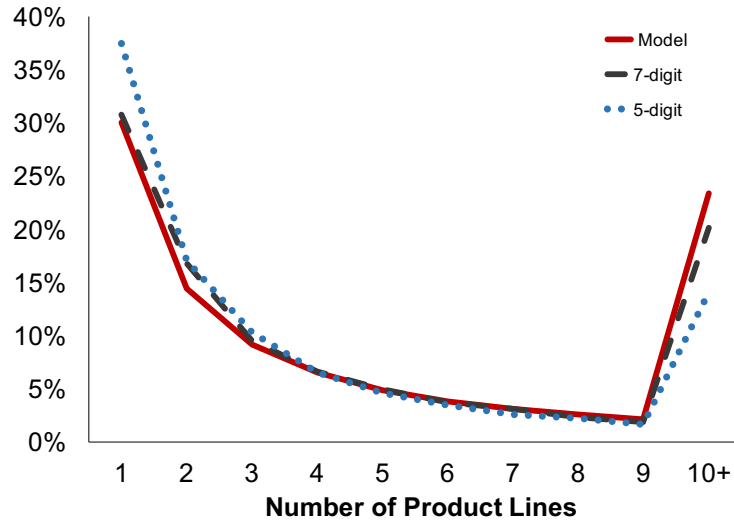
Figure 18: Product Line Distribution

Product information for firms is taken from the Product Trailers to the Census of Manufacturers. Our model generates a product line distribution that is almost identical to the 7-digit product distribution in the data. In addition, we plot 5-digit product distribution which is not too different from our model either. Panel D of Table 20 also shows that the unweighted rate at which firms add and drop products over five year periods in the model and data are reasonably aligned and in accordance with Bernard et al. (2010). This comparability for product count distributions and firm-level adjustments is encouraging since information on these product line distributions are not used in our estimation.

We next follow Foster et al. (2001), Bartelsman and Doms (2000), and Lentz and Mortensen (2008) and perform a simple growth decomposition according to the following

identity,

$$\Delta\Theta_t = \underbrace{\sum_{i\in\mathbb{C}_t} s_{it-1}\Delta\hat{Y}_{it}}_{within} + \underbrace{\sum_{i\in\mathbb{C}_t} \left(\hat{Y}_{it-1} - \Theta_{t-1}\right)\Delta s_{it}}_{between}$$

$$+ \underbrace{\sum_{i\in\mathbb{C}_t} \Delta s_{it}\Delta\hat{Y}_{it}}_{cross} + \underbrace{\sum_{i\in\mathbb{E}_t} s_{it}\left(\hat{Y}_{it} - \Theta_{t-1}\right)}_{entry} - \underbrace{\sum_{i\in\mathbb{X}_t} s_{it-1}\left(\hat{Y}_{it-1} - \Theta_{t-1}\right)}_{exit} \quad (3.28)$$

where Y_{it} is value added for firm i at time t, N_{it} is the number of employees of the firm, and $\mathbb{C}_t$, $\mathbb{E}_t$ and $\mathbb{X}_t$, respectively, denote the set of continuing, entering and exiting firms. In addition, we have $\Theta_t = \sum_i s_{it}\hat{Y}_{it}$, $\hat{Y}_{it} = Y_{it}/N_{it}$, and $s_{it} = N_{it}/\sum_i N_{it}$. We report each of the five components in this decomposition both in the data and from our model in Table 21.

	Model	Data	St Error
Within Share	0.607	0.999	(0.176)
Between Share	-0.024	-0.049	(0.057)
Cross Share	0.239	-0.305	(0.176)
Entry Share	0.175	0.192	(0.062)
Exit Share	0.003	0.164	(0.033)
Net Entry Share	0.178	0.356	(0.074)
10-year Cumulative Growth	0.254	0.261	(0.051)

Table 21: Decomposition

The model and data both show the largest component of growth coming from the within-firm labor productivity growth term, and the signs and magnitudes mostly line up over the components too. The one exception is in the cross term, where the model finds growing share of employment connected to growing labor productivity, whereas the data finds a negative correlation. This discrepancy is not a very robust feature, however; the cross term becomes positive, for example, when we look at the 1987-1992 subsample.

3.5 Policy Experiments and Efficiency

In this section, we perform counterfactual policy analysis to gain insight on both the implications of different types of industrial policies and the form of optimal policy in this economy. Before turning to our analysis of optimal policy, we first show how incumbent R&D subsidies, fixed cost subsidies and entry subsidies impact the equilibrium.[61]

3.5.1 Incumbent R&D Subsidy

The results from subsidizing the R&D of incumbents are shown in Table 22. As in other policy experiments, we choose the subsidy rate to be equivalent to 1% of GDP, and also show the key equilibrium objects from our baseline economy (from Table 19) in Panel A for comparison.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.51	25.90	38.13	55.04	6.28	147.26	130.33	19.86	17.16	2.26	100.00
				Panel B. 1% of GDP ($s_i = 14\%$)						
0.46	27.39	40.73	53.01	6.85	150.97	133.78	21.75	17.78	2.34	100.63

Table 22: Incumbent R&D Subsidy

A subsidy equivalent to 1% of GDP translates into a 14% subsidy on R&D spending of continuing firms. Unsurprisingly, this leads to higher R&D by these incumbents. Low-type incumbents increase their innovation rate from 25.9% to 27.4%, while high-type incumbents go from 38.1% to 40.7%—both of these are about 6% higher than the baseline. However, the overall impact on innovation and growth is much less than this direct effect. The average rate of creative destruction, τ, increases only by 3%, for instance. This

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[61]To focus on the key economic implications of our model in the clearest fashion, we abstract from the costs of raising taxes. In any case, we will see below that optimal policies typically involve taxes on the operation of continuing firms, thus raising rather than reducing revenues to tax authorities.

is for two reasons. First, at a given skilled wage, greater incumbent R&D would increase creative destruction and thus discourage entry. Second, and more important, the greater demand for skilled workers from incumbent R&D increases the skilled wage. This reduces R&D by entrants from 0.51% to 0.46%, and also modestly reduces the amount of skilled labor allocated to operations and thus raises the ratio of skilled labor employed in R&D from 19.9% to 21.7%. The overall result is a modest increase in growth 2.26% to 2.34%, and aggregate welfare goes up by 0.6% (in consumption equivalent terms).

3.5.2 Subsidy to Operating Costs

We next consider an industrial policy subsidizing the continued operation of incumbents by subsidizing their fixed costs of operations $w^s\phi$, which approximates policies that support large firms that are in economic trouble.[62] A subsidy equivalent to 1% of GDP in this case corresponds to a 4% subsidy on the fixed costs of operation of continuing firms.

Panel B of Table 23 shows that this subsidy discourages exit, increasing the fraction of active product lines (Panel A again gives the baseline for comparison). It also leads to modest declines in the innovation rates of entrants, low-type incumbents and high-type incumbents. In particular, because now more firms are operating, the demand for skilled labor increases, the skill wage goes up and fewer skilled workers perform R&D (the fraction of skilled workers allocated to R&D goes down modestly, from 19.9% to 19.4%). Because low-type firms are overrepresented among those at the margin of obsolescence (recall Figure 17), this policy also induces further negative selection: the share of product lines operated by low-type firms in the economy increases from 55.0% to 55.6%, while the share operated by high-type firms declines from 6.3% to 6.1%. As a consequence of all of these negative effects, the growth rate of the economy declines from 2.26% to 2.24%, and

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[62]Or equivalently, their exit is taxed or some combination thereof. We consider subsidies or taxes on the fixed cost of operations rather than on all costs or on accounting profits, because these alternative policies would also affect markups, partly confounding the main effect we are interested in. All the same, such subsidies or taxes have broadly similar impacts.

aggregate welfare declines by 0.2%.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.51	25.90	38.13	55.04	6.28	147.26	130.33	19.86	17.16	2.26	100.00
				Panel B. 1% of GDP ($s_o = 4\%$)						
0.50	25.57	37.58	55.63	6.05	146.27	129.30	19.39	17.00	2.24	99.80

Table 23: Operation Subsidy

In sum, a subsidy to the operating costs of incumbents reduces growth and welfare because it causes a negative selection effect, increasing the share of product lines controlled by low-type firms, as low-type firms tend to benefit more from this subsidy which is directed to low-productivity product lines.

3.5.3 Entry Subsidy

Finally, for comparison, we also consider the implications of an entry subsidy equivalent to 1% of GDP. The results are reported in Table 24.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.51	25.90	38.13	55.04	6.28	147.26	130.33	19.86	17.16	2.26	100.00
				Panel B. 1% of GDP ($s_e = 65\%$)						
1.35	24.59	35.08	49.50	10.16	151.30	138.46	22.02	17.09	2.25	98.36

Table 24: Entry Subsidy

The direct effect of the subsidy is to increase entry. In Panel B we see that the innovation effort of entrants increases from 0.51% to 1.35%, but now there is a decline in the innovation rates of continuing firms. The total effect is a modest reduction in the average creative destruction rate of the economy from 17.2% to 17.1%. This in turn leads to slightly lower growth and aggregate welfare.

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3.5.4 Social Planner

The results of the previous subsection show only small effects from subsidies to incumbent R&D, entrant R&D and operations. We will see now, however, that the social planner can significantly increase welfare. Since we are not interested in monopoly distortions per se, we restrict the social planner to the same production and pricing decisions as the equilibrium, and only allow her to control the entry, exit and R&D margins of different firms. It is straightforward to see that the social planner will choose the same per product R&D for all high-type firms and also the same R&D for all low-type product lines. Then, we can represent the problem of the planner as choosing $\{\hat{q}_{h,\min}, \hat{q}_{l,\min}, x^h, x^l\}$ to maximize representative household welfare (3.26) subject to the skilled labor market-clearing condition (3.24). Table 25 summarizes the allocation implied by social planner's choices.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.51	25.90	38.13	55.04	6.28	147.26	130.33	19.86	17.16	2.26	100.00
				Panel B. Social Planner						
0.60	25.42	45.34	5.64	44.70	240.42	27.80	34.21	22.30	2.94	104.47

Table 25: Social Planner

The social planner improves growth and welfare quite significantly. Growth increases from 2.26% to 2.94%. Welfare increases by 4.47%, underscoring that the equilibrium was far from optimal in the baseline model, and the limited consequences of the subsidy policies considered so far stemmed from the fact that each was by itself ineffective in triggering a reallocation of resources towards R&D by high-type firms. How the social planner is achieving such a reallocation can also be seen from Table 25, which illustrates the form of the optimal allocation. Most notably, the exit threshold for low-type firms, $\hat{q}_{l,\min}$, increases substantially (from 1.47 to 2.40) whereas the threshold for high-type firms, $\hat{q}_{h,\min}$, actually decreases (from 1.30 to 0.28). The social planner also differentially increases R&D

by firm type: high-type incumbents increase from $x^h = 0.38$ to 0.45, while R&D for low-type firms remains essentially unchanged (there is also a modest increase in the entry rate). The combined effect of the large increase in the exit threshold for low-type firms and increased R&D for high-type firms is a significant change in the selection effect—the ratio of high- to low-type firms (Φ^h/Φ^l) increases from 0.11 to 7.93.

Table 26 further dissects how the social planner is improving welfare relative to the baseline economy. Row 3 shows that if the social planner can only change the entry and innovation rates (keeping the exit thresholds at their baseline equilibrium values, $\hat{q}_{l,\min}$ and $\hat{q}_{h,\min}$), there is essentially no effect on welfare. On the other hand, when she only controls the exit thresholds (keeping the innovation and entry rates at their baseline equilibrium values), she achieves most of the selection gains and can increase welfare by 1.58% in consumption-equivalent terms. Naturally, when the two margins are combined, she can achieve much greater growth and welfare gains as we have seen in Table 25.

	x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	g	Wel
1. Baseline	0.51	25.90	38.13	55.04	6.28	147.26	130.33	2.26	100.00
2. Social Planner (SP)	0.60	25.42	45.34	5.64	44.70	240.42	27.80	2.94	104.47
3. SP choosing innovation	0.52	25.63	38.71	54.45	6.91	147.26	130.33	2.26	100.00
4. SP choosing $\hat{q}_{\min}$	0.94	25.90	38.13	39.74	18.92	161.16	29.91	2.43	101.58

Table 26: Restricted Social Planner

3.5.5 Uniform Optimal Policy

The social planner's allocation discussed in the previous subsection relied on choosing the exit thresholds and R&D rates of different types of firms. In practice, policies cannot be directly conditioned on type (at least not without also specifying relevant incentive compatibility constraints).[63] Motivated by this restriction, in this subsection we study how much of the gap between the baseline equilibrium allocation and the social

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[63]See Scotchmer (2004), Hopenhayn et al. (2006), and Akcigit et al. (2016b) on the design of policies to encourage innovation under asymmetric information.

planner's allocation characterized in the previous subsection can be closed with uniform policies. In Table 27, we start by looking at the optimal choice of each one of the three policies already introduced previously.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.51	25.90	38.13	55.04	6.28	147.26	130.33	19.86	17.16	2.26	100.00
				Panel B. Incumbent R&D ($s_i = 39\%$)						
0.38	30.74	46.54	47.67	8.65	160.07	142.83	26.40	19.06	2.51	101.22
				Panel C. Operation ($s_o = -69\%$)						
0.61	30.78	46.04	45.95	9.84	161.50	145.72	27.08	19.29	2.54	101.42
				Panel D. Entry ($s_e = 18\%$)						
0.62	25.74	37.69	54.26	6.95	147.58	131.35	20.00	17.20	2.27	100.04
				Panel E. Incumbent R&D and Operation ($s_i = -3\%, s_o = -74\%$)						
0.63	30.74	45.94	45.90	9.90	161.50	145.81	27.07	19.29	2.54	101.42

Table 27: Uniform Policies

Panel A again depicts the baseline equilibrium for comparison. In Panel B, we show that the optimal rate of incumbent R&D subsidy (by itself) would be $s_i = 39\%$, which is higher than what we considered in Table 22 above, but has similar implications. In Panel C, we turn to taxes/subsidies on operations. Here, we see that the optimal policy is a rather large tax (instead of the subsidy considered in Table 23 above). With this optimal tax rate of $s_o = -69\%$,[64] we can obtain a significant increase in growth, achieving $g = 2.54\%$. As with the social planner's allocation, this is made possible by increasing the exit thresholds and generating a significant selection effect—the fraction of product lines operated by high-type firms increases from 10% to 18%. Finally, Panel D shows that entry subsidies have a very small effect.

In sum, the results of single uniform tax/subsidy policies Panels A-D in Table 27 suggest that taxes on the operations (or the fixed costs of operations) may be the most

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[64]Recall that this is a tax on the fixed costs of operation, $w^s\phi$, not on all costs or revenues of firms. The 69% tax on the fixed costs of operation of incumbents is equivalent to an average tax of 8% on the revenues of incumbents.

potent policies.

We next analyze the optimal combination of these uniform policies, with the results presented in Panel E of Table 27. Panel D already showed that entry subsidies are not very effective, and it turns out that conditional on using incumbent R&D subsidies and operation taxes, there is no further gain from using entry subsidies. So, Panel E focuses on the optimal combination of incumbent R&D subsidies and taxes on the fixed costs for continuing firms. The optimal combination of these uniform policies involves a large tax on fixed costs ($s_o = -74\%$) and perhaps surprisingly also a small tax on incumbent R&D ($s_i = -3\%$). The resulting allocation increases the growth rate of the economy to 2.54% and secures a 1.42% consumption-equivalent welfare gain. This gain is achieved by substantially increasing the exit threshold for low-type firms, which then increases the ratio of product lines operated by high-type firms to those operated by low-type firms from 11% in the baseline to 22%. With the skilled labor freed from operations, overall R&D investments also increase, though because these are uniform policies, R&D investments by both types of firms increase in tandem.

3.6   Robustness

The broad pattern of estimation results and policy analyses reported so far is quite robust. In this section, we illustrate this by considering a number of variations on our sample and model. In each case, we report the baseline equilibrium moments, the social planner's allocation and the allocation that results from the optimal choice of uniform incumbent R&D subsidies and taxes on operations.

3.6.1 Employment-weighted Sample

Our baseline estimation targets unweighted moments. Our first variation shows that targeting moments weighted by beginning of period employment (which means that we are using such weighted moments both in the model and the data) makes little difference. The results are shown in Table 28, where we see similar values for most key equilibrium objects. The social planner's allocation reported in Panel B is also very similar to the baseline, though the increase in the growth rate is a little more modest—from 2.22% to 2.54%, with a corresponding 1.25% consumption-equivalent welfare gain. The implications of the optimal uniform policies are also similar (and these policies again involve a large tax on operations and in this case, no tax or subsidy on incumbent R&D), increasing the growth rate to 2.39%, with a consumption-equivalent welfare gain of 0.56%. The main reason for the smaller gains from both the social planner's allocation and the optimal uniform policies is that the ratio of product lines operated by high-type firms to low-type firms is not as low in this case as in our baseline estimation, thus limiting the extent of the selection effects that optimal policies leverage.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.52	25.26	47.76	63.37	11.62	126.53	89.02	23.86	22.08	2.22	100.00
				Panel B. Social Planner						
0.58	24.43	52.99	38.08	28.90	152.47	42.06	31.99	25.20	2.54	101.25
			Panel C. Incumbent R&D and Operation ($s_i = 0\%, s_o = -47\%$)							
0.57	27.28	51.53	53.96	16.41	138.78	109.77	28.54	23.74	2.39	100.56

Table 28: Employment-weighted Estimation

3.6.2 Organic Sample that Excludes M&A Activities

Our baseline sample includes "inorganic" entry and exit, taking the form of mergers and acquisitions (M&A) and spinouts (where part of an existing firm becomes a new legal

entity). We next reestimate the model after removing all observations we determined to be potentially influenced by inorganic activity on these margins.[65] The results from this exercise are reported in Table 29. The broad patterns of various policy implications remain very similar to the baseline—for example, the social planner is now able to increase growth from 2.24% to 2.90%, with a 4.17% consumption-equivalent welfare gain, and the optimal policies once again involve a substantial tax on operations of continuing firms and no tax or subsidy on incumbent R&D.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.46	27.24	33.97	55.48	2.41	154.57	146.64	18.33	16.39	2.24	100.00
				Panel B. Social Planner						
0.58	28.84	44.41	3.17	44.38	269.34	29.39	32.91	21.20	2.90	104.17
			Panel C. Incumbent R&D and Operation ($s_i = -4\%, s_o = -84\%$)							
0.60	33.99	43.00	48.80	3.49	168.45	160.26	26.22	18.69	2.56	101.82

Table 29: Excluding M&A Activities

3.6.3 Manufacturing Sample

Because of our reliance on R&D moments, our baseline sample includes continuously-innovative firms as explained in Section 3.3.1, and thus excludes most manufacturing firms. We believe that the same dynamics should apply to many firms that do not report R&D but still engage in innovation-type activities to take over product lines currently operated by competitors.[66] To investigate this issue, we first reestimated our model drop-

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[65]We identify these cases following the procedures of Haltiwanger et al. (2013b). We use the establishment identifiers, which are distinct from firm identifiers, to identify cases where an establishment exists before or after the associated firm id. We flag as being a potentially inorganic birth the cases where more than 10% of the firm's initial employment appears to come from a pre-existing establishment owned by another firm in the prior year; similarly, a potential inorganic exit is flagged when more than 10% of the exiting firm's employment is in a plant that transfers to a new firm in the following year. This 10% bar is aggressive, but also serves well to test the issues. About 19% of births, 30% of exits, and 41% of firms overall show some measure of inorganic activity in our innovative firm sample. Excluding these firms leaves a sample size of 9,854 firm-period observations.

[66]See National Research Council (2004) and Corrado et al. (2005) on the range of innovation activities not recorded in R&D surveys.

ping all R&D moments and calculating the remaining 15 moments using the universe of manufacturing firms (982,559 firm-period observations). We weight each firm such that the firm size distribution matches that of our core sample using 16 size bins. The results of this estimation are reported in Panel A of Table 30.[67]

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
					Panel A. Baseline					
1.71	5.08	29.69	22.92	4.63	215.50	118.64	25.47	4.25	1.92	100.00
					Panel B. Social Planner					
1.95	5.29	35.03	16.93	6.63	256.76	53.92	36.29	5.17	2.34	101.88
				Panel C. Incumbent R&D and Operation ($s_i = 7\%, s_o = -41\%$)						
1.80	5.72	33.80	20.34	5.10	233.44	149.89	31.17	4.69	2.12	100.75

Table 30: Full Manufacturing (Non-innovating Firms Included)

The overall patterns are similar to our baseline, though with lower innovation rates and aggregate growth, likely reflecting the inclusion of less innovative firms in the sample. Panel B shows that the social planner's allocation can again increase growth significantly (from 1.92% to 2.34%), and achieves this once again by leveraging the selection effects. The implications of optimal uniform policies in Panel C are also similar, though now there is a small subsidy to incumbent innovation too.

3.6.4 Model with Unskilled Overhead Labor

In this subsection, we return to our initial sample but modify our baseline model to allow for the fixed operations cost to consist of both skilled and unskilled labor. Namely, we assume that a β fraction of the overhead labor ϕ has to be skilled, and the remaining $1 - \beta$ fraction is from unskilled labor. This leads to a simple generalization of our setup,

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[67]We also verified that dropping the R&D moments in our baseline sample leads to similar estimation results and policy conclusions.

with the Bellman equation for a $k-$type firm now taking the form

$$r\tilde{V}_k\left(\hat{Q}\right) - \dot{\tilde{V}}_k\left(\hat{Q}\right) = \max_{x_k \geq 0}\left\{\begin{array}{c} \sum_{\hat{q}\in\hat{Q}}\left[\begin{array}{c}\tilde{\pi}(\hat{q}) - \phi\left[\beta\tilde{w}_s + (1-\beta)\tilde{w}_u\right] \\ +\tau\left[\tilde{V}_k\left(\hat{Q}\backslash\{\hat{q}\}\right) - \tilde{V}_k\left(\hat{Q}\right)\right]\end{array}\right] \\ -n\tilde{w}^s G(x,\theta^k) + nx_k\left[\mathbb{E}\tilde{V}_k\left(\hat{Q}\cup\{\hat{q}+\lambda\bar{\hat{q}}\}\right) - \tilde{V}_k\left(\hat{Q}\right)\right] \\ +\varphi\left[0 - \tilde{V}_k\left(\hat{Q}\right)\right] + \mathbb{I}_{k=h}\nu\left[\tilde{V}_l\left(\hat{Q}\right) - \tilde{V}_h\left(\hat{Q}\right)\right]\end{array}\right\}.$$

The labor-market clearing conditions are then modified to accommodate the use of both skilled and unskilled labor in operations as follows

$$L^S = L^{RD} + \Phi\beta\phi \text{ and } 1 = L^P + \Phi(1-\beta)\phi.$$

We also set the parameter β to match the fraction of managers who have a college degree or above, which is 45.7%. The results are reported in Table 31.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.56	21.48	36.46	54.08	10.26	134.58	104.20	28.70	15.91	2.23	100.00
				Panel B. Social Planner						
0.59	19.77	39.08	38.48	22.23	151.12	30.07	32.71	16.89	2.37	100.56
			Panel C. Incumbent R&D and Operation ($s_i = -4\%, s_o = -9\%$)							
0.59	21.76	36.92	52.64	10.93	136.93	107.82	29.54	16.08	2.26	100.02

Table 31: Model with Unskilled Overhead Labor

The baseline estimation leads to very similar results. The implications of the social planner's allocation and optimal uniform policies are also similar, but generate smaller gains relative to the baseline—in large part because the ability of these policies to free up skilled labor from operations is now more limited. All the same, the qualitative patterns are similar, and both the social planner's direct intervention and the optimal uniform policies again leverage the selection effect.[68]

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[68]Perhaps the most important difference is that the tax on the operations of continuing firms is now smaller, 9%, as opposed to the taxes that were around 70% in our other samples and variations.

3.6.5 Model with Reallocation Cost

Our baseline model does not incorporate any costs for reallocating labor from the original firm operating a product line to a new one taking it over. In practice, there may be several types of reallocation costs, both because some workers might go through unemployment and also because some employees may need to be retrained to work for their new employers or with new technologies. Here, we investigate the implications of allowing for these types of reallocation costs by introducing them in a reduced-form manner. Namely, we assume that hiring new workers entails training costs, and training each type of worker requires v workers of the same type for training. As a result, when a new firm hires l new unskilled workers and ϕ skilled workers for operations, it incurs an additional cost of $v\left[\tilde{w}^u l + \tilde{w}^s \phi\right]$ (the reallocation of R&D inputs is assumed to be costless). This modification leads to a small modification in the Bellman equations, which now take the form

$$
r\tilde{V}_k\left(\hat{Q}\right) - \dot{\tilde{V}}_k\left(\hat{Q}\right) = \max_{x_k \geq 0} \left\{ \begin{array}{c} \sum_{\hat{q} \in \hat{Q}} \left[\tilde{\pi}(\hat{q}) - \tilde{w}^s \phi + \tau \left[\tilde{V}_k\left(\hat{Q} \setminus \{\hat{q}\}\right) - \tilde{V}_k\left(\hat{Q}\right)\right]\right] \\ -n\tilde{w}^s G(x, \theta^k) \\ +nx_k \left[\begin{array}{c} \mathbb{E}\tilde{V}_k\left(\hat{Q} \cup \{\hat{q} + \lambda\bar{\hat{q}}\}\right) - \tilde{V}_k\left(\hat{Q}\right) \\ -v\tilde{w}^u \mathbb{E}\left[l(\hat{q} + \lambda\bar{\hat{q}})\right] - v\tilde{w}^s \phi \end{array} \right] \\ +\varphi\left[0 - \tilde{V}_k\left(\hat{Q}\right)\right] + \mathbb{I}_{k=h} v\left[\tilde{V}_l\left(\hat{Q}\right) - \tilde{V}_h\left(\hat{Q}\right)\right] \end{array} \right\},
$$

where we have imposed that the reallocation costs are paid when the firm expands by taking over a product line from another incumbent. Because in equilibrium reallocation costs are incurred at the rate of average creative destruction τ, the labor-market clearing conditions become

$$
L_{\text{supply}}^S = L_{\text{demand}}^S + v\tau\phi \qquad \text{and} \qquad L_{\text{supply}}^P = L_{\text{demand}}^P + \tau v L_{\text{demand}}^P.
$$

We identify the new cost parameter v using estimates from Bloom et al. (2014) on the costs of training as equivalent to one month of a worker's time, which translates into $v = 1/12$. The resulting baseline equilibrium values and policy experiments are reported in Table 32, which shows very similar results to the baseline.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
				Panel A. Baseline						
0.41	23.49	43.60	60.26	4.89	137.43	106.61	19.57	16.70	2.25	100.00
				Panel B. Social Planner						
0.49	24.53	52.38	31.57	23.58	173.87	30.47	31.65	20.58	2.77	102.55
			Panel C. Incumbent R&D and Operation ($s_i = 7\%, s_o = -73\%$)							
0.46	26.55	52.72	48.15	10.81	154.34	127.68	27.05	18.94	2.55	101.44

Table 32: Model with Reallocation Cost

For example, the social planner's allocation increases the growth rate from 2.25% to 2.77%, with a consumption-equivalent welfare gain of 2.55%. Optimal uniform policies again impose a substantial tax operations and achieve a 1.44% consumption-equivalent welfare gain.

3.6.6 Model with Three Types of Firms

We next verify that our results are not unduly sensitive to assuming two types of firms by extending the model to three types of firms. The estimation results show that the innovative capacities of high-type and middle-type firms are estimated to be similar. Unsurprisingly in view of this, we find Table 33 that the policy implications also remain similar. For example, the social planner's allocation increases the growth rate from 2.20% to 2.94%, with a consumption-equivalent welfare gain of 5.6%. Optimal uniform policies again substantially tax the fixed cost of operations for continuing firms and achieve 1.81% consumption-equivalent welfare gain.

x^{entry}	x^l	x^m	x^h	Φ^l	Φ^m	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{m,\min}$	$\hat{q}_{h,\min}$	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
					Panel A. Baseline								
0.51	25.83	38.19	40.85	52.32	5.07	1.12	153.14	136.78	132.04	19.11	16.41	2.20	100.00
					Panel B. Social Planner								
0.34	23.23	45.83	46.97	2.30	24.21	21.19	277.38	0.25	95.08	34.04	21.92	2.94	105.60
				Panel C. Incumbent R&D and Operation ($s_i = 6\%, s_o = -69\%$)									
0.59	31.35	46.90	50.83	42.30	6.50	3.78	169.22	154.47	149.30	27.29	18.82	2.52	101.81

Table 33: Model with Three Types $\left(\theta^H, \theta^M, \theta^L\right)$

3.6.7 Model with Endogenous Supply of Skills

Finally, we extend our model to endogenize the supply of skilled workers. Specifically, we adapt our framework to an overlapping generations setup where each individual faces a constant death rate of ζ, and a measure ζ of new agents arrive at each instant, so that total population remains constant. In addition, each agent has a type indexed by κ. Upon entry into the economy, agents have a decision to acquire skills. Each agent can supply one unit of unskilled labor without any investment, and can also supply one unit of skilled labor if they acquire education, which is assumed to last a^* years for everybody. Education requires some of the skilled workers to be allocated to teaching, and we assume that an agent with type κ requires the services of $1/\kappa$ teachers during his education. Thus, the costs of education are higher for agents with low κ, and because these agents will have to bear this cost of education, they are less likely to become skilled.

We take the distribution of κ to be truncated Pareto,

$$\kappa \sim A\kappa^{\chi-1},$$

for convenience, where $\chi < 1$, $\kappa \in [0, \bar{\kappa}]$ and $A = \chi\bar{\kappa}^{-\chi}$.

Education decisions are some of the most heavily subsidized activities in practice. In our model too the social planner will face a strong incentive to subsidize education

because skilled workers create positive externalities when they perform R&D. If we rule out such subsidies, then other optimal policies would try to mimic them, potentially distorting the results of our policy analysis. For this reason, we also introduce an education subsidy at the rate $s_{edu} \in [0,1]$ that reduces the cost of education faced by the agents. Incorporating this subsidy, we can see that an agent of type κ will acquire education if

$$
\underbrace{\frac{e^{-(r-g+\zeta)a^*}w_S}{r-g+\zeta}}_{\text{present value of skilled worker}} - \underbrace{(1 - s_{edu})\frac{1}{\kappa}w_S \int_0^{a^*} e^{-(r-g+\zeta)t}dt}_{\text{present value of education cost}} > \underbrace{\frac{w_U}{r-g+\zeta}}_{\text{present value of unskilled worker}} .
$$

The right-hand side of this expression is the present discounted value of working as an unskilled worker, taking into account that the unskilled wage at the moment, w_U, will grow at the rate g, and that the agent has an effective discount rate of $r + \zeta$. The first term on the left-hand side is the present discounted value of working as a skilled labor, which recognizes that skilled workers will have no earnings during the first a^* years of their lives. Finally, the second term on the left-and side is the subsidized cost of education for a worker of type κ. This comparison gives a threshold for κ,

$$
\kappa^* = (1 - s_{edu})\left[1 - e^{-(r-g+\zeta)a^*}\right]\left(e^{-(r-g+\zeta)a^*} - \frac{w_U}{w_S}\right)^{-1},
$$

such that only those with $\kappa > \kappa^*$ will become skilled.

We denote the total population by L, which comprises unskilled labor (L^P), skilled R&D labor (L^{RD}), skilled labor working in operations (L^F), skilled teachers (L^T), and students still in the education process (L^E). Given the exponential age structure (due to the constant death rate), the fraction of workers becoming skilled who are still below the age of a^* is $1 - e^{-\zeta a^*}$, which implies that in the stationary equilibrium, the masses of teachers and students are, respectively,

$$
L^T = L\left(1 - e^{-\zeta a^*}\right)\int_{\kappa^*}^{\bar{\kappa}}\frac{1}{\kappa}dF(\kappa) \quad\text{and}\quad L^E = L\left(1 - e^{-\zeta a^*}\right)(1 - F(\kappa^*)).
$$

Incorporating the employment of skilled workers as teachers, the labor market-clearing conditions become

$$L^{RD} + L^F = L \left[e^{-\zeta a^*} \left(1 - \frac{A}{\chi}(\kappa^*)^\chi \right) - \left(1 - e^{-\zeta a^*} \right) \frac{A}{\chi - 1} \left(\bar{\kappa}^{\chi - 1} - (\kappa^*)^{\chi - 1} \right) \right]$$

$$L^P = L \frac{A}{\chi}(\kappa^*)^\chi .$$

To estimate this extended model with endogenous supply of skills, we choose the parameter ζ as 35 years to approximate the working life of skilled workers, and set $a^* = 4$ as the length of post-secondary education. We then choose $\chi = 0.035$, $\bar{\kappa} = 95.55$, and $L = 1.193$ so that this extended model replicates the supply of skilled and unskilled labor in our benchmark economy ($L^{RD} + L^F = 0.166$ and $L^U = 1$) and 0.6% of total employment ($= L^T/(L^{RD} + L^F + L^P + L^T)$) being devoted to post-secondary teaching as in the US economy. By construction, the estimates for the remaining parameters are identical to our baseline estimates reported in Table 17 (because $L^{RD} + L^F = 0.166$ and $L^U = 1$ as before).

Table 34 reports the results of our policy analysis in this case. The baseline allocation without the education subsidy is identical to our benchmark results by construction and is reported in Panel A for comparison. Panel B shows that introducing an optimal education subsidy, at the rate $s_{edu} = 0.81$, increases the growth rate from 2.26 to 2.69, and secures a 11% improvement in welfare. This sizable welfare effect reflects the severe underprovision of skilled labor in the benchmark allocation. Panel C provides the social planner's optimal allocation, which exploits the same selection effect as in our baseline model and increases the growth rate further by another 0.59 percentage points to 3.28 and welfare by an additional 3.46% relative to the allocation with optimal education subsidy. The additional welfare and growth gains from the social planner's allocation over the one with just education subsidies are similar to the gains from the social planner's allocation in our benchmark economy.

x^{entry}	x^l	x^h	Φ^l	Φ^h	$\hat{q}_{l,\min}$	$\hat{q}_{h,\min}$	L^S	$\frac{L^{R\&D}}{L^S}$	τ	g	Wel
					Panel A. Baseline						
0.51	25.90	38.13	55.04	6.28	147.26	130.33	16.55	19.86	17.16	2.26	100.00
					Panel B. Baseline with Optimal Education Subsidy $(s_{edu} = 81\%)$						
0.55	27.46	40.98	60.02	8.26	133.97	114.56	18.94	22.03	20.41	2.69	110.96
					Panel C. Social Planner						
0.63	26.81	47.73	16.13	41.68	188.74	28.86	18.93	33.93	24.84	3.28	114.42
					Panel D. Incumbent R&D, Operation Cost and Education Policies $(s_i = -3\%, s_o = -62\%, s_{edu} = 92\%)$						
0.65	31.32	47.22	50.58	12.36	147.01	129.78	18.94	28.10	22.33	2.94	112.08

Table 34: Model with Endogenous Supply of Skills

Panel D shows that the same mix of uniform policies as before—incumbent R&D subsidy and tax on operation costs—but now combined with education subsidies lead to somewhat smaller gains than the social planner, but again achieve this by leveraging the selection effect. In particular, in addition to a higher education subsidy, $s_{edu} = 0.92$, we have a tax on operations, which has a very similar magnitude to our baseline results ($s_o = -0.62$), and a small tax on incumbent R&D ($s_i = -0.03$). These policies again increase the exit thresholds, and especially for low-type firms, and increase the growth rates to 2.94% and lead to 12.08% improvement in consumption-equivalent welfare. Thus overall, we conclude that our policy conclusions are robust to endogenizing the supply of skilled labor.

3.7   Conclusions

In this paper we build a micro-founded model of firm innovation and growth. The model enables us to examine the forces jointly driving innovation, productivity growth and reallocation. We estimate the parameters of the model using simulated method of moments on detailed US Census Bureau micro data on employment, output, R&D and patenting. Our model fits the key moments from microdata reasonably well, and also performs well on non-targeted moments and is in line with the range of micro estimates

in the literature.

We use the model to investigate the implications of several types of industrial policies on long-run growth and welfare. We find that industrial policies (subsidies to incumbent R&D or to their operating costs) reduce growth and welfare, and entry subsidies are also ineffective. These small effects are not because the equilibrium of our model is approximately optimal. On the contrary, a social planner limited to affecting only R&D, entry and exit decisions can increase growth from 2.26% to 2.94%, and increase welfare by 4.47%. The social planner achieves this by strongly leveraging the selection effect. She forces low-type incumbents to exit at a very high rate, reduces their R&D, and increases the R&D of high-type incumbents.

Our general equilibrium model, which incorporates both reallocation and selection effects, also highlights the potential pitfalls of industrial policies supporting incumbents. Though there is substantial underinvestment in R&D, the optimal policy is not to subsidize R&D-type activities, because such subsidies increase R&D investments by both low-type and high-type firms. Instead, optimal policy should free up resources from the operations of low-type firms to be used for R&D by high-type firms, and this can be achieved by encouraging the exit of low-productivity firms, for example by taxing the operations of all firms.

Several further topics of inquiry are left for future research. First, it would be interesting to extend our analysis to incorporate an endogenous selection between non-innovation and innovation, and also to incorporate reallocation of other resources (unskilled labor and capital). Second, our analysis has been confined to comparisons of stationary equilibria (balanced growth paths), thus ignoring transition costs, which could be nontrivial. Third, and related, our baseline model did not incorporate any reallocation costs, though we allowed for such costs in a reduced-form manner in our extensions. A more systematic investigation of such costs would necessitate an micro-founded model of closely allocation of resources, for example via search (see Lentz and Mortensen, 2010)

for a complementary approach on this question). Fourth, an interesting possible extension of our framework would be to model the joint dynamics of innovation, reallocation and unemployment, which can enrich the analysis of the effects of various policies, and also enable us to incorporate some of the potential unemployment benefits of supporting incumbent producers. Fifth, we have also abstracted from political constraints. It would be important to consider the political economy of different types of industrial policies, which have often been politically difficult to manage and prone to capture. Sixth, our model can also be used to study mergers between high- and low-type firms which might be able to make more efficient use of the existing knowledge stock of low-type firms in certain circumstances. Finally, supplementing our approach with more direct estimation of the costs and benefits of different types of policies targeted at R&D by incumbents is a major area for future research.

Appendix A

Appendix to Chapter 1

A.1 Derivations for Static Product Market and Labour Market

By using the production function final goods sector, together with optimal price and quantity at the intermediate firm level, we can get a relation between wage rate and average quality as

$$Y = \frac{L^\beta}{1-\beta} \int_{\mathcal{N}} q_j \left(\left[(1-\beta)\frac{\bar{q}}{w} \right]^{\frac{1}{\beta}} L q_j \right)^{1-\beta} dj$$

$$Y = L(1-\beta)^{\frac{1-2\beta}{\beta}} \left[\frac{\bar{q}}{w} \right]^{\frac{1-\beta}{\beta}} \bar{q}\Phi \tag{A.1}$$

Also, final goods' producer profit needs to be zero (with aggregate price index normalized to 1). In other words, we have the following condition:

$$Y = \int_{\mathcal{N}} p_j k_j dj + wL$$

$$Y = \frac{1}{(1-\beta)} \frac{w}{\bar{q}} \left[(1-\beta) \frac{\bar{q}}{w} \right]^{\frac{1}{\beta}} L \int_{\mathcal{N}} q_j dj + wL$$

$$Y = (1-\beta)^{\frac{1-\beta}{\beta}} \left[\frac{\bar{q}}{w} \right]^{\frac{1-\beta}{\beta}} L\bar{q}\Phi + wL \tag{A.2}$$

Using A.1 and A.2, we obtain

$$L(1-\beta)^{\frac{1-2\beta}{\beta}} \left[\frac{\bar{q}}{w} \right]^{\frac{1-\beta}{\beta}} \bar{q}\Phi = (1-\beta)^{\frac{1-\beta}{\beta}} \left[\frac{\bar{q}}{w} \right]^{\frac{1-\beta}{\beta}} L\bar{q}\Phi + wL$$

$$\beta(1-\beta)^{\frac{1-2\beta}{\beta}} \Phi = \left[\frac{w}{\bar{q}} \right]^{\frac{1}{\beta}}$$

$$\frac{w}{\bar{q}} = \beta^{\beta}(1-\beta)^{1-2\beta}\Phi^{\beta}$$

$$w = \tilde{\beta}\bar{q} \tag{A.3}$$

where $\tilde{\beta} = \beta^{\beta}(1-\beta)^{1-2\beta}\Phi^{\beta}$. So wage is proportional to average quality in the economy. Incorporating the equilibrium wage rate, the profits simplify to

$$\pi(q_j) = \beta \left[(1-\beta) \right]^{\frac{1-\beta}{\beta}} \left(\beta^{\beta}(1-\beta)^{1-2\beta}\Phi^{\beta} \right)^{\frac{\beta-1}{\beta}} L q_j$$

$$\pi(q_j) = \beta^{\beta}(1-\beta)^{(1-2\beta)\frac{\beta-1}{\beta} - \frac{1-\beta}{\beta}}\Phi^{\beta-1} L q_j$$

$$= \frac{\beta^{\beta}(1-\beta)^{2-2\beta}}{\Phi^{1-\beta}} L q_j$$

This last expression makes it clear that the higher the firm mass, the lower the profits. This is because more firms imply higher wages, given the constant supply of workers, which reduces the profits. Furthermore, by combining A.1 with A.5, we can show that

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output is linear in $\bar{q}$

$$Y = L(1-\beta)^{\frac{1-2\beta}{\beta}} \left[\beta^\beta (1-\beta)^{1-2\beta} \Phi^\beta \right]^{\frac{\beta-1}{\beta}} \bar{q}\Phi$$

$$= \beta^{\beta-1}(1-\beta)^{(\beta-1)\frac{1-2\beta}{\beta}+\frac{1-2\beta}{\beta}} \Phi^{\beta-1}\Phi L\bar{q}$$

$$= \frac{(1-\beta)^{1-2\beta}}{\beta^{1-\beta}} \Phi^\beta L\bar{q}$$

Finally, by combining 1.5 and A.5, we can find L as

$$\tilde{L} = \int_\mathcal{N} l_i dj$$

$$= \int_\mathcal{N} \left[(1-\beta)\frac{\bar{q}}{w} \right]^{\frac{1}{\beta}} Lq_j \frac{1}{\bar{q}} dj$$

$$= \left[(1-\beta)\frac{1}{\tilde{\beta}} \right]^{\frac{1}{\beta}} L\Phi.$$

Labor market clearing implies that

$$1 = L + \tilde{L} \tag{A.4}$$

$$1 = L + \left[(1-\beta)\frac{1}{\tilde{\beta}} \right]^{\frac{1}{\beta}} L\Phi$$

$$L = \frac{\beta}{\beta + (1-\beta)^2}$$

Note that mass of firms does not affect L. If we substitute this to the profit, we get

$$\pi(q_j) = \frac{\beta^\beta (1-\beta)^{2-2\beta}}{\Phi^{1-\beta}} \frac{\beta}{\beta + (1-\beta)^2} q_j.$$

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A.2 Quality Distributions

Denote q as relative qualities. The density of q at BGP satisfies the following Kolmogorov Forward Equation (KFE):[69]

$$0 = (gqf)_q + x_{q-\eta}f(q-\eta) \times |1 - \eta_q| - x_q f(q) + x_e \Psi(q) - \kappa f(q)$$

where x_q is the expansion rate of the firm with quality q, x_e is the entry rate, $\Psi(q)$ is the entrant's quality distribution. $\eta = \eta(q, \bar{q})$ is related to the inverse jump amplitude such that

$$q = \xi + J(\xi, \bar{q})$$

is the new state value corresponding to the old state value ξ, such that

$$\eta(q, \bar{q}) = J(\xi, \bar{q})$$

assuming J is monotonic in ξ so that J is invertible with respect to ξ, that the Jacobian

$$(1 - \eta_q) = 1 - \frac{\partial \eta(q, \bar{q})}{\partial q}$$

is non-vanishing, and that the inverse transformation from ξ to q maps $(-\infty, +\infty)$ onto $(-\infty, +\infty)$. Let's consider a parametric form for J of the form

$$J(q, \bar{q}) = \lambda \left(\omega \bar{q} + (1 - \omega)q \right), \quad \omega \in [0, 1].$$

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[69]Here, for ease of exposition, I consider the simple case where firms are not allowed to switch between legal forms. In this case, KFE given here is valid for any firm type and legal form.

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Given this form, the previous state is given by

$$q = \xi + \lambda \left(\omega \bar{q} + (1 - \omega)\xi \right)$$
$$\xi = \frac{q - \lambda \omega \bar{q}}{1 + \lambda(1 - \omega)}$$

which gives

$$\eta(q, \bar{q}) = \lambda \left(\omega \bar{q} + (1 - \omega) \frac{q - \lambda \omega \bar{q}}{1 + \lambda(1 - \omega)} \right)$$
$$= \frac{\lambda \omega \bar{q}}{1 + \lambda(1 - \omega)} + \frac{\lambda(1 - \omega)q}{1 + \lambda(1 - \omega)}.$$

Finally we have

$$|1 - \eta_q(q, \bar{q})| = 1 - \frac{\lambda(1 - \omega)}{1 + \lambda(1 - \omega)}.$$
$$= \frac{1}{1 + \lambda(1 - \omega)}.$$

Therefore the density $f()$ is given by

$$0 = (gqf)_q + x_{q-\eta} f(q - \eta) \frac{1}{1 + \lambda(1 - \omega)} - x_q f(q) + x_e \Psi(q) - \kappa f(q) \tag{A.5}$$

$$gqf_q = (x_q + \kappa - g)f(q) - x_{q-\eta}f(q - \eta)\frac{1}{1 + \lambda(1 - \omega)} - x_e \Psi(q) \tag{A.6}$$

with $f(q) = 0$ for $q < q_{min}$ where q_{min} is solved from value function. Integrating over the domain $[q_{min}, \infty)$, we get

$$gq_{min}f(q_{min}) + \kappa\Phi = x_e \tag{A.7}$$

under the assumption that the density is integrable, i.e. $\lim_{q \to \infty} f(q) = 0$. Above equation simply implies that the amount of qualities going under q_{min} plus exits due to κ should be equal to the amount entering the system so that total mass is stable in stationary distribution.

Next let's look at the tail of the distribution, which will help us to solve the distri-

bution. First note that as q goes to infinity, x_q becomes constant (see Appendix A.3). We start with guessing that the distribution tail has a Pareto shape of the form $Cq^{-\zeta-1}$ as $q \to \infty$. Substituting this guess into the equation for the density delivers

$$-\zeta g C q^{-\zeta-1} + \bar{x}\left[C\left(\frac{q - \lambda \omega \bar{q}}{1 + \lambda(1 - \omega)}\right)^{-\zeta-1}\frac{1}{1 + \lambda(1 - \omega)} - Cq^{-\zeta-1}\right] + x_e \Psi(q) - \kappa C q^{-\zeta-1} = 0$$

$$-\zeta g - \kappa + \bar{x}\left[q^{\zeta+1}\left(\frac{q - \lambda \omega \bar{q}}{1 + \lambda(1 - \omega)}\right)^{-\zeta-1}\frac{1}{1 + \lambda(1 - \omega)} - 1\right] + x_e \frac{\Psi(q)}{Cq^{-\zeta-1}} = 0.$$

Now assume that entry distribution has a thinner tail, i.e.$\lim_{q\to\infty}\frac{\Psi(q)}{Cq^{-\zeta-1}} = 0$. Then we have

$$\left[(1 + \lambda(1 - \omega))^\zeta - 1\right] = \frac{\zeta g + \kappa}{\bar{x}}. \tag{A.8}$$

Here one solution for ζ is zero which yields a degenerate solution. The next result partially characterize the non-degenerate solution.

Lemma 3 *The solution to ζ described in (A.8) is non-decreasing in ω and g and non-increasing in λ and τ for $\zeta \geq 1$. Moreover $\zeta = 1$ is a solution whenever $\lambda(1 - \omega)\bar{x} = g + \kappa$ is satisfied. Finally $\lim_{\omega\to 1}\zeta(\omega) = \infty$.*

A.3 Boundary Behavior of Firm Value

We can show that value function is linear in q as q goes to infinity when profits are is linear in q. First notice that as $q \to \infty$, $\bar{q}$ and fixed cost of operation becomes insignificant, therefore

$$\lim_{q\to\infty} q + J(q, \bar{q}) = q \times (1 + \lambda(1 - \omega))$$

When ω is equal to one, there is no benefit of innovating as q gets very large. Now lets guess that the value function for incorporated firms is of the form $v = Cq$. By substituting

this in (1.12), we get

$$(\rho + \kappa)Cq = \max_{x} \left\{ \Pi q - q \left(\frac{x}{\theta} \right)^2 - gqC + xC \left[q \times (1 + \lambda(1 - \omega)) - q \right] \right\}$$

This gives optimal expansion rate as

$$x_* = \frac{\theta^2}{2} C \lambda (1 - \omega)$$

which is a constant. By substituting this above, we get

$$(\rho + \kappa)Cq = \Pi q - q \left(\frac{x_*}{\theta} \right)^2 - gqC + x_* C \left[q \times (1 + \lambda(1 - \omega)) - q \right]$$

$$0 = C^2 \frac{(\lambda(1 - \omega))^2}{\theta^2} - (\rho + \kappa + g)C + \Pi$$

which solves the constant. The roots are

$$C_{-+} = \frac{\rho + \kappa + g \pm \sqrt{(\rho + \kappa + g)^2 - 4 \frac{(\lambda(1-\omega))^2}{\theta^2} \Pi}}{2 \frac{(\lambda(1-\omega))^2}{\theta^2}}$$

Note that C_+ is never a solution we are looking for because it makes net profit negative. Therefore the slope of the value function is given by C_-. A similar expression can be obtained for unincorporated firms as

$$C_{-+} = \frac{\rho + \kappa + g \pm \sqrt{(\rho + \kappa + g)^2 - 4 \frac{(\lambda(1-\omega))^2}{\theta^2} (\Pi - \kappa c_E)}}{2 \frac{(\lambda(1-\omega))^2}{\theta^2}}.$$

Appendix B

Appendix to Chapter 2

B.1 Theoretical Appendix

B.1.1 Firm Size Distribution

Let $v_{n,t}^H$ denote the share of high-type firms with n products, and F_t^j be the number of firms of type j. Then, firm size distribution of the economy can be represented by the following differential equations:

$$\frac{\partial F_t^H v_{1,t}^H}{\partial t} = z_t \times \delta - F^{H,t} v_{1,t}^H \tau_{H,t} \tag{B.1}$$

$$\frac{\partial F_t^H v_{n,t}^H}{\partial t} = \left[v_{n-1,t}^H (n-1) x_{n-1,t} + v_{n+1,t}^H \tau_{H,t} (n+1) - v_{n,t}^H n (\tau_{H,t} + x_{n,t}) \right] \times F_t^H. \tag{B.2}$$

$$\frac{\partial F_t^L}{\partial t} = z_t \times (1-\delta) - F^{L,t} \tau_{L,t}. \tag{B.3}$$

and the requirement that $v_{n,t}^H$ be a proper distribution, $\sum_{n=1}^{\infty} v_{n,t}^H = 1$.

Equation (B.1) states that the number of one-product high type firms is given by the difference between entering high-type firms and exiting high-type firms. Recall that $\tau_{j,t}$

denotes the rate at which a firm of type j loses a given product at each point in time. Similarly, equation (B.2) is an accounting equation for the net-change in the number of high type firms with n products. Finally, (B.3) is the analogue of (B.1) for low-type firms, which always have a single product.

Proposition 3. *Consider a stationary equilibrium and let the flow of entry z and high-type firms' expansion rates $\{x_n\}_{n=1}^{\infty}$ at stationary equilibrium be given. The distribution of high-type firms is*

$$v_n^H = \frac{n^{-1} \frac{\tau_H}{x_n} \prod_{j=1}^{n} \left(\frac{x_j}{\tau_H}\right)}{\sum_{s=1}^{\infty} s^{-1} \frac{\tau_H}{x_s} \prod_{j=1}^{s} \left(\frac{x_j}{\tau_H}\right)},$$ (B.4)

the measure of high- and low-type firms is

$$F^H = \frac{\delta z}{\tau_H} \times \left[\sum_{n=1}^{\infty} \frac{\tau_H}{n x_n} \prod_{j=1}^{n} \left(\frac{x_j}{\tau_H}\right) \right] \quad and \quad F^L = \frac{(1-\delta)z}{\tau_L},$$ (B.5)

the aggregate rate of creative destruction is

$$\tau = z \times \left[\delta \sum_{s=1}^{\infty} \prod_{j=1}^{s} \left(\frac{x_j}{\tau_H}\right) + 1 \right],$$ (B.6)

and the type-specific creative destruction rates are

$$\tau_H = \tau - z(1-\delta)\left(\frac{\beta-1}{\beta}\right) \quad and \quad \tau_L = \beta\tau - z(1-\delta)(\beta-1).$$ (B.7)

Proof. By setting the time derivatives to zero in (B.1), (B.2) and (B.3), stationary firm size distribution is described by the following equations

$$F^H v_1^H \tau_H = z \times \delta$$ (B.8)

$$v_n^H n (\tau_H + x_n) = v_{n-1}^H (n-1) x_{n-1} + v_{n+1}^H \tau_H (n+1)$$ (B.9)

$$F^L \tau_L = z \times (1-\delta)$$ (B.10)

Let v_1^H and τ be given. First note that consistency requires that the total amount of

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innovation has to be equal to the total rate of creative destruction:

$$\tau = \tau_H(1 - F^L) + \tau_L F^L \tag{B.11}$$

Then, by using (B.10), (B.11) and $\tau_L = \beta \tau_H$, we get

$$\tau_H = \tau - z(1 - \delta)\left(\frac{\beta - 1}{\beta}\right) \quad \text{and} \quad \tau_L = \beta \tau - z(1 - \delta)(\beta - 1). \tag{B.12}$$

Next, by using (B.8) - (B.10), we calculate F^L, F^H, and $\{v_n\}_{n=2}^{\infty}$.

Lemma 3. *The distribution of high types takes the following form*

$$v_n^H n = \frac{\prod_{j=1}^{n} x_j}{\tau_H^n} \frac{\tau_H}{x_n} v_1^H. \tag{B.13}$$

Proof. Substituting (B.13) in (B.8) - (B.10) shows that if v_n^H satisfies (B.13), it satisfies all the flow equations in (B.8) - (B.10). □

This implies that $1 = \sum_{n=1}^{\infty} v_n^H = v_1^H \sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau_H}{x_n} \prod_{j=1}^{n} \left(\frac{x_j}{\tau_H}\right)$, so that (B.13) reads

$$v_n^H = \frac{1}{n} \frac{\prod_{j=1}^{n} x_j}{\tau_H^n} \frac{\tau_H}{x_n} \frac{1}{\sum_{s=1}^{\infty} \frac{1}{s} \frac{\tau_H}{x_s} \prod_{j=1}^{s} \left(\frac{x_j}{\tau_H}\right)}. \tag{B.14}$$

Then, from (B.8) and (B.10), we have

$$F^H = \frac{\delta z}{\tau_H} \times \left[\sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau_H}{x_n} \prod_{j=1}^{n} \left(\frac{x_j}{\tau_H}\right)\right] \quad \text{and} \quad F^L = \frac{(1 - \delta)z}{\tau_L}.$$

Hence, we only need to determine τ, which we get from (2.20) as

$$\tau = \sum_{n=1}^{\infty} n x_n v_n^H F^H + z = \left[\sum_{n=1}^{\infty} \delta \left(\prod_{j=1}^{n} \left(\frac{x_j}{\tau_H}\right)\right) + 1\right] z. \tag{B.15}$$

Together with (B.12), one can show that (B.15) has a unique solution for τ. □

B.1.2 Derivation of Equation (2.22)

We can express $\ln Q_t$ after an instant Δt as

$$\ln Q_{t+\Delta t} = \int_0^1 \left[\tau_t \Delta t \ln \left(\gamma_t q_{jt} \right) + (1 - \tau_t \Delta t) \ln q_{jt} \right] dj$$

$$= \tau_t \Delta t \ln \left(\gamma_t \right) + \ln Q_t$$

where second and higher order terms in Δt are omitted. By subtracting $\ln Q_t$ from both sides, dividing by Δt, and taking the limit as $\Delta t \to 0$, we get

$$g_t = \frac{\dot{Q}_t}{Q_t} = \lim_{\Delta t \to 0} \frac{\ln Q_{t+\Delta t} - \ln Q_t}{\Delta t} = \ln \left(\gamma_t \right) \tau_t.$$

B.1.3 Transitional Dynamics with Stationary Firm Size Distribution

Proposition 4. *Suppose that the firm-size distribution at time t coincides with the stationary distribution characterized in Proposition 3. Then, for any path of the step size γ_t, there is an equilibrium path, where (i) the firm size distribution remains stationary, (ii) all aggregate variables grow at the same rate $\ln(\gamma_t)\tau_{BGP}$, where τ_{BGP} is the constant rate of creative destruction rate at the stationary equilibrium.*

Proof. Note that in the stationary equilibrium of the model described in Appendix B.1.6, the step size γ_t does not affect any expressions. Hence, we need to show that there exists an interest rate path r_t such that C_t, Q_t and Y_t grow at the same rate during the transition. If this was the case, firms' innovation and entry choices would not change and the distribution would remain stationary. It is easy to see that interest rate path

$$r_t = \ln(\gamma_t)\tau_{BGP} + \rho$$

serves the purpose. Recall that consumption decisions of the household yield the usual

Euler equation which implies that

$$r_t = g_{C,t} + \rho$$

so that under the proposed interest rate path, $g_{C,t} = \ln(\gamma_t)\tau_{BGP}$. Moreover $g_{Q,t} = \ln(\gamma_t)\tau_{BGP}$ as shown in Appendix B.1.2. Lastly we have $Y_t = Q_t\mathcal{M}_t L_{P,t}$. Since $\mathcal{M}_t$ and $L_{P,t}$ are constant at the proposed equilibrium, this implies that $g_{Y,t} = g_{Q,t}$. Therefore all growing variables grows at the same rate. □

B.1.4 Static Equilibrium

Consider the equilibrium in the product market. At each point in time, each product line j is produced by a single firm with productivity q_{jt}. We normalize the price of aggregate output Y to one. As firms set a price equal to $p_{jf} = q_{jf}^{-1} w_t$ we get that

$$\ln(Y) = \int_0^1 \ln(y_j)dj = \int_0^1 \ln(p_j y_j)dj - \int_0^1 \ln(p_j)dj = \ln(Y) - \ln(w_P) + \int_0^1 \ln(q_j)dj$$

which implies $w_P = Q \equiv \exp\left[\int_0^1 \ln q_j dj\right]$. The production function [see equation (3.7)] also implies that

$$L^P = \int_0^1 l_j dj = \int_0^1 \frac{y_j p_j}{q_j p_j} dj = \frac{Y}{w} \int_0^1 \mu_j^{-1} dj, \tag{B.16}$$

where L^P is the aggregate demand for production labor. Using that $\mu_j = \frac{1}{1 - e(n_j)^\sigma}$, where n_j is the number of products the producer of product j has in its portfolio, (B.16) implies that $L^P = \frac{1}{\mathcal{M}}\frac{1}{\omega_P}$, where $\omega_P = \frac{w_P}{Y}$ and $\mathcal{M}$ is given by

$$\mathcal{M} = \left[1 - \sum_{n=1}^\infty (e(n))^\sigma \times n \times \left(v_n^H F^H + v_n^L F^L\right)\right]^{-1},$$

where function $e(.)$ is defined in (2.7), v_n^i and F^i are the size distribution and the measure of i-type firms, $i \in \{H, L\}$, respectively (see Proposition 3).

B.1.5 A Simple Microfoundation for α

In this section, we provide a simple example of how α could depend on various institutional parameters in an economy. Please note that none of the analysis in the main text depends on this particular example. This example is provided to fix ideas.

Suppose that both managers and entrepreneurs each have one unit of time at their disposal. While the latter can provide T units of effort during that time interval, managers can provide 1 unit of effort. Suppose that the provision of managerial effort is subject to contractual frictions. For simplicity, assume that the manager can decide to either provide effort or shirk, in which case he adds no usable services to the firm. The firms can translate each unit of managerial effort into η units of managerial services.

While the manager's effort choice is not contractible, the entrepreneur can monitor the manager to prevent him from shirking. If the entrepreneur spends s units of her time monitoring the manager, she will catch a shirking manager with probability s. Whenever the manager shirks and gets caught, the entrepreneur can go to court and sue the manager for the managerial wage w. In particular, the court (rightly) decides in the entrepreneur's favor with probability κ. Hence one can think of κ as parameterizing the efficiency of the legal system. Finally, the demand for shirking arises because shirking carries a private benefit bw, where $b < 1.$[70]

It is straightforward to characterize the equilibrium of this simple game. If the entrepreneur spends s units of her time monitoring the manager, the manager does not

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[70]The necessity for the private benefit being proportional to the wage arises in order to make the contract stationary.

shirk if and only if

$$w \geq bw + w\left(1 - \kappa s\right),$$

where $(1 - \kappa s)$ is the probability that the manager gets paid despite having shirked. Clearly the owner will never employ a manager without inducing effort. Hence, the owner will spend $s = b/\kappa$ units of time monitoring the manager. The overall amount of managerial services in product line j is therefore given by[71]

$$e_j = \frac{T}{n} - m_j s + \eta m_j \quad = \quad \frac{T}{n} + \left(\eta - \frac{b}{\kappa}\right) \times m_j = \frac{T}{n} + \alpha\left(\kappa, \eta, b\right) \times m_j. \qquad \text{(B.17)}$$

Hence, α measures precisely the net increase in managerial services through delegation. In particular, the delegation efficiency is increasing in the firm's efficiency to employ managers (η) and in the state of the contractual environment (κ), because monitoring and the strength of the legal system are substitutes. Note also that the whole purpose of delegation is to increase a firm's managerial resources, so that firms will never hire a manager if $\alpha\left(\kappa, \eta\right) \leq 0$. Hence, whenever managers are sufficiently unproductive or the quality of legal systems is sufficiently low, firms will never want to hire outside managers because owners need to spend more of their own time to prevent the opportunistic behavior of managers than they gain in return.

B.1.6 Stationary Equilibrium of the Model

In this section, we describe the stationary equilibrium of the model in detail. To do so, we proceed in two steps.

Step 1 Fix $s \equiv (n^*, \omega_P)$ where n^* and ω_P are delegation cut-off and normalized wage rate for production workers, respectively. By using (B.6) and (B.7), we can write the rate

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[71]Note that we do not require that $s < T$, i.e., we do not require the owner to perform the monitoring himself. We rather think of managerial efficiency units to be perfect substitutes within the firm, i.e., an owner can hire a manager to monitor other managers.

of destruction for high types $\tau_H(s)$ as

$$\tau_H(s) = z(s) \times \left\{ \left[\delta \sum_{h=1}^{\infty} \prod_{j=1}^{h} \left(\frac{x_j(s)}{\tau_H(s)} \right) \right] + 1 - (1-\delta) \left(\frac{\beta-1}{\beta} \right) \right\}, \qquad \text{(B.18)}$$

where $[x_j(s)]_{j=1}^{\infty}$ is the optimal innovation policy by high types implicitly defined in (2.14) and $z(s)$ is the optimal entry rate. We focus on a solution where $x_j < \tau_H$ for all τ_H. This is a sufficient condition for a stationary solution.[72] We will show below that such a solution exists for all s provided that θ_E is large enough.

Let $v_H(n)$ be normalized value function (normalized with Y_t) of a high-type firm depicted in (2.14).[73] At BGP where both C_t and Y_t grows at the same rate and $\dot{v}_{H,t} = 0$, it can be written as

$$\rho v_H(n) = \max_{x_n} \left\{ \tilde{\pi}(n;n^*) - \omega_p \theta^{-\frac{1}{\zeta}} n x_n^{\frac{1}{\zeta}} + x_n n \left[v_H(n+1) - v_H(n) \right] \right\}$$

$$+ \tau_H n \left[v_H(n-1) - v_H(n) \right].$$

where we use the fact that $w_p = Q$ to substitute $\frac{Q}{Y}$ with ω_P and $r = \rho + g$ from household problem.[74] By rearranging terms and explicitly imposing the restriction $x_j < \tau_H$, we can write v_H as

$$v_H(n) \;\; = \;\; n \times \max_{x_n < \tau_H} \left\{ \frac{\frac{\tilde{\pi}(n;n^*)}{n} - \omega_p \theta^{-\frac{1}{\zeta}} x_n^{\frac{1}{\zeta}} + x_n v_H(n+1) + \tau_H v_H(n-1)}{\rho + (x_n + \tau_H)n} \right\}.$$

Now consider the function $b(n) \equiv \frac{v_H(n)}{n}$, which - by using the above equation - can be written as

$$b(n) = \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H)n} b(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H)n} b(n-1) \right\}, \quad \text{(B.19)}$$

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[72]A necessary condition is that there exists $\hat{n}$ with $x_j < \tau_H$ for all $j > \hat{n}$.
[73]We drop the dependence of the value function on s for notational clarity.
[74]See Section B.1.4 for details.

where $h(n, x_n) \equiv \frac{\frac{\tilde{\pi}(n;n^*)}{n} - \omega_p \theta^{-\frac{1}{\zeta}} x_n^{\frac{1}{\zeta}}}{\rho + (x_n + \tau_H)n}$.

We will show that the right-hand side of (B.19) satisfies Blackwell's sufficient conditions for a contraction. To see this, define the operator T by

$$(Tf)(n) \equiv \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H)n} f(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H)n} f(n-1) \right\}. \text{(B.20)}$$

Hence, b can be defined as a fixed point of T, i.e., a function such that $(Tb)(n) = b(n)$. First, note that $h(n, x_n)$ is bounded [see (2.12)] so that T maps the space of continuous bounded functions into itself (Berge's Maximum Theorem). Moreover, for any continuous bounded functions f, g with $f(n) \leq g(n)$ for all $n \in Z^{++}$, we have

$$\begin{aligned}
(Tf)(n) &= \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H)n} f(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H)n} f(n-1) \right\} \\
&\leq \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H)n} g(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H)n} g(n-1) \right\} \\
&= (Tg)(n),
\end{aligned}$$

so that the monotonicity condition is satisfied. Lastly, for any continuous bounded function f and $a \geq 0$,

$$\begin{aligned}
&(T[f + a])(n) \\
&= \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H)n} [f(n+1) + a] + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H)n} [f(n-1) + a] \right\} \\
&\leq \max_{x_n < \tau_H} \left\{ h(n, x_n) + \frac{x_n(n+1)}{\rho + (x_n + \tau_H)n} f(n+1) + \frac{\tau_H(n-1)}{\rho + (x_n + \tau_H)n} f(n-1) \right\} + \Omega a \\
&= (TF)(n) + \Omega a
\end{aligned}$$

where

$$\Omega \equiv \max_{x_n < \tau_H} \left\{ \frac{(x_n + \tau_H)n}{\rho + (x_n + \tau_H)n} + \frac{x_n - \tau_H}{\rho + (x_n + \tau_H)n} \right\} < 1.$$

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Hence, the operator T satisfies the discounting condition, so that T is a contraction mapping and therefore possesses a unique fixed point [Stokey et al. (1989)], which is continuous in s and τ_H. Moreover, the expression inside the max operator in (B.20) is continuous in x_n and strictly concave so that Berge's Maximum Theorem implies that the set of maximizers x_n^* is a continuous function of s and τ_H. The equilibrium entry rate z is fully determined from v_H and v_L [see (2.18)] and hence also a continuous function of s and τ_H.[75]

Hence, equation (B.18) is continuous in τ_H. To see that there exists a fixed point for τ_H, note that the RHS is bounded away from zero because $z(s) > 0$ and that it is bounded from above. To see that, note that $\sum_{h=1}^{\infty} \prod_{j=1}^{h} \left(\frac{x_j(s)}{\tau_H(s)} \right)$ is bounded in a stationary equilibrium and that z is bounded [see (2.18)]. Hence, there exits a fixed point for τ_H. Moreover, because z is increasing in θ_E for a given s and τ_H, (B.18) implies that for each s there is θ_E large enough such that this fixed point satisfies $\tau_H > x_n$.

Step 2 We can now represent the whole model in terms of labor market clearing conditions. The Cobb-Douglas final good production function together with the market structure described in Section 2.2.1 implies that the total number of production workers hired for variety j by a producer, who is active in n markets, is given by[76]

$$l_j = [\omega_P \mu(e)]^{-1} = \omega_P^{-1} \times (1 - e(n)^\sigma).$$

Using firms' optimal delegation policy and aggregating over the firm size distribution yields the aggregate demand for production workers is given by

$$H^P = \left[1 - \sum_{n=1}^{\infty} \left(\max \left\{ \frac{T}{n}, \frac{T}{n^*} \right\} \right)^\sigma \times n \times \varphi_n \right] \times \omega_P^{-1} \tag{B.21}$$

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[75]Recall that $v_L(1) = \frac{\pi(1)}{\rho + \tau_L}$, where $\tau_L = \beta \times \tau_H$.
[76]To see this, note that $Y = p_j y_j = \frac{w_P}{q_j} q_j \mu(e_j) l_j$ and $\omega_P = w_P / Y$.

Similarly, firms' managerial demand function implies that the aggregate demand for managers is given by

$$H^M = \sum_{n \geq n^*}^{\infty} n \times m(n) \times \varphi_n = \left(\frac{\sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}} \alpha^{\frac{\sigma}{1-\sigma}} \sum_{n \geq n^*}^{\infty} n \varphi_n - \frac{T}{\alpha} \sum_{n \geq n^*}^{\infty} \varphi_n. \qquad \text{(B.22)}$$

Given Step 1, we can calculate the firm size distribution $\varphi_n(s) = v_n^H(s) F^H(s) + v_n^L(s) F^L(s)$ from Proposition 3. From (2.25), (B.21), and (B.22), the labor market clearing conditions for managers and production workers can then be written by

$$0 = \left(\frac{\vartheta - 1}{\vartheta} \mu_M \right)^{\vartheta} \left(\frac{(n^*)^{1-\sigma} \sigma \alpha}{T^{1-\sigma} \omega_P} \right)^{\vartheta - 1} \frac{\vartheta}{\vartheta - 1} - \frac{T}{\alpha} \sum_{n > n*} \left(\frac{1}{n^*} - \frac{1}{n} \right) n \varphi_n(s) \qquad \text{(B.23)}$$

$$0 = 1 - \left(\frac{\vartheta - 1}{\vartheta} \mu_M \right)^{\vartheta} \left(\frac{(n^*)^{1-\sigma} \sigma \alpha}{T^{1-\sigma} \omega_P} \right)^{\vartheta} - \frac{1}{\omega_P} \left[1 - \sum_{n=1}^{\infty} \left(\max \left\{ \frac{T}{n}, \frac{T}{n^*} \right\} \right)^{\sigma} n \varphi_n(s) \right] \qquad \text{(B.24)}$$

where two equations depend only on $s \equiv (n^*, \omega_P)$. Note that $\varphi_n(s)$ is continuous in z, τ_H and x_n. Therefore, from Step 1, left-hand-side of both equations are continuous in (n^*, ω_P). Solution to the system of equation given by (B.23) and (B.24) constitutes an equilibrium for our economy.

B.2   Empirical Appendix

B.2.1   Data

In this section we provide more information about our data sources.

Establishment- and Firm-level Information for the US   We use data from the Business Dynamics Statistics (BDS). BDS is a product of the US Census Bureau. The BDS data are compiled from the Longitudinal Business Database (LBD). The LBD is a longitudinal database of business establishments and firms covering the years between 1976 and

2012. We focus on the manufacturing sector in 2012. The data are publicly available at http://www.census.gov/ces/dataproducts/bds/.

For our analysis, we utilize the following four moments from the US data: (i) the cross-sectional relationship between age and size, which we refer to as the life-cycle, (ii) the aggregate employment share by age, (iii) the exit rate as a function of age *conditional on size*, and (iv) the rate of entry. For our main analysis we focus on establishments. The BDS reports both aggregate employment and the number of establishments by age. This allows us to calculate the first two moments. The BDS also directly reports both entry and exit rates for each size-age bin. The entry rate at the establishment level is calculated as the number of new establishments at time t relative to the average number of establishments in t and $t-1$. Similarly, the exit rate at the establishment level is calculated as the number of exiting establishments in t relative to the average number of establishments in t and $t-1$. The corresponding information is also reported at the firm level. In particular, the BDS reports the number of exiting firms for different size-age bin. Note that all establishments owned by the firm must exit for the firm to be considered an exiting firm. As for firm entry, we treat firms of age 0 as an entering firm. Because a firm's age is derived from the age of its establishments, this implies that we treat firms as entering firms only if all their establishments are new. In Section B.2.7 in the Appendix we provide detailed descriptive statistics about the dynamic process at both the firm- and establishment-level.

Establishment-Level Information for India   As explained in the main body of the text, we construct a representative sample of the Indian manufacturing sector by combining data from the Annual Survey of Industries (ASI) and the National Sample Survey (NSS), which - every five years - has a special module to measure unorganized manufacturing establishments. We use cross-sectional data from 2010. In contrast to the US, both the ASI and NSS are based on establishments and we cannot link establishments to firms. With the majority of employment being accounted for by very small producers, multi-

establishment firms are unlikely to be important for the aggregate in India. Firms in the NSS account for 99.2% of all establishments and for 76% of manufacturing employment. In Table 35 we report the size distribution of establishments in the NSS. More than 80% of plants have at most 2 employees and only 5% have more than 5 employees. Note that the NSS data contains some large firms: 1.5% of plants have more than 10 employees and roughly 0.25% have more than 20 employees. These plants are sampled in what is called "Segment 9" of the data, which is reserved for such large firms.

			Number of employees				
1-2	3-5	6-9	10-14	15-19	20-24	25-49	>50
82.10%	13.49%	2.90%	0.87%	0.36%	0.11%	0.10%	0.05%

Notes: The table reports the share of firms in the respective size category in the NSS data in 2010. We use the sampling weights provided in the data to aggregate the number of firms.

Table 35: The employment distribution in the NSS

Comparison of the NSS/ASI Data with the Economic Census   In our analysis we follow the literature to treat the combination of the NSS and ASI data as measuring the population of firms (see for example Hsieh and Olken (2014) or Hsieh and Klenow (2014a)). To provide further evidence for the validity of this choice, we now compare this data to the Indian Economic Census (EC). The EC is a complete count of all economic units in the country. While the ASI/NSS is collected in the year 2010, no EC was conducted in 2010. We therefore report a comparison with the EC in 2005 and 2013. Given that the ASI/NSS focuses on manufacturing plants, we also select manufacturing firms from the EC.

In Table 36 we compare the firm size distribution as measured by these three datasets. We report the share of plants, the share of employment and the average plant size for different size categories. The main take-away from Table 36 is that the distributions from the EC and our ASI/NSS are very similar. There are slightly more firms with 1-4 employees in our ASI/NSS sample and hence their aggregate employment share is consequentially also larger. Note however, that the ASI/NSS sample contain less firms

and therefore less employment in the 5-9 category. The share of firms and employment in firms with less than 10 employees is almost identical between the EC and the NSS/ASI data. Also note that the distribution of average firm size within size classes is very similar.

| Size	Share of firms			Share of employment			Average firm size		
	EC '05	EC '13	ASI/NSS	EC '05	EC '13	ASI/NSS	EC '05	EC '13	ASI/NSS
1-4	89.82%	90.08%	92.98%	49.01%	49.26%	54.75%	1.7	1.6	1.6
5-9	8.24%	7.88%	4.91%	17.24%	16.12%	11.61%	6.5	6.0	6.3
10-19	0.93%	1.05%	1.42%	3.92%	4.54%	6.96%	13.1	12.8	13.0
20-49	0.55%	0.60%	0.41%	5.19%	5.95%	4.55%	29.3	29.5	29.2
50-99	0.24%	0.22%	0.14%	5.19%	4.91%	3.57%	67.6	67.0	69.9
100-249	0.16%	0.11%	0.09%	7.45%	5.66%	4.89%	142.0	146.5	149.3
250-499	0.04%	0.03%	0.03%	4.70%	3.83%	3.58%	329.8	336.1	346.7
500-999	0.01%	0.01%	0.01%	2.87%	3.35%	3.43%	664.3	678.7	683.9
1000+	0.01%	0.01%	0.01%	4.43%	6.38%	6.65%	2208.1	2256.3	2452.7

Notes: This table contains summary statistics of the firm size distribution as measured by the NSS/ASI in 2010, the Economic Census in 2005 and the Economic Census in 2013. For the NSS/ASI sample we use the sampling weights provided in the data.

Table 36: Comparison of NSS/ASI and Economic Census

Non-homothetic Demand for Outside Managers In Figure 19 we provide additional evidence for the non-homothetic pattern of managerial demand reported in Table 8. While Table 8 is based on firm-level data, Figure 19 uses individual data from IPUMS (for India) and the Current Population Survey (the US). In both datasets we observe individuals occupation, whether they work as a wage worker and the size of the firm in which they work. Hence, we can compute the share of people who are classified as outside managers conditional on working in firms in a particular size bin.

The left panel of Figure 19 shows that the non-homotheticity of managerial demand is not only present in the firm-level data but also pervasive in the data from IPUMS. The results are also (roughly) quantitatively in line with our measurement from the firm-level data reported in Table 8. The right panel documents the same relationship for the US. Again, we find robust evidence for managerial demand to be non-homothetic. Note that the share of outside managers in the CPS data is quantitatively similar to what we measure IPUMS. There we found a managerial share of 12.4%. Note also that our

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model predicts that the non-homotheticity should be less pronounced in the US, where the delegation efficiency α is high relative to the owners' managerial supply T - see e.g., equation (2.10).
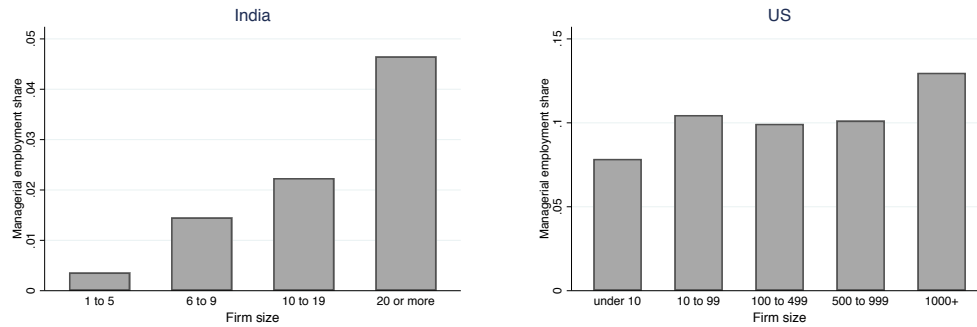
Notes: The left (right) panel shows the share of workers working as outside managers for different firm size bin in India (the US). The India data stems from IPUMS in 2004. The US data stems from the CPS and we averaged the annual data for 2005-2016.

Figure 19: Non-homothetic Demand for Outside Managers

Data on Managerial Compensation and Profits for the US We identify σ from the share of managerial compensation in aggregate profits *before* managerial payments [see equation (B.25)]. To estimate this moment, we use two data sources. From NIPA we can retrieve a measure of aggregate profits in the manufacturing industry. Specifically, we start with aggregate corporate profits, which are directly measured in NIPA. The BEA's featured measure of corporate profits -profits from current production - provides a comprehensive and consistent economic measure of the income earned by all US corporations. As such, it is unaffected by changes in tax laws, and it is adjusted for non- and misreported income. We then add to this measure non-farm proprietors' income in the manufacturing sector, which provides a comprehensive and consistent economic measure of the income earned by all US unincorporated non-farm businesses.

To measure managerial wages, we augment the information in NIPA from information in the census. While NIPA reports compensation for workers, managerial payments are not directly recorded in NIPA. To calculate the managerial wage bill, we therefore use

the US census data. In the census we have micro data on labor compensation and occupations at the micro level. Hence, we calculate the *share* of managerial payments in the total wage bill and apply that share to the aggregate compensation data in NIPA. According to the census, managerial compensation amounts to roughly 20% of total wages. Recall that the managerial employment share in the US is about 12% so that managerial wages are relatively high. We then calculate the share of managerial compensation (CSM) in aggregate profits net of managerial wages as

$$CSM = \frac{\text{Managerial Compensation}}{\text{Corporate Profits} + \text{Nonfarm Proprietor's Income} + \text{Managerial Compensation}},$$

where "Managerial Compensation" is simply 20% of the total labor compensation in NIPA. We also calculate a second measure of CSM, where we do not include "Nonfarm Proprietor's Income." We calculate CSM before the Great Recession, because we were concerned about corporate profits being very low during the financial crisis. CSM is quite volatile. It ranges from 65% in 2001 to 33% in 2006. For our calibration we focus on the average across the years 2000 - 2007, which is 51%. If we do not include "Nonfarm Proprietor's Income", the numbers are very similar and only slightly larger, ranging from 69% in 2001 to 35% in 2006. Hence, it is not essential for us to take "Nonfarm Proprietor's Income" into account.

Data on Managerial Employment and Earning: To measure managerial employment and earnings in the US and India, we employ national Census data from the IPUMS project. We focus on the most recent year, which is 2010 for the US and 2004 for India. For each country we get a sample from the census, which has detailed information about personnel characteristics. In particular we observe each respondent's education, occupation, employment status, sex, and industry of employment. We focus on male workers in the manufacturing industry working in private-sector jobs.

The list of occupations according to ISCO is contained in Table 37. To qualify as a

manager in the sense of our theory, two characteristics have to be satisfied. First, the respective individual has to work as a "Legislator, senior official, and manager." In order to focus on managers, which are agents of a firm owner, i.e., outside managers, we *also* require workers to be wage workers and not working on their own account or to be unpaid family members. This information is also contained in the IPUMS census data in the variable "worker type." As we showed in Table 7 above, it is important to take these differences into account as poor countries have a higher share of people working on their own account (or as a family member) *conditional* on being classified as a manager according to ISCO.

Legislators, senior officials, and managers	Plant and machine operators and assemblers
Professionals	Elementary occupations
Technicians and associate professionals	Armed forces
Clerks	Other occupations, unspecified or n.e.c.
Service workers and shop and market sales	Response suppressed
Skilled agricultural and fishery workers	Unknown
Crafts and related trades workers	NIU (not in universe)

Notes: Table 37 contains the occupational categories available in the IPUMS data. A necessary condition for someone to be classified as an outside manager is to be assigned the occupational title "Legislators, senior officials, and managers." See the main body of the text for the additional requirements.

Table 37: List of Occupations according to ISCO

B.2.2 Identification of the Model

We will now discuss the identification of our model in more detail. In total, there are 11 parameters to identify[77]:

$$\Omega \equiv \{\alpha, \sigma, T, \mu_M, \vartheta, \theta, \theta_E, \delta, \beta, \gamma^{US}, \lambda\}.$$

In Section B.1.1, we discussed how the distribution of firm size is determined given the optimal innovation and entry rates $\{x_n\}_{n=1}^{\infty}$ and z. More specifically, $\{x_n\}_{n=1}^{\infty}$ and z deter-

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[77]Recall that we calibrate ζ and ρ outside of the model.

mine the aggregate innovation rate τ and these three objects together uniquely pin down the joint distribution of age and size, i.e., the entire process of firm-dynamics. The four parameters that affect this process directly are $(\theta, \theta_E, \beta, \delta)$. We therefore use the following four firm-level moments to calibrate these parameters: (i) the life cycle, i.e., the relative size of firms of age 21-25 to firms of age 1-5, (ii) the share of aggregate employment accounted for by firms of age 21-25, (iii) the relative exit rate of 1-5 year old firms relative firms of age 21-25 conditional on size, and (iv) the entry rate. Intuitively, the slope of the life-cycle is informative about θ, which determines the level of incumbent's innovation effort. As β effectively controls the size of old cohorts (by determining the speed with which high-type firms exit), it is related to the aggregate importance of old cohorts in the economy, i.e., the relative employment share of old firms. The exit hazard conditional on size is informative about the degree of selection. If there was no type heterogeneity, the exit rate would only be a function of size. To the extent that older firms are positively selected, they are less likely to exit conditional on size. The ex-ante heterogeneity δ determines how strong this effect can be. Finally, the entry rate is informative about θ_E.

We then use several moments related to managerial employment patterns - namely the compensation of managers relative to corporate profits, the entrepreneurial share in total compensation, the dispersion of managerial wages, and managerial employment shares - to identify σ, T, ϑ, α and μ_M. Consider first σ, the elasticity of profits with respect to managerial services.[78] In the model, the total compensation for managerial personnel relative to aggregate profits (before managerial payments) is given by

$$\frac{w_M H^M}{\Pi + w_M H^M} = \frac{\sum_{n=1}^{\infty} w_M \times n \times m(n) \times \varphi_n}{\sum_{n=1}^{\infty} e(n)^{\sigma} Y \times n \times \varphi_n},$$

where $\varphi_n = F^H v_n^H$ and $\varphi_1 = F^H v_1^H + F^L$ is the endogenous firm size distribution. By using $m(n) = T\alpha^{-1} \times \max\left\{0, (n^*)^{-1} - (n)^{-1}\right\}$, $\omega_M \equiv \frac{w_M}{Y} = \sigma\alpha\left(\frac{n^*}{T}\right)^{1-\sigma}$ and $e(n) =$

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[78]Although the specific ordering of parameters in the identification discussion is not essential, it facilitates the argument.

$T \max \left\{ n^{-1}, (n^*)^{-1} \right\}$, we get that

$$\frac{w_M H^M}{\Pi + w_M H^M} = \sigma \frac{\sum_{n=1}^{\infty} (n^*)^{1-\sigma} \left(\max \left\{ 0, \frac{1}{n^*} - \frac{1}{n} \right\} \right) \times n \times \varphi_n}{\sum_{n=1}^{\infty} \left(\max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\} \right)^{\sigma} \times n \times \varphi_n}. \tag{B.25}$$

Hence, conditional on n^* and the firm size distribution, (B.25) only depends on σ.

To determine T, we target the share of income accruing to entrepreneurs after paying for their factors of production. As entrepreneurs are the residual claimants on firm profits, this moment is simply given by

$$
\begin{aligned}
\frac{\Pi}{Y} &= \sum_{n=1}^{\infty} \left[e(n)^{\sigma} - \omega_M m(n) \right] \times n \times \varphi_n \\
&= T^{\sigma} \sum_{n=1}^{\infty} \left[\left(\max \left\{ n^{-1}, (n^*)^{-1} \right\} \right)^{\sigma} - \sigma n^* \max \left\{ 0, \frac{1}{n^*} - \frac{1}{n} \right\} \right] \times n \times \varphi_n,
\end{aligned}
$$

which is directly informative about T for given n^*, φ_n, and σ.

The shape parameter of skill distribution ϑ can be identified directly from the dispersion of managerial earnings. To see this, note that the earnings of a manager with relative skill h is $w_M h$. The distribution of managerial earning is therefore given by

$$P \left[w_M h > x | h \geq \frac{w_P}{w_M} \right] = \left(\frac{w_P / w_M}{x / w_M} \right)^{\vartheta} = \left(\frac{w_P}{x} \right)^{\vartheta},$$

which is pareto with shape ϑ and location w_P. Defining the relative managerial earnings $y \equiv \ln \left(\frac{w_M h}{w_P} \right)$, we get $P(y \leq y_0) = 1 - e^{-\vartheta y_0}$, so that

$$var(y) = var \left(ln \left(\frac{w_M h}{w_P} \right) \right) = var \left(ln \left(w_M h \right) \right) = \vartheta^{-2}.$$

Hence, we can calibrate ϑ directly to the variance of log managerial earnings.

Finally, we identify α and μ_M by using the share of managers in the whole economy *and* among Indian immigrants to the US economy. Let χ denote the equilibrium

managerial employment share which is given by

$$\chi = P\left[h_M w_M \geq w_P\right] = \left(\frac{\frac{\vartheta-1}{\vartheta}\mu_M}{w_P/w_M}\right)^{\vartheta} = \left(\frac{\vartheta-1}{\vartheta}\mu_M \frac{\sigma\alpha}{\omega_P}\left(\frac{n^*}{T}\right)^{1-\sigma}\right)^{\vartheta}.$$

Using the expression for total managerial demand, the equilibrium condition for the managerial labor market can be written as

$$\mu_M \alpha = (\chi)^{-\frac{\vartheta-1}{\vartheta}} \times \sum_{n \geq n^*}^{\infty} T\left(\frac{1}{n^*} - \frac{1}{n}\right) \times n \times \varphi_n. \tag{B.26}$$

Hence, given n^*, T, ϑ, and φ_n, we can directly determine $\mu_M \times \alpha$ from the data on the share of managers in the whole population (i.e., χ). To separate the effect of managerial human capital (μ_M) from delegation efficiency (α), we use data on managerial employment pattern of Indian immigrants. Because our approach uses additional data and because all allocations in the model only depend on $\mu_M \times \alpha$, we discuss the details of our strategy in Section B.2.4. Once we identify μ_M, we get α from (B.26).

Lastly we use moments regarding aggregate dynamics of the economies to pin down γ and λ. In particular, we calibrate the step-size for US, γ^{US}, to fit the aggregate growth rate as $g = \ln\left(\gamma^{US}\right)\tau$ and US is assumed to be on the balanced growth path. For India, step size is partly determined by the productivity gap between US and India and λ parametrizes the importance of this channel on step size [see (2.31)]. By using (2.22) and (2.31), we can write the *change* of relative productivity differences $Z_t \equiv \frac{Q_{US,t}}{Q_{IND,t}}$ as

$$g_{Z,t} = \frac{\dot{Z}_t}{Z_t} = \left\{\ln(\gamma^{US}\tau_{US,t} - \tau_{IND,t}\left[\ln(\gamma^{US}) + \lambda\ln(Z_t)\right]\right\} \tag{B.27}$$

Therefore, given γ^{US} and the aggregate rates creative destruction for US and India, we can infer λ from the dynamics of relative productivity differences between the US and India.

To relate Z_t to the data, note that empirically we observe total factor productivity as

implied by the Penn World Tables. Given that total population size is normalized to unity, our model implies that TPF is given by $TFP = Y = Q\mathcal{M}L^P$ (see (2.6)). Hence, relative TFP is given by

$$\frac{TFP_{t,US}}{TFP_{t,IND}} = Z_t \times \frac{\mathcal{M}_{t,US}L^P_{t,US}}{\mathcal{M}_{t,IND}L^P_{t,IND}}.$$

Note that if the firm-size distribution is stationary, both $\mathcal{M}_{t,c}$ and the sectoral allocation of labor $L^P_{t,US}$ are constant. Hence, the change in measured relative TFP, $TFP_{t,US}/TFP_{t,IND}$, is exactly $g_{Z,t}$ given in (B.27) and hence can be used to calibrate λ.

In Figure 20 we depict the evolution of relative TFP levels between the US and India between 1985 and 2005. It is clearly seen that India is catching up as relative TFP differences decline from 4 in 1985 to roughly 3.5 in 2005. We therefore calibrate λ and level of relative productivity in 1985, Z_{1985}, to minimize the distance (as measured by the sum of squared residuals) between the model and the data. The resulting fit is also displayed in Figure 20.
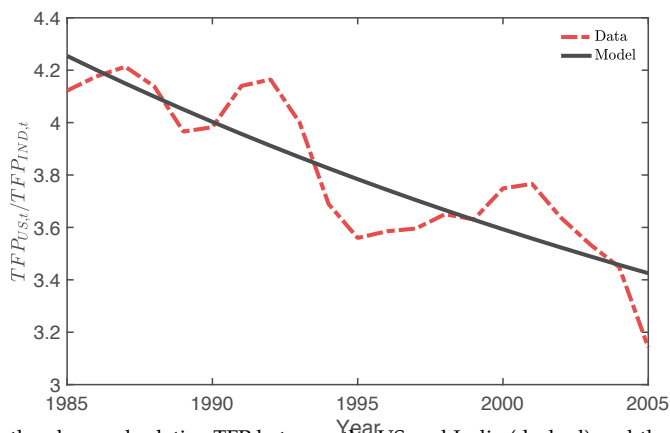
Notes: The figure shows the observed relative TFP between the US and India (dashed) and the one implied by the model (solid).

Figure 20: Identification of λ: TFP Differences between the US and India

B.2.3 Identifying the managerial output elasticity σ

In this section we describe in detail how we estimate the managerial output elasticity σ using indirect inference. As explained in Section 2.3.2, our measure of firms' managerial environment is their total managerial services $e = T/n + \alpha \times m$ (see (2.7)). This object is endogenous through firms' choice of outside managers m. While e is not directly observable, we assume that it is related to the observable share of managerial practices firms adopt. We refer to the share of practices firm f adopts as MP_f. In particular, we assume that e and MP_f are related via the measurement equation $e_f = vMP_f^\varrho$. As explained in Section 2.3.2, we can use the pre-treatment information on the share of adopted practices in the US and India and the model-implied differences in e in our US and India calibration to identify ϱ. Given ϱ, we can then express the model-implied change in total managerial services e due to the treatment, e_{IND}^{Treat}, as a function of observables $(MP_{IND}, MP_{US}, MP_{IND}^{Treat})$ and the equilibrium objects in our calibration (e_{IND}, e_{US}) - see equation (2.28). In our baseline calibration, we infer that the treatment increased total managerial efficiency among treatment plants by 26% (see footnote 32).

Because e is endogenous, we have to take a stand *how* the experiment induced firms to increase e by 26%, i.e., which structural parameter changed. We assume that the experiment increases the total efficiency of managerial services e by a multiple $\xi > 1$. Hence, if a treatment firm hires m units of managerial human capital on the market, it generates $\xi e = \xi(T/n + \alpha \times m)$ units of managerial services in the firm. This formalization captures the main spirit of the experiment in that the intervention provided information about how to make management more efficient via the provision of consulting services, but left the actual adoption of such managerial practices up to the treatment firms.

In practice we implement this procedure in the following way. Given the partial equilibrium nature of the experiment, treatment firms chose their optimal quantity of efficiency units of outside managers according to (2.8) taking the higher return to man-

agerial services ξ as given. Formally, the optimal number of outside managers treatment firms hire, $m(\xi)$, is implicitly defined by

$$[m_j(\xi)]_{j=1}^n = \underset{m_j \geq 0}{\text{argmax}} \sum_{j=1}^n \left\{ \left(\xi \left(\frac{T}{n} + \alpha m_j \right) \right)^\sigma Y - w_M m_j \right\}. \tag{B.28}$$

The solution to this problem is given by (see (2.10))

$$m(n; \xi) = \left(\frac{\sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}} (\xi \alpha)^{\frac{\sigma}{1-\sigma}} - \frac{1}{\alpha} \frac{T}{n} \tag{B.29}$$

and the associated number of managerial services, $e(n; \xi)$ is given by

$$e(n; \xi) = \xi(T/n + \alpha m(n; \xi)) = \left(\frac{\xi \alpha \sigma}{\omega_M} \right)^{\frac{1}{1-\sigma}}. \tag{B.30}$$

This implies that

$$\frac{e_{IND}^{Treat}}{e_{IND}} = \frac{e(n; \xi)}{e(n)} = \frac{(\xi \alpha \sigma / \omega_M)^{\frac{1}{1-\sigma}}}{(\alpha \sigma / \omega_M)^{\frac{1}{1-\sigma}}} = \xi^{1/(1-\sigma)}, \tag{B.31}$$

so that the required productivity increase ξ for treatment firms to increase their level of managerial efficiency from e_{IND} to e_{IND}^{Treat} is given by $\xi = \left(\frac{e_{IND}^{Treat}}{e_{IND}} \right)^{1-\sigma}$.

To understand our strategy to estimate the σ, suppose that all other structural parameters were given. In this hypothetical case, where we would only estimate σ, our algorithm would be the following:

1. Guess a value of σ and solve the equilibrium of the model.

2. The model then implies equilibrium values for e_{IND} and e_{US}.

3. Given (e_{IND}, e_{US}), an assumption on the adoption of such managerial practices in the US, MP_{US} and the estimated increase in managerial practices for treatment firms, $\frac{MP_{IND}^{Treat}}{MP_{IND}}$, we can use (2.28) to calculate e_{IND}^{Treat}

4. Given e_{IND}^{Treat}, we can calculate ξ according to $\xi = \left(\frac{e_{IND}^{Treat}}{e_{IND}} \right)^{1-\sigma}$

5. Given ξ we then perform the management experience in our model.

(a) We select 100 firms (50 for the treatment and 50 for the control group) from the top 0.01% of the size distribution from our India calibration. This selection procedure based on size mimics the selection procedure in Bloom et al. (2013), who note that the experimental firms had "about 270 employees, assets of 13 million, and sales of 7.5 million a year. Compared to US manufacturing firms, these firms would be in the top 2% by employment and the top 4% by sales, and compared to India manufacturing they are in the top 1% by both employment and sales (Hsieh and Klenow 2010)" (Bloom et al., 2013, p. 9). Because we calibrate our model to the population of Indian firms (i.e. including firms in the NSS), firms with 270+ employees correspond to the top 0.01% of the firm size distribution. Our calibrated model implies that this set of firms coincides with firms of $n = 7$ products.

(b) We then scale the total managerial efficiency of treatment firms by ξ to induce the required increase in managerial efficiency e and simulate their life-cycle for 100 weeks. Note that treatment firms are free to change their number of outside managers at any point at the equilibrium wage rate w_M of the baseline economy to mimic the partial equilibrium nature of the experiment. For the entire 100 weeks, managerial services in treatment firms have a productivity advantage of ξ.

(c) We then measure profits for all 100 weeks according to (2.8) for both treatment and control firms. For control firms, profits gross of innovation spending are given by (2.26). For treatment firms, profits are given by

$$\tilde{\pi}^{Treat}(n) = (1 - \sigma) e (n; \xi)^{\sigma} n + e (n; \xi)^{-(1-\sigma)} \sigma \xi T.$$

Hence, treatment firms have higher profits for three reasons: (1) they hire more

managerial service given their size ($m\left(n;\xi\right) > m\left(n\right)$), (2) they receive a direct benefit of being able to use e more efficiently ($\xi > 1$) and (3) they will on average be larger as their innovation incentives increase. While (1) and (2) are static effects, (3) is a dynamic effect

6. Given the model-generated data on $\tilde{\pi}^{Treat}(n)$ and $\tilde{\pi}(n)$ we then run the regression in (2.27), i.e. we estimate the specification

$$\ln \tilde{\pi}_{i,t} = \beta_0 + \beta_1 \times TREAT_{i,t} + \epsilon_{i,t} \tag{B.32}$$

and recover the treatment effect $\hat{\beta}_1$. Note that in our regression there is no need to use firm-fixed effects as all firms with $n > 1$ are high-type firms and all firms have the same size n. As explained in Section 2.3.2 we choose profits as our measure of firm-performance, while Bloom et al. (2013) focus on physical output. Bloom et al. (2013) do not estimate a treatment effect based on profits.

7. To average out the sampling variation in our estimate, we replicate this procedure 250 times and calculate the model-implied treatment effect

$$\hat{\beta}^{Treat} = \frac{1}{250} \sum_{i=1}^{250} \hat{\beta}_1^{(i)}. \tag{B.33}$$

8. If $\hat{\beta}^{Treat}$ is equal to the empirically observed value of 9%, we stop. Otherwise we go back to step 1 with a different guess for σ.

Recall that in order to infer e_{IND}^{Treat}, we had to assume a particular value for the share of practices adopted by firms in the US, MP_{US} (see (2.28)). For our baseline calibration, we assumed that firms in the US adopt all such practices as these practices "have been standard for decades in the developed world" (Bloom et al., 2013, p. 43). From the experimental micro-data, we can provide some additional evidence for this assumption. In the experimental data for Indian firms, we observe two objects related to the firms'

managerial environment: the share of particular practices the firm implements and the management score from Bloom and Van Reenen (2007). The management score is only measured pre-treatment but the practices are observed pre- and post-treatment. Using the pre-treatment variation of managerial practices and managerial scores across the Indian firms and the estimated changes in managerial practices due to the treatment, we can predict the average change in the firms' managerial score induced by the intervention. More specifically, we first run the cross-sectional regression

$$BVR_f = \beta + \gamma \times MP_f + \epsilon_f, \tag{B.34}$$

where BVR_f is the management score from Bloom and Van Reenen (2007) and MP_f is the share of adopted managerial practices. We then predict the change in the BVR score due to the treatment according to

$$E[BVR_f|Treatment] = E[BVR_f] + \hat{\gamma} \times \left(E[MP_f|Treatment] - E[MP_f] \right), \tag{B.35}$$

where $\hat{\gamma}$ is estimated coefficient from (B.35). The average BVR score among Indian firms before the treatment is 2.6. Using the estimated coefficient $\hat{\gamma}$ and the change in managerial practices due to the treatment $E[MP_f|Treatment] - E[MP_f]$, we find that the treatment increases the BVR score among treatment firms, $E[BVR_f|Treatment]$, depending on how we treat outliers in the regression, to 2.84 on the low end and 3.12 on the high end. The average BVR score among US firms is equal to 3.28. Hence, this exercise suggests that the treatment closes the "management gap" as measured by BVR scores by $\frac{2.84-2.6}{3.28-2.6} = 35\%$ on the low end and $\frac{3.12-2.6}{3.28-2.6} = 76\%$ on the high end.

We can compare this number to the implications of our model. Our baseline calibration implies that the treatment increases e from $e_{IND} = 0.201$ by 26% to $e_{IND}^{Treat} = 0.252$. Our calibration also implies that $e_{US} = 0.283$. Hence, Indian firms use 71% the amount of managerial services as firms in the US and the treatment increases managerial services to 89% of the US level. Hence, the treatment reduces the "management gap" by

$\frac{0.252-0.201}{0.283-0.201} \approx 62\%$.

B.2.4 Identifying Managerial Skill Supplies μ_M

To decompose differences in the managerial environment in India and the US into supply and demand factors, we start out with 4 parameters: $(\mu_{M,US}, \alpha_{US}, \mu_{M,IND}, \alpha_{IND})$. Without loss of generality we can normalize $\mu_{M,US} = 1$. Since $\mu_{M,c} \times \alpha_c$ is identified from the equilibrium managerial employment shares [see (B.26)], we require one additional equation to determine the relative managerial human capital in India, $\mu_{M,IND}$. To do so, we use data on employment patterns of immigrants from India to the US

Let χ_c be the managerial share of the native population in country c. Let χ_{IND}^M be the managerial employment share in the population of Indian migrants in India (i.e., pre-migration). Let χ_{US}^M be the managerial employment share in the population of Indian migrants in the US (i.e., post-migration). Suppose that the distribution of managerial ability of Indians who migrate to the US is distributed Pareto with shape ϑ and mean $\hat{\mu}_{M,IND}$. If $\hat{\mu}_{M,IND} = \mu_{M,IND}$, migration is orthogonal to managerial skills. If $\hat{\mu}_{M,IND} > \mu_{M,IND}$, migrants have, on average, a comparative advantage in managerial work. Given these assumptions it follows that

$$\chi_c = \tilde{\vartheta} \left(\omega_M^c\right)^\vartheta \left(\mu_{M,c}\right)^\vartheta \quad \text{and} \quad \chi_c^M = \tilde{\vartheta} \left(\omega_M^c\right)^\vartheta \left(\hat{\mu}_{M,c}\right)^\vartheta$$

where $\tilde{\vartheta} = \left(\frac{\vartheta-1}{\vartheta}\right)^\vartheta$ and ω_M^c is the relative managerial wage $\frac{w_M}{w_P}$ in country c. Hence,

$$\frac{\mu_{M,IND}}{\mu_{M,US}} = \underbrace{\left(\frac{\chi_{US}^M}{\chi_{US}}\right)^{1/\vartheta}}_{uncorrected\ ratio} \times \underbrace{\left(\frac{\chi_{IND}}{\chi_{IND}^M}\right)^{1/\vartheta}}_{selection\ correction\ term} . \tag{B.36}$$

The first term in (B.36) compares migrants and US natives in the US economy, i.e., holding α constant. Differences in managerial employment are therefore interpreted as differences

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in human capital. The second term accounts for selection into migration: if immigrants are positively selected on their managerial skills, i.e., $\chi_{IND}^M > \chi_{IND}$, the observed differences in outcomes in the US *under*estimate the differences in skills in the population. The last term in equation (B.36) corrects for that potential selection.

We want to note that this identification strategy relies on occupational sorting being based on skills - both before and after migrating. If for example Indian migrants face excessive frictions to enter managerial positions (relative to other jobs), their observed managerial employment share is lower than their skills warrant. In that case we would conclude that they have relatively little human capital. See for example Hsieh et al. (2013) for an elaboration of this point. Alternatively, migrants could have been *more* likely to work as managers prior to migrating relative to their innate skills.[79] If, for example, migrants stem from families, which are richer and more likely to own a business, migrants might have worked as managers before simply because of their family connection. In that case migrants might not be selected on their managerial skill but rather representative of the population at large. If that was the case, we would erroneously conclude that the US population had a comparative advantage in managerial occupations. Again we want to stress that our identification strategy will correctly recover $\alpha \times \mu$. The information in (B.36) is only used to separately identify α and μ.

Given that we already calibrated ϑ and we already used χ_{IND} and χ_{US} in our calibration. χ_{US}^M is directly observable in the US Census, because we see the employment structure among recent Indian immigrants. Finally, χ_{IND}^M can be estimated from the New Immigration Study, which explicitly asks immigrants about the occupations *prior* to migration [see Hendricks and Schoellman (2016)].

The data to quantify (B.36) is contained in Table 38. Column 1 and 3 report the managerial share in the US and India, respectively. In column 2 we report the managerial share among Indian immigrants in the US To ensure that this population is informative

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[79]We are grateful to one of our referees to suggest this possibility.

about the human capital of recent Indian migrants, we restrict the sample to migrants that arrived in the US within the last 5 years. The managerial share in this population is given by 12.9%. In the last column we exploit information from the New Immigration Study to measure the share of migrants that used to work as managers in India. We find that roughly 6% of them worked as outside manager.

	US		India	
Sample	Male, 20-60 years, employed			
Population	US population	Indian migrants	Indian population	Indian migrants
	χ_{US}	χ_{US}^M	χ_{IND}	χ_{IND}^M
Managerial share	12.4 %	12.9 %	1.7%	6.1%
Data source	US Census	US Census	Indian Census	New Immigration Study

Notes The table contains estimates for the managerial employment share in the native population of the US (column 1), the population Indian immigrants in the US (column 2), the native population in India (column 3), and the sample of Indian migrants to the US *in* India (column 4). For the definition of outsider managers, see Table 7 and the discussion there. χ_{US} and χ_{US}^M are calculated from the US census and χ_{IND} from the Indian census. χ_{IND}^M is calculated from the data of the New Immigration Study. We refer to Hendricks and Schoellman (2016) for a detailed description of the data. For the New Immigration Study we use the occupational codes "10 to 430: executive, administrative and managerial" and "500 to 950: management related" as referring to managers. We also insist on the individual having received a salary (instead of, for example, being self-employed).

Table 38: Identification of Managerial Skills: Managerial Employment Shares

The sample size for estimating the managerial share of migrants in India, χ_{IND}^M, is only 403, i.e., quite small. To judge the robustness of our results, we report the implied differences in delegation quality $\frac{\alpha_{US}}{\alpha_{IND}}$ as a function of the point estimate of χ_{IND}^M. We treat the other empirical objects in (B.36), as fixed as these are precisely estimated. We construct the confidence intervals for $\frac{\alpha_{US}}{\alpha_{IND}}$ using a Bootstrap procedure, where we repeatedly draw samples with replacement from the New Immigration Study data and calculate χ_{IND}^M. The results of this exercise are contained in Figure 21. We find that the confidence interval [1.7,3.2] contains the relative delegation efficiency of the US with 90% probability. We also want to stress that this uncertainty *only* affects the decomposition of the implied counterfactual into the human capital and the delegation efficiency component, as all allocation only depend on $\mu_{M,c}\alpha_c$.
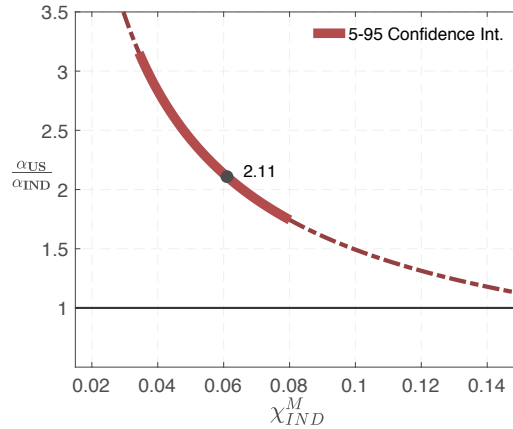
Notes: The figure depicts the resulting $\frac{\alpha_{US}}{\alpha_{IND}}$ as a function of χ^M_{IND}. Our point estimate for the immigrants' managerial share in India (6.1%) yields a relative delegation quality of 2.11. The 5-to-95 confidence interval around that value ranges from about 1.7 to 3.2.

Figure 21: Calibrating $\frac{\alpha_{US}}{\alpha_{IND}}$

B.2.5 Moment Sensitivity

In Table 39 we report a sensitivity matrix, which contains the elasticity of each moment used in the internal calibration (rows) with respect to the parameters of the model (columns). Specifically, we report percentage change in the moment for a 1% change in the parameter from its benchmark calibrated value, while keeping the rest of the parameters at their benchmark values. We report the average elasticities based on +1% and -1% changes. This provides useful information about how the parameters influence the model counterpart of targeted moments. For brevity, we report the matrix for our India calibration. The sensitivity matrix for the US calibration is available upon request.

B.2.6 Reduced-Form Evidence based on Variation across Indian Establishments

In Section 2.4.2, we reported some basic patterns on managerial hiring and firm size from the Indian micro data and discussed how they relate to our theory. This section describes this analysis in more detail.

	δ	β	T	$\alpha \times \mu$	θ	θ_E	σ
M1. Entry rate	-0.02	-0.03	0.59	0.05	0.06	1.25	-0.90
M2. Mean empl. of 21-25-year-old firms	0.09	0.12	0.17	0.001	0.27	0.04	-0.31
M3. Empl. share of 21-25-year-old firms	-0.06	-0.42	-0.23	-0.04	-0.37	-0.16	0.50
M4. Rel. exit rate of small 21-25-year-old firms	0.10	0.18	0.08	0.01	0.01	0.17	-0.12
M5. Share of managers	0.02	0.33	0.24	1.07	0.56	-0.44	-0.89
M6. Share of entrepreneurial profit	-0.001	-0.13	0.41	-0.03	-0.33	0.18	-0.43
M7. Treatment effect of Bloom et al. (2013)	0.08	2.01	-1.65	-1.57	3.26	-2.63	-0.80

Notes: The table presents the elasticity for each moment used the internal calibration for India with respect to the parameters of the model. In particular, we report percentage change in the moment for a 1% change in the parameter from its benchmark value in the Indian calibration, while keeping the rest of the parameters at their benchmark values. We report the average elasticities based on +1% and -1% changes. To identify α and μ separately, we use the manager share among Indian migrants before and after emigrating to the US See Section B.2.4 for more information.

Table 39: Moment Sensitivity

Our empirical investigation mainly focuses on the implications of the two parameters of our model: (i) entrepreneur's time endowment T and (ii) delegation efficiency α. In the theory, time endowment of entrepreneurs T has the interpretation that it can neither be sold on the market, nor is there any need to monitor. The NSS data for 1995 contain information on the size of the family of the establishment's owner. As long as family members require less monitoring time than outside managers, we can think of family size as inducing variation in the time endowment T. As for the delegation efficiency α, we will rely on the variation in trust across 22 Indian states. The Indian micro data contain information about the state in which the respective establishment is located. Additionally, we extract information on the general level of trust between people at the state level from the World Value Surveys. The World Values Survey is a collection of surveys based on representative samples of individuals and provides an index of trust in different regions of India. The primary index we use is derived from the answers to the question "Generally speaking, would you say that most people can be trusted, or that you can not be too careful in dealing with people?". Following Bloom et al. (2012) and La Porta et al. (1997), the regional trust index is constructed as the percentage of people providing the answer "Most people can be trusted" within the state where the firm is located. This is the most common measure of trust used in the literature. While this variable is not directly aimed

at eliciting the (perceived) quality of the prevailing legal environment, it fits well into our theoretical framework as long as trust reduces the required time the owner needs to spend to incentivize outside managers. See also Bloom et al. (2012), who also use this variable to proxy the efficiency with which decisions can be delegated.

In Table 40, we look at some of the implications of our theory based on the above-mentioned proxies. We first focus on the extensive margin of managerial hiring. In the model, a firm hires an outside manager only when its size n is above a certain (endogenous) threshold which we denote as n^*

$$n^* \equiv T \times \left(\frac{\omega_M}{\sigma \alpha} \right)^{\frac{1}{1-\sigma}} .$$

For the purpose of the empirical analysis, in addition to firm size n, suppose that firms also differ in (i) owner's time endowment T and (ii) delegation efficiency α. Then, the extensive margin of managerial hiring decision for firm f can be summarized as

$$
\begin{aligned}
\mathbb{1}\left[Manager_f > 0 \right] &= \mathbb{1}\left[n_f \geq n_f^* \right] \\
&= \mathbb{1}\left[n_f \geq T_f \times \left(\frac{\omega_M}{\sigma \alpha_f} \right)^{\frac{1}{1-\sigma}} \right] \\
&= \mathbb{1}\left[\log n_f - \log T_f + \frac{1}{1-\sigma} \times \log \alpha_f + const. \geq 0 \right],
\end{aligned}
$$

where subscript f indicates firm specific values and *const.* includes all terms that are not firm specific. This relation can be converted to an *estimable* one by introducing some stochasticity. In particular, by introducing a uniformly distributed random variable, which can be considered as measurement error, to the RHS of the above equation and taking the expectation of both sides, we get

$$\mathbb{P}\left(Manager_f > 0 \right) = \beta_0 + \beta_1 \log n_f - \beta_2 \log T_f + \beta_3 \log \alpha_f. \tag{B.37}$$

This equation implies that the likelihood of hiring a manager should be increasing in firm size and delegation efficiency and declining in the owner's time endowment. To test these predictions empirically, we estimate the coefficients of (B.37) by using the proxy variables mentioned above.[80] Column 1 of Table 40 summarizes the results. It suggests that the predictions of the model regarding extensive margin of managerial hiring are in line with the data: empirically large firms and firms in states with favorable trust measures are more likely to hire outside managers, while firms with larger families abstain from hiring outside managerial personnel holding firm size constant.

| | Dependent Variable | | | | |
	Manager > 0	Log empl (Manager > 0)		Log empl	
Log Empl	0.039***				
	(0.003)				
Log HH Size	-0.003**	0.927***	0.812***	0.224***	0.235***
	(0.001)	(0.306)	(0.278)	(0.033)	(0.032)
Trust	0.013**	3.264**		0.094	
	(0.006)	(1.628)		(0.174)	
Log HH Size* Trust		-1.694**	-1.329*	0.036	0.028
		(0.818)	(0.758)	(0.093)	(0.090)
State FE	N	N	Y	N	Y
N	178,999	2,350	2,350	178,999	178,999
R^2	0.04	0.42	0.50	0.18	0.20

Notes: Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.10$. All regressions include 2-digit fixed effects, the age of the establishment, year dummies, and a dummy variable for the establishment to be in a rural area as control variables. For the regressions that do not include state-level fixed effects, log GDP per capita at the state level is included as a control variable. "Log Empl" denotes the (log of) total employment at the establishment. "Log HH size" denotes the (log of) the size of the household of the establishment's owner. This variable is only available for the NSS data. "Trust" is the measure of trust at the state level, which we calculate from the World Value Surveys. The dependent variables are: an indicator of managerial hiring (column 1), log employment conditional on managerial hiring (columns 2 - 3), log employment (columns 4-5).

Table 40: Managerial Hiring, Firms Size and Growth in India

These static determinants of managerial hiring have dynamic implications relating to firms' expansion incentives and hence firm size. In particular, conditional on hiring managers, growth incentives and hence firm size are increasing in delegation efficiency. Our theory implies that delegation efficiency α and the owner's time endowment T are

---

[80]Note that (B.37) implies a linear probability model and its parameters can be estimated using OLS. We also include additional control variables in the regression. Details are given in the notes under Table 40.

substitutes, i.e., we should expect a tighter link between family size and firm size in low-trust regions. Columns 2 and 3 show that this is the case. First, similar to Bloom et al. (2013), we also find a tight relationship between firm size and family size. We interpret this correlation as family members substituting for the scarcity of available outside managers. Furthermore, the coefficient on the interaction term is negative, which means that the positive relationship between firm size and family size is weaker in regions where trust is higher and hence delegation is more efficient.[81] In column 3, we replicate these results with state-fixed effects to control for all time-invariant regional characteristics.

In columns 4 and 5, we redo the analysis of columns 2 and 3 for the whole sample of firms, i.e., we do not condition on delegation. Again we find a positive correlation between the size of the family and firm size. Note that the effect of trust for the entire sample of firms is much weaker. This is consistent with our theory, which implies that delegation efficiency only matters for the firms that actually delegate. For firms without outside managers (i.e., firms with $n < n^*$), growth incentives are only determined by the owner's time endowment T.

Finally, we replicated the entire analysis of Table 40, which controlled for 2-digit sector fixed effects, with 3-sector fixed effects. The results are contained in Table 41. It is seen that results are similar. The only exception are the results in columns 2 and 3, which are conditioned on managerial hiring and hence have a small sample size[82]. While all point estimates are of the same sign, they are not significantly different from zero.

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[81]In a separate regression, not shown here, we also control for the assets of the firm as both family size and the level of regional trust could be correlated with the supply of capital to the firm. The results are very similar.

[82]Given the small sample size, finer controls for sector fixed effect leave less variation in the data for the relations we are interested in.

	Dependent Variable				
	Manager > 0	Log empl (Manager > 0)		Log empl	
Log Empl	0.040***				
	(0.003)				
Log HH Size	-0.004***	0.389	0.394*	0.207***	0.220***
	(0.001)	(0.248)	(0.231)	(0.030)	(0.030)
Trust	0.012*	0.570		-0.008	
	(0.006)	(1.300)		(0.160)	
Log HH Size* Trust		-0.443	-0.359	0.062	0.040
		(0.658)	(0.614)	(0.086)	(0.084)
State FE	N	N	Y	N	Y
N	178,999	2,350	2,350	178,999	178,999
R^2	0.05	0.58	0.63	0.28	0.30

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10. All regressions include 3-digit fixed effects, the age of the establishment, and a dummy variable for the establishment to be in a rural area as control variables. For the regressions that do not include state level fixed effects, log GDP per capita at the state-level is included as a control variable. "Log Empl" denotes the (log of) total employment at the establishment. "Log HH size" denotes the (log of) the size of the household of the establishment's owner. This variable is only available for the NSS data. "Trust" is the measure of trust at the state level, which we calculate from the World Value Surveys. The dependent variables are: an indicator of managerial hiring (column 1), log employment conditional on managerial hiring (columns 2 - 3), log employment (columns 4-5).

Table 41: Managerial Hiring, Firms Size and Growth in India: Robustness

B.2.7   Firms vs. Establishments in the US Manufacturing Sector

In this section we compare the process of firm-dynamics across US manufacturing firms and establishments. Table 42 provides some summary statistics about the size-distribution of firms and establishments in the US The average manufacturing firm in the US has 51 employees, while the average establishment only 43. It is also the case that large firms have multiple establishments (firms with more than 1000 employees have on average 13) so that large firms account for half of total employment. There is a lower concentration at the establishment level in that establishments with more than 1000 employees account for less than one-fifth of aggregate employment in manufacturing in the US

We now turn to the implied dynamics. Because we focus on cross-sectional data, the information on firm (establishment) age is crucial for us. For establishments, the definition of age is straightforward. Birth year is defined as the year a establishment first reports positive employment in the LBD. Establishment age is computed by taking the difference

Size	Firms					Establishments			
	No.	Avg. Employment	Agg. Share	No. of Establishments	Exit rate	No.	Avg. Employment	Agg. Share	Exit rate
1-4	86936	2.30	1.65	1.00	13.22	93038	2.31	1.78	16.50
5-9	48178	6.68	2.66	1.00	3.46	54281	6.73	3.02	4.20
10-19	37942	13.80	4.33	1.01	2.66	45803	14.01	5.30	3.10
20-49	32555	30.92	8.31	1.05	2.27	44085	31.90	11.62	2.40
50-99	13516	67.94	7.58	1.21	2.03	21582	71.54	12.75	1.90
100-249	8914	139.90	10.30	1.61	1.59	16476	155.76	21.20	1.00
250-499	3167	280.96	7.35	2.47	0.92	5444	348.72	15.68	0.50
500-999	1720	503.49	7.15	3.94	0.29	2120	677.19	11.86	0.30
1000+	2423	2531.92	50.67	12.68	0.25	984	2068.2	16.81	0.30
Aggregate	235351	51.44	100		6.53	283813	42.66	100	7.3

Notes: This table contains summary statistics for US manufacturing firms and establishments in 2012. The data are taken from the BDS.

Table 42: Descriptive Statistics: US Micro Data

between the current year of operation and the birth year. Given that the LBD series starts in 1976, the observed age is by construction left censored at 1975. In contrast, firm age is computed from the age of the establishments belonging to that particular firm. A firm is assigned an initial age by determining the age of the oldest establishment that belongs to the firm at the time of birth. Firm age accumulates with every additional year after that. In Figure 22 we show the cross-sectional age-size relationship for establishments (left panel) and firms (right panel) in the US
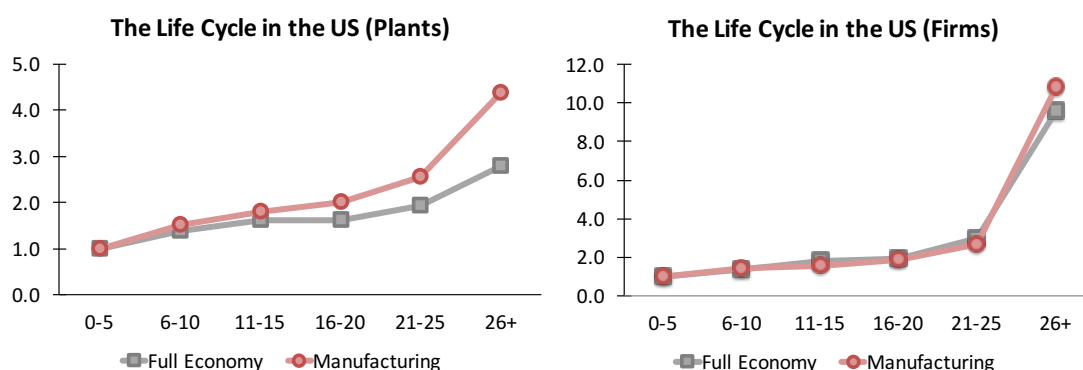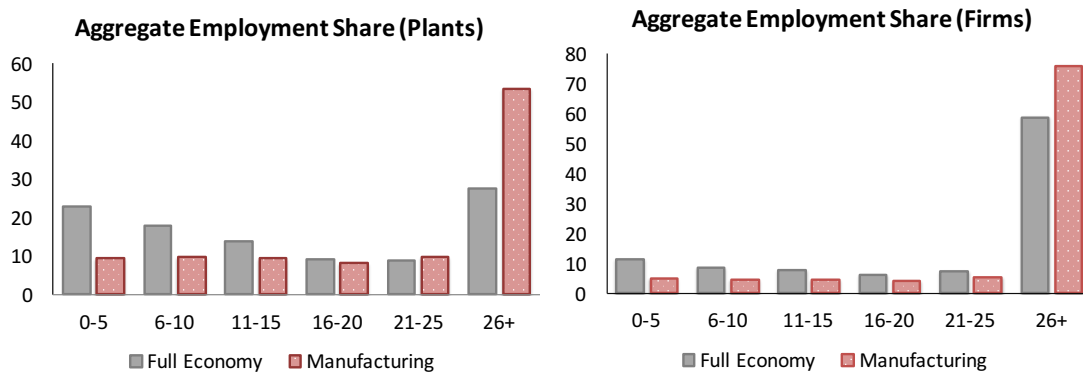

Notes: The figure contains the cross-sectional age-size relationship for establishments (left panel) and firms (right panel) in the US The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

Figure 22: Life Cycle of Establishments and firms in the US

Not surprisingly, the life-cycle is much steeper for firms, especially for +26-year-old firms, as firms grow both on the intensive margin at the establishment level and the extensive margin of adding establishments to their operation.

In Figure 23 we show the aggregate employment share of establishments and firms of different ages. As suggested by the life-cycle patterns in Figure 22, old firms account for the bulk of employment in the US However, the relative importance of old establishments/firms is somewhat less pronounced because of exit, i.e., while the average firm/establishment grows substantially by age conditional on survival, many firms / establishments have already exited by the time they would have been 20 years old. Nevertheless, firms (establishments) older than 25 years account for 76% (53%) of employment in the manufacturing sector.
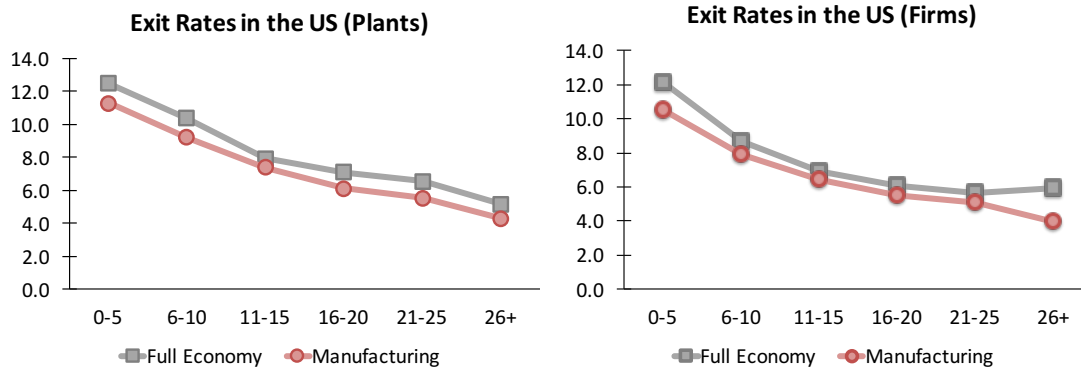

Notes: The figure contains the aggregate employment share of establishments (left panel) and firms (right panel) in the US as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

Figure 23: The employment share by age of establishments and firms in the US

This pattern of exit is depicted in Figure 24. There we show annual exit rates for firms and establishments as a function of age. The declining exit hazard is very much suggestive of a model of creative destruction, whereby firms and establishments grow as they age (conditional on survival) and exit rates are lower for bigger firms/establishments.

An important moment for us is the age-specific exit rate conditional on size. It is
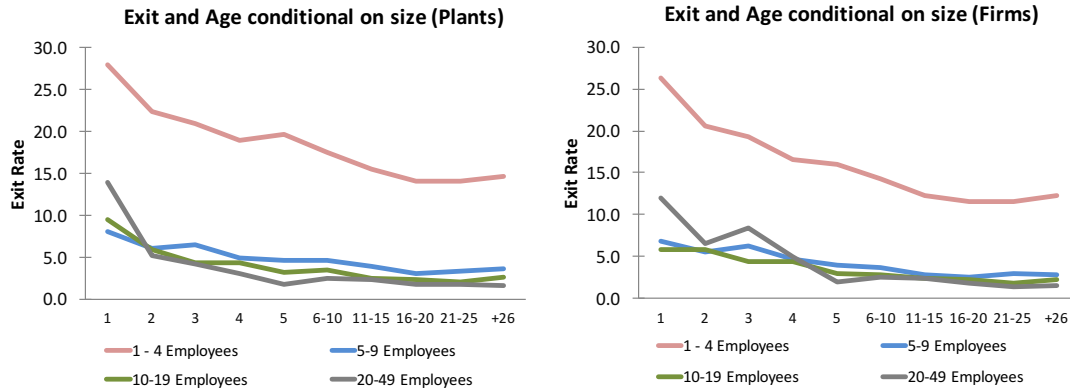
Notes: The figure contains the exit rates of establishments (left panel) and firms (right panel) in the US as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

Figure 24: The Exit Rates of Establishments and Firms in the US by Age

this moment that will identify the importance of selection. In a model without heterogeneity, size will be a sufficient statistic for future performance, so that age should not predict exit conditional on size. However, if the economy consists of high- and low-type entrepreneurs, old firms are more likely to be composed of high types conditional on size. Hence, the size-specific exit rate by age is monotone in the share of high types by age. In Figure 25 we report this schedule for both establishments and firms. The data show a large degree of age-dependence (conditional on size). The schedules for small firms and establishments look almost identical. This is reassuring because small firms are almost surely single-establishment firms, so that a firm-exit will also be a establishment-exit and vice versa.

B.2.8 Establishments in the Indian Manufacturing Sector

In this section we provide more descriptive evidence about the underlying process of firm dynamics in the manufacturing sector in India. Table 43 contains descriptive statistics for our sample of Indian manufacturing establishments. For comparison, we organize

Exit and Age conditional on size (Plants)   Exit and Age conditional on size (Firms)

1 - 4 Employees   5-9 Employees
10-19 Employees   20-49 Employees

Notes: The figure contains the conditional exit rates by size of establishments (left panel) and firms (right panel) in the US as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for the manufacturing sector.

Figure 25: Size-dependent exit rates of establishments and firms in the US by age

the data in the same way as in the left panel of Table 42, which contains the results for manufacturing establishments in the US It is clearly seen that the establishment-size distribution in India is concentrated on very small firms. The average establishment has fewer than 3 employees and more than 50% of aggregate employment is concentrated in establishments with at most 4 employees. Such establishments account for 93% of all establishments in the Indian manufacturing sector. A comparison of establishment size distribution for the years 1995 and 2010 in Table 44 suggests that these patterns are stable over time.

Figure 26 reports the aggregate employment share by age for Indian manufacturing establishments and is hence comparable to Figure 23 for the US

It is clearly seen that the aggregate importance of old firms is very small in India. While firms that are older than 25 years account for 55% of employment in the US, the corresponding number is less than 20% in India. This is a reflection of the shallow life-cycle in India and not of there being fewer old firms in the Indian economy.

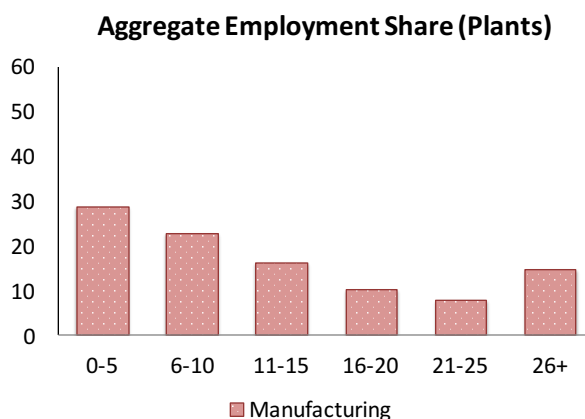| Size	No.	Avg. Employment	Aggregate Employment Share
1-4	15957296	1.56	54.76
5-9	843091	6.26	11.61
10-19	243868	12.98	6.96
20-49	70834	29.22	4.55
50-99	23242	69.89	3.57
100-249	14898	149.31	4.89
250-499	4701	346.69	3.58
500-999	2283	683.86	3.43
1000+	1232	2452.65	6.65
Aggregate	17161445	2.65	100.00

Notes: This table contains summary statistics for establishments in the Indian manufacturing sector in 2010. The data are taken from the ASI and the NSS. To calculate the number of firms, we use the sampling weights provided in the data.

Table 43: Descriptive Statistics: Indian Micro Data

	Plant Size				
	1-4	**5-9**	**10-19**	**20-49**	**50+**
1995	0.9171	0.0631	0.0143	0.0035	0.0020
2010	0.9297	0.0491	0.0143	0.0042	0.0027

Notes: This table presents the share of establishments for different size bins in India, for the years 1995 and 2010. Size bins are constructed based on number of employees.

Table 44: Establishment Size Distribution in India

Notes: The figure contains the aggregate employment share of manufacturing establishments in India as a function of age. The data are taken from the ASI and the NSS and we focus on the data for 2010. We combine the two data sets using the sampling weights provided in the micro data.

Figure 26: The employment share by age of establishments in India

Appendix C

Appendix to Chapter 3

Proof of Lemma 1. Consider the low-type firms and conjecture $\tilde{V}_l\left(\hat{\mathcal{Q}}\right) = \sum\limits_{\hat{q}\in\hat{\mathcal{Q}}} Y^l\left(\hat{q}\right)$:

$$r\sum_{\hat{q}\in\hat{\mathcal{Q}}} Y^l\left(\hat{q}\right) = \sum_{\hat{q}\in\hat{\mathcal{Q}}} \max\left\{0, \max_{x\geq 0}\left[\begin{array}{c} \tilde{\pi}\left(\hat{q}_j\right) - \tilde{w}^s\phi^s - \tilde{w}^sG\left(x,\theta^L\right) + \frac{\partial Y^l(\hat{q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} \\ +x\mathbb{E}Y^l\left(\hat{q} + \lambda\bar{\hat{q}}\right) - \left(\tau + \varphi\right)Y^l\left(\hat{q}\right) \end{array}\right]\right\},$$

which implies

$$rY^l\left(\hat{q}\right) = \max\left\{0, \left\{\begin{array}{c} \tilde{\pi}\left(\hat{q}\right) - \tilde{w}^s\phi^s + \frac{\partial Y^l(\hat{q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} - \left(\tau + \varphi\right)Y^l\left(\hat{q}\right) \\ + \max_{x\geq 0}\left[x\mathbb{E}Y^l\left(\hat{q} + \lambda\bar{\hat{q}}\right) - \tilde{w}^sG\left(x,\theta^L\right)\right] \end{array}\right\}\right\},$$

where we also use the fact that a firm can choose not to operate an individual product line.

Next consider the high-type firms and conjecture $\tilde{V}_h\left(\hat{\mathcal{Q}}\right) = \sum\limits_{\hat{q}\in\hat{\mathcal{Q}}} Y^h\left(\hat{q}\right)$:

$$r\sum_{\hat{q}\in\hat{\mathcal{Q}}} Y^h\left(\hat{q}\right) = \sum_{\hat{q}\in\hat{\mathcal{Q}}} \max\left\{0, \max_{x\geq 0}\left[\begin{array}{c} \tilde{\pi}\left(\hat{q}\right) - \tilde{w}^s\phi^s - \tilde{w}^sG\left(x,\theta^H\right) + \frac{\partial Y^h(\hat{q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} \\ +x\mathbb{E}Y^h\left(\hat{q} + \lambda\bar{\hat{q}}\right) \\ -\left(\tau + \varphi\right)Y^h\left(\hat{q}\right) + \nu\left[\mathbb{I}_{\hat{q}>\hat{q}_{l,\min}}\cdot Y^l\left(\hat{q}\right) - Y^h\left(\hat{q}\right)\right] \end{array}\right]\right\},$$

which similarly implies

$$
rY^h(\hat{q}) = \max \left\{ 0, \max_{x \geq 0} \left[\begin{array}{c} \tilde{\pi}(\hat{q}) - \tilde{w}^s \phi^s - \tilde{w}^s G\left(x, \theta^H\right) + \frac{\partial Y^h(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t} + \\ x \mathbb{E} Y^h\left(\hat{q} + \lambda \tilde{\hat{q}}\right) \\ -\left(\tau + \varphi\right) Y^h(\hat{q}) + \nu\left[\mathbb{I}_{\hat{q} > \hat{q}_{l,\min}} \cdot Y^l(\hat{q}) - Y^h(\hat{q}) \right] \end{array} \right] \right\}.
$$

Monotonicity follows from the fact that the per-period return function is increasing in $\hat{q}$. $\qquad \square$

Proof of Proposition 1. First note that $\tilde{\pi}(q) = \left(\frac{\varepsilon-1}{\varepsilon}\right)^\varepsilon \frac{1}{\varepsilon-1} \hat{q}^{\varepsilon-1} = \Pi \hat{q}^{\varepsilon-1}$. Then, defining $\Psi \equiv r + \tau + \varphi$, equation (3.19) can be written as the following linear differential equation

$$
\Psi Y^l(\hat{q}) + g\hat{q}\frac{\partial Y^l(\hat{q})}{\partial \hat{q}} = \Pi \hat{q}^{\varepsilon-1} + \Omega^l - \tilde{w}^s \phi \text{ if } \hat{q} > \hat{q}_{l,\min}
$$

or

$$
\xi_1 \hat{q}^{-1} Y^l(\hat{q}) + \frac{\partial Y^l(\hat{q})}{\partial \hat{q}} = \xi_2 \hat{q}^{\varepsilon-2} - \xi_3 \hat{q}^{-1}, \tag{C.1}
$$

where $\xi_1 \equiv \frac{\Psi}{g}$, $\xi_2 \equiv \frac{\Pi}{g}$ and $\xi_3 \equiv \frac{\tilde{w}^s \phi - \Omega^l}{g}$. Then the solution to (C.1) can be written as

$$
Y^l(\hat{q}) = \hat{q}^{-\xi_1}\left(\int\left[\xi_2 t^{\xi_1+\varepsilon-2} - \xi_3 t^{\xi_1-1}\right] dt + D\right) = \frac{\xi_2 \hat{q}^{\varepsilon-1}}{\xi_1+\varepsilon-1} - \frac{\xi_3}{\xi_1} + D\hat{q}^{-\xi_1}.
$$

Imposing the boundary condition $Y^l(\hat{q}_{l,\min}) = 0$, we can solve out for the constant of integration D, obtaining

$$
\begin{aligned}
Y^l(\hat{q}) &= \frac{\xi_2 \hat{q}^{\varepsilon-1}}{\xi_1+\varepsilon-1} - \frac{\xi_3}{\xi_1} + \left(\frac{\xi_3 \hat{q}_{l,\min}^{\xi_1}}{\xi_1} - \frac{\xi_2 \hat{q}_{l,\min}^{\xi_1+\varepsilon-1}}{\xi_1+\varepsilon-1}\right)\hat{q}^{-\xi_1} \\
&= \frac{\Pi \hat{q}^{\varepsilon-1}}{\Psi+(\varepsilon-1)g}\left(1 - \left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi}{g}+\varepsilon-1}\right) + \frac{\Omega^l - \tilde{w}^s \phi}{\Psi}\left(1 - \left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi}{g}}\right).
\end{aligned} \tag{C.2}
$$

We next provide the derivation of the value for a high-type product line. Let us rewrite

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the expression in (C.2) as

$$Y^l(\hat{q}) = \xi_4 \hat{q}^{\varepsilon-1} + \xi_5 \hat{q}^{-\frac{\Psi}{g}} - \xi_6,$$

where

$$\xi_4 \equiv \frac{\Pi}{\Psi + (\varepsilon-1)g}, \quad \xi_5 = \frac{\left(\tilde{w}^s\phi - \Omega^l\right)\hat{q}_{l,\min}^{\frac{\Psi}{g}}}{\Psi} - \frac{\Pi\hat{q}_{l,\min}^{\frac{\Psi}{g}+\varepsilon-1}}{\Psi + g(\varepsilon-1)}, \text{ and } \xi_6 = \frac{\tilde{w}^s\phi - \Omega^l}{\Psi}.$$

Recall the value of a product line of a high-type firm

$$(\Psi + \nu) Y^h(\hat{q}) + \frac{\partial Y^h(\hat{q})}{\partial \hat{q}} g\hat{q}$$

$$= \Pi\hat{q}^{\varepsilon-1} + \Omega^h - \tilde{w}^s\phi + \nu\left(\xi_4\hat{q}^{\varepsilon-1} + \xi_5\hat{q}^{-\frac{\Psi}{g}} - \xi_6\right) \text{ for } \hat{q} \geq \hat{q}_{l,\min}$$

$$= \Pi\hat{q}^{\varepsilon-1} + \Omega^h - \tilde{w}^s\phi \text{ for } \hat{q}_{l,\min} > \hat{q} \geq \hat{q}_{h,\min},$$

which can be rewritten as

$$K_1 Y^h(\hat{q}) \hat{q}^{-1} + \frac{\partial Y^h(\hat{q})}{\partial \hat{q}} = K_2\hat{q}^{\varepsilon-2} + K_3\hat{q}^{-\frac{\Psi+g}{g}} - K_4\hat{q}^{-1},$$

where

$$K_1 \equiv \frac{\Psi+\nu}{g}, \ K_2 \equiv \frac{\Pi+\nu\xi_4}{g}, \ K_3 \equiv \frac{\nu\xi_5}{g}, \ K_4 \equiv \frac{\nu\xi_6 + \tilde{w}^s\phi - \Omega^h}{g} \text{ for } \hat{q} \geq \hat{q}_{l,\min} \text{ (C.3)}$$

$$K_1 \equiv \frac{\Psi+\nu}{g}, \ K_2 \equiv \frac{\Pi}{g}, \ K_3 \equiv 0, \ K_4 \equiv \frac{\tilde{w}^s\phi - \Omega^h}{g} \text{ for } \hat{q}_{l,\min} > \hat{q} \geq \hat{q}_{h,\min}. \qquad \text{(C.4)}$$

Then we can express the general solution for the high-type value function as

$$Y^h(\hat{q}) = \hat{q}^{-K_1}\left(\int\left[K_2\hat{q}^{K_1+\varepsilon-2} + K_3\hat{q}^{K_1-\frac{\Psi+g}{g}} - K_4\hat{q}^{K_1-1}\right]d\hat{q} + D\right)$$

$$= \frac{K_2\hat{q}^{\varepsilon-1}}{K_1+\varepsilon-1} + \frac{K_3\hat{q}^{1-\frac{\Psi+g}{g}}}{K_1+1-\frac{\Psi+g}{g}} - \frac{K_4}{K_1} + D\hat{q}^{-K_1}. \qquad \text{(C.5)}$$

To find the constant of integration D, we use $Y^h\left(\hat{q}_{h,\min}\right) = 0$, which yields

$$D = -\frac{K_2\hat{q}_{h,\min}^{K_1+\varepsilon-1}}{K_1+\varepsilon-1} - \frac{K_3\hat{q}_{h,\min}^{K_1+1-\frac{\Psi+g}{g}}}{K_1+1-\frac{\Psi+g}{g}} + \frac{K_4\hat{q}_{h,\min}^{K_1}}{K_1} \text{ for } \hat{q} \in \left[\hat{q}_{h,\min}, \hat{q}_{l,\min}\right].$$

Then we can express the value function as

$$Y^h\left(\hat{q}\right) = \left\{ \begin{array}{l} \frac{K_2\hat{q}^{\varepsilon-1}}{K_1+\varepsilon-1} + \frac{K_3\hat{q}^{1-\frac{\Psi+g}{g}}}{K_1+1-\frac{\Psi+g}{g}} - \frac{K_4}{K_1} \\[4mm] + \left[-\frac{K_2\hat{q}_{h,\min}^{\varepsilon-1}}{K_1+\varepsilon-1} - \frac{K_3\hat{q}_{h,\min}^{1-\frac{\Psi+g}{g}}}{K_1+1-\frac{\Psi+g}{g}} + \frac{K_4}{K_1} \right]\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{K_1} \end{array} \right\} = \begin{array}{l} \frac{K_2\hat{q}^{\varepsilon-1}}{K_1+\varepsilon-1}\left[1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{K_1+\varepsilon-1}\right] \\[4mm] +\frac{K_3\hat{q}^{-\frac{\Psi}{g}}}{K_1-\frac{\Psi}{g}}\left[1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{K_1-\frac{\Psi}{g}}\right] \\[4mm] -\frac{K_4}{K_1}\left[1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{K_1}\right] \end{array}.$$

Then from (C.4), we have that for $\hat{q} \in \left[\hat{q}_{h,\min}, \hat{q}_{l,\min}\right]$,

$$Y^h\left(\hat{q}\right) = \frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+\nu+(\varepsilon-1)g}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}\right) + \frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu}{g}}\right).$$

Intuitively, because product lines with relative quality $\hat{q} \in \left[\hat{q}_{h,\min}, \hat{q}_{l,\min}\right]$ immediately become obsolete when operated by low-type firms, but not by high-type firms, the flow rate of transitioning from high-type to low-type, ν, becomes part of the effective discount rate in this range.

For $\hat{q} \geq \hat{q}_{l,\min}$, the appropriate values for K's from (C.3) delivers (C.5) as

$$Y^h\left(\hat{q}\right) = \frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+(\varepsilon-1)g}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi+(\varepsilon-1)g}{g}}\right) + \frac{\Omega^l-\tilde{w}^s\phi}{\Psi}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi}{g}}\right)$$
$$+ \frac{\Omega^h-\Omega^l}{\Psi+\nu} + D\hat{q}^{-\frac{\Psi+\nu}{g}}$$

We also have the boundary condition

$$Y^h\left(\hat{q}_{l,\min}\right) = \frac{\Pi\hat{q}_{l,\min}^{\varepsilon-1}}{\Psi+\nu+(\varepsilon-1)g}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}_{l,\min}}\right)^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}\right)+\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}_{l,\min}}\right)^{\frac{\Psi+\nu}{g}}\right).$$

$$\text{(C.6)}$$

Hence, the constant of integration for $\hat{q}\geq\hat{q}_{l,\min}$ must satisfy $(C.6)$. Next using $(C.3)$ and $(C.5)$, $Y^h\left(\hat{q}_{l,\min}\right)$ for $\hat{q}\geq\hat{q}_{l,\min}$ can be computed as

$$
\begin{aligned}
Y^h\left(\hat{q}_{l,\min}\right) &= \frac{K_2\hat{q}_{l,\min}^{\varepsilon-1}}{K_1+\varepsilon-1}+\frac{K_3\hat{q}_{l,\min}^{1-\frac{\Psi+g}{g}}}{K_1+1-\frac{\Psi+g}{g}}-\frac{K_4}{K_1}+D\hat{q}_{l,\min}^{-K_1} \\
&= \frac{(\Pi+\nu\zeta_4)\hat{q}_{l,\min}^{\varepsilon-1}}{\Psi+\nu+g(\varepsilon-1)}+\zeta_5\hat{q}_{l,\min}^{-\frac{\Psi}{g}}-\frac{\nu\zeta_6+\tilde{w}^s\phi-\Omega^h}{\Psi+\nu}+D\hat{q}_{l,\min}^{-\frac{\Psi+\nu}{g}}, \quad \text{(C.7)}
\end{aligned}
$$

which must be equal to $(C.6)$. Equating $(C.6)$ to $(C.7)$, we get

$$
D = \left\{
\begin{array}{c}
-\frac{\Pi}{\Psi+\nu+(\varepsilon-1)g}\hat{q}_{h,\min}^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}+\frac{\tilde{w}^s\phi-\Omega^h}{\Psi+\nu}\hat{q}_{h,\min}^{\frac{\Psi+\nu}{g}} \\
-\frac{\nu\zeta_4}{\Psi+\nu+g(\varepsilon-1)}\hat{q}_{l,\min}^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}-\zeta_5\hat{q}_{l,\min}^{\frac{\nu}{g}}+\frac{\nu\zeta_6}{\Psi+\nu}\hat{q}_{l,\min}^{\frac{\Psi+\nu}{g}}
\end{array}
\right\}.
$$

Hence

$$
\hat{q}^{-\frac{\Psi+\nu}{g}}D = \left\{
\begin{array}{c}
\frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+\nu+g(\varepsilon-1)}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}\right)-\frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+\nu+g(\varepsilon-1)}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}\right) \\
+\frac{\Omega^l-\Omega^h}{\Psi+\nu} \\
-\frac{\tilde{w}^s\phi-\Omega^h}{\Psi+\nu}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu}{g}}\right)+\frac{\tilde{w}^s\phi-\Omega^l}{\Psi+\nu}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu}{g}}\right)
\end{array}
\right\}.
$$

Therefore, for $\hat{q}\geq\hat{q}_{l,\min}$ we have

$$
Y^h\left(\hat{q}\right) = \left\{
\begin{array}{c}
\frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+\nu+g(\varepsilon-1)}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}\right)+\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}\left(1-\left(\frac{\hat{q}_{h,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu}{g}}\right) \\
\frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+(\varepsilon-1)g}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi+(\varepsilon-1)g}{g}}\right)+\frac{\Omega^l-\tilde{w}^s\phi}{\Psi}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi}{g}}\right) \\
-\left(\frac{\Pi\hat{q}^{\varepsilon-1}}{\Psi+\nu+g(\varepsilon-1)}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}}\right)+\frac{\Omega^l-\tilde{w}^s\phi}{\Psi+\nu}\left(1-\left(\frac{\hat{q}_{l,\min}}{\hat{q}}\right)^{\frac{\Psi+\nu}{g}}\right)\right)
\end{array}
\right\}.
$$

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Finally, we need to determine the values for the exit thresholds $\hat{q}_{l,\min}$ and $\hat{q}_{h,\min}$. Using the above differential equations we get

$$\left. \frac{\partial \Upsilon^l(\hat{q})}{\partial t} \right|_{\hat{q}=\hat{q}_{l,\min}} = \frac{1}{g} \left(\Pi \hat{q}_{l,\min}^{\varepsilon-2} + \frac{\Omega^l - \tilde{w}^s \phi}{\hat{q}_{l,\min}} \right).$$

From the smooth-pasting condition we get

$$\left. \frac{\partial \Upsilon^l(\hat{q})}{\partial \hat{q}} \right|_{\hat{q}=\hat{q}_{l,\min}} = 0 \implies \hat{q}_{l,\min} = \left(\frac{\tilde{w}^s \phi - \Omega^l}{\Pi} \right)^{\frac{1}{\varepsilon-1}}.$$

Similarly, we also have

$$\left. \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}} \right|_{\hat{q}=\hat{q}_{h,\min}} = \frac{\Pi}{\Psi + \nu + (\varepsilon-1)g} \left((\varepsilon-1) \hat{q}^{\varepsilon-2} + \frac{\Psi + \nu}{g} \hat{q}_{h,\min}^{\frac{\Psi+\nu+(\varepsilon-1)g}{g}} \hat{q}^{-\frac{\Psi+\nu}{g}-1} \right)$$
$$- \frac{\tilde{w}^s \phi - \Omega^h}{g} \hat{q}_{h,\min}^{\frac{\Psi+\nu}{g}} \hat{q}^{-\frac{\Psi+\nu}{g}-1}$$

and $\left. \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}} \right|_{\hat{q}=\hat{q}_{h,\min}} = 0$ implies

$$\hat{q}_{h,\min} = \left(\frac{\tilde{w}^s \phi - \Omega^h}{\Pi} \right)^{\frac{1}{\varepsilon-1}}.$$

□

Lemma 4. *Let F denote the overall relative productivity distribution, including both active and inactive product lines. In stationary equilibrium, it satisfies the following differential equation:*

$$g\hat{q} f(\hat{q}) = \tau \left[F(\hat{q}) - F(\hat{q} - \lambda \bar{\hat{q}}) \right],$$

where $\tau = \Phi^h x^h + \Phi^l x^l + x^{entry}$ and $\bar{\hat{q}} = \int_0^\infty \hat{q} f(\hat{q}) \, d\hat{q}$. Moreover let $\tilde{F}_k$ denote the (unnormalized) distribution of relative productivities of active product lines, owned by type $k \in \{h, l\}$.

In stationary equilibrium, they satisfy

$$g\hat{q}\tilde{f}_h(\hat{q}) = g\hat{q}_{h,\min}\tilde{f}_h(\hat{q}_{h,\min}) + \left(\tau^l + \varphi + \nu\right)\tilde{F}_h(\hat{q}) - \tau^h\left[F(\hat{q} - \lambda\bar{\hat{q}}) - F(\hat{q}_{h,\min} - \lambda\bar{\hat{q}}) - \tilde{F}_h(\hat{q})\right]$$

$$g\hat{q}\tilde{f}_l(\hat{q}) = g\hat{q}_{l,\min}\tilde{f}_l(\hat{q}_{l,\min}) + \left(\tau^h + \varphi\right)\tilde{F}_l(\hat{q}) - \tau^l\left[F(\hat{q} - \lambda\bar{\hat{q}}) - F(\hat{q}_{l,\min} - \lambda\bar{\hat{q}}) - \tilde{F}_l(\hat{q})\right]$$

$$- \nu\left[\tilde{F}_h(\hat{q}) - \tilde{F}_h(\hat{q}_{l,\min})\right],$$

where $\tau^l = \Phi^l x^l + (1 - \alpha)x^{entry}$ and $\tau^h = \Phi^h x^h + \alpha x^{entry}$. The measure of active product lines are given by

$$\Phi^k = \tilde{F}_k(\infty),\ k \in \{h, l\}.$$

Proof of Lemma 4. In a stationary equilibrium inflows and outflows into different parts of the distributions have to be equal. First consider overall productivity distribution F. Given a time interval of Δt, this implies that $F_t(\hat{q}) = F_{t+\Delta t}(\hat{q})$,

$$F_t(\hat{q}) = F_t(\hat{q}(1 + g\Delta t)) - \tau\Delta t\left[F_t(\hat{q}) - F_t(\hat{q} - \lambda\bar{\hat{q}})\right]$$

Next, subtract $F_t(\hat{q}(1 + g\Delta t))$ from both sides, multiply both sides by -1, divide again sides by Δt, and take the limit as $\Delta t \to 0$, so that

$$\lim_{\Delta t \to 0}\frac{F(\hat{q}(1 + g\Delta t)) - F(\hat{q})}{\Delta t} = g\hat{q}f(\hat{q}).$$

Using this last expression delivers

$$g\hat{q}f(\hat{q}) = \tau\left[F(\hat{q}) - F(\hat{q} - \lambda\bar{\hat{q}})\right].$$

Similarly, for active product line distributions $\tilde{F}_k$, we can write

$$\tilde{F}_{h,t}(\hat{q}) = \tilde{F}_{h,t}(\hat{q}(1+g\Delta t)) - \tilde{F}_{h,t}(\hat{q}_{h,\min}(1+g\Delta t))$$
$$+ \tau^h \Delta t \left[F_t(\hat{q} - \lambda\bar{\hat{q}}) - \tilde{F}_{h,t}(\hat{q}) - F_t(\hat{q}_{h,\min} - \lambda\bar{\hat{q}}) \right]$$
$$- \left(\tau^l + \varphi + \nu \right) \Delta t \tilde{F}_{h,t}(\hat{q})$$
$$\tilde{F}_{l,t}(\hat{q}) = \tilde{F}_{l,t}(\hat{q}(1+g\Delta t)) - \tilde{F}_{l,t}(\hat{q}_{l,\min}(1+g\Delta t))$$
$$+ \tau^l \Delta t \left[F_t(\hat{q} - \lambda\bar{\hat{q}}) - \tilde{F}_{l,t}(\hat{q}) - F_t(\hat{q}_{l,\min} - \lambda\bar{\hat{q}}) \right]$$
$$- \left(\tau^h + \varphi \right) \Delta t \tilde{F}_{l,t}(\hat{q}) + \nu\Delta t \left[\tilde{F}_{h,t}(\hat{q}) - \tilde{F}_{h,t}(\hat{q}_{l,\min}) \right].$$

Again, by subtracting $\tilde{F}_{k,t}\left(\hat{q}\,(1+g\Delta t) \right) - \tilde{F}_{k,t}(\hat{q}_{k,\min}(1+g\Delta t))$ from both sides, dividing by $-\Delta t$, and taking the limit as $\Delta t \to 0$, we get the desired equations for $k \in \{h, l\}$ in Lemma 4. □

Proof of Proposition 2. As shown in Lemma 4, overall productivity distribution satisfies

$$\hat{q}f(\hat{q}) = \frac{\tau}{g} \left[F(\hat{q}) - F\left(\hat{q} - \lambda\bar{\hat{q}} \right) \right]$$

By integrating both sides over the domain, we get

$$\mathbb{E}(\hat{q}) \equiv \int_0^\infty \hat{q}f(\hat{q})d\hat{q} = \frac{\tau}{g} \int_0^\infty \left[F(\hat{q}) - F\left(\hat{q} - \lambda\bar{\hat{q}} \right) \right] d\hat{q}$$

We can write above equation as follows

$$\mathbb{E}(\hat{q}) = \frac{\frac{\tau}{g}}{1 + \frac{\tau}{g}} \int_0^\infty \left[1 - F\left(\hat{q} - \lambda\bar{\hat{q}} \right) \right] d\hat{q}.$$

as $\int_0^\infty \left[1 - F(\hat{q}) \right] d\hat{q} = \mathbb{E}(\hat{q})$.

By changing of variable as $x = \hat{q} - \lambda\bar{\hat{q}}$, which implies $dx = d\hat{q}$, we have

$$\mathbb{E}(\hat{q}) = \frac{\frac{\tau}{g}}{1 + \frac{\tau}{g}} \int_{-\lambda\bar{\hat{q}}}^{\infty} [1 - F(x)] \, dx = \frac{\tau}{g}\lambda\bar{\hat{q}}$$

Last equality follows from the fact that $F(x) = 0$ for $x \leq 0$. In equilibrium we have, $\bar{\hat{q}} = \mathbb{E}(\hat{q})$. Therefore

$$g = \tau\lambda.$$

$\square$

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