

## Controlled multi-scale turbulence through the use of Laser Sintered Sierpinski Pyramids

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### Abstract

The research presented here is the result of a new collaboration between the Centre for Advanced Additive Manufacturing (AdAM) and the Thermofluids group at The University of Sheffield, regarding the use of fractal geometries for the control and influence of fluid flow. It is believed that the use of multiscale objects can be used to introduce many different orders of turbulence into a flow. However, whilst substantial simulations have been carried out in this area, the complexity of the physical geometries means that to date these have not been validated via physical testing.

In this work, varying orders of Sierpinski pyramids were produced using Laser Sintered PA2200 and analysed in a wind tunnel with regards to their effects on air flow through the structures. As predicted by theoretical analyses, the coarsest pyramids induced large vortices into the air-stream, whereas the more complex orders induced vortices at a number of different scales, rapidly developing into a standard turbulent flow. Further investigations are planned to isolate the effects of the smaller-scale turbulence in this situation.

### Aim

This paper presents preliminary research investigating fluid flow through 3D fractal geometries. Previous, theoretical, research makes predictions regarding the effects of different order fractals, but inability to efficiently produce physical models has prevented substantial experimental work. This ongoing research utilises the geometric freedom of Laser Sintering to provide experimental validation of these theoretical studies.

### Characteristics of turbulent flow

Fully-developed turbulent flow comprises a range of vortices of varying sizes, similar to those shown in Figure 1, all of which have an impact on energy transfer.

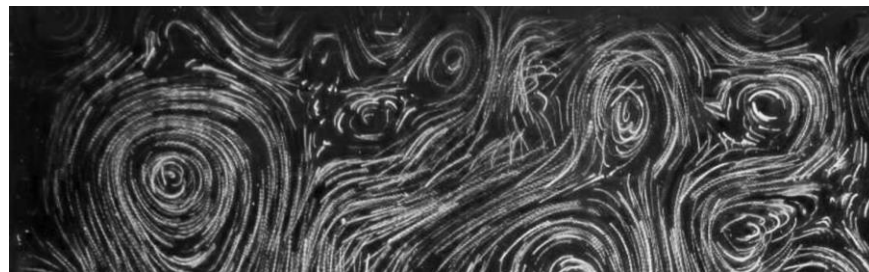


Figure 1 - Example of varied size vortices for turbulence<sup>1</sup>

Whenever a disruption occurs within a flow-stream (for example a physical object such as a valve, or simply a bend in a pipe), this flow structure is likely to change, requiring a certain distance/time before the flow returns to its fully-developed state. For example, a solid object placed within the flow would be expected to lead to the generation of mainly large vortices, which would gradually break down into the normal range of sizes.

In many cases, minimisation of the distance before flow returns to its fully-developed state is required, with the theoretical optimum being the case where the object appears ‘invisible’ to the flow. Whilst difficult to achieve in practice, theoretically this would mean that the flow immediately following the obstruction is indistinguishable from the flow entering it.

In other cases, it is more important to control the flow characteristics of a fluid. For example, the accuracy of standard orifice plate flow measurement devices can be sensitive to fluctuations in flow (e.g. asymmetry, flow not being fully-developed), requiring the use of a separate flow conditioner<sup>2</sup>. This involves the deliberate placement of an additional object within the flow, upstream of the measurement device, in order to ensure the flow contains the required characteristics.

### Fractal structures

Whilst the exact origins of fractal-related concepts are debatable, the term ‘fractal’ itself was first proposed by Benoit Mandelbrot<sup>3</sup>. Although there are various strict mathematical definitions, at its most simplistic a fractal can be described as a repeating pattern that is self-similar across different length scales. Fractal structures are commonly found in nature (for example the human lung, or the branches of a tree), as shown in Figure 2.

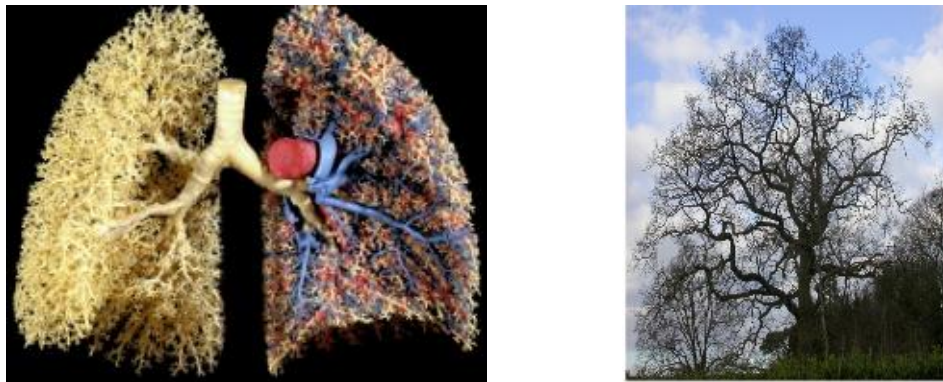


Figure 2 - Fractals in Nature - human lung, branches of a tree<sup>4</sup>

These structures have also developed applications across a number of sectors, ranging from the aesthetic use of mathematical forms<sup>5</sup> through to cooling systems<sup>6</sup> or computer games<sup>7</sup>.

One of the earliest examples of a fractal geometry, the Koch snowflake<sup>8</sup>, is shown in Figure 3.



Figure 3 - Koch Snowflake<sup>9</sup>

The base of this particular geometry is an equilateral triangle. To produce the second order fractal, the centre third of each side is replaced with two sides of an additional equilateral triangle. Whilst only four different orders of the Koch Snowflake are shown in Figure 3, this process can be repeated an infinite number of times, in each case replacing the centre third of each line with a new equilateral triangle.

### **Fractals and turbulence**

A number of authors have studied the effects of fractal geometries within a flowstream, whether by experimentation within a wind/water tunnel<sup>10,11</sup> or via theoretical modelling<sup>12</sup>. Of particular relevance here is research into the use of 2D fractal plates as flow conditioners, in combination with an orifice plate flow measurement device, which showed decreased sensitivity to upstream flow disturbances. Simulation work within the same research also demonstrated that the use of these fractal plates could reduce the length of pipe required to return a flow to its fully-developed state<sup>13</sup>.

Additional work investigated pressure drops across fractal plates, demonstrating their potential for use in flowmeters themselves, in place of the standard circular cross-sections<sup>14</sup>.

In order to build on this, and other 2D fractal work, the research presented in this paper investigated the effects of several different orders of 3D fractal on downstream air-flow.

### **Experimental work**

#### **Geometry selection**

For this work, the geometry selected for investigation was the Sierpinski pyramid, an example of which can be seen in Figure 4.

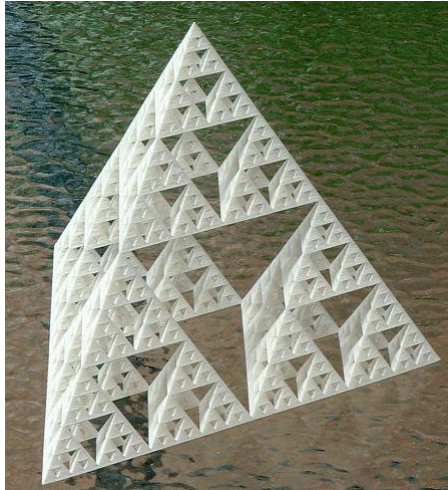


Figure 4 - Sierpinski pyramid<sup>15</sup>

The Sierpinski pyramid is a 3D geometry based around the triangular structures used in the Sierpinski sieve<sup>16</sup>, as shown in Figure 5.

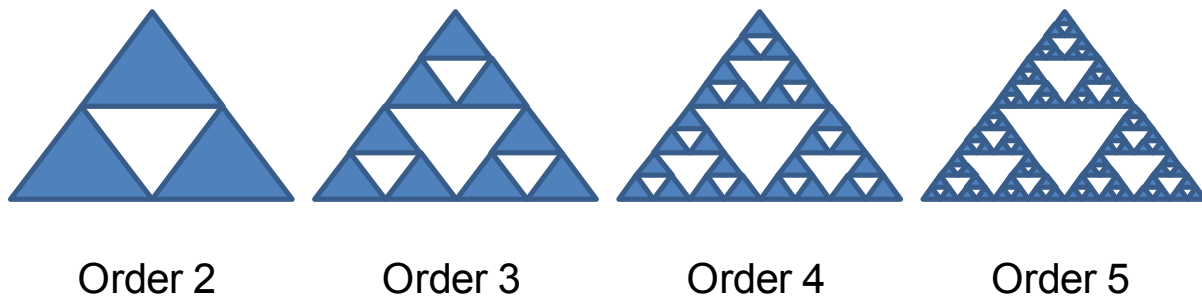


Figure 5 - Sierpinski triangle generation

The structure begins with a solid triangle (order 1). This can then be divided into four triangles of equal size, and the centre one removed to create the structure seen in the order 2 triangle. This process can then be repeated as many times as required, in each case replacing every solid triangle with this new structure.

It is clear that a change in the order of fractal used would necessarily cause a change in the apparent flow area of the geometry. However, for this work the focus was on the effect of different fractal iterations, rather than the effect of variations in flow area. For this reason, the geometries were designed to ensure a constant flow area for each sample. As it was not possible to maintain a constant overall size and a constant flow area simultaneously, this meant that each pyramid was produced using different overall dimensions, as shown in Table 1.

Order/n	Base of total(cm)	Base of unit(cm)	Height of total(cm)	Height of unit(cm)	Total volume (cm <sup>3</sup> )
2	6.5	3.2	5.6	2.8	49
3	7.6	1.9	6.6	1.6	49
4	8.9	1.1	7.7	1.0	49
5	10.4	0.6	9.0	0.6	49

Table 1 - Size of Sierpinski pyramids

### Part production

The pyramid geometries were produced in PA2200 (50:50 virgin:used powder) on an EOS Formiga P100 system, using the parameters shown in Table 2.

Parameter	Value
Laser Power (contour)	16.0 W
Scan Speed (contour)	1500 mm/s
Laser Power (hatching)	21.0 W
Scan Speed (hatching)	2500 mm/s
Scan spacing	0.15 mm
Layer thickness	0.10 mm

Table 2 - Laser Sintering parameters

### Testing

Each sample was mounted within a wind tunnel, and fluid flow achieved by drawing air through the fan shown on the right in Figure 6. Smoke was generated and incorporated into the air flow, to provide a visible indication of the flow, and a laser/camera system was included to allow easier visualisation of flow and recording of video footage.

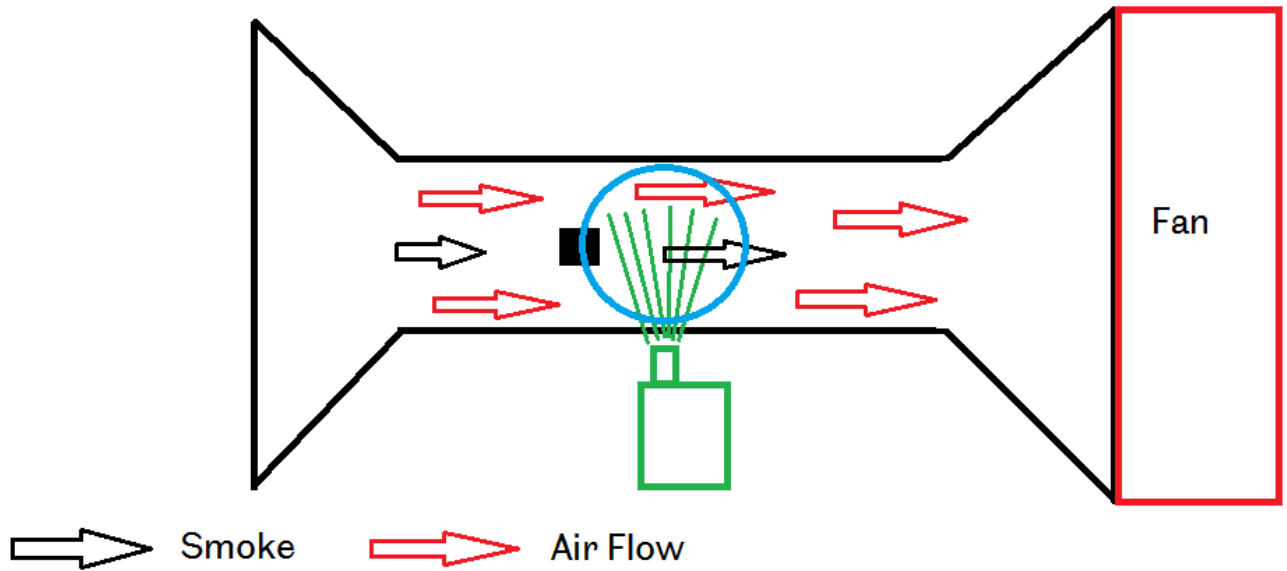
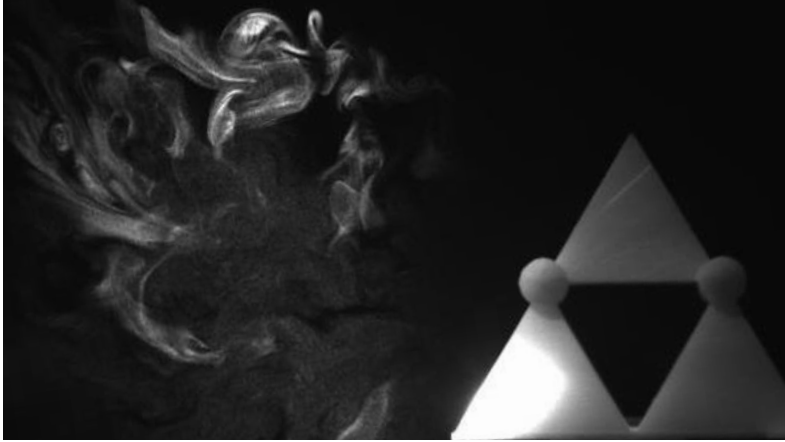


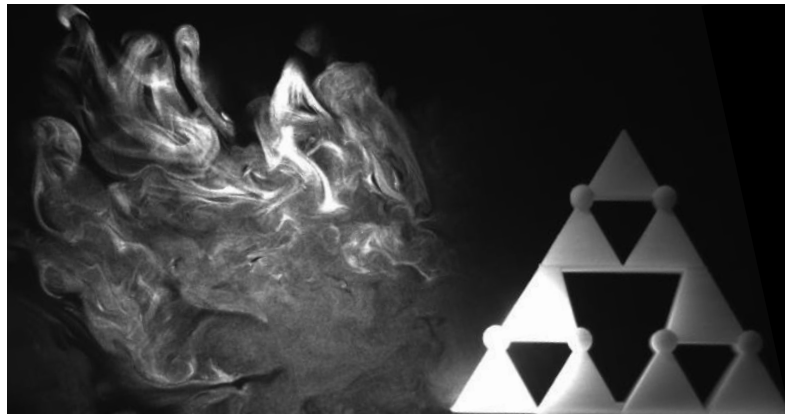
Figure 6 - Experimental rig set-up

## Results

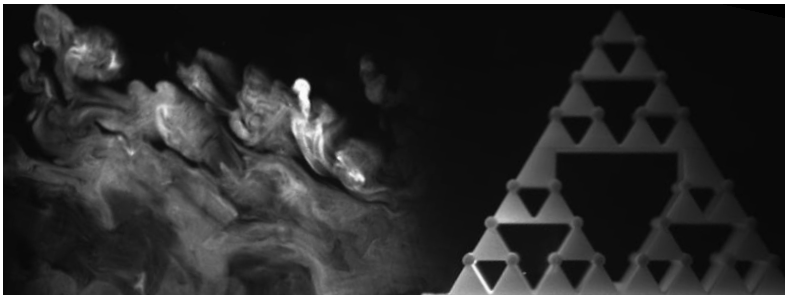
The purpose of this work was to investigate the flow characteristics immediately following the object. Figure 7 shows examples of camera images for each order of pyramid. However, as it is somewhat difficult to observe detail from these still images, Figure 8 provides a schematic of the observed effects.



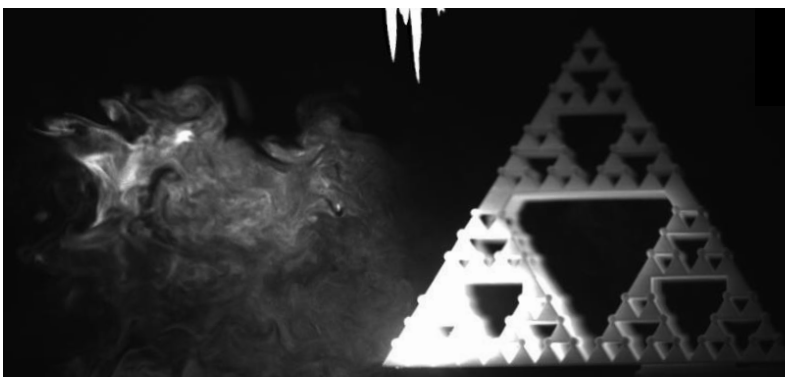
Order 2



Order 3

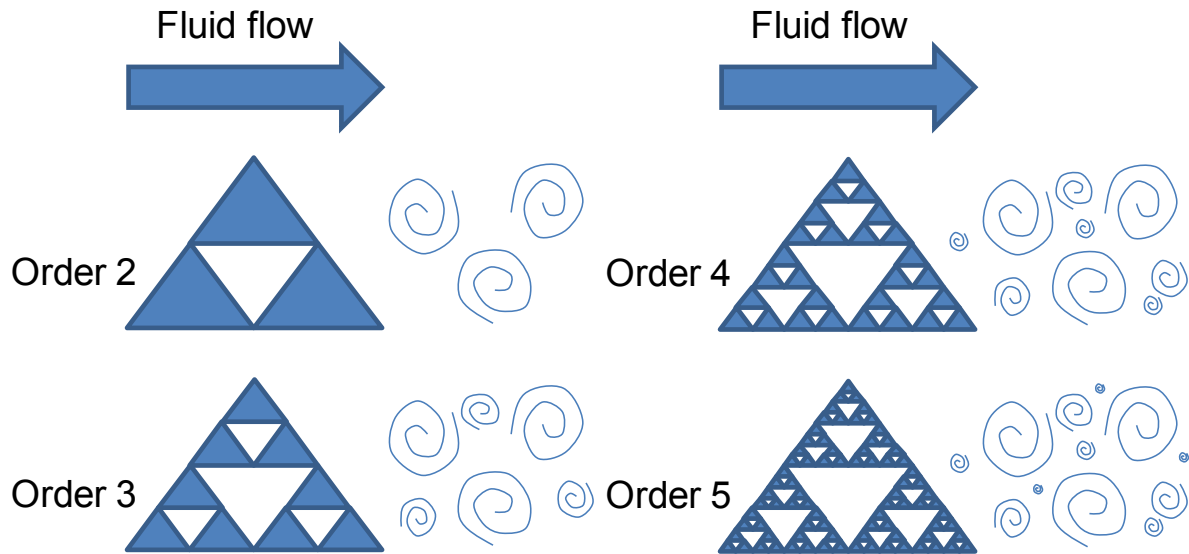


Order 4



Order 5

Figure 7 - Still images of flow through each geometry



**Figure 8 - Schematic of observed effects**

When observing the Order 2 geometry, it can be seen that the flow immediately following the object is comprised almost entirely of large vortices, of a size comparable with the flow areas (holes) within the pyramid. As the fractal order of the pyramid was increased, so too the range of vortex sizes increased, again of comparable sizes to the flow areas. In general, the higher order fractals produced downstream flow closer to a fully-developed state than the lower order fractals.

Whilst further, in-depth research is planned to investigate this area in more detail, these results demonstrate the potential for the use of this type of 3D fractal to control and or optimise flow characteristics.

### **Future work**

The work presented in this paper represents the initial stages of an ongoing investigation into the use of 3D fractals to influence and control fluid flow. Further, more in-depth research is now required in order to establish the capabilities and limitations of these geometries for flow control applications. The planned next stages are:

- Investigations of different types of fractal geometries

Whilst Sierpinski pyramids were selected for this work, there are a wide variety of other designs which may show even greater potential for control or optimisation of fluid flow. Initial stages will focus on a selection of different 3D fractals, in order to assess their effects on the characteristics of downstream flow. These results will then be applied in order to condition the flowstream either into a fully-developed turbulent flow, or to deliberately induce specific flow characteristics as required.



- Investigations of specific scales of turbulence

In this work, fluid flow has been investigated through several orders of 3D fractal. Whilst the initial focus was on the size, and range of sizes, of vortices within the flow, further information can also be gained. Measurement and comparison of downstream velocity fluctuations in flow through fractals of different orders will allow the determination of information related to each specific turbulence scale.

- Testing of Kolmogorov Limit

The Kolmogorov Limit is a theoretical size limit below which no additional turbulence scales will be produced, and standard turbulence models break down. However, despite much discussion, this limit has yet to be confirmed. Again, by testing and comparing increasing orders of fractals, it may be possible to experimentally prove or disprove this theory by physical determination of the point at which no further scales of turbulence are observed.

As the research focuses on higher fractal orders, increasingly small feature sizes will be required. An additional feature of this project stage will therefore be the determination of the most suitable AM technologies to conduct this work.

## References

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