GRAPH THEORETICAL MODELLING OF ELECTRICAL DISTRIBUTION GRIDS

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ABSTRACT

Graph Theoretical Modelling of Electrical Distribution Grids

Iris Kohler

This thesis deals with the applications of graph theory towards the electrical distribution networks that transmit electricity from the generators that produce it and the consumers that use it. Specifically, we establish the substation and bus network as graph theoretical models for this major piece of electrical infrastructure. We also generate substation and bus networks for a wide range of existing data from both synthetic and real grids and show several properties of these graphs, such as density, degeneracy, and planarity. We also motivate future research into the definition of a graph family containing bus and substation networks and the classification of that family as having polynomial expansion.

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Chapter 1

INTRODUCTION

Modern society heavily relies on electricity. Lights, air conditioning, heating, computers, internet access, food storage, and a growing number of vehicles all require electricity to function. Furthermore, key components of modern infrastructure, including water, medical services, and telecommunications, rely on consistent access to electricity [51]. This heavy reliance on the electrical system means it is critical to make sure this infrastructure keeps running correctly.

With computer technology advancing rapidly, computer tools are used more and more to maintain grid performance and quickly solve problems. These tools and the models they use must be flexible, as electrical infrastructure constantly grows and new sources of energy are introduced. In this thesis, we propose the use of a graph-theoretical model. Graph theory is a subject of math with applications in many domains, such as transportation networks. Since electrical infrastructure involves routing power through different components with clear starting and ending nodes (power generators and consumers respectively), it appears to be well-suited for this type of model.

This thesis collects relevant information to hopefully allow for future research in the graph-theoretical applications for power grids. In Chapter 2, we give relevant background information about both about electrical infrastructure and graph theoretical properties of interest. In Chapter 3, we discuss road networks, which is the result of applying graph theory to roads, and existing non-graph-theoretical tools and data used in power systems analysis. In Chapter 4, we establish a graph-theoretical model for power systems and prove some properties about it. In Chapter 5, we examine properties of graphs we created from pre-existing data. Finally, in Chapter 6, we propose several possible directions to continue this research.

Chapter 2

BACKGROUND

2.1 Electrical Grids

The *electrical distribution grid* refers to the infrastructure that transmits electrical power from generators and distributes it to consumers [51]. *Electrical power*, measured in Watts, has an associated voltage and current. *Current*, measured in Amperes, is the speed of electrons flowing through a circuit, and *voltage*, measured in Volts, is the force applied on each electron. The relationship between power, current, and voltage can be shown with the equation $P = IV$, where P represents power, I represents current, and V represents voltage.

There are two ways that electricity flows: alternating and direct current [64]. Alternating current, or AC, refers to electrical generation and transmission circuits where the direction of flow of electricity quickly switches direction. The direction usually changes 100 or 120 times per second, which results in 50 or 60 cycles per second (50/60 Hz). Direct current (DC), on the other hand, refers to electrical generation and transmission circuits where the direction of the flow of electricity stays constant. Historically, power was generated and transmitted using DC equipment [51]. However, AC is used for most generation and transmission today, as it is more cost-efficient. However, DC circuitry is still used to transfer electricity over long distances.

In the United States, electricity is typically generated between 5 to 34.5 kilovolts (kV). Because many power generators are located far away from consumers, that electricity has to travel a long distance. However, when travelling at long distances, electricity meets a lot of resistance, causing a loss in power. Because of this, the voltage of electrical power is increased, or stepped up, to overcome that resistance. During transmission, the voltage is anywhere from 69 to 765 kV, depending on how far the electricity needs to travel. It is important to note that electricity does not necessarily stay at one voltage during transmission. Electric power may be split off, and each split comes with a decrease in voltage; this may happen multiple times throughout transmission. At consumer level distribution networks,

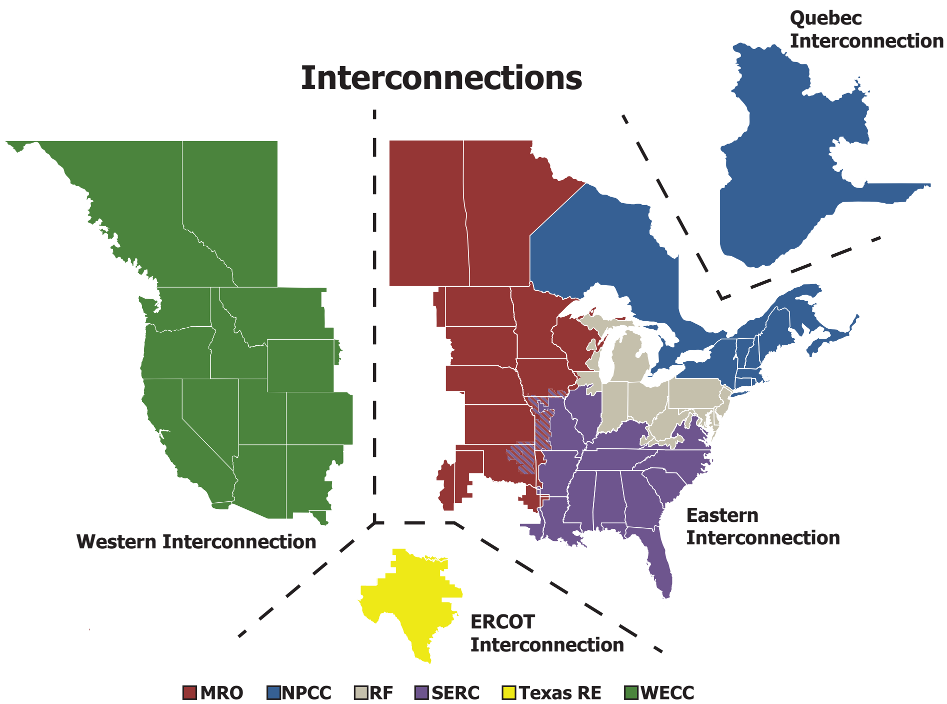


Figure 2.1: The interconnections of the North American bulk power system from the North American Electric Reliability Corporation [50]

where distances are much smaller, electricity is stepped down to much safer levels–usually between 15 and 34.5 kV.

A major independent grid where all electrical infrastructure is connected together is known as an *interconnection*. Often, geographically-close interconnections may be connected using DC circuitry. The electrical grids of the continental United States and Canada and part of the Mexican grid are connected together in 4 distinct interconnections.

Substations are components of the electrical grid that are in charge of, among other important tasks, stepping the voltage of electric power up or down, collecting multiple sources into one output, and distributing power through one or more outputs [51]. This can be generalized to substations taking one or more input power sources and having one or more output power sources, with each input and output having an associated voltage. In other words, substations handle stepping up, stepping down, splitting, and/or combining sources of electrical power. An electrical grid consists of power generators passing power to one or more substations, each of which then pass power to one or more substations, and so on until the power is finally transferred to distribution networks.

Figure 2.2: Conceptual flow chart of the electricity supply chain from the U.S. Department of Energy [51]

Substations are composed of multiple *buses*, which handle power coming in or out [64]. Specifically, a *transmission bus* routes power into a substation and a *distribution bus* routes power out of a substation. As power moves from transmission buses and distribution buses in a substation, its voltage may be stepped down or stepped up as described before.

Figure 2.3 shows a 240-bus model based on the WECC interconnection mapped to its geographic area. This small example shows the flow of electricity from generators to consumers, and it also demonstrates how AC voltage is stepped down. The two DC lines show electrical transfer over long distances.

2.2 Graph Properties

Since electrical distribution grids are formed of buses and substations with transmission lines between them, they are structurally suited for a graph-theoretical representation. Furthermore, given that these systems exist in real life, there are physical limits on the size of the network. Every time an input is split into multiple outputs, the voltage across each output drops to maintain current. This means there will only be so many transmission lines attached to each substation and to each bus.

For this reason, we are interested in the properties of sparse graphs. Graph sparsity refers to how the numbers of edges of graphs in that family grow as their numbers of vertices get larger [18]. A graph is considered sparse if it has some sort of bound on the ratio of edges to vertices. Many graph theoretical algorithms involve graph traversal, which involves visiting vertices and travelling along edges to the next vertex. For this reason, algorithms on sparse graphs can often take advantage of that sparsity in order to solve problems faster.

The following sections give definitions of graph sparsity and provide examples of properties that can be taken advantage of by algorithms on sparse graphs. We also provide planar graphs as an example of sparse graphs and highlight the algorithmic improvements that can be obtained by restricting algorithms to planar graphs.

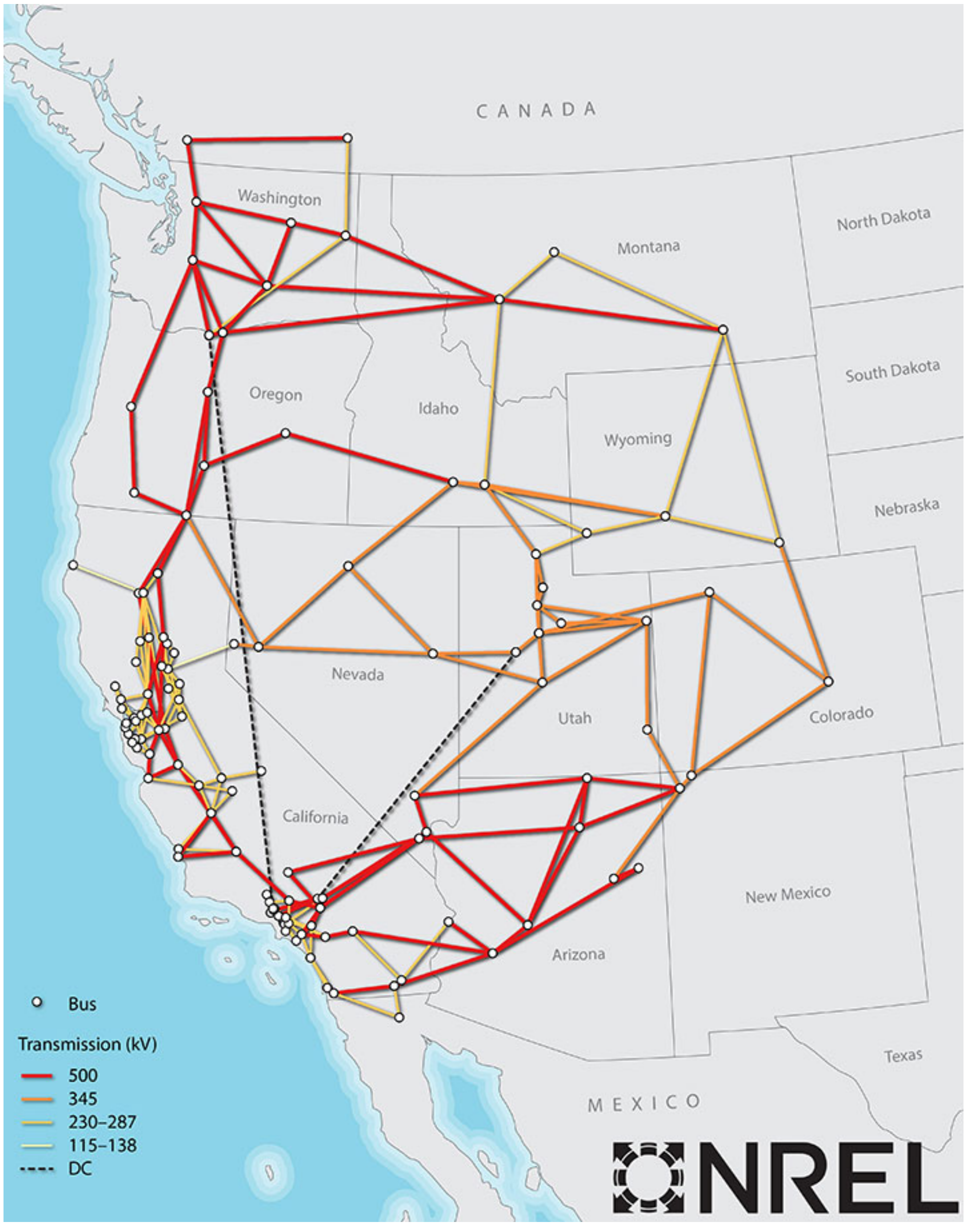


Figure 2.3: A 240-bus model of the WECC interconnection published by the National Renewable Energy Laboratory [47]

2.2.1 Bounded Expansion

Let G and H be graphs. H is considered a *minor* of G if H can be formed by deleting or contracting edges in G or removing vertices. H is a *k-shallow minor* if it is formed by contracting pairwise-disjoint subgraphs whose minimum distance between any two points is at most k. Note that G is a 0-shallow minor of itself, as is every subgraph of G.

For some graph G, define $\triangledown_k(G)$ as follows:

$$\triangledown_k(G) = \max\left\{\frac{E(H)}{V(H)} \; : \; H \text{ is a } k\text{-shallow minor of } G\right\}$$

In other words, $\triangledown_k(G)$ gives the largest density of any k-shallow minor of G.

A family of graphs is said to have *bounded expansion* if there exists a function $f : \mathbb{Z}^+ \to \mathbb{R}^+$ such that $\triangledown_k(G) \leq f(k)$ for all $k \geq 0$ and for all G in the family.

The family of graphs with a constant bound on the maximum degree of all vertices is a graph with bounded expansion [48]. Further examples of families with bounded expansion are given in Sections 2.2.6 and 2.2.6.3.

2.2.2 Graph Separators

Let G be a graph with n vertices and vertex partitions A, B, and C such that, for some function $f : \mathbb{N} \to \mathbb{R}^+$ and constant $0 < k < 1$:

1. $|A| \leq kn$
2. $|B| \leq kn$
3. $|C| \leq f(n)$
4. There is no edge in G incident at a vertex in A and a vertex in B

We call C a *k-separator of size $O(f(n))$*.

A k-separator C that produces partitions A and B is considered *balanced* if $|A \setminus B| \leq \frac{2}{3}n$ and $|B \setminus A| \leq \frac{2}{3}n$. Every graph in every class of graphs with bounded expansion has balanced separators.

A family of graphs with balanced separators is said to have *sublinear separators* if there exists some $c \geq 1$ and $0 < d \leq 1$ such that, for every graph G, G has some separator C such that $|C| \leq cn^{1-d}$.

2.2.3 Polynomial Expansion

A family with bounded expansion is said to have *polynomial expansion* if the function f that bounds $\triangledown_k$ is polynomial [18].

In 2001, Plotkin, Rao, and Smith proved the following [54]:

Theorem 1. All families of graphs with polynomial expansion have strongly sublinear separators.

In 2015, Dvŏrák and Norin expanded on that result by proving the following [18]:

Theorem 2. A family of graphs has polynomial expansion if and only if it has strongly sublinear separators.

2.2.4 Minor-closed Graph Families

A family of graphs is called *minor-closed* if, for every graph in the family, every one of its minors is also contained in the family. In 2006, Lovász proved the following theorem: [42]

Theorem 3. Every minor-closed family of graphs can be characterized by a finite set of graphs that cannot be members or minors of any members of that family.

Graphs in this set are known as the *excluded minors* or *forbidden minors* of the graph family. These minors are forbidden based on the structural characteristics that define that graph family. Section 2.2.6 shows an example of forbidden minors in the family of planar graphs.

Kawarabayashi and Reed proved the following in 2010 [36]:

Theorem 4. Let $\mathcal{F}$ be a minor-closed family of graphs and t be the size of an excluded minor of $\mathcal{F}$. $\mathcal{F}$ has $O(t\sqrt{n})$ separators.

Note that $1 \leq t \leq n$. Taking c and d from the definition of sublinear separators to be t and $\frac{1}{2}$ respectively gives the following result:

Theorem 5. Every minor-closed family of graphs has sublinear separators.

Theorems 2 and 5 together result in the following conclusion:

Theorem 6. Every minor-closed graph family has polynomial expansion.

2.2.5 Separator Decompositions

Recall that, for a graph G with separator C, the removal of C creates two edge-disjoint subgraphs A and B. Further recall that subgraphs of G are 0-shallow minors of G. This means the removal of a separator from a graph produces two minors of that graph that are not connected.

Suppose that G is a member of a minor-closed family. Now, the subgraphs A and B have their own sublinear separators by Theorem 5, and removal of those separators results in two more edge-disjoint subgraphs of each minor.

This recursive removal of separators leads to the the idea of a *k-separator decomposition of a graph* [28]. This decomposition is created by identifying a k-separator in a graph, removing the separator, and repeating the process recursively on the two partitions, with a base case reached when a k-separator can no longer be created from a subgraph. This creates a *decomposition tree*, which is a binary tree metagraph where each node represents a subgraph of G, and a node v has children u, w if the subgraphs represented by u and w are created by removing a k-separator from the subgraph represented by v. The root of this decomposition tree is the original graph.

Figure 2.4 shows an example separator decomposition tree. Section 2.2.6.1 demonstrates applications of separator decomposition.

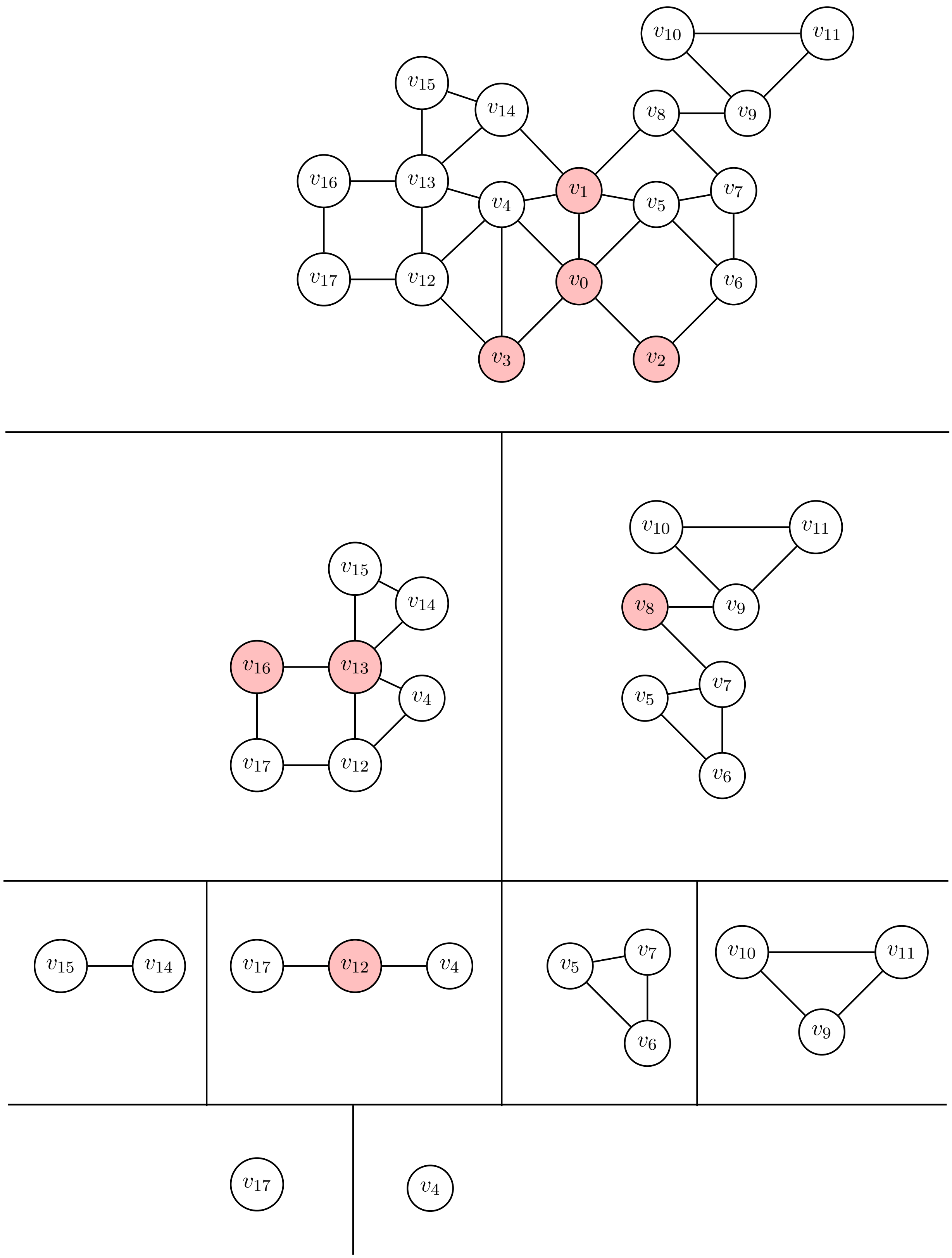


Figure 2.4: An example separator decomposition, with the root graph at the top. The vertices of each graph's separator are highlighted.

2.2.6 Planar Graphs

A graph is *planar* if it can be drawn in $\mathbb{R}^2$ such that none of its edges cross each other without intersecting at a vertex.

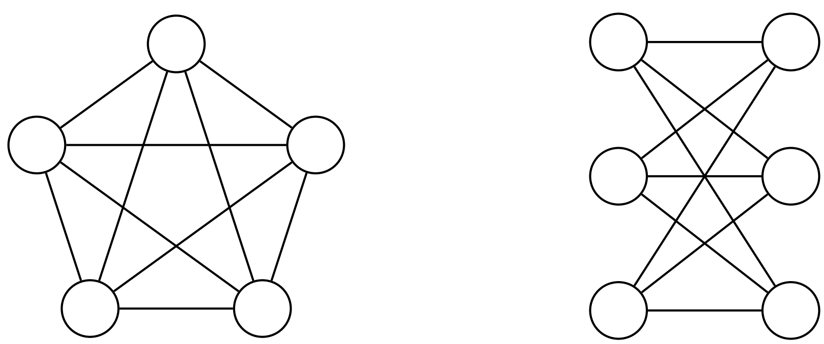
In 1930, Kuratowski proved the following [39]:

Theorem 7 (Kuratowski's Theorem)**.** A finite graph is planar if and only if it does not contain as a subgraph any subdivision of K_5 or $K_{3,3}$.

In 1937, Wager proved the following: [65]

Theorem 8 (Wagner's Theorem)**.** A finite graph is planar if and only if it has neither K_5 nor $K_{3,3}$ as minors.

By this result and Theorem 3, the family of planar graphs is minor-closed. Theorem 6 shows that the family of planar graphs has polynomial expansion.



(a) K_5 (b) $K_{3,3}$

Figure 2.5: The two graphs referenced in Kuratowski's Theorem

2.2.6.1 Planar Separator Decomposition

Since the family of planar graphs is minor-closed, by Theorem 5, we can expect planar graphs to have strongly sublinear separators. In 1979, Lipton and Tarjan proved the following result on planar graphs: [41]

Theorem 9 (Planar Separator Theorem)**.** Let G be a planar graph. G has a $\frac{2}{3}$-separator of size $O(\sqrt{n})$.

In 1995, Goodrich described an algorithm to, in $O(n)$ time, construct a $\frac{2}{3}$-separator decomposition with separators of size $O(\sqrt{n})$ [28]. This decomposition allows for efficient divide-and-conquer algorithms on planar graphs, some of which are described below.

2.2.6.2 Shortest Path on Planar Graphs

An example of the utility of planar graphs (and polymorphic expansion in general) is the shortest path problem applied to planar graphs. The *shortest path problem* (also known as the single-pair shortest path problem) is defined on a graph with real-valued weights on its edges. Given two vertices u and v, the shortest path problem finds a path from u to v with the lowest sum of edge weights compared to any other path from u to v if v is reachable from u. Several variations of the shortest path problem exist. The *single-source shortest path problem* calculates the shortest path from one vertex v to all other vertices in the graph reachable from v. The *multiple-source shortest path problem* calculates, for every vertex in the graph, the shortest path to every reachable vertex from that graph. In other words, the multiple-source shortest path problem solves the single-source shortest path problem for every vertex in the graph.

For this section, we will discuss the application of the problem to directed graphs. When edge weights are required, we will consider graphs with nonnegative edge weights.

Klein *et al.* showed that, using planar separator decomposition, the single-source shortest path problem can be calculated on planar graphs in linear time over the number of vertices in the graph [38]. In the nonplanar case, Thorup showed an $O(m + n \log\log \min\{n, C\})$ algorithm, where m is the number of edges and C is the maximal edge weight [63]. This pseudo-polynomial algorithm uses Fibonacci heap-style integer priority queues. Note that, even in the case where $m = O(n)$, Thorup's algorithm is less efficient than that proposed by Klein *et al.* in the case of planar graphs.

2.2.6.3 Near-planarity

Near-planarity refers to graphs with certain bounds on the number of times edges can cross. Some of these bounds are strong enough to create families with bounded or polynomial expansion.

A graph is *k-planar* if any edge is allowed to cross at most k other edges for some constant k [23]. The family of k-planar graphs has bounded expansion [48]. The family of graphs with a constant bound on the total number of crossings has polynomial expansion, and the family of graphs where the total number of crossings has a linear relationship with the number of vertices has bounded expansion.

2.2.6.3.1 Crossing Minimization Problem The *metro-line crossing minimization problem* takes a graph as an input and outputs an embedding of the graph in $\mathbb{R}^2$ with the minimum number of edges crossings [44]. This problem was defined by Marek-Sadowska and Sarrafzadeh in 1995, and the authors proposed an $O(nm^2 + n\epsilon^{3/2})$-time algorithm, where n is the number of vertices, m the number of edges, and ϵ is the crossing number. Chen and Lee presented an $O(m(n + \epsilon))$-time algorithm in 1998 [11].

2.2.6.3.2 Planarization Given G, a nonplanar graph embedded in $\mathbb{R}^2$, we can *planarize* G by, for every pair of edges (v_1, v_2) and (u_1, u_2) that cross without intersecting at a vertex, removing those edges, introducing a new vertex w, and adding edges (v_1, w), (v_2, w), (u_1, w), and (u_2, w). This procedure effectively places a vertex at each edge crossing, resulting in what is known as a *planarization of* G.

In 2010, Epstein *et al.* proved that, when a graph has $O\left(\frac{n}{\log^{(c)} n}\right)$ crossings for some constant c (known as *restrained graphs*), that graph can be planarized in linear time [21]. The authors also describe an algorithm to use the separator decomposition of the planarization of such a graph can be used to create a $O(\sqrt{n})$-sized separator decomposition of the original graph.

2.3 Relevance

Modern society depends on a reliable electrical infrastructure [51]. This involves balancing how much generators can produce, how much consumers use, and the physical limits on the equipment that transfer power between. Failing to take this into account can lead to not enough power being generated for all users who need it. Any improvement to the algorithms used on electrical distribution networks could mean faster responses to power system faults, less people left without power when things go wrong, or more accurate collection of grid information.

Being able to show that electrical distribution grids have polynomial expansion would mean that they have strongly sublinear separators. This would mean it may be possible to take advantage of separator decomposition-based algorithms. As demonstrated above and in Section 3.1, these algorithms tend to provide large improvements over algorithms that do not rely on separator decomposition.

Chapter 3

RELATED WORKS

3.1 Road Networks

The graph-theoretical classification of road networks is of particular interest for the efforts to classify electrical grid networks. Both are networks that have physical limits on the size of their graphs–road intersections can have so many roads connected to them, and substations can only connect to so many substations.

Road networks are graphs where the vertices represent intersections between two or more roads or dead ends and edges represent sections of road in between two intersections. Due to spacial limitations and a need for roads to be human-friendly, the maximum degree of a road network based on any existing road system is incredibly small. These networks are also nonplanar, as any bridge over or tunnel under another road becomes an edge intersection in their graph-theoretical representation.

Initial attempts at using graph theory to model road networks worked with the assumption that these networks are planar or near-planar. This means that many problems, such as shortest path, can benefit from fast planar graph algorithms. Due to the aforementioned nonplanarities, research on road networks focused on finding a useful categorization of road networks and motivating efficient algorithms given said categorization. Attempts to classify road networks as near-planar, such as k-genus (able to be embedded without crossings in a surface with at most k holes) and k-planar failed, as there are no bounds on the number of crossings a single road can have [23].

Eppstein and Goodrich showed that road networks are a subgraphs of disk intersection graphs [20]. They start by considering road networks as *geometric graphs*, where every vertex is associated with a point in $\mathbb{R}^2$. In particular, the longitude and latitude of every intersection and dead end of a particular road network are used as the coordinates in $\mathbb{R}^2$ for the geometric graph representation. Then, disks in $\mathbb{R}^2$ are created with a center at each vertex and radius equal to half the length of the longest stretch of road attached to the intersection. Eppstein and Goodrich showed that, except for a few exceptional high-radius disks, most disks in these neighborhoods are low ply. In other words, most points in $\mathbb{R}^2$ are covered by at most some constant number of disks. Then, they created disk intersection graphs, where each disk is represented by a vertex with an edge between two vertices existing when their associated disks intersect. The authors discovered that the resulting disk intersection graphs are supergraphs of the road networks they were built from. This information along with an algorithm for linear time $O(\sqrt{n})$-separator decomposition previously created for k-ply disk neighborhood systems by Eppstein, Miller, and Teng [24] allows for efficient algorithms on road networks such as Voroni diagrams and shortest path [20]. However, the exceptional disks present a problem. In theory, it is not impossible to have disk neighborhood systems with dense high-radius disk clusters, even if no real-world road network would ever produce such a neighborhood.

In an effort to make a more accurate classification of road networks, Eppstein and Gupta focused on the crossing graphs of road networks [23]. Given a graph G embedded in $\mathbb{R}^2$, the crossing graph of G is a graph where every edge in G has a vertex in the crossing graph and two vertices in the crossing graph share an edge if the associated edges in G cross each other. The authors also considered the degeneracy of road network crossing graphs. The *degeneracy* of a graph is the maximum of the minimum degree over all subgraphs of that graph. A d-degenerate graph also contains a d-core, and d is the largest value for which the graph has a core. They showed empirically that crossing graphs of real-world networks are very sparse and have a bounded constant degeneracy before proving that the family of graphs whose crossing graphs have bounded degeneracy has polynomial expansion. The authors finally investigated planarizing road networks by replacing crossings with vertices and discovered that this process introduces a sublinear number of new vertices. Using the previously-described linear-time planarization algorithm [21] and linear-time planar separator decomposition [28], the authors showed how planar graph algorithms can be applied to road networks.

3.2 Power Systems Research

Given the importance of power systems to society, much research has been done on power systems. This has lead to the creation of several tools and much synthetic network data.

3.2.1 Tools

In 1997, MATPOWER, an open source power systems simulation tool, was initially published in 1997 [69]. It has since grown into a large MATLAB library. It has built-in libraries to solve optimization problems, including optimal power flow, on electrical grids. Zimmerman *et. al* described the system in their 2011 publication.

RTE, the main transmisison grid operator in France, created and maintains a tool known as Convergeance [59]. It is the main network analysis tool of the company and provides simulation for over the French transmission grid [34].

The Pan European Grid Advanced Simulation and state Estimation (PEGASE) project was a four-year project from the European Commission meant to build a representation of the high and extra high voltage transmission networks in the European mainland developed between 2008 and 2012 [13]. It defined state estimation, optimization, and simulation frameworks for this pan-European grid and provided funding for researchers working on the data. The result was a number of models for dynamic simulation over the network designed to help operators make decisions based on current load.

The European Commission also created the Innovative Tools for Electrical System Security within Large Areas (ITESLA) project from January 1st, 2012, to March 31st, 2016 [14]. Its purpose was to increase coordination among the many network operators throughout Europe, define security limits of the pan-European network, and develop tools to assess the security of the network and dynamically model parts of the grid. The result is online and offline platforms for time-based simulation, a model-validation software, and toolboxes for security assessment of networks.

3.2.2 Synthetic Electrical Grids

Real-world electrical grid data is often confidential and therefore hard to obtain. Because of this, much research has focused on generating synthetic representations of electrical grids as well as developing validation criteria to determine how well those grids represent reality. A *synthetic grid* refers to a synthetically-generated electrical grid that is in some way representative of a real-life electrical grid.

In 1979, the Application of Probability Methods Subcommittee of the IEEE Power Systems Engineering Committee published a 24-bus synthetic grid known as the IEEE Reliability Test System (later called RTS-79) [56]. This was created to help test methods of power system reliability testing. RTS-79 was expanded by Allan *et. al* in 1986 to create RTS-86 [1]. This modernized the test system as well as added additional data about power generation to the system. Grigg *et. al* added another update, RTS-96, in 1996 [31]. This again modernized RTS data, added operating cost and constraint data to the power generators, and added different methods of power transfer to the system. In 2019, Barrows *et. al* through the Grid Modernization Lab Consortium added more modifications to IEEE-96, creating IEEE RTS-GMLC [6]. They modernized generator data to account to account for changes in the amount of nuclear, wind, and solar production as well as energy storage.

Fliscouhakis *et al.* published a modified synthetic version of the PEGASE network similar in size and complexity to the original [26].

In 2017, Birchfield *et al.* presented a creation and validation methodology for generating synthetic grid test cases [9, 10]. To create these synthetic grids, substations are placed geographically based on publicly-available information about populations and locations of electricity generators before adding buses and creating the connections in between them. The authors describe several structural characteristics that characterize real power grids and can be used to verify large synthetic grids.

MATPOWER contains a number of test cases based on both syntehtic and real grids [69]. As additional test cases have been published, they have been included in MATPOWER's collection of test data, including the iTesla, PEGASE, RTE, RTS-79, RTS-GMLC, and ACTIVSg data. The full list of data included in the MATPOWER is described in Section 5.2. In 2016, Josz *et al.* published synthetic data from the iTesla, PEGASE, and RTE projects described above in the MATPOWER data format [34].

Chapter 4

THEORY

4.1 Substation Networks

Definition 1. The *substation network* of an electrical grid is a directed graph where each vertex represents a substation and an edge (u, v) exists if power flows from the substation represented by u to the substation represented by v.

Each edge in a substation network is weighted with the voltage carried between the associated substations. This network represents a significant part of an electrical grid, since substations do the operations necessary to make power usable by consumers. For this reason, substation networks will show the entire flow of power from power generators to the substations that convert power for consumer use. By the definition of an substation network, any vertices with indegree 0 represent substations connected directly to power sources, and any vertices with outdegree 0 represent substations connected directly to distribution grids.

4.2 Bus Networks

Definition 2. The *bus network* of an electrical grid is a directed graph where each vertex represents a bus and an edge (u, v) exists if power flows from the bus represented by u to the bus represented by v.

Like a substation network, each edge is weighted with the voltage carried between buses. Each bus is also labeled with the substation it belongs to.

Figures 4.1 and 4.2 show the substation graph and and bus graph, respectively, of the 200-bus ACTIVSg200 synthetic grid [9, 10]. Buses close to each other of the same color in Figure 4.2 belong to the same substation, which is the substation that shares the same color in Figure 4.1.

Theorem 10. Given an electrical grid, the substation network G_S representing that grid, and the bus network G_B representing that grid, G_S is a minor of G_B.

Proof. Color the vertices in G_B such that two vertices u and v in G_B have the same color if and only if u and v represent buses belonging to the same substation. Then, for each edge (w, x) such that w and x have the same color, contract that edge. Finally, remove any resulting loops and multiple edges. The result of is process of a simple directed graph G'_B, a minor of G_B, such that every vertex color is assigned to exactly one vertex. By definition of the vertex coloring, this means there is exactly one vertex in G'_B for each substation, and the edge (u', v') exists in G'_B if and only if the substation that u' represents transfers power to the substation that v' represents. Thus, by definition, G'_B is the substation network of the grid. ∎

Theorem 11. Let G_S be a substation network and G_B be a bus network representing the same grid. If G_B is planar, then G_S is planar.

Proof. Suppose G_B is planar. By Theorem 8, the family of planar graphs has a finite set of excluded minors. By Theorem 3, the family of planar graphs is minor-closed. Because G_S is a minor of G_B by Theorem 10, G_B is planar, and the family of planar graphs is minor-closed, G_S must be planar. ∎

4.3 Discussion

The result of Theorem 10 shows that, for any electrical grid, its substation network is a minor of its bus network. Because of this, any minor-closed graph family that contains all bus networks will also contain all substation networks.

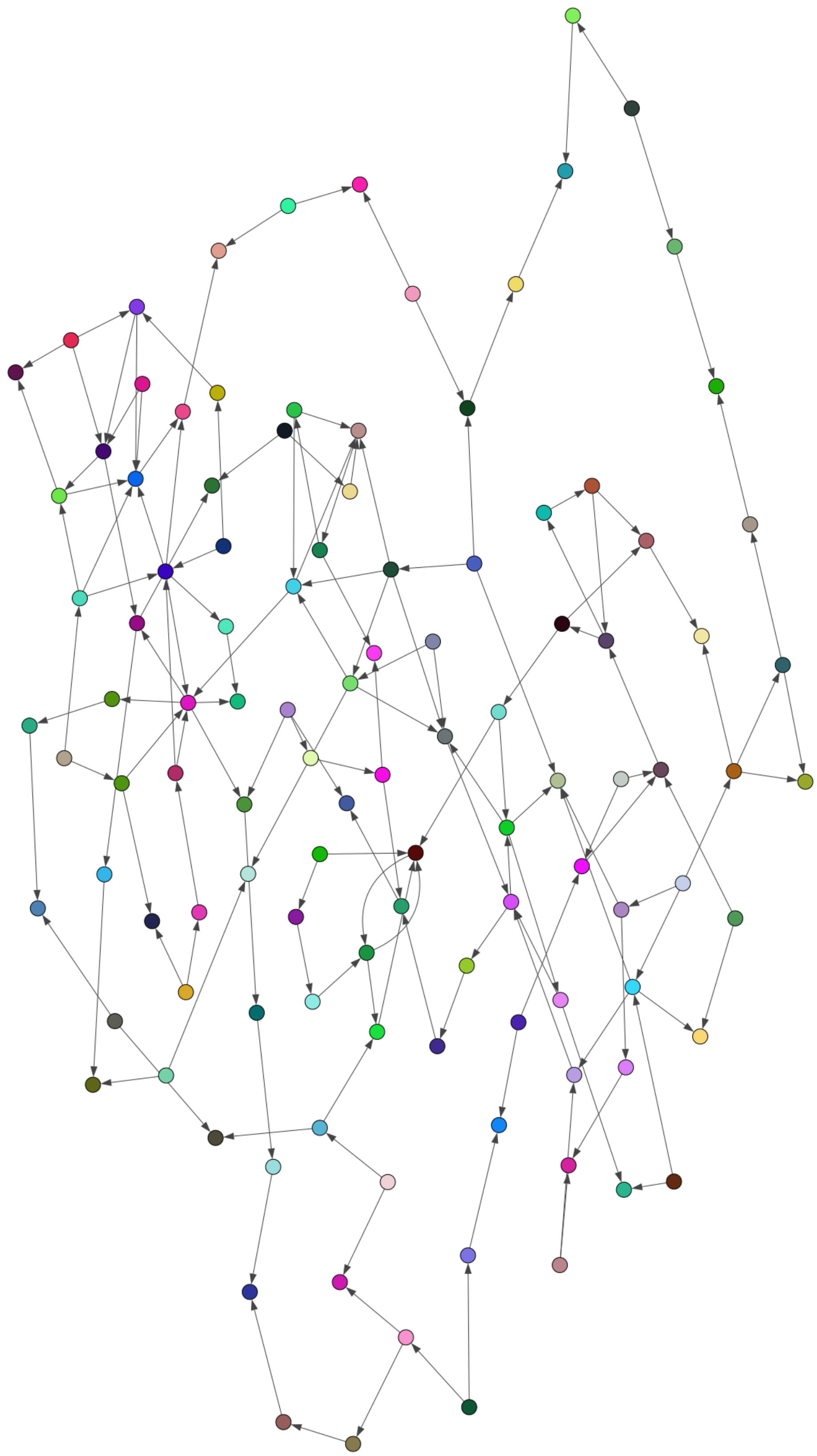


Figure 4.1: The substation network based on ACTIVSg200 [9, 10]

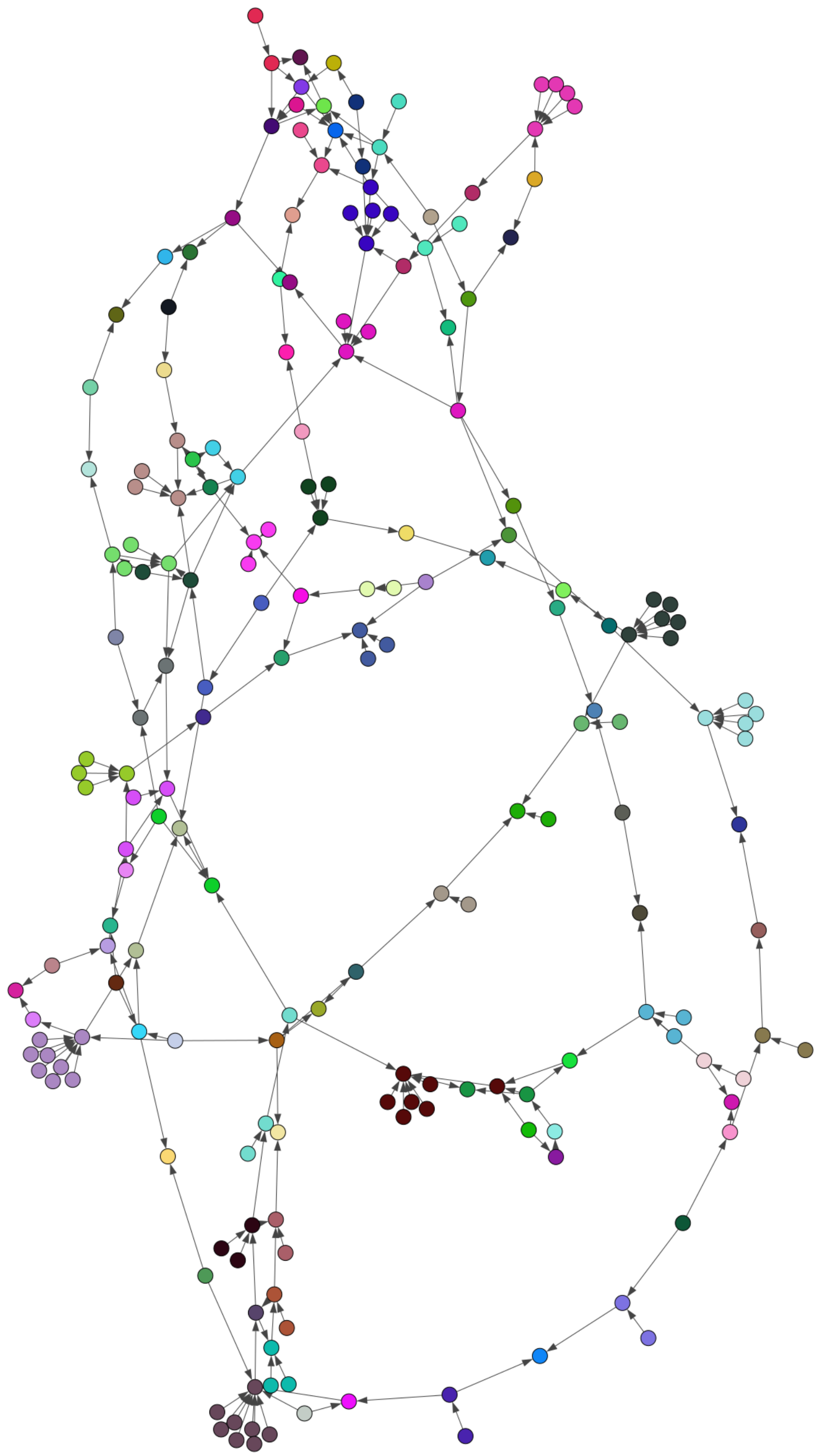


Figure 4.2: The bus network based on ACTIVSg200 [9, 10]

Chapter 5

EXPERIMENTS

With the graph-theoretical models defined, we created the bus and substation networks for existing grids (both real and synthetic). We analyzed every grid's average vertex degree, maximum vertex degree, degeneracy, density, and planarity. Each of these properties could give an idea on how sparse or dense the graphs may be.

5.1 Tools

For graph property analysis, we used the python-igraph network analysis package [33]. This is a graph theory package that allows for the analysis of graph properties and transformation of graphs.

For statistical analysis, we used the spreadsheet program LibreOffice Calc.

5.2 Data

The following is a description of all data used in the thesis. All 72 test cases were sourced from MATPOWER's collection of test cases, which combines grid data from many real life and synthetic grids [69]. We refer to the names of each network as given in the MATPOWER test collection. The ACTIVSg networks are synthetic grids based on several regions of the United States, with SyntheticUSA being representative of the national grid [9, 10]. The PEGASE networks come from the PEGASE project described before [34]. The RTE data comes from RTE's Convergence tool [34]. The Polish grid snapshots were provided by Roman Korab and published by the MATPOWER maintainers [69]. The Caracas, Venezuela data is a distribution network published by Khodr *et. al* [37]. Ramalinga Raju *et. al* published a portion of the East Indian Power Distribution System [58]. The New England data was published by Bills *et. al*, and Pai made modifications using data from Athay *et. al* [3, 8, 52, 55]. The portion of the PG&E distribution system was published by Baran and Wu [5]. The Power Systems Test Case Archive is an archive of grid data and contains several test cases [19]. The RTS-79 and RTS-GMLC networks are as described in Section 3.2.2 [56, 6]. Alsac and Stott published a 30-bus case, and Ferrero, Shahidehpour, and Ramesh added generator costs and limits [2, 25]. All other networks were published by the authors cited in Tables 5.1 to 5.3

Out of the test cases, we could only find substation data for ACTIVSg70k and SyntheticUSA. No other test cases had substation data, so most of our experiments will focus on just the bus networks for each grid.

5.3 Maximum Degree

The maximum degree of a graph is the largest number of edges that can be connected to each vertex. As discussed in Section 2.2.1, the family of graphs with bounded maximum degree has bounded expansion.

9241pegase and 13659pegase had the highest maximum degree of 46. The average was 9.694, and the median was 6.5. Some graphs with large $|V|$ had larger maximum degrees. For example, the 9241pegase network had a maximum degree of 46 with 9,241 vertices. However, this increase is not guaranteed; for example, the 3120sp network had 3120 vertices and a maximum degree of 9. SyntheticUSA had a maximum degree of 23 with 82,000 vertices.

The Pearson correlation coefficient between the factors was 0.444, implying a moderate correlation. This suggests that, while a high $|V|$ allows for the possibility of a high maximum degree, it is not guaranteed. There is no evidence to show that the maximum degree has a constant bound.

The full results are listed in Table 5.1.

5.4 Average Degree

The average degree of a graph gives an idea of how many edges will be connected to most of its vertices. A bounded average degree could suggest sparse graphs, as it would show that these networks maintain a relatively low number of edges connected to each vertex as the number of vertices grows.

The network with the highest average degree was Caracas, Venezuela (141) network, with 6.248. The mean was 2.499 and the median was 2.440. Note that a high average degree does not necessarily mean a high maximum degree and vice versa; 9241pegase and 13659pegase have average degrees of 3.473 and 2.997, respectively.

Calculating the Pearson correlation coefficient between $|V|$ and the average degree resulted in 0.0541, implying an extremely weak correlation between $|V|$ and average degree. This suggests a possible constant bound on the average degree of bus networks.

The full results are listed in Table 5.1.

5.5 Degeneracy

9241pegase and 13659pegase had the maximum degeneracy over all graphs, with 22. All others had a degeneracy less than or equal to 7. The average degeneracy was 3.32, and the median was 2. The Pearson coefficient correlation between $|V|$ and graph degeneracy was 0.370, implying a moderate correlation. However, this is not enough evidence to suggest a bound on degeneracy.

The full results are listed in Table 5.1.

Table 5.1: Average and max degree and degeneracy grouped by data source

Network	$\|V\|$	$\|E\|$	average degree	max degree	degeneracy
ACTIVSg [9, 10]					
ACTIVSg200	200	245	2.45	11	2
ACTIVSg500	500	597	2.388	16	2
ACTIVSg2000	2000	3206	3.206	17	9
ACTIVSg10k	10000	12706	2.5412	20	5
ACTIVSg25k	25000	32230	2.5784	23	6
ACTIVSg70k	70000	88207	2.5202	23	7
SyntheticUSA	82000	104121	2.53954	23	9
PEGASE [34]					
89pegase	89	210	4.7191	15	8
1354pegase	1354	1991	2.94092	17	5
2869pegase	2869	4582	3.19414	17	8

Network	$\|V\|$	$\|E\|$	average degree	max degree	degeneracy
9241pegase	9241	16049	3.47343	46	22
13659pegase	13659	20467	2.99685	46	22
RTE [34]					
1951rte	1951	2596	2.6612	17	5
1888rte	1888	2531	1.88889	4	1
2848rte	2848	3776	2.65169	17	5
2868rte	2868	3808	2.65551	17	5
6468rte	6468	9000	2.78293	17	5
6470rte	6470	9005	2.78362	17	5
6495rte	6495	9019	2.77721	17	5
6515rte	6515	9037	2.77421	17	5
Polish grid snapshots [69]					
2383wp	2383	2896	2.43055	9	2
2746wp	2746	3514	2.55936	10	2
2746wop	2746	3514	2.55936	10	2
2737sop	2737	3506	2.56193	10	2
2736sp	2736	3504	2.5614	10	2
3012wp	3012	3572	2.37185	10	2
3120sp	3120	3693	2.36731	9	2
3375wp	3374	4161	2.46651	13	5
Caracas, Venezuela [37]					
141	141	140	6.24828	20	7
Portion of East Indian Power Distribution System [58]					
22	22	21	1.90909	3	1
New England [8, 52, 3, 55]					
39	39	46	2.35897	5	2
Portion of the PG&E distribution system [5]					
69	69	68	1.97101	4	1
Power Systems Test Case Archive [19]					
14	14	20	1.98582	4	1
ieee30	30	41	2.73333	7	2
57	57	80	2.80702	6	2
118	118	186	3.15254	12	3

Network	$\|V\|$	$\|E\|$	average degree	max degree	degeneracy
145	145	453	2.85714	5	2
300	300	411	2.74	12	2
RTS-79 [56]					
24_ieee_rts	24	38	3.16667	5	2
RTS-GMLC [6]					
RTS_GMLC	73	120	3.28767	6	2
Grainger and Stevenson [30]					
4gs	4	4	2	2	2
Zimmerman *et. al* [69]					
4_dist	4	3	1.5	2	1
Li and Bo [40]					
5	5	6	2.4	3	2
Wood and Wollenberg [66]					
6ww	6	11	3.66667	5	3
Schulz, Turner, and Ewart[61]					
9	9	9	2	3	2
9Q	9	9	2	3	2
9target	9	9	2	3	2
Baghzouz and Ertem [4]					
10ba	10	9	1.8	2	1
Das, Nagi, and Kothari [17]					
12da	12	11	1.83333	2	1
28da	28	27	1.92857	3	1
Das, Kothari, and Kalam [16]					
15da	15	14	1.86667	4	1
16am	15	14	1.86667	4	1
85	85	84	1.97647	4	1
Battu, Abhyankar, and Senroy [7]					
15nbr	15	14	1.86667	4	1
18nbr	18	17	1.88889	3	1
Civanlar *et. al* [12, 68]					
16ci	16	16	2	3	2
Mendoza *et. al* [45]					

Network	$\|V\|$	$\|E\|$	average degree	max degree	degeneracy
17me	17	16	1.88235	3	1
Grady, Samotyj, and Noyola [29]					
18	18	17	2.68114	17	5
Alsac & Stott and Ferrero, Shahidehpour, & Ramesh [2, 25]					
30	30	41	2.73333	7	2
30pwl	30	41	2.73333	7	2
30Q	30	41	2.73333	7	2
Baran and Wu [5]					
33bw	33	37	2.24242	3	2
Kashem *et. al* [35]					
33mg	33	37	2.24242	3	2
Salama and Chikhani [60]					
34sa	34	33	1.94118	3	1
Singh and Misra [62]					
38si	38	37	1.94737	3	1
Gampa and Das [27]					
51ga	51	50	1.96078	3	1
Hengsritawat *et. al* [32]					
51he	51	50	1.96078	3	1
Das [15]					
70da	70	76	2.17143	3	2
Myint and Naing [46]					
74ds	74	73	1.97297	3	1
Pires, Antunes, and Martins [53]					
94pi	94	93	1.97872	4	1
Zhang *et. al* [67]					
118zh	118	132	2.23729	4	2
Mantovani *et. al* [43]					
136ma	136	156	2.29412	8	2

5.6 Density

Density is calculated by dividing the total number of edges in each network by the total possible number of edges on a directed graph with the same number of vertices using the equation

$$D_e = \frac{|E|}{2\binom{|V|}{2}}$$

This results in densities as high as 0.36... and as low as 1.5×10^{-5}. However, we observed that networks with fewer buses tend to have a higher density than networks with more buses. We hypothesized that $D_e \sim \dfrac{1}{|V|}$.

To confirm this hypothesis, we considered D_e^{-1} for each graph. Doing this allowed us to run a linear correlation test on our data. The Pearson correlation coefficient comparing $|V|$ with D_e^{-1} was 0.99949, implying a high correlation between the number of vertices with D_e^{-1}. This then implies a correlation between a large $|V|$ and a small D_e.

With this knowledge, we created a trend line to predict the edge density of a bus network based on $|V|$. We found the trend line $D_e^{-1} = 0.787261|V| - 78.30860$, with $R^2 = 0.99898$ showing that this predicts D_e^{-1} extremely well. Thus, we can predict edge density with 99.898% certainty using

$$D_e = (0.787261|V| - 78.30860)^{-1} = \frac{1.270226774}{|V| - 99.468670322}$$

Interestingly, for cases based on real life data or synthetic data derived from real networks, density measurements seem to be very similar across the world. For example, 9241pegase, a 9,241-bus network, has a density of 1.880×10^{-4}; ACTIVSg10k, a 10,000-bus network, has a density of 1.2707×10^{-4}; and 13659pegase, a 13,659-bus network, has a density of 1.097×10^{-4}. ACTIVSg10k's number of vertices is between 9421pegase's and 13659pegase's, and its density is between 13659pegase's and 9421pegase's. Similarly, 89pegase has 89 vertices and a density of 2.681×10^{-2}, the Carcas, Venezuela case has 141 vertices and a density of 7.092×10^{-3}, and ACTIVSg200 has 200 vertices and a density of 6.156×10^{-3}. Similarly, the East Indian case has 22 vertices with a density of 4.545×10^{-2}, the New England case has 39 vertices and a density of 3.104×10^{-2}, and the PG&E case has 69 vertices and a density of 1.449×10^{-2}. The fact that this relationship is present seemingly independent of geographic location is very promising, as it suggests that this may be a universal property of real-life electrical grids.

The full results of edge density testing are listed in Table 5.2.

Table 5.2: Edge density and inverse edge density grouped by data source

Network	$\|V\|$	$\|E\|$	D_e	D_e^{-1}
ACTIVSg [9, 10]				
ACTIVSg200	200	245	0.006155778894472	162.448979591837
ACTIVSg500	500	597	0.002392785571142	417.922948073703
ACTIVSg2000	2000	3206	0.000801900950475	1247.03680598877
ACTIVSg10k	10000	12706	0.000127072707271	7869.51046749568
ACTIVSg25k	25000	32230	5.15700628025121E-05	19391.09525287
ACTIVSg70k	70000	88207	1.80016857383677E-05	55550.3531465756
SyntheticUSA	82000	104121	1.54851680225726E-05	64577.923761777
PEGASE [34]				
89pegase	89	210	0.026813074565884	37.295238095238
1354pegase	1354	1991	0.001086812936076	920.121546961322
2869pegase	2869	4582	0.000556859187788	1795.78611959843
9241pegase	9241	16049	0.000187956364236	5320.38382453735
13659pegase	13659	20467	0.000109710495407	9114.89822641325
RTE [34]				
1951rte	1951	2596	0.000682358816649	1465.50462249615
1888rte	1888	2531	0.000710425031213	1407.60806005532
2848rte	2848	3776	0.000465698172332	2147.31355932204
2868rte	2868	3808	0.000463116335115	2159.28466386555
6468rte	6468	9000	0.00021516401379	4647.61733333334
6470rte	6470	9005	0.000215150463165	4647.91004997224
6495rte	6495	9019	0.000213829168537	4676.63044683446
6515rte	6515	9037	0.000212942382085	4696.10600863118
Polish grid snapshots [69]				
2383wp	2383	2896	0.000510190958697	1960.05041436464
2746wp	2746	3514	0.000466185622538	2145.06829823563
2746wop	2746	3514	0.000466185622538	2145.06829823563
2737sop	2737	3506	0.000468188801073	2135.89047347405

Network	$\|V\|$	$\|E\|$	D_e	D_e^{-1}
2736sp	2736	3504	0.000468263895571	2135.54794520548
3012wp	3012	3572	0.000393863492118	2538.95072788354
3120sp	3120	3693	0.000379497866673	2635.06092607636
3375wp	3374	4161	0.000365625347634	2735.04013458303
Caracas, Venezuela [37]				
141	141	140	0.00709219858156	141
Portion of East Indian Power Distribution System [58]				
22	22	21	0.045454545454546	22
New England [8, 52, 3, 55]				
39	39	46	0.031039136302294	32.2173913043478
Portion of the PG&E distribution system [5]				
69	69	68	0.014492753623188	69
Power Systems Test Case Archive [19]				
14	14	20	0.10989010989011	9.09999999999999
ieee30	30	41	0.047126436781609	21.2195121951219
57	57	80	0.025062656641604	39.9
118	118	186	0.013472403302912	74.2258064516128
145	145	453	0.021695402298851	46.092715231788
300	300	411	0.004581939799331	218.248175182482
RTS-79 [56]				
24_ieee_rts	24	38	0.068840579710145	14.5263157894737
RTS-GMLC [6]				
RTS_GMLC	73	120	0.022831050228311	43.8
Grainger and Stevenson [30]				
4gs	4	4	0.333333333333333	3
Zimmerman *et. al* [69]				
4_dist	4	3	0.25	4
Li and Bo [40]				
5	5	6	0.3	3.33333333333333
Wood and Wollenberg [66]				
6ww	6	11	0.366666666666667	2.72727272727272
Schulz, Turner, and Ewart[61]				
9	9	9	0.125	8

Network	$\|V\|$	$\|E\|$	D_e	D_e^{-1}
9Q	9	9	0.125	8
9target	9	9	0.125	8
Baghzouz and Ertem [4]				
10ba	10	9	0.1	10
Das, Nagi, and Kothari [17]				
12da	12	11	0.083333333333333	12
28da	28	27	0.035714285714286	28
Das, Kothari, and Kalam [16]				
15da	15	14	0.066666666666667	15
16am	15	14	0.066666666666667	15
85	85	84	0.011764705882353	85.0000000000003
Battu, Abhyankar, and Senroy [7]				
15nbr	15	14	0.066666666666667	15
18nbr	18	17	0.055555555555556	18
Civanlar *et. al* [12, 68]				
16ci	16	16	0.066666666666667	15
Mendoza *et. al* [45]				
17me	17	16	0.058823529411765	17
Grady, Samotyj, and Noyola [29]				
18	18	17	0.055555555555556	18
Alsac & Stott and Ferrero, Shahidehpour, & Ramesh [2, 25]				
30	30	41	0.047126436781609	21.2195121951219
30pwl	30	41	0.047126436781609	21.2195121951219
30Q	30	41	0.047126436781609	21.2195121951219
Baran and Wu [5]				
33bw	33	37	0.035037878787879	28.5405405405405
Kashem *et. al* [35]				
33mg	33	37	0.035037878787879	28.5405405405405
Salama and Chikhani [60]				
34sa	34	33	0.029411764705882	34.0000000000001
Singh and Misra [62]				
38si	38	37	0.026315789473684	38
Gampa and Das [27]				

Network	$\lvert V\rvert$	$\lvert E\rvert$	D_e	D_e^{-1}
51ga	51	50	0.019607843137255	51
Hengsritawat *et. al* [32]				
51he	51	50	0.019607843137255	51
Das [15]				
70da	70	76	0.015734989648033	63.5526315789475
Myint and Naing [46]				
74ds	74	73	0.013513513513514	74.0000000000001
Pires, Antunes, and Martins [53]				
94pi	94	93	0.01063829787234	94.0000000000002
Zhang *et. al* [67]				
118zh	118	132	0.009561060408518	104.590909090909
Mantovani *et. al* [43]				
136ma	136	156	0.008496732026144	117.692307692308

5.7 Planarity

Out of all the bus networks, fifty-five are planar and seventeen are nonplanar. By Theorem 11, we knew the substation networks associated with those planar bus networks had to be planar as well. Out of the seventeen nonplanar bus networks, we only had the substation data for ACTIVSg70k and SyntheticUSA; we found that those networks are also not planar. For the remaining fifteen nonplanar bus networks, we could not experimentally test the planarity of their associated substation networks, as we did not have access to substation data for those cases. Overall, there is no guarantee that bus and substation networks will always be planar or nonplanar.

The full results of planarity testing are listed in Table 5.3.

Table 5.3: Bus and substation network planarity grouped by data source

Network	$\lvert V \rvert$	$\lvert E \rvert$	Bus planar?	Substation planar?
ACTIVSg [9, 10]				
ACTIVSg200	200	245	True	True*
ACTIVSg500	500	597	True	True*
ACTIVSg2000	2000	3206	True	True*
ACTIVSg10k	10000	12706	True	True*
ACTIVSg25k	25000	32230	True	True*
ACTIVSg70k	70000	88207	False	False†
SyntheticUSA	82000	104121	False	False†
PEGASE [34]				
89pegase	89	210	False	**?**
1354pegase	1354	1991	True	True*
2869pegase	2869	4582	False	**?**
9241pegase	9241	16049	False	**?**
13659pegase	13659	20467	False	**?**
RTE [34]				
1951rte	1951	2596	True	True*
1888rte	1888	2531	True	True*
2848rte	2848	3776	False	**?**
2868rte	2868	3808	True	True*
6468rte	6468	9000	False	**?**
6470rte	6470	9005	False	**?**
6495rte	6495	9019	False	**?**
6515rte	6515	9037	False	**?**
Polish grid snapshots [69]				
2383wp	2383	2896	True	True*
2746wp	2746	3514	False	**?**
2746wop	2746	3514	False	**?**
2737sop	2737	3506	False	**?**
2736sp	2736	3504	False	**?**

* determined by Theorem 11

† determined by testing substation data

? unable to find substation data for this network or apply Theorem 11

Network	$\|V\|$	$\|E\|$	Bus planar?	Substation planar?
3012wp	3012	3572	True	True*
3120sp	3120	3693	True	True*
3375wp	3374	4161	False	**?**
Caracas, Venezuela [37]				
141	141	140	True	True*
Portion of East Indian Power Distribution System [58]				
22	22	21	True	True*
New England [8, 52, 3, 55]				
39	39	46	True	True*
Portion of the PG&E distribution system [5]				
69	69	68	True	True*
Power Systems Test Case Archive [19]				
14	14	20	True	True*
ieee30	30	41	True	True*
57	57	80	True	True*
118	118	186	True	True*
145	145	453	False	**?**
300	300	411	True	True*
RTS-79 [56]				
24_ieee_rts	24	38	True	True*
RTS-GMLC [6]				
RTS_GMLC	73	120	True	True*
Grainger and Stevenson [30]				
4gs	4	4	True	True*
Zimmerman *et. al* [69]				
4_dist	4	3	True	True*
Li and Bo [40]				
5	5	6	True	True*
Wood and Wollenberg [66]				
6ww	6	11	True	True*
* determined by Theorem 11				
† determined by testing substation data				
? unable to find substation data for this network or apply Theorem 11				

Network	$\|V\|$	$\|E\|$	Bus planar?	Substation planar?
Schulz, Turner, and Ewart[61]				
9	9	9	True	True*
9Q	9	9	True	True*
9target	9	9	True	True*
Baghzouz and Ertem [4]				
10ba	10	9	True	True*
Das, Nagi, and Kothari [17]				
12da	12	11	True	True*
28da	28	27	True	True*
Das, Kothari, and Kalam [16]				
15da	15	14	True	True*
16am	15	14	True	True*
85	85	84	True	True*
Battu, Abhyankar, and Senroy [7]				
15nbr	15	14	True	True*
18nbr	18	17	True	True*
Civanlar *et. al* [12, 68]				
16ci	16	16	True	True*
Mendoza *et. al* [45]				
17me	17	16	True	True*
Grady, Samotyj, and Noyola [29]				
18	18	17	True	True*
Alsac & Stott and Ferrero, Shahidehpour, & Ramesh [2, 25]				
30	30	41	True	True*
30pwl	30	41	True	True*
30Q	30	41	True	True*
Baran and Wu [5]				
33bw	33	37	True	True*
Kashem *et. al* [35]				
33mg	33	37	True	True*

* determined by Theorem 11

† determined by testing substation data

? unable to find substation data for this network or apply Theorem 11

Network	$|V|$	$|E|$	Bus planar?	Substation planar?
Salama and Chikhani [60]				
34sa	34	33	True	True*
Singh and Misra [62]				
38si	38	37	True	True*
Gampa and Das [27]				
51ga	51	50	True	True*
Hengsritawat *et. al* [32]				
51he	51	50	True	True*
Das [15]				
70da	70	76	True	True*
Myint and Naing [46]				
74ds	74	73	True	True*
Pires, Antunes, and Martins [53]				
94pi	94	93	True	True*
Zhang *et. al* [67]				
118zh	118	132	True	True*
Mantovani *et. al* [43]				
136ma	136	156	True	True*
* determined by Theorem 11				
† determined by testing substation data				
? unable to find substation data for this network or apply Theorem 11				

5.8 Discussion

We hypothesize that bus networks are either a minor-closed family or can be shown to be part of some existing minor-closed family. The heavily-decreasing edge density and bounded average degree suggest that they could belong to a family with bounded expansion, possibly with polynomial expansion. Table 5.3 clearly shows that bus networks cannot universally be included in the family of planar graphs, so other properties must be considered to provide a classification.

Chapter 6

CONCLUSIONS

We have established the bus network and substation network as graph-theoretical models for electrical distribution grids and proved that the substation network of an electrical distribution grid is a minor of the bus network of the same grid. We have also created bus and substation networks from existing grid data and analyzed properties of those graphs.

Establishing a graph-theoretical model for electrical distribution grids provides several benefits. As demonstrated in Sections 2.2 and 3.1, graph theoretical analysis can allow for efficient algorithms. As we learn more about the properties of these networks, this could lead to the fast solving of problems on electrical distribution grids.

6.1 Future Research

Proving that bus and substation networks are part of a graph family with polynomial expansion would be extremely helpful for future research, as demonstrated in Sections 2.2.1 and 2.2.6. The possibly constant bound average degree and sharply decreasing edge density as $|V|$ grows both show promise for bounded or even polynomial expansion. Though most networks did not have substation data, Theorem 10 shows that any minor-closed graph family that contains all bus networks will also contain all substation networks. This means it is not necessary to have substation network data for any research regarding graph classification. Generating and testing the properties of all k-shallow minors of all the test case would be helpful in describing a graph family containing bus networks.

Among the graph families that would be helpful to examine is the family of graphs with crossing graphs with bounded degeneracy described by Eppstein and Gupta [23]. The road datasets used in their study contained real-world coordinates that allowed them to greedily find the crossing number of their road networks and create crossing graphs. The fact that the test data we used did not have associated location information prevents us from using the same technique as Eppstine and Gupta. As mentioned in Section 2.2.6.3.1, this is a hard problem, and we did not have access to enough computational power to test the bus networks and their minors for bounded degeneracy in crossing graphs.

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