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Corrections to Classical Kinetic and Transport Theory for a Two-Temperature, Fully Ionized Plasma in Electromagnetic Fields

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Abstract: Sets of lower order and higher order kinetic and macroscopic equations are developed for a plasma where collisions are important but electrons and ions are allowed to have different temperatures when transports, due to gradients and fields, set in. Solving the lower order kinetic equations and taking appropriate velocity moments we show that usual classical transports emerge. From the higher order kinetic equations special notice is taken of some new correction terms to the classical transports. These corrections are linear in gradients and fields, some of which are found in a two-temperature state only.

I. Introduction

In kinetic and transport theories of collision-dominated, fully-ionized plasmas expressions for electron- and ion transports and transport coefficients have been derived using various methods by Braginskii [1], for instance, for a two-temperature plasma and by Robinson and Bernstein [2] for a one-temperature plasma. The so-called classical transports of mass, momentum and energy which are linear in gradients and fields, and corresponding derived transport coefficients, emerge from kinetic equations on a level of approximation corresponding to that of Chapman and Enskog [3]. These theories may be extended to include corrections from the next higher level of approximation corresponding to that of Burnett [4]. The effects on the transports due to derivatives and products of gradients would then be taken into account. However, especially for a two-temperature plasma proper corrections may be more subtle and revealed only after applying a refined perturbation procedure on kinetic and macroscopic equations. In this paper we apply the multiple time scale method to obtain such corrections for a two-component plasma model where ε, which measures the weakness of the gradients and fields as compared to a mean free path, and α, the square root electron-to-ion mass ratio, are treated as of the same order of magnitude. In a previous paper by Naze Tjøtta and Øien [5], a study of the evolution from the "kinetic" to the "hydrodynamic" regimes, Bogoliubov [6], of such a model was made. The equations of that study were all within the frame of the Chapman-Enskog approximation. Looking at the transport aspect of the model in the present paper, and particularly at how corrections to the classical transports may be done, we have to go to higher order

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kinetic and macroscopic equations than dealt with in [5]. It turns out that of special interest in these equations and the derived transports are not terms on the typical Burnett level of approximation, but new terms, some of which are found in the two-temperature state only, that formally belong to the Chapman-Enskog level of approximation, since they are linear in gradients and fields. They represent new mechanisms in transport processes not found in earlier theories.

Because of the different purpose of the present work as compared to [5] it is most convenient to make the presentation self-consistent and therefore we briefly outline in section II the underlying assumptions of the model and deal with the kinetic and macroscopic equations used in the study. Section III takes up what we shall call the lower order equations, i.e. we state properties of the zeroth, first and second order equations that will put our results into a right perspective. These lower order equations comprise the frame of equations used in classical transport theories and extend also into the Burnett approximation. The properties of these equations are also essential for setting up the bulk of higher (third) order equations in section IV. The higher order kinetic equations contain parts that extend from the Chapman-Enskog to beyond the Burnett level of approximation. We extract terms that give rise to new transports linear in gradients and fields in section V.

II. Assumptions and Basic Equations.

To begin with we briefly sketch the plasma model presented in [5] and make some further notes relevant for the study we are taking up in this paper.

Particles have masses m_1 and m_2 and charges e_1 and e_2 where subscript 1 refers to electrons and 2 to ions. Denoting the effective mean free path and collision frequency for electrons by λ_1 and $1/\tau$ and the scale for (all) inhomogenities by L, the plasma has a certain weakness of inhomogenities and fields characterized by

$$\frac{\lambda_1}{L} \approx \frac{e_1 E \lambda_1}{kT_1} \approx \varepsilon << 1$$

$$\tau\Omega_1 \approx 1$$

where $\underline{E}$ and $\Omega_1 = |e_1|B/m_1$ denote the macroscopic electric field and electron cyclotron frequency due to the magnetic field $\underline{B}$. k is Boltzmann's constant and T_1 the electron temperature. These assumptions may characterize a state not too far from equilibrium. In accordance with this we also assume

$$\tfrac{1}{2} m_1 \bar{c}_1^2 \approx \tfrac{1}{2} m_2 \bar{c}_2^2$$

i.e. the electron and ion temperatures are of the same order of magnitude. Then for the electron and ion mean speeds we have

$$\frac{\bar{c}_2}{\bar{c}_1} \approx \left(\frac{m_1}{m_2}\right)^{\frac{1}{2}} = \alpha << 1$$

Using the parameters ε and α introduced above we are able to parametrize the kinetic equations for the plasma: Let

$$\underline{C}_i = \underline{c}_i - \underline{c}_o \ , \quad i = 1,2,$$

denote the peculiar velocities of electrons and ions, i.e., the velocities as seen from points moving with the mass velocity $\underline{c}_o$ (see Eq. (5) below). We order velocities in the following manner:

$|\underline{C}_1| \cong \bar{c}_1$

$|\underline{C}_2| \cong \alpha\bar{c}_1$

$|\underline{c}_o| \cong \alpha\bar{c}_1$

The kinetic equations for distribution functions $f_i = f_i(\underline{C}_i,\underline{r},t)$, [3], $i = 1,2$, assuming weak (two particle) collisions, then may be parametrized as follows:

$$\frac{\partial f_1}{\partial t} + \varepsilon\underline{C}_1\cdot\frac{\partial f_1}{\partial \underline{r}} + \varepsilon\alpha\underline{c}_o\cdot\frac{\partial f_1}{\partial \underline{r}} + \varepsilon\,\frac{e_1}{m_1}\,\underline{E}\cdot\frac{\partial f_1}{\partial \underline{C}_1} - \alpha\left(\frac{\partial \underline{c}_o}{\partial t} + \varepsilon\alpha\underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\right)\cdot\frac{\partial f_1}{\partial \underline{C}_1} +$$

$$+ \alpha\,\frac{e_1}{m_1}\,\underline{c}_o \times \underline{B}\cdot\frac{\partial f_1}{\partial \underline{C}_1} + \frac{e_1}{m_1}\,\underline{C}_1 \times \underline{B}\cdot\frac{\partial f_1}{\partial \underline{C}_1} - \varepsilon\alpha\,\frac{\partial f_1}{\partial \underline{C}_1}\,\underline{C}_1 : \frac{\partial \underline{c}_o}{\partial \underline{r}} = \tag{1}$$

$$= \frac{1}{m_1^2}\frac{\partial}{\partial \underline{C}_1}\cdot\int d\underline{C}_1'\,\underset{\sim}{\Phi}^{(11)}(\underline{C}_1-\underline{C}_1')\cdot\left(\frac{\partial}{\partial \underline{C}_1} - \frac{\partial}{\partial \underline{C}_1'}\right) f_1(\underline{C}_1,\underline{r},t) f_1(\underline{C}_1',\underline{r},t) +$$

$$+ \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1}\cdot\int d\underline{C}_2\,\underset{\sim}{\Phi}^{(12)}(\underline{C}_1-\alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2}\right) f_1(\underline{C}_1,\underline{r},t) f_2(\underline{C}_2,\underline{r},t)$$

$$\frac{\partial f_2}{\partial t} + \varepsilon\alpha\underline{C}_2\cdot\frac{\partial f_2}{\partial \underline{r}} + \varepsilon\alpha\underline{c}_o\cdot\frac{\partial f_2}{\partial \underline{r}} + \varepsilon\alpha\,\frac{e_2}{m_2}\,\underline{E}\cdot\frac{\partial f_2}{\partial \underline{C}_2} - \left(\frac{\partial \underline{c}_o}{\partial t} + \varepsilon\alpha\underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\right)\cdot\frac{\partial f_2}{\partial \underline{C}_2} +$$

$$+ \alpha^2\,\frac{e_2}{m_2}\,\underline{c}_o \times \underline{B}\cdot\frac{\partial f_2}{\partial \underline{C}_2} + \alpha^2\,\frac{e_2}{m_2}\,\underline{C}_2 \times \underline{B}\cdot\frac{\partial f_2}{\partial \underline{C}_2} - \varepsilon\alpha\,\frac{\partial f_2}{\partial \underline{C}_2}\,\underline{C}_2 : \frac{\partial \underline{c}_o}{\partial \underline{r}} = \tag{2}$$

$$= \alpha\,\frac{1}{m_2^2}\frac{\partial}{\partial \underline{C}_2}\cdot\int d\underline{C}_2'\,\underset{\sim}{\Phi}^{(22)}(\underline{C}_2-\underline{C}_2')\cdot\left(\frac{\partial}{\partial \underline{C}_2} - \frac{\partial}{\partial \underline{C}_2'}\right) f_2(\underline{C}_2,\underline{r},t) f_2(\underline{C}_2',\underline{r},t) -$$

$$- \alpha\,\frac{1}{m_2}\frac{\partial}{\partial \underline{C}_2}\cdot\int d\underline{C}_1\,\underset{\sim}{\Phi}^{(12)}(\underline{C}_1-\alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2}\right) f_1(\underline{C}_1,\underline{r},t) f_2(\underline{C}_2,\underline{r},t)$$

Here t is a time variable on time scale τ. The collision tensors $\underset{\sim}{\Phi}^{(ij)}(\underline{C}_i - \underline{C}_j)$, $i = 1,2,$ are given by

$$\underset{\sim}{\Phi}^{(ij)}(\underline{c}_i-\underline{c}_j) = \int d\underline{x}\, \frac{\partial\varphi_{ij}}{\partial\underline{x}}(|\underline{x}|) \int_0^\infty d\tau\, \frac{\partial\varphi_{ij}}{\partial\underline{x}'}(|\underline{x}' = \underline{x} - (\underline{c}_i-\underline{c}_j)\tau|) =$$

$$= \kappa_{ij}\, \frac{\underset{\sim}{I}|\underline{c}_i-\underline{c}_j|^2-(\underline{c}_i-\underline{c}_j)(\underline{c}_i-\underline{c}_j)}{|\underline{c}_i-\underline{c}_j|^3} \quad , \; i,j = 1,2 \qquad (3)$$

$$\kappa_{ij} = 2\pi c^4 e_i^2 e_j^2 \ln \frac{\lambda_D}{\lambda_L}$$

where φ_{ij} are Coulomb interaction potentials with suitable cut-offs, Landau [7], giving the last equality in Eq. (3). $\lambda_D = kT_1/4\pi c^2 n_1 e_1^2$ and $\lambda_L = e_1^2 c^2/kT_1$ are the Debye and Landau lengths. This parametrization of Eqs. (1) and (2) permits the evolution of various plasma models to be discussed, depending on how ε and α are related to one another when one solves Eqs. (1) and (2) successively. Treating ε as a small expansion parameter while α is taken as of order one we are close to models in [3], where electrons and ions have equal temperatures. However, treating α also as an expansion parameter will make a split up of collision terms in a successive approximation procedure that will allow for a description of a two-temperature plasma. In fact, we shall treat ε and α as small and of the same order of magnitude, i.e.

$$\varepsilon \approx \alpha \; , \quad \varepsilon^2 \approx \varepsilon\cdot\alpha \approx \alpha^2 \quad \text{etc.} \qquad (4)$$

This coincides with the ordering of ε and α used in [5]. However, there the motivation for this was quite different than the present one: In [5] it primarily had to do with the different ordering of magnetic force terms in the kinetic equations. That the study in [5] and this one are possible using Eq. (4) increases the relevance of the corresponding model.

As in [5] we need various macroscopic or moment equations too: Taking appropriate velocity moments of Eqs. (1) and (2) we get equations for the total density n and for the electron and ion densities n_1 and n_2 (or corresponding mass densities ρ, ρ_1, ρ_2), electron- and ion diffusions $\underline{\overline{C}}_1$ and $\underline{\overline{C}}_2$

$$n_i \underline{\overline{C}}_i = \int d\underline{C}_i \underline{C}_i f(\underline{C}_i, \underline{r}, t) \quad , \quad i = 1,2$$

and for the electron and ion temperatures T_1 and T_2

$$\frac{3}{2} n_i \; kT_i = \int d\underline{C}_i \; \tfrac{1}{2} m_i C_i^2 f_i (\underline{C}_i, \underline{r}, t) \quad , \quad i = 1,2$$

The moment equations are parametrized according to the parametrization of Eqs. (1) and (2). We also need the equation for the mass velocity $\underline{c}_o$

$$\underline{c}_o = \frac{1}{\rho} \sum_{i=1}^{2} \int d\underline{c}_i m_i \underline{c}_i f_i (\underline{c}_i, \underline{r}, t) \tag{5}$$

Note that f_i here is a function of $\underline{c}_i$. A consequence of defining $\underline{c}_o$ as in Eq. (5) is the following condition

$$\alpha n_1 m_1 \underline{\overline{C}}_1 + n_2 m_2 \underline{\overline{C}}_2 = \underline{0} \tag{6}$$

which we have parametized too, [5].

In the velocity moment equations the heat flux vectors $\underline{q}_1$ and $\underline{q}_2$ and kinetic pressure tensors $\underset{\sim}{P}_1$ and $\underset{\sim}{P}_2$ show up

$$\underline{q}_i = \int d\underline{C}_i \; \tfrac{1}{2} m_i C_i^2 \underline{C}_i f_i (\underline{C}_i, \underline{r}, t) \; , \; i = 1,2$$

$$\underset{\sim}{P}_i = \int d\underline{C}_i m_i \underline{C}_i \underline{C}_i f_i (\underline{C}_i, \underline{r}, t) \; , \quad i = 1,2$$

Together with $\underline{\overline{C}}_i$, $i = 1,2$, the evaluation of the quantities $\underline{q}_i$ and $\underset{\sim}{P}_i$, $i = 1,2$, is the principal aim with the present study. This involves the kinetic and macroscopic equations mentioned above. To prepare these further we shall use the multiple-time-scale method. This method was also applied in [5]. In accordance with Eq. (3) we make expansions such as

$$f_i = f_i^o + \varepsilon f_i^1 + \varepsilon^2 f_i^2 + \cdots \quad , \quad i = 1,2$$

where the superscripts on the functions denote the order of approximation. The macroscopic quantities are expanded accordingly. Also the time derivative is expanded

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_o} + \varepsilon \frac{\partial}{\partial t_1} + \varepsilon^2 \frac{\partial}{\partial t_2} + \cdots$$

where t_o, t_1, t_2 etc. are time variables on longer and longer time scales $\tau_o = \tau$, $\tau_1 = \tau_o/\varepsilon \approx \tau_o/\alpha$, $\tau_2 = \tau_o/\varepsilon^2 \approx \tau_o/\varepsilon\cdot\alpha \approx \tau_o/\alpha^2$ etc. Besides these expansions, which are directly connected to the multiple-time-scale method, we also expand the collision tensor $\underset{\sim}{\Phi}^{(12)}(\underline{C}_1 - \alpha\underline{C}_2)$ of Eqs. (1) and (2) in a Taylor series around $\underline{C}_1$, assuming the series is distributionally convergent. This will contribute in the splitting up of the collision terms of Eqs. (1) and (2) necessary for a two-temperature description. All these expansions are substituted into the kinetic- and moment equations which then split up into sets of zeroth-, first-, second-order equations etc. These sets are successively solved according to the prosedure of the multiple-time-scale method. In [5] the electron and ion kinetic equations to zeroth- and first orders in the limits $t_o = \infty$ and $t_1 = \infty$ (for f_i^o, $i = 1,2$, and f_1^1) and the second order ion kinetic equation in the same time limits (for f_2^1) as well as moment equations to zeroth-, first- and second orders were obtained. Solving

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III. Equations to Zeroth- First- and Second Orders.

Consider first the relaxations of f_1^o and f_2^o:

From the zeroth order electron kinetic equation

$$\frac{\partial f_1^o}{\partial t_o} + \frac{e_1}{m_1} \underline{C}_1 \times \underline{B} \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} = FP_{11}\left[f_1^o(\underline{C}_1) f_1^o(\underline{C}_1')\right] + D_1\left[f_1^o(\underline{C}_1)\right]$$

where FP_{11} and D_1 are collision operators given from

$$FP_{ii} = \frac{1}{m_i^2} \frac{\partial}{\partial \underline{C}_i} \cdot \int d\underline{C}_i' \; \underset{\sim}{\Phi}^{(ii)}(\underline{C}_i - \underline{C}_i') \cdot \left(\frac{\partial}{\partial \underline{C}_i} - \frac{\partial}{\partial \underline{C}_i'}\right), \quad i = 1,2$$

$$D_1 = \frac{n_2^o}{m_1^2} \frac{\partial}{\partial \underline{C}_1} \cdot \left(\underset{\sim}{\Phi}^{(12)}(\underline{C}_1) \cdot \frac{\partial}{\partial \underline{C}_1}\right)$$

it follows that

$$f_1^o \to f_{1M}^o = n_1 \left(\frac{m_1}{2\pi k T_1^o} \right)^{3/2} \exp \left(- \frac{m_1 C_1^2}{2kT_1^o} \right) \quad \text{as} \quad t_o \to \infty \tag{7}$$

i.e., f_1^o tends towards a local, isotropic Maxwellian on the τ_o time scale. (The subscript "M" for quantities means these quantities in the limit $t_o = \infty$, and later on in the limit $t_1 = \infty$ also). The ion kinetic equation to zeroth order reduces to

$$\frac{\partial f_2^o}{\partial t_o} = 0$$

so that f_2^o is stationary on the τ_o time scale. However, the first order ion kinetic equation in the limit $t_o = \infty$ reduces to

$$\frac{\partial f_2^o}{\partial t_1} = FP_{22} \left[f_2^o(\underline{C}_2) f_2^o(\underline{C}_2') \right]$$

which, together with the zeroth order condition Eq.(6)

$$n_2^o m_2 \overline{\underline{C}}_2^o = \underline{0}$$

has the property to Maxwellize f_2^o :

$$f_2^o \to f_{2M}^o = n_2^o \left(\frac{m_2}{2\pi k T_2^o} \right)^{3/2} \exp \left(- \frac{m_2 C_2^2}{2kT_2^o} \right) \quad \text{as} \quad t_1 \to \infty \tag{8}$$

We note that the relaxation of f_1^o and f_2^o towards the (local) equilibra takes place on different time scales as well as in different ways: While the electrons tend to an isotropic Maxwellian colliding among themselves (through FP_{11}) and with ions "at rest" (through D_1) the ions go to a corresponding equilibrium colliding only among themselves given that they have a zero mean drift relative to $\underline{c}_o^o$. This difference in collision operators in electron and ion kinetic equations will be observed over and over again in

what follows. Thus the first order electron kinetic equation can be shown (cf. Appendix B of [5]) to have the property that f_1^1 on the τ_o time scale tends towards a solution of

$$FP_{11}\left[f^o_{1M}(\underline{C}_1)f^1_{1M}(\underline{C}_1') + f^1_{1M}(\underline{C}_1)f^o_{1M}(\underline{C}_1')\right] + D_1\left[f^1_{1M}\right] - \frac{e_1}{m_1}\underline{C}_1 \times \underline{B}\cdot\frac{\partial f^1_{1M}}{\partial \underline{C}_1} =$$
$$= f^o_{1M}\left[\left(\frac{m_1 C_1^2}{2kT_1^o} - \frac{5}{2}\right)\frac{1}{T_1^o}\frac{\partial T_1^o}{\partial \underline{r}} - \frac{e_1}{kT_1^o}\left(\underline{E} + \underline{c}^o_o \times \underline{B} - \frac{kT_1^o}{e_1}\frac{\partial}{\partial \underline{r}}\ln p_1^o\right)\right]\cdot \underline{C}_1 \tag{9}$$

while the second order ion kinetic equation in the limits $t_o = \infty$ and $t_1 = \infty$ reduces to the following equation for f^1_{2M} :

$$FP_{22}\left[f^o_{2M}(\underline{C}_2)f^1_{2M}(\underline{C}_2') + f^1_{2M}(\underline{C}_2)f^o_{2M}(\underline{C}_2')\right] = f^o_{2M}\,\underline{C}_2\cdot\left(\frac{m_2 C_2^2}{2kT_2^o} - \frac{5}{2}\right)\frac{1}{T_2^o}\frac{\partial T_2^o}{\partial \underline{r}} +$$
$$+ \frac{m_2}{kT_2^o} f^o_{2M}\,\underline{C}^o_2\underline{C}_2 : \left(\frac{\partial \underline{c}^o_o}{\partial \underline{r}} + \frac{1}{m_2 k}\frac{T_2^o - T_1^o}{T_1^o T_2^o}\int d\underline{C}_1\, \underset{\sim}{\Phi}^{(12)}(\underline{C}_1) f^o_{1M}(\underline{C}_1)\right) \tag{10}$$

which further reduces to

$$FP_{22}\left[f^o_{2M}(\underline{C}_2)f^1_{2M}(\underline{C}_2') + f^1_{2M}(\underline{C}_2)f^o_{2M}(\underline{C}_2')\right] =$$
$$= f^o_{2M}\left[\left(\frac{m_2 C_2^2}{2kT_2^o} - \frac{5}{2}\right)\underline{C}_2\cdot\frac{1}{T_2^o}\frac{\partial T_2^o}{\partial \underline{r}} + \frac{m_2}{kT_2^o}\underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}}\right] \tag{11}$$

since the product between the traceless tensor $\underline{C}^o_2\underline{C}_2 = \underline{C}_2\underline{C}_2 - \frac{1}{3}C_2^2\underset{\sim}{I}$ and the tensor $\int d\underline{C}_1\,\underset{\sim}{\Phi}^{(12)}(\underline{C}_1)f^o_{1M}$, which is proportional to the unit tensor $\underset{\sim}{I}$, vanishes identically. Eqs.(9) and (11) are fundamental in a transport - kinetic theory: From them classical transports and transport coefficients can be derived. For later reference we also note some properties of Eqs.(9) and (11): The solutions of the associated homogeneous equations to Eqs.(9) and (11) are respectively

$$f^o_{1M}(\alpha_1 + \gamma_1 \tfrac{1}{2}m_1C_1^2) \tag{12}$$

and

$$f^o_{2M}(\alpha_2 + \underline{\beta}_2 \cdot m_2\underline{C}_2 + \gamma_2 \tfrac{1}{2}m_2C_2^2) \tag{13}$$

where α_1, γ_1, α_2, $\underline{\beta}_2$, γ_2 are $\underline{C}_1$ and $\underline{C}_2$ - independent, arbitrary parameters. Necessary conditions for existence of solutions of Eqs. (9) and (11) are that the source terms on the right hand sides of Eqs. (9) and (11), which we for shorthand denote by h_1 and h_2 respectively, are orthogonal to the solutions of the associated homogeneous equations, i.e.

$$\int d\underline{C}_1(\alpha_1 + \gamma_1 \tfrac{1}{2}m_1C_1^2)h_1 = 0 \tag{14}$$

$$\int d\underline{C}_2(\alpha_2 + \underline{\beta}_2 \cdot m_2\underline{C}_2 + \gamma_2 \tfrac{1}{2}m_2C_2^2)h_2 = 0 \tag{15}$$

That these requirements are fulfilled is quite easy to show. The solution of Eq. (9) is then a sum of Eq. (12) and the solution set up by the source terms of Eq. (9), with a corresponding construction for the solution of Eq. (11), [3]. It is convenient in these solutions to choose the parameters α_1, γ_1, α_2 and γ_2 so that

$$\begin{aligned} n_1^1 &= n_2^1 = 0 \\ T_1^1 &= T_2^1 = 0 \end{aligned} \tag{16}$$

Due to these choices and to similar choices to higher orders, we shall identify n_i^o (ρ_i^o), and T_i^o , $i = 1,2$ by n_i (ρ_i) and T_i , $i = 1,2$, thus also simplifying the notation a bit. The parameter $\underline{\beta}_2$ has to be determined so that the condition Eq. (6) to first order in the limits $t_o = \infty$ and $t_1 = \infty$ is fulfilled:

$$n_2^o m_2 \underline{\overline{C}}_{2M}^1 = \underline{0} \tag{17}$$

For a further mathematical study of Eqs.(9) and (11) we refer to Leversen and Naze Tjøtta, [8].

The densities, mass transport and temperatures showing up in Eqs.(9) and (11) are all evaluated in the limits $t_o = \infty$ and $t_1 = \infty$ and their first approximation variations on the τ_2-time scale are given from the macroscopic equations to second order in the $t_o = \infty$ and $t_1 = \infty$ limits:

$$\frac{\partial \rho_1}{\partial t_2} + \frac{\partial}{\partial \underline{r}}\cdot(\rho_1 \underline{c}_o^o) + \frac{\partial}{\partial \underline{r}}\cdot(\rho_1 \underline{\overline{C}}_1)^1 = 0 \tag{18}$$

$$\frac{\partial \rho_2}{\partial t_2} + \frac{\partial}{\partial \underline{r}}\cdot(\rho_2 \underline{c}_o^o) = 0 \tag{19}$$

$$\rho\left(\frac{\partial \underline{c}_o^o}{\partial t_2} + \underline{c}_o^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}}\right) = -\frac{\partial p}{\partial \underline{r}} + \rho_e \underline{c}_o^o \times \underline{B} + n_1 e_1 \underline{\overline{C}}_{1M}^1 \times \underline{B} + \rho_e \underline{E} \tag{20}$$

$$\frac{3}{2} n_1 k\left(\frac{\partial T_1}{\partial t_2} + \underline{c}_o^o\cdot\frac{\partial T_1}{\partial \underline{r}}\right) = \frac{3}{2} k T_1 \frac{\partial}{\partial \underline{r}}\cdot(n_1 \underline{\overline{C}}_1)_M^1 + n_1 e_1 \underline{\overline{C}}_{1M}^1\cdot\underline{E} +$$

$$+ n_1 e_1 \underline{\overline{C}}_{1M}^1\cdot(\underline{c}_o^o \times \underline{B}) - \frac{\partial}{\partial \underline{r}}\cdot\underline{q}_{1M}^1 - p_1 \frac{\partial}{\partial \underline{r}}\cdot\underline{c}_o^o - \tag{21}$$

$$- \frac{n_1 n_2}{m_2}\left(\frac{m_1}{2\pi k}\right)^{3/2} \frac{T_1 - T_2}{T_1^{5/2}} \int d\underline{C}_1\, \underset{\sim}{\Phi}^{(12)}(\underline{C}_1) : \underset{\sim}{I} \exp\left(-\frac{m_1 C_1^2}{2kT_1}\right)$$

$$\frac{3}{2} n_2 k\left(\frac{\partial T_2}{\partial t_2} + \underline{c}_o^o\cdot\frac{\partial T_2}{\partial \underline{r}}\right) = -p_2 \frac{\partial}{\partial \underline{r}}\cdot\underline{c}_o^o +$$

$$+ \frac{n_1 n_2}{m_2}\left(\frac{m_1}{2\pi k}\right)^{3/2} \frac{T_1 - T_2}{T_1^{5/2}} \int d\underline{C}_1\, \underset{\sim}{\Phi}^{(12)}(\underline{C}_1) : \underset{\sim}{I} \exp\left(-\frac{m_1 C_1^2}{2kT_1}\right) \tag{22}$$

Here scalar pressures $p = p_1 + p_2$, $p_i = n_i k T_i$, $i = 1,2$, show up,

and $\rho_e = n_1 e_1 + n_2 e_2$ is the space charge density. The integral terms of Eqs.(21) and (22) take care of energy transfer between electrons and ions due to collision. Using for $\underset{\sim}{\Phi}^{(12)}$ the Landau expression Eq.(3) these terms take the familiar form as derived in [7] for a homogeneous gas. We note that Eqs.(19) and (22) have been used already to derive the form Eq.(10), and all equations (18)-(22) will be used later when deriving higher order kinetic equations.

We complete the equations to second order by including the electron kinetic equation to this order in the limits $t_o = \infty$ and $t_1 = \infty$. The equation (for f^2_{1M}) can be transformed into

$$
\begin{aligned}
&FP_{11}\left[f^o_{1M}(\underline{C}_1)f^2_{1M}(\underline{C}'_1) + f^2_{1M}(\underline{C}_1)f^o_{1M}(\underline{C}'_1)\right] + D_1\left[f^2_{1M}(\underline{C}_1)\right] - \frac{e_1}{m_1}\underline{C}_1\times\underline{B}\cdot\frac{\partial f^2_{1M}}{\partial\underline{C}_1} = \\
&= f^o_{1M}\frac{m_1}{kT_1}\underline{C}^o_1\underline{C}_1 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}} + f^o_{1M}\left(\frac{m_1 C_1^2}{2kT_1} - \frac{5}{2}\right)\frac{1}{n_1}\frac{\partial}{\partial\underline{r}}\cdot\left(n_1\,\overline{\underline{C}}^1_{1M}\right) + \\
&+ f^o_{1M}\left(\frac{m_1 C_1^2}{3kT_1} - 1\right)\frac{1}{p_1}\left[(n_1 e_1\underline{E} + n_1 e_1 \underline{c}^o_o \times \underline{B})\cdot\overline{\underline{C}}^1_{1M} - \frac{\partial}{\partial\underline{r}}\cdot\underline{q}^1_{1M}\right] - \\
&- f^o_{1M}\left(\frac{m_1 C_1^2}{3kT_1} - 1\right)\frac{1}{p_1}\frac{n_1 n_2}{m_2}\left(\frac{m_1}{2\pi k}\right)^{3/2}\frac{8\pi\kappa_{12}k}{m_1}\frac{T_1 - T_2}{T_1^{3/2}} + \underline{C}_1\cdot\frac{\partial f^1_{1M}}{\partial\underline{r}} + \\
&+ \frac{e_1}{m_1}\underline{E}\cdot\frac{\partial f^1_{1M}}{\partial\underline{C}_1} + \frac{e_1}{m_1}\underline{c}^o_o\times\underline{B}\cdot\frac{\partial f^1_{1M}}{\partial\underline{C}_1} - \frac{e_1}{kT_1}f^o_{1M}\,\underline{c}^1_{oM}\times\underline{B}\cdot\underline{C}_1 - \\
&- \frac{2n_2\kappa_{12}}{m_1 m_2}\frac{T_1 - T_2}{T_1}\left(4\pi\delta(\underline{C}_1) - \frac{m_1}{kT_1}\frac{1}{C_1}\right)f^o_{1M} - FP_{11}\left[f^1_{1M}(\underline{C}_1)f^1_{1M}(\underline{C}'_1)\right]
\end{aligned}
\tag{23}
$$

The operator on the left hand side of Eq.(23) is the same as in Eq.(9) and it is straight forward to show that the orthogonality condition Eq.(14) is fulfilled with the new source terms of Eq.(23). Concerning these source terms we note

the appearence of the Dirac delta function $\delta(\underline{C}_1)$. We also notice f^1_{1M} and derivations of it among the source terms. This will give rise to typical Burnett approximation terms in f^2_{1M}, but we shall not pay special attention to these later. More important are source terms that will give contribution to f^2_{1M} on the Chapman-Enskog level of approximation, for instance the first one on the right hand side of Eq.(23). Of special interest in our context is the observation of the source terms with the factor $T_1 - T_2$. Terms like that in Eq.(10) we showed vanished identically. Not so in Eq.(23). However, though these terms will contribute to f^2_{1M}, they have no effect on the transports derived from f^2_{1M}, as later will be shown.

The equations up to second order discussed up to now, which we may call the lower order equations, constitute a frame of a classical kinetic-transport theory of a two-component, two-temperature plasma. From Eqs.(9), (11) and (23) classical transports, linear in gradients and fields, can be derived. One might think that higher order equations would contribute nothing but complicated Burnett corrections to the classical transports, as Eq.(23) may already indicate. However, the nature of the two-temperature state is such as to hide more simple corrections among these higher order equations. They will subsequently be revealed. Also, generally speaking, the inclusion of the third order equations makes the whole theory for this model more complete.

IV. Third order equations.

The first thing we need is the third order macroscopic equa-

tions in the limits $t_o = \infty$ and $t_1 = \infty$. They can be shown to reduce to

$$\frac{\partial \rho_1}{\partial t_3}(t_2,t_3,\cdot\cdot) + \frac{\partial}{\partial \underline{r}}\cdot(\rho_1 \underline{c}^1_{oM}) + \frac{\partial}{\partial \underline{r}}\cdot(\rho_1 \overline{\underline{C}}^2_{1M}) = 0 \tag{24}$$

$$\frac{\partial \rho_2}{\partial t_3}(t_2,t_3,\cdot\cdot) + \frac{\partial}{\partial \underline{r}}\cdot(\rho_2 \underline{c}^1_{oM}) = 0 \tag{25}$$

$$\rho\left(\frac{\partial \underline{c}^1_{oM}}{\partial t_2} + \frac{\partial \underline{c}^o_o}{\partial t_3}(t_2,t_3,\cdot\cdot) + \underline{c}_o\cdot\frac{\partial \underline{c}^1_{oM}}{\partial \underline{r}} + \underline{c}^1_{oM}\cdot\frac{\partial \underline{c}^o_o}{\partial \underline{r}}\right) = -\frac{\partial}{\partial \underline{r}}\cdot \underset{\sim}{p}^1_{2M} +$$
$$+ \rho_e \underline{c}^1_{oM}\times \underline{B} + n_1 e_1 \overline{\underline{C}}^2_{1M}\times \underline{B} \tag{26}$$

$$\frac{3}{2} n_1 k\left(\frac{\partial T_1}{\partial t_3}(t_2,t_3,\cdot\cdot) + \underline{c}^1_{oM}\cdot\frac{\partial T_1}{\partial \underline{r}}\right) = \frac{3}{2} kT_1 \frac{\partial}{\partial \underline{r}}\cdot(n_1\overline{\underline{C}}_1)^2_M + n_1 e_1 \overline{\underline{C}}^2_{1M}\cdot\underline{E} +$$
$$+ n_1 e_1 \overline{\underline{C}}^2_{1M}\cdot(\underline{c}^o_o\times\underline{B}) + n_1 e_1 \overline{\underline{C}}^1_{1M}\cdot(\underline{c}^1_{oM}\times\underline{B}) - \frac{\partial}{\partial \underline{r}}\cdot\underline{q}^2_{1M} - p_1\frac{\partial}{\partial \underline{r}}\cdot\underline{c}^1_{oM} \tag{27}$$

$$\frac{3}{2} n_2 k\left(\frac{\partial T_2}{\partial t_3}(t_2,t_3,\cdot\cdot) + \underline{c}^1_{oM}\frac{\partial T^o_2}{\partial \underline{r}}\right) =$$
$$= -\frac{\partial}{\partial \underline{r}}\cdot\underline{q}^1_{2M} - p_2\frac{\partial}{\partial \underline{r}}\cdot\underline{c}^1_{oM} - \underset{\sim}{p}^1_{2M} : \frac{\partial \underline{c}^o_o}{\partial \underline{r}} \tag{28}$$

Observe the derivatives on the τ_3-time scale: These equations are correcting equations to the corresponding second order equations (18) - (22), to which they eventually should be added. One striking feature worth noticing is the absence of any electron - ion energy transfer term in Eqs.(27) and (28). This was unforseen and strengthens our confidence in the energy exchange terms of the second order equations (21) and (22) as an accurate description of that process.

Turning now to the third order ion kinetic equation in the limits $t_o = \infty$ and $t_1 = \infty$ we are left with the following form after various transformations which include use of both the second and third order sets of macroscopic equations:

$$
\begin{aligned}
&FP_{22}\left[f^o_{2M}(\underline{C}_2) f^2_{2M}(\underline{C}'_2) + f^2_{2M}(\underline{C}_2) f^o_{2M}(\underline{C}'_2) \right] = \frac{\partial f^1_{2M}}{\partial t_2} + \underline{c}^o_o \cdot \frac{\partial f^1_{2M}}{\partial \underline{r}} + \underline{C}_2 \cdot \frac{\partial f^1_{2M}}{\partial \underline{r}} + \\
&+ \frac{1}{\rho_2} \frac{\partial p_2}{\partial \underline{r}} \cdot \frac{\partial f^1_{2M}}{\partial \underline{C}_2} + \frac{e_2}{m_2} \underline{C}_2 \times \underline{B} \cdot \frac{\partial f^1_{2M}}{\partial \underline{C}_2} - \frac{\partial f^1_{2M}}{\partial \underline{C}_2} \underline{C}_2 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}} + f^o_{2M}\left[-\frac{2}{3p_2}\left(\frac{m_2 C_2^2}{2kT_2} - \frac{3}{2} \right) \times \right. \\
&\times \left(\frac{\partial}{\partial \underline{r}} \cdot \underline{q}^1_{2M} + \underset{\sim}{P}^1_{2M} : \frac{\partial \underline{c}^o_o}{\partial \underline{r}} \right) - \frac{1}{p_2} \frac{\partial}{\partial \underline{r}} \cdot \underset{\sim}{P}^1_{2M} \cdot \underline{C}_2 + \left. \frac{m_2}{kT_2} \underline{C}^o_2 \underline{C}_2 : \frac{\partial \underline{c}^1_{oM}}{\partial \underline{r}} \right] - \\
&- FP_{22}\left[f^1_{2M}(\underline{C}_2) f^1_{2M}(\underline{C}'_2) \right] - \frac{4\kappa_{12} n_1}{3m_2^2} \left(\frac{m_1}{2\pi k T_1} \right)^{\frac{1}{2}} \frac{\partial^2}{\partial \underline{C}_2^2} f^1_{2M} - \\
&- \frac{8\pi\kappa_{12} n_1}{3m_1 m_2} \left(\frac{m_1}{2\pi k T_1} \right)^{3/2} \frac{\partial}{\partial \underline{C}_2} \cdot \left[\underline{C}_2 f^1_{2M} \right]
\end{aligned} \tag{29}
$$

The operator on the left hand side is the same as in Eq. (11), and the orthogonality condition Eq. (15) with the source term from Eq. (29) can be seen to be fulfilled. When solving Eq. (29) we have to get the condition Eq. (6) to second order fulfilled, i.e.

$$n_1 m_1 \overline{\underline{C}}^1_{1M} + n_2 m_2 \overline{\underline{C}}^2_{2M} = \underline{0} \tag{30}$$

which is possible since we have a parameter $\underline{\beta}_2$ as in Eq. (13) at our disposal in the solution of f^2_{2M}.

Nearly every source term contains f^1_{2M} or derivations of it. One could therefore be tempted to think that these source terms would give rise to Burnett terms only in f^2_{2M}. However, there are important exceptions, for instance the last two terms on the right hand side, and also the combination $\partial f^1_{2M}/\partial t_2 + \underline{c}^o_o \cdot \partial f^1_{2M}/\partial \underline{r}$

contains parts which we later shall study particularly.

We end this section giving the third order electron kinetic equation in the limits $t_o = \infty$ and $t_1 = \infty$, thereby completing the bulk of equations on the third level of approximation:

$$
\begin{aligned}
&FP_{11}\left[f^o_{1M}(\underline{C}_1)f^3_{1M}(\underline{C}'_1) + f^3_{1M}(\underline{C}_1)f^o_{1M}(\underline{C}'_1)\right] + D_1\left[f^3_{1M}(\underline{C}_1)\right] - \frac{e_1}{m_1}\underline{C}_1\times\underline{B}\cdot\frac{\partial f^3_{1M}}{\partial\underline{C}_1} = \\
&= \frac{\partial f^1_{1M}}{\partial t_2} + \underline{c}^o_o\cdot\frac{\partial f^1_{1M}}{\partial\underline{r}} - \frac{\partial f^1_{1M}}{\partial\underline{C}_1}\underline{C}_1 : \frac{\partial\underline{c}^o_o}{\partial\underline{r}} - \frac{1}{n_1}\frac{\partial}{\partial\underline{r}}\cdot(n_1\overline{\underline{C}}^2_{1M})f^o_{1M} + \\
&+ \frac{1}{p_1}\left(\frac{m_1C^2_1}{3kT_1} - 1\right)f^o_{1M}\left[\frac{3}{2}kT_1\frac{\partial}{\partial\underline{r}}\cdot(n_1\overline{\underline{C}}^2_{1M}) + n_1e_1\overline{\underline{C}}^2_{1M}\cdot(\underline{E} + \underline{c}^o_o\times\underline{B}) + \right. \\
&\left.+ n_1e_1\overline{\underline{C}}^1_{1M}\cdot(\underline{c}^1_{oM}\times\underline{B}) - \frac{\partial}{\partial\underline{r}}\cdot\underline{q}^2_{1M}\right] + \frac{m_1}{kT_1}f^o_{1M}\underline{C}^o_1\underline{C}_1 : \frac{\partial\underline{c}^1_{oM}}{\partial\underline{r}} + \underline{C}_1\cdot\frac{\partial f^2_{1M}}{\partial\underline{r}} + \\
&+ \frac{e_1}{m_1}(\underline{E} + \underline{c}^o_o\times\underline{B})\cdot\frac{\partial f^2_{1M}}{\partial\underline{C}_1} - \frac{e_1}{kT_1}f^o_{1M}\,\underline{c}^2_{oM}\times\underline{B}\cdot\underline{C}_1 + \frac{e_1}{m_1}\underline{c}^1_{oM}\times\underline{B}\cdot\frac{\partial f^1_{1M}}{\partial\underline{C}_1} + \\
&+ \frac{m_1}{kT_1\rho}f^o_{1M}\underline{C}_1\cdot\left[-\frac{\partial p}{\partial\underline{r}} + n_1e_1\overline{\underline{C}}^1_{1M}\times\underline{B} + \rho_e(\underline{E} + \underline{c}^o_o\times\underline{B})\right] - \\
&- FP_{11}\left[f^1_{1M}(\underline{C}_1)f^2_{1M}(\underline{C}'_1) + f^2_{1M}(\underline{C}_1)f^1_{1M}(\underline{C}'_1)\right] - \frac{2\kappa_{12}n_2}{m_1m_2}\left[\frac{1}{C^3_1}\underline{C}_1\cdot\right. \\
&\left.\cdot\underline{\mathfrak{g}}(C_1,\underline{r},t_2,\cdots) + \frac{1}{C^2_1}\frac{\partial\underline{\mathfrak{g}}}{\partial C_1}\cdot\underline{C}_1\right] + \frac{2\kappa_{12}n_1}{m_2kT_1}f^o_{1M}\frac{1}{C^3_1}\underline{C}_1\cdot\overline{\underline{C}}^1_{1M} - \frac{n_2kT_2}{2m^2_1m_2}\frac{\partial^2}{\partial\underline{C}_1\partial\underline{C}_1} : \\
&:\left[\frac{\partial\underset{\sim}{\Phi}^{(12)}}{\partial\underline{C}_1}(\underline{C}_1)\cdot\left(\underline{\mathfrak{g}}(C_1,\underline{r},t_2,\cdots) + \frac{\underline{C}_1\underline{C}_1}{C_1}\cdot\frac{\partial\underline{\mathfrak{g}}}{\partial C_1}\right)\right] + \frac{n_2kT_2}{2m^2_1m_2}\frac{\partial}{\partial\underline{C}_1}\cdot \\
&\cdot\left[\frac{\partial\underset{\sim}{\Phi}^{(12)}}{\partial\underline{C}_1}(\underline{C}_1) : \frac{\partial}{\partial\underline{C}_1}\left(\underline{\mathfrak{g}}(C_1,\underline{R},t_2,\cdots) + \frac{\underline{C}_1\underline{C}_1}{C_1}\cdot\frac{\partial\underline{\mathfrak{g}}}{\partial C_1}\right)\right] + \\
&+ \frac{1}{2m_1m_2kT_1}\frac{\partial}{\partial\underline{C}_1}\cdot\left[\underset{\sim}{P}^1_{2M} : \frac{\partial^2\underset{\sim}{\Phi}^{(12)}}{d\underline{C}_1\partial\underline{C}_1}(\underline{C}_1)\cdot\underline{C}_1f^o_{1M}\right] \qquad (31)
\end{aligned}
$$

We observe Eq. (31) is of the same form as Eqs. (9) and (23) concerning

the operator on the left hand side. Also the orthogonality condition Eq.(14) is fulfilled with h_1 denoting the right hand side of Eq.(31). In these source terms the function $\underset{\sim}{g}$ appearing stems from the solution of Eq.(9) which it is possible to write in the symbolic form

$$f^1_{1M} = \underset{\sim}{g}(C_1, \underline{r}, t_2, \cdot\cdot)\cdot\underline{C}_1$$

when the parameters α_1 and γ_1 have been set equal to zero. The $\partial/\partial\underline{C}_1$ - and $\partial^2/\partial\underline{C}_1\partial\underline{C}_1$ - operators in the last terms on the right hand side are operators in the distributional sence. As expected the complexity of the source terms has grown substantially, however, there are exceptions also here: We mention the combination of the two first terms on the right hand side that contains parts similar to the ones we get from Eq.(29). We also make a note here concerning the last terms on the right hand side, all of which contain derivatives of $\underset{\sim}{\Phi}^{(12)}$: None of these are Burnett terms and they contribute to f^3_{1M} with electron-ion collisional effects. Now this effect is taken account of also in the D_1 - operator on the left hand side. In this term, however, the ions are considered at rest. This is not so for all the concerned terms on the right hand side, for instance the last one. Terms like that describe ion thermal effects in collision with electrons for the case when the ions have a different temperature than the electrons.

V. Some solutions.

The procedure for solving the electron and ion kinetic equations, Eqs.(9), (11) and (23) and Eqs.(29) and (31) may be as follows: Expressing the velocity variable in spherical coordinates, the

unknown in each equation is expanded in a series of spherical harmonics $Y_\ell^m(\theta,\varphi)$ with coeffisients that are functions of the particle speed and macroscopic quantities. The derived equations for these coeffisients are subsequently solved by further expansions.

Since we here primarily are interested in pointing out the effects and mechanisms that are new we postpone exact numerical calculations and substitute for the collision operators on the left hand sides of the electron kinetic equations the operator $-\nu_1(\ell)$ where $\nu_1(\ell)$, a collisions frequency, is given by

$$\nu_1(\ell) = \frac{\kappa_{11} b_{11}(\ell)}{m_1^{\frac{1}{2}}(3k)^{3/2}} \frac{n_1}{T_1^{3/2}} + \frac{\kappa_{12} b_{12}(\ell)}{m_1^{\frac{1}{2}}(3k)^{3/2}} \frac{n_2}{T_1^{3/2}} \tag{32}$$

and for the operators on the left hand sides of the ion kinetic equations the operator $-\nu_2(\ell)$, where the collision frequency $\nu_2(\ell)$ is given by

$$\nu_2(\ell) = \frac{\kappa_{22} b_{22}(\ell)}{m_2^{\frac{1}{2}}(3k)^{3/2}} \frac{n_2}{T_2^{3/2}} \tag{33}$$

In Eq. (32) both the electron - electron and electron - ion collison frequencies (with ions at rest) are included. $b_{ij}(\ell)$, $i,j = 1,2$, are numbers that may change with ℓ ($= 0,1,2,\cdots$), the order of anisotropy (in terms of spherical harmonics). Though independent of velocities each part of the collision frequencies have correct dependencies on densities and temperatures.

Qualitative expressions for the distribution functions f_{1M}^1 and f_{2M}^1 thus are :

$$f^1_{1M} = - f^o_{1M}\left[\frac{1}{\nu_1(1)}\, \underline{C}_{1\parallel} + \frac{\frac{1}{\nu_1(1)}}{1+(\frac{\Omega_1}{\nu_1(1)})^2}\left(\underline{C}_{1\perp} - \frac{\Omega_1}{\nu_1(1)B}\ \ \underline{C}_1 \times \underline{B}\right)\right] \cdot$$

$$\cdot \left[\left(\frac{m_1 C_1^2}{2kT_1} - \frac{5}{2}\right)\frac{1}{T_1}\frac{\partial T_1}{\partial \underline{r}} - \frac{e_1}{kT_1}\left(\underline{E} + \underline{c}^o_o \times \underline{B} - \frac{kT_1}{e_1}\frac{\partial}{\partial \underline{r}} \ln p_1\right)\right] \tag{34}$$

$$f^1_{2M} = - f^o_{2M}\left[\frac{1}{\nu_2(1)}\left(\frac{m_2 C_2^2}{2kT_2} - \frac{5}{2}\right)\frac{1}{T_2}\frac{\partial T_2}{\partial \underline{r}} \cdot \underline{C}_2 + \frac{1}{\nu_2(2)}\frac{m_2}{kT_2}\,\underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}}\right] \tag{35}$$

The similarity with more exact solutions is striking. They fulfill Eq.(16), and Eq.(35) fulfills also the condition Eq.(17). Choosing the numbers $b_{22}(1)$ and $b_{22}(2)$ so that $\frac{b_{22}(2)}{b_{22}(1)} = \frac{3}{2}$ and $\mu\nu_2(2) = n_2 kT_2$, where μ is a first approximation to the coefficient of viscosity ([3] for the case with the Boltzmann collision operator), Eq.(35) is indeed equal to a first approximation to f^1_{2M} from Eq.(11). Taking appropriate velocity moments classical transports emerge. Of these we note for later reference the ion heat transport and kinetic pressure tensor

$$\underline{q}^1_{2M} = - \frac{5}{2}\ \frac{(3k)^{3/2} k^2}{m_2^{\frac{1}{2}}\ \kappa_{22} b_{22}(1)}\ \ T_2^{5/2}\ \ \frac{\partial T_2}{\partial \underline{r}} \tag{36}$$

$$\underset{\sim}{p}^1_{2M} = - \frac{(3k)^{3/2} k m_2^{\frac{1}{2}}}{\kappa_{22} b_{22}(2)}\, T_2^{\ 5/2}\left[\frac{\partial \underline{c}^o_o}{\partial \underline{r}} + \left(\frac{\partial \underline{c}^o_o}{\partial \underline{r}}\right)^T - \frac{2}{3}\left(\frac{\partial}{\partial \underline{r}} \cdot \underline{c}^o_o\right) \underset{\sim}{I}\right] \tag{37}$$

The superscript T denotes transpose.

Concerning the solution of Eq.(23) we only discuss the effect of source terms with the factor $T_1 - T_2$. Since these source terms are of zeroth order of anisotropy we simply get as their

qualitative contribution to f^2_{1M}

$$f^2_{1M} \approx - \frac{1}{\nu_1(0)} f^o_{1M} \left\{ -\left(\frac{m_1 C_1^2}{3kT_1} - 1 \right) \frac{1}{p_1} \frac{8\pi\kappa_{12}k}{m_1} \left(\frac{m_1}{2\pi k} \right)^{3/2} \frac{n_1 n_2}{m_2} \frac{T_1 - T_2}{T_1^{3/2}} - \right.$$
$$\left. - \frac{2\kappa_{12}}{m_1 m_2} n_2 \frac{T_1 - T_2}{T_1} \left(4\pi\delta(\underline{C}_1) - \frac{m_1}{kT_1} \frac{1}{C_1} \right) \right\} \qquad (38)$$

Taking appropriate velocity moments of these terms no contributions to $\underline{C}^2_{1M}$, $\underline{q}^2_{1M}$ or $\underset{\sim}{P}^2_{1M}$ are observed. This is physically reasonable since the terms of Eq.(38) contain no gradients or forces that we consider necessary for transports to be set up. An exact treatment of Eq.(23) would not change this: The concerned terms would still be of zeroth order of anisotropy and obviously contribute nothing to $\overline{\underline{C}}^2_{1M}$, $\underline{q}^2_{1M}$ and the off-diagonal terms of $\underset{\sim}{P}^2_{1M}$. The diagonal terms of $\underset{\sim}{P}^2_{1M}$ always vanish due to our choice of the parameters α_1 and γ_1 so that $n_1^2 = 0$ and $T_1^2 = 0$. Thus the kinetic equations up to and including the second order in our perturbation procedure only give rise to classical transports if we neglect the Burnett corrections introduced by Eq.(23). In particular the electron-ion temperature difference has no effect on the transports to this order of approximation.

In the third order kinetic equations, however, necessary driving mechanisms for transports are present in connection with the factor $T_1 - T_2$: An interesting thing about these transports is that they are linear in gradients and fields. We shall limit the further discussion mainly to Eq.(29) and extract all source terms giving rise to linear transports which are not found in earlier theories. Using f^1_{2M} from Eq.(35) in the source terms of Eq.(29) we get an exact evaluation of all these to first order

(choosing $b_{22}(1)$ and $b_{22}(2)$ as above). Of these terms we single out the part of $\partial f^1_{2M}/\partial t_2 + \underline{c}^o_o \cdot \partial f^1_{2M}/\partial \underline{r}$ where temperature difference terms show up, and the two last terms on the right hand side. We qualitatively evaluate their contribution to f^2_{2M} and add to this the solution of the associated homogeneous equation to Eq. (29). Thus we obtain

$$
\begin{aligned}
f^2_{2M} \approx\ & f^o_{2M}\left(\alpha_2 + \underline{\beta}_2 \cdot m_2\underline{C}_2 + \gamma_2\, \tfrac{1}{2} m_2 C_2^2\right) + \frac{1}{\nu_2^2(1)}\, f^o_{2M}\left[\frac{m_2C_2^2}{2kT_2^3}\left(\frac{m_2C_2^2}{2kT_2} - \frac{9}{2}\right) + \right. \\
& \left. + \frac{5}{2T_2^2}\right] \frac{8\kappa_{12} n_1}{3m_2 k}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} \frac{T_1 - T_2}{T_1^{3/2}}\, \underline{C}_2 \cdot \frac{\partial T_2}{\partial \underline{r}} + \frac{1}{\nu_2^2(1)}\, f^o_{2M}\, \frac{1}{T_2}\left(\frac{m_2C_2^2}{2kT_2} - \frac{5}{2}\right)\underline{C}_2 \cdot \\
& \cdot \frac{\partial}{\partial \underline{r}}\left[\frac{8\kappa_{12} n_1}{3m_2 k}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} \frac{T_1 - T_2}{T_1^{3/2}}\right] + \frac{1}{\nu_2^2(2)}\, f^o_{2M}\, \frac{m_2}{kT_2^2}\left(\frac{m_2C_2^2}{2kT_2} - \right. \\
& \left. - 1\right)\frac{8\kappa_{12} n_1}{3m_2 k}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} \frac{T_1 - T_2}{T_1^{3/2}}\, \underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}} + \frac{1}{\nu_2^2(1)}\, f^o_{2M}\, \frac{4\kappa_{12} n_1}{3m_2^2}\left(\frac{m_1}{2\pi kT_1}\right)^{\frac{1}{2}} \frac{m_2}{kT_2^2}\Bigg[\\
& \left[\left(\frac{m_2C_2^2}{2kT_2} - \frac{7}{2}\right)\left(7 - \frac{m_2C_2^2}{kT_2}\right) + 7\right]\underline{C}_2 \cdot \frac{\partial T_2}{\partial \underline{r}} + \qquad\qquad (39) \\
& + \frac{1}{\nu_2^2(2)}\, f^o_{2M}\, \frac{4\kappa_{12} n_1}{3m_2^2}\left(\frac{m_1}{2\pi kT_1}\right)^{\frac{1}{2}}\left(\frac{m_2}{kT_2}\right)^2\left(7 - \frac{m_2C_2^2}{kT_2}\right)\underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}} - \\
& - \frac{1}{\nu_2^2(1)}\, f^o_{2M}\, \frac{8\pi\kappa_{12} n_1}{3m_1 m_2}\left(\frac{m_1}{2\pi kT_1}\right)^{3/2} \frac{1}{T_2}\left[\left(\frac{m_2C_2^2}{2kT_2} - \frac{7}{2}\right)\left(4 - \frac{m_2C_2^2}{kT_2}\right) + 4\right]\underline{C}_2 \cdot \frac{\partial T_2}{\partial \underline{r}} - \\
& - \frac{1}{\nu_2^2(2)}\, f^o_{2M}\, \frac{8\pi\kappa_{12} n_1}{3m_1 m_2}\left(\frac{m_1}{2\pi kT_1}\right)^{3/2} \frac{m_2}{kT_2}\left(5 - \frac{m_2C_2^2}{kT_2}\right)\underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_o}{\partial \underline{r}}
\end{aligned}
$$

This expression for f^2_{2M} consists of 8 main parts. The source terms from which the last seven terms stem are simply these seven terms with the factors $1/\nu_2^2(1)$ and $1/\nu_2^2(2)$ replaced by $-1/\nu_2(1)$ and $-1/\nu_2(2)$. In spite of this simplification the

expression Eq. (39) contains the essential information concerning the transports which we are going to evaluate by taking appropriate moments. This is so because the above mentioned source terms can be shown to be orthogonal to $\underline{C}_2$ and C_2^2 and then, comparing with the method of [3] in connection with the "third" approximation to derive transports directly from the kinetic equation, we find we will end up with the same results to first order using either method. Choosing the parameters α_2 and γ_2 both equal to zero in Eq. (39) give $n_2^2 = T_2^2 = 0$. To fulfil the condition Eq. (30) we must choose

$$\underline{\beta}_2 = - \frac{\rho_1}{m_2 p_2} \overline{\underline{C}}{}^1_{1M}$$

where $\overline{\underline{C}}{}^1_{1M}$ is obtained from f^1_{1M} by taking the appropriate velocity moment. Thus $\overline{\underline{C}}{}^2_{2M}$ from Eq. (39) is given by

$$n_2 \overline{\underline{C}}{}^2_{2M} = - \frac{m_1}{m_2} n_1 \overline{\underline{C}}{}^1_{1M}$$

in accordance with Eq. (30).

With α_2 , γ_2 and $\underline{\beta}_2$ evaluated as above we turn to the ion heat transports and pressure tensors that may be derived from Eq. (39). To $\underline{q}^2_{2M}$ the first, second, third and seventh terms contribute, while the fifth, though of first order of anisotropy, does not. In this order we get

$$\underline{q}^2_{2M} = \sum_{i=1}^{4} \underline{q}^2_{2i}$$

where

$$\underline{q}^2_{21} = \frac{5}{2} n_2 (kT_2)^2 \underline{\beta}_2 = - \frac{5}{2} p_2 \frac{\rho_1}{\rho_2} \overline{\underline{C}}{}^1_{1M} \tag{40}$$

$$\underline{q}_{22}^{2} = K_{22}^{2} \frac{\partial T_2}{\partial \underline{r}} \tag{41}$$

where

$$K_{22}^{2} = \left(\frac{5}{2}\right)^2 \frac{(3k)^3 k^2}{b_{22}^{2}(1)\kappa_{22}^{2}} \frac{T_2^3}{n_2}\left(\frac{\partial T_2}{\partial t_2}\right)_c = \left(\frac{5}{2}\right)^2 \frac{8\kappa_{12}(3k)^3 k}{3b_{22}^{2}(1)\kappa_{22}^{2}} \frac{1}{m_2}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}}\frac{n_1}{n_2}\left(T_1 - T_2\right)\frac{T_2^3}{T_1^{3/2}} \tag{42}$$

$$\underline{q}_{23}^{2} = \frac{5(3k)^3 k^2}{2b_{22}^{2}(1)\kappa_{22}} \frac{T_2^4}{n_2} \frac{\partial}{\partial \underline{r}}\left[\left(\frac{\partial T_2}{\partial t_2}\right)_c\right] = \frac{5}{2} \frac{8\kappa_{12}(3k)^3 k}{3b_{22}^{2}(1)\kappa_{22}^{2} m_2} \left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} \frac{T_2^4}{n_2} \frac{\partial}{\partial \underline{r}}\left[n_1 \frac{T_1 - T_2}{T_1^{3/2}}\right] \tag{43}$$

and

$$\underline{q}_{24}^{2} = K_{24}^{2} \frac{\partial T_2}{\partial \underline{r}} \tag{44}$$

where

$$K_{24}^{2} = \frac{5}{2} \frac{4\kappa_{12}(3k)^3 k}{b_{22}^{2}(1)\kappa_{22}^{2}} \frac{1}{m_2}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} \frac{n_1}{n_2}\left(\frac{T_2}{T_1}\right)^{3/2} T_2^{5/2}$$

In Eqs.(42) and (43) $(\partial T_2/\partial t_2)_c$ denotes the change of T_2 per unit time due to ion-electron collisions, i.e.

$$\left(\frac{\partial T_2}{\partial t_2}\right)_c = \frac{8\kappa_{12}}{3m_2 k}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} n_1 \frac{T_1 - T_2}{T_1^{3/2}} \tag{45}$$

$\underline{q}_{21}^{2}$ is the ion heat flux corresponding to the diffusion $\underline{C}_{2M}^{2}$ above. $\underline{q}_{22}^{2}$ and $\underline{q}_{23}^{2}$ may be of more interest: Originating from the same group of source terms they can be added and written more compactly:

$$\underline{q}_{22}^{2} + \underline{q}_{23}^{2} = \frac{5}{2} \frac{(3k)^3 k^2}{b_{22}^{2}(1)\kappa_{22}^{2}} \frac{T_2^{3/2}}{n_2} \frac{\partial}{\partial \underline{r}}\left[T_2^{5/2}\left(\frac{\partial T_2}{\partial t_2}\right)_c\right] =$$

$$= \frac{5}{2} \frac{8\kappa_{12}(3k)^3 k}{3b_{22}^{2}(1)\kappa_{22}^{2} m_2} \left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}} \frac{T_2^{3/2}}{n_2} \frac{\partial}{\partial \underline{r}}\left[n_1 T_2 \left(\frac{T_2}{T_1}\right)^{3/2}(T_1 - T_2)\right] \tag{46}$$

Thus a heat transport is set up in the direction of the gradient of the product of $T_2^{5/2}$ and $(\partial T_2/\partial t_2)_c$. Depending on the situation this temperature-difference-driven heat transport may either act to weaken or strengthen the classical transport Eq. (36) by a certain amount. We may express this mechanism as follows: "A temperature-difference-driven heat transport is set up opposing changes in the classical transport (Eq. (36))." Thus, for instance when $(\partial T_2/\partial t_2)_c$ is uniform and positive, say, $\underline{q}_{2M}^1$ from Eq. (36) will tend to increase after a short time since the factor $T_2^{5/2}$ increases because of electron to ion energy transfer. The heat transport Eq. (46) is "induced" to oppose this change. Unlike $\underline{q}_{22}^2 + \underline{q}_{23}^2$, the heat transport $\underline{q}_{24}^2$ is non-vanishing also for a one temperature plasma. For a one temperature plasma $\underline{q}_{24}^2$ shows the classical temperature dependency. However, it is always directed opposite to $\underline{q}_{2M}^1$ and is a correction to this because of the electrons. We note that $\underline{q}_{24}^2$ comes from a part of the ion-electron collision term on the right hand side of Eq. (29). In a one-temperature plasma theory where the full ion-ion and ion-electron collision terms appear on the left hand side of equations like Eq. (29), the quantity corresponding to $\underline{q}_{24}^2$ may be inherent in what corresponds to $\underline{q}_{2M}^1$, the classical transport. Thus $\underline{q}_{24}^2$ may be looked upon as an extension of a classical, one-temperature plasma-effect into the two-temperature regime.

Turning to the kinetic pressure tensor we get contributions from the fourth and eigth terms of Eq. (39), while the contribution from the sixth term vanishes. In this order we have

$$\underset{\sim}{P}^2_{2M} = \sum_{i=1}^{2} \underset{\sim}{P}^2_{2i}$$

$$\underset{\sim}{P}^2_{21} = - \nu^2_{21}\left[\frac{\partial \underline{c}^o_o}{\partial \underline{r}} + \left(\frac{\partial \underline{c}^o_o}{\partial \underline{r}}\right)^T - \frac{2}{3}\left(\frac{\partial}{\partial \underline{r}}\cdot \underline{c}^o_o\right)\underset{\sim}{I}\right] \tag{47}$$

where

$$\nu^2_{21} = - \frac{5}{2}\,\frac{8\kappa_{12}(3k)^3}{3b^2_{22}(2)\kappa^2_{22}}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}}\frac{n_1}{n_2}\left(T_1 - T_2\right)\frac{T^3_2}{T^{3/2}_1}$$

and

$$\underset{\sim}{P}^2_{22} = \nu^2_{22}\left[\frac{\partial \underline{c}^o_o}{\partial \underline{r}} + \left(\frac{\partial \underline{c}^o_o}{\partial \underline{r}}\right)^T - \frac{2}{3}\left(\frac{\partial}{\partial \underline{r}}\cdot \underline{c}^o_o\right)\underset{\sim}{I}\right] \tag{48}$$

where

$$\nu^2_{22} = \frac{8\kappa_{12}(3k)^3}{3b^2_{22}(2)\kappa^2_{22}}\left(\frac{m_1}{2\pi k}\right)^{\frac{1}{2}}\frac{n_1}{n_2}\left(\frac{T_2}{T_1}\right)^{3/2} T^{5/2}_2$$

Commenting on $\underset{\sim}{P}^2_{21}$ we note that when $T_2 > T_1$, i.e. $(\partial T_2/\partial t_2)_c < 0$, $\underset{\sim}{P}^2_{21}$ acts to enforce the classical pressure $\underset{\sim}{P}^1_{2M}$ given by Eq. (37); however, when $T_2 < T_1$ we have the opposite effect. We may express this mechanism also as an "induction" opposing changes in the corresponding classical expression given by Eq. (37) because of electron-ion energy transfer. The pressure component $\underset{\sim}{P}^2_{22}$, on the other hand, corresponding to $\underline{q}^2_{24}$ above, comes from a part of the ion-electron collision term on the right side of Eq. (29); it always reduces the classical result given by Eq. (37) and shows the classical temperature dependency when $T_1 = T_2$. Like $\underline{q}^2_{24}$, $\underset{\sim}{P}^2_{22}$ may be looked upon as an extension into the two-temperature regime of an effect that may be inherent in a one-temperature classical transport theory.

Concerning the solution of the third order electron kinetic equation we note that the temperature difference between electrons and ions appears on the right hand side of Eq. (31) only in the four terms $\partial f^1_{1M}/\partial t_2 + \underline{c}^o_o \cdot \partial f^1_{1M}/\partial \underline{r}$, $\underline{C}_{1M} \cdot \partial f^2_{1M}/\partial \underline{r}$, $e_1/m_1(\underline{E} + \underline{c}^o_o \times B)\cdot \partial f^2_{1M}/\partial \underline{C}_1$ and $-FP_{11}\left[f^1_{1M}(\underline{C}_1)f^2_{1M}(\underline{C}_1') + f^2_{1M}(\underline{C}_1)f^1_{1M}(\underline{C}_1')\right]$. It is easily

shown that all source terms where the temperature difference appears are of odd order of anistropy, and therefore, they may contribute to $\underline{C}^3_{1M}$ and $\underline{q}^3_{1M}$, not to $\underset{\sim}{P}^3_{1M}$. The temperature-difference-driven heat transports which are due to the first of the above-written source terms, we expect contain at least one part corresponding to the temperature difference driven ion heat transport studied in the foregoing section. A closer examination shows this to be true giving an electron heat transport of the same form as Eq.(46). However, for the electrons various new temperature difference driven terms show up to this order of approximation, both for diffusion and heat transport.

As a final note it is interesting to resume the relative strengths of influence the temperature difference between electrons and ions has on the various electron and ion transports. Let

$$Q = Q^0 + \varepsilon Q^1 + \varepsilon^2 Q^2 + \cdots$$

where Q shall represent electron and ion diffusions, heat flux vectors and kinetic pressure tensors, and Q^0 the classical expression in each case. For electron diffusion the temperature difference appears for the first time in Q^2 , and from the condition Eq.(6) to fourth order it follows that the corresponding ion diffusion shows up in Q^2 for the first time. Note that a diffusion Q^2 for ions is two orders of magnitude smaller than a diffusion Q^2 for electrons in this theory. Turning to the electron heat transport, temperature difference terms for the first time appear in Q^2 , while such terms for the ion heat transport already

appear in Q^1 . Here the electron "Q^2" and the ion "Q^1" are of the same order of magnitude. For the kinetic pressure tensors temperature difference terms for the electrons emerge from the fourth order kinetic equation and therefore appear for the first time in Q^2 , while for the ions they show up already in Q^1 . Here the electron "Q^2" is one order of magnitude smaller than the ion "Q^1" .

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