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Essays In Asset Pricing

Yicheng Zhu
University of Pennsylvania

Essays In Asset Pricing

Abstract

In the first chapter, ``A Unified Theory of the Term Structure and the Beta Anomaly'', I propose a novel generalized framework which allows for disentangling agent's risk aversion, elasticity of intertemporal substitution, and the agent's preference for early or late resolution of uncertainty. I apply this framework to a consumption-based asset pricing model in which the representative agent's consumption process is subject to rare but large disasters. The calibrated model matches major asset pricing moments, while higher exposure to systematic risks may lead to lower risk premia. This is consistent with empirical finding, while existing consumption-based asset pricing models fail to deliver.

The second chapter, ``A Model of Two Days: Discrete News and Asset Prices'', co-authored with Jessica A. Wachter, provides a quantitative model to address the macro-announcement premium. Empirical studies demonstrate striking patterns in stock returns related to scheduled macroeconomic announcements. A large proportion of the total equity premium is realized on days with macroeconomic announcements. The relation between market betas and expected returns is far stronger on announcement days as compared with non-announcement days. Finally, these results hold for fixed-income investments as well as for stocks. We present a model in which agents learn the probability of an adverse economic state on announcement days. We show that the model quantitatively accounts for the empirical findings. Evidence from options data provides support for the model's mechanism.

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Supervisor of Dissertation

_____

Jessica A. Wachter, Richard B. Worley Professor of Financial Management

Graduate Group Chairperson

_____

Nancy Zhang, Ge Li and Ning Zhao Professor, Professor of Statistics

Dissertation Committee

Jules H. van Binsbergen, The Nippon Life Professor in Finance

Winston Wei Dou, Assistant Professor of Finance

Itamar Drechsler, Associate Professor of Finance

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Yicheng Zhu

ABSTRACT

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Jessica A. Wachter

In the first chapter, "A Unified Theory of the Term Structure and the Beta Anomaly", I propose a novel generalized framework which allows for disentangling agent's risk aversion, elasticity of intertemporal substitution, and the agent's preference for early or late resolution of uncertainty. I apply this framework to a consumption-based asset pricing model in which the representative agent's consumption process is subject to rare but large disasters. The calibrated model matches major asset pricing moments, while higher exposure to systematic risks may lead to lower risk premia. This is consistent with empirical finding, while existing consumption-based asset pricing models fail to deliver.

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CHAPTER 1 : A Unified Theory of the Term Structure and the Beta Anomaly

1.1　Introduction

The trade-off between risk and return is at the center of asset pricing research. It is commonly believed that assets with large and positive exposure to systematic risks should carry a higher risk premium. It is thus important to find what these systematic risks are. The capital asset pricing model (CAPM) by Sharpe (1964) and Lintner (1965)—one of the seminal works on asset pricing—concludes that the market portfolio should serve as a proxy of systematic risks. However, the CAPM has since been refuted by various empirical works. Specifically, Black et al. (1972) show that the security market line—which describes the connection between risk premium and exposure to the market portfolio—is too flat compared to the benchmark predicted by the CAPM.

Additionally, a large body of macro-finance literature tries to connect risks in the financial market to aggregate economic fundamentals. Leading models include the external habit model by Campbell and Cochrane (1999), long-run risk model by Bansal and Yaron (2004), and disaster risk model by Barro and Ursúa (2009) and Wachter (2013). These models try to jointly explain equity premium, risk-free rate, and observed market volatility.

One important framework for these models is the recursive utility introduced by Epstein and Zin (1991) and Weil (1990). The utility specification allows researchers to characterize the agent's risk aversion and elasticity of intertemporal substitution (EIS) separately, while standard CRRA expected utility imposes that production of the two quantities should be one. It turns out this separation is critical for jointly explaining the equity premium and riskfree rate puzzles. Specifically, the representative agent is allowed to feature a high risk aversion and a high EIS, and thus in equilibrium charges high equity premium and a low riskfree rate in models.

While disentangling the agent's risk aversion and elasticity of intertemporal substitution,

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the Epstein-Zin-Weil recursive utility introduces the agent's preference for early or late resolution of uncertainty. In the framework of Epstein-Zin-Weil, such preference is perfectly determined by the agent's risk aversion and EIS. In most existing models, it is automatically implied that the agent has a preference for early resolution of uncertainty, meaning that the agent would want to know everything about his life as early as possible.

However, such implication has been widely challenged, especially in the space of term structure. Figure 1.3 plots the implied yield curve at the steady state from one of the leading models mentioned. It is clear that the implied real yield curve is downward sloping. Meanwhile, Figure 1.1 depicts a time series of the US Treasury Inflation-Protected Securities (TIPS) yield curve from January 2013 to December 2018, while Figure 1.2 shows a time series of the difference between 10-year and 5-year TIPS yields. Both figures clearly show that the real yield curve should be upward-sloping. It turns out that, a preference for late resolution of uncertainty is necessary for a general equilibrium model to feature an upward-sloping real yield curve.

There are other indirect evidences. Papanikolaou (2011) studies the role of shocks to investment opportunities, and uses the difference in the risk premium of consumption-good and investment-good production firms to identify the price of risk for shocks to investment opportunities. The price of risk if negative, which also implies that the agent has a preference for late resolution of uncertainty.

For those models, they feature shocks to the future distribution of consumption growth, or news shocks as sometimes called by macro economists. Long-maturity zero-coupon bonds provide hedges against such shocks, while short-maturity zero-coupon bonds do not. When the agent has a preference for early resolution of uncertainty, the demand for long-maturity bonds is higher—resulting in higher prices and lower yields. In fact, the slope of the yield curve implies that the agent should have a preference for late resolution of uncertainty in existing consumption-based asset pricing models.

In short, it appears that although the existing literature requires the agent to have a preference for early resolution of uncertainty, while evidences, especially in cross-section, suggest the opposite.

A conventional way of tackling such puzzles with a theoretical model is to design a model with a more complicated consumption process or incomplete market. However, such models should feature a higher risk in the short run and contradict the intuition of long-run risk or disaster models.

In this article, I aim to tackle these puzzles by focusing on the agent's preference while working with a standard friction-less consumption model. The model features a representative agent, a straightforward aggregate consumption process, and a unified explanation for the puzzles being addressed. The agent's preference for early or late resolution of uncertainty is disentangled from his risk aversion and EIS, allowing potentially addressing the existing puzzles in a unified framework.

The main innovation of this paper is the Generalized Recursive Utility model, which is a natural extension of the popular Epstein-Zin-Weil recursive utility model. The generalized recursive utility allows the agent to show various degrees of aversion toward different types of shocks, and this is achieved by permitting the agent to process different shocks recursively. Empirically, various experiments document that agents' risk aversion changes when they face different types of shocks (Holt and Laury (2002), Coble and Lusk (2010)). The generalization also disconnects the agent's preference for early or late resolution of uncertainty from the agent's risk aversion and elasticity of intertemporal substitution, and such disconnection has critical asset pricing implication.

One implication of this model is that the marginal distribution of continuation value no longer serves as a sufficient statistic for its certainty equivalence. Such a characteristic implies that the agent is not an expected utility agent within each period. Specifically, the agent violates the independence axiom enforced by von Neumann and Morgenstern (1944).

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However, empirical evidence presented by Allais (1953) suggests that the independence axiom has been commonly violated.

A common criticism of the generalization is that researchers lack discipline. I provide a solution to this by categorizing the shocks into two classes: shocks to instantaneous consumption growth and shocks to the long-run future consumption growth. The latter is termed "news shocks" or "information" in the literature on macroeconomics and finance. Grant et al. (1998) demonstrate that such distinction can help characterize agents' intrinsic preference for information.

As for asset pricing implication, the preference for early or late resolution of uncertainty no longer determines the risk-pricing of shocks. Through generalized recursive utility, I illustrate that an agent may prefer the existence of early information, but need not want the information immediately. This feature contradicts the prediction of Epstein-Zin-Weil recursive utility and is depicted as a determinant of the prices of risks for long-run future shocks. Such disconnection then allows the model to feature a high equity premium, low risk-free rate, and solve the puzzles raised before.

The Generalized Recursive Utility has the Epstein-Zin-Weil recursive utility and the constant relative risk aversion (CRRA) expected utility as special cases, while featuring tractability and usefulness. Following Hansen et al. (2008) and Ju and Miao (2012), I incorporate a homothetic utility specification and demarcate a clear representation of the pricing kernel. I also illustrate that with specific parametric restrictions, the generalization allows for a closed-form solution, which facilitates analysis and insights regarding the economic mechanism.

After formally defining generalized recursive utility, I apply it on an aggregate consumption process which is subject to rare but large disasters. The disaster probability is time varying, and shocks to the probability are considered as long-run future shocks. The model can be solved in closed form, thus facilitating the identification of elements necessary for explaining

the puzzles described. Following that, I calibrate the model and show that the model can generate reasonable quantitative moments.

In equilibrium, there are three distinct priced risks: shocks to consumption growth, disaster, and shocks to disaster probability. The disaster is modeled to be conditionally independent, which implies that assets with different maturities should have identical exposure to disasters. The maturity of a zero-coupon bond is proportional to the expected number of disasters realized, meaning that long-maturity bonds have higher positive loading on disaster probability. In equilibrium, the shocks to disasters carry a negative price of risk, and long-maturity zero-coupon bonds have higher risk premia.

I also show that this paper can address various other asset pricing puzzles in the equity market. Binsbergen et al. (2012) illustrate that claims to long-maturity dividends—which are potentially more exposed to systematic risk—carry lower risk premia. Weber (2018) demonstrates that risk premia decreases with assets' cash flow duration. Similarly, Giglio et al. (2015) deploy evidence from the real estate market to show that the agent's discount rate for risky cash flows decreases with respect to the cash flows' maturities. As high-beta firms and long-maturity dividends are more exposed to variation in disaster risk, similar intuition can be applied and address why long-maturity risky claims could have lower risk premia.

I also show that the model can address the challenges to pricing the claims of volatility. Bollerslev et al. (2009) documents that exposure to short-term realized volatility carries negative risk premia. This was further elucidated by Drechsler (2013) and Drechsler and Yaron (2011). In their models, volatility is counter-cyclical and exposure to volatility facilitates hedging, and thus carries a negative risk premium. However, Dew-Becker et al. (2017) show that, while short-term realized volatility contracts carry a negative premium, volatility forward contracts with longer maturity carry a weak positive risk premium. In the model, variation in expected volatility is driven by shocks to disaster probability, while realized volatility provides additional insurance against disaster realization. This addresses

the fundamental difference between the realized volatility and volatility forward contracts, and qualitatively explains why a long position in future expected volatility carries a positive Sharpe Ratio.

This paper is closely related to that of Andries et al. (2018). They specify a recursive utility specification, which allows the agent to show lower risk aversion inspired by future shocks. However, when the shocks actually hit, the agent's risk aversion is high, rendering it time inconsistent. I focus on shocks being realized in the present but affecting the long-run future and provide a time-consistent framework. Ju and Miao (2012) consider a recursive utility specification that allows smooth aversions from uncertainty with different natures. However, I focus on uncertainty led by physical shocks only. Skiadas (2013) points out that the smooth ambiguity aversion would quantitatively diminish in the continuous-time limit, but the model I present would survive such criticism.

In what follows, I will first develop the theory of the Generalized Recursive Utility. Next, I will demonstrate that the Generalized Recursive Utility disconnects the agent's risk aversion, elasticity of intertemporal substitution (EIS), and preference for early (late) resolution of uncertainty. Following that, I will present a representative-agent consumption-based asset pricing model, with time-varying rare disaster probability. I provide a closed-form solution to the asset-pricing moments and analyze the asset-pricing properties of the model. Finally, I will calibrate the model and show that the model can successfully match macro-finance asset pricing moments, including the equity premium, risk-free rates, the slope of real yield curves, and other cross-sectional moments.

1.2 The Generalized Recursive Utility

This section constructs the Generalized Recursive Utility in an endowment economy. Similar arguments can be also applied to a production economy.

Since the Generalized Recursive Utility is a natural extension of the Epstein-Zin-Weil recursive utility, I start with a quick review of the construction of the Epstein-Zin-Weil utility. Later it is shown that the construction of the Generalized Recursive Utility shares a very similar spirit.

1.2.1 From the CRRA Expected Utility to the Generalized Recursive Utility

The Epstein-Zin-Weil Utility

Consider the CRRA expected utility specification:

$$V_t = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} f_\gamma(C_s) \right], \tag{1.1}$$

where C_t is the agent's consumption at time t, β is the agent's subjective time discount rate, and $f_\gamma(C_t)$ is the von Neumann-Morgenstern utility function, with

$$f_\gamma(x) \equiv \frac{1}{1-\gamma} x^{1-\gamma},$$

where γ is the agent's risk aversion.

Let $\mathcal{F}_t$ be the filtration process that represents the agent's information. The Law of Iterated

Expectations implies

$$V_t = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} f_\gamma(C_s) \right]$$

$$= f_\gamma(C_t) + \beta E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-(t+1)} f_\gamma(C_s) \right]$$

$$= f_\gamma(C_t) + \beta E_t \left[E_{t+1} \left[\sum_{s=t+1}^{\infty} \beta^{s-(t+1)} f_\gamma(C_s) \right] \right]$$

$$= f_\gamma(C_t) + \beta E[V_{t+1}|\mathcal{F}_t], \tag{1.2}$$

which is the well-known recursive form of the CRRA expected utility.

Define

$$U_t \equiv f_\gamma^{-1} \left((1-\beta)V_t \right). \tag{1.3}$$

As the operator $f_\gamma^{-1}((1-\beta)\cdot)$ is strictly increasing, V_t and U_t represent the same preferences. Combining (1.2) and (1.3) yields the recursive equation that characterizes U_t:

$$U_t = f_\gamma^{-1} \left((1-\beta)f_\gamma(C_t) + \beta E\left[f_\gamma(U_{t+1})|\mathcal{F}_t \right] \right). \tag{1.4}$$

We can further apply an operator $f_\gamma \left(f_\gamma^{-1}(\cdot) \right)$ on the expectation term in Equation 1.4. The operator is the identity transformation, so the equation still holds:

$$U_t = f_\gamma^{-1} \left((1-\beta)f_\gamma(C_t) + \beta f_\gamma \left(f_\gamma^{-1} \left(E\left[f_\gamma(U_{t+1})|\mathcal{F}_t \right] \right) \right) \right). \tag{1.5}$$

Define $\nu(U; u(\cdot), \Omega)$ as

$$\nu(U; u(\cdot), \Omega) \equiv u^{-1} \left(E\left[u(U)|\Omega \right] \right), \tag{1.6}$$

where U is a random variable, Ω is a σ-algebra or σ-field, and $u(\cdot)$ is an increasing function.

Equation 1.5 then can be rewritten as

$$\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t) = f_\gamma^{-1}\big(E\left[f_\gamma(U_{t+1})|\mathcal{F}_t\right]\big) \tag{1.7}$$

$$U_t = f_\gamma^{-1}\bigg((1-\beta)f_\gamma(C_t) + \beta f_\gamma\big(\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t)\big)\bigg). \tag{1.8}$$

Equations 1.7 and 1.8 show that the recursive computation of the CRRA expected utility each period can be decomposed into two steps. First, the agent computes $\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t)$, and second the agent aggregates $\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t)$ with C_t using a Constant Elasticity of Substitution (CES) aggregator.

U_t is homogeneous of degree 1. As a result, we can interpret a drop of 1% in the utility level as the effect of a 1% drop in all future consumption levels.

The decomposition allows for distinguishing two different economic mechanisms. If U_{t+1} is constant given $\mathcal{F}_t$, $\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t)$ yields U_{t+1}; the value is lower than $E[U_{t+1}|\mathcal{F}_t]$ if U_{t+1} is random and $f_\gamma(\cdot)$ is concave. In fact, this operator computes the continuation value the agent is indifferent to if there is no uncertainty for the next period, and captures the agent's risk aversion. Indeed, this operator is called the Certainty Equivalence (CE) operator.

The second step, however, aggregates the contemporaneous consumption C_t and the certainty equivalence. This step connects quantities from two periods and characterizes how the agent makes choices intertemporally.

One key observation is that the two steps both involve the concave functions with the same concavity parameter γ. However, they stand for two different economic mechanisms. We can relax the restriction by introducing ψ to the intertemporal aggregation step, and get

$$\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t) = f_\gamma^{-1}\big(E\left[f_\gamma(U_{t+1})|\mathcal{F}_t\right]\big). \tag{1.9}$$

$$U_t = f_{1/\psi}^{-1}\bigg((1-\beta)f_{1/\psi}(C_t) + \beta f_{1/\psi}\big(\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t)\big)\bigg) \tag{1.10}$$

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The aggregator function in (1.10) is now a CES aggregator with an elasticity of substitution ψ, and an agent represented by the utility specification above shows an EIS of ψ. Also, the certainty equivalence employs a concavity parameter γ, and the agent would show a relative risk aversion γ. The change disconnects the EIS and risk aversion and leads to the recursive utility proposed by Epstein and Zin (1991) and Weil (1990).

The Idea of the Generalized Recursive Utility

In what follows, I first show that the certainty equivalence operator implies a restriction that the agent would have the same risk aversion toward shocks of different types. I then show that the restriction can be relaxed in a manner similar to the development of Epstein-Zin-Weil utility. This procedure leads to the Generalized Recursive Utility.

Consider an endowment economy. Define $\mathcal{G}_{t+1}$ as

$$\mathcal{G}_{t+1} \equiv \sigma(\mathcal{F}_t, \{C_{t+1}\}), \tag{1.11}$$

where $\sigma(\Omega, \omega)$ is an operator which generates the smallest σ-field such that

$$\Omega \subseteq \sigma(\Omega, \omega)$$
$$\omega \in \sigma(\Omega, \omega).$$

It follows that
$$\mathcal{F}_t \subseteq \mathcal{G}_{t+1} \subseteq \mathcal{F}_{t+1}.$$

As a result, $\mathcal{G}_{t+1}$ decomposes the uncertainty resolved from time t to $t+1$ into two parts. The first part, which is between $\mathcal{F}_t$ and $\mathcal{G}_{t+1}$, is about the uncertainty of instantaneous consumption growth – the uncertainty about the short-run future. The second part captures the uncertainty resolved for future consumption growth, the risks appearing as 'information' for the long-run future.

The certainty equivalence operator satisfies

$$
\begin{aligned}
\nu(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t) &= f_\gamma^{-1}\big(E\left[f_\gamma(U_{t+1})|\mathcal{F}_t\right]\big) \\
&= f_\gamma^{-1}\Big(E\big[E\left[f_\gamma(U_{t+1})|\mathcal{G}_{t+1}\right]\big|\mathcal{F}_t\big]\Big) \\
&= f_\gamma^{-1}\Big(E\big[f_\gamma\big(f_\gamma^{-1}\big(E\left[f_\gamma(U_{t+1})|\mathcal{G}_{t+1}\right]\big)\big)\big|\mathcal{F}_t\big]\Big).
\end{aligned}
\tag{1.12}
$$

The second equality holds due to the Law of Iterated Expectations. The third equality holds as the operator $f_\gamma\left(f_\gamma^{-1}(\cdot)\right)$ is an identity transformation.

The calculation above suggests that the computation of certainty equivalence can again be understood as a two-step procedure. The first step, $f_\gamma^{-1}\big(E\left[f_\gamma(U_{t+1})|\mathcal{G}_{t+1}\right]\big)$, computes the certainty equivalence for the agent conditioning on the information set $\mathcal{G}_{t+1}$. This step reflects how the agent would avoid uncertainty conditioning on C_{t+1}, or the uncertainty about the long-run future. The second step captures the uncertainty that drives variation in $\mathcal{G}_{t+1}$, or the uncertainty of instantaneous consumption growth. Again, the two steps facilitate the same concave function $f_\gamma(\cdot)$. This restriction implies that the agent shows the same level of aversion toward both types of shocks.

The restriction that the agent has same levels of risk aversion toward two types shocks can be relaxed by introducing η to the first step of the certainty equivalence computation. The change then allows the agent to show distinct levels of aversion to the two types of uncertainty. Clearly, the generalization has the Epstein-Zin-Weil certainty equivalence operator as a special case. As a result, I call this the Generalized Certainty Equivalence Operator, and the recursive utility function that facilitates this is called the Generalized Recursive Utility.

1.2.2 Formal Definition

In what follows, I give a formal definition of the Generalized Recursive Utility.

Definition 1.2.1 (The Generalized Recursive Utility). *Let $\{\mathcal{F}_t\}_{t=0,1,2,\ldots}$ be the filtration*

process which represents the information flow to the agent . In addition, the agent has an intermediate σ-field process: $\mathcal{G}_{t+1}$, such that

$$\mathcal{G}_{t+1} = \sigma(\mathcal{F}_t, \{C_{t+1}\}). \tag{1.13}$$

Then the Generalized Recursive Utility satisfies the recursion

$$U_t = f_{1/\psi}^{-1}\left((1-\beta)f_{1/\psi}(C_t) + \beta f_{1/\psi}\left(\nu\left(\nu(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1}); f_\gamma(\cdot), \mathcal{F}_t\right)\right)\right) \tag{1.14}$$

$$\nu\left(\nu(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1}); f_\gamma(\cdot), \mathcal{F}_t\right) = f_\gamma^{-1}\left(E\left[f_\gamma\left(\nu(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1})\right)\middle|\mathcal{F}_t\right]\right) \tag{1.15}$$

$$\nu(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1}) = f_\eta^{-1}\left(E\left[f_\eta(U_{t+1})|\mathcal{G}_{t+1}\right]\right), \tag{1.16}$$

where

$$f_\alpha(x) = \frac{1}{1-\alpha}x^{1-\alpha}, \ \alpha \neq 1 \tag{1.17}$$

$$f_\alpha(x) = \log(x), \ \alpha = 1, \tag{1.18}$$

for $\alpha = \eta, \gamma$ and $1/\psi$.

Remark 1.2.1. If the consumption growth process is i.i.d, the Generalized Recursive Utility is observationally equivalent to the Epstein-Zin-Weil Recursive Utility.

If the consumption growth is i.i.d, $\mathcal{F}_{t+1}$ only contains information from $\mathcal{F}_t$ and about C_{t+1}. As a result, $\mathcal{G}_{t+1} = \mathcal{F}_{t+1}$, and

$$\nu\left(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1}\right) = \nu\left(U_{t+1}; f_\eta(\cdot), \mathcal{F}_{t+1}\right) = U_{t+1}.$$

As a result, the recursion reduces to

$$U_t = f_{1/\psi}^{-1}\left((1-\beta)f_{1/\psi}(C_t) + \beta f_{1/\psi}\left(\nu\left(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t\right)\right)\right)$$
$$\nu\left(U_{t+1}; f_\gamma(\cdot), \mathcal{F}_t\right) = f_\gamma^{-1}E\left[f_\gamma(U_{t+1})\middle|\mathcal{F}_t\right], \tag{1.19}$$

which is the Epstein-Zin-Weil recursive utility. □

1.2.3 Connection between the Generalized Recursive Utility and the Epstein-Zin-Weil Utility

It is straightforward to show that the Generalized Recursive Utility nests the Epstein-Zin-Weil recursive utility.

Remark 1.2.2. If $\eta = \gamma$, the Generalized Recursive Utility reduces to the Epstein-Zin-Weil recursive utility. □

Common questions for the generalization of the utility function include what assumptions of the existing literature are violated and what feature of human behavior can motivate such violation.

For the Generalized Recursive Utility, the most significant part is that the agent is no longer an expected utility agent *each period*. This deviates from the framework of Kreps and Porteus (1978) as they assume the agent makes decisions like an expect-utility agent for uncertainty resolved each period. Specifically, the Generalized Recursive Utility agent violates the independence axiom per von Neumann and Morgenstern (1944), as a compounded lottery is not equivalent to the reduced lottery with the same distribution of final payoff. However, the famous Allais Paradox shows that people widely violate the independence axiom.

In addition, Grant et al. (1998) show that variation in risk aversion could be closely connected to people's intrinsic preference for information. They show that if uncertainty is resolved as early information, and the agent prefers that, then the agent's risk aversion toward early information uncertainty must be lower than that toward instantaneous consumption growth risks. Meanwhile, experimental studies by Ahlbrecht and Weber (1996) show that people do not appear to be indifferent toward the existence of the information, which supports the separation suggested by the Generalized Recursive Utility.

1.2.4 The Preference of the Generalized Recursive Utility Agent

This section shows that an Generalized Recursive Utility disconnects risk aversion, elasticity of intertemporal substitution, and the preference for early (late) resolution of uncertainty. The agent could prefer the existence of early information, but a late resolution (arrival) of that. In Section 1.3, I will show that this is critical in addressing the asset pricing puzzles raised before.

Remark 1.2.3. The Elasticity of Intertemporal Substitution is given by ψ. $\qquad\square$

Unsurprisingly, the intertemporal decision of the agent is characterized by the CES intertemporal aggregator (1.14). As a result, the Elasticity of Intertemporal Substitution is given by ψ.

Remark 1.2.4. In a one-period case, the agent is risk averse if and only if $\gamma > 0$. $\qquad\square$

For a one-period lottery, the agent's continuation value is entirely determined by the financial payoff of the lottery. As a result, the first step of the Generalized Recursive Certainty Equivalence computation collapses, and whether the agent is averse to the uncertainty of the lottery is characterized by the concavity of the function $f_\gamma(\cdot)$.

The following remark implies the key difference between the Generalized Recursive Utility and the Epstein-Zin-Weil recursive utility. As the Generalized Recursive Utility distinguishes the shock types, the 'existence of early information' and 'timing of early information' could be considered as two different concepts.

Remark 1.2.5. If $\gamma \geq \eta > 1/\psi$, the agent exhibits a preference for early resolution of uncertainty and would want the uncertainty resolved as early as possible; if $\gamma \geq 1/\psi > \eta$, the agent prefers the existence of information, but prefers later realization of that. $\qquad\square$

This remark can be better illustrated by comparing the following consumption plans.

$$
\begin{array}{ll}
\text{Plan 1} \quad C_0 = 1 \Big<
\begin{array}{l}
C_1 = 1 \text{\textemdash} C_2 = 1 \text{\textemdash} C_3 = 10 \\
C_1 = 1 \text{\textemdash} C_2 = 1 \text{\textemdash} C_3 = 1
\end{array}
\end{array}
$$

$$
\text{Plan 2} \quad C_0 = 1 \text{\textemdash} C_1 = 1 \Big<
\begin{array}{l}
C_2 = 1 \text{\textemdash} C_3 = 10 \\
C_2 = 1 \text{\textemdash} C_3 = 1
\end{array}
$$

$$
\text{Plan 3} \quad C_0 = 1 \text{\textemdash} C_1 = 1 \text{\textemdash} C_2 = 1 \Big<
\begin{array}{l}
C_3 = 10 \\
C_3 = 1
\end{array}
$$
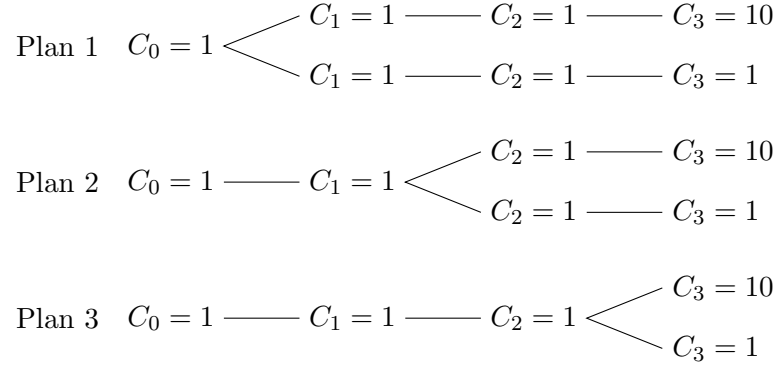
The distributions of consumption in all three plans are identical. The only difference is the time when the agent knows the value of C_3, the only source of uncertainty.

Both plans 2 and 3 feature early information for C_3. However, the timing for the resolution of uncertainty in plan 1 is earlier.

Under the Epstein-Zin-Weil utility, if $\gamma > 1/\psi$, the preference order will be Plan 1 $\succ$ Plan 2 $\succ$ Plan 3, while the agent would strictly prefer Plan 3 to Plan 2 to Plan 1 when $\gamma < 1/\psi$.

Appendix A.1.1 shows that, under the Generalized Recursive Utility, the agent would prefer Plan 2 the most if $\gamma > 1/\psi > \eta$. The fact that $\gamma > \eta$ implies that the agent would have an intrinsic preference of information, and this implies that both Plan 1 and 2 and would be preferred to Plan 3.

The preference order between Plan 1 and Plan 2, however, depends on the net effect between intertemporal substitution and uncertainty aversion. When $\eta < 1/\psi$, compared to uncertainty aversion, the inter-temporal smoothing is so costly that the agent would ex-ante prefer to avoid the inter-temporal smoothing in bad states by having the uncertainty resolved later.

What is the intuition behind such preference? Think about a crystal ball, which would tell all the details about an agent's life, while the agent could do nothing. An Epstein-Zin-Weil agent with a preference for early resolution of uncertainty would love to have the crystal ball. For such an agent, the information should always arrive as early as possible.

An expected utility agent would be indifferent to the crystal ball, while for a Generalized Recursive Utility agent, the best case would be to have the crystal ball chopped into pieces, while the agent is allowed to pick the piece for next period only every time.

1.3 The Term Structure and the Beta Anomaly

In this section, I investigate the asset pricing implications of the Generalized Recursive Utility.

As discussed in the introduction, the upward sloping real yield curve in data contradicts a large number of consumption-based asset pricing models. Additionally, there are puzzles in the equity market, including the beta anomaly, the term structure of equity, and the risk premium of volatility forward contracts.

In what follows, I will show some empirical results regarding the real yield curve, the beta anomaly, and volatility forward contracts. I will provide a discussion on why they are puzzles for existing literature. I then proceed with a consumption-based asset pricing model featuring rare disasters, and show that with the Generalized Recursive Utility, the puzzles can be jointly solved.

1.3.1 Empirical Facts

Data and Methodology

The data used in this paper is mostly collected from CRSP. I collect the monthly returns of all stocks traded in NYSE / AMEX / NASDAQ from January 1961 to December 2017 and exclude companies of utility (SIC code 4900-4949) and financial (SIC code 6000-6799) industries. I use the value-weighted average of NYSE / AMEX / NASDAQ returns as a proxy for the market portfolio return. The 1-month risk-free rates in CRSP Risk-Free Rates File are used as nominal risk-free rates. Real risk-free rates are not directly observable, so I deduct realized inflation rate from nominal risk-free rates to obtain real quantities. The values are not the real risk-free rate but serve well as proxies.

The excess returns of individual stocks and the market are computed as the difference between the monthly returns and the 1-month risk-free rates at the end of the previous month.

With the stock return data, I create ten beta-sorted portfolios. I estimate the stocks' CAPM betas at the end of each month with 60-month rolling windows and create deciles. In order to ensure the accuracy of beta estimates, a stock can be included if and only if there are at least 36 months' monthly returns data available for beta estimation. I then compute the value-weighted average of the stock excess returns in each decile and get the monthly excess returns of the beta-sorted portfolios.

I use TIPS yield as the proxy for the real yield curve. The data is provided by US Department of the Treasury.

The prices of the variance forward contracts are provided by Dew-Becker et al. (2017) and are available on Ian Dew-Becker's website. I assume that the investors enter a forward contract at the beginning of each trading month, and then settle the position one month after. The difference in the forward prices would then be realized upon the maturity of the contracts and can be understood as the excess returns of the investment strategy. I compute the time series of the excess returns for the forward contracts with maturities from 1 to 12 months. Specifically, the contracts with a maturity of 1 month provide the investors with exposure to realized volatility. Then with the time series of excess returns, I compute the Sharpe Ratios for investing in variance forward contracts with different maturities.

Empirical Results

The main results are showed in Figures 1.1, 1.2, 1.4 and 1.5.

From the figures, the following conclusions can be obtained:

- The TIPS yield curve is, for most of the time, upward sloping. With the fact that the nominal yield curve is also upward-sloping for most of the time, there is strong empirical evidence supporting a upward-sloping real yield curve.

- The security market line with beta-sorted portfolios is too flat compared to the CAPM benchmark. As the market is the value-weighted average of the beta-sorted portfolios,

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the security market line can get very close to the market portfolio; also, the intersect of the regression line is large and positive.

- Only exposure to realized volatility provides a significantly negative Sharpe Ratio. Forward contracts to volatility carry tiny negative or even positive Sharpe Ratios.

1.3.2 The Model

In what follows, I present a consumption-based asset pricing model with rare disasters and its solution with the Generalized Recursive Utility.

The Aggregate Consumption Process

There is an infinitely-lived representative agent endowed with the aggregate consumption process $\{C_t\}_t$ given by

$$c_{t+1} - c_t = \mu_C + \sigma_C B_{C,t+1} - \sum_{n=1}^{\Delta N_{t+1}} Z_{n,t+1}. \tag{1.20}$$

Here $c_t \equiv \log C_t$ is the log consumption. $Z_{n,t} > 0$ is an i.i.d process, which captures the distribution of disaster shocks. I use ν to denote the distribution of $Z_{n,t}$. N_t is a counting process, with the conditional distribution of $\Delta N_{t+1} \equiv N_{t+1} - N_t$ being Poisson. The conditional distribution of ΔN_{t+1} is given by $E(\Delta N_{t+1}) = p_t$, where p_t is the time-varying jump intensity process:

$$p_{t+1} - p_t = -\rho_p(p_t - \bar{p}) + \sigma_p \sqrt{p_t} B_{p,t+1}. \tag{1.21}$$

Equation 1.21 does not guarantee that p_t is positive with probability 1. However, with reasonable calibration, we can keep the unconditional probability of $p_t < 0$ negligibly low; the process has the CIR process by Cox et al. (1985), which is bounded above zero, as its continuous-time limit. For simplicity, I assume that $B_{p,t}$ and $B_{C,t}$ are independent. In addition, I assume that $0 < \rho_p < 1$ to guarantee the stationarity of p_t.

The Generalized Recursive Utility represents the representative agent's preference. I assume that $\gamma > 0$ and $\eta \geq 0$, meaning that the agent is not uncertainty loving.

It can be shown that when $\psi = 1$, a closed-form solution to the pricing kernel in the economy can be obtained; when $\psi \neq 1$, a first-order log-linear approximation can be used to compute the kernel. The following theorem provides the pricing kernel of the economy.

Theorem 1.1. *The one-period pricing kernel* $M_{t,t+1}$, *at time-t is approximated by*

$$
M_{t,t+1} \approx \beta e^{-\mu_C/\psi - \frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2 \sigma_p^2 p_t - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 - \frac{1/\psi-\gamma}{1-\gamma} E_\nu \left[e^{(\gamma-1)Z_{n,t+1}} - 1 \right] p_t}
$$

$$
\times\, e^{-\gamma \sigma_C B_{C,t+1} + \gamma \sum\limits_{n=1}^{N_{t+1}} Z_{n,t+1} + (1/\psi-\eta)b_p \sigma_p \sqrt{p_t} B_{p,t+1}}, \tag{1.22}
$$

where b_p *is characterized by following equations:*

$$
a = \frac{1}{1 - 1/\psi} \log \left((1 - \beta) + \beta e^{(1-1/\psi)m} \right) - \beta \frac{e^{(1-1/\psi)m}}{1 - \beta + \beta e^{(1-1/\psi)m}} n\bar{p})
$$

$$
b_p = \beta \frac{e^{(1-1/\psi)m}}{1 - \beta + \beta e^{(1-1/\psi)m}} n
$$

$$
m = a + \mu_C + b_p \rho_p \bar{p} + \frac{1}{2}(1 - \gamma)\sigma_C^2 + \left(b_p(1 - \rho_p) + \frac{1}{2}(1 - \eta)b_p^2 \sigma_p^2 + \frac{1}{1 - \gamma} E_\nu \left[e^{(\gamma-1)Z_{n,t}} - 1 \right] \right) \bar{p}
$$

$$
n = b_p(1 - \rho_p) + \frac{1}{2}(1 - \eta)b_p^2 \sigma_p^2 + \frac{1}{1 - \gamma} E_\nu \left[e^{(\gamma-1)Z_{n,t}} - 1 \right],
$$

with

$$
b_p = \frac{1 - \beta^*(1 - \rho_p) - \sqrt{[\beta^*(1 - \rho_p) - 1]^2 - 2\beta^{*2}\frac{1-\eta}{1-\gamma}\sigma_p^2 E_\nu \left[e^{(\gamma-1)Z_{n,t}} - 1 \right]}}{\beta^*(1 - \eta)\sigma_p^2}
$$

$$
\beta^* = \frac{\beta e^{(1-1/\psi)m}}{1 - \beta + \beta e^{(1-1/\psi)m}}.
$$

In addition, when $\psi = 1$, *the approximation is exact.*

Proof. In Appendix A.1.2, we show that one-period pricing kernel can be written as

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi} \left(\frac{\nu^*(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1})}{\nu(\nu^*; f_\gamma(\cdot), \mathcal{F}_t)}\right)^{1/\psi-\eta} \left(\frac{U_{t+1}}{\nu^*(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1})}\right)^{1/\psi-\gamma},$$

(1.23)

where U_{t+1} is the representative agent's continuation value, ν^* is the certainty equivalence in the first stage, and ν is the certainty equivalence in the second stage. From (A.1.9) we know

$$\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi} = e^{-\mu_C/\psi - \sigma_C/\psi B_{C,t+1} + 1/\psi \sum\limits_{n=1}^{N_{t+1}} Z_{n,t+1}}$$

$$\left(\frac{\nu^*(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1})}{(\nu^*; f_\gamma(\cdot), \mathcal{F}_t)}\right)^{1/\psi-\eta} = e^{(1/\psi-\eta)b_p\sigma_p\sqrt{p_t}B_{p,t+1} - \frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2\sigma_p^2 p_t}$$

$$\left(\frac{U_{t+1}}{\nu^*(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1})}\right) = e^{(1/\psi-\gamma)\sigma_C B_{C,t+1} + (\gamma-1/\psi)\sum\limits_{n=1}^{N_{t+1}} Z_{n,t+1} - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 - \frac{1/\psi-\gamma}{1-\gamma}E_\nu\left[e^{(\gamma-1)Z_{n,t+1}}-1\right]p}$$

(1.24)

Combining (1.23) and (1.24), we get Equation 1.22. □

$B_{C,t}$ and rare disasters both shock the immediate consumption growth, but not future growth rates. Thus the price of risk is given by the risk aversion of the agent, γ. This result implies that an agent's marginal utility will increase when there is an unexpected decrease in consumption.

Appendix A.1.3 shows that b_p captures how the agent's continuation value varies as a function of p_t. The following corollary shows that the agent's continuation value decreases when p_t increases.

Corollary 1.1. *b_p, as the sensitivity of the agent's continuation value to variation in p_t, is negative.*

Proof. When $\eta \neq 1$, we have

$$b_p = \frac{1 - \beta^*(1 - \rho_p) - \sqrt{[\beta^*(1 - \rho_p) - 1]^2 - 2\beta^{*2}\frac{1-\eta}{1-\gamma}\sigma_p^2 E_\nu\left[e^{(\gamma-1)Z_{n,t}} - 1\right]}}{\beta^*(1 - \eta)\sigma_p^2}$$

$$\beta^* = \frac{\beta e^{(1-1/\psi)m}}{1 - \beta + \beta e^{(1-1/\psi)m}}.$$

As $0 < \beta^* < 1$, we know $1 - \beta^*(1 - \rho_p) > 0$. In addition, $\frac{1}{1-\gamma}\sigma_p^2 E_\nu\left[e^{(\gamma-1)Z_{n,t}} - 1\right] < 0$, as a result $b_p < 0$.

When $\eta = 1$,

$$b_p = \frac{\beta^*}{1 - \beta^*(1 - \rho_p)} \times \frac{1}{1 - \gamma}\sigma_p^2 E_\nu\left[e^{(\gamma-1)Z_{n,t}} - 1\right] < 0.$$

$\square$

For an increase in p_t, the first-order effect is the decrease in expected future consumption. Furthermore, an increase in p_t implies higher future volatility, while its effect on the agent's continuation value depends on the agent's preference for the timing of information. However, the effect is secondary, and the net effect must be negative, as implied by the corollary.

$B_{p,t+1}$ affects the distribution of agent's future consumption growth. The effect on the agent's marginal utility, however, depends on the difference between η and $1/\psi$, or the agent's preference toward the timing for the resolution of $B_{p,t+1}$. Combining the results of Theorem 1.1 and Corollary 1.1, we can see that $B_{p,t+1}$ shocks carry a positive price of risk if and only if $\eta < 1/\psi$. This is critical to the qualitative and quantitative results of the model.

Pricing Equity and Zero-coupon Bonds

In what follows, I define and then solve for the prices of equity and real zero-coupon bonds.

Equity is defined as the claim to a dividend process $\{D_t\}_{t=1,2,3,...}$, and the dividend process

is modeled as a levered consumption process:

$$d_{t+1} - d_t = \mu_D + \sigma_D B_{C,t+1} - \varphi \sum_{n=1}^{N_{t+1}} Z_{n,t+1}, \tag{1.25}$$

where $d_t \equiv \log D_t$ is the log dividend in period t, and φ is the leverage of the dividend with respect to aggregate consumption when a disaster hits.

Following Lettau and Wachter (2011), I solve for the prices of the dividend claims with different maturity recursively. This also allows me to extract the term structure of equity premium.

The following theorem characterizes the pricing of claim to future dividends.

Theorem 1.2. *The time-t price of a dividend, or equity strip, maturing at time-$t + s$ is given by*

$$F_t(D_t, p_t, s) = D_t \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t), \tag{1.26}$$

where $a_\varphi(0) = b_{\varphi,p}(0) = 0$, and $a_\varphi(s)$, $b_{\varphi,p}(s)$, $s \geq 1$ are recursively given by

$$a_\varphi(s) = a_\varphi(s-1) + b_\varphi(s-1)\rho_p\bar{p} + \mu_D + \log\beta - \mu_C/\psi - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 + \frac{1}{2}(\sigma_D - \gamma\sigma_C)^2 \tag{1.27}$$

$$b_\varphi(s) = b_{\varphi,p}(s-1)(1-\rho_p) - \frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2\sigma_p^2 + \frac{1}{2}((1/\psi-\eta)b_p + b_{\varphi,p}(s-1))^2\sigma_p^2$$
$$+ E_\nu\left[e^{(\gamma-\varphi)Z_{n,t+1}} - 1\right] - \frac{1/\psi-\gamma}{1-\gamma}E_\nu\left[e^{(\gamma-1)Z_{n,t+1}} - 1\right]. \tag{1.28}$$

Proof. Conjecture that the time-t price of D_{t+s} is given by

$$F(D_t, p_t, s) = D_t \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t). \tag{1.29}$$

Since $F(D_t, p_t, 0) = D_t$, this implies that $a_\varphi(0) = b_{\varphi,p}(0) = 0$.

Suppose that for $s - 1 \geq 0$, Equation 1.29 holds, then we have

$$F(D_t, p_t, s) = E_t \left[M_{t,t+1} F(D_{t+1}, p_{t+1}, s-1) \right]$$

$$= E_t \left[\beta e^{-\mu_C/\psi - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 - \frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2\sigma_p^2 p_t - \frac{1/\psi-\gamma}{1-\gamma} E_\nu \left[e^{(\gamma-1)Z_{n,t+1}} - 1 \right] p_t} \right.$$

$$\times e^{-\gamma\sigma_C B_{C,t+1} + (1/\psi-\eta)b_p\sigma_p\sqrt{p_t}B_{p,t+1} + \gamma \sum_{n=1}^{N_{t+1}} Z_{n,t+1}}$$

$$\left. \times D_t e^{\mu_D + \sigma_D B_{C,t+1} + a_\varphi(s-1) - \varphi \sum_{n=1}^{N_{t+1}} Z_{n,t+1} + b_{\varphi,p}(s-1)[p_t - \rho_p(p_t - \bar{p}) + \sigma_p\sqrt{p_t}B_{p,t+1}]} \right]$$

$$= D_t e^{a_\varphi(s-1) + b_\varphi(s-1)\rho_p\bar{p} + \mu_D + \log\beta - \mu_C/\psi - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 + \frac{1}{2}(\sigma_D - \gamma\sigma_C)^2}$$

$$\times e^{-\frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2\sigma_p^2 p_t + \frac{1}{2}((1/\psi-\eta)b_p + b_{\varphi,p}(s-1))^2\sigma_p^2 p_t}$$

$$\times e^{E_\nu \left[e^{(\gamma-\varphi)Z_{n,t+1}} - 1 \right] p_t - \frac{1/\psi-\gamma}{1-\gamma} E_\nu \left[e^{(\gamma-1)Z_{n,t+1}} - 1 \right] p_t + b_{\varphi,p}(s-1)(1-\rho_p)p_t},$$

$$(1.30)$$

which implies that

$$a_\varphi(s) = a_\varphi(s-1) + b_\varphi(s-1)\rho_p\bar{p} + \mu_D + \log\beta - \mu_C/\psi - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 + \frac{1}{2}(\sigma_D - \gamma\sigma_C)^2$$

$$b_\varphi(s) = b_{\varphi,p}(s-1)(1-\rho_p) - \frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2\sigma_p^2 + \frac{1}{2}((1/\psi-\eta)b_p + b_{\varphi,p}(s-1))^2\sigma_p^2$$

$$+ E_\nu \left[e^{(\gamma-\varphi)Z_{n,t+1}} - 1 \right] - \frac{1/\psi-\gamma}{1-\gamma} E_\nu \left[e^{(\gamma-1)Z_{n,t+1}} - 1 \right].$$

$\square$

The zero-coupon bonds are defined as claims to one unit of consumption good at specified time. Their prices are given by the following theorem.

Theorem 1.3. *The time-t price of a zero-coupon bond maturing in s period (or at time t + s) is given by*

$$B(p_t, s) = \exp\left(a_b(s) + b_{b,p}(s)p_t \right), \tag{1.31}$$

where $a_b(0) = b_{b,p}(0) = 0$, and $a_b(s)$ and $b_{b,p}(s)$, $s \geq 1$, are recursively given by

$$a_b(s) = a_b(s-1) + \log\beta - \mu_C/\psi - \frac{1}{2}(1/\psi - (1+1/\psi)\gamma)\sigma_C^2 + \rho_p\bar{p}b_{b,p}(s-1)$$

$$b_{b,p}(s) = b_{b,p}(s-1)(1-\rho_p)$$

$$+ (1/\psi - \eta)b_p b_{b,p}(s-1)\sigma_p^2 + \frac{1}{2}(1/\psi - 1)(1/\psi - \eta)b_p^2\sigma_p^2 + \frac{1}{2}b_{b,p}(s-1)^2\sigma_p^2$$

$$+ E_\nu\left[e^{\gamma Z_{n,t}} - 1\right] - \frac{1/\psi - \gamma}{1-\gamma}E_t\left[e^{(\gamma-1)Z_{n,t}} - 1\right].$$

$$(1.32)$$

Proof. We show the pricing by induction.

1. When $s = 0$,

$$B(p_t, 0) = 1 \leftrightarrow a_b(0) = b_{b,p}(0) = 0,$$

and the theorem holds.

2. Suppose that for bonds with maturity no greater than $s - 1$ periods, the recursion defined before holds. Then

$$B(p_t, s) = E_t\left[M_{t,t+1}B(p_{t+1}, s-1)\right]$$

$$= E_t\left[\beta e^{-\mu_C/\psi - \frac{1}{2}(1-\gamma)(1/\psi-\gamma)\sigma_C^2 - \frac{1}{2}(1-\eta)(1/\psi-\eta)b_p^2\sigma_p^2 p_t - \frac{1/\psi-\gamma}{1-\gamma}E_\nu\left[e^{(\gamma-1)Z_{n,t+1}}-1\right]p_t}\right.$$

$$\times e^{-\gamma\sigma_C B_{C,t+1} + (1/\psi-\eta)b_p\sigma_p\sqrt{p_t}B_{p,t+1} + \gamma\sum\limits_{n=1}^{N_{t+1}} Z_{n,t+1}}$$

$$\left. \times e^{a_b(s-1)+b_{b,p}(s-1)[p_t - \rho_p(p_t-\bar{p})+\sigma_p\sqrt{p_t}B_{p,t+1}]}\right]$$

Matching coefficients, we get that Equation 1.32 holds for s. By the property of deduction, the theorem holds for $s \geq 0$.

$\square$

An interesting quantity is the evolution of $b_{b,p}(s)$. From Equation 1.32 we can get

$$
\begin{aligned}
b_{b,p}(s) - b_{b,(s-1)} = &-\rho_p b_{b_p}(s-1) \\
&+ (1/\psi - \eta)b_p b_{b,p}(s-1)\sigma_p^2 + \frac{1}{2}(1/\psi - 1)(1/\psi - \eta)b_p^2\sigma_p^2 + \frac{1}{2}b_{b,p}(s-1)^2\sigma_p^2 \\
&+ E_\nu\left[e^{\gamma Z_{n,t}} - 1\right] - \frac{1/\psi - \gamma}{1 - \gamma}E_t\left[e^{(\gamma-1)Z_{n,t}} - 1\right]. \quad (1.33)
\end{aligned}
$$

The third line of Equation 1.33 captures the bond's role as hedging toward the disasters. As the disasters are modeled as i.i.d, the agent is willing to pay the same insurance fee for each additional unit of maturity time. The first line captures the fact that p_t is roughly an AR(1) process. The second line captures the joint effect of variation in p_t: the shocks to p_t are priced, so there is a premium associated in the expected discount rate; Also, there is a Jensen's Inequality term that affects the expected value of bond price because of the exposure to p_t.

1.3.3 The Term Structure of Real Yield Curve and Equity

In this section, I discuss the term structure of yield curve for real zero-coupon bonds and provide the intuition why the Generalized Recursive Utility, under some parametric restrictions, can yield an upward sloping yield curve. I consider the special case $1/\psi = 1$ so that the intuition can be better explained. When $1/\psi \neq 1$, the argument still follows, but the analysis will not be exact.

When $1/\psi = 1$,

$$
\begin{aligned}
&E_\nu\left[e^{\gamma Z_{n,t}} - 1\right] - \frac{1/\psi - \gamma}{1 - \gamma}E_t\left[e^{(\gamma-1)Z_{n,t}} - 1\right] \\
=&E_\nu\left[e^{\gamma Z_{n,t}} - e^{(\gamma-1)Z_{n,t}}\right] > 0.
\end{aligned}
$$

This equation implies the first-order effect of change in p_t on the price of real zero coupon bonds: with higher p_t, there is a higher probability of rare disasters, and hence higher demand for hedging in the economy, driving up the price of zero-coupon bonds.

Consider the case then $p_t = \bar{p}$, or when the economy is at the steady state of p_t. The following equation holds for the difference in log price of zero-coupon real bonds.

$$
\begin{aligned}
&\log\left(B(p_t, s)\right) - \log\left(B(p_t, s-1)\right) \\
&= \underbrace{\log \beta - \mu_C/\psi - \frac{1}{2}(1-2\gamma)\sigma_C^2 + E_\nu\left[e^{\gamma Z_{n,t}} - e^{(\gamma-1)Z_{n,t}}\right]\bar{p}}_{(1.34.1)} \\
&\quad + \underbrace{(1-\eta)b_p b_{b,p}(s-1)\sigma_p^2\bar{p}}_{(1.34.2)} + \underbrace{\frac{1}{2}b_{b,p}(s-1)^2\sigma_p^2\bar{p}}_{(1.34.3)}.
\end{aligned}
\tag{1.34}
$$

(1.34.1) captures the first order marginal effect of maturity on bond price: zero-coupon bonds provide insurance against rare disasters and consumption growth risks, and at the steady state, the quantity of risk is proportional to maturity. In addition, the economy features constant consumption growth and time-preference, making the hedging demand proportional to maturity. When transformed into yield space, this would result in a flat yield curve at the steady state.

(1.34.2) captures the effect of the variation in p_t. It could be showed that, for certain s, the loading of $\log\left(B(p_t, s)\right)$ on p_t, $b_{b,p}(s)$, is always positive, meaning that the price of bond would increase when p_t increases. However, when $\eta < 1/\psi$, shocks to p_t carry positive price of risk. When $b_{b,p}(s)$ is increasing with maturity, the premium required also is increasing in maturity, driving down the price of bonds further. This premium then yields the upward slope of real yield curve, and is the key mechanism in this model. (1.34.3) stands for Jensen's Inequality effect, and is generally small for short maturities.

Similar arguments follow for the term structure of equity. Prices of long-maturity dividends drop more than short-maturity dividends with an increase in p_t. However, this implies that the agent is willing to pay a lower risk premium for such exposure. In equilibrium, the long-maturity dividend claims carry lower risk premium relative to the short-maturity claims, resulting in a downward sloping term structure of equity premium.

1.3.4 Pricing Volatility Future

The realized volatility of the market portfolio over period $t+1$ is defined as

$$RV_{t+1} = \sum_{n=1}^{N} rx^2_{t+\frac{n-1}{N}, t+\frac{n}{N}}, \tag{1.35}$$

where $rx_{t+\frac{n-1}{N}, t+\frac{n}{N}}$ is the log return of the market portfolio during sub-period from time $t + \frac{n-1}{N}$ to $t + \frac{n}{N}$.

The volatility forward contract is an agreement such that one party pays a fixed amount while the other party pays the value of realized volatility of a certain period in the future. The volatility forward price is the fixed leg amount such that the volatility forward contract has value zero.

As the realized volatility is defined using log return, it is helpful to derive a formula for the log return of an equity asset in the model.

The (post-dividend) price of an equity asset is given by

$$
\begin{aligned}
P(D_t, p_t) &= \sum_{s=1}^{\infty} F(D_t, p_t, s) \\
&= D_t \sum_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)
\end{aligned}
\tag{1.36}
$$

The one-period return of the market portfolio is then given by

$$R_{\varphi,t+1} = \frac{D_{t+1} \sum\limits_{s=0}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_{t+1})}{D_t \sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)}. \tag{1.37}$$

As a result the log return of the market portfolio is

$$
\log\left(R_\varphi(p_t)\right) = \log\left(\frac{D_{t+1}}{D_t}\right) + \log\left(\frac{1 + \sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_{t+1})}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)}\right)
$$

$$
= \mu_D + \sigma_D B_{C,t} + \varphi \sum_{n=1}^{N_{t+1}} Z_{n,t+1} + \log\left(\frac{1 + \sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_{t+1})}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)}\right).
$$

$$(1.38)$$

The equation above indicates the shocks that affect the realized returns: while $B_{C,t+1}$ and $Z_{n,t+1}$ affect the dividend level, shocks to p_t will affect the realized return through changing the price-dividend ratio. Define

$$
\zeta(p_t) = \frac{\sum\limits_{s=1}^{\infty} \exp\left(a_\varphi(s) + b_{\varphi,p}(s)(p_t - \rho_p(p_t - \bar{p}))\right)}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)}.
$$

Then with a first-order Taylor expansion with respect to $\sigma_p\sqrt{p_t}B_{p,t+1}$, a linear approximation of the log price-dividend ratio variation can be obtained:

$$
\log\left(\frac{1 + \sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_{t+1})}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)}\right)
$$

$$
\approx \log(\zeta(p_t)) + \frac{1}{\zeta(p_t)} \times \frac{\sum\limits_{s=1}^{\infty} \exp\left(a_\varphi(s) + b_{\varphi,p}(s)(p_t - \rho_p(p_t - \bar{p}))\right)b_{\varphi,p}(s)}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)} \times \sigma_p\sqrt{p_t}B_{p,t+1}.
$$

$$(1.39)$$

Approximating

$$
\frac{1}{\zeta(p_t)} \times \frac{1 + \sum\limits_{s=1}^{\infty} \exp\left(a_\varphi(s) + b_{\varphi,p}(s)(p_t - \rho_p(p_t - \bar{p}))\right)b_{\varphi,p}(s)}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)p_t)}
$$

as a constant, with $p_t = \bar{p}$, yields a linear approximation of the log return:

$$\log\left(R_\varphi(p_t)\right) \approx \mu_D + \log(\zeta(p_t)) + \sigma_D B_{C,t} + \varphi \sum_{n=1}^{\Delta N_{t+1}} Z_{n,t+1} + \zeta_p \sigma_p \sqrt{p_t} B_{p,t+1}, \qquad (1.40)$$

where

$$\zeta_p = \frac{1}{\zeta(\bar{p})} \times \frac{1 + \sum\limits_{s=1}^{\infty} \exp\left(a_\varphi(s) + b_{\varphi,p}(s)zu\bar{p}\right) b_{\varphi,p}(s)}{\sum\limits_{s=1}^{\infty} \exp(a_\varphi(s) + b_{\varphi,p}(s)\bar{p})}. \qquad (1.41)$$

Then following Drechsler and Yaron (2011), the realized volatility from t to $t+1$ must be

$$RV_{t+1} = \sigma_D^2 + \zeta_p^2 \sigma_p^2 p_t + \varphi^2 \sum_{n=1}^{\Delta N_{t+1}} Z_{n,t+1}^2. \qquad (1.42)$$

The time t forward price of realized volatility from $t+s-1$ to $t+s$ must satisfy

$$F_t^{VOL}(p_t, s) = E_t^Q[RV_{t+s}], \qquad (1.43)$$

where Q refers to the risk-neutral probability of the economy.

The following theorem characterizes the pricing of the volatility future.

Theorem 1.4. *The time-t forward price of realized volatility from $t+s$ to $t+s+1$ is given by*

$$F_t^{VOL}(p_t, s) = a_{RV}(s) + b_{RV}(S)p_t, \qquad (1.44)$$

where

$$a_{RV}(1) = \sigma_D^2 \qquad (1.45)$$

$$b_{RV}(1) = \zeta_p^2 \sigma_p^2 + \varphi^2 E_\nu^Q\left[Z_{n,t+1}^2\right] E_\nu\left[e^{\gamma Z_{n,t+1}}\right], \qquad (1.46)$$

and $b_{RV}(s)$, $b_{RV}(s)$, $s \geq 2$ are recursively given by

$$a_{RV}(s) = a_{RV}(s-1) + b_{RV}(s-1)\rho_p \bar{p} \tag{1.47}$$

$$b_{RV}(s) = b_{RV}(s-1)\left(1 - \rho_p + (1/\psi - \eta)b_p\sigma_p^2\right). \tag{1.48}$$

Proof. See Appendix A.1.3. □

After obtaining the forward price of realized volatility, the risk premium associated with the volatility forward contracts can be computed.

First, let's consider the forward contract maturing in 1 period. This contract provides a long position of the realized volatility of period $t+1$. The forward price is given at time t, thus leading to the following expected pay-off:

$$
\begin{aligned}
&E_t^P[RV_{t+1}] - F_t^{VOL}(p_t, 1) \\
&= \sigma_D^2 + \zeta_p^2 \sigma_p^2 p_t + \varphi^2 p_t E_\nu^P[Z_{n,t+1}^2] - F_t^{VOL}(p_t, 1) \\
&= p_t \varphi^2 \left(E_\nu^P[Z_{n,t+1}^2] - E_\nu^Q[Z_{n,t+1}^2] E_\nu\left[e^{\gamma Z_{n,t+1}}\right]\right).
\end{aligned}
\tag{1.49}
$$

As $Z_{n,t+1} < 0$, the characterization of the risk-neutral measure implies that $E_\nu^Q[Z_{n,t+1}^2] > E_\nu^P[Z_{n,t+1}^2] > 0$, as the risk neutral probability bias toward realizations of $Z_{n,t+1}$ with higher absolute value. In addition, $E_\nu[e^{\gamma Z_{n,t+1}}] > 1$. As a result,

$$E_t^P[RV_{t+1}] - F_t^{VOL}(p_t, 1) < 0, \tag{1.50}$$

or there is a negative premium associated to exposure to realized volatility.

Next, let's consider the holding return for entering a volatility forward contract.

At time t, the investor enters the contract by committing to pay $F_t^{VOL}(p_t, s)$ at time $t+s$, and receive RV_{t+s}. At time $t+1$, the contract would have value $E_{t+1}^Q[RV_{t+s}] - F_t^{VOL}(p_t, s)$.

Note that, $E_{t+1}^Q[RV_{t+s}] = F_{t+1}^{VOL}(s-1)$, and $F_t^{VOL}(p_t, s) = E_t^Q[F_{t+1}^{VOL}(s-1)]$. Then again

the expected pay-off of holding a forward contact would be

$$E_t^P \left[F_{t+1}^{VOL}(s-1) \right] - E_t^Q \left[F_{t+1}^{VOL}(s-1) \right]$$

$$= E_t^P \left[a_{RV}(s-1) + b_{RV}(s-1)p_{t+1} \right] - E_t^Q \left[a_{RV}(s-1) + b_{RV}(s-1)p_{t+1} \right] \qquad (1.51)$$

$$= - b_{RV}(s-1)(1/\psi - \eta)b_p\sigma_p\sqrt{p_t},$$

which is positive when $1/\psi - \eta > 0$.

The fundamental difference between realized volatility and volatility forward contracts beyond one period is that, only realized volatility provides hedge against the rare disasters on consumption level. As a result, exposure to realized volatility carries negative premium. However, volatility forward contracts provide positive exposure to conditional volatility p_t, resulting in a positive risk premium and Sharpe Ratio.

1.3.5 Calibration and Quantitative Results

I calibrate the model as described below. The consumption process, absent of disasters, is calibrated the same as in Wachter (2013). I choose the unconditional disaster probability to be 3.6%, as in Barro and Ursúa (2008). The disaster distribution is multinomial, with the data provided by Barro and Ursua. Other parameters for the dynamics of p_t process follow Wachter (2013). The model in Wachter (2013) is in continuous time, and I discretized them so that the process is consistent in the discrete-time framework.

The market consists of eight equity assets, with medium leverage $\varphi = 3.5$. I also use an equity asset with $\varphi = 3.5$ to approximate the market for the simplicity of computation. Following Wachter and Zhu (2019a), I assume that the dividend growth rate equals consumption growth rate, or $\mu_D = \mu_C$.

I choose a time-preference parameter $\beta = 0.97$, and an EIS $\psi = 0.5$. In addition, I choose risk-aversion $\gamma = 3.3$. All the parameters are consistent with a large body of existing literature. I choose $\eta = 0$, which implies that the agent is indifferent to the variation in p_t.

This is a fairly strong assumption, but I would like to see how far the model can help to resolve the puzzles described above without assuming that the agent is uncertainty-loving. The details of the calibration are summarized in Table 1.1.

I compute the pricing parameters in the model. Recall that the model is written in discrete-time, so the monotonicity of $b_{\varphi p}(s)$ and $b_{bp}(s)$ can not be guaranteed. I resolve this issue by allowing for finer intervals in the numerical exercise. The actual frequency is monthly in computation.

The main result for the real yield curve is reported in Figure 1.6. In the figure, I plot the real yield curve at the steady state ($p_t = \bar{p}$). The figure features an upward sloping yield curve, whith is consistent with data but contradicts existing literature.

To explore the quantitative implication of the model on the equity market, I simulate 500 parallel samples, each of length 57 years (12×57 periods). To obtain a stationary distribution for the state variables, I simulate a burn-in period of 5 years. The market portfolio is defined as the value-weighted average of individual equity assets. I report the summary statistics of the market portfolio, the risk-free rate, and claims to short- and long-maturity of aggregate dividends in Table 1.2. I define the short maturity dividend claims as claims for dividends maturing within 5 years, and long-maturity dividend as dividends as the remaining part of the equity.

The model can match empirical moments in the data: the mean equity excess return and the mean and volatility of the risk-free rate from data all fall within 90% confidence interval generated from the simulation.

Figure 1.7 shows that the model is able to explain the beta anomaly. Why is that the case? For portfolios with higher leverage φ, the equity premium from rare disasters is higher. They also have higher negative exposure to p_t, the disaster probability, and such exposure generates negative risk premium, flattening the security market line across assets.

The same intuition can be applied when investigating the term structure of equity premium. Binsbergen et al. (2012) show that the term structure of equity premium, defined as the risk premium of equity strips as a function of maturity, is downward sloping. Figure 1.8 confirms that this is the case in the model. Table 1.2 shows that long-maturity dividends have weakly higher CAPM betas, but lower risk premia.

Figure 1.9 illustrates the results for the variance forward contacts. The contracts with 1-month maturity, or realized volatility contracts, carry a negative Sharpe Ratio in the model, as realized volatility jumps up when a disaster hits. The variance forward contracts, however, feature small positive Sharpe Ratios. The intuition has already been discussed.

In conclusion, by allowing the agent to show lower aversion toward shocks about the long-run future, the model is able to match a large body of puzzles in both fixed income and equity markets.

1.4   Conclusion

The expected utility hypothesis and recursive utility have been workhorse models for macro-economic and financial research. However, they both imply restrictions on how the agent would treat smoothing across time and different states. In addition, they both have strong implications for asset returns in the cross-section, and strongly contradict data.

This paper proposes a generalized recursive utility function that further relaxes the restriction implied by the Epstein-Zin-Weil recursive utility. The utility allows agents to show various levels of aversion to two different types of shocks and then disconnects risk aversion, Elasticity of Intertemporal Substitution, and preference for early resolution of uncertainty. The new utility specification is then applied to a representative agent model with a frictionless market. I show that with certain parametric restrictions, the model can qualitatively explain the empirical puzzles.

I then calibrate the model with parameter values consistent with a large body of literature. I show that the model not only provides qualitative explanation to the puzzles found in empirical work but also can quantitatively match the empirical patterns.

While the current generalization focuses on distinguishing the short-run future and long-run future shocks, similar approaches can be applied to distinguish other types of uncertainty (e.g., Bayesian posterior uncertainty, ambiguity, etc.) It would be interesting to combine the current framework with other literatures and investigate further implications.

Table 1.1: Model parameters

Panel I: Consumption Growth	
Normal time log consumption growth $\mu_C(\%)$	2.50
Volatility of consumption growth $\sigma_C(\%)$	2.00
Panel II: Dynamics of p_t	
Average probability of disaster $\bar{p}(\%)$	2.86
Mean reversion ρ_p	0.07
Volatility σ_p	0.067
Panel III: Preference	
Time preference β	0.97
Elasticity of Intertemporal Substitution ψ	0.50
Aversion to consumption risk γ	3.30
Aversion to variation in p_t η	0.00
Panel IV: Equity Asset	
Normal time log dividend growth $\mu_D(\%)$	2.90
Leverage ϕ	3.5
Volatility of dividend growth σ_D (%)	5.0
Panel V: Simulation parameters	
Number of Samples	500
Sample Length	57 years
Cut-off for short- and long-end of equity maturities	5 years

Note: Parameters for the calibration of the disaster risk model in Section 1.3. The quantities are reported in annualized terms.

Table 1.2: Simulation results: market portfolio

Panel I: Equity excess returns				
Statistics	Data	Sim. Mean	Sim. Median	90 % CI
Mean	6.37	5.57	4.93	$[1.42, 12.23]$
Volatility	15.20	10.17	10.20	$[5.50, 15.75]$
Panel II: Risk-free rate				
Statistics	Data	Sim. Mean	Sim. Median	90 % CI
Mean	0.74	2.58	3.38	$[-4.48, 6.58]$
Volatility	1.14	1.20	1.06	$[0.45, 2.40]$
Panel III: Difference in long- and short-maturity dividend claims				
Statistics	Data	Sim. Mean	Sim. Median	90 % CI
Mean excess returns		-0.20	-0.12	$[-0.76, 0.13]$
CAPM Beta		0.01	-0.01	$[-0.04, 0.14]$

Note: In data simulated from the model, I compute the mean excess returns and unconditional volatility of the market portfolio for each simulation sample path; I also compute the mean and unconditional volatility of the risk-free rates. The long-maturity dividend claim is defined as the claim to dividends realized after 8 years or longer. The first column reports empirical moments; the second and third column report the median and 90% confidence interval computed using simulation. The unit is percentage per annum.

Table 1.3: Simulation results: cross-section

| Panel A: Mean Excess Returns				
Portfolio	1	2	3	4
Median	4.08	4.53	4.91	5.17
90% CI	$[0.98, 11.01]$	$[1.32, 11.58]$	$[1.55, 12.33]$	$[1.73, 12.86]$
Portfolio	5	6	7	8
Median	5.31	5.61	5.86	5.99
90% CI	$[1.82, 13.20]$	$[1.79, 13.61]$	$[1.78, 14.20]$	$[1.69, 14.84]$
Panel B: CAPM beta				
Portfolio	1	2	3	4
Median	0.63	0.78	0.91	1.00
90% CI	$[0.50, 0.82]$	$[0.68, 0.91]$	$[0.82, 1.02]$	$[0.92, 1.13]$
Portfolio	5	6	7	8
Median	1.07	1.23	1.36	1.50
90% CI	$[1.01, 1.23]$	$[1.09, 1.48]$	$[1.12, 1.67]$	$[1.15, 1.88]$

Note: In data simulated from the model, I compute the mean excess returns and unconditional volatility of the market portfolio for each simulation sample path; I also compute the mean and unconditional volatility of the risk-free rates. The long-maturity dividend claim is defined as the claim to dividends realized after 8 years or longer. The first column reports empirical moments; the second and third column report the median and 90% confidence interval computed using simulation. The unit is percentage per annum.

Figure 1.1: Time series of US TIPS yields
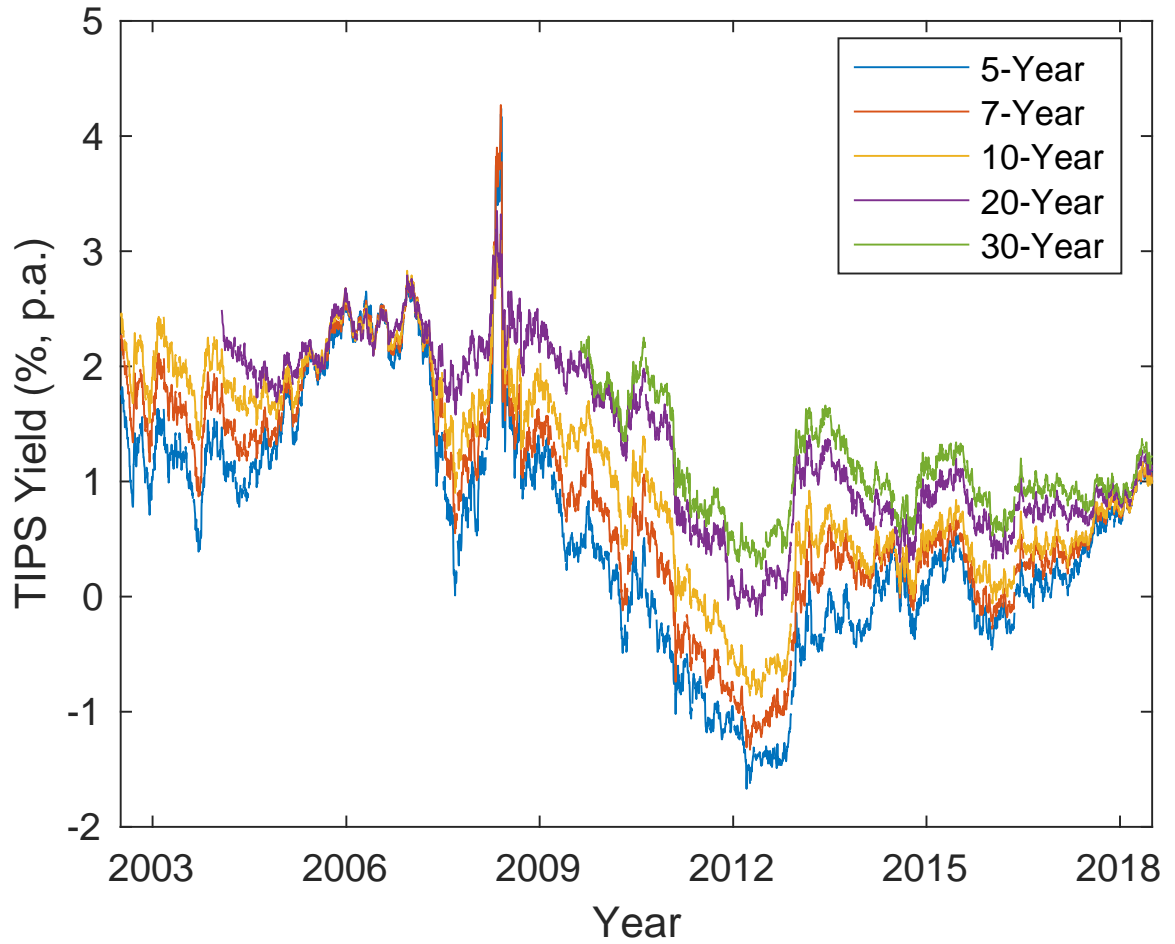
Note: This figure plots time series of US TIPS (Treasury Inflation Protected Securities) yields from 2003.01 - 2017.10. The legend shows the corresponding maturities. The unit is % p.a.

Figure 1.2: Time series of 5- and 10-year yield spread of US TIPS
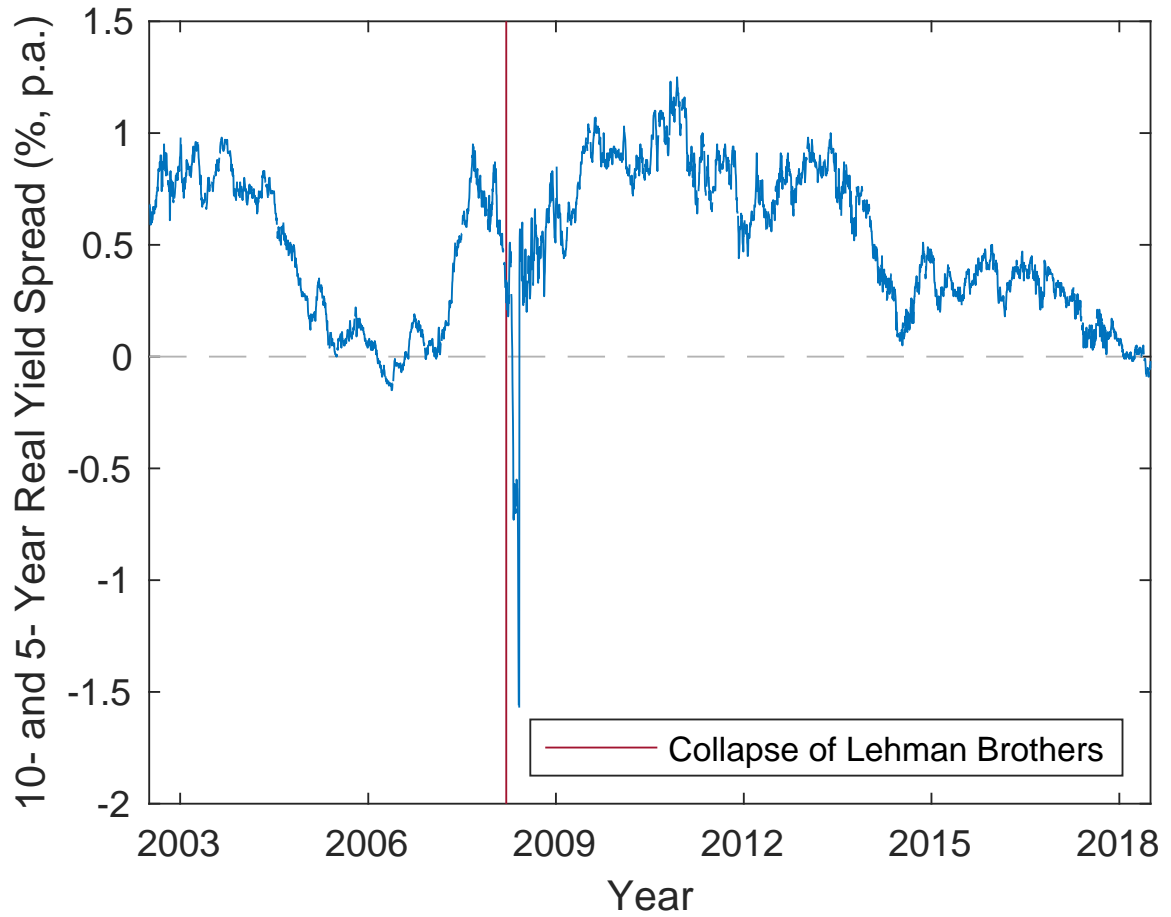
Note: This figure plots time series of the difference between the yields of 10- and 5-year US TIPS (Treasury Inflation Protected Securities) I also point out the day when the 2008 financial crisis started (the collapse of Lehman Brothers.) .

Figure 1.3: Implied real risk-free yield curve at $p_t = \bar{p}$, Epstein-Zin-Weil Utility
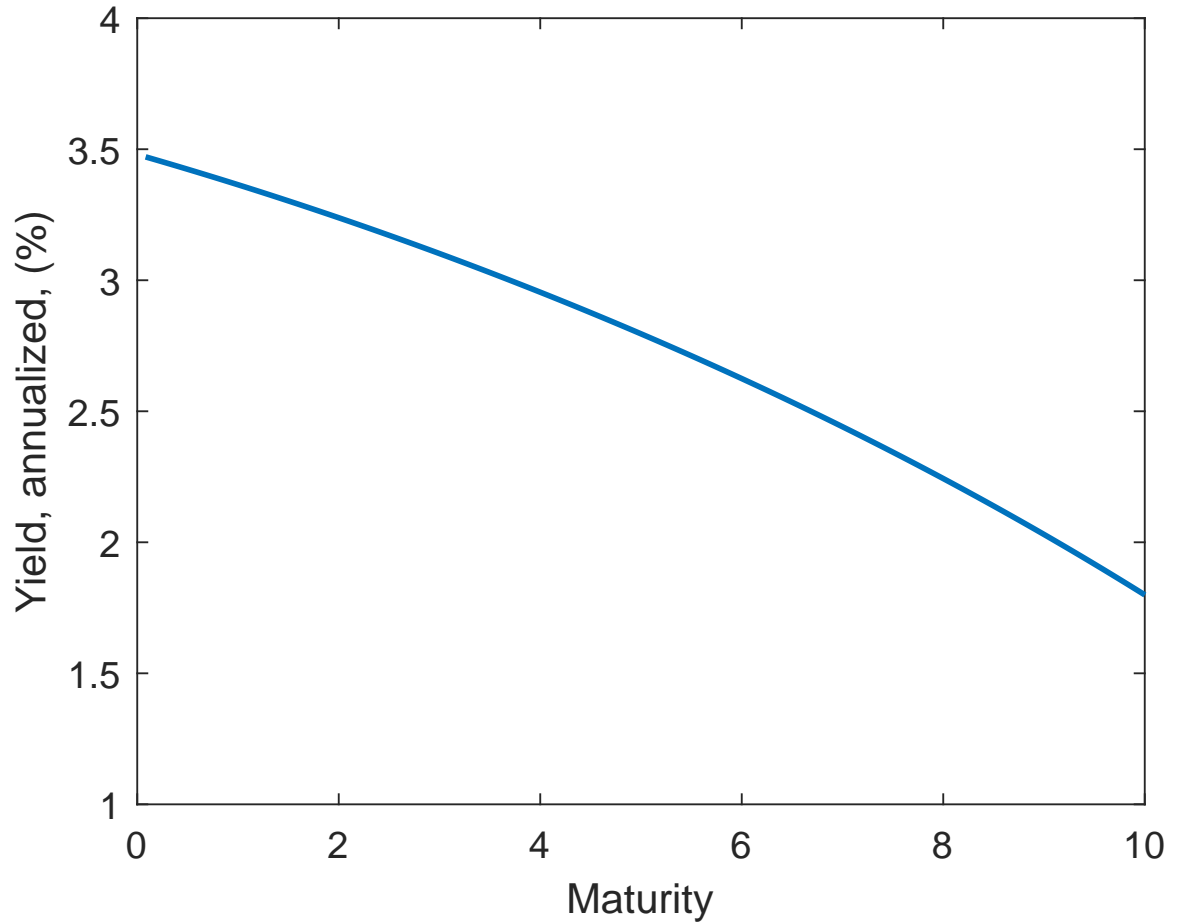
Note: This figure plots the implied yield curve for real zero-coupon bond at the steady state $p_t = \bar{p}$. The agent's preference is represented by the Epstein-Zin-Weil Utility, with risk aversion $\gamma = 3.3$, and EIS $\psi = 0.5$.

Figure 1.4: Mean excess returns and unconditional CAPM beta for beta-sorted portfolios
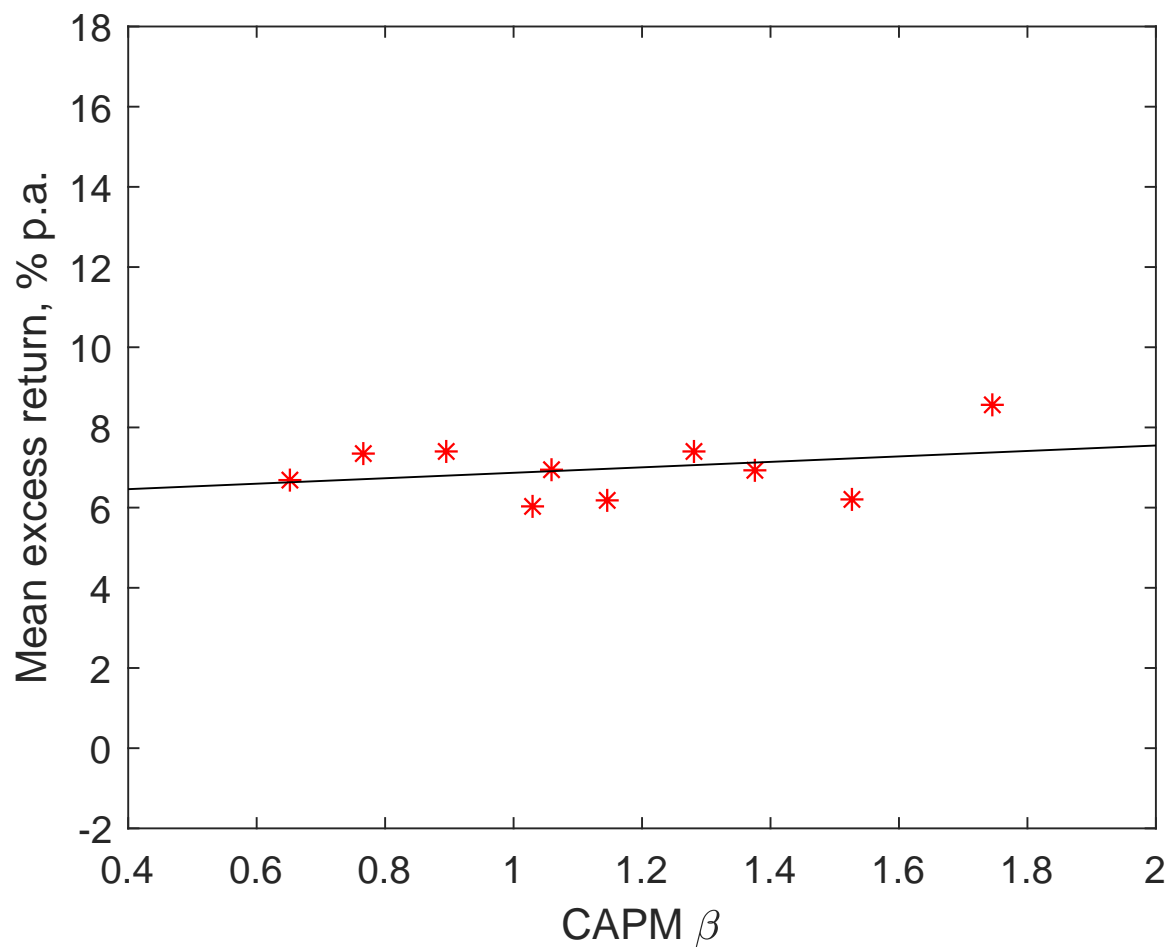
Note: This figure plots the mean excess returns and unconditional CAPM beta for the 10 beta-sorted portfolios from 1961.01 - 2017.12. Also shown is the regression line of mean excess returns against CAPM beta, or the Security Market Line (SML).

Figure 1.5: Sharpe Ratio of Investing in Variance Forwards

Note: The figure shows the realized Sharpe Ratios of investing in variance forward contracts with different maturities from 1996.01 to 2013.09. Each star stands for forward contracts with one certainty maturity. On the horizontal axis is the maturity (in month), while on the vertical axis is the (annualized) Sharpe Ratio. The forward contract with 1-month maturity provides exposure to realized volatility.

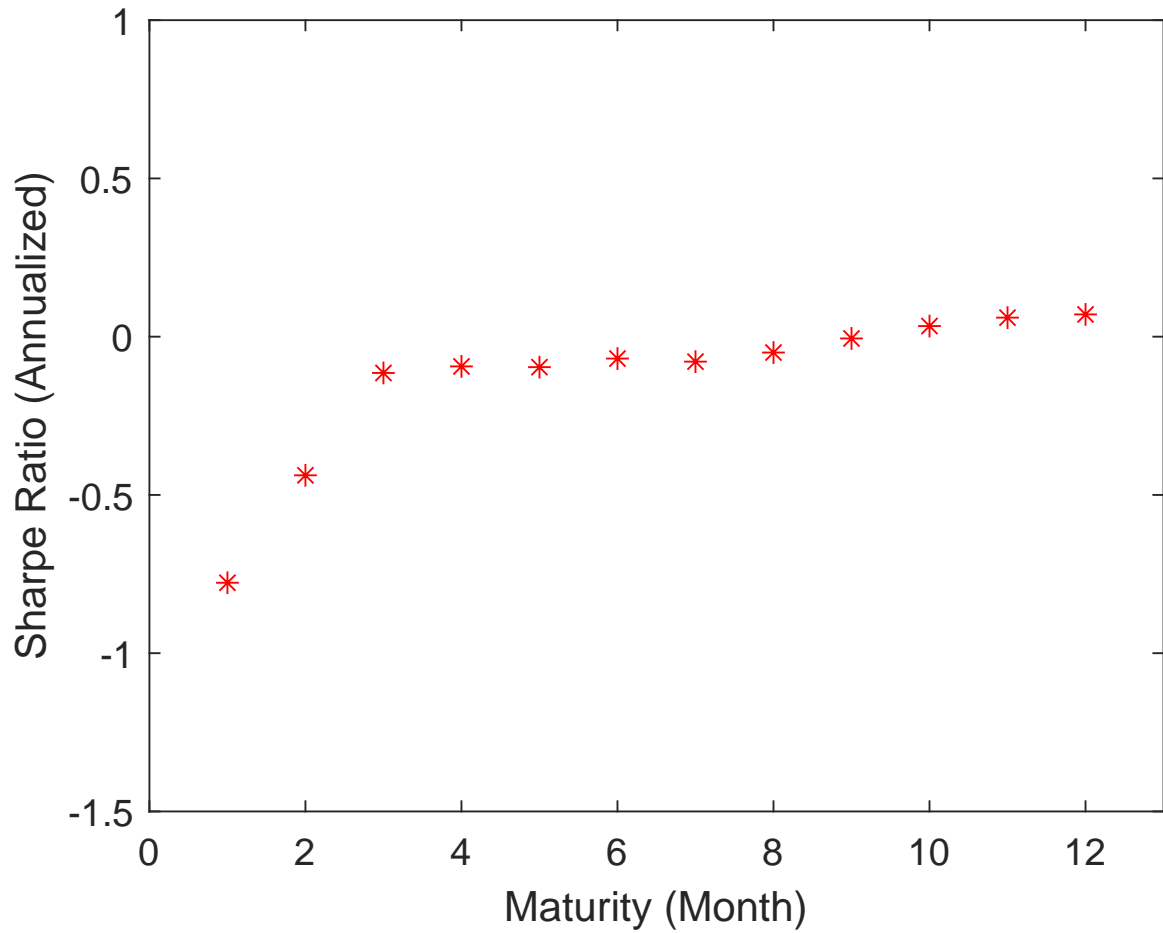Figure 1.6: Implied real risk-free yield curve at $p_t = \bar{p}$
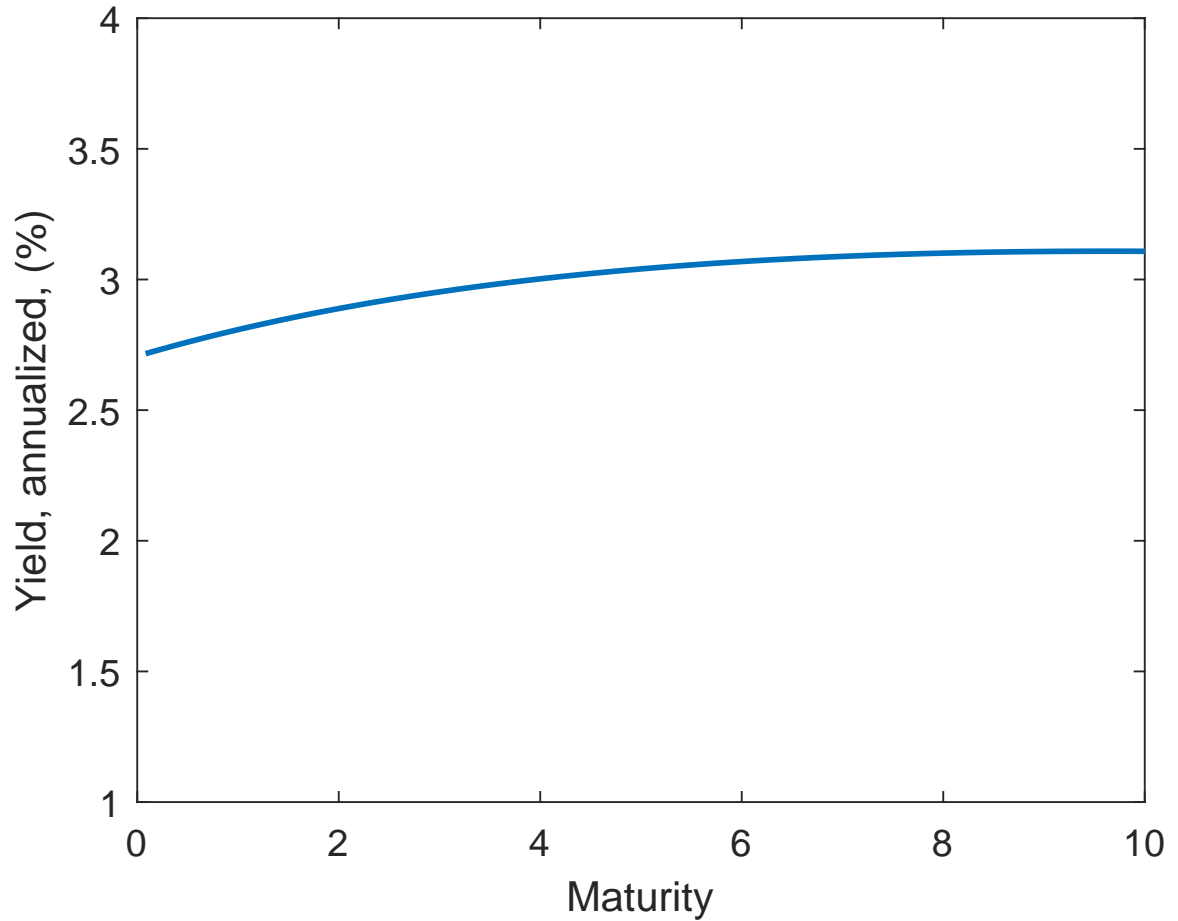
Note: This figure plots the implied yield curve for real zero-coupon bond at the steady state $p_t = \bar{p}$. The agent's preference is represented by the Generalized Recursive Utility, with risk aversion $\gamma = 3.3$, and EIS $\psi = 0.5$, and aversion to long-run future shocks $\eta = 0$.

Figure 1.7: Simulated equity portfolio CAPM betas and excess returns
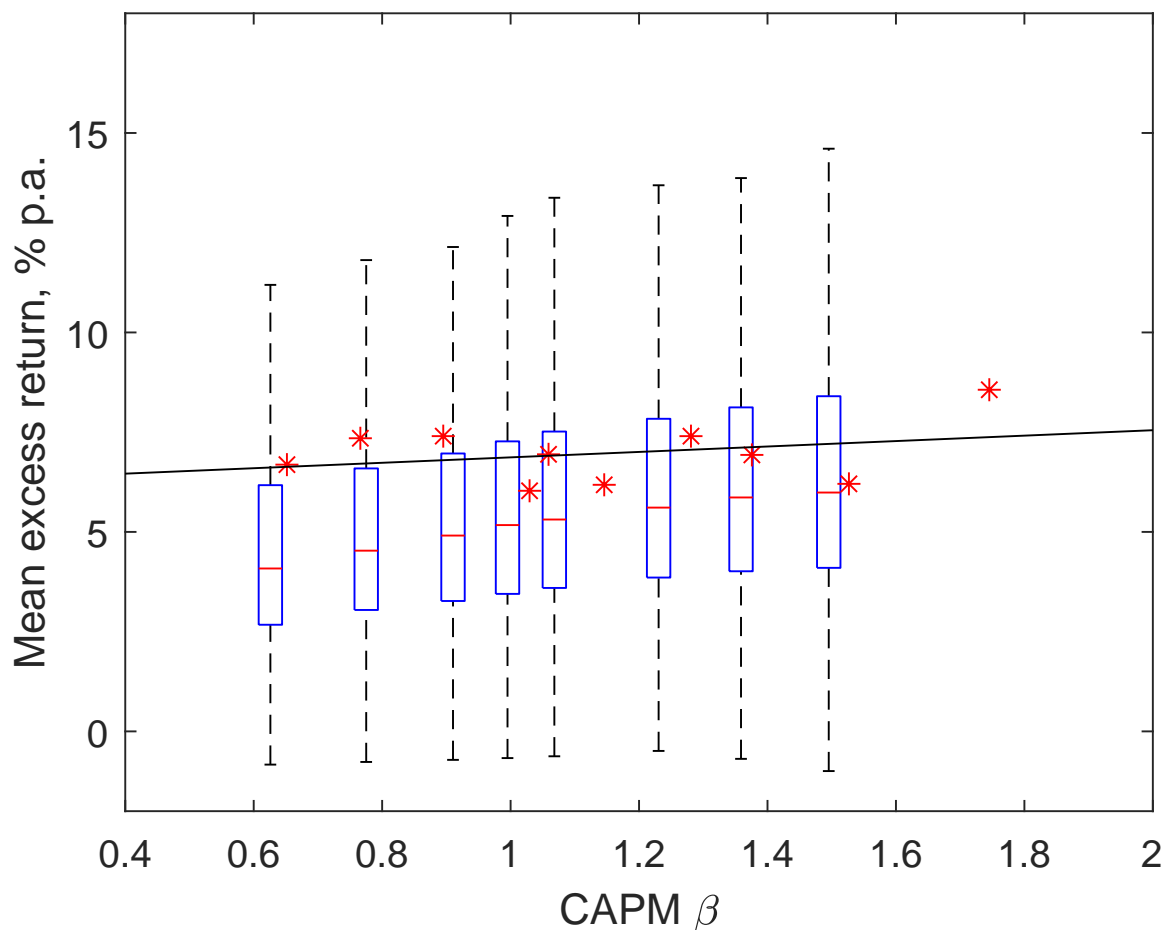
Note: This figure shows the mean excess returns on beta-sorted portfolios in daily data from 1961.01-2017.12 as a function of the CAPM beta. Also showed on the figure are moments generated from simulation data. I calculate average excess returns and CAPM betas for a cross-section of assets in data simulated from the model. The red lines stand for the median for each portfolio across samples; the box show the interquartile range (IQR), and the whiskers show the highest and lowest data point within 1.5 × IQR of the highest and lowest quartiles. The returns are plotted against the median CAPM beta across samples. The red stars correspond to the estimated CAPM beta and mean excess returns for beta-sorted portfolios from empirical data.

Figure 1.8: Implied term structure of equity premium
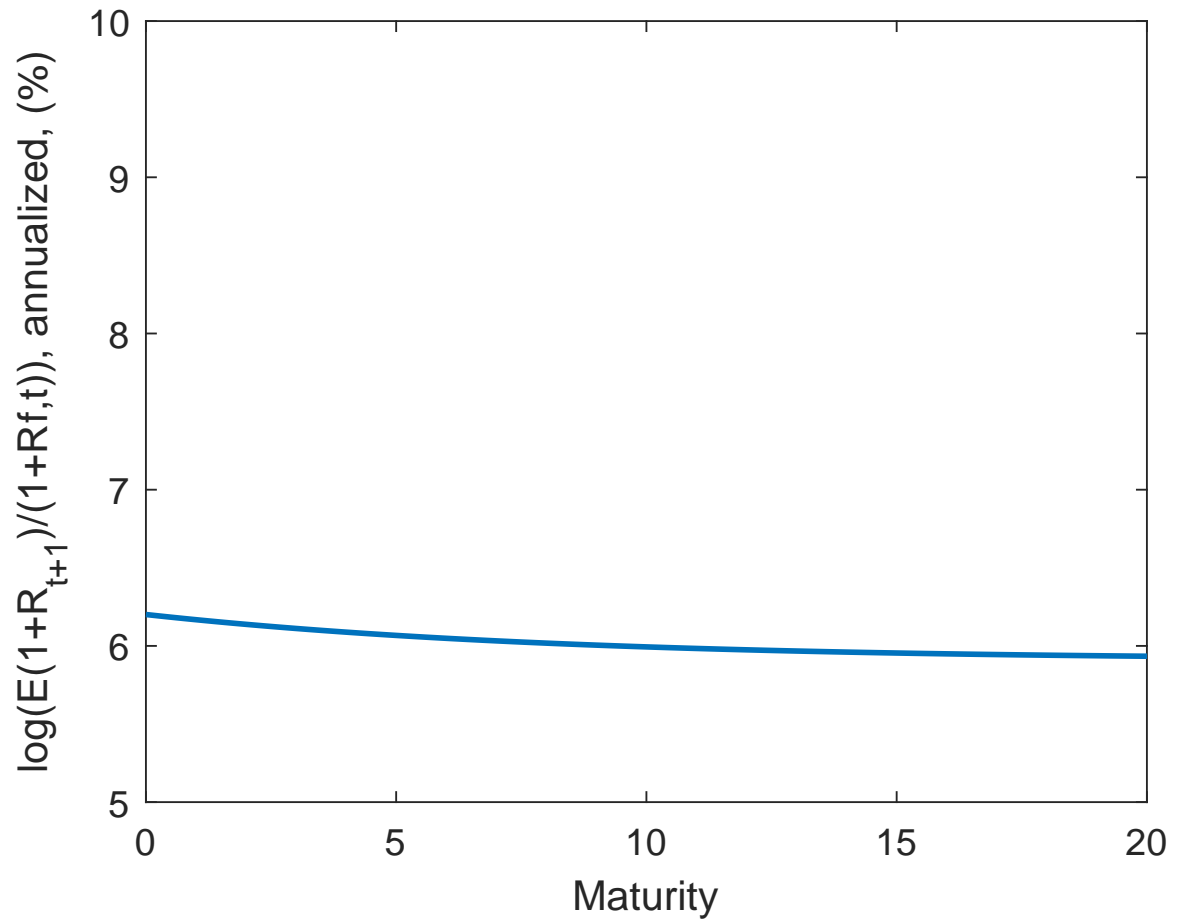
Note: This figure shows the implied term structure of equity premium at the steady state $p_t = \bar{p}$, with the Generalized Recursive Utility. I use an equity asset with leverage $\varphi = 3.5$ as the proxy for market portfolio.

Figure 1.9: Simulated mean return of entering a volatility future contract
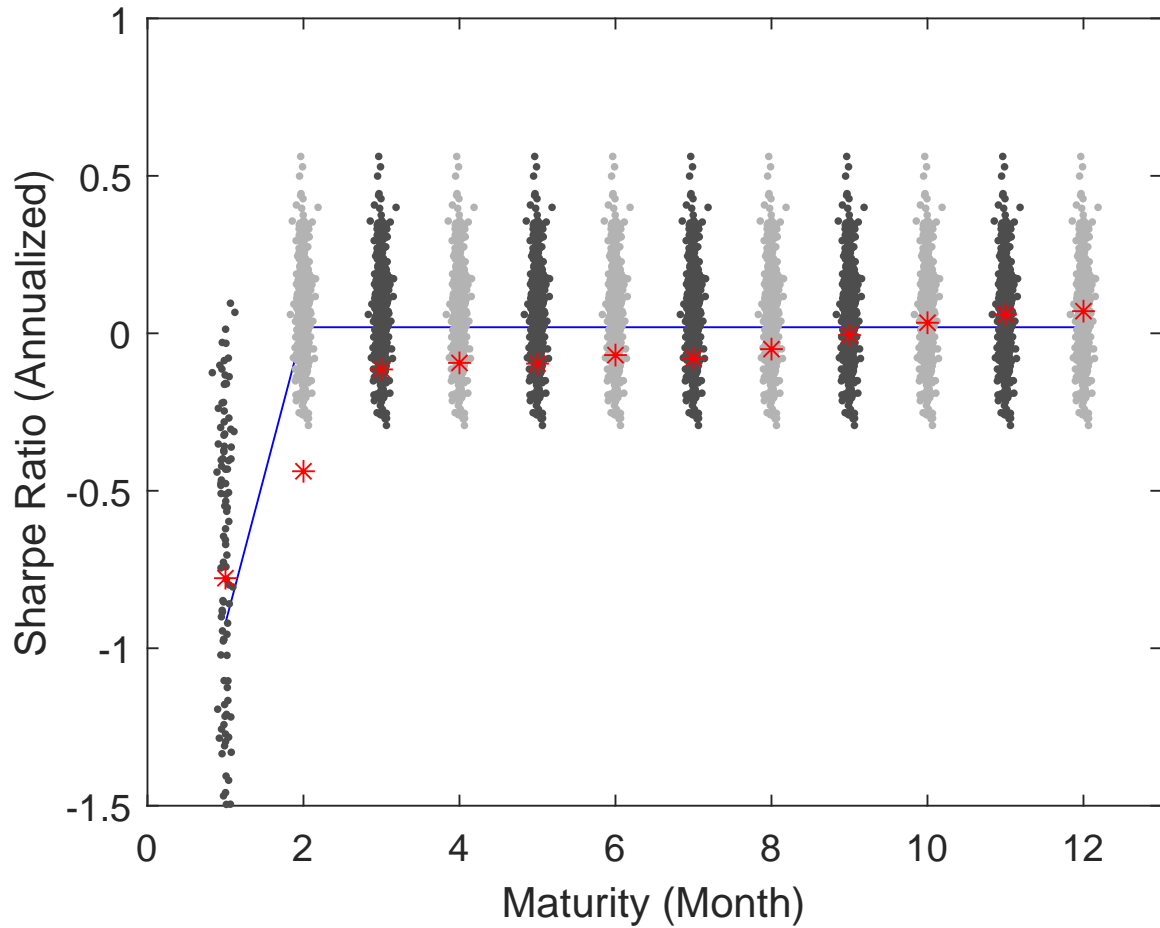
Note: The figure shows the Sharpe Ratios of investing in variance forward contracts with different maturities from 1996.01 to 2013.09. I also show the realized Sharpe Ratios from the simulated data. For each simulated sample, I add a small noise to the horizontal position of the dots to better illustrate the distribution of Sharpe Ratios across simulation samples.

CHAPTER 2 : A Model of Two Days: Discrete News and Asset Prices

2.1 Introduction

Since the work of Sharpe (1964) and Lintner (1965), the Capital Asset Pricing Model (CAPM) has been the benchmark model of the cross-section of asset returns. While the literature has explored many generalizations, the CAPM, with its simple and compelling structure and tight empirical predictions, remains the major theoretical framework for understanding the relation between risk and return. Recently, Savor and Wilson (2014) document a striking fact about the fit of the CAPM. Despite its poor performance in explaining the cross section overall, the CAPM does quite well on a subset of trading days, namely those days in which the Federal Open Market Committee (FOMC) or the Bureau of Labor Statistics (BLS) releases macroeconomic news.

Figure 2.1 reproduces the main result of Savor and Wilson (2014) using updated data. We sort stocks into portfolios based on market beta (the covariance with the market divided by market variance) computed using rolling windows. The figure shows the relation between portfolio beta and expected returns on announcement days and non-announcement days in the data. This relation is known as the security market line. On non-announcement days (the majority), the slope is indistinguishable from zero. That is, there appears to be no relation between beta and expected returns. This result holds unconditionally, and is responsible for the widely-held view of the poor performance of the CAPM. However, on announcement days, a strong positive relation between betas and expected returns appears. Moreover, portfolios line up well against the security market line, suggesting that the relation is not only strong, but that the total explanatory power is high. Finally, these results appear even stronger for fixed-income investments than for equities.

We summarize the facts as follows:

1. The equity premium is much higher on announcement days as opposed to non-

announcement days

2. The slope of the security market line is higher on announcement days than on non-announcement days. The difference is economically and statistically significant.

3. The security market line is essentially flat on non-announcement days.

4. Results 1 and 2 hold for Treasury bonds as well as for stocks.

In this paper, we build a frictionless model with rational investors that explains these findings. Our model is relatively simple and solved in closed form, allowing us to clearly elucidate the elements of the theory that are necessary to explain these results. Nonetheless, the model is quantitatively realistic, in that we explain not only these findings above, but also the overall risk and return of the aggregate stock market.

One important aspect of our model is that, despite the lack of frictions, investors do not have full information. Macroeconomic announcements matter for stock prices because they reveal information to investors concerning underlying shocks that have already occurred.[1] The information that is revealed matters to investors, which is why a premium is required to hold stocks on announcement days (the first finding). In our model, the information concerns the likelihood of economic disaster similar to the Great Depression or what many countries suffered following the 2008 financial crisis. We assume that this latent probability follows a Markov process.

We further assume that stocks have differential exposure to macroeconomic risk. We endogenously derive the exposure on stock returns from the exposure of the underlying cash flows. We also assume, plausibly, that there is some variability in the probability of disaster that is not revealed in the macroeconomic announcements. Stocks with greater exposure have endogenously higher betas, both on announcement and non-announcement days, than those with lower exposure. They have much higher returns, in line with the data, on

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[1]Another possibility is that macroeconomic announcements themselves create the risk perhaps because they reflect on the competence of the Federal Reserve. We do not consider that possibility here.

announcement days, because that is when a disproportionate amount of information is revealed (the second finding). Finally, the presence of disasters and of time-varying disaster risk implies that a linear relation between expected returns and betas does not hold. Stocks can have high variances, and covariances with the market, driven by time-varying disaster risk, without exposure to the actual disasters rising in proportion. This explains the third finding.

An extension of the model to bonds explains the fourth finding. We assume that some information that is revealed on announcements is informative about expected inflation. Bonds are exposed to announcements to a greater extent than equities. In the model, as well as in the data, betas on bonds rise dramatically on announcement days (they are near zero on non-announcement days), while equity betas do not.

We find that the presence of shifts in regime breaks the traditional relation between risk and return. This is important, because conventional measures of risk such as variance and covariance do not appear markedly higher on announcement days. Our model is consistent with this finding, because of the asymmetric nature of the shift in regime. Most likely, investors will learn that the economy continues to be in good shape and the risk of disaster remains low. There is a small probability, however, that they will learn that the economy is in worse shape than believed. In any given sample, positive announcements could easily appear in greater proportion than they would in population. A prediction of the model, then, is that implied volatilities should fall following announcements, even if realized volatilities do not (this is because implied volatilities measure the ex ante volatility assumed by investors). Moreover, implied volatilities should fall more for out-of-the-money put options than for at-the-money options. We confirm both of these predictions in the data.

While we focus on macroeconomic announcements, the techniques we employ could be used to address other types of predictable releases of discrete news (i.e. announcements). There is a vast empirical literature on announcement effects (La Porta et al., 1997; Fama, 1970), of which the literature on macro-announcements is a part. In this paper, we develop a set

of theoretical tools to handle the fact that announcements occur at deterministic intervals, and that a finite amount of information is released over a vanishingly small period of time. In so doing, we complement and extend findings of Ai and Bansal (2018), who derive necessary conditions on a utility function for the existence of an announcement premium as well as closed-form expressions for risk premia in continuous time under the assumption of conditional lognormality. As in their work, time just before and just after the announcement is connected through intertemporal optimization conditions. We show that these conditions form a set of boundary conditions for the dynamic evolution of prices in the interval between announcements. It is this insight that allows us to compute a cross-section of stock prices in closed form.

There is a very recent literature on modeling the macroannouncement premium, focusing on the Bansal and Yaron (2004) long-run-risk setting.[2] In work contemporaneous to the present paper, Ai et al. (2018) aims to rationalize the relative performance of the CAPM on announcement days in a production economy. In their model, total factor productivity follows an AR(1) process, about which agents receive normally distributed signals on announcement days. The fact that all shocks are normal simplifies the filtering problem. However, the evidence that daily returns exhibit no greater volatility on announcement versus non-announcement days, together with the option pricing results, is more in line with the regime-shift model that we propose. Ai et al. (2019) show that stocks whose implied volatilities change more around announcement days also have higher announcement premium, a result consistent with our model. In earlier work Andersen et al. (2003) show that foreign exchange markets respond more to negative announcements than positive ones, which is also consistent with our model. Cocoma (2018) builds a model to explain the Lucca and Moench (2015) evidence that much of the premium is realized prior to announcements.

The rest of the paper proceeds as follows. Section 2.2 discusses the model. Section 2.3 discusses the fit of the model to the data, and Section 2.4 concludes.

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[2]Savor and Wilson (2013) describe qualitatively how a long-run risk model might account for a macroannouncement premium.

2.2 A model of asset prices with macroeconomic announcements

In the section that follows, we describe the model. Section 2.2.1 gives the endowment and preferences, Section 2.2.2 the relation between cash flows and announcements, Section 2.2.3 describes state prices, Section 2.2.4 equity prices, and Section 2.2.5 nominal bonds. Unless otherwise stated, proofs are contained in the Appendices.

2.2.1 Endowment and preferences

We assume an endowment economy with an infinitely-lived representative agent. Aggregate consumption (the endowment) follows the stochastic process

$$\frac{dC_t}{C_{t^-}} = \mu dt + \sigma dB_{Ct} + \left(e^{-Z_t} - 1\right) dN_t, \tag{2.1}$$

where B_{Ct} is a standard Brownian motion and where N_t is a Poisson process. The diffusion term $\mu dt + \sigma dB_{Ct}$ represents the behavior of consumption during normal times. The Poisson term $\left(e^{-Z_t} - 1\right) dN_t$ represents rare disasters. The random variable $Z_t > 0$, is the decline in log consumption, given a disaster. We assume, for tractability, that Z_t has a time-invariant distribution, which we call ν; that is, Z_t is iid over time, and independent of all other shocks. We use the notation $\mathbb{E}_\nu$ to denote expectations taken over ν.

We assume the representative agent has recursive utility with EIS equal to 1, which gives us closed-form solutions up to ordinary differential equations. We use the continuous-time characterization of Epstein and Zin (1989) derived by Duffie and Epstein (1992). The following recursion characterizes utility V_t:

$$V_t = \max \mathbb{E}_t \int_t^\infty f(C_s, V_s) ds, \tag{2.2}$$

where

$$f(C_t, V_t) = \beta(1 - \gamma)V_t \left(\log C_t - \frac{1}{1 - \gamma} \log[(1 - \gamma)V_t] \right). \tag{2.3}$$

Here β represents the rate of time preference, and γ represents relative risk aversion. The case of $\gamma = 1$ collapses to time-additive (log) utility. When $\gamma \neq 1$, preferences satisfy risk-sensitivity, the characteristic that Ai and Bansal (2018) show is a necessary and sufficient condition for a nonzero announcement premium.

2.2.2   Scheduled announcements and the disaster probability

We assume that scheduled announcements convey information about the probability of a rare disaster (in what follows, we use the terminology probability and intensity interchangeably). The probability may also vary over time for exogenous reasons; this creates volatility in stock prices in periods that do not contain announcements.

To parsimoniously capture these features in the model, we assume the intensity of N_t is a sum of two processes, λ_{1t} and λ_{2t}.[3] The intensity λ_{1t} follows a latent Markov switching process. Following Benzoni et al. (2011), we assume two states, $\lambda_{1t} = \lambda^L$ (low) and $\lambda_{1t} = \lambda^H$ (high), with $0 \leq \lambda^L < \lambda^H$, and

$$P(\lambda_{1,t+dt} = \lambda^L | \lambda_{1t} = \lambda^H) = \phi_{H \to L} dt$$
$$P(\lambda_{1,t+dt} = \lambda^H | \lambda_{1t} = \lambda^L) = \phi_{L \to H} dt, \tag{2.4}$$

with $\phi_{H \to L}, \phi_{L \to H} > 0$. Note that $\phi_{H \to L}$ can be interpreted as the probability (per unit of time) of a switch from the high-risk state to the low-risk state and $\phi_{L \to H}$ is similarly, the probability of a switch from the low-risk state to the high-risk state.

Announcements convey information about λ_{1t}. Let T be the length of time between an-

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[3]Equivalently, decompose, N_t as

$$N_t = N_{1t} + N_{2t},$$

where N_{jt}, for $j = 1, 2$, has intensity λ_{jt}.

nouncements.[4] Define

$$\mathcal{A} \equiv \{t : t \bmod T = 0\},$$

$$\mathcal{N} \equiv \{t : t \bmod T \neq 0\}. \tag{2.5}$$

That is, $\mathcal{A}$ is the set of announcement times, and $\mathcal{N}$ is the set of non-announcement times. Note that $\mathcal{N}$ is an open set, so we can take derivatives of functions evaluated at times $t \in \mathcal{N}$.

Let p_t denote the probability that the representative agent places on $\lambda_{1t} = \lambda^H$. For $t \in \mathcal{N}$, assume

$$dp_t = ((1 - p_t)\phi_{L \to H} - p_t \phi_{H \to L})\, dt = (\phi_{L \to H} - p_t(\phi_{H \to L} + \phi_{L \to H}))\, dt. \tag{2.6}$$

This assumption implies that the agent learns only from announcements.[5] Outside of announcement periods, the agent updates based on (2.4). If the economy is in a low-risk state, which it is with probability $1 - p_t$, the chance of a shift to a high-risk over the next instant is $\phi_{L \to H}\, dt$. If the economy is in a high-risk state, which is with probability p_t, the chance of a shift to the low-risk state over the next instant is $\phi_{H \to L}\, dt$. Define

$$\bar{\lambda}_1(p_t) \equiv p_t \lambda^H + (1 - p_t)\lambda^L,$$

as the agent's posterior value of λ_{1t}. For simplicity, we assume announcements convey full information, that is, they perfectly reveal λ_{1t}.[6] We refer to announcements revealing λ_{1t} to

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[4]In the data, announcements are periodic, but, depending on the type of announcement, the period length is not precisely the same. Our assumption of an equal period length is a convenient simplification that has little effect on our results.

[5]Bayesian learning implies

$$dp_t = p_{t-} \left(\frac{\lambda^H - \bar{\lambda}_1(p_{t-})}{\bar{\lambda}_1(p_{t-})} \right) dN_{1t} + \left(-(p_{t-})(\lambda^H - \bar{\lambda}_1(p_{t-})) - (p_{t-})\phi_{H \to L} + (1 - p_{t-})\phi_{L \to H} \right) dt$$

(Wachter and Zhu, 2019b). The first term multiplying N_{1t} corresponds to the actual effect of disasters. The term $-p(\lambda^H - \bar{\lambda}_1(p))$ in the drift corresponds to the effect of no disasters. We abstract from these effects in (2.6). Because disasters will be very unlikely, the term $-p(\lambda^H - \bar{\lambda}_1(p))$ is small (agents do not learn much from the fact that disasters do not occur). In what follows, we compare the data to simulations that do not contain disasters. Therefore ignoring the Poisson term can be understood as an implementation of realization utility, defined by Cogley and Sargent (2008). We allow agents to learn from disasters; however, they do not forecast that they will learn from disasters.

[6]In effect, we assume the government body issuing the announcement has better information, perhaps because of superior access to data. Stein and Sunderam (2015) model the strategic problem of the announcer

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be λ^L as positive and those revealing it to be λ^H as negative. As we will show, this language is precise in the sense that the risk averse agent's utility rises when the announcement is positive.

It is useful to keep track of the content of the most recent announcement, because of the information it conveys about the evolution of the disaster probability. Define τ as the time elapsed since the most recent announcement:

$$\tau \equiv t \bmod T,$$

Let

$$\chi_t \equiv p_{t-\tau}. \tag{2.7}$$

That is, χ_t is the revealed probability of a high-risk state at the most recent announcement. By definition, $\chi \in \{0, 1\}$. The process for p_t is right-continuous with left limits. In the instant just before the announcement it is governed by (2.6). On the announcement itself, it jumps to 0 or 1 depending on the true (latent) value of λ_{1t}.

Under these assumptions, p_t has takes a simple form:

Lemma 2.1. *The probability assigned to the high-risk state satisfies $p_t = p(\tau; \chi_t)$, where $\tau \in [0, T)$, $\chi \in \{0, 1\}$ and*

$$p(\tau; \chi) = \chi e^{-(\phi_{H \to L} + \phi_{L \to H})\tau} + \frac{\phi_{L \to H}}{\phi_{H \to L} + \phi_{L \to H}} (1 - e^{-(\phi_{H \to L} + \phi_{L \to H})\tau}). \tag{2.8}$$

Proof. Equation 2.6 implies that p_t is deterministic between announcements. Moreover, p_t is memoryless in that it contains no information prior to the most recent announcement. Because the information revealed at the most recent announcement is summarized in χ, any solution for (2.6) takes the form $p_t = p(\tau; \chi)$, where $\tau = t \bmod T$ and $\chi \in \{0, 1\}$. It follows

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and investors, and show that announcements might reveal more information than a naive interpretation would suggest.

directly from (2.6) that

$$\frac{d}{d\tau}p(\tau;\chi) = -p(\tau;\chi)(\phi_{H\to L} + \phi_{L\to H}) + \phi_{L\to H}, \qquad \tau \in [0,T). \tag{2.9}$$

This has a general solution:

$$p(\tau;\chi) = K_\chi e^{-(\phi_{H\to L} + \phi_{L\to H})\tau} + \frac{\phi_{L\to H}}{\phi_{H\to L} + \phi_{L\to H}}, \tag{2.10}$$

where K_χ is a constant that depends on χ. The boundary condition $p(0;\chi) = \chi$ determines K_χ. $\qquad\square$

Equation 2.8 shows that p_t is a weighted average of χ, the probability of the high-risk state revealed in the most recent announcement, and $\frac{\phi_{L\to H}}{\phi_{L\to H} + \phi_{H\to L}}$, the unconditional probability of the high-risk state. As τ, the time elapsed since the announcement, goes from 0 to T, the agent's weight shifts from the former of these probabilities to the latter.

Agents forecast the outcome of the announcement based on p_t. The optimality conditions connecting the instant before the announcement to the instant after are crucial determinants of equilibrium. It is thus useful to define notation for p_t just before the announcement. Let

$$p_\chi^* = \lim_{\tau\uparrow T} p(\tau;\chi) \qquad \chi = 0,1 \tag{2.11}$$

Then p_0^* is the probability that the agent assigns to a negative announcement just before the announcement is realized, if the previous announcement was positive. If the previous announcement was negative, then the agent assigns probability p_1^*. The values of p_0^* and p_1^* follow from (2.8):

$$p_\chi^* = \chi e^{-(\phi_{H\to L} + \phi_{L\to H})T} + \frac{\phi_{L\to H}}{\phi_{H\to L} + \phi_{L\to H}}(1 - e^{-(\phi_{H\to L} + \phi_{L\to H})T}). \tag{2.12}$$

Not surprisingly, $0 < p_0^* < p_1^* < 1$:

Finally, we assume investors observe λ_{2t}, which follows

$$d\lambda_{2t} = \kappa(\bar{\lambda}_2 - \lambda_{2t})dt + \sigma_\lambda\sqrt{\lambda_{2t}}dB_{\lambda t}, \tag{2.13}$$

with $B_{\lambda t}$ a Brownian motion independent of B_{Ct}. The process for λ_{2t} is the same as the one assumed for the disaster probability in Wachter (2013).

In what follows, all expectations should be understood to be taken with respect to the agent's posterior distribution, unless noted otherwise.

2.2.3 Equilibrium state prices

In what follows, we will separate quantities into a component that remains constant over announcement intervals and a component that jumps over the announcement interval. This separation allows us to focus our theoretical results on the behavior of asset prices around announcements.[7] This separation also implies that the results in this section could in principle be applied to any underlying model for the equity premium, provided that it is based on the revelation of latent regimes on announcement days.

We first solve for the value function of the representative agent. We then use this result to solve for the stochastic discount factor, and finally to price assets. We show that the value function depends on five state variables: consumption C_t, probability of the high-risk state p_t, time since the announcement τ, the previously announced state χ, and the observed component of the disaster probability λ_2. The state variable p_t is technically redundant, as it is a function of χ and τ. However, separating it out helps to gain economic intuition.

Theorem 2.1. *In equilibrium, the agent's continuation value $V_t = J(C_t, p_t, \lambda_{2t}, \tau; \chi_t)$, with $\tau = t \bmod T$. Continuation value takes the form:*

$$J(C_t, p_t, \lambda_{2t}, \tau; \chi_t) = \frac{1}{1-\gamma}C_t^{1-\gamma}I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma}, \tag{2.14}$$

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[7]Quantitative implications depend on the behavior of the model at all time, however, and for this reason a full solution of the model is given in the Appendix.

with

$$I(p_t, \lambda_{2t}, \tau; \chi_t) = I_{\mathcal{A}}(p_t, \tau; \chi_t) I_{\mathcal{N}}(\lambda_{2t}) \tag{2.15}$$

for $I_{\mathcal{N}}$ constant over the announcement interval, and

$$I_{\mathcal{A}}(p_t, \tau; \chi_t) = e^{\zeta_{\chi} e^{\beta \tau} + b_p p_t}, \tag{2.16}$$

where

$$b_p = \frac{(\lambda^H - \lambda^L)\mathbb{E}_\nu \left[e^{(\gamma-1)Z_t} - 1 \right]}{(1 - \gamma)(\beta + \phi_{H \to L} + \phi_{L \to H})}, \tag{2.17}$$

and where ζ_χ, $\chi = 0, 1$ satisfy

$$e^{(1-\gamma)(\zeta_\chi e^{\beta T} + b_p p_\chi^*)} = p_\chi^* e^{(1-\gamma)(\zeta_1 + b_p)} + (1 - p_\chi^*) e^{(1-\gamma)\zeta_0}, \tag{2.18}$$

with probabilities p^* satisfying (2.12).

Each term in the value function has an economic interpretation. Note that the coefficient (2.17) multiplies the probability of a high-risk. This term depends on the difference between the disaster probability in the two states and the expected outcome for utility should a disaster occur. It also depends on $\beta + \phi_{H \to L} + \phi_{L \to H}$, which captures the persistence of effect on investor utility. The more patient the investor (the lower is β), and the more persistent the states (the lower the transition probabilities), the greater the effect.

The value function also depends on the announced state and the time since the last announcement. The recursion (2.18) derives from the condition that the value function prior to the announcement must equal its expected value following the announcement.

Using the analytical expressions in Theorem 2.1, we can show that the agent is always worse off should the high-risk state prevail. The proof follows from the condition (2.18), and the result that $\zeta_0 > \zeta_1 + b_p$ (which we prove in the Appendix).

Corollary 2.1. *For all risk averse agents, utility increases for positive announcements and*

decreases for negative ones. That is, for $\gamma > 0$, $I_{\mathcal{A}}$ increases when the announcement is positive and decreases when it is negative:

$$I_{\mathcal{A}}(1, 0; 1) < \lim_{\tau \uparrow T} I_{\mathcal{A}}(p_\chi^*, \tau; \chi) < I_{\mathcal{A}}(0, 0; 0) \qquad \chi = 0, 1.$$

Duffie and Skiadas (1994) link the equilibrium value function to the state-price density:

$$\pi_t = \exp\left\{ \int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds \right\} \frac{\partial}{\partial C} f(C_t, V_t). \tag{2.19}$$

We can think of π_t as the process for marginal utility. Standard calculations (see Lemma A.2.3) imply that

$$\pi_t = \beta \exp\left\{ \int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds \right\} C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma}, \tag{2.20}$$

Now define the function

$$M(\chi, \chi_-) \equiv \left(\frac{\exp\{\zeta_\chi + b_p \chi\}}{\exp\{e^{\beta T} \zeta_{\chi_-} + b_p p_{\chi_-}^*\}} \right)^{1-\gamma} \tag{2.21}$$

It follows from (2.20) and Theorem 2.1 that (2.21) is, with probability 1, the change in the state-price density over the announcement interval:

$$M(\chi_t, \chi_{t-}) = \frac{\pi_t}{\pi_{t-}}. \tag{2.22}$$

Following Ai and Bansal (2018), we refer to M as the announcement stochastic discount factor, or the announcement SDF.[8] To summarize:

Theorem 2.2 (Announcement SDF)**.** *The change in state-price density over the announce-*

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[8]There is a theoretical possibility of a disaster co-occurring with an announcement, in which case (2.22) would not hold. Because announcements occur on a set of measure zero, this is a zero probability event, and we can ignore it when calculating expectations and therefore prices and returns.

ment interval equals

$$M(\chi, \chi_-) = \left(\frac{\exp\{\zeta_\chi + b_p \chi\}}{\exp\{e^{\beta T} \zeta_{\chi_-} + b_p p^*_{\chi_-}\}} \right)^{1-\gamma}, \tag{2.23}$$

where (2.17) defines b_p and where ζ satisfies (2.18). We refer to $M(\chi, \chi_-)$ as the announcement SDF.

Negative announcements decrease utility for all risk averse agents. However, negative announcements only affect *marginal utility*, and hence the SDF, for agents with a preference for the timing of the resolution of uncertainty.:

Corollary 2.2. *The announcement SDF is > 1 for negative announcements and < 1 for positive ones, if $\gamma > 1$. If $\gamma < 1$, the inequalities reverse.*

A preference for early or late resolution of uncertainty is a special case of risk-sensitivity, as defined by Ai and Bansal (2018). In their setting, as in ours, risk-sensitivity is a necessary and sufficient condition for a nonzero announcement premium.

Using the announcement SDF, we can define risk-neutral probabilities of negative announcements, just before the announcement occurs.

$$\tilde{p}^*_\chi \equiv M(1, \chi) p^*_\chi \qquad \chi = 0, 1 \tag{2.24}$$

These are the risk-neutral counterparts of (2.11). When $\chi = 0$, (2.24) is the risk-neutral probability of a negative announcement, given that the previous announcement was positive. when $\chi = 1$, (2.24) is the risk-neutral probability of a negative announcement given that the previous announcement was negative.

Provided that $\gamma > 1$, risk-neutral probabilities of a negative announcements are higher than physical probabilities because of the effect of the announcement on state prices. Perhaps less obvious is the fact that, regardless of the value of γ, the risk-neutral probability of a negative announcement following a previous negative announcement is higher than the

risk-neutral probability of a negative announcement following a positive one. This means that a negative announcement is bad news in a dynamic sense: it affects not only dividends that are about to be realized, but also the agents' beliefs about future cash flows. This insight is important for equity pricing.

Theorem 2.3. *Let $\tilde{p}_1^*$ be the risk-neutral probability of a negative announcement, just prior to the announcement occurring, provided that the previous announcement was negative, and $\tilde{p}_0^*$ be the analogous quantity, provided that the previous announcement was positive. Then*

$$\tilde{p}_1^* > \tilde{p}_0^*.$$

This section shows that the stochastic discount factor undergoes a discrete change at the instant of an announcement, provided that there is a preference for the timing of the resolution of uncertainty. As we will show, any asset whose price undergoes a discrete change at the instant of an announcement will carry an announcement premium: investors must be compensated for the risk of holding the asset over any interval containing the announcement. If the price change is in the same direction as the SDF change, then the announcement premium is positive. Unlike the risk premium due to diffusion or Poisson risk, the announcement premium does not scale with the length of time over which the premium is measured. Even though the announcement occurs at an infinitesimal point in time, the premium is bounded away from zero.

Savor and Wilson (2014) document announcement premia, as measured by the slope of the security market line, for equities and bonds. In what follows, these are the focus of our analysis. We endogenously derive the price change for equities and for nominal bonds upon announcements and show that the magnitude of the effect matches the Savor and Wilson evidence. Savor and Wilson also document an announcement premium in foreign exchange markets. When currencies are sorted into portfolios based on interest rate differentials, higher beta portfolios have higher returns on announcement days, but there is no relation

(or a negative relation) on non-announcement days. While a full explanation of this finding is outside the scope of our paper, the above reasoning suggests that if high interest rate differential portfolios are those exposed to macroeconomic disasters (as captured in the $\lambda_1(t)$ regime) then these would have high announcement day returns, with little or no relation between risk and return on non-announcement days.

2.2.4 Equities

In this section, we derive properties of claims to dividends (that is, equity claims). Dividends follow a process that is similar to that of consumption:

$$\frac{dD_t}{D_{t^-}} = \mu dt + \sigma dB_{Ct} + (e^{-\varphi Z_t} - 1)dN_t. \tag{2.25}$$

To reduce the number of free parameters, we assume dividends have the same drift rate as consumption and the same loading on Brownian risk.[9] We allow dividends to display additional disaster sensitivity, where the parameter φ determines the degree of this sensitivity.[10] The parsimonious structure (2.25) will allow us to define a cross-section of equity claims in a simple and transparent way.

It is useful to first consider the price of a claim that pays a dividend at a fixed point in time (an equity strip). Let F denote the ratio of the price of this claim to the current dividend. By the Markov property,

$$F(p_t, \lambda_{2t}, \tau, s; \chi_t) = \mathbb{E}_t \left[\frac{\pi_{t+s}}{\pi_t} \frac{D_{t+s}}{D_t} \right]. \tag{2.26}$$

Theorem 2.4. *The price of an s-period equity strip (scaled by the current dividend) takes*

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[9]This specification does imply that dividends in the model will, during normal times, feature the same volatility as consumption. In the data, dividends are more volatile than consumption, but the normal-times correlation between dividends and consumption is low. Adding unpriced dividend risk would make it easier to explain the volatility of returns but would leave the results otherwise unchanged.

[10] Longstaff and Piazzesi (2004) show that earnings were far more affected than consumption during the Great Depression. Bianchi (2015), Bai et al. (2019) and Lu and Murray (2017) find that disaster sensitivity is an important determinant of risk and return in the cross-section.

the following form:

$$F(p_t, \lambda_{2t}, \tau, s; \chi_t) = F_{\mathcal{A}}(p_t, \tau, s; \chi_t) F_{\mathcal{N}}(\lambda_{2t}, s), \qquad (2.27)$$

where $F_{\mathcal{N}}$ is constant over the announcement interval, where

$$F_{\mathcal{A}}(p_t, \tau, s; \chi_t) = \exp\{g(\tau + s; \chi_t) + b_{\varphi p}(s)p_t\}, \qquad (2.28)$$

with

$$b_{\varphi p}(s) = \frac{(\lambda^H - \lambda^L)\mathbb{E}_\nu \left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right]}{\phi_{H \to L} + \phi_{L \to H}} \left(1 - e^{-(\phi_{H \to L} + \phi_{L \to H})s}\right), \quad s \geq 0. \qquad (2.29)$$

The function $g : \mathbf{R}_+ \times \{0, 1\}$ is the unique solution to the system of equations

$$e^{g(u;\chi) + b_{\varphi p}(u-T)p_\chi^*} = \tilde{p}_\chi^* e^{g(u-T;1) + b_{\varphi p}(u-T)} + (1 - \tilde{p}_\chi^*)e^{g(u-T;0)} \qquad (2.30)$$

with boundary condition $g(u; \cdot) = 0$, $u \in [0, T)$, for risk neutral probabilities $\tilde{p}_\chi^*$ satisfying (2.24).

Theorem 2.4 decomposes the price of an equity strip into a component affected by the announcement, and a component that is unaffected (which we describe in the Appendix). The component affected by the announcement depends on the probability of a high-risk state, the time since the announcement, the maturity of the strip, and the previous announcement. When an announcement occurs, the time since the last announcement jumps from T back to 0, the probability of a high risk state jumps to either 0 or 1, and the content of the previous announcement is updated to the content of the current announcement.

We can gain some intuition from the form of prices in Theorem 2.4. First, provided that $\varphi > 1$, $-\varphi Z_t < -Z_t$, implying that $b_{\varphi p}(s)$ is strictly negative and decreasing in s. The greater is the probability that the economy is in the high-risk state, the lower is the price, and the longer the maturity of the claim, the more pronounced the effect. Dividing this

term is the sum of the transition probabilities; thus, the more persistent the state, the greater the effect on the price.

Second, consider (2.30). This equation arises from the fact that the price just prior to the announcement must be the expected value of the price just after the announcement, under the risk-neutral probabilities. Essentially, the function g depends on the number of announcements until maturity, and the most recent announcement.[11] It keeps track of the cumulative effects of anticipated future announcements on the price. It also enables us to establish that equities increase in price upon positive announcements and decrease in price upon negative ones.

Corollary 2.3. *Assume $\varphi > 1$. Then the price of an equity strip with positive maturity on the announcement date increases when the announcement is positive and decreases when the announcement is negative. That is*

$$F_{\mathcal{A}}(1,0,s;1) < \lim_{\tau \uparrow T} F_{\mathcal{A}}(p_{\chi},\tau,s;\chi) < F_{\mathcal{A}}(0,0,s;0) \qquad \chi = 0,1,$$

for $s > 0$.

This result relies on the dynamic and recursive implications of announcements, as expressed in the risk-neutral probabilities of Theorem 2.3. When $\varphi > 1$, a higher probability of disaster lowers the value of the dividend claim (this effect operates through $b_{\phi p}$). Consider first the claim with one announcement prior to maturity. Clearly, this claim will fall in price if the announcement is negative and rise if it is positive. Now consider the the claim with two announcements prior to maturity. Note that $\tilde{p}_1^* > \tilde{p}_0^*$, in other words, the probability of a negative announcement is higher if the previous announcement was negative than if it was positive. Thus the asset will fall in price if the second-to-last announcement prior to maturity is negative and rise if it is positive. Iteratively applying this reasoning (see

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[11]The fact that g depends on the sum $s + \tau$ rather than s and τ by themselves indicates that it does not matter how far away in time the next announcement is. As time goes by, τ increases, s decreases, so that $s + \tau$ remains constant until (upon the announcement) τ jumps back to zero.

Appendix A.2.2 for details) leads to the result above.

We apply these ideas to compute the announcement premium. Consider first the change in price (the return) for an equity strip around an announcement. This announcement return depends on the announced state χ, the previously announced state χ_-, and the strip maturity s:

$$r_{\mathcal{A}}(\chi, \chi_-, s) \equiv \frac{F_{\mathcal{A}}(\chi, 0, s; \chi)}{\lim_{\tau \uparrow T} F_{\mathcal{A}}(p^*_{\chi_-}, \tau, s; \chi_-)} = \frac{e^{g(s; \chi) + b_{\phi p}(s)\chi}}{e^{g(T+s; \chi_-) + b_{\phi p}(s)p^*_{\chi_-}}} \tag{2.31}$$

Note that the change in price reduces to (2.31) because of the decomposition (2.27), and because s and D_t are continuous variables (with probability 1). Equations 2.30 and 2.31 together imply the intuitive conclusion that the expected (gross) announcement return under the risk-neutral probability must equal 1:

$$\tilde{p}^*_{\chi_-} r_{\mathcal{A}}(1, \chi_-, s) + (1 - \tilde{p}^*_{\chi_-}) r_{\mathcal{A}}(0, \chi_-, s) = 1. \tag{2.32}$$

Now consider the expected announcement return under the physical probability:

$$\bar{r}_{\mathcal{A}}(\chi_-, s) \equiv p^*_{\chi_-} r_{\mathcal{A}}(1, \chi_-, s) + (1 - p^*_{\chi_-}) r_{\mathcal{A}}(0, \chi_-, s) \tag{2.33}$$

Subtracting (2.33) from (2.32) implies the following expression for the announcement premium:

$$\begin{aligned} \bar{r}_{\mathcal{A}}(\chi_-, s) - 1 &= (\tilde{p}^*_{\chi_-} - p^*_{\chi_-})(r_{\mathcal{A}}(0, \chi_-, s) - r_{\mathcal{A}}(1, \chi_-, s)) \\ &= (\tilde{p}^*_{\chi_-} - p^*_{\chi_-}) \frac{e^{g(s; 0)} - e^{g(s; 1) + b_{\phi p}(s)}}{e^{g(T+s; \chi_-) + b_{\phi p}(s)p^*_{\chi_-}}}, \end{aligned} \tag{2.34}$$

where (2.34) follows from (2.31). As long as the risk-neutral probability of a negative announcement is greater than the physical probability, the announcement premium is positive. Corollary 2.2 and (2.24) show that this will be the case as long as $\gamma > 1$ (namely, if the agent has a preference for early resolution of uncertainty). This corresponds to the finding, in Ai

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and Bansal (2018), that risk-sensitive preferences are a necessary and sufficient condition for a nonzero announcement premium.

Another way to write the announcement premium is in terms of the co-movement of the price with the SDF around announcements:

Corollary 2.4. *The announcement premium on the s-period equity strip equals*

$$
\begin{aligned}
\mathbb{E}_{t^-}\left[r_{\mathcal{A}}(\chi_t, \chi_{t^-}, s) - 1\right] &= -\mathbb{E}_{t^-}\left[(r_{\mathcal{A}}(\chi_t, \chi_{t^-}, s) - 1)\left(M(\chi_t, \chi_{t^-}) - 1\right)\right] \quad (2.35) \\
&= -\mathrm{Cov}_t(r_{\mathcal{A}}(\chi_t, \chi_{t^-}, s), M(\chi_t, \chi_{t^-})).
\end{aligned}
$$

Moreover, provided $s > T - \tau$,

1. *The announcement premium is strictly positive if $\varphi > 1$ and $\gamma > 1$, or if $\varphi < 1$ and $\gamma < 1$.*

2. *The announcement premium is strictly negative if $\varphi < 1$ and $\gamma > 1$, or if $\varphi > 1$ and $\gamma < 1$.*

3. *The announcement premium is equal ot zero if either γ or φ equals 1.*

It may first appear that Corollary 2.4 and (2.34) refer merely to the existence of an announcement premium; it appears to say nothing of the magnitude. However, implicit in Corollary 2.4 is a very strong statement about the magnitude of the announcement premium. Equation 2.35 gives an absolute number; it does not scale with the size of the interval containing the announcement. By contrast, the risk premium on the equity strip (or on any other asset) at a non-announcement time is proportional to the time interval, and is infinitesimal over infinitesimal intervals. The key difference between the announcement day and the non-announcement day is that the announcement day provides a discrete amount of news: the agent anticipates receiving news on this day with probability 1. At any other day, there is either a tiny amount of news for sure (in the case of Brownian risk), or a large amount of news with a tiny probability (in the case of Poisson risk). The Brownian and

Poisson shocks provide risk that is continuous, whereas announcement news is discrete.

Because of the discrete quantity of news released on the announcement day, the daily return on an announcement day can easily be an order of magnitude higher than on a non-announcement day. Our numerical evaluation in the next section makes this statement precise. In this numerical evaluation, we will consider claims to continuous streams of dividends. These will represent stock prices; we will consider a cross-section with varying parameters φ. For the remainder of this section, we specify how pricing works for a fixed φ, and postpone discussion of the cross-section until Section 2.3. No-arbitrage gives us the value of the stock:

$$S_t = \mathbb{E}_t \int_t^\infty \frac{\pi_s}{\pi_t} D_s \, ds = \int_t^\infty \mathbb{E}_t \frac{\pi_s}{\pi_t} D_s \, ds. \tag{2.36}$$

Clearly, the price of the stock is an integral of the prices of the underlying strips.

Lemma 2.2. *Let S_t be the time-t price of an asset paying the dividend process (2.25). Then*

$$S\big(D_t, p_t, \lambda_{2t}, \tau; \chi_t\big) = \int_0^\infty D_t F(p_t, \lambda_{2t}, \tau, s; \chi_t) ds, \tag{2.37}$$

Proof. The result follows directly from Theorem 2.4 and the no-arbitrage condition (2.36).

□

The stock price moves in the same direction as the underlying strips, given an announcement:

Corollary 2.5. *Assume that $\varphi > 1$. Then $S\big(D_t, p_t, \lambda_{2t}, \tau; \chi_t\big)$ increases when the announcement is positive and decreases when the announcement is negative. That is,*

$$S(D, 1, \lambda_2, 0; 1) < \lim_{\tau \uparrow T} S(D, p_{t^-}, \lambda_2, \tau; \chi_{t^-}) < S(D, 0, \lambda_2, 0; 0).$$

Proof. The result follows directly from Corollary 2.3 and from Lemma 2.2. □

The expression for announcement premium on the stock is necessarily more complicated

than the announcement premium on the equity strip. However, the sign of the premium is clearly the same.

Corollary 2.6. *Consider an asset paying dividends given by (2.25),*

1. *The announcement premium is strictly positive if $\varphi > 1$ and $\gamma > 1$, or if $\varphi < 1$ and $\gamma < 1$.*

2. *The announcement premium is strictly negative if $\varphi < 1$ and $\gamma > 1$, or if $\varphi > 1$ and $\gamma < 1$.*

3. *The announcement premium is equal ot zero if either γ or φ equals 1.*

Proof. Corollaries 2.2 and 2.5 show that increases in S coincide with $M > 1$ in case 1, whereas increases in S coincide with $M < 1$ in case 2. Finally, in case 3, either $M = 1$ or S does not change given an announcement. $\square$

2.2.5 Nominal bonds

The pricing of nominal bonds requires an assumption on inflation. For simplicity, in event of disaster we assume that inflation rises by the same amount – in percentage terms – that consumption declines. Thus, in event of disaster, bonds will suffer a loss equal to the percent decline in consumption. The price level P_t follows

$$\frac{dP_t}{P_{t^-}} = q_t dt + \sigma_p dB_{Pt} + \left(e^{Z_t} - 1\right) dN_t. \tag{2.38}$$

Expected normal-times inflation, q_t, follows a mean-reverting process:

$$dq_t = \kappa_q(\bar{q}_t - q_t)dt + \sigma_q dB_{qt}, \tag{2.39}$$

where B_{Pt} and B_{qt} are independent Brownian motion processes that are also independent of B_{Ct} and $B_{\lambda t}$, and where $\kappa_q > 0$.

Equation 2.39 implies that expected inflation mean-reverts to a time-varying $\bar{q}_t$, which follows a Markov-switching process. Consistent with the data (Dergunov et al., 2018), we assume that high risk to consumption and elevated expected inflation co-occur. That is, $\bar{q}_t = \bar{q}^H$ when $\lambda_{1t} = \lambda^H$ and $\bar{q}_t = \bar{q}^L$ when $\lambda_{1t} = \lambda^L$, with $\bar{q}^H > \bar{q}^L$. This implies that the macro-announcements, which reveal the latent disaster-probability state, also reveal expected inflation. Given that macro-announcements are often ostensibly about inflation, this seems reasonable.[12]

The nominal state-price density, which prices payoffs written in nominal terms, equals

$$\pi_t^\$ = \frac{\pi_t}{P_t}. \tag{2.40}$$

Thus if $F^\$(p_t, q_t, \tau, s; \chi_t)$ denotes the price of a default-free nominal bond with s years to maturity and a face value of 1, no-arbitrage implies

$$F^\$(p_t, q_t, \tau, s; \chi_t) = \mathbb{E}_t \left[\frac{\pi_{t+s}^\$}{\pi_t^\$} \right]. \tag{2.41}$$

Note that realized inflation stays constant over the announcement interval, so the nominal announcement SDF equals the real announcement SDF.

Theorem 2.5. *The nominal price of an s-period nominal bond satisfies the following decomposition*

$$F^\$\left(p_t, q_t, \tau, s; \chi_t\right) = F_\mathcal{A}^\$(p_t, \tau, s; \chi_t) F_\mathcal{N}^\$(q_t, s) \tag{2.42}$$

where $F_\mathcal{N}^\$$ is constant over the announcement interval, and where

$$F_\mathcal{A}^\$(p_t, \tau, s; \chi_t) = \exp\left\{ g^\$(\tau + s; \chi_t) + b_p^\$(s) p_t \right\} \tag{2.43}$$

---

[12]We continue to assume that the agent infers the state only from announcements, and not from inflation observations.

with $b_p^\$$ satisfying

$$\frac{d}{ds}b_p^\$(s) = -(\phi_{H\to L} + \phi_{L\to H})b_p^\$(s) + \kappa_q \left(\bar{q}^H - \bar{q}^L\right) b_q^\$(s), \tag{2.44}$$

with boundary condition $b_p^\$(0) = 0$, and with $b_q^\$(s) = \frac{1}{\kappa_q}(e^{-\kappa_q s} - 1)$. The function $g^\$$:
$\mathbf{R}_+ \times \{0,1\}$ *is the unique solution to the system of equations*

$$e^{g^\$(u;\chi)+b_p^\$(u-T)p_\chi^*} = \tilde{p}_\chi^* e^{g^\$(u-T;1)+b_p^\$(u-T)} + (1 - \tilde{p}_\chi^*)e^{g^\$(u-T;0)}, \tag{2.45}$$

with boundary condition $g^\$(u; \cdot) = 0$, $u \in [0, T)$, for risk neutral probabilities $\tilde{p}_\chi^*$ satisfying
(2.24).

An increase in risk coincides with an increase in inflation. For this reason, bond prices fall
when the announcement is negative and rise when it is positive:

Corollary 2.7. *The price of a zero-coupon bond with positive maturity on the announce-*
ment date increases when the announcement is positive and decreases when the announce-
ment is negative. That is

$$F_{\mathcal{A}}^\$(1, 0, s; 1) < \lim_{\tau \uparrow T} F_{\mathcal{A}}^\$(p_\chi, \tau, s; \chi) < F_{\mathcal{A}}^\$(0, 0, s; 0) \qquad \chi = 0, 1,$$

for $s > 0$.

Because bond prices fall when the announcement is negative, bonds have an announcement
premium, provided that there is a preference for early resolution of uncertainty. Define the
announcement return on the s-period bond as:

$$r_{\mathcal{A}}^\$(\chi, \chi_-, s) = \frac{F_{\mathcal{A}}^\$(p_\chi^*, 0, s; \chi)}{\lim_{\tau \uparrow T} F_{\mathcal{A}}^\$(p_{\chi_-}^*, \tau, s; \chi_-)}$$

Note that, with probability 1, realized inflation does not change over the announcement
interval, and therefore the nominal announcement SDF can be treated as if it were identical

to the announcement SDF defined in (2.23). The remainder of the analysis proceeds in a manner analogous to that of equities.

Corollary 2.8. *The announcement premium on the s-period nominal bond equals*

$$
\begin{aligned}
\mathbb{E}_{t-}\left[r_{\mathcal{A}}^{\$}(\chi_t, \chi_{t-}, s) - 1\right] &= -\mathbb{E}_{t-}\left[\left(r_{\mathcal{A}}^{\$}(\chi_t, \chi_{t-}, s) - 1\right)\left(M(\chi_t, \chi_{t-}) - 1\right)\right] \quad (2.46) \\
&= \mathrm{Cov}_t(r_{\mathcal{A}}^{\$}(\chi_t, \chi_{t-}, s), M(\chi_t, \chi_{t-}))
\end{aligned}
$$

Moreover, provided $s > T - \tau$, the bond announcement premium is positive if $\gamma > 1$, negative if $\gamma < 1$, and zero if $\gamma = 1$.

2.3 Quantitative results

We start by replicating the evidence of Savor and Wilson (2014) in an extended sample. Section 2.3.1 describes the data and Section 2.3.2 the empirical findings. We then simulate repeated samples from the model described in the previous section. Section 2.3.3 describes the calibration of our model and Section 2.3.4, the simulation results.

2.3.1 Data

We obtain daily stock returns from the Center for Research in Security Prices (CRSP). We consider individual stocks traded on NYSE, AMEX, NASDAQ and ARCA from January 1961 to September 2016. In addition, we also use the daily market excess returns and risk-free rate provided by Kenneth French. Data for bond returns comes from the CRSP fixed-term indices file. Each month, for each target maturity, we choose a Treasury bond with a maturity closest to the target maturity and compute daily returns on this bond. The scheduled announcement dates before 2010 are provided by Savor and Wilson (2014). Following their approach, we add target-rate announcements of the FOMC and inflation and employment announcements of the BLS for the remaining dates.

We define the daily excess return to be the daily (level) return of a stock (or bond) in excess

of the daily return on the 1-month Treasury bill. We estimate covariances on individual stock returns with the market return using daily data and 12-month rolling windows. We include stocks which are available for trading on 90% or more of the trading days. At the start of each trading month, we sort stocks by estimated betas, and create deciles. We then form value-weighted portfolios of the stocks in each deciles, and compute daily excess returns.

2.3.2   Empirical findings

Table 2.1 presents summary statistics on the ten beta-sorted portfolios. For each portfolio j, $j = 1, \ldots, 10$, we use the notation $E[RX^j]$ to denote the mean excess return, σ^j the volatility of the excess return, and β^j the covariance with the value-weighted market portfolio divided by the variance of the market portfolio. Table 2.1 shows statistics for daily returns computed over the full sample, over announcement days, and over non-announcement days.[13] There is a weak positive relation between full-sample returns and market betas. On non-announcement days, there is virtually no relation between betas and expected returns. However, on announcement days, there is a strong relation between beta and expected returns.

Figure 2.1 shows average daily excess returns in each of the ten portfolios, plotted against the betas on the portfolios for announcement days (diamonds) and non-announcement days (squares). Also shown is the fitted line on both days. This relation, known as the security market line, is strongly upward-sloping on announcement days, but virtually flat on non-announcement days.

Table 2.2 shows that Treasury bonds also feature much higher returns on announcement days. On non-announcement days, the beta on Treasury bond returns with respect to the market is negative, and there is no discernable relation between risk and return. How-

---

[13]Betas and volatilities are computed in the standard way, as central second moments. An announcement-day volatility therefore is computed as the mean squared difference between the announcement return and the mean announcement return. Announcement-day betas are computed analogously.

ever, this beta is strongly positive on announcement days, and a clear security market line emerges.

2.3.3 Calibration

We now describe the calibration of the model in Section 2.2. We choose preference parameters, normal-times consumption parameters, the mean reversion for λ_{2t} (κ), and the volatility parameter (σ_λ) as in Wachter (2013). For simplicity, we assume that, when the economy is in the low-risk state, the intensity λ_{1t} is zero, that is $\lambda^L = 0$. We choose $\phi_{L \to H} = 0.10$ (that is, 10% per annum) so that switching to the high-risk state is unusual. Should the high-risk state occur, it is persistent – there is a 33% chance of switching back to the low state ($\phi_{H \to L} = 0.33$). The unconditional probability of the high-risk state in our calibration is $\phi_{L \to H}/(\phi_{L \to H} + \phi_{H \to L}) = 23\%$. We then choose $\bar{\lambda}_2$ and λ^H so that the average disaster probability is 3.6%, as in Barro and Ursúa (2008). The values $\bar{\lambda}_2 = 2.1\%$ and $\lambda^H = 6.2\%$ satisfy this requirement, while ensuring a reasonable announcement premium and equity premium. The model implies that the regime switch process (namely λ_{1t}) is responsible for 40% of disasters. We assume a multinomial distribution for the outcomes Z_t. This multinomial distribution, which also comes from Barro and Ursúa (2008), is the same as in Wachter (2013).

We choose the disaster sensitivities φ_j to obtain a reasonable spread in betas, and so that the average exposure to disasters is three times the consumption claim (this is a standard calibration, see, e.g. Bansal and Yaron (2004)). We use the fact that betas depend primarily on the exposure to $\lambda_2(t)$, which, as we show in Appendix A.2.2, is approximately proportional to $\mathbb{E}_\nu\left[e^{\gamma Z}(e^{-\varphi Z} - e^{-Z})\right]$.[14] We solve for φ_j such that

$$\frac{\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi_j Z_t} - e^{-Z_t})\right]}{\mathbb{E}_\nu\left[e^{\gamma Z}(e^{-3Z_t} - e^{-Z_t})\right]} = k, \qquad k \in \{0.2, 0.35, \ldots, 1.85\}.$$

---

[14]Note that $\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right]$ is the last term in the ordinary differential equation (A.2.49) for the sensitivity $b_{\varphi\lambda}(s)$. It therefore determines the magnitude of this sensitivity as φ varies.

This yields 12 firm types, and a spread in betas that is sufficiently wide to compare model with data.

Normal-times inflation parameters, σ_q, σ_P, and κ_q, are as in Tsai (2016). These roughly determine the volatility of inflation, the persistence, and the volatility and persistence of the nominal interest rate. Given these parameters, we choose expected inflation in each regime to match normal-times expected inflation in the data. Table 2.4 reports parameter choices.

2.3.4 Simulation method

To evaluate the fit of the model, we simulate 500 artificial histories, each of length 50 years (240×50 days). We assume that announcements occur every 10 trading days. For each history, we simulate a burn-in period, so that we start the history from a draw from the stationary distribution of the state variables. We simulate the model using the true (as opposed to the agents') distribution. We report statistics for the full set of sample paths.

While time is continuous in our analytical model, it is necessarily discrete in our simulations. We simulate the model at a daily frequency to match the frequency of the data. We compute end-of-day prices, and assume the announcement occurs in the middle of a trading day. We will use the notation a and n to denote announcement and non-announcement days respectively.

Given a series of state variables and of shocks, we compute returns as follows. For each asset j, define the price-dividend ratio

$$G^j(p_t, \lambda_{2t}, \tau; \chi_t) = \frac{S^j(D_t, p_t, \lambda_{2t}, \tau; \chi_t)}{D_t} = \int_0^\infty F^j(p_t, \lambda_{2t}, \tau, s; \chi_t)\, ds$$

We approximate the daily return as

$$
\begin{aligned}
R_{t,t+\Delta t}^{j} &\approx \frac{S_{t+\Delta t}^{j} + D_{t+\Delta t}^{j}\Delta t}{S_{t}^{j}} \\
&= \frac{D_{t+\Delta t}^{j}G_{t+\Delta t}^{j} + D_{t+\Delta t}^{j}\Delta t}{D_{t}^{j}G_{t}^{j}} \\
&= \frac{D_{t+\Delta t}^{j}}{D_{t}^{j}}\frac{G_{t+\Delta t}^{j} + \Delta t}{G_{t}^{j}} \\
&\approx \exp\left\{\mu\Delta t - \frac{1}{2}\sigma^2\Delta t + \sigma(B_{C,t+\Delta t} - B_{C,t}) + \phi Z(N_{t+\Delta t} - N_t)\right\}\frac{G_{t+\Delta t}^{j} + \Delta t}{G_{t}^{j}},
\end{aligned}
$$

$$(2.47)$$

where Z is drawn from the specified multinomial distribution, $\Delta t = 1/240$, and where $N_{t+\Delta t} - N_t = 1$ with probability $(\lambda_{1t} + \lambda_{2t})\Delta t$ and zero otherwise. The risk free rate is approximated by

$$
R_{ft} = \exp(r_{ft}\Delta t). \tag{2.48}
$$

The daily excess return of asset j is then

$$
RX_{t,t+\Delta t}^{j} = R_{t,t+\Delta t}^{j} - R_{ft}. \tag{2.49}
$$

We define the value-weighted market return just as in the data, namely we take a value-weighted portfolio of returns. We assume that the assets have the same value at the beginning of the sample. Because the assets all have the same loading on the Brownian shock and the same drift, and conditional on a history not containing rare events, the model implies a stationary distribution of portfolio weights. Given a time series of excess returns on firms (which, because we have no idiosyncratic risk, we take as analogous to portfolios), and a time series of excess returns on the market, we compute statistics exactly as in the data.

Before discussing the implications of our model for returns around announcement days, we confirm that the model replicates the main findings in Wachter (2013): namely that it can match the equity premium, the average riskfree rate, and the predictability in stock returns. We show these and their data equivalents in Table 2.10. The main difference between this

model and the earlier one is that this model produces negative return skewness, as in the data.[15]

2.3.5 The equity premium and the riskfree rate on announcement and non-announcement days

The model captures the time series result that most of the equity premium is realized on announcement days (Savor and Wilson, 2013; Lucca and Moench, 2015).[16] Table 2.5 shows that the average market return is far higher on announcement days versus non-announcement days, both in the model and in the data. On the other hand, the increase in volatility is small. While the median increase in volatility is greater in the model than in the data, the data is well-within the 90 percent confidence intervals, reflecting the fact that a substantial fraction of the samples feature no increase in volatility on non-announcement days at all.[17]

Savor and Wilson (2013) also show that riskfree interest rates are lower on announcement days as compared with non-announcement days. Our model can account for the sign and magnitude of this result. The interest rate in the model equals:

$$r_t = \beta + \mu - \gamma\sigma^2 + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right)\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-Z_t} - 1)\right].$$

and is a decreasing function of the disaster probability. Bonds of non-infinitesimal maturity are a hedge against disaster risk (because they go up in price when the interest rate declines

---

[15]One might ask whether this difference comes from the imperfect information or from the regime-switching process, since these are both ways in which the current model differs from that of Wachter (2013). Under our calibration, it comes from the regime-switching process. Figures 5 and 6 of the Online Appendix show that results for full-sample moments do not change in the limit as the model approaches full information.

[16]Lucca and Moench (2015) focus on a later sample period and on scheduled FOMC announcements. They show that the premium is realized on the announcement day, but before the actual announcement. While outside the scope of our model, this finding could be rationalized in a similar model in which information about the disaster regime leaks with some probability in the interval prior to the announcement, and then is fully realized on the announcement itself.

[17]The fact that announcement-day volatility does not increase is a key feature, along with options evidence to be presented in Section 2.3.8, that distinguishes our model from competing risk-premium-based explanations. Any explanation based on normally-distributed risk would imply much greater volatility on announcement days.

(see Section 3.2 of the Online Appendix). They therefore feature a negative risk premium that, through the same mechanism as equities, is greater in magnitude on announcement days as compared with non-announcement days. The difference in the 30-day yield between announcements and non-announcements is 40 basis points in the model, as compared with 80 basis points in the data. To summarize: short-term interest rates decline in the model, as in the data, and the declines are of similar magnitude.[18]

2.3.6 The cross-section of beta-sorted portfolios on announcement and non-announcement days

Figure 2.2 shows our main result: the model's ability to match the differential beta-return relation on announcement days. We overlay the simulated statistics on the empirical statistics from Figure 2.1.[19] Each dot on the figure represents a statistic for one firm, for one simulated sample. Blue dots show pairs of average excess returns and betas on announcement days, while grey dots show pairs on non-announcement days. The figure shows that average returns on announcement days in the model are much higher than on non-announcement days. Furthermore, average returns vary with beta on announcement days in the model, whereas they do not on non-announcement days.

Figure 2.4 further clarifies the relation between the announcement and non-announcement days in the model by showing medians and interquartile ranges from the full set of simulated samples. Median returns closely match the data, whereas interquartile ranges show that the vast majority of samples with announcements can be clearly distinguished from those of non-announcements.

How is it that the model can explain these findings? Announcements convey important news about the distribution of future outcomes in the economy. On that day, it is possible

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[18]We ignore, for simplicity, the effect of inflation uncertainty on a short-term Treasury bill. This is the approach usually taken in the literature. The presence of an average inflation term would not affect this calculation.

[19]This figure reports simulated statistics from samples without disasters. As we show below, this does not affect inference from the model.

that a high-risk state of the economy could be revealed. If the high-risk state is realized, not only will asset values be affected, but the marginal utility of economic agents will rise. Thus investors require a premium to hold assets over the risky announcement period.

In our model, some assets have cash flows that are more sensitive than others. The sensitivity parameter φ_j, while not the same as the beta, is closely related. Assets with high φ_j have a greater dividend response to disasters. Their prices thus move more with changes in the disaster probability, and in particular with λ_{1t} and λ_{2t}.[20] The value-weighted market portfolio also moves with the disaster probability, and thus the higher is φ_j (over the relevant range), the higher is the return beta with the market, both on non-announcement days (which reveal information about λ_{2t}, and on announcement days, which reveal additional information about λ_{1t}.

Panel A of Table 2.6 shows the security market line for equities on announcement and non-announcement days. We run the regression

$$\hat{E}[RX_t^j \mid t \in i] = \delta_i \beta_i^j + \text{error}, \tag{2.50}$$

where $i = a$ (announcement days) or n (non-announcement days). The regression slope δ_i is the slope of the security market line. It is simultaneously a measure of risk and return, and a measure of the daily market risk premium. Table 2.6 shows an economically significant difference between the slope on announcement and non-announcement days in the data, a difference that is matched in the model.[21] Thus the model predicts a relation between risk and return on both announcement and non-announcement days, but because the risk is so much greater on announcement days, the premium, and therefore the spread in expected

---

[20]This implies that more volatile stocks should have higher announcement-day returns. Savor and Wilson (2013) show that this true empirically. Table 2.7 shows that it is true in the model.

[21] One concern is that there might be a positive correlation between the slope difference and the volatility difference (Table 2.5 shows that the volatility difference is within a 90% confidence interval implied by the model) in data simulated from the data. If this were true, samples with a large difference in SML slopes would also be those with a (counterfactually) high difference in volatility. In fact, there is almost no correlation between these statistics, and a joint test (see Figure 3 of the Online Appendix) shows that the data has a p-value of greater than 0.5.

returns between low and high-sensitivity assets, will also be much greater.

Thus one reason for the differential slope in the SML is the difference in the announcement premium. There is another reason for the difference in the slope, however. Table 2.6 shows that the slope of the security market line for equities predicted by the model is about half the size of the equity premium on non-announcement days. That is, while the model predicts that the premium is far lower on non-announcement days, as compared with announcement days, it also implies that the slope of the security market line is below even the premium on non-announcement days. Furthermore, consider Figure 2.4. The relation between beta and expected return implied by the model is linear on announcement days, but concave on non-announcement days, just as in the data.

The reason is that, on announcement days, there is a single source of variation driving both the risk premium and the covariance. This is variation due to the disaster probability p_t. The greater the response to a change in p_t, the greater the covariance and the greater the risk premium. This relation is approximately linear. However, on non-announcement days, there are two sources of covariance: the disaster probability and disasters themselves. It is exposure to disasters themselves that explain most of the risk premium, but it is covariation with the disaster probability that determines the beta. The resulting error-in-variables problem leads to a flattened beta-return relation, both in non-announcement periods, and in the full sample, thus partially explaining the beta anomaly.[22] It also implies that a conditional CAPM does not hold on non-announcement days.[23]

2.3.7 Bond returns on announcement and non-announcement days

Table 2.6 repeats the regression (2.50 for bonds with various maturities. For bonds, the data reveal a slightly negative slope on non-announcement days. The slope on announcement days is strongly positive.

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[22]Figure 4 in the Online Appendix shows the unconditional security market line in the model and in the data.

[23]This reasoning could explain why, as Ai et al. (2019) show, implied volatilities from option prices explain announcement returns better than betas.

A crucial difference between bonds and equities is that equities are, by virtue of their cash flows, exposed to aggregate stock market risk. For bonds, this need not be the case. Indeed, Table 2.2 shows that betas on bonds are close to zero on average. It is well-known that the covariances between Treasury bonds and stocks are unstable (Campbell et al., 2017), suggesting that the the beta does not reveal much about the risk in bonds. This makes it all the more striking that bonds exhibit positive betas on announcement days, and that these betas line up with the expected returns. To summarize: both equities and bonds exhibit a strong relation between risk and return on announcement days. Bonds, unlike equities exhibit no relation on non-announcement days. Furthermore, betas for bonds change substantially on announcement days versus non-announcement days.[24]

What does the model have to say about these findings? Section 2.2 shows that, on non-announcement days, the true instantaneous covariance between bonds and stocks is equal to zero. This implies that the true security market line is undefined on non-announcement days. Thus the model is consistent both with negative observed betas on non-announcement days, and the fact that these betas exhibit no relation with expected returns. On the other hand, macro-announcements directly reveal news about bond cash flows, because they are informative about inflation. In our model, news of higher inflation is interpreted as indicating macroeconomic stability. Losses on bonds therefore coincide with losses on the stock market. Thus the model predicts both positive betas on bonds on announcement days, and a strong risk-return relation. Table 2.8 shows that, indeed, bonds have much higher betas on announcement days in simulated data. In contrast, equity betas can increase or decrease, with confidence intervals generally containing zero.

Because betas on announcement days are higher in the model than in the data, the model does not succeed in capturing the full magnitude of the announcement-day slope. The model does succeed, however, in capturing the fact that bond returns contain substantial market risk on announcement days, and no measurable market risk on non-announcement

---

[24]For further discussion of the properties of bond returns around announcements, see Jones et al. (1998) and Balduzzi and Moneta (2017).

days. In the model, news about disaster directly correlates with that of expected inflation. Stated differently, the announcements are concerned with inflation; investors perhaps infer that information concerning inflation also is informative about disasters. Moreover, because inflation tends to rise when the probability of a disaster rises, news about inflation is priced. The greater the bond maturity, the greater the impact of this news, and the greater is the expected return.

2.3.8 Changes in index-option prices around announcements

Our explanation for announcement day anomalies focuses on a resolution of uncertainty at announcements, and specifically, a resolution of uncertainty regarding tail events. One place to look for direct evidence on resolution of uncertainty is from options markets.

In Figure 2.3, we show implied volatility of index put options at the close on announcement days, and at the close of the day prior to the announcement day. Implied volatilities come from OptionMetrics, which reports these as functions of option Delta, namely the change in price of the option with respect to the change in price of the index. The lower the magnitude is Delta, the further out-of-the-money are the put options that go into the implied volatility calculation. Options with low Deltas best represent insurance against low-probability crash events. The slope of the implied volatility curve represents, roughly speaking, the risk-neutral probability of these states relative to a benchmark lognormal model.

Figure 2.3 shows a downward shift in implied volatilities following a scheduled announcement. Thus even the options market, with sophisticated traders, prices in a decline in uncertainty following announcements. Figure 2.3 also shows that the effect is strongest for options with the lowest Deltas, namely those that are furthest out-of-the-money. The implied volatility curve flattens after announcements; even after controlling for the quantity of uncertainty (which is reduced across nearly all moneyness levels), it is the uncertainty about tail risk that is reduced the most. Table 2.3 shows that the decline in the slope is statistically significant at the 5% level. This is direct evidence in favor of the mechanism

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in our model.

One could further ask whether the decline in uncertainty in options data is of the correct magnitude, given the model. In Table 2.9 we report the decline in the VIX, a measure of risk-neutral standard deviation, around announcement days. As a risk-neutral moment, the VIX has a closed-form solution in the model, which we describe in Section 3.1 of the Internet Appendix. Table 2.9 shows that the decline in the VIX computed in the model is about 1 percentage point. This is of similar economic magnitude as the decline in VIX in the data (0.3 percentage points).

2.4 Conclusion

The Capital Asset Pricing Model has been a major focus of research in financial economics, and the benchmark model in financial practice for over fifty years. Despite its pre-eminent status, years of empirical research has found little support for the CAPM. That is, until quite recently. The CAPM predicts a tight relation between market beta and expected return, known as the security market line. Recent research has shown that this security market line, seemingly absent on most days, appears on days with macro-economic announcements (Savor and Wilson, 2014).

This paper builds a general equilibrium model to explain why the security market line appears on macroeconomic announcement days, but is hard to discern on others. The model derives the result from underlying economic principles in a frictionless environment. For this reason, we can explain why the relation between risk and return is not asset-class specific. It holds for both bonds and equities. Days with scheduled announcements provide a discrete amount of news, leading to a risk premium that does not, unlike a risk premium for Brownian or Poisson risk, does not scale with the time interval. This risk premium can be an order of magnitude greater than the risk premium realized on other days.

Our model also makes use of a preference for early resolution of uncertainty, implying risk

sensitive preferences (Ai and Bansal, 2018). Because investors have a preference for early resolution of uncertainty, they require a risk premium for bearing assets that fall in price on adverse economic news (as opposed to simply adverse economic events themselves, as would be the case with time-additive utility). Quantitatively matching the model to the data also appears to require an asymmetry in the release of bad versus good economic news. In the data, the risk in equity returns appears about the same in good and bad economic times. Our model explains this finding through the result that the release of bad news is relatively unusual; most of the time, investors learn what they expect, which is that economic fundamentals are sound. Occasionally, they learn that the economy is facing higher risk; this possibility is sufficient to produce a risk premium, even if the risk does not always realize.

While our focus in this paper is on macro-announcements, the methodology can be applied to scheduled announcements more generally, and understanding the rich array of empirical facts that the announcement literature has uncovered.

Figure 2.1: Portfolio excess returns against CAPM betas
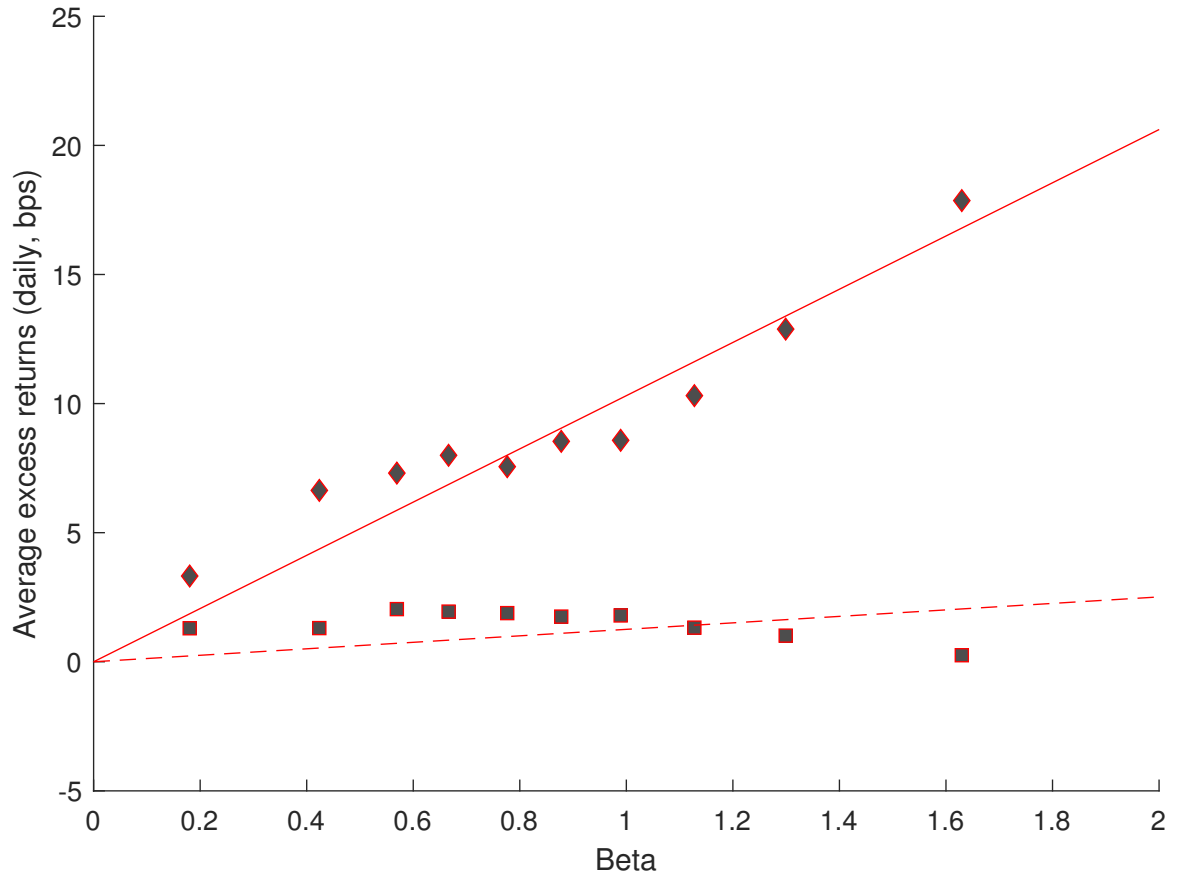
Notes: The figure shows average excess returns on announcement days (diamonds) and non-announcement days (squares) on beta-sorted portfolios in daily data from 1961.01-2016.09. On the horizontal axis is CAPM beta. Also shown are estimated regression lines for announcement day returns against beta (solid red) and non-announcement day returns against beta (dashed red).

Figure 2.2: Portfolio excess returns against CAPM betas on announcement and non-announcement days
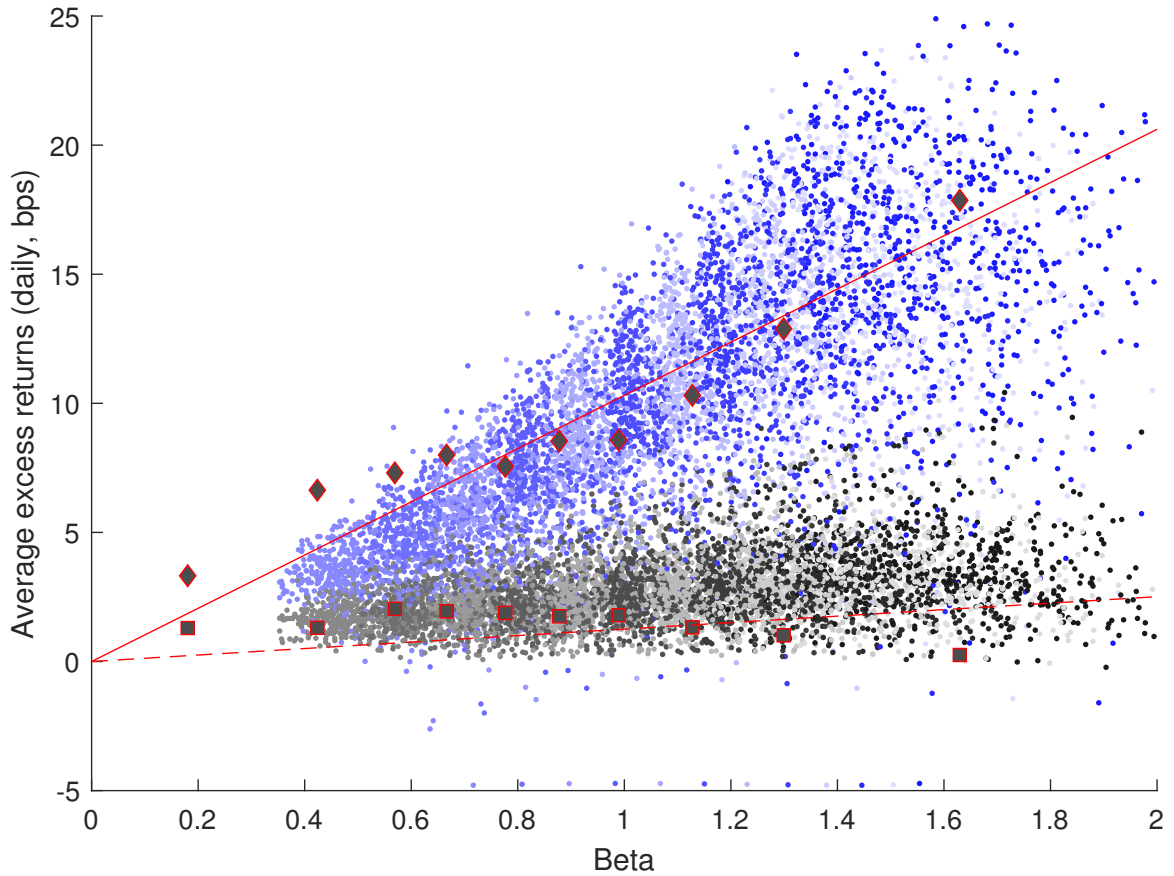
Notes: The figure shows average excess returns on announcement days (diamonds) and non-announcement days (squares) on beta-sorted portfolios in daily data from 1961.01-2016.09 as a function of the CAPM beta. Also shown are estimated regression lines for announcement day returns against beta (solid red) and non-announcement day returns against beta (dashed red). We simulate 500 samples of artificial data from the model, each containing a cross-section of firms. The blue and grey dots show average announcement day and non-announcement day returns for each sample as a function of beta, respectively.

Figure 2.3: Annualized implied volatilities at announcement day closes and prior to announcements
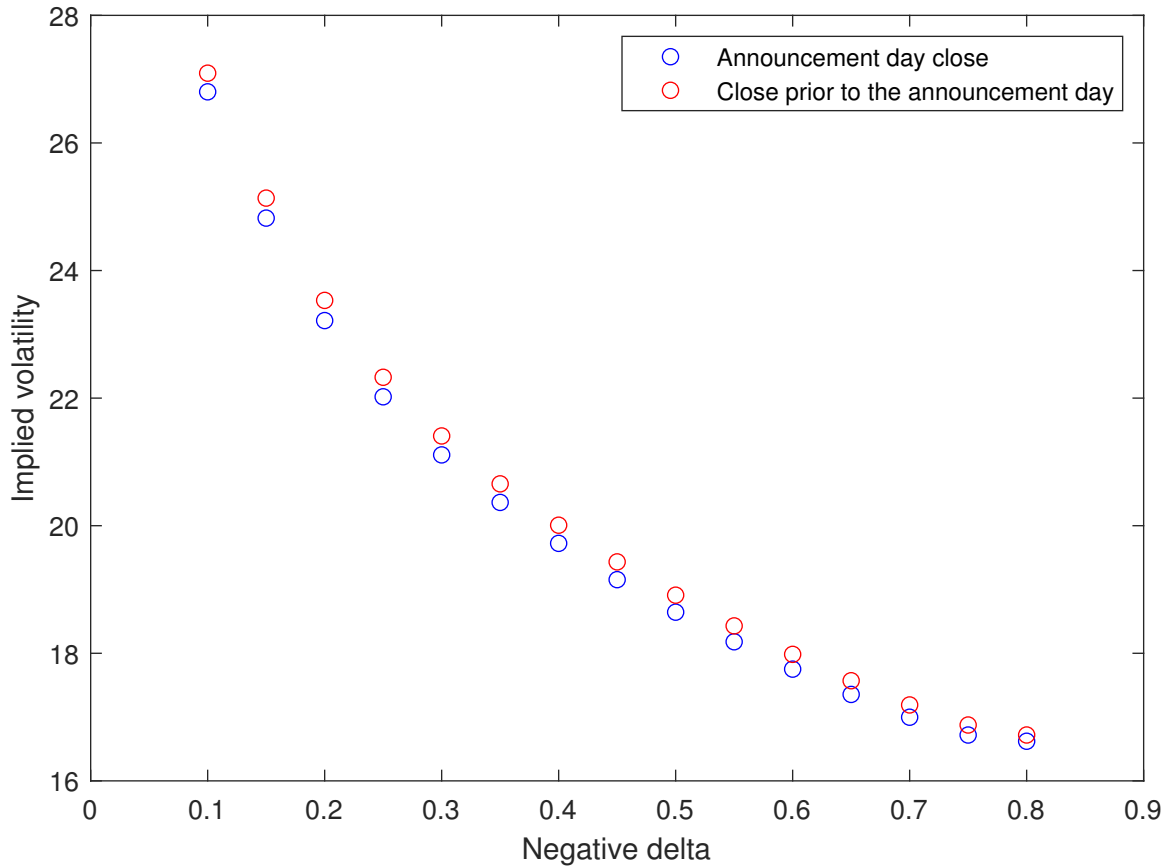
Notes: We plot the average implied volatility surfaces computed from put option prices against the negative of the options' delta. The option's delta is defined as the sensitivity of the option price relative to the underlying asset, or the change in option price per unit change of underlying asset price. The blue circles stand for the average implied volatilities at close on the announcement days, while the red circles are the average implied volatilities at the close prior to announcements. The sample period is 1996.01 to 2016.12.

Figure 2.4: Boxplots of simulated portfolio average excess returns on announcement and non-announcement days
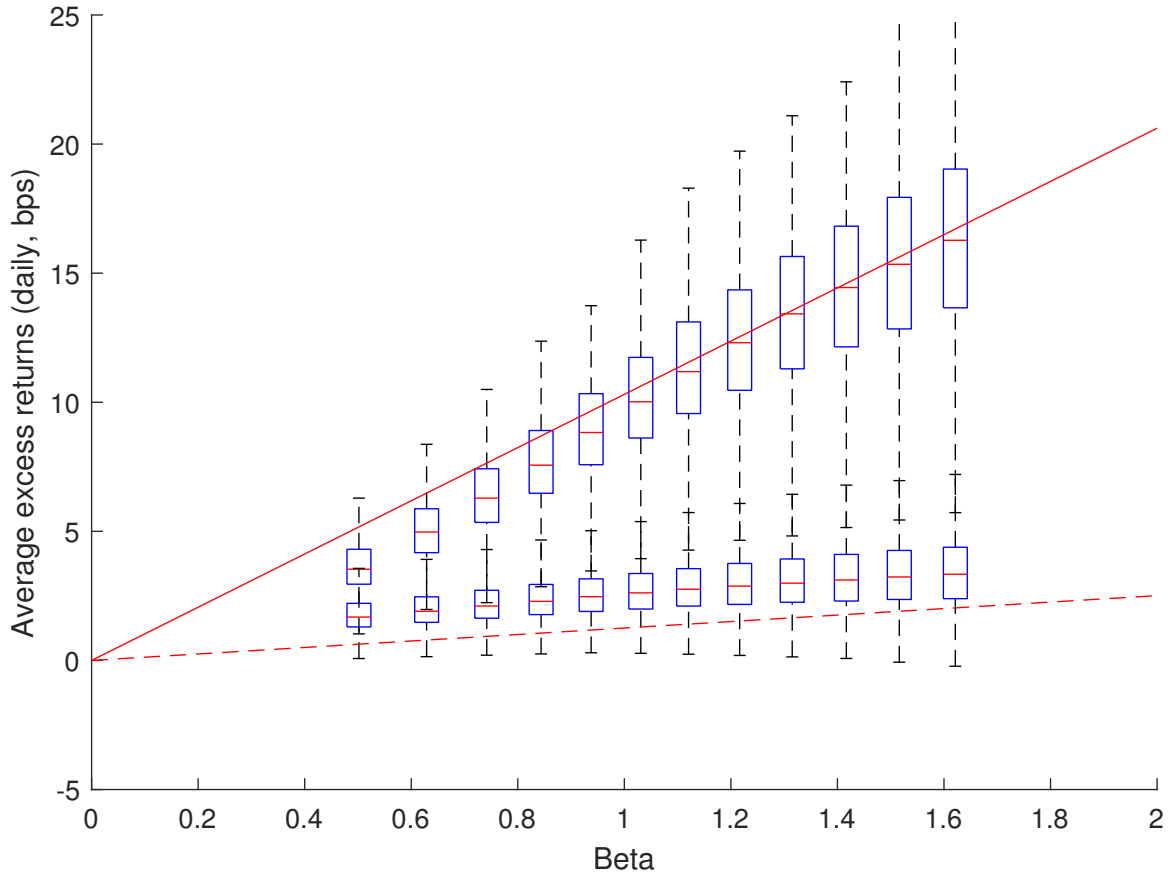
Notes: We compute average excess returns on announcement and non-announcement days for a cross-section of assets in data simulated from the model. The red line shows the median for each portfolio across samples; the box corresponds to the interquartile range (IQR), and the whiskers correspond to the highest and lowest data value within $1.5 \times \mathrm{IQR}$ of the highest and lowest quartile. We plot returns against the median CAPM beta across samples for each portfolio. The red solid and dashed lines are the empirical regression lines of portfolio mean excess returns against market beta on announcement and non-announcement days, respectively.

Table 2.1: Statistics on excess returns of 10 beta-sorted portfolios

k	Unconditional			Announcement day			Non-announcement day		
	$E[RX^k]$	σ^k	β^k	$E[RX^k]$	σ^k	β^k	$E[RX^k]$	σ^k	β^k
1	1.53	53.1	0.20	3.32	52.8	0.18	1.30	53.2	0.20
2	1.91	59.2	0.44	6.64	58.8	0.42	1.30	59.2	0.44
3	2.64	69.2	0.57	7.31	70.8	0.57	2.04	69.0	0.58
4	2.63	77.4	0.69	8.00	77.1	0.67	1.94	77.4	0.69
5	2.53	87.9	0.81	7.56	87.6	0.78	1.88	87.9	0.81
6	2.52	96.2	0.90	8.54	96.7	0.88	1.75	96.1	0.91
7	2.56	105.4	1.00	8.58	107.5	0.99	1.79	105.1	1.00
8	2.34	118.9	1.14	10.31	121.8	1.13	1.32	118.5	1.14
9	2.36	136.5	1.31	12.88	139.1	1.30	1.01	136.2	1.31
10	2.25	176.2	1.67	17.86	176.9	1.63	0.25	176.0	1.67

Notes: Sample statistics for excess returns of ten beta-sorted portfolios. The sample period is 1961.01-2016.09. We show the sample mean excess returns ($E[RX^k]$), and CAPM beta (β^k). Each portfolio is labelled by k. Column 1-3 report estimates with all data available. Column 4-6 and column 7-9 use returns on announcement and non-announcement days, respectively. The unit is basis points per day.

Table 2.2: Statistics on excess bond returns

Maturity	Unconditional		Announcement day		Non-announcement day	
k	$E[RX^k]$	β^k	$E[RX^k]$	β^k	$E[RX^k]$	β^k
1	0.363	0.000	−0.043	0.007	0.415	−0.001
5	0.855	−0.007	3.211	0.029	0.549	−0.013
10	0.779	−0.010	3.882	0.051	0.376	−0.019
20	1.122	−0.021	4.988	0.060	0.620	−0.033
30	0.986	−0.045	5.219	0.046	0.437	−0.058

Notes: Sample statistics for excess returns on Treasury bonds. The sample period is 1961.01-2016.09. We show the sample mean excess returns ($E[RX^k]$) and CAPM beta (β^k). The excess returns are computed using as the difference between CRSP nominal bond returns and the CRSP riskfree rates. Returns and betas are computed using the full sample (first two columns), announcement days (second two columns), and non-announcement days (last two columns). Maturity is in units of years; returns are in units of basis points per day.

Table 2.3: Statistics on implied volatility surface

Negative Delta	0.1	0.3	0.6	0.8	Slope
Announ. days	26.80	21.11	17.75	16.62	10.18
Pre-announ. days	27.09	21.41	17.98	16.72	10.37
Change	−0.29	−0.30	−0.23	−0.10	−0.19
t-stat	[−3.73]	[−4.34]	[−3.30]	[−0.80]	[−2.03]

Notes: We report the summary statistics of the average 30-day implied volatility surface computed using put options on the index. The surfaces are computed using the closing prices of each trading day. The pre-announcement days are the trading days right before the pre-scheduled macro-economic announcements. The option's delta is defined as the sensitivity of the option price relative to the underlying asset, or the change in option price per unit change of underlying asset price. The implied volatility slope is defined as the difference between the implied volatilities of options with delta -0.8 and -0.1. The volatilities are in units of percentage per annum. The sample period is 1996.01 to 2016.12.

Table 2.4: Parameter values for the simulated model

Panel A: Basic parameters	
Expected normal-times growth in dividends and consumption μ, (%)	2.52
Volatility of consumption growth σ, %	2.00
Rate of time preference β	0.012
Relative risk aversion γ	3.00
Average leverage φ	3.00
Panel B: The process for λ_{1t}	
Poisson intensity in the low-risk state λ^L	0
Poisson intensity in the high-risk state λ^H	0.062
Probability of switching to the high-risk state $\phi_{L\to H}$	0.10
Probability of switching to the low-risk state $\phi_{H\to L}$	0.33
Panel C: The process for λ_{2t}	
Average probability of disaster $\bar{\lambda}_2$	0.021
Mean reversion in disaster probability κ	0.08
Volatility for disaster probability σ_λ	0.067
Panel D: Inflation	
Expected inflation in the low-risk state $\bar{q}^L$	0.014
Expected inflation in the high-risk state $\bar{q}^H$	0.070
Mean reversion in expected inflation κ_q	0.09
Volatility for expected inflation σ_q	0.013
Volatility for realized inflation σ_P	0.008

Notes: Parameter values for the calibrated model, expressed in annual terms.

Table 2.5: The equity premium and volatility on announcement and non-announcement days

Statistic	Data	Simulation Median	90 % CI
$E_a[RX_t^{\text{mkt}}]$	10.79	8.86	$[3.48, 13.10]$
$\text{std}_a[RX_t^{\text{mkt}}]$	101.2	99.3	$[62.0, 176.1]$
$E_n[RX_t^{\text{mkt}}]$	1.16	2.39	$[0.87, 4.61]$
$\text{std}_n[RX_t^{\text{mkt}}]$	97.8	72.0	$[34.6, 110.0]$
$E_a[RX_t^{\text{mkt}}] - E_n[RX_t^{\text{mkt}}]$	9.63	6.48	$[0.45, 10.73]$
$\text{std}_a[RX_t^{\text{mkt}}] - \text{std}_n[RX_t^{\text{mkt}}]$	3.4	29.9	$[-29.3, 113.8]$

Notes: $E_a[RX_t^{\text{mkt}}]$ and $E_n[RX_t^{\text{mkt}}]$ denote the average excess return on the market portfolio on announcement days and non-announcement days respectively. $\text{std}_a[RX_t^{\text{mkt}}]$ and $\text{std}_n[RX_t^{\text{mkt}}]$ denote analogous statistics for the standard deviation. The first column reports the empirical estimate. The second column reports the median across samples simulated from the model. The third column reports the two-sided 90% confidence intervals from simulated samples. The units are in basis points per day.

Table 2.6: Cross-sectional regressions on announcement and non-announcement days

Panel A: Equity Portfolios			
Coefficient	Data	Simulation Median	90 % CI
δ_a	10.30	8.37	$[3.57, 13.32]$
δ_n	1.23	1.71	$[0.27, 4.22]$
$\delta_a - \delta_n$	9.07	6.83	$[0.49, 11.34]$

Panel B: Nominal Bonds			
Coefficient	Data	Simulation Median	90 % CI
δ_a	93.33	9.12	$[1.53, 31.74]$
δ_n	-0.51	-2.05	$[-671.32, 407.88]$
$\delta_a - \delta_n$	93.84	13.56	$[-402.98, 696.80]$

Notes: For each sample, the regression $E[RX_t^k \,|\, t \in i] = \delta_i \beta_i^k + \eta_i^k$ is estimated, where $i = a, n$ stands for sets of announcement and non-announcement days, respectively. These regressions are estimated for beta-sorted equity portfolios (Panel A) and for Treasury bonds (Panel B). The first column reports regression slopes in daily data from 1961.01-2016.09. The second column reports medians in simulated samples. The third column reports 90% confidence intervals computed using simulations.

Table 2.7: Summary statistics for simulated equity assets

Panel A: Mean excess returns: announcement days						
Portfolio	1	2	3	4	5	6
Median	4.20	6.90	9.38	11.65	13.81	15.67
90% CI	[1.85, 6.42]	[3.37, 10.07]	[4.79, 13.22]	[5.86, 16.12]	[6.77, 18.79]	[7.38, 21.33]
Panel B: Mean excess returns: non-announcement days						
Portfolio	1	2	3	4	5	6
Median	1.80	2.19	2.53	2.79	3.02	3.23
90% CI	[0.73, 3.54]	[0.91, 4.25]	[1.07, 4.89]	[1.17, 5.39]	[1.15, 5.96]	[1.12, 6.41]
Panel C: Volatility: announcement days						
Portfolio	1	2	3	4	5	6
Median	43.02	74.77	102.91	130.52	155.38	179.35
90% CI	[31.87, 93.11]	[52.88, 131.16]	[71.59, 171.70]	[90.01, 215.44]	[107.18, 260.90]	[122.94, 308.12]
Panel D: Volatility: non-announcement days						
Portfolio	1	2	3	4	5	6
Median	42.46	57.35	70.35	84.43	97.68	111.68
90% CI	[22.01, 82.53]	[29.23, 101.41]	[34.32, 118.79]	[38.33, 137.78]	[41.69, 156.31]	[44.61, 174.42]

Notes: In this table, we report the summary statistics of the equity assets from simulated data. We report the distribution of mean excess returns and volatility of the assets on announcement and non-announcement days across simulated samples. The units are in basis points per day.

Table 2.8: Difference in announcement and non-announcement day betas in simulated data

				Panel A: Equity Portfolios		
Portfolio	1	2	3	4	5	6
Median	-0.19	-0.07	0.03	0.11	0.19	0.26
90% CI	$[-0.31, -0.05]$	$[-0.22, 0.12]$	$[-0.14, 0.30]$	$[-0.05, 0.44]$	$[-0.01, 0.58]$	$[-0.04, 0.78]$

			Panel B: Bonds			
Maturity	1	3	5	7	10	
Median	0.01	0.21	0.48	0.81	0.95	
90% CI	$[0.00, 0.02]$	$[0.05, 0.30]$	$[0.12, 0.67]$	$[0.20, 1.11]$	$[0.24, 1.30]$	

Notes: In data simulated from the model, we compute betas on announcement days and non-announcement days. We do this for beta-sorted equity portfolios (Panel A) and for zero-coupon bonds (Panel B). The table reports the median difference and 90% confidence intervals for the difference.

Table 2.9: VIX in the model in simulated data

	Data	Simulation Median	90 % CI
Post-announcement	19.82	29.73	$[23.95, 40.14]$
Pre-announcement	20.1	31.0	$[25.5, 41.1]$
Change on announcement days	-0.29	-1.26	$[-1.59, -0.91]$

Notes: We report the average VIX pre- and on announcement days in data and from data simulated from the model. We report the closing VIX of the trading days, and pre-announcement VIX is defined as the closing VIX one day before the announcement days. The unit is in percentage per annum.

Table 2.10: Aggregate market and predictive regression moments

		Panel A: Summary statistics		
		Simulation quantiles		
	Data	0.05	0.5	0.95
$E[R_{ft}]$	4.59	−3.23	0.20	1.92
$\sigma(R_{ft})$	3.20	1.97	3.00	4.61
$E[R_{t+1}^{mkt} - R_{ft}]$	6.73	4.22	7.55	11.62
$\sigma(R_{t+1}^{mkt} - R_{ft})$	17.50	7.88	12.36	18.17
Sharpe Ratio	0.38	0.27	0.60	1.06
Skewness	−0.67	−3.64	−1.21	1.01
$\exp(E(pd))$	36.38	14.40	16.94	21.32
$\sigma(pd)$	0.40	0.07	0.14	0.30
$AR1(pd)$	0.91	0.60	0.85	0.96

		Panel B: predictive regressions: 1-year ahead excess returns		
		Simulation quantiles		
	Data	0.05	0.5	0.95
b	0.07	−0.28	0.19	0.63
R^2	0.03	0.00	0.05	0.45

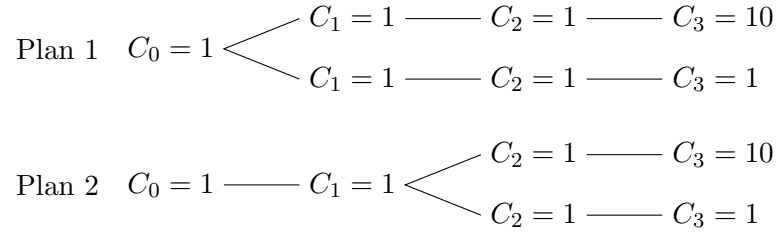|		Panel C: predictive regressions: 5-year ahead excess returns		
		Simulation quantiles		
	Data	0.05	0.5	0.95
b	0.19	−1.31	0.66	1.82
R^2	0.06	0.00	0.19	0.76

Notes: In panel A, we report the moments for the aggregate market including the means and standard deviations of riskfree rates, equity premium, and log price-dividend ratios. In panel B, we report the moments for predictive regressions. Specifically, we run the regression $\log R_{t:t+k}^{mkt} - r_{ft} = a + b \times pd_t + \varepsilon_{t+1}$, where $R_{t:t+k}^{mkt}$ is the realized return of the equity market from time t to $t+k$, and pd_t is defined as the log price-dividend ratio of the equity market at time t. The prediction horizon is one year or five years. The units are percentage per annum.

APPENDIX

A.1 Appendix for Chapter 1

A.1.1 Computing the Utility Level with the Generalized Recursive Utility

Consider Plans 1 and 2.

$$\text{Plan 1} \quad C_0 = 1 \begin{cases} C_1 = 1 \text{——} C_2 = 1 \text{——} C_3 = 10 \\ C_1 = 1 \text{——} C_2 = 1 \text{——} C_3 = 1 \end{cases}$$

$$\text{Plan 2} \quad C_0 = 1 \text{——} C_1 = 1 \begin{cases} C_2 = 1 \text{——} C_3 = 10 \\ C_2 = 1 \text{——} C_3 = 1 \end{cases}$$

At time-0, the agent knows that the two plans are the same at time 2. As a result we can use U_2 to denote the time-2 utility for both plans.

With plan 1, at time 1, the agent knows perfectly about his time-2 utility level. As a result, the agent's utility level at time 1 with plan 1 is given by

$$U_1^1 = f_{1/\psi}^{-1} \left((1 - \beta) f_{1/\psi}(C_1) + \beta f_{1/\psi}(U_2) \right).$$

The uncertainty is resolved at time-1, and is about consumption for the long-run future (period 3). As a result, the certainty equivalence of U_1^1 at time 0 is given by

$$\nu_0^1 = f_\eta^{-1} \left(E \left[f_\eta(U_1^1) \right] \right)$$
$$= f_\eta^{-1} \left(E \left[f_\eta(f_{1/\psi}^{-1} \left((1 - \beta) f_{1/\psi}(C_1) + \beta f_{1/\psi}(U_2) \right)) \right] \right).$$

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Similarly, the certainty equivalence of U_1 at time 0 for plan 2 is given by

$$\nu_0^2 = U_1^2$$
$$= f_{1/\psi}^{-1}\left((1-\beta)f_{1/\psi}(C_1) + \beta f_{1/\psi}(f_\eta^{-1}E\left[f_\eta(U_2)\right])\right).$$

It is straightforward to show that

$$\nu_0^1 < \nu_0^2$$
$$\Leftrightarrow f_\eta(\nu_0^1) < f_\eta(\nu_0^2).$$
$$\Leftrightarrow E\left[f_\eta(f_{1/\psi}^{-1}\left((1-\beta)f_{1/\psi}(C_1) + \beta f_{1/\psi}(f_\eta^{-1}\left(f_\eta(U_2)\right))\right))\right]$$
$$< f_\eta\left(f_{1/\psi}^{-1}\left((1-\beta)f_{1/\psi}(C_1) + \beta f_{1/\psi}(f_\eta^{-1}E\left[f_\eta(U_2)\right])\right)\right).$$

By Jensen's Inequality, the inequality holds if and only if the operator

$$f_\eta\left(f_{1/\psi}^{-1}\left((1-\beta)f_{1/\psi}(C_1) + \beta f_{1/\psi}(f_\eta^{-1}(\cdot))\right)\right)$$

is concave. It then can be showed that the operator is concave if and only if $\eta < 1/\psi$.

As a result, the agent prefers Plan 2 to Plan 1 if and only if $\eta < 1/\psi$. When $\gamma > 1/\psi > \eta$, it can be further showed that Plan 1 is preferred compared to Plan 3.

A.1.2 Pricing Kernel with the Generalized Recursive Utility

In what follows, I use subscripts to denote the information set the expectation operator is conditioning on. The utility is defined recursively by

$$U_t(C_t, \nu(U_{t+1})) = f_{1/\psi}^{-1} \left[(1-\beta)f_{1/\psi}(C_t) + \beta f_{1/\psi}(\nu(U_{t+1}))\right], \tag{A.1.1}$$

which is homogeneous of degree 1. $\nu(U_{t+1})$ is the certainty equivalence function defined in Section 1.2.2, which is also homogeneous of degree 1.

As a result, by slightly abusing notations, we have the following recursive form of utility:

$$U_t(C_t, U_{t+1}) = f_{1/\psi}^{-1} \left[(1-\beta)f_{1/\psi}(C_t) + \beta f_{1/\psi}(\nu(U_{t+1}))\right]. \tag{A.1.2}$$

The difference is that the second argument now is next period utility as a random variable, instead of a certainty equivalence. The function itself is again homogenous of degree 1. As a result

$$kU_t = U_t(kC_t, kU_{t+1}), \forall k > 0. \tag{A.1.3}$$

Noting that, with $f_\alpha(x) = \frac{x^{1-\alpha}}{1-\alpha}$, we have

$$\frac{\partial f_\alpha(x)}{\partial x} = x^{-\alpha},$$
$$\frac{\partial f_\alpha^{-1}(x)}{\partial x} = \left(f_\alpha^{-1}(x)\right)^\alpha,$$

Take the derivative of k on both sides, and let $k = 1$, we get

$$
\begin{aligned}
U_t &= (1-\beta)\left(\frac{U_t}{C_t}\right)^{1/\psi} C_t + \beta \left(\frac{U_t}{\nu(U_{t+1})}\right)^{1/\psi} \nu(U_{t+1})^{\eta} E_{\mathcal{F}_t} \nu_1(U_{t+1})^{-\eta} \nu^*(U_{t+1})^{\gamma} E_{\mathcal{G}_{t+1}}\left(U_{t+1}^{-\gamma} U_{t+1}\right) \\
&= (1-\beta)\left(\frac{U_t}{C_t}\right)^{1/\psi} C_t + \beta E_{\mathcal{F}_t}\left[\left(\frac{U_t}{\nu(U_{t+1})}\right)^{1/\psi}\left(\frac{\nu(U_{t+1})}{\nu^*(U_{t+1})}\right)^{\eta} E_{\mathcal{G}_{t+1}}\left[\left(\frac{\nu^*(U_{t+1})}{U_{t+1}}\right)^{\gamma} U_{t+1}\right]\right] \\
&= (1-\beta)\left(\frac{U_t}{C_t}\right)^{1/\psi} C_t + \beta E_{\mathcal{F}_t}\left[E_{\mathcal{G}_{t+1}}\left[\left(\frac{U_t}{\nu(U_{t+1})}\right)^{1/\psi}\left(\frac{\nu(U_{t+1})}{\nu^*(U_{t+1})}\right)^{\eta}\left(\frac{\nu^*(U_{t+1})}{U_{t+1}}\right)^{\gamma} U_{t+1}\right]\right] \\
&= (1-\beta)\left(\frac{U_t}{C_t}\right)^{1/\psi} C_t + \beta E_{\mathcal{F}_t}\left[\left(\frac{U_t}{\nu(U_{t+1})}\right)^{1/\psi}\left(\frac{\nu(U_{t+1})}{\nu^*(U_{t+1})}\right)^{\eta}\left(\frac{\nu^*(U_{t+1})}{U_{t+1}}\right)^{\gamma} U_{t+1}\right],
\end{aligned}
$$

$$\text{(A.1.4)}$$

where $\nu(U_{t+1})$ and $\nu^*(U_{t+1})$ are the certainty equivalence given $\mathcal{F}_t$ and $\mathcal{G}_{t+1}$, respectively.

Let

$$
MC_t = (1-\beta)\left(\frac{U_t}{C_t}\right)^{1/\psi}
$$

$$
MV_{t+1} = \beta\left(\frac{U_t}{\nu(U_{t+1})}\right)^{1/\psi}\left(\frac{\nu(U_{t+1})}{\nu^*(U_{t+1})}\right)^{\eta}\left(\frac{\nu^*(U_{t+1})}{U_{t+1}}\right)^{\gamma},
$$

Then (A.1.4) can be re-written as

$$
U_t = MC_t C_t + E_{\mathcal{F}_t} MV_{t+1} U_{t+1} \tag{A.1.5}
$$

If we let C_t be the numeraire, and divide both by MC_t, we get

$$
W_t = C_t + E_{\mathcal{F}_t}\left[\frac{MC_{t+1} MV_{t+1}}{MC_t} W_{t+1}\right], \tag{A.1.6}
$$

where W_t is the representative agent's wealth at time t. Then the stochastic discount factor is given by

$$
\begin{aligned}
M_{t+1} &= \frac{MC_{t+1} MV_{t+1}}{MC_t} \\
&= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left(\frac{\nu^*(U_{t+1})}{\nu(U_{t+1})}\right)^{1/\psi-\eta}\left(\frac{U_{t+1}}{\nu^*(U_{t+1})}\right)^{1/\psi-\gamma}.
\end{aligned} \qquad \text{(A.1.7)}
$$

A.1.3 Solving the Disaster Risk Model with the Generalized Recursive Utility

Lemma A.1.1. *The representative agent's continuation value is given by*

$$U_t(C_t, p_t) = C_t \exp(a + b_p p_t), \tag{A.1.8}$$

where b_p can be solved with the following equations:

$$a = \frac{1}{1 - 1/\psi} \log\left((1 - \beta) + \beta e^{(1-1/\psi)m}\right) - \beta \frac{e^{(1-1/\psi)m}}{1 - \beta + \beta e^{(1-1/\psi)m}} n\bar{p})$$

$$b_p = \beta \frac{e^{(1-1/\psi)m}}{1 - \beta + \beta e^{(1-1/\psi)m}} n$$

$$m = a + \mu_C + b_p \rho_p \bar{p} + \frac{1}{2}(1 - \gamma)\sigma_C^2 + \left(b_p(1 - \rho_p) + \frac{1}{2}(1 - \eta)b_p^2 \sigma_p^2 + \frac{1}{1 - \gamma} E_\nu \left[e^{(1-\gamma)Z_{n,t}} - 1\right]\right)\bar{p}$$

$$n = b_p(1 - \rho_p) + \frac{1}{2}(1 - \eta)b_p^2 \sigma_p^2 + \frac{1}{1 - \gamma} E_\nu \left[e^{(1-\gamma)Z_{n,t}} - 1\right],$$

Proof. We solve the function by conjecture and verify the fixed point of the recursion. Conjecture that

$$U_t = C_t \exp(a + b_p p_t).$$

Then we have [1]

$$\nu^*(U_{t+1}; f_\eta(\cdot), \mathcal{G}_{t+1}) = C_t e^{a + \mu_C + \frac{1}{2}(1-\gamma)\sigma_C^2 + b_p p_{t+1} + p_t \frac{1}{1-\gamma} E_\nu \left[e^{(1-\gamma)Z_{n,t}} - 1\right]}$$

$$\nu(\nu^*; f_\gamma(\cdot), \mathcal{F}_t) = C_t e^{a + \mu_C + \frac{1}{2}(1-\gamma)\sigma_C^2 + b_p(p_t - \rho_p(p_t - \bar{p})) + \frac{1}{2}(1-\eta)b_p^2 \sigma_p^2 p_t + p_t \frac{1}{1-\gamma} E_\nu \left[e^{(1-\gamma)Z_{n,t}} - 1\right]}$$

$$= C_t e^{a + \mu_C + b_p \rho_p \bar{p} + \frac{1}{2}(1-\gamma)\sigma_C^2 + \left(b_p(1-\rho_p) + \frac{1}{2}(1-\eta)b_p^2 \sigma_p^2 + \frac{1}{1-\gamma} E_\nu \left[e^{(1-\gamma)Z_{n,t}} - 1\right]\right)p_t}.$$

---

[1] When $\gamma = 1$, we consider the limit case and $\lim_{\gamma \to 1} \frac{1}{1-\gamma} E_\nu \left[e^{(1-\gamma)Z_{n,t}} - 1\right] = E_\nu \left[Z_{n,t+1}\right]$.

which implies the following log-linearization approximation

$$U_t = \left((1-\beta)C_t^{1-1/\psi} + \beta C_t^{1-1/\psi}e^{(1-1/\psi)m+(1-1/\psi)n(p_t-\bar{p})}\right)^{\frac{1}{1-1/\psi}}$$

$$= C_t e^m \left((1-\beta)e^{-(1-1/\psi)m} + \beta e^{(1-1/\psi)n(p_t-\bar{p})}\right)^{\frac{1}{1-1/\psi}}$$

$$\approx C_t e^{\frac{1}{1-1/\psi}\log\left((1-\beta)+\beta e^{(1-1/\psi)m}\right)+\beta\frac{e^{(1-1/\psi)m}}{1-\beta+\beta e^{(1-1/\psi)m}}n(p_t-\bar{p})},$$

where

$$m = a + \mu_C + b_p\rho_p\bar{p} + \frac{1}{2}(1-\gamma)\sigma_C^2 + \left(b_p(1-\rho_p) + \frac{1}{2}(1-\eta)b_p^2\sigma_p^2 + \frac{1}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right]\right)\bar{p}$$

$$n = b_p(1-\rho_p) + \frac{1}{2}(1-\eta)b_p^2\sigma_p^2 + \frac{1}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right].$$

This implies a four-equation system for a, b_p, m and n:

$$a = \frac{1}{1-1/\psi}\log\left((1-\beta) + \beta e^{(1-1/\psi)m}\right) - \beta\frac{e^{(1-1/\psi)m}}{1-\beta+\beta e^{(1-1/\psi)m}}n\bar{p})$$

$$b_p = \beta\frac{e^{(1-1/\psi)m}}{1-\beta+\beta e^{(1-1/\psi)m}}n$$

$$m = a + \mu_C + b_p\rho_p\bar{p} + \frac{1}{2}(1-\gamma)\sigma_C^2 + \left(b_p(1-\rho_p) + \frac{1}{2}(1-\eta)b_p^2\sigma_p^2 + \frac{1}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right]\right)\bar{p}$$

$$n = b_p(1-\rho_p) + \frac{1}{2}(1-\eta)b_p^2\sigma_p^2 + \frac{1}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right],$$

$$(\text{A.1.9})$$

which can be numerically solved. Specificatlly, Let

$$\beta^* = \frac{\beta e^{(1-1/\psi)m}}{1-\beta+\beta e^{(1-1/\psi)m}},$$

we can show that $0 < \beta^* < 1$ when $0 < \beta < 1$.

When $\eta \neq 1$, b_p is the solution to the following quadratic function:

$$b_p = \beta^*\left(b_p(1-\rho_p) + \frac{1}{2}(1-\eta)b_p^2\sigma_p^2 + \frac{1}{1-\gamma}E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right]\right),$$

or

$$b_p = \frac{1 - \beta^*(1 - \rho_p) \pm \sqrt{[\beta^*(1 - \rho_p) - 1]^2 - 2\beta^{*2}\frac{1-\eta}{1-\gamma}\sigma_p^2 E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right]}}{\beta^*(1 - \eta)\sigma_p^2}.$$

Following Wachter (2013), I choose the solution according to the limit case when $\Pr(Z_{n,t+1} = 0) = 1$. When $\Pr(Z_{n,t+1} = 0) = 1$, disasters do not affect agent's consumption at all so the agent's continuation value should not be affected by p_t. This is the case when

$$b_p = \frac{1 - \beta^*(1 - \rho_p) - \sqrt{[\beta^*(1 - \rho_p) - 1]^2 - 2\beta^{*2}\frac{1-\eta}{1-\gamma}\sigma_p^2 E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right]}}{\beta^*(1 - \eta)\sigma_p^2}.$$

as $1 - \beta^*(1 - \rho_p) > 0$.

When $\eta = 1$, b_p is the solution to the following linear function:

$$b_p = \beta^* \left(b_p(1 - \rho_p) + \frac{1}{1 - \gamma} E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right]\right),$$

or

$$b_p = \frac{\beta^*}{1 - \beta^*(1 - \rho_p)} \times \frac{1}{1 - \gamma} E_\nu\left[e^{(1-\gamma)Z_{n,t}} - 1\right].$$

$\square$

Proof of Theorem 1.4. We prove the theorem by induction. Theorem 1.1 implies the following Radon-Nikodym derivative:

$$\frac{dQ}{dP} = e^{-\gamma\sigma_C B_{C,t+1} - \frac{1}{2}\gamma^2\sigma_C^2} \times e^{(1/\psi-\eta)b_p\sigma_p\sqrt{p_t}B_{p,t+1} - \frac{1}{2}(1/\psi-\eta)^2 b_p^2\sigma_p^2 p_t} \times e^{\gamma\sum_{n=1}^{\Delta N_{t+1}} Z_{n,t+1} - p_t\left(E_\nu\left[e^{\gamma Z_{n,t+1}} - 1\right]\right)}.$$

$$(A.1.10)$$

This implies that the compound Poisson process

$$\sum_{n=1}^{\Delta N_{t+1}} Z_{n,t+1},$$

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is also a compound Poisson process, but with conditional jump intensity

$$p_t E_\nu \left[e^{\gamma Z_{n,t+1}} \right],$$

(A.1.11)

and the p.d.f of $Z_{n,t+1}$ given by

$$f(Z_{n,t+1}) \frac{e^{\gamma Z_{n,t+1}}}{E_\nu \left[e^{\gamma Z_{n,t+1}} \right]},$$

(A.1.12)

under risk-neutral measure.

The time-t risk neutral expectation of RV_{t+1} is then given by

$$
\begin{aligned}
E_t^Q[RV_{t+1}] &= \sigma_D^2 + C^2 \sigma_p^2 p_t + \varphi^2 E_t^Q \left[\sum_{n=1}^{N_{t+1}} Z_{n,t+1}^2 \right] \\
&= \sigma_D^2 + C^2 \sigma_p^2 p_t + \varphi^2 p_t^Q E_\nu^Q \left[Z_{n,t+1}^2 \right] \\
&= \sigma_D^2 + \left(C^2 \sigma_p^2 + \varphi^2 E_\nu^Q \left[Z_{n,t+1}^2 \right] E_\nu \left[e^{\gamma Z_{n,t+1}} \right] \right) p_t
\end{aligned}
$$

(A.1.13)

which is strictly increasing in p_t as $p_t \geq 0$.

The second term of the Radon-Nikodym derivative implies that under risk-neutral probability, $B_{p,t+1}$ has mean

$$(1/\psi - \eta) b_p \sigma_p \sqrt{p_t}.$$

This would imply that the time t expectation of p_{t+1} under risk neutral probability is

$$
\begin{aligned}
E_t^Q(p_{t+1}) &= p_t - \rho_p(p_t - \bar{p}) + (1/\psi - \eta) b_p \sigma_p^2 p_t \\
&= (1 - \rho_p + (1/\psi - \eta) b_p \sigma_p^2) p_t + \rho_p \bar{p}.
\end{aligned}
$$

(A.1.14)

Now consider the future price of the realized volatility. We want to show that the future price of realized volatility is given by (1.44).

- $s = 1$, (A.1.13) implies that

$$a_{RV}(1) = \sigma_D^2$$

$$b_{RV}(1) = \left(C^2\sigma_p^2 + \varphi^2 E_\nu^Q \left[Z_{n,t+1}^2\right] E_\nu \left[e^{\gamma Z_{n,t+1}}\right]\right),$$

and (1.44) holds.

- Suppose (1.44) holds for $s - 1$, then

$$E_t\left[RV_{t+s}\right] = E_t\left[E_{t+1}\left[RV_{t+s-1}\right]\right]$$

$$= E_t\left[a_{RV}(s-1) + b_{RV}(s-1)p_{t+1}\right]$$

$$= a_{RV}(s-1) + b_{RV}(s-1)\left((1 - \rho_p + (1/\psi - \eta)b_p\sigma_p^2)p_t + \rho_p\bar{p}\right)$$

$$= a_{RV}(s-1) + b_{RV}(s-1)\rho_p\bar{p} + b_{RV}(s-1)(1 - \rho_p + (1/\psi - \eta)b_p\sigma_p^2)p_t.$$

Let

$$a_{RV}(s) = a_{RV}(s-1) + b_{RV}(s-1)\rho_p\bar{p}$$

$$b_{RV}(s) = b_{RV}(s-1)(1 - \rho_p + (1/\psi - \eta)b_p\sigma_p^2).$$

Then (1.44) holds for s. By the property of induction, (1.44) holds for any positive integer s.

$\square$

A.2 Appendix for Chapter 2

A.2.1 The value function and the state-price density

For the remainder of the Appendix, define the vector Brownian motion

$$dB_t \equiv [dB_{Ct}, dB_{\lambda t}]^\top. \tag{A.2.1}$$

Lemma A.2.1. *In equilibrium, the representative agent's continuation value takes the form*

$$J(C_t, p_t, \lambda_{2t}, \tau; \chi_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma}, \tag{A.2.2}$$

with

$$I(p_t, \lambda_{2t}, \tau; \chi_t) = e^{a(\tau; \chi_t) + b_p p_t + b_\lambda \lambda_{2t}}, \tag{A.2.3}$$

and

$$b_p = \frac{(\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right]}{(1-\gamma)(\beta + \phi_{H \to L} + \phi_{L \to H})}, \tag{A.2.4}$$

$$b_\lambda = \frac{1}{(1-\gamma)\sigma_\lambda^2}\left(\beta + \kappa - \sqrt{(\beta + \kappa)^2 - 2\sigma_\lambda^2 \mathbb{E}_\nu[e^{(\gamma-1)Z_t} - 1]}\right). \tag{A.2.5}$$

for a function $a : [0, T) \times \{0, 1\} \to \mathbf{R}$ satisfying

$$a(\tau; \chi_t) = \zeta_{\chi_t} e^{\beta \tau} +$$
$$\frac{1}{\beta}\left(\mu - \frac{1}{2}\gamma\sigma^2 + b_p \phi_{L \to H} + b_\lambda \kappa \bar{\lambda}_2 + \frac{\lambda^L}{1-\gamma}\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right]\right), \tag{A.2.6}$$

for scalars ζ_0, ζ_1 solving a system of two equations in two unknowns.

Proof. Along the optimal path, and over intervals not containing announcements, the value function must satisfy the usual Hamilton-Jacobi-Bellman equation. That is:

$$f(C_t, J_t) + \frac{\partial J}{\partial \tau} + \frac{\partial J}{\partial C} C_t \mu + \frac{\partial J}{\partial p}(\phi_{L \to H} - p_t(\phi_{H \to L} + \phi_{L \to H})) - \frac{\partial J}{\partial \lambda}\kappa(\lambda_{2t} - \bar{\lambda}_2)$$
$$+ \frac{1}{2}\frac{\partial^2 J}{\partial C^2}C_t^2 \sigma^2 + \frac{1}{2}\frac{\partial^2 J}{\partial \lambda^2}\lambda_{2t}\sigma_\lambda^2$$
$$+ \left(p_t \lambda^H + (1 - p_t)\lambda^L + \lambda_{2t}\right) J \mathbb{E}_\nu\left[\frac{J(Ce^{-Z}, \cdot) - J(C, \cdot)}{J(C, \cdot)}\right] = 0. \tag{A.2.7}$$

Given the conjecture (A.2.2),

$$\frac{1}{J}(J(Ce^{-Z}, \cdot) - J(C, \cdot)) = e^{(\gamma-1)Z} - 1. \tag{A.2.8}$$

Further conjecturing (A.2.3), and using (2.3) and (A.2.8), we find

$$
\begin{aligned}
&- \beta(1-\gamma)(a(\tau;\chi_t) + b_p p_t + b_\lambda \lambda_{2t}) \\
&+ (1-\gamma)\frac{da}{d\tau} + (1-\gamma)\mu + (1-\gamma)(\phi_{L\to H} - p_t(\phi_{H\to L} + \phi_{L\to H}))b_p - (1-\gamma)b_\lambda \kappa(\lambda_{2t} - \bar{\lambda}_2) \\
&- \frac{1}{2}\gamma(1-\gamma)\sigma^2 + \frac{1}{2}(1-\gamma)^2 b_\lambda^2 \sigma_\lambda^2 \lambda_{2t} \\
&+ p_t(\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right] + \lambda^L \mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right] + \lambda_{2t}\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right] = 0.
\end{aligned}
$$
(A.2.9)

Matching coefficients on λ_{2t}, p_t, and on the constant term implies:

$$
-\beta(1-\gamma)b_\lambda - (1-\gamma)b_\lambda \kappa + \frac{1}{2}(1-\gamma)^2 b_\lambda^2 \sigma_\lambda^2 + \mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right] = 0 \quad (A.2.10)
$$

$$
-\beta(1-\gamma)b_p - (1-\gamma)(\phi_{H\to L} + \phi_{L\to H})b_p + (\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right] = 0 \quad (A.2.11)
$$

and

$$
\frac{da}{d\tau} = \beta a(\tau;\chi_t) - \mu + \frac{1}{2}\gamma\sigma^2 - b_p \phi_{L\to H} - b_\lambda \kappa \bar{\lambda}_2 - \frac{\lambda^L}{1-\gamma}\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right]. \quad (A.2.12)
$$

This verifies the conjecture (A.2.3) over non-announcement intervals. Furthermore, (A.2.5–A.2.6) solve (A.2.10–A.2.12).[2]

It remains to verify (A.2.3) over announcement intervals. Along the optimal path, continuation value must satisfy

$$
\begin{aligned}
V_{t-} &= \mathbb{E}_{t-}\left[\int_t^\infty f(C_s, V_s)ds\right] \\
&= \mathbb{E}_{t-}[V_t].
\end{aligned}
$$
(A.2.13)

Applying (A.2.13) for $t \in \mathcal{A}$, we obtain

$$
\lim_{\tau \uparrow T} J(C_{t-}, p_{t-}, \lambda_{2,t-}, \tau; \chi_{t-}) = \mathbb{E}_{t-}\left[J(C_t, p_t, \lambda_{2t}, 0; \chi_t)\right]. \quad (A.2.14)
$$

---

[2]Equation A.2.10 as two solutions. Equation A.2.5 represents the economically reasonable one in that zero disaster risk implies zero impact of disasters on the value function.

That is, the value function on the instant before the announcement must equal the expectation of its value just after the announcement. Furthermore, because C_t and λ_{2t} are continuous at t with probability 1,

$$\lim_{\tau \uparrow T} J(C_t, p_{t-}, \lambda_{2t}, \tau; \chi_{t-}) = \mathbb{E}_{t-}\left[J(C_t, p_t, \lambda_{2t}, 0; \chi_t)\right]. \tag{A.2.15}$$

A solution of the form (A.2.2) will satisfy (A.2.13) provided that

$$\lim_{\tau \uparrow T} I(p_{t-}, \lambda_{2t}, \tau; \chi_{t-}) = \mathbb{E}_{t-}\left[I(p_t, \lambda_{2t}, 0; \chi_t)\right]. \tag{A.2.16}$$

because, almost surely, C_t does not change on announcements or on any other specific time t. Moreover, (A.2.3) and (A.2.16) imply a set of two equations in the two unknowns ζ_0 and ζ_1, verifying (A.2.2) and (A.2.3) over announcement intervals. $\qquad\square$

Proof of Theorem 2.1. Define the function $I_{\mathcal{A}} : [0, 1] \times [0, T) \times \{0, 1\} \rightarrow \mathbf{R}$ as follows:

$$I_{\mathcal{A}}(p_t, \tau; \chi_t) = e^{\zeta_{\chi_t} e^{\beta\tau} + b_p p_t}. \tag{A.2.17}$$

The form of the function I (Equation A.2.3) then implies the multiplicative decomposition:

$$I(p_t, \lambda_{2t}, \tau; \chi_t) = I_{\mathcal{A}}(p_t, \tau; \chi_t) I_{\mathcal{N}}(\lambda_{2t}), \tag{A.2.18}$$

for $I_{\mathcal{N}}(\cdot)$ a function of λ_{2t}. Substituting (A.2.2), (A.2.3) and (A.2.17) into (A.2.15) leads to

$$\lim_{\tau \uparrow T} I_{\mathcal{A}}(p_{t-}, \tau; \chi_{t-}) = \mathbb{E}_{t-}\left[I_{\mathcal{A}}(p_t, 0; \chi_t)\right]. \tag{A.2.19}$$

Equation 2.18 then follows from substituting (A.2.17) into (A.2.19), using the definition of p^*. $\qquad\square$

Lemma A.2.2. *Define ζ_0, ζ_1, and b_p as in Theorem 2.1. Then $b_p < 0$ and*

$$\zeta_0 > \zeta_1 + b_p. \tag{A.2.20}$$

Proof. Suppose by contradiction that

$$\zeta_0 \le \zeta_1 + b_p. \tag{A.2.21}$$

Recall the following pair of equations which determine ζ_0 and ζ_1:

$$
\begin{aligned}
e^{(1-\gamma)\left(\zeta_0 e^{\beta T} + b_p p_0^*\right)} &= p_0^* e^{(1-\gamma)(\zeta_1 + b_p)} + (1 - p_0^*) e^{(1-\gamma)\zeta_0} \\
e^{(1-\gamma)\left(\zeta_1 e^{\beta T} + b_p p_1^*\right)} &= p_1^* e^{(1-\gamma)(\zeta_1 + b_p)} + (1 - p_1^*) e^{(1-\gamma)\zeta_0},
\end{aligned}
\tag{A.2.22}
$$

The expressions on the left hand side of (A.2.22) are weighted averages of $e^{(1-\gamma)(\zeta_1 + b_p)}$ and $e^{(1-\gamma)\zeta_0}$ with weights between 0 and 1. Thus they must lie between these two terms. Because the exponential function is strictly increasing, it follows that

$$
\begin{aligned}
\zeta_0 &\le \zeta_0 e^{\beta T} + b_p p_0^* \\
\zeta_1 e^{\beta T} + b_p p_1^* &\le \zeta_1 + b_p.
\end{aligned}
\tag{A.2.23}
$$

However, (A.2.23) implies

$$
\begin{aligned}
\zeta_0(1 - e^{\beta T}) &\le b_p p_0^* < 0 \\
\zeta_1(e^{\beta T} - 1) &\le b_p(1 - p_1^*) < 0,
\end{aligned}
$$

because $b_p < 0$. Therefore $\zeta_0 > 0$ and $\zeta_1 < 0$, contradicting (A.2.21).

$\square$

Proof of Corollary 2.1. Utility prior to the announcement must equal its expectation

just after the announcement (see Equation A.2.19). That is:

$$\lim_{\tau \uparrow T} I_{\mathcal{A}}(p_\chi^*, \tau; \chi) = p_\chi^* I_{\mathcal{A}}(1,0;1) + (1 - p_\chi^*) I_{\mathcal{A}}(0,0;0), \qquad \text{(A.2.24)}$$

for $\chi = 0, 1$, where p_χ^* is the probability of a negative announcement for the previous announcement being positive ($\chi = 0$) or negative ($\chi = 1$). It follows from Lemma A.2.2 and the form of $I_{\mathcal{A}}$ that

$$I_{\mathcal{A}}(1,0;1) < I_{\mathcal{A}}(0,0;0),$$

namely, utility is lower for a negative announcement than for a positive one. Utility just before the announcement is a weighted average of the utility for the announcement outcomes as (A.2.24) shows. Thus it must lie strictly between the two. It follows that utility falls when the announcement is negative and rises when it is positive.

$\square$

Lemma A.2.3. *The state-price density π_t takes the form*

$$\pi_t = \beta \exp\left\{ \int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds \right\} C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma}, \qquad \text{(A.2.25)}$$

with $I(p_t, \lambda_{2t}, \tau; \chi_t)$ equal to (A.2.3).

Proof. Duffie and Skiadas (1994) show that

$$\pi_t = \exp\left\{ \int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds \right\} \frac{\partial}{\partial C} f(C_t, V_t). \qquad \text{(A.2.26)}$$

The form of f implies

$$
\begin{aligned}
\frac{\partial}{\partial C} f(C_t, V_t) &= \beta(1-\gamma)\frac{V_t}{C_t} \\
&= \beta(1-\gamma)(1-\gamma)^{-1} C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma} \qquad \text{(A.2.27)} \\
&= \beta C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma}.
\end{aligned}
$$

Combining (A.2.26) and (A.2.27) implies

$$
\pi_t = \beta \exp\left\{ \int_0^t \frac{\partial}{\partial V} f(C_s, V_s) ds \right\} C_t^{-\gamma} I(p_t, \lambda_{2t}, \tau; \chi_t)^{1-\gamma}.
$$

$\square$

Proof of Theorem 2.2. We compute the instantaneous change in π_t over an infinitesimal interval containing an announcement. With probability 1, a disaster does not coincide with an announcement. Therefore, it follows from (A.2.25) that

$$
\frac{\pi_t}{\pi_{t-}} = \lim_{\tau \uparrow T} \frac{I_{\mathcal{A}}(p_t, 0; \chi_t)}{I_{\mathcal{A}}(p_{t-}, \tau; \chi_{t-})} = \lim_{\tau \uparrow T} \frac{I_{\mathcal{A}}(\chi_t, 0; \chi_t)}{I_{\mathcal{A}}(p^*_{\chi_{t-}}, \tau; \chi_{t-})}, \quad t \in \mathcal{A}. \qquad \text{(A.2.28)}
$$

The second equality follows from the definition of p^* and of χ_t. We substitute in for $I_{\mathcal{A}}$ using (A.2.17) to find

$$
\lim_{\tau \uparrow T} \frac{I_{\mathcal{A}}(\chi_t, 0; \chi_t)}{I_{\mathcal{A}}(p^*_{\chi_{t-}}, \tau; \chi_{t-})} = \frac{e^{(1-\gamma)(\zeta_{\chi_t} + b_p \chi_t)}}{e^{(1-\gamma)(\zeta_{\chi_{t-}} e^{\beta T} + b_p p^*_{\chi_{t-}})}}. \qquad \text{(A.2.29)}
$$

This shows that the change in the state-price density equals the right hand side of (2.23). Finally (2.23) follows from the definition of M as the change in the state-price density around announcements. $\square$

Proof of Corollary 2.2. The result follows directly from Lemma A.2.2 and the fact that the denominator of (2.23) is a weighted average of two terms, with weights strictly between 0 and 1, as given in (2.18). $\square$

Proof of Theorem 2.3. We show the result for $\gamma > 1$. The proof for $\gamma < 1$ is similar and easier. Recall that $M(\chi, \chi_-)$ is the announcement SDF for previously announced probability χ_- and current announcement χ. It follows from (2.23) that

$$\frac{M(1,1)}{M(0,1)} = \frac{M(1,0)}{M(0,0)}. \tag{A.2.30}$$

Define

$$x = \frac{M(0,0)}{M(1,0) - M(0,0)} = \frac{M(0,1)}{M(1,1) - M(0,1)}.$$

It follows from

$$p_\chi^* M(1,\chi) + (1 - p_\chi^*)M(0,\chi) = 1$$

and (A.2.30) that

$$\frac{M(1,0)}{M(1,1)} = \frac{p_1^* + x}{p_0^* + x} < \frac{p_1^*}{p_0^*}.$$

The second inequality follows from the fact that $\frac{p_1^* + x}{p_0^* + x}$ is decreasing in x for $p_1^* > p_0^*$. Therefore,

$$\tilde{p}_1^* = p_1^* M(1,1) > p_0^* M(1,0) = \tilde{p}_0^*.$$

$\square$

Lemma A.2.4. *Over non-announcement intervals $(t \in \mathcal{N})$, the state-price density π_t follows the stochastic process*

$$\frac{d\pi_t}{\pi_{t-}} = -\left(r_t + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) \mathbb{E}_\nu \left[e^{\gamma Z_t} - 1\right]\right)dt$$

$$- \gamma\sigma dB_{Ct} + (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}dB_{\lambda t} + (e^{\gamma Z_t} - 1)dN_t, \quad \text{(A.2.31)}$$

where b_λ is given by (A.2.5) and where r_t is the instantaneous riskless interest rate:

$$r_t = \beta + \mu - \gamma\sigma^2 + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right) \mathbb{E}_\nu \left[e^{\gamma Z_t}(e^{-Z_t} - 1)\right]. \tag{A.2.32}$$

Proof. Consider $t \in \mathcal{N}$. Ito's Lemma and Lemma A.2.3 imply

$$\frac{d\pi_t}{\pi_{t-}} = \mu_{\pi t}\, dt + \sigma_{\pi t}\, dB_t + \frac{\pi_t - \pi_{t-}}{\pi_{t-}} dN_t, \qquad \text{(A.2.33)}$$

for a scalar process $\mu_{\pi t}$ and a 1×2 vector process $\sigma_{\pi t}$.[3] It follows from (A.2.25) and Ito's Lemma that

$$\sigma_{\pi t} = [-\gamma\sigma, (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}], \qquad \text{(A.2.34)}$$

and that, for $t_i = \inf\{t|N_t = i\}$,

$$\frac{\pi_{t_i} - \pi_{t_i^-}}{\pi_{t_i^-}} = e^{\gamma Z_{t_i}} - 1. \qquad \text{(A.2.35)}$$

It follows from no-arbitrage that

$$\mathbb{E}_{t-}\left[\frac{d\pi_t}{\pi_{t-}}\right] = -r_{t-}\, dt.$$

It follows from the definition of an intensity that

$$\mathbb{E}_{t-}\left[\frac{d\pi_t}{\pi_{t-}}\right] = \mu_{\pi t} + \left(\bar\lambda_1(p_t) + \lambda_{2t}\right)\mathbb{E}_\nu[e^{\gamma Z_t} - 1],$$

implying

$$\mu_{\pi t} = -r_t - \left(\bar\lambda_1(p_t) + \lambda_{2t}\right)\mathbb{E}_\nu[e^{\gamma Z_t} - 1], \qquad \text{(A.2.36)}$$

where $r_t = r_{t-}$ because $\mu_{\pi t}$, and λ_{2t} are continuous.

---

[3]Lemma A.2.3 also implies the continuity of $\mu_{\pi t}$ and $\sigma_{\pi t}$ on non-announcement dates. This allows us to use t rather than t^- to subscript these variables in (A.2.33) and elsewhere.

Finally, we show (A.2.32). Note that

$$\frac{\partial}{\partial V} f(C_t, V_t) = \frac{\partial}{\partial V}\left(\beta(1-\gamma)V_t \log C_t - \beta V_t \log[(1-\gamma)V_t]\right)$$

$$= \beta(1-\gamma)\log C_t - \beta \log[(1-\gamma)V_t] - \beta \tag{A.2.37}$$

$$= -\beta\left(1 + (1-\gamma)[a(\tau;\chi_t) + b_p p + b_\lambda \lambda_{2t}]\right).$$

It follows from (A.2.25) and Ito's Lemma that

$$\mu_{\pi t} = \left(-\beta\left[1 + (1-\gamma)a(\tau;\chi_t) + (1-\gamma)b_p p_t + (1-\gamma)b_\lambda \lambda_{2t}\right] + (1-\gamma)\frac{\partial a}{\partial \tau}\right)$$

$$- \gamma\mu + (1-\gamma)b_p\left[-p_t\phi_{H\to L} + (1-p_t)\phi_{L\to H}\right] - (1-\gamma)b_\lambda\kappa(\lambda_{2t} - \bar{\lambda}_2)$$

$$+ \frac{1}{2}\gamma(\gamma+1)\sigma^2 + \frac{1}{2}(1-\gamma)^2 b_\lambda^2 \sigma_\lambda^2 \lambda_{2t}.$$

Collecting terms and subsituting in for $a(\tau;\chi_t)$, b_p, and b_λ using (A.2.4–A.2.6) implies

$$\mu_{\pi t} = -\left(\beta + \mu - \gamma\sigma^2 + \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right)\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right]\right). \tag{A.2.38}$$

The result then follows from (A.2.36).

$$\square$$

A.2.2   Equity prices

Lemma A.2.5. *Define the function*

$$H(D_t, p_t, \lambda_{2t}, \tau, s; \chi_t) = \mathbb{E}_t\left[\frac{\pi_{t+s}}{\pi_t} D_{t+s}\right]. \tag{A.2.39}$$

Then H represents the price of an equity strip of maturity s. Moreover,

$$H(D_t, p_t, \lambda_{2t}, \tau, s; \chi_t) = D_t F(p_t, \lambda_{2t}, \tau, s; \chi_t), \tag{A.2.40}$$

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where

$$F(p_t, \lambda_{2t}, \tau, s; \chi_t) = \mathbb{E}_t \left[\frac{\pi_{t+s}}{\pi_t} \frac{D_{t+s}}{D_t} \right]. \tag{A.2.41}$$

Proof. The validity of (A.2.39) follows from the Markov property for π_t and D_t. The fact that (A.2.39) represents the price of an equity strip follows from the absence of arbitrage. Finally (A.2.40) again follows from the Markov property, and (A.2.41) is by definition, given (2.26). □

Lemma A.2.6. *Define $H_t = H(D_t, p_t, \lambda_{2t}, \tau, \bar{t} - t; \chi_t)$, so that H_t is the time-t price of the equity strip maturing at date $\bar{t}$. Then, for $t \in \mathcal{N}$, H_t satisfies*

$$\frac{dH_t}{H_{t-}} = \mu_{Ht} dt + \sigma_{Ht} dB_t + (e^{-\varphi Z_t} - 1) dN_t, \tag{A.2.42}$$

with scalar μ_{Ht} and (row) vector σ_{Ht} satisfying

$$\mu_{Ht} + \mu_{\pi t} + \sigma_{Ht} \sigma_{\pi t}^\top + \left(\bar{\lambda}_1(p_t) + \lambda_{2t} \right) \mathbb{E}_\nu \left[e^{(\gamma - \varphi) Z_t} - 1 \right] = 0, \tag{A.2.43}$$

with μ_π as in (A.2.38) and σ_π as in (A.2.34).

Proof. It follows from (A.2.40) and (2.25) that

$$\frac{1}{H} \left(H(De^{-\varphi Z}, \cdot) - H(D, \cdot) \right) = e^{-\varphi Z_t} - 1. \tag{A.2.44}$$

Then (A.2.42) follows from Ito's Lemma.

Equation A.2.39 implies that $\pi_t H_t$ is a martingale. Consider $t \in \mathcal{N}$ and chose Δt sufficiently small so that the interval $[t, t + \Delta t]$ does not contain an announcement. It follows from

(A.2.42) that

$$H_{t+\Delta t}\pi_{t+\Delta t} = H_t\pi_t + \int_t^{t+\Delta t} \pi_u H_u(\mu_{Hu} + \mu_{\pi u} + \sigma_{Hu}\sigma_{\pi u}^\top)du + \int_t^{t+\Delta t} \pi_u H_u(\sigma_{Hu} + \sigma_{\pi u})dB_u$$

$$+ \sum_{t < u_i \leq t+\Delta t} (\pi_{u_i}H_{u_i} - \pi_{u_{i^-}}H_{u_{i^-}}), \quad \text{(A.2.45)}$$

where $u_i = \inf\{u : N_u = i\}$. Rewriting, we have:

$$H_{t+\Delta t}\pi_{t+\Delta t} = H_t\pi_t + \underbrace{\int_t^{t+\Delta t} \pi_u H_u \left(\mu_{Hu} + \mu_{\pi u} + \sigma_{Hu}\sigma_{\pi u}^\top + \left(\bar{\lambda}_1(p_u) + \lambda_{2u}\right)\mathbb{E}_\nu\left[e^{(\gamma-\varphi)Z} - 1\right]\right)du}_{(1)}$$

$$+ \underbrace{\int_t^{t+\Delta t} \pi_u H_u(\sigma_{Hu} + \sigma_{\pi u})dB_u}_{(2)}$$

$$+ \underbrace{\sum_{t < u_i \leq t+\Delta t} (\pi_{u_i}H_{u_i} - \pi_{u_{i^-}}H_{u_{i^-}}) - \int_t^{t+\Delta t} \pi_u H_u \left(\bar{\lambda}_1(p_u) + \lambda_{2u}\right)\mathbb{E}_\nu\left[e^{(\gamma-\varphi)Z} - 1\right]du}_{(3)}.$$

$$\text{(A.2.46)}$$

Since $H_t\pi_t$ is a martingale, the time-t expectation of $H_{t+\Delta t}\pi_{t+\Delta t}$ must be $H_t\pi_t$. In (A.2.46), (2) and (3) equal zero in expectation, so that the integrand in (1) must be zero.[4] We obtain (A.2.43). □

Corollary A.2.1. *The price of an equity strip with maturity s satisfies:*

$$H(D_t, p_t, \lambda_{2t}, \tau, s; \chi_t) = D_t \exp\left\{a_\varphi(\tau, s; \chi_t) + b_{\varphi p}(s)p_t + b_{\varphi \lambda}(s)\lambda_{2t}\right\} \qquad \text{(A.2.47)}$$

---

[4] Note that $\pi_t H_t$ follows a jump diffusion with intensity $\bar{\lambda}_1(p_t) + \lambda_{2t}$ and jump size

$$\frac{\pi_{u_i}H_{u_i} - \pi_{u_i^-}H_{u_i^-}}{\pi_{u_i^-}H_{u_i^-}} = e^{(\gamma-\varphi)Z_{u_i}} - 1.$$

It follows that the term (3) in (A.2.46) equals zero.

with

$$b_{\varphi p}(s) = \frac{(\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right]}{\phi_{H \to L} + \phi_{L \to H}}\left(1 - e^{-(\phi_{H \to L} + \phi_{L \to H})s}\right), \quad s \geq 0, \quad \text{(A.2.48)}$$

where $b_{\varphi\lambda}(s)$ solves

$$\frac{db_{\varphi\lambda}}{ds} = \frac{1}{2}\sigma_\lambda^2 b_{\varphi\lambda}(s)^2 + \left((1-\gamma)b_\lambda \sigma_\lambda^2 - \kappa\right)b_{\varphi\lambda}(s) + \mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right], \quad \text{(A.2.49)}$$

with boundary condition $b_{\varphi\lambda}(0) = 0$, and where $a_\varphi : [0,T) \times [0,\infty) \times \{0,1\} \to \mathbf{R}$ satisfies

$$a_\varphi(\tau, s; \chi_t) = g(\tau + s; \chi_t) +$$
$$\int_0^s \left(-\beta + \lambda^L \mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right] + \kappa\bar{\lambda}_2 b_{\varphi\lambda}(u) + \phi_{L \to H} b_{\varphi p}(u)\right) du \quad \text{(A.2.50)}$$

for a function $g : \mathbf{R}_+ \times \{0,1\} \to \mathbf{R}$.

Proof. No-arbitrage applied to the zero-maturity claim implies the following boundary condition:

$$H(D, p, \lambda_2, \tau, 0; \chi) = D.$$

Thus

$$a_\varphi(\tau, 0; \chi) = b_{\varphi p}(0) = b_{\varphi\lambda}(0) = 0. \quad \text{(A.2.51)}$$

Define μ_{Ht} and σ_{Ht} as in Lemma A.2.6. Applying Ito's Lemma to the conjecture (A.2.47) implies

$$\mu_{Ht} = \mu + \frac{\partial a_\varphi}{\partial \tau} - \frac{\partial a_\varphi}{\partial s} + b_{\varphi p}(s)\phi_{L \to H} + b_{\varphi\lambda}(s)\kappa\bar{\lambda}_2$$
$$+ \left(-\frac{\partial b_{\varphi p}}{\partial s} - b_{\varphi p}(s)(\phi_{H \to L} + \phi_{L \to H})\right)p_t + \left(-\frac{\partial b_{\varphi\lambda}}{\partial s} + \frac{1}{2}b_{\varphi\lambda}(s)^2 \sigma_\lambda^2 - \kappa b_{\varphi\lambda}(s)\right)\lambda_{2t},$$
$$\text{(A.2.52)}$$

and

$$\sigma_{Ht} = \left[\sigma, b_{\varphi\lambda}(s)\sigma_\lambda\sqrt{\lambda_{2t}}\right]. \tag{A.2.53}$$

Substituting (A.2.52), (A.2.53), (A.2.34), and (A.2.38) into (A.2.43) and matching coefficients implies

$$-\frac{\partial b_{\varphi p}}{\partial s} - (\phi_{H \to L} + \phi_{L \to H})b_{\varphi p}(s) + (\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right] = 0 \tag{A.2.54}$$

$$-\frac{db_{\varphi\lambda}}{ds} + \frac{1}{2}\sigma_\lambda^2 b_{\varphi\lambda}(s)^2 + \left((1-\gamma)b_\lambda\sigma_\lambda^2 - \kappa\right)b_{\varphi\lambda}(s) + \mathbb{E}_\nu\left(e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right] = 0, \tag{A.2.55}$$

and

$$\frac{\partial a_\varphi}{\partial \tau} - \frac{\partial a_\varphi}{\partial s} = \beta - \lambda^L\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right] - \kappa\bar{\lambda}_2 b_{\varphi\lambda}(s) - \phi_{L \to H}b_{\varphi p}(s). \tag{A.2.56}$$

Then (A.2.48) uniquely solves (A.2.54) together with the boundary condition (A.2.51). Moreover, (A.2.56) and (A.2.51) ensure that that a_φ takes the form (A.2.50).

$\square$

Proof of Theorem 2.4. Corollary A.2.1, and specifically (A.2.47), implies that there exists a decomposition (2.27), where $F_{\mathcal{A}} : [0,1] \times [0,T) \times \{0,1\}$ takes the form

$$F_{\mathcal{A}}(p_t, \tau, s; \chi_t) = \exp\{g(\tau + s; \chi_t) + b_{\varphi p}(s)p_t\},$$

with $b_{\varphi p}(s)$ in (A.2.48), and with $g : \mathbf{R}_+ \times \{0,1\} \to \mathbf{R}$. Note that

$$H(D_t, p_t, \lambda_{2t}, \tau, s; \chi_t) = D_t F_{\mathcal{A}}(p_t, \tau, s; \chi_t)F_{\mathcal{N}}(\lambda_2). \tag{A.2.57}$$

We now show (2.30). We apply (A.2.39) over an interval containing an announcement:[5]

$$\lim_{\tau \uparrow T} H(D_{t-}, p_{t-}, \lambda_{2t}, \tau, s; \chi_{t-}) = \mathbb{E}_{t-} \left[\frac{\pi_t}{\pi_{t-}} H(D_t, p_t, \lambda_{2t}, 0, s; \chi_t) \right]. \tag{A.2.58}$$

for $t \in \mathcal{A}$. Almost surely, D_t does not change over a sufficiently short announcement interval. We substitute (A.2.57) into (A.2.58) to obtain

$$\lim_{\tau \uparrow T} F_{\mathcal{A}}(p_{t-}, \tau, s; \chi_{t-}) = \mathbb{E}_{t-} \left[\frac{\pi_t}{\pi_{t-}} F_{\mathcal{A}}(p_t, 0, s; \chi_t) \right], \qquad t \in \mathcal{A}.$$

We use Theorem 2.2 to substitute in for the change in the state-price density:

$$\lim_{\tau \uparrow T} F_{\mathcal{A}}(p^*_{\chi_{t-}}, \tau, s; \chi_{t-}) = \mathbb{E}_{t-} \left[M(\chi_t, \chi_{t-}) F_{\mathcal{A}}(\chi_t, 0, s; \chi_t) \right], \tag{A.2.59}$$

where we have also applied the definition of p^* and χ_t. Substituting in for $F_{\mathcal{A}}$ using (2.28) yields

$$e^{g(\tau + s; \chi_{t-}) + b_{\varphi p}(s) p^*_{\chi_{t-}}} = \mathbb{E}_{t-} \left[M(\chi_t, \chi_{t-}) e^{g(s; \chi_t) + b_{\varphi p}(s) \chi_t} \right].$$

Then (2.30) follows from the definition of $\tilde{p}^*$.

We now show that (2.30) uniquely characterizes g. In the process, we provide a recursive algorithm for computing g. Define $u = s + \tau$. For $u < T$, $g(u, \cdot) = 0$ uniquely solves (2.30). Let

$$n = \left\lfloor \frac{u}{T} \right\rfloor$$

equal the number of announcements prior to maturity. We prove uniqueness by induction on n. Assume g is unique for $u \in [(n-1)T, nT)$. Note that (2.30) defines $g(u, \cdot)$ in terms of $g(u - T, \chi)$, $\chi = 0, 1$. Consider $u \in [n, (n+1)T)$. Then $u - T \in [(n-1)T, nT)$. It follows that the right hand side of (2.30) is unique. Therefore, for each $u \in [nT, (n+1)T)$, (2.30),

---

[5]Note that

$$H(D_t, p_t, \lambda_{2t}, \tau, s; \chi_t) = \mathbb{E}_t \left[\frac{\pi_u}{\pi_t} H(D_u, p_u, \lambda_{2u}, \tau + t - u, s - (t - u); \chi_u) \right]$$

for $u \geq t$.

applied at $\chi = 0, 1$, gives the value of $g(u, \chi)$ on the left hand side. Thus $g(u, \cdot)$ is unique for $u \in [n, (n+1)T)$, and hence for all $u > 0$. $\qquad \square$

Proof of Corollary 2.3. It follows from Theorem 2.4 that the equity strip price just prior to an announcement is a weighted average of its possible values just after the announcement, with the weights given by the risk-neutral probabilities (which are strictly between zero and one). Thus the value prior to the announcement must lie between the post-announcement values. It therefore suffices to show that the equity strip price is higher when the announcement is positive as compared to when it is negative. That is, we need to show:

$$g(s; 0) > g(s; 1) + b_{\varphi p}(s) \tag{A.2.60}$$

for $s > 0$.

When $s < T$, (A.2.60) follows from $g(s; 1) = g(s; 0) = 0$ and $b_{\varphi p}(s) < 0$ (recall that we assume $\varphi > 1$). We now show (A.2.60) for general $s \geq T$ using induction on the number of announcements prior to maturity:

$$n = \left\lfloor \frac{u}{T} \right\rfloor.$$

Assume for $s \in [(n-1)T, nT)$, a weaker condition holds:

$$g(s; 0) \geq g(s; 1) + b_{\varphi p}(s).$$

Consider $s \in [nT, (n+1)T)$. It is helpful to write (2.30) out more explicitly:

$$e^{g(s;0)+b_{\varphi p}(s-T)p_0^*} = \tilde{p}_0^* e^{g(s-T;1)+b_{\varphi p}(s-T)} + (1 - \tilde{p}_0^*)e^{g(s-T;0)} \tag{A.2.61}$$

$$e^{g(s;1)+b_{\varphi p}(s-T)p_1^*} = \tilde{p}_1^* e^{g(s-T;1)+b_{\varphi p}(s-T)} + (1 - \tilde{p}_1^*)e^{g(s-T;0)}. \tag{A.2.62}$$

By Theorem 2.3, $\tilde{p}_1^* > \tilde{p}_0^*$. That is, under the risk-neutral measure, when the previous announcement was negative, the probability that the high-risk state will prevail in the next

period is higher. However, by the induction step, we know that the equity price, in the next period, is (weakly) lower, if the high-risk state occurs. That is,

$$g(s - T; 0) \geq g(s - T; 1) + b_{\varphi p}(s - T).$$

Because the right hand side of (A.2.61) puts greater weight on the state with higher prices, as compared with (A.2.62), the left hand side of (A.2.61) is bigger than the left hand side of (A.2.62). That is,

$$g(s; 0) + b_{\varphi p}(s - T)p_0^* \geq g(s; 1) + b_{\varphi p}(s - T)p_1^*.$$

Finally,

$$
\begin{aligned}
g(s; 0) &\geq g(s; 0) + b_{\varphi p}(s - T)p_0^* \\
&\geq g(s; 1) + b_{\varphi p}(s - T)p_1^* \\
&\geq g(s; 1) + b_{\varphi p}(s - T) \\
&> g(s; 1) + b_{\varphi p}(s).
\end{aligned}
$$

The last inequality follows because $b_{\varphi p}$ is a strictly decreasing function. Thus (A.2.60) holds for $s \in [nT, (n + 1)T)$, and therefore for all $s > 0$, completing the proof. $\qquad\square$

Proof of Corollary 2.4. It follows from the definition of the announcement return (2.31), and the instantaneous Euler equation for the price around announcements (A.2.59) that

$$\mathbb{E}_{t-} \left[M(\chi_t, \chi_{t-}) r_{\mathcal{A}}(\chi_t, \chi_{t-}, s) \right] = 1. \tag{A.2.63}$$

Moreover, it follows from (A.2.19), and (A.2.28) that

$$\mathbb{E}_{t-} \left[M(\chi_t, \chi_{t-}) \right] = 1 \tag{A.2.64}$$

Then, (2.35) follows from (A.2.63), (A.2.64), and algebraic manipulation.

Statement 1 of the corollary follows from the fact that, under the stated conditions, the announcement return and the announcement SDF are in opposite positions relative to 1. (see Corollaries 2.2 and 2.3. Statement 2 follows from the fact that, under the stated conditions, they are in the same position relative to 1. Statement 3 follows from the fact that, under the stated conditions, either M or $r_\mathcal{A}$ equal 1. $\qquad\square$

A.2.3 Nominal bond prices

Define the vector Brownian motion

$$dB_t^\$ = [dB_t^\top, dB_{Pt}, dB_{qt}]^\top,$$

with dB_t defined in (A.2.1).

We first show the validity of the nominal stochastic discount factor.

Lemma A.2.7. *Let P_t denote a process for the price level, and let $F_t^\$$ denote a time-t nominal price of a non-dividend paying asset. Then absence of arbitrage implies that there exists a nominal state-price density $\pi_t^\$ = \pi_t/P_t$, such that*

$$\pi_t^\$ F_t^\$ = \mathbb{E}_t\left[\pi_u^\$ F_u^\$\right], \qquad u \geq t. \tag{A.2.65}$$

Proof. The time-t real price of the asset equals $F_t^\$/P_t$. Absence of arbitrage implies that

$$\pi_t \frac{F_t^\$}{P_t} = \mathbb{E}_t\left[\pi_u \frac{F_u^\$}{P_u}\right], \qquad u \geq t. \tag{A.2.66}$$

Define $\pi_t^\$ = \pi_t/P_t$, then (A.2.66) is equivalent to (A.2.65), implying that $\pi_t^\$$ is the nominal stochastic discount factor process. $\qquad\square$

Corollary A.2.2. *For* $t \in \mathcal{N}$, *the nominal state-price density* $\pi_t^\$$ *evolves according to*

$$\frac{d\pi_t^\$}{\pi_{t-}^\$} = -(r_t^\$ + (\bar{\lambda}_1(p_t) + \lambda_{2t}) E_\nu \left[e^{(\gamma-1)Z_t} - 1 \right]) dt$$

$$- \gamma\sigma dB_{Ct} + (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}dB_{\lambda t} - \sigma_P dB_{Pt}$$

$$+ (e^{(\gamma-1)Z_t} - 1)dN_t, \quad \text{(A.2.67)}$$

where $r_t^\$$, *the nominal riskless rate, equals*

$$r_t^\$ = r_t + q_t - \sigma_P^2 - (\bar{\lambda}_{1t} + \lambda_{2t})E_\nu \left[e^{-\gamma Z_t}(e^{Z_t} - 1) \right], \quad \text{(A.2.68)}$$

for r_t *the real riskless rate in (A.2.32), and where* b_λ *equals (A.2.5).*[6]

Proof. Applying Itô's Lemma to

$$\pi_t^\$ = \frac{\pi_t}{P_t} \quad \text{(A.2.69)}$$

implies that there exists a (scalar) process $\mu_{\pi t}^\$$ and (row vector) process $\sigma_{\pi t}^\$$ such that

$$\frac{d\pi_t^\$}{\pi_{t-}^\$} = \mu_{\pi t}^\$ \, dt + \sigma_{\pi t}^\$ \, dB_t^\$ + \frac{\pi_t^\$ - \pi_{t-}^\$}{\pi_{t-}^\$} dN_t. \quad \text{(A.2.70)}$$

Given (A.2.34) and (2.38), it follows that

$$\sigma_{\pi t}^\$ = [-\gamma\sigma, (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}, -\sigma_P, 0]. \quad \text{(A.2.71)}$$

Furthermore, (A.2.35) and (2.38) together imply that, for $t_i = \inf\{t|N_t = i\}$,

$$\frac{\pi_{t_i}^\$ - \pi_{t_i^-}^\$}{\pi_{t_i^-}^\$} = e^{(\gamma-1)Z_{t_i}} - 1. \quad \text{(A.2.72)}$$

Finally, the drift of $\pi_t^\$$, together with (A.2.68), arise from (A.2.36) and the drift of P_t given

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[6]The nominal riskless interest rate is the nominal return on the asset that is instantaneously riskfree when payoffs are expressed in nominal terms.

in (2.38). Substituting in for r_t using (A.2.32) implies

$$\mu_{\pi t}^{\$} = -\beta - \mu + \gamma\sigma^2 - q_t + \sigma_P^2 - \left(\bar{\lambda}_1(p_t) + \lambda_{2t}\right)\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right]. \tag{A.2.73}$$

$\square$

Lemma A.2.8. *Define the function*

$$F^{\$}(p_t, q_t, \lambda_{2t}, \tau, s; \chi_t) = \mathbb{E}_t\left[\frac{\pi_{t+s}^{\$}}{\pi_t^{\$}}\right]. \tag{A.2.74}$$

Then $F^{\$}$ represents the price of a nominal bond with maturity s.

Proof. The validity of (A.2.74) follows from the Markov property of $\pi_t^{\$}$. The fact that (A.2.74) equals the nominal bond price follows from the absense of arbitrage. $\square$

Lemma A.2.9. *Define $F_t^{\$} = F^{\$}(p_t, q_t, \lambda_{2t}, \tau, \bar{t} - t; \chi_t)$, so that $F_t^{\$}$ is the time-t nominal price of the nominal bond maturing at date $\bar{t}$. Then, for $t \in \mathcal{N}$, $F_t^{\$}$ satisfies*

$$\frac{dF_t^{\$}}{F_{t^-}^{\$}} = \mu_{Ft}^{\$}\, dt + \sigma_{Ft}^{\$}\, dB_t^{\$}, \tag{A.2.75}$$

with scalar $\mu_{Ft}^{\$}$ and (row) vector $\sigma_{Ft}^{\$}$ satisfying

$$\mu_{\pi t}^{\$} + \mu_{Ft}^{\$} + \sigma_{\pi t}^{\$}(\sigma_{Ft}^{\$})^\top + (\bar{\lambda}_1(p_t) + \lambda_{2t})\mathbb{E}_\nu\left[e^{(\gamma-1)Z_t} - 1\right] = 0, \tag{A.2.76}$$

with $\mu_{\pi t}^{\$}$ as in (A.2.73) and $\sigma_{\pi t}^{\$}$ as in (A.2.71)

Proof. Equation A.2.75 follows from Ito's Lemma. Equation A.2.74 implies that $\pi_t^{\$} F_t^{\$}$ is a martingale. Moreover, it follows from (A.2.67) that for $t_i = \inf\{t | N_t = i\}$,

$$\frac{\pi_{t_i}^{\$} F_{t_i}^{\$} - \pi_{t_i^-}^{\$} F_{t_i^-}^{\$}}{\pi_{t_i^-}^{\$} F_{t_i^-}^{\$}} = \frac{\pi_{t_i}^{\$} - \pi_{t_i^-}^{\$}}{\pi_{t_i^-}^{\$}} = e^{(\gamma-1)Z_{t_i}} - 1.$$

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The remainder of the proof follows that of Lemma A.2.6.

□

Corollary A.2.3. *The time-t nominal price of a nominal zero-coupon bond with maturity s satisfies*

$$F^{\$}\left(p_t, q_t, \tau, s; \chi_t\right) = \exp\left\{a^{\$}(\tau, s; \chi_t) + b_p^{\$}(s)p_t + b_q^{\$}(s)q_t\right\}, \qquad (A.2.77)$$

with

$$b_q^{\$}(s) = \frac{1}{\kappa_q}(e^{-\kappa_q s} - 1), \qquad (A.2.78)$$

where $b_p^{\$}(s)$ solves

$$\frac{db_p^{\$}}{ds} = -(\phi_{H\to L} + \phi_{L\to H})b_p^{\$}(s) + b_q^{\$}(s)\kappa_q\left(\bar{q}^H - \bar{q}^L\right) \qquad (A.2.79)$$

with boundary condition $b_p^{\$}(0) = 0$, and where $a_\varphi^{\$} : [0,T) \times [0,\infty) \times \{0,1\} \to \mathbf{R}$ takes the form

$$a^{\$}(\tau, s; \chi_t) = g^{\$}(\tau + s; \chi_t) +$$
$$\int_0^s (-\beta - \mu + \gamma\sigma^2 + \sigma_p^2 + b_q^{\$}(u)\kappa_q\bar{q}^L + b_p^{\$}(u)\phi_{L\to H} + \frac{1}{2}b_q^{\$}(u)^2\sigma_q^2)du, \quad (A.2.80)$$

with $g : \mathbf{R}_+ \times \{0,1\} \to \mathbf{R}$.

Proof. No-arbitrage applied to the zero-maturity claim implies the following boundary condition

$$\exp(a^{\$}(\tau, 0; \chi_t) + b_p^{\$}(0)p_t + b_q^{\$}(0)q_t) = 1.$$

Thus

$$a^{\$}(\tau, 0; \chi_t) = b_p^{\$}(0) = b_q^{\$}(0) = 0. \qquad (A.2.81)$$

Define $\mu_{Ft}^{\$}$ and $\sigma_{Ft}^{\$}$ as in Lemma A.2.9. Applying Ito's Lemma to the conjecture (A.2.77)

implies

$$\mu_{Ft}^{\$} = \frac{\partial a^{\$}}{\partial \tau} - \frac{\partial a^{\$}}{\partial s} + b_p^{\$}\phi_{L\to H} + b_q^{\$}(s)\kappa_q \bar{q}^L$$
$$+ \left(-\frac{\partial b_p^{\$}}{\partial s} - (\phi_{H\to L} + \phi_{L\to H})b_p^{\$}(s) \right) p_t + \left(-\frac{\partial b_q^{\$}}{\partial s} - \kappa_q b_q^{\$}(s) \right) q_t, \quad (\text{A.2.82})$$

and

$$\sigma_{Ft}^{\$} = \left[\sigma, 0, 0, b_q^{\$}(s)\sigma_q \right]. \quad (\text{A.2.83})$$

Substituting (A.2.82), (A.2.83), (A.2.73) and (A.2.71) into (A.2.76) and matching coefficients implies

$$0 = \frac{\partial a^{\$}}{\partial \tau} - \frac{\partial a^{\$}}{\partial s} + b_p^{\$}(s)\phi_{L\to H} + b_q^{\$}(s)\kappa_q \bar{q}^L + \frac{1}{2}b_q^{\$}(s)^2\sigma_q^2 - \beta - \mu + \gamma\sigma^2 + \sigma_p^2 \quad (\text{A.2.84})$$

$$0 = -\frac{db_p^{\$}}{ds} - (\phi_{H\to L} + \phi_{L\to H})b_p^{\$}(s) + b_q^{\$}(s)\kappa_q \left(\bar{q}^H - \bar{q}^L \right) \quad (\text{A.2.85})$$

$$0 = -\frac{db_q^{\$}}{ds} - b_q^{\$}(s)\kappa_q - 1. \quad (\text{A.2.86})$$

Then (A.2.78) uniquely solves (A.2.86) together with the boundary condition (A.2.81). Moreover, (A.2.84) and (A.2.81) ensure that that $a^{\$}$ takes the form (A.2.80). □

Proof of Theorem 2.5. Given the foregoing results, this proof follows closely along the lines of that of Theorem 2.4. □

Lemma A.2.10. $b_p^{\$}(s) \leq 0$, and the inequality is strict when $s > 0$.

Proof. We prove the lemma by contradiction.

Substituting the boundary conditions (A.2.81) into (A.2.85) yields

$$\left. \frac{\partial b_p^{\$}}{\partial s} \right|_{s=0} = 0. \quad (\text{A.2.87})$$

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In addition, (A.2.78) implies

$$\kappa_q(\bar{q}^H - \bar{q}^L)b_q^{\$}(s) < 0, \ s > 0. \tag{A.2.88}$$

It follows that there is a sufficiently small but positive s_1, such that

$$b_p^{\$}(s_1) < 0.$$

Suppose there exists $s_2 > 0$, such that $b_p^{\$}(s_2) \geq 0$. Then there must exists $s^* \in [s_1, s_2]$, such that

$$b_p^{\$}(s^*) = 0 \tag{A.2.89}$$

$$\left.\frac{db_p^{\$}}{ds}\right|_{s=s^*} \geq 0 \tag{A.2.90}$$

However, substituting (A.2.89) into (2.44) yields

$$\left.\frac{\partial b_p^{\$}}{\partial s}\right|_{s=s^*} = \kappa_q(\bar{q}^H - \bar{q}^L)b_q^{\$}(s_3) < 0. \tag{A.2.91}$$

which contradicts (A.2.90). $\qquad\qquad\square$

Lemma A.2.11. *Suppose that $\bar{q}^H > \bar{q}^L$, $0 < \kappa_q < 1$, $\phi_{H \to L} > 0$ and $\phi_{L \to H} > 0$. Then*

$$\frac{db_p^{\$}(s)}{ds} < 0, \ s > 0.$$

Proof. We prove the lemma by contradiction. Suppose there exists s^*, such that $\left.\frac{db_p^{\$}}{ds}\right|_{s=s^*} \geq$

0. Define $f^*(s)$ as the solution to the following O.D.E:

$$\frac{df^*(s)}{ds} = -(\phi_{H \to L} + \phi_{L \to H})f^*(s) + b_q^\$(s^*)\kappa_q(\bar{q}^H - \bar{q}^H)$$

$$f^*(0) = 0.$$

Then

$$f^*(s) = \frac{b_q^\$(s^*)\kappa_q(\bar{q}^H - \bar{q}^H)}{\phi_{H \to L} + \phi_{L \to H}}\left(e^{-(\phi_{H \to L} + \phi_{L \to H})s} + 1\right) > \frac{b_q^\$(s^*)\kappa_q(\bar{q}^H - \bar{q}^H)}{\phi_{H \to L} + \phi_{L \to H}}.$$

Specifically,

$$f^*(s^*) > \frac{b_q^\$(s^*)\kappa_q(\bar{q}^H - \bar{q}^H)}{\phi_{H \to L} + \phi_{L \to H}}.$$

As $\frac{db_p^\$(s^*)}{ds} \geq 0$, from (A.2.85) we have

$$-(\phi_{H \to L} + \phi_{L \to H})b_p^\$(s^*) + \kappa_q\left(\bar{q}^H - \bar{q}^L\right)b_q^\$(s^*) \geq 0. \tag{A.2.92}$$

Reorder, and we get

$$b_p^\$(s^*) \leq \frac{\kappa_q(\bar{q}^H - \bar{q}^H)b_q^\$(s^*)}{\phi_{H \to L} + \phi_{L \to H}} < f^*(s^*). \tag{A.2.93}$$

Then we have

$$\frac{d}{ds}\left(f^*(s) - b_p^\$(s)\right) = \frac{df^*}{ds} - \frac{db_p^\$}{ds}$$

$$= -(\phi_{H \to L} + \phi_{L \to H})\left(f^*(s) - b_p^\$(s)\right) + \kappa_q(\bar{q}^H - \bar{q}^L)\left(b_q^\$(s^*) - b_q^\$(s)\right) \tag{A.2.94}$$

Specifically, for $s \in (0, s^*)$, $b_q^\$(s^*) < b_q^\$(s)$, and

$$\kappa_q(\bar{q}^H - \bar{q}^L)\left(b_q^\$(s^*) - b_q^\$(s)\right) < 0.$$

Then with the proof similar to that of Lemma A.2.10, we have that $f^*(s) - b_p^\$(s) < 0, 0 < s \leq s^*$. However (A.2.93) implies that $f^*(s^*) - b_p^\$(s^*) > 0$, which is a contradiction. $\qquad \square$

Proof of Corollary 2.7. Using (A.2.77), (A.2.80) and the almost-sure continuity of all variables around announcements, with the exception of p_t and χ_t, it suffices to show that nominal zero-coupon bond price is higher when the announcement is positive as compared to when it is negative. That is, we need to show:

$$g^{\$}(s;0) > g^{\$}(s;1) + b_p^{\$}(s) \tag{A.2.95}$$

for $s > 0$.

When $s < T$, (A.2.95) follows from $g^{\$}(s;0) = g^{\$}(s;1) = 0$ and $b_p^{\$}(s) < 0$ from Lemma A.2.10.

We now show (A.2.95) holds for $s \geq T$. We prove this by using induction on the number of announcements prior to maturity:

$$n = \left\lfloor \frac{u}{T} \right\rfloor.$$

Assume for $s \in [(n-1)T, nT)$, $n = 1, 2, 3, \ldots$, the following weaker condition holds:

$$g^{\$}(s;0) \geq g^{\$}(s;1) + b_p^{\$}(s). \tag{A.2.96}$$

Equation 2.45 suggests

$$e^{g^{\$}(s;0)+b_p^{\$}(s-T)p_0^*} = \tilde{p}_0^* e^{g^{\$}(s-T;1)+b^{\$}(s-T)} + (1-\tilde{p}_0^*)e^{g^{\$}(s-T;0)}$$

$$e^{g^{\$}(s;1)+b_p^{\$}(s-T)p_1^*} = \tilde{p}_1^* e^{g^{\$}(s-T;1)+b^{\$}(s-T)} + (1-\tilde{p}_1^*)e^{g^{\$}(s-T;0)}.$$

Theorem 2.3 shows that $\tilde{p}_1^* > \tilde{p}_0^*$. However, by the induction step, we know that the equity price, in the next period, is (weakly) lower, if the high-risk state occurs. That is,

$$g^{\$}(s-T;0) \geq g^{\$}(s-T;1) + b^{\$}(s-T).$$

Therefore, it follows that

$$g^{\$}(s;0) + b_p^{\$}(s-T)p_0^* \geq g^{\$}(s;1) + b_p^{\$}(s-T)p_1^*.$$

Finally,

$$g^{\$}(s;0) \geq g^{\$}(s;0) + b_p^{\$}(s-T)p_0^*$$
$$\geq g^{\$}(s;1) + b_p^{\$}(s-T)p_1^*$$
$$\geq g^{\$}(s;1) + b_p^{\$}(s-T)$$
$$> g^{\$}(s;1) + b_p^{\$}(s).$$

The last inequality follows because $b_p^{\$}(s)$ is strictly decreasing from Lemma A.2.11. Thus (A.2.95) holds for $s \in [nT, (n+1)T]$, and therefore for all $s > 0$, completing the proof. $\square$

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