Supersymmetric Cascade Decays at the LHC

by

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To Bæ-Bæ

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Preface

The research presented in this thesis is the result of work done as a PhD student at the Department of Physics and Technology, University of Bergen, in the period 2004-2007, and during a year spent in the CERN Theory Unit as a Marie Curie Early Stage Research Training Fellow in 2006.

The research focuses on extending the CERN Large Hadron Collider (LHC) potential for the discovery of supersymmetry in a wide range of possible supersymmetry models, and in investigating the LHC potential for the measurement of model parameters should a discovery be made, such as the masses of supersymmetric particles (sparticles). This is facilitated by exploring several benchmark models of supersymmetry, some suggested by other authors [1, 2], and mainly by looking at the cascade decays of sparticles.

The view taken in these investigations focuses on the phenomenology of the various *a priori* possible models of supersymmetry at the LHC. This is partially in contrast and certainly in complement to an experimental view focusing on the details of the detectors and their construction in this pre-LHC period, or on the experimental data itself during the running of the LHC, and it is also in complement to the more theoretical model building from which these models of supersymmetry have surfaced. To discover supersymmetry at the LHC it is not enough to know how the detectors of the experiments work and what supersymmetry is, but we need also to know what supersymmetry will look like at the LHC, in all of its many disguises. To do this we take advantage of Monte Carlo event generators to model proton–proton collisions, and in many cases use fast simulations of generic LHC detectors to get an idea of the potential of the LHC experiments.

This thesis presents the research in the form of a few introductory chapters on the physics involved, and four papers, three of which have been published in refereed journals. Below follows a short summary of these papers:[1]

Paper 1: We investigate the improvements made possible in measuring the masses of sparticles from the invariant mass distributions of cascade decay products, by considering not only the endpoints of these distributions, but also their full shapes. The shapes are derived for generic decay chains with two intermediate sparticles, most of them for the first time. We find that knowledge of the shapes helps cure several problems with the endpoint method, such as multiple solutions for masses and the existence of feet: small features in the distributions close to the endpoints that are easily mistaken for background

[1]Papers 1 and 3 are reprints of papers published in the Journal of High Energy Physics [3, 4], reproduced with the kind permission of SISSA, ©2006 SISSA. Paper 2, published in Physical Review D [5], is reprinted with permission from the Americal Physical Society, ©2006 by the Americal Physical Society. Paper 4 [6] has been submitted for publication to the Journal of High Energy Physics.

or smearing effects in an experiment.

Paper 2: This paper explores the possibility of detecting the scalar partner of top quarks (stops) predicted in supersymmetry models, when these are relatively light (lighter than the top quark). Such scenarios are preferred in models of baryogenesis and can help explain the value of the dark matter density of the universe. They also have the added advantage of less fine tuning for the Higgs mass. The idea of the paper is to look for events with a pair of same-sign top quarks, the result of the decays of a pair of gluinos into pairs of top and stop. We show that the signal is observable in a wide area of the parameter space, and for gluino masses up to 900 GeV. Using the shapes of the invariant mass distributions for the decay products of the top and the stop we can also constrain the masses of the sparticles involved.

Paper 3: Here we investigate models where the gravitino is the lightest sparticle and a dark matter candidate. Due to the gravitino's weak gravitational coupling, the next-to-lightest sparticle will have a long lifetime, and in the cosmologically preferred scenarios this is the charged scalar partner of the tau, the stau. This leads to distinctive signatures for the LHC, consisting of a pair of "muon"-like tracks made by two staus that are relatively slow due to the high mass of the staus when compared to muons. Similar signatures can be found in other models of supersymmetry, featuring long-lived sparticles, and indeed in many non-supersymmetric models as well. We find that such particles are easily discovered and that their masses can be measured to a very high precision, down to 0.1% in the best case scenarios. From the staus we successfully reconstruct heavier sparticles, partially with the help of invariant mass distributions.

Paper 4: The last paper deals with measuring sparticle masses in decay chains with exclusively hadronic final states, involving also the decays of massive bosons. We use the k_T jet-algorithm to reconstruct the bosons, finding that it effectively suppresses background in the cases where the bosons are highly boosted and the decay product jets are collimated. This allows us to constrain the sparticle masses involved by using the end-points of invariant mass distributions formed from jets in the events.

Contents

Chapter 1

Introduction

'Where shall I begin, please your Majesty?' he asked
'Begin at the beginning,' the King said, gravely, 'and go on till you come to the end: then stop.'

Lewis Carroll
Alice's Adventures in Wonderland

At the end of 2007 the Large Hadron Collider (LHC) at CERN, Geneva, is set to see its first proton–proton collisions. Due to its unprecedented high energy regime and high collision rate, the results of the LHC experiments are eagerly awaited by the particle physics community.

Among the most exciting areas under study is the search for supersymmetry (SUSY), a symmetry between the boson force carriers and the fermion matter particles of the universe. First suggested as an extension of the so-called Standard Model (SM) as early as the seventies [7], its presence at the energy scales accessible at the LHC is believed to hold the solution to several of the more vexing problems in today's high energy physics. It can incorporate natural candidates for dark matter consistent with the recent spectacular measurements of the dark matter density of the universe, based in part on data from the WMAP satellite [8–10]. Supersymmetry further points to the unification of the strong and electroweak forces at a high energy scale [11–13]. In addition, it reduces the fine tuning of the SM required to give a mass, consistent with measurements of other electroweak parameters, for the Higgs boson [14–16], the one missing particle of the SM and another candidate for discovery at the LHC.

If hints of supersymmetry are indeed found at the LHC they may come in the form of a discovery of one or more of the plethora of new particles predicted even in the most minimal of SUSY models. However, supersymmetry may also manifest itself only indirectly in the properties of SM particles measured at the LHC. Whichever choice Nature has made, the LHC experiments will be sensitive to the effects up to energy scales in the multi-TeV range. If the conclusions should be negative, this weakens the case for supersymmetry, even at energy scales accessible to us in the distant future, as it removes the central underpinning of reduced fine tuning and makes the prospects of a supersymmetric dark matter candidate dim. This implies that the LHC constitutes a significant test of low-energy supersymmetry.

The LHC experiments have a variety of search strategies for supersymmetry. If the symmetry known as R-parity holds for a SUSY model, this implies that the decay of a supersymmetric particle (sparticle) must result in another sparticle among the decay products, and it implies that the sparticles produced in high energy collisions must come in pairs. In turn this predicts the existence of a stable sparticle, as we will see in Chapter 3. If uncharged, this particle is a good dark matter candidate, but it will escape the LHC detectors. The production of heavier sparticles thus result in decay chains down to the lightest supersymmetric particle (LSP): a *cascade decay*. Summing the momenta of the detected particles in such an event will give an imbalance due to the missing LSPs. This missing momentum, combined with energetic jets and/or leptons from the cascade, constitute the main search channels for SUSY at the LHC.

With escaping LSPs the standard technique of measuring the masses of new particles by reconstruction from their decay products becomes impossible. One must instead rely on the distributions of the detectable particles and their relation to the mass of the sparticles involved in the cascade decay. This will be one of the main subjects of this thesis. However, missing energy is not the unique direct signal of SUSY that may appear at the LHC. There are scenarios where the next-to-lightest supersymmetric particle (NLSP) is long lived and charged, so that the SUSY signature is a pair of tracks from massive charged particles. This intriguing possibility is also discussed. Another alternative is that R-parity is broken, leaving some sparticles to decay solely to SM particles, and possibly to be produced singly. While presenting important search channels for the LHC, these scenarios will not be discussed extensively here due to their different nature.

Following this Introduction we will start with a discussion of the present Standard Model of particle physics in Chapter 2, first looking at its great successes in predicting the behaviour of the universe on the most fundamental scales, and then why we, despite these successes, have to look further for a more complete model of the world we inhabit. This is followed by a short introduction to supersymmetry in Chapter 3, and how it can answer some of the questions raised in the previous Chapter. Chapter 4 focuses on the phenomenology of the detection and mass measurement of sparticles at the LHC, and gives an introduction to the papers presented in the following chapters.

Chapter 2

The Standard Model

These metaphysics of magicians,
And necromantic books are heavenly;
Lines, circles, letters and characters;

Christopher Marlowe
The Tragical History of Doctor Faustus

Our unceasing efforts to understand the world around us in terms of its most fundamental laws and basic components have led us from paradigm to paradigm in the physical sciences, advancing into worlds very different from the experiences of our everyday lives. We describe these worlds with mathematical models, and from the models we make testable predictions to confirm our beliefs. From developments that may arguably reach back to the ancient Greek philosophers, we have arrived at a Standard Model (SM) of particle physics that successfully describes the physics of the very small distances and high energies. Here we will give a condensed, but necessarily incomplete exposition of the SM, hopefully sufficient for the discussions to follow.

2.1 A Brief History of the Standard Model

The question of the nature of matter was contended among the ancient Greeks. On one side were those that believed that matter was infinitely divisible and continuous, on the other was the view represented by Democritus and Leucippus, that there were indivisible units of matter, in ancient Greek *atomos*. While not popular in its own time, the idea was strengthened in the 19th century by the work of people like John Dalton, Amedo Avogadro and Dimitri Mendeleev. Their developments in chemistry led to the description of all matter as being built out of a finite number of elementary particles, atoms. While the atoms of the periodic table of the elements still constitute the basic building blocks of chemistry, the atom has long since been split into more fundamental components.

The first sub-atomic particle, the electron, was discovered by J.J. Thomson in 1897 [17] by cathode-ray experiments, and in 1911 his one-time student Ernest Rutherford proposed [18], on the basis of scattering experiments of α particles and electrons on gold foil, that the structure of the atom is that of a positively charged nucleus containing the bulk of its mass, surrounded by electrons. Further developments led to the discovery of the

Leptons			Quarks		
Flavour	Mass	Charge	Flavour	Mass	Charge
ν_e	< 2 eV	0	u	$1.5 - 3.0$ MeV	$\frac{2}{3}$
e	0.51099892(4) MeV	-1	d	$3 - 7$ MeV	$-\frac{1}{3}$
ν_μ	< 2 eV	0	c	1.25(9) GeV	$\frac{2}{3}$
μ	105.658369(9) MeV	-1	s	95(25) MeV	$-\frac{1}{3}$
ν_τ	< 2 eV	0	t	174.2(3.3) GeV	$\frac{2}{3}$
τ	$1776.99\pm^{0.29}_{0.26}$ MeV	-1	b	4.20(7) GeV	$-\frac{1}{3}$

Table 2.1: The spin-$\frac{1}{2}$ matter particles of the SM with masses and charges adapted from [26]. Errors for masses are given in parenthesis where symmetric. The neutrino masses are unknown, but are believed to be non-zero due to the existence of neutrino flavour oscillations. The given limit is taken from tritium decays. With the exception of the top quark the quark masses are $\overline{MS}$ masses, for the light quarks (u, d and s) they are so-called current quark masses.

proton in 1919 [19] and the neutron in 1932 [20], that make up the nucleus. The meaning of atom was stretched even further in 1964 with the proposal that nucleons have structure themselves in the form of constituent quarks [21, 22], to explain the veritable zoo of heavier particles found as the result of collider experiments at ever increasing energies.

Meanwhile Wolfgang Pauli had proposed the existence of a neutral and very light particle, the neutrino, in 1930 to explain the missing energy in β decays. However, it was not found experimentally until 1953 by Clyde Cowan and Frederick Reines [23]. Also, the theories of Paul Dirac on quantum mechanics had lead him to postulate the existence of anti-particles in 1928 [24], in turn confirmed by Carl Anderson [25] by the observation of the positron, the anti-particle of the electron, in 1933. A particle and its anti-particle have the exact same masses in the SM, but opposite charges. However, it is worth noticing that elementary particles can also be classified according to a quantum number called spin, and that both the electron and the positron are fermions — particles with half-integer spins — and have spin-$\frac{1}{2}$, as have all the other matter particles we have seen.

Over time it also became clear that one generation of leptons — the electron and neutrino — and one generation of quarks — the up and down quarks — were not enough to explain all the exotic beasts observed in experiments. Three generations with different masses are needed in the SM, and experimental searches indicate that there are no more.[1] This leads to the prediction of six leptons: the electron, muon and tau with their corresponding neutrinos, and six quarks: the up, down, strange, charm, bottom and top. All of these have now been found in high energy experiments together with their anti-particles. A table showing the matter particles of the SM with their masses and electric charges can be found in Table 2.1.

In the SM the interactions of elementary matter particles are represented by the exchange of force mediating particles that are quantisations of the fields of the forces. The

[1] E.g. the number of generations of neutrinos was measured by the LEP experiments to be 2.994 ± 0.012 [26]. If more neutrinos should exist they would either need to be very heavy so that they do not contribute in the LEP measurement, or their couplings to other particles need to be different from the SM neutrino couplings.

Force	Particle	Mass	Charge
Electromagnetic	γ	0	0
Weak	Z^0	91.1876(21) GeV	0
	$W^{\pm}$	80.403(29) GeV	± 1
Strong	g	0	0

Table 2.2: The spin-1 force mediating particles of the SM with masses and charges adapted from [26]. Errors for masses are given in parenthesis.

electromagnetic field has been known since James Maxwell's 1864 paper [27] and its quantisation is the photon of Quantum Electro Dynamics (QED), the so-called gauge theory of electrodynamics which we shall discuss further below. In addition to the electromagnetic force the SM describes the two nuclear forces: the weak and the strong nuclear force. Inspired by the gauge theory nature of QED, the description of the weak force was developed in the combined electroweak framework by Sidney Glashow, Abdus Salam and Steven Weinberg during the 1960's [28–30]. The electro-weak force is mediated by the massive vector gauge bosons $W^{\pm}$ and Z^0, discovered at CERN in 1983 [31–34], in addition to the massless photon. The strong force is not unified with the other two forces, and it is mediated by the massless gluon, g. However, the force-mediating particles share a common feature in that they are bosons — particles of integer spin — with spin-1. We show some of the properties of the force mediating particles in the SM in Table 2.2.

Parallel to the experimental discoveries theoretical models were naturally being built, describing what had been found and predicting what should be out there. At times experimental findings have demanded theoretical explanation, as can be seen in the case of the particle zoo and the quark sub-structure of the nucleons, at other times theorists have been more or less certain of the existence of particles before they are found experimentally. We turn now to describing these theoretical frameworks, on which the SM is built.

2.2 Quantum Electro Dynamics

Attempts to understand the nature of light played a central role in the striking developments in physics at the beginning of the 20th century. The particle–wave debate, going back to Isaac Newton's corpusculars and Christiaan Huygen's waves in the 1600s, was finally resolved in the particle–wave duality of quantum mechanics, laying the foundations for the field theoretical description of electromagnetic interaction in Quantum Electro Dynamics (QED) by people such as Freeman Dyson, Richard Feynman, Julian Schwinger and Sin-Itiro Tomonaga [35–38], in the late 1940s. Here we will focus on the gauge aspects of QED.

Maxwell's successful field description of electromagnetism ruled the ground at the end of the 19th century, but two major problems were on the horizon. The radiation predicted from a black body was found to be ultraviolet divergent, i.e. divergent at short wavelengths. Additionally, light was assumed to propagate in some medium, dubbed the *aether*, but experimental searches for medium effects were negative. The suggestion by Max Plack in 1901 of a quantised electromagnetic energy [39] to explain the black-body spectrum, together with Albert Einstein's model for the photoelectric effect in 1905 [40], favoured a particle description of light. At the same time Einstein's theory of special relativity [41] did away with an absolute frame of reference, and as a consequence the necessity of the aether, at the expense of some added mathematical work in transforming between frames of reference.

Developments in quantum mechanics at the beginning of the 20th century by the likes of Bohr, de Broglie, Heisenberg, Pauli and Schrödinger extended the quantisation of Planck into a particle–wave duality and culminated in the first relativistic quantum mechanical description of a spin-$\frac{1}{2}$ particle by Dirac in 1928 [24]. Here a free particle with mass m is described by the Lagrangian density

$$\mathcal{L}_0 = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x), \tag{2.1}$$

where $\psi(x)$ is the four-component spinor representation of the spin-$\frac{1}{2}$ particle and γ^μ are gamma matrices (see Appendix A). The interaction of this free particle with the electromagnetic field can correctly be described by the so-called *minimal substitution*

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu(x), \tag{2.2}$$

where q is the charge of the particle and $A_\mu(x) = (\phi(x), \mathbf{A}(x))$ is a covariant description of the electric and magnetic fields of Maxwell's equations given by

$$\mathbf{E}(x) = -\nabla\phi(x) - \frac{\partial \mathbf{A}(x)}{\partial t} \quad \text{and} \quad \mathbf{B}(\mathrm{x}) = \nabla \times \mathbf{A}(\mathrm{x}). \tag{2.3}$$

D_μ is sometimes called the *covariant derivative*. It is important to notice that there is a group of transformations of A_μ that leave the electric and magnetic fields unchanged for all space-time coordinates x, namely

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu f(x), \tag{2.4}$$

where $f(x)$ is some analytic function. This can be shown by insertion into Eq. (2.3). A corresponding phase transformation of the $\psi(x)$

$$\psi(x) \rightarrow \psi(x)e^{-iqf(x)}, \tag{2.5}$$

will leave the Lagrangian, including the interaction term created by (2.2), invariant under the combined effects of both transformations. Because the phase of $\psi(x)$ is not an observable in quantum mechanics, the physics of the particle is invariant under this transformation, as the electric and magnetic fields are under (2.4). The transformations of Eqs. (2.4) and (2.5) are called *local gauge transformations*, and reflect a symmetry in the system described by the Lagrangian, which is called a *local gauge symmetry*. Here local indicates the dependence of the transformations on the space-time coordinate x. We say that the phase transformation of (2.5) is a representation of the $U(1)$ group of unitary transformations, since the exponential involved is just a one-dimensional unitary matrix.

Alternatively one could demand the invariance of Eq. (2.1) under the gauge transformation of (2.5). This would imply that the covariant derivative should have the form given in (2.2), with a field $A_\mu(x)$ transforming as in Eq. (2.4), and thus give the coupling of a matter particle with charge q under the unitary gauge transformation of (2.5) to the gauge field $A_\mu(x)$.

Inserting the minimal substitution of Eq. (2.2) into the Lagrangian in Eq. (2.1) and adding a term for the free electromagnetic field results in the complete QED Lagrangian

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{2.6}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2.7}$$

is the electromagnetic field strength tensor, which is obviously invariant under Eq. (2.4) for a well-behaved $f(x)$. Interpreting $\psi(x)$ as the field of the spin-$\frac{1}{2}$ particle, introducing creation and annihilation operators both for the matter particle and the photons through second quantisation, and using renormalisation techniques to deal with the infinities predicted by the theory, this Lagrangian gives a complete quantum field theoretical description of a spin-$\frac{1}{2}$ particle and its interaction with the electromagnetic field, where the interaction comes about through a gauge mechanism. The elegance of this approach inspired tremendous efforts to give similar descriptions of the other forces between the elementary particles.

2.3 The Electroweak Theory

By the mid-1950s weak *charged* interactions had been observed in nuclear decays, e.g. the neutron conversion to proton, electron and anti-neutrino: $n \rightarrow p + e^- + \bar{\nu}_e$, leading to the β decays of nuclei, from which Pauli had predicted the existence of the neutrino. In 1956 it was observed that weak processes were parity violating [42], meaning that they are not invariant under the sign reversal of all three spatial coordinates. The consequence of this breaking of space symmetry is that only certain components of particle (anti-particle) states $\psi(x)$, which are denoted left (right) handed, are involved in weak interactions. We can introduce the chirality operators P_L and P_R (see Eqs. (A.11) and (A.12) of Appendix A) that by construction project out the left- and right-handed components of a state:

$$\psi^{L/R}(x) = P_{L/R}\psi(x). \tag{2.8}$$

It is easiest to write the gauge transformation of the weak interactions by the use of weak isospinors, combining the left-handed neutrino and lepton states in doublets, and treating the right-handed states as singlets:

$$\Psi_l^L(x) = \begin{pmatrix} \psi_{\nu_l}^L(x) \\ \psi_l^L(x) \end{pmatrix}, \quad \psi_{\nu_l}^R(x), \quad \psi_l^R(x). \tag{2.9}$$

While we will only consider leptons here for reasons of simplicity in notation, corresponding results hold for quarks, with doublets consisting of left-handed up and down family quarks, as long as one carefully takes into account the differences in charge. We start from the free-lepton Lagrangian corresponding to (2.1), using Eq. (A.7) to simplify notation,[2]

$$\mathcal{L}_0 = i\bar{\Psi}_l^L(x)\partial\!\!\!/\,\Psi_l^L(x) + i\bar{\psi}_l^R(x)\partial\!\!\!/\,\psi_l^R(x) + i\bar{\psi}_{\nu_l}^R(x)\partial\!\!\!/\,\psi_{\nu_l}^R(x). \tag{2.10}$$

[2]Note that no mass terms are included in this Lagrangian. We will later demonstrate that lepton mass terms are not gauge invariant in the electroweak model.

The gauge transformation of weak interactions is then given by

$$\Psi_l^L(x) \rightarrow U(\omega(x))\Psi_l^L(x), \tag{2.11}$$
$$\psi_l^R(x) \rightarrow \psi_l^R(x), \tag{2.12}$$

where $U(\omega(x))$ is a local $SU(2)$ transformation by a unitary matrix with $\det U(\omega(x)) = 1$. The representation of $U(\omega(x))$ acting on the weak isospinor doublets is

$$U(\omega(x)) = e^{ig\omega(x)\sigma/2}, \tag{2.13}$$

where σ_i are the Pauli matrices (see Appendix A), $\omega_i(x)$ is a vector of real-valued differentiable functions and g is identified with the coupling constant of weak interactions, corresponding to the charge in QED. We could continue the procedure of Section 2.2 for making the Lagrangian invariant under this gauge transformation by introducing a covariant derivative which couples the leptons to gauge fields and by giving the corresponding transformations for the gauge fields. However, the resulting model of weak interactions has the unfortunate property that it is ruled out by experiment.

The solution is to introduce a $U(1)$ gauge transformation similar to that of QED. If we require that all the fields of the Lagrangian in (2.10) are also invariant under the $U(1)$ transformation

$$\psi(x) \rightarrow e^{-ig'Yf(x)}\psi(x), \tag{2.14}$$

where Y is a charge called *hypercharge*, g' is the second coupling constant of a combined electroweak theory and $f(x)$ is a real-valued differentiable function, then the gauge invariant leptonic Lagrangian density for the combined electroweak gauge theory is given by

$$\mathcal{L} = i\bar{\Psi}_l^L(x)\not{D}\,\Psi_l^L(x) + i\bar{\psi}_l^R(x)\not{D}\,\psi_l^R(x) + i\bar{\psi}_{\nu_l}^R(x)\not{D}\,\psi_{\nu_l}^R(x). \tag{2.15}$$

where

$$D^\mu\Psi_l^L(x) = [\partial^\mu + ig\omega_j(x)W_j^\mu(x)/2 + ig'YB^\mu(x)]\Psi_l^L(x), \tag{2.16}$$
$$D^\mu\psi_l^R(x) = [\partial^\mu + ig'YB^\mu(x)]\psi_l^R(x), \tag{2.17}$$
$$D^\mu\psi_{\nu_l}^R(x) = [\partial^\mu + ig'YB^\mu(x)]\psi_{\nu_l}^R(x). \tag{2.18}$$

This has coupled the leptons to four gauge fields $W_i^\mu(x)$ and $B^\mu(x)$ that must transform as[3]

$$W_i^\mu(x) \rightarrow W_i^\mu(x) - \partial^\mu\omega_i(x) - g\epsilon_{ijk}\omega_j(x)W_k^\mu(x) \tag{2.19}$$
$$B^\mu(x) \rightarrow B^\mu(x) - \partial^\mu f(x) \tag{2.20}$$

to ensure gauge invariance.

From Noether's theorem [43], stating that all symmetries of the Lagrangian density under continuous transformations imply corresponding conserved currents and charges, one can show, using global gauge transformations, that the hypercharge is given in terms of the electric charge q and the weak isocharge I_3^W (the charge under the $SU(2)$ gauge group), as

$$Y = q - I_3^W. \tag{2.21}$$

[3]For simplicity we give the transformations for infinitesimal values of $\omega_i(x)$.

The weak isocharge I_3^W takes the values $(\frac{1}{2}, -\frac{1}{2})$ for the doublets and 0 for the singlets, giving hypercharges of $(-\frac{1}{2}, -1, 0)$ for the fields $\Psi_l^L(x)$, $\psi_l^R(x)$ and $\psi_{\nu_l}^R(x)$ respectively.

The resulting electroweak theory has $SU(2)_L \times U(1)_Y$ as its gauge group, where the indices denote the chirality of the weak interactions and the charge of the $U(1)$ gauge group. The natural question is then how to interpret the gauge fields. We change the basis of the fields, introducing the four fields $W_\mu(x)$, $W_\mu^\dagger(x)$, $Z_\mu(x)$ and $A_\mu(x)$, by the substitutions

$$W_{1\mu}(x) = \frac{1}{\sqrt{2}}[W_\mu(x) + W_\mu^\dagger(x)], \tag{2.22}$$

$$W_{2\mu}(x) = \frac{i}{\sqrt{2}}[W_\mu(x) - W_\mu^\dagger(x)], \tag{2.23}$$

$$W_{3\mu}(x) = \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x), \tag{2.24}$$

$$B_\mu(x) = -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x), \tag{2.25}$$

where θ_W is the so-called *Weinberg angle*. By further demanding that the gauge field $A_\mu(x)$ is the electromagnetic field with the usual coupling to a charge q, which amounts to requiring

$$g\sin\theta_W = g'\cos\theta_W = q, \tag{2.26}$$

one can show that the total Lagrangian density, sans terms for free field gauge bosons and gauge boson self-interactions, becomes

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I, \tag{2.27}$$

where the interaction term $\mathcal{L}_I$ is given by

$$\begin{aligned}\mathcal{L}_I = & -qs^\mu(x)A_\mu(x) - \frac{g}{2\sqrt{2}}[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)] \\ & -\frac{g}{\cos\theta_W}[J_3^\mu(x) - \sin^2\theta_W s^\mu(x)]Z_\mu(x).\end{aligned} \tag{2.28}$$

In the quantisation of the fields we interpret the $W(x)$ fields as the charged $W^\pm$ vector bosons and the $Z(x)$ as the neutral vector boson Z^0. The covariant quantities $s^\mu(x)$, $J^\mu(x)$, $J^{\mu\dagger}(x)$ and $J_3^\mu(x)$ are linear combinations of conserved currents from the gauge symmetries, and are given in terms of the fields by

$$s^\mu(x) = -\bar{\psi}_l(x)\gamma^\mu\psi_l(x), \tag{2.29}$$

$$J^\mu(x) = \bar{\psi}_l^L(x)\gamma^\mu\psi_{\nu_l}^L(x), \tag{2.30}$$

$$J^{\mu\dagger}(x) = \bar{\psi}_{\nu_l}^L(x)\gamma^\mu\psi_l^L(x), \tag{2.31}$$

$$J_3^\mu(x) = \frac{1}{2}[\bar{\psi}_{\nu_l}^L(x)\gamma^\mu\psi_{\nu_l}^L(x) - \bar{\psi}_l^L(x)\gamma^\mu\psi_l^L(x)]. \tag{2.32}$$

The first of these is the electromagnetic current that also appears in QED, the next two are the expected charged currents that should govern the charged weak interactions, but the fourth current is a *neutral current* in the sense that it can couple neutral leptons. The existence of a neutral current in weak interactions was a prediction of the electroweak theory, and it was a great confirmation of the theory when this current was subsequently found in 1973 [44].

There remains, however, one problem with the electroweak model. The bosons in an un-broken gauge theory must be massless, since mass terms in the Lagrangian of the type

$$\frac{1}{2}m^2 X_\mu(x)X^\mu(x), \tag{2.33}$$

for a gauge field $X_\mu(x)$, are not invariant under gauge transformations such as (2.19) and (2.20). The lack of mass implies that the forces carried by the gauge bosons could have infinite range, as the electromagnetic interactions have. This is in clear contradiction to the experimental evidence for weak interactions that limit them to the small distance scales of a nucleus and indicate that the force-mediating particles should be very massive. Furthermore, for the electroweak theory mass terms for the leptons are also not gauge invariant. This can be easily seen by rewriting the potential lepton mass term

$$-m_l\bar{\psi}_l(x)\psi_l(x) = -m_l[\bar{\psi}_l^L(x)\psi_l^R(x) + \bar{\psi}_l^R(x)\psi_l^L(x)], \tag{2.34}$$

where we have used the completeness property (A.15) of the chirality operators. The right-hand side is clearly not gauge invariant under $SU(2)_L \times U(1)_Y$, since the terms are a mixture of isoscalars and isospinors that transform differently. In addition to these problems, models with massive bosons are not normalisable, meaning that infinities that are not removable occur in the theory when calculating physical observables.

Early attempts at constructing a gauge theory for the weak interactions had stranded at these problems of massless particles. We shall see in Section 2.5 how this can be resolved by breaking the $SU(2)_L \times U(1)_Y$ gauge symmetry using the Higgs mechanism, but we will first give a short description of the last piece of the SM force puzzle, the strong nuclear force.

2.4 Quantum Chromo Dynamics

Early in the development of a model of strong interactions the quark constituent model of the nucleons proposed by Murray Gell-Man and George Zweig in 1964 [21,22], to explain the large number of new particles found in collider experiments, faced difficulty from the observation of one particle, the Δ^{++}, that could only be explained by three quarks in the same spin state. Quarks were assumed to be spin-$\frac{1}{2}$ fermions and Pauli's exclusion principle explicitly forbids two fermions to occupy the same quantum state simultaneously. The solution to the problem was the introduction of a new quantum charge for quarks, the colour charge — thereby the name of the emerging theory: Quantum Chromo Dynamics (QCD) — independently by Moo-Young Han and Yoichiro Nambu, and by Oscar W. Greenberg [45, 46]. The assignment of a colour charge to each quark, out of three possible, avoids the exclusion principle. The coloured quarks then form a basis for the three-dimensional representation of the symmetry group $SU(3)$ operating on their colour charges. The gauge theory resulting from this group was subsequently found to be able to provide a theory for the strong interactions. Below we give a brief overview of the structure of QCD on the basis of its Lagrangian.

If we denote the quarks by three-component spinors $\psi_q^i(x)$, where q is one of the six quark flavors and where the components, indexed by i, represent the colour content of the quark, we can again start from the free-field Lagrangian density, this time for quarks,

$$\mathcal{L}_0 = \sum_q \bar{\psi}_q^i(x)(i\partial\!\!\!/ - m_q)\psi_q^i(x). \tag{2.35}$$

We require invariance under the local $SU(3)$ gauge transformation

$$\psi_q(x) \rightarrow e^{-ig_s\theta_a(x)\lambda_a/2}\psi_q(x), \tag{2.36}$$

where λ_a are the eight 3×3 matrix generators of $SU(3)$ given in (A.22), g_s is identified with the strong coupling constant and $\theta_a(x)$ are eight differentiable functions.[4] One can show that the resulting QCD Lagrangian is given by

$$\mathcal{L} = \sum_q \bar{\psi}_q(x)(iD\!\!\!\!/ - m_q I)\psi_q(x) - \frac{1}{4}F_{a\mu\nu}(x)F_a^{\mu\nu}(x). \tag{2.37}$$

Here

$$F_a^{\mu\nu}(x) = \partial^\mu G_a^\nu(x) - \partial^\nu G_a^\mu(x) - g_s f_{abc} G_b^\mu(x) G_c^\nu(x) \tag{2.38}$$

are the field strength tensors for the gluon fields $G_a^\mu(x)$ and f_{abc} are the structure constants of the $SU(3)$ gauge group (see Appendix A). The covariant derivatives $D_\mu(x)$ are given by

$$D^\mu(x) = I\partial^\mu + ig_s G_a^\mu(x)\lambda_a/2, \tag{2.39}$$

giving the quark–gluon interaction when the gluon is interpreted as the quantum of the field $G_a^\mu(x)$.

Since there are eight gluon fields there are eight gluon colour charges that transform under $SU(3)$, albeit in a different representation than the quarks. This is strikingly different from photons, which have no electromagnetic charge, and the gluon colour charge is believed to result in *confinement*, i.e. that colour charges cannot be isolated due to the structure of gluon fields that form around them. However, despite strong experimental evidence, confinement in QCD still lacks analytic proof. A consequence of colour confinement is that it would allow us to live with massless gluons, because the range of the strong force would be very limited by the confinement mechanism, in agreement with experiment.

The physics interpretation of the last term in (2.38) is that the gluons, unlike photons, have self-interactions. The implication is that the strong coupling is small at high energies due to its running with energy being dominated by the gluon, it has *asymptotic freedom* as first showed by David Gross, David Politzer and Frank Wilczek in 1973 [47, 48]. This means that we can reliably calculate interactions of quarks and gluons at high energies using perturbation theory, expanding the quantities in terms of power series of the coupling strength.

[4]For clarity we suppress color indices, and use the identity matrix I where appropriate.

2.5 The Higgs Mechanism

As we saw in Section 2.3, the gauge theory for electroweak interactions predicts zero masses for both the gauge bosons and the leptons, in contradiction with experimental evidence. The question is then where the masses of the elementary particles originate.[5] To solve the problem of masses in gauge theories the so-called Higgs mechanism of spontaneous symmetry breaking was independently suggested by a number of people around 1964 [49–51].

In its application to electroweak theory the Higgs mechanism consists of introducing an extra weak isospin doublet,

$$\Phi(x) = \begin{pmatrix} \phi_a(x) \\ \phi_b(x) \end{pmatrix}, \tag{2.40}$$

with complex scalar spin-0 fields $\phi_a(x)$ and $\phi_b(x)$ as components and the same transformations, (2.11) and (2.14), under $SU(2)_L \times U(1)_Y$ as the other weak isospin doublets. This is called the Higgs doublet. The additional terms $\mathcal{L}_H$ introduced in the Lagrangian that contain the Higgs field are

$$\mathcal{L}_H = [D^\mu \Phi(x)]^\dagger [D_\mu \Phi(x)] - \mu^2 \Phi^\dagger(x)\Phi(x) - \lambda[\Phi^\dagger(x)\Phi(x)]^2, \tag{2.41}$$

where $\lambda > 0$ and $\mu^2 < 0$. To keep the gauge invariance of the Lagrangian density the covariant derivative acting on $\Phi(x)$ is defined analogously to (2.16) with hypercharge $Y = \frac{1}{2}$.[6] The last two terms of (2.41) are the so-called Higgs potential. The shape of the Higgs potential

$$V(\Phi(x)) = \mu^2 \Phi^\dagger(x)\Phi(x) + \lambda[\Phi^\dagger(x)\Phi(x)]^2, \tag{2.42}$$

for the given range of λ and μ, is illustrated in Fig. 2.1.

In a field theory, the state of lowest energy is given by the stable vacuum, corresponding to the lowest potential. This explains why we must require $\lambda > 0$, so that the potential is bounded from below. For the Higgs potential (2.42), the classical minimum is realised by a field with

$$\Phi^\dagger \Phi = \frac{-\mu^2}{2\lambda}, \tag{2.43}$$

and the vacuum is degenerate, i.e. a global phase transformation

$$\Phi(x) \rightarrow e^{i\alpha}\Phi(x) \tag{2.44}$$

of the field leaves the potential unchanged, which can be seen as a rotation around the origin in Fig. 2.1. Breaking this symmetry of the potential, by fixing the vacuum expectation value for one of the fields is the fundamental mechanism behind spontaneous

[5] Note that this is not the question of where most of the mass around us originates from. We know that while the atoms of our world take their masses mainly from the protons and neutrons that constitute them, their masses are in turn not made up from the masses of the quarks, but rather their binding energy.

[6] From (2.21) it then follows that the $\phi_b(x)$ field is electrically neutral. Since the $\phi_b(x)$ component will be the source of the spontaneous symmetry breaking, the symmetry breaking will occur only for the neutral component of the vacuum, leaving charge conserved and the photon massless.

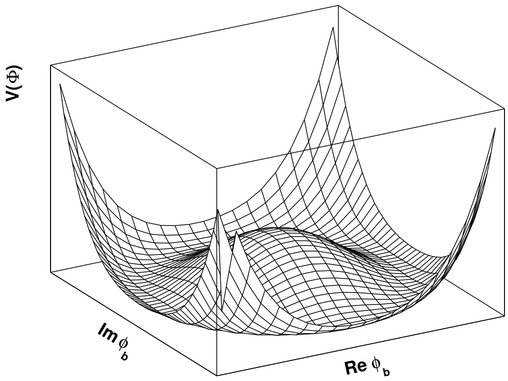

Figure 2.1: Illustration of the Higgs potential (2.42) for $\lambda > 0$ and $\mu^2 < 0$, setting $\phi_a = 0$.

symmetry breaking. Without loss of generality we can choose the vacuum to be given by the vacuum expectation values (VEVs) $\langle 0|\phi_a(x)|0\rangle = 0$ and $\langle 0|\phi_b(x)|0\rangle = v/\sqrt{2}$ of the Higgs fields, where $v = (-\mu^2/\lambda)^{1/2}$. The Higgs doublet can then be parametrised as a perturbation of the vacuum by

$$\Phi(x) = \begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ \frac{1}{\sqrt{2}}[v + \sigma(x) + i\eta_3(x)] \end{pmatrix}, \tag{2.45}$$

in terms of four real fields $\eta_i(x)$ and $\sigma(x)$. Using the gauge freedom of the $SU(2)_L \times U(1)_Y$ group we can apply (2.11) to transform this isospinor into a spinor with only a lower component and (2.14) to remove the imaginary part of the remaining component. Thus there exists a gauge so that

$$\Phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}[v + \sigma(x)] \end{pmatrix}, \tag{2.46}$$

where we identify the $\sigma(x)$ field with the Higgs particle. The leptons can now be given masses by the addition to the Lagrangian of so-called *Yukawa terms* $\mathcal{L}_Y$, invariant under the $SU_L(2) \times U(1)_Y$ gauge transformations, and involving the Higgs field:

$$\mathcal{L}_Y = -\lambda_l[\bar{\Psi}_l^L(x)\psi_l^R(x)\Phi(x) + \Phi^\dagger(x)\bar{\psi}_l^R(x)\Psi_l^L(x)], \tag{2.47}$$

where the λ_l are the dimensionless Yukawa coupling constants. Similar terms can be added for neutrinos to arrive at non-zero neutrino masses. When (2.46) is substituted into the sum of the Lagrangian in (2.27), and the terms of (2.41) and (2.47), a somewhat tedious calculation gives the complete Lagrangian density $\mathcal{L}$ of the electroweak theory, with mass terms for both leptons and bosons. We write

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I, \tag{2.48}$$

where the free field terms $\mathcal{L}_0$ for the leptons, gauge bosons and the real scalar Higgs field $\sigma(x)$, are given by

$$\mathcal{L}_0 = \bar{\psi}_l(x)(i\partial\!\!\!/ - m_l)\psi_l(x)$$

$$\begin{aligned}
&-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)\\
&-\frac{1}{2}F^{\dagger}_{W\mu\nu}(x)F^{\mu\nu}_{W}(x)+m_W^2W^{\dagger}_{\mu}(x)W^{\mu}(x)\\
&-\frac{1}{4}Z_{\mu\nu}(x)Z^{\mu\nu}(x)+\frac{1}{2}m_Z^2Z_{\mu}(x)Z^{\mu}(x)\\
&+\frac{1}{2}\partial^{\mu}\sigma(x)\partial_{\mu}\sigma(x)-\frac{1}{2}m_h^2\sigma^2(x).
\end{aligned} \tag{2.49}$$

The resulting mass terms are given in terms of the parameters of the Higgs potential's minimum, the Yukawa couplings and the gauge couplings:

$$m_l=\frac{1}{\sqrt{2}}v\lambda_l,\quad m_W=\frac{1}{2}vg,\quad m_Z=\frac{m_W}{\cos\theta_W},\quad m_h=\sqrt{-2\mu^2} \tag{2.50}$$

We omit the even more lengthy interaction terms of the complete theory, the interested reader can find these in e.g. Chapter 14 of [52].

Since the parameter v can be expressed in terms of the Fermi coupling constant, and the gauge coupling constants are given in terms of the Weinberg angle (2.26), both of which are experimentally well known, the masses of the W and Z bosons could be predicted from the Higgs mechanism, and their later discovery at these masses is one of the greatest triumphs of the SM. However, the parameter λ that enters in the Higgs mass is unknown. As the issues surrounding the value of the Higgs mass and our failure to discover the quanta of the Higgs field — the Higgs particle — is both one of the major problems and most exciting challenges of physics today, we will discuss this further in the next section.

2.6 Beyond the Standard Model

While it has had fantastic success in explaining observations on a wide range of energy scales accessible to current experiments, the Standard Model is by no means a complete theory of everything. Most importantly perhaps, it does not include gravity, which any theory that pretends to describe physics up to energies around the Planck scale ($\sim 10^{19}$ GeV) must, since we know that quantum effects of gravity will become important at or before this scale. So the SM cannot, and does not claim to describe physics at all scales.

There is little if any direct and significant evidence that contradicts the SM on the energy scales probed up to now. [7] However, in addition to the absence of gravity, there are a handful of very good reasons for extending the SM, as we shall see below. Furthermore, we also have reason to believe — as opposed to gravity — that these may show up in the near future, at energy scales that will be accessible to us at the LHC.

2.6.1 Tuning the Higgs Mass and a Hierarchy Problem

In (2.50) we gave the tree level (no quantum loop corrections) Higgs mass resulting from the VEV of the Higgs field. If one assumes that the Higgs potential coupling λ is naturally

[7] One possible exception here are the recent results for the anomalous magnetic moment of the muon, for which there seem to be at present at 3.3σ deviation from the SM [53–55].

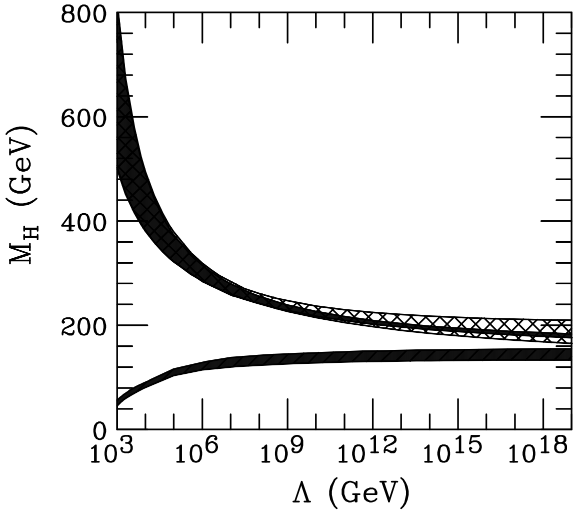

Figure 2.2: Bounds on the Higgs mass at two loops from the self-consistency of the SM [56]. The upper bound is from the perturbativity of the coupling λ. The lower bound is the requirement that the electroweak minimum remains the actual minimum after loop corrections. For both bounds a top mass of $m_t = 175$ GeV is assumed. The hatched area of the upper bound shows the dependence on varying the top mass between 150 and 200 GeV.

of $\mathcal{O}(1)$, this results in a Higgs mass of $\mathcal{O}(100\text{ GeV})$. While, as stated in the previous Section, λ cannot be directly determined in terms of measured electroweak parameters, bounds can be found from quantum corrections to the electroweak masses and the Higgs potential. Requiring that the SM Higgs sector is self-consistent, Fig. 2.2 shows the bounds from the perturbativity of λ and the minimum of the electroweak potential at some energy scale Λ.

While this still allows for masses above the direct search limits set by the LEP experiment,[8] the Higgs mass has a disturbing sensitivity to quantum loop corrections from every particle it couples to. From a lepton[9] it gets a correction

$$m_h^2 = -\frac{|\lambda_l|^2}{8\pi^2}\Lambda_{\text{cut}}^2 + \ldots, \tag{2.51}$$

where Λ_{cut} is a cutoff used to make the corresponding loop integral finite, and which represents the scale at which some new physics enters to regulate the high-energy behaviour of the theory. The dots represent less divergent terms $\propto \ln(\Lambda_{\text{cut}})$. If no new physics is present between the electroweak scale and the Planck scale this correction is 16 orders of magnitude greater than the naive expectation for the Higgs mass. This is often called a *hierarchy problem*, which can be more generically described to occur when the scale of parameters used to explain other parameters — the Planck scale explaining the Higgs mass — are vastly different from the scale of the parameters to be explained.[10]

[8] For a SM Higgs boson a combined but preliminary fit gives $m_h > 114.4$ GeV at the 95% confidence level [57].

[9] This holds more generally for any Dirac fermion.

[10] Or in other words, why are elephants afraid of mice?

For bosons with a coupling λ_B to the Higgs the loop correction is given by

$$m_h^2 = \frac{\lambda_B}{16\pi^2}\Lambda_{\text{cut}}^2 + \ldots, \tag{2.52}$$

where λ_B must be positive for the potential to be bounded from below. Although the contributions from bosons and fermions have opposite signs, a very fine tuning of the SM masses is needed to arrive at a mass within the narrow limits that can be found from electroweak precision measurements, considering the enormous size of the individual corrections.

While quantum corrections to the SM fermion and gauge boson masses from the cutoff scale are only indirect through the Higgs, with a logarithmic dependence on the Higgs mass, the effects on precision measurements of electroweak observables, mainly W and top masses, and Z decay data, allows the Higgs mass to be predicted to be $m_h = 85^{+39}_{-28}$ GeV [58], assuming a top quark mass of $m_t = 171.4 \pm 2.1$ GeV. Comparing with Fig. 2.2 this is still consistent with the SM being valid up to GUT scales, but only barely.

What is needed to avoid this fantastic fine-tuning of the cancellations is a symmetry that predicts the existence of two bosons for every SM fermion, with couplings $\lambda_B = |\lambda_l|^2$. Such a symmetry exists, and the exact cancellation of all quadratic divergences to all orders is unavoidable once unbroken supersymmetry is assumed. We shall see in Chapter 3 that supersymmetry must be broken to some degree, i.e. the supersymmetric particles cannot have the same masses as their SM partners. This will lead to further corrections, but of a logarithmic nature. To avoid reintroducing the fine-tuning problem of the SM the supersymmetric masses should at most be an order of magnitude above the electroweak scale, and should thus be within reach of the LHC.

2.6.2 Unification of Forces

The Standard Model has 19 free parameters that need to be determined experimentally. For a fundamental theory of nature this does seem a lot. It is tempting to search for a more unified theory which can explain some — if not all — of these parameters. The term Grand Unified Theory (GUT) describes such a theory that unifies the gauge groups of the SM: the weak, hypercharge and colour gauges. The idea first suggested by Howard Georgi and Sheldon Glashow [59] in 1974, is that since the coupling constants of the three gauge groups change with energy scale due to the effects of renormalisation, they could unify at some high energy scale — the GUT scale — to the coupling constant of a larger group G containing the SM gauge groups[11]

$$SU(3)_c \times SU(2)_L \times U(1)_Y \subset G. \tag{2.53}$$

One can calculate the behaviour of the coupling constants as a function of energy in a given model and its particle content using Renormalisation Group Equations (RGEs). The result for the SM is shown in Fig. 2.3a and for the Minimal Supersymmetric Standard Model (MSSM), which we will discuss further in Section 3.4, the result is shown in

[11] Originally Georgi and Glashow suggested $SU(5)$ as the unification group, but this predicts proton decay at rates that are incompatible with experimental limits.

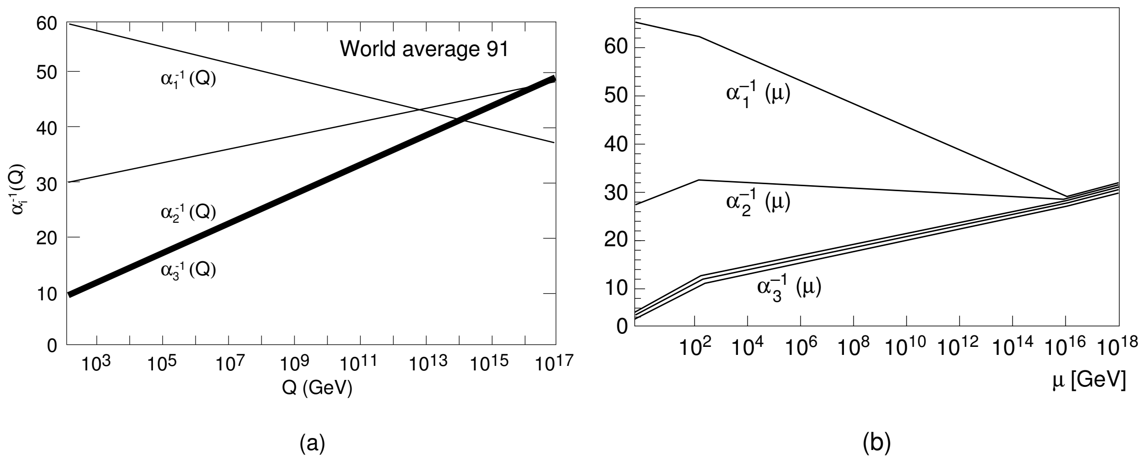

Figure 2.3: Running of $\alpha_i^{-1} = 4\pi/g_i^2$ with energy scale Q/μ, where g_i are the coupling constants of the SM gauge groups. In (a) we display the result for the SM model, in (b) for the MSSM. The error bands show the propagation of the experimental uncertainty from measurements of the coupling constants at low energy. The figures are taken from [60].

Fig. 2.3b. While the SM shows little sign of unification, the MSSM is very promising if one assumes that the supersymmetric particles enter into the running slightly above the electroweak scale (seen as kinks in Fig. 2.3b), up to ~ 1 TeV. Again these are energy scales that will be fully explored by the LHC.

2.6.3 Dark Matter

Dark matter (DM) was first observed as unexplained mass in the Coma galaxy cluster in 1933 by Fritz Zwicky [61], estimating the mass of the cluster by the movement of galaxies at its edges. The majority of a galaxy's mass that can be described within the SM comes from baryons, mainly protons and neutrons in atomic nuclei, that make up the stars and the large amounts of hot gas present in most galaxies. Since then many other clusters and single galaxies have been observed to contain more mass than can be explained by baryonic matter.

While direct observations of the gravitational dynamics of galaxies is difficult and fraught with possible sources of errors, there is also very strong indirect evidence for cold dark matter[12] in combined fits to Cosmic Microwave Background (CMB) data from the WMAP satellite and other astrophysical data on e.g. supernovae [8–10]. In particular from fits to the ΛCDM model of cosmology, using the WMAP third year data, [10] arrives at a cold dark matter density Ω_{CDM}:

$$\Omega_{\mathrm{CDM}} h^2 = 0.111 \pm 0.006, \tag{2.54}$$

where h is the Hubble constant.[13]

[12] By "cold" we mean that the DM is composed of non-relativistic particles.

[13] The model also predicts an acceleration of the expansion of the universe, represented by a non-zero cosmological constant Λ, or more generally so-called dark energy. This has very interesting cosmological

To understand what could constitute this mass, which has about five times the abundance of the baryonic matter we know today and describe in the SM, we can look at the properties of DM. First of all it must be electrically neutral as we have very strict limits on charged dark matter from cosmic ray searches. It must be stable on the scale of the age of the universe, and it must also only interact weakly with matter. If the DM had electromagnetic interactions it would not be dark, and strong interactions are very disfavoured due to direct searches with massive detectors. The only remaining SM candidates are massive neutrinos, unfortunately these will be very relativistic due to their low masses (see Table 2.1), which has consequences for structure formation in the early universe. This allows very stringent limits to be put on their contribution to the dark matter. The suggestion then is to suppose the existence of some new Weakly Interacting Massive Particle (WIMP).

One can calculate the current density of a DM candidate χ, originally in thermal equilibrium with the SM particles at the end of inflation, which subsequently freeze out to a constant density as the universe expands. Kolb and Turner [62] give the approximation

$$\Omega_\chi h^2 \simeq \frac{0.1\text{pb} \cdot \text{c}}{\langle \sigma v \rangle}, \tag{2.55}$$

where σ is the total annihilation cross section of a pair of DM particles, v is their relative velocity in the centre of mass reference frame, and the brackets denote an average over the velocity distribution at the freeze-out temperature. The remarkable feature of this estimate is that a $m_{\text{DM}} \sim 100$ GeV weakly interacting particle will have an averaged cross section of the order

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{8\pi m_{\text{DM}}^2} \sim 0.1\text{pb} \cdot \text{c}. \tag{2.56}$$

Thus a WIMP with a mass sitting right at the edge of the energies explored by previous and current high energy collider experiments such as LEP and the Tevatron, and well within the reach of the LHC seems to be a very likely DM candidate. We shall see in the next Chapter that supersymmetry has such a WIMP candidate.

2.6.4 Genesis

There is wide agreement on a hot and dense Big Bang model for the early cosmology of the universe. However, such a model should naively predict the existence of equal amounts of matter and anti-matter. Yet the observable universe seems to prefer baryons over anti-baryons to an astounding degree. To explain this difference Andrei Sakharov proposed a set of three necessary conditions in 1967 [63] for a model of *baryogenesis*:

- There must exist processes that violate baryon number conservation.

- There must be a sufficient violation of CP-symmetry.

consequences, but will not be dealt with further in this thesis. The fact that the cosmological constant is very small compared to the scales of the parameters one is tempted to explain it with, is another example of a hierarchy problem.

- The baryon-asymmetry generating processes must be out of thermal equilibrium, i.e. the rate of the reaction must be less than the rate of the expansion of the universe.

The first of these conditions is the rather obvious requirement that there are processes which actually create more baryons than anti-baryons. The second requirement is needed because on the basis of CP-symmetry one can show that for any process that creates an excess of baryons there will be a complementary process that creates more anti-baryons. The final criterion is necessary since a particle in thermal equilibrium would be destroyed and recreated through pair annihilation, washing out the asymmetry.

The baryon number and CP-violation found in the SM is too small to account for the present-day dominance of baryons, which extensions like supersymmetry may be able to explain. One of the most studied models is *electroweak baryogenesis* where Sakharov's third criterion can be satisfied during the electroweak phase transition, which occurs at the energy scale where the electroweak symmetry is broken. The requirement is that the electroweak phase transition is a strongly first order phase transition. Detailed calculations show that for this to occur in the MSSM, both the Higgs and the supersymmetric partner of the top quark, the scalar top, must be light [64–71]. The scalar top should have a mass well within the reach of the LHC. We will return to this point in Chapter 4.

Further models for baryogenesis have also been discussed. In *GUT baryogenesis* the asymmetry can come about from the decay of some super-heavy particle at GUT scale. Other models feature a *leptogenesis* mechanism that creates a lepton/anti-lepton asymmetry, which in turn is converted to a baryon asymmetry through so-called sphaleron processes. However, also in this case the SM is unable to quantitatively explain the observed asymmetry.

Chapter 3

Supersymmetry

Tiger, Tiger, Burning Bright
In the Forest of the Night
What Immortal Hand or Eye
Could Frame Thy Fearful
Symmetry

William Blake
Tiger

In the previous chapter we saw that the development of the Standard Model of particle physics over the last century has focused on symmetries, and in particular on the symmetry groups $SU(2)_L \times U(1)_Y$ and $SU(3)_C$, that are the basis of the gauge field theory descriptions of the electroweak and strong forces respectively. In addition to these internal symmetries of quantum charges — weak, hypercharge, colour — the external space-time symmetries of the Poincaré group, introduced by Einstein in the special theory of relativity [41], i.e. those of rotations and translation of four-dimensional space-time, have been central to the development of particle physics.

It is then natural to pose the question whether the internal and external symmetries could be united in a larger symmetry group, giving a more fundamental description of nature — perhaps even reducing the number of free parameters in the SM. Unfortunately, in 1967 Coleman and Mandula showed the so-called "no-go" theorem [72], that under reasonable assumptions[1] about the structure of the larger theory it can at best have a symmetry group that is the direct product of the Poincaré group and an internal symmetry group. As a result of the direct product structure the Poincaré group and the internal symmetry group act on particle states independent of each other and foils the attempt at any real unification of internal and external symmetries.

Not to be defeated by a mathematical proof physicists found a way around the "no-go" theorem by redefining the problem. In 1975 Haag, Lopuszanski and Sohnius [73] showed that if the conditions on the generators of the symmetry groups are relaxed we can indeed have a non-trivial unification of internal and external symmetries. What

[1] The underlying assumptions are those of locality, causality, positive energy and a finite number of particles, and that the theory is a quantum field theory. In addition there are also some assumptions on the generators of the symmetry.

was done was to extend the notion of Lie algebras that define the basic structure of the symmetry groups, to graded Lie algebras, also called Lie superalgebras.[2] In this way the algebra of the Poincaré group can be extended to the Poincaré superalgebra, the basis of supersymmetry. This was the algebraic birth of supersymmetry.

In this chapter we will first look at the algebraic structure of supersymmetry in Sections 3.1 and 3.2, giving the full supersymmetry algebra and the properties of its representations. We will then turn to constructing a supersymmetric Lagrangian in Sections 3.3 and 3.4 with the help of superfield formalism, finally arriving at the Lagrangian of the Minimal Supersymmetric Standard Model (MSSM), the basis of most current phenomenological investigations of supersymmetry.

3.1 The Poincaré Superalgebra

The Poincaré group is the basis of any relativistic field theory. It is the symmetry group for all transformations of Minkowski space-time coordinates that leave the distance between two points in Minkowski space invariant. Thus it describes all the symmetries of special relativity: translations, rotations and boosts. We discuss the generators of the Poincaré group in Appendix B, and give the Poincaré algebra consisting of these generators in Eqs. (B.6)–(B.8).

The Poincaré algebra is a Lie algebra, and we make the observation in Appendix B that a Lie superalgebra is the direct sum of a Lie algebra and a vector space. This opens up the possibility of extending the Poincaré algebra into a Lie superalgebra. We do this by construction, letting the Poincaré algebra play the role of the original Lie algebra, and taking its direct sum with a vector space L_1 spanned by a set of generators Q_a, $a \in \{1, 2, 3, 4\}$. We take these generators to be Majorana spinors with the following algebraic structure:

$$\{Q_a, \bar{Q}_b\} = 2\gamma^\mu_{ab} P_\mu, \tag{3.1}$$

$$[Q_a, P_\mu] = 0, \tag{3.2}$$

$$[Q_a, M^{\mu\nu}] = \sigma^{\mu\nu}_{ab} Q_b. \tag{3.3}$$

The central point here is that the definition of composition between two elements of the algebra is done by anti-commutators in (3.1). This allows the resulting algebra to satisfy the definition of a superalgebra, given in Section B.2, which requires one of the two vector spaces in the direct sum to have a symmetric composition rule and the other to have one that is anti-symmetric. To check that both properties 1. and 2. of Section B.2 hold is straight forward. We have thus constructed a Poincaré superalgebra, the algebraic basis of supersymmetry.

This is, however, not the most general supersymmetry extension we can construct from the Poincaré algebra. The content of a theorem by Haag, Lopuszanski and Sohnius in [73] is that the most general symmetry group one can have based on superalgebra extensions of the Poincaré algebra has N sets of spinor generators Q^α_a, $\alpha \in \{1, \ldots, N\}$, that themselves transform under an internal symmetry group with generators B_l, according to

$$\{Q^\alpha_a, Q^\beta_b\} = \{\bar{Q}^\alpha_a, \bar{Q}^\beta_b\} = 0 \tag{3.4}$$

[2]For a more technical discussion of Lie algebras and superalgebras see Appendix B.

$$\begin{aligned}
\{Q_a^\alpha, \bar{Q}_b^\beta\} &= 2\delta^{\alpha\beta}\gamma_{ab}^\mu P_\mu && (3.5)\\
[Q_a^\alpha, P_\mu] &= [\bar{Q}_a^\alpha, P_\mu] = 0 && (3.6)\\
[Q_a^\alpha, M^{\mu\nu}] &= \sigma_{ab}^{\mu\nu} Q_b^\alpha && (3.7)\\
[Q_a^\alpha, B_l] &= iS_l^{\alpha\beta} Q_a^\beta && (3.8)\\
[B_k, B_l] &= ic_{klm} B_m, && (3.9)
\end{aligned}$$

where c_{lm}^k are the structure constants of the internal symmetry group and $S_l^{\alpha\beta}$ are the matrices of its representation. One can also have a further generalisation by introducing central charges $Z^{\alpha\beta}$, objects that commute with everything in a Lie algebra, so that

$$\begin{aligned}
\{Q_a^\alpha, Q_b^\beta\} &= \epsilon_{ab} Z^{\alpha\beta}, && (3.10)\\
Z^{\alpha\beta} &= -Z^{\beta\alpha}, && (3.11)\\
\left[Z^{\alpha\beta}, B_l\right] &= 0. && (3.12)
\end{aligned}$$

What we constructed in (3.1)–(3.3) from the Poincaré algebra is the $N = 1$ supersymmetry algebra or the minimal supersymmetric algebra, where the central charges disappear along with the internal symmetry group. This is the algebra that we will use to construct a supersymmetric Lagrangian, as it seems to be the one that might have physical relevance, at least at low energies.[3] One may perhaps see some irony in that the dream of unifying internal and external symmetries ends up with extending and complicating the symmetry group of the external symmetries, but leaving internal symmetries out.

3.2 Representations of the Supersymmetry Algebra

Having constructed the $N = 1$ supersymmetry algebra (3.1)–(3.3), which fixes the symmetry group of our supersymmetric theory, we turn to looking at the properties of particle states in this theory by looking at the possible representations of the algebra.

The irreducible representations of an algebra are characterised by the eigenvalues of its Casimir operators, i.e. operators that commute with every element in the algebra. In what follows we shall restrict the discussion to massive particles in their rest frame.[4] For the case of the $N = 1$ supersymmetry algebra these can be shown to be the operators P^2 and $J^2 = J_k J^k$, where J_k is given by

$$mJ_k = mS_k + \frac{1}{8}\bar{Q}\gamma_k\gamma_5 Q, \quad (3.13)$$

in terms of the spin operator S_k (see Section B.3 of the Appendix) and the mass of the particle m, and where we have suppressed the spinor indices of the Q.

[3] Typically higher N supersymmetry algebras are not phenomenologically viable because they predict a multitude of particles, of which many should have been observed experimentally.

[4] Massless states can be treated in a similar manner, see e.g. page 16 of [74]. However, restricting the discussion to the rest frame is not completely trivial. We will only find the representations of the sub-group that leaves momentum unchanged and have to rely on the method of induced representation, i.e. that the properties of a particle are entirely determined by its properties in a given frame. See e.g. page 31 of [75].

In the rest frame the action of P^μ on a state is given by $P^2 = m^2$. The J_k can be shown to fulfil the $SU(2)$ algebra, given in (A.19) for reference. This implies that J_k is an angular momentum operator and that there exists a representation of the algebra where J^2 has eigenvalues of the form $j(j+1)$, where the j are half-integral. Thus we can specify the irreducible representations of the $N = 1$ supersymmetry algebra by the values of the mass m and the spin eigenvalues $j(j+1)$. For each representation there are at least $2j+1$ different states, corresponding to the eigenvalues j_3 of the J_3 operator, which as an angular momentum operator can assume the values $j_3 \in \{-j, -j+1, \ldots, j\}$. However, there are more states. Using (3.1) and (3.13), and letting the four supersymmetry generators Q_a act on a state $|m, j, j_3\rangle$ with definite eigenvalues of m, j and j_3, can be shown to yield four different states with eigenvalues $s_3 = j_3$ (two states), and $s_3 = j_3 \pm \frac{1}{2}$ of the spin operator S_3. There are only four different states because of the Majorana nature of the supersymmetry generators. Thus, for a given irreducible representation of the algebra, characterised by values of m and j, we have $4(2j+1)$ states. Independent of whether j_3 is of integral or half-integral value, this gives us two states of half-integral spin, fermions, and two of integral spin, bosons. The representations of a larger N supersymmetry algebra will of course have more states, because they have more supersymmetry generators Q.

Two important properties of the representations should be noticed. Firstly that all the states in a given irreducible representation of supersymmetry have the same mass. This is blatantly not the case in nature. There are no spin-0 partners of our fermions with the same masses. Thus if supersymmetry is realised there must also be a mechanism for breaking the symmetry, so that a sensible spectrum of masses results. This will be dealt with in Section 3.3.6. The second property is the equal number of bosonic and fermionic states in a supersymmetric theory. We have already seen that this is the case for the $N = 1$ supersymmetry algebra, but this can be shown to hold for any N.

3.3 Towards a Supersymmetry Lagrangian

We would now like to formulate a supersymmetry Lagrangian suitable for phenomenological investigations. It turns out to be very useful to introduce so-called *superfields* to aid in the construction, a formalism first described by Salam and Strathdee [76].

3.3.1 Superfields

Before we can describe superfields we must comment on the generalised coordinates on which they are defined, the coordinates of *superspace*. Superspace is formally an 8-dimensional manifold parametrised by the coordinates x^μ of ordinary Minkowski-space and four anti-commuting Grassmann numbers θ. In Appendix C we give a short superspace calculus, with the most central definitions and properties used in this thesis, and we discuss the notation for the Grassmann numbers in terms of Weyl spinors θ^A and $\bar{\theta}^{\dot{A}}$ used here.

A general superfield is an operator-valued function Φ on superspace, $\Phi(x, \theta, \bar{\theta})$. We

Component field	Type
$f(x)$, $n(x)$, $m(x)$	Complex scalar or pseudo-scalar fields
$\psi_A(x)$, $\phi_A(x)$	Left-handed Weyl spinor fields
$\bar{\chi}(x)^{\dot{A}}$, $\bar{\lambda}(x)^{\dot{A}}$	Right-handed Weyl spinor field
$V_\mu(x)$	Lorentz 4-component field
$d(x)$	Scalar field

Table 3.1: Properties of superfield component fields.

can expand a superfield as a power series in terms of the Grassmann variables[5]

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= f(x) + \theta^A\phi_A(x) + \bar{\theta}_{\dot{A}}\bar{\chi}^{\dot{A}}(x) + (\theta\theta)m(x) + (\bar{\theta}\bar{\theta})n(x) \\ &\quad +(\theta\sigma^\mu\bar{\theta})V_\mu(x) + (\theta\theta)\bar{\theta}_{\dot{A}}\bar{\lambda}^{\dot{A}}(x) + (\bar{\theta}\bar{\theta})\theta^A\psi_A(x) + (\theta\theta)(\bar{\theta}\bar{\theta})d(x), \quad (3.14)\end{aligned}$$

where the $f(x)$, $\phi_A(x)$, *et cetera* are component fields. This expansion is complete. We have here all possible terms since higher powers than two of θ and $\bar{\theta}$ will vanish due to the anti-commutation of Grassmann numbers, and the σ^μ constitute a basis for the space of 2×2-matrices so that $(\theta\sigma^\mu\bar{\theta})V_\mu(x)$ completes the possible combinations of θ and $\bar{\theta}$. Note that all superfields commute since the anti-commuting Weyl spinors operate in pairs.

The properties of the component fields can be deduced from requiring that $\Phi(x,\theta,\bar{\theta})$ is a Lorentz scalar or pseudo-scalar field. The results are listed in Table 3.1. We can see that superfields are a concise way of treating a multiplet of different fields, all at once. It is useful to classify the fields in terms of the following properties, using the covariant derivatives D_A and $\bar{D}_{\dot{A}}$ defined in (C.19) and (C.20),

$$\bar{D}_{\dot{A}}\Phi(x,\theta,\bar{\theta}) = 0 \quad (3.15)$$
$$D_A\Phi^\dagger(x,\theta,\bar{\theta}) = 0 \quad (3.16)$$
$$\Phi(x,\theta,\bar{\theta}) = \Phi^\dagger(x,\theta,\bar{\theta}). \quad (3.17)$$

Superfields that satisfy the first two restrictions are respectively called left- and right-handed chiral or scalar superfields, while the third equation describes a vector superfield.

The above constraints (3.15)–(3.17) can be shown to all result in superfields that form linear representations of the supersymmetry algebra.[6] The different constraints are then ways of categorising the representations, an alternative to using the Casimir invariants demonstrated in Section 3.2. We show how to build supersymmetric Lagrangians out of these superfields, but first we will briefly discuss the properties of the two types: the scalar and vector superfields.[7]

[5]For simplicity of notation we will often suppress Lorentz and Weyl spinor indices, in particular when contracting, when we are reasonably sure no confusion can arise.

[6]See e.g. Section 6 of [77].

[7]This may seem somewhat *ad hoc*, since the above restrictions (3.15)–(3.17) appear to lack a clear connection to particle representation. However the component fields that will result from equations (3.15)–(3.17) have been directly derived from irreducible particle representations of the supersymmetry algebra. See page 397 of [77] and its reference to [78].

3.3.2 Scalar Superfields

To construct phenomenologically sensible Lagrangians we need to know the field content of the scalar and vector superfields. By a change of variables

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}, \tag{3.18}$$

we can write the covariant derivatives as

$$D_A = \frac{\partial}{\partial\theta^A}, \tag{3.19}$$

$$\bar{D}_{\dot{A}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{A}}} - 2i(\theta\sigma^\mu)_{\dot{A}}\frac{\partial}{\partial y^\mu}. \tag{3.20}$$

We see from (3.16) and (3.19) that in the new coordinates the right-handed chiral superfield is independent of θ. Thus in a power series expansion we can write the most general right-handed chiral superfield as

$$\Phi^\dagger(y,\bar{\theta}) = A^*(y) + \sqrt{2}\bar{\theta}\bar{\psi}(y) + (\bar{\theta}\bar{\theta})F^*(y), \tag{3.21}$$

where we have three independent component fields, the two complex scalar fields A^* and F^*, and the right-handed[8] chiral Weyl spinor field $\bar{\psi}$. As expected we have an equal number of bosonic and fermionic degrees of freedom in the scalar fields and the Weyl spinor, respectively. Having identified the field content of the right-handed superfields, we can change back to ordinary Minkowski space coordinates:

$$\begin{aligned}\Phi^\dagger(x,\theta,\bar{\theta}) &= A^*(x) - i(\theta\sigma^\mu\bar{\theta})\partial_\mu A^*(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box A^*(x) \\ &\quad + \sqrt{2}\bar{\theta}\bar{\psi}(x) + \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\theta\sigma^\mu\partial_\mu\bar{\psi}(x) + (\bar{\theta}\bar{\theta})F^*(x).\end{aligned} \tag{3.22}$$

The left-handed chiral superfield can by a similar change of coordinates be found to depend solely on θ:

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)F(y), \tag{3.23}$$

or in x^μ coordinates

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= A(x) + i(\theta\sigma^\mu\bar{\theta})\partial_\mu A(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box A(x) \\ &\quad + \sqrt{2}\theta\psi(x) + \frac{i}{\sqrt{2}}(\theta\theta)\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + (\theta\theta)F(x).\end{aligned} \tag{3.24}$$

As the notation suggests the right- and left-handed superfields are Hermitian conjugates. We can show that the product of two or more scalar superfields of the same "handedness" is again a scalar superfield. Since the covariant derivative is a linear differential operator, we have for two left-handed superfields Φ_i and Φ_j:

$$D_A(\Phi_i\Phi_j) = D_A(\Phi_i)\Phi_j + \Phi_i D_A(\Phi_j) = 0, \tag{3.25}$$

and thus the product of left-handed superfields satisfy the restriction (3.15) of a left-handed superfield.

[8]Hence the name *right-handed chiral* superfield.

Component field	Type
$f(x)$, $d(x)$	Real scalar fields
$\phi(x)$, $\lambda(x)$	Weyl spinor fields
$V_\mu(x)$	Real vector field
$m(x)$	Complex scalar field

Table 3.2: Properties of the vector superfield's component fields.

3.3.3 Vector Superfields

By taking the Hermitian conjugate on the component field expansion of the generic superfield (3.14) we see that the condition (3.17) for a vector superfield requires the following relations for the component fields:

$$\begin{aligned} f(x) &= f^*(x), & (3.26)\\ \phi_A(x) &= \chi_A(x), & (3.27)\\ m(x) &= n^*(x), & (3.28)\\ V_\mu(x) &= V_\mu^*(x), & (3.29)\\ \lambda_A(x) &= \psi_A(x), & (3.30)\\ d(x) &= d^*(x). & (3.31) \end{aligned}$$

Thus a generic vector superfield, $V(x,\theta,\bar\theta)$, has a component field expansion of

$$\begin{aligned} V(x,\theta,\bar\theta) &= f(x) + \theta\phi(x) + \bar\theta\bar\phi(x) + (\theta\theta)m(x) + (\bar\theta\bar\theta)m^*(x) \\ &\quad +(\theta\sigma^\mu\bar\theta)V_\mu(x) + (\theta\theta)\bar\theta\bar\lambda(x) + (\bar\theta\bar\theta)\theta\lambda(x) + (\theta\theta)(\bar\theta\bar\theta)d(x), \qquad (3.32) \end{aligned}$$

where the component fields are of the types listed in Table 3.2.

Again the degrees of freedom of the bosonic fields $f(x)$, $d(x)$, $V_\mu(x)$ and $m(x)$, equal those of the fermionic fields $\phi_A(x)$ and $\lambda_A(x)$. We also note that from the definition (3.17) and the fact that superfields commute it is easy to see that the product of two vector superfields is also a vector superfield. The same is true for the sum of two vector fields.

For a vector superfield we define the supersymmetric field strength in analogy with SM field strengths. To be general, we include the self couplings of a possibly non-abelian gauge group with generators t^a, under which a vector multiplet V^a transforms. We write the field strength as a Weyl spinor,

$$\begin{aligned} W_A &\equiv -\frac{1}{4}\bar D\bar D e^{-qt^aV^a} D_A e^{qt^aV^a}, & (3.33)\\ \bar W_{\dot A} &\equiv -\frac{1}{4}DDe^{-qt^aV^a}\bar D_{\dot A} e^{qt^aV^a}, & (3.34) \end{aligned}$$

It is easy to check that each component of these spinors is itself a scalar superfield. From (C.23) we get

$$\begin{aligned} \bar D_{\dot A} W_A &= 0, & (3.35)\\ D_A \bar W_{\dot A} &= 0, & (3.36) \end{aligned}$$

so that W_A and $\bar{W}_{\dot{A}}$ are left- and right-handed chiral superfields respectively. An explicit component expansion of the W_A and $\bar{W}_{\dot{A}}$ will turn up a SM-like field strength tensor

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + qf^{abc}V^b_\mu V^c_\nu, \tag{3.37}$$

as one component field, where f^{abc} are the structure constants of the gauge group.

3.3.4 Lagrangian Densities in Terms of Superfields

To construct a Lagrangian density $\mathcal{L}$ for a supersymmetry model one needs to be able to write down terms that result in an action invariant under supersymmetry transformations, among others. The ordinary action S is an integral over some space-time region R where the fields that make up $\mathcal{L}$ are defined,

$$S = \int_R d^4x\, \mathcal{L}(x). \tag{3.38}$$

This action is invariant if the Lagrangian transforms as a total derivative, i.e. that under a transformation the new Lagrangian $\mathcal{L}'(x)$ is given by

$$\mathcal{L}'(x) = \mathcal{L}(x) + \partial^\mu f_\mu(x) \tag{3.39}$$

for some well behaved[9] function $f_\mu(x)$, necessarily also a function of the fields of the model.

Instead of writing down a Lagrangian consisting of explicit component fields we wish to use superfields. The question is then what combinations of superfields, scalar and vector, result in Lagrangian densities that transform into themselves plus a total derivative under supersymmetry transformations. It can be shown that only the highest order component field of a superfield, in a θ and $\bar{\theta}$ expansion, behaves in this manner.[10] Thus for all superfield terms we want to have in the Lagrangian density we must project out the highest-order component field.

The machinery for doing this can be found in the short superspace calculus of Appendix C, and consists of integrals over superspace. From (C.13) and (C.16)–(C.18) we see that we can write the action as

$$S = \int_R d^4x \int d^4\theta\ \left[\mathcal{L}_{\theta\theta\bar{\theta}\bar{\theta}} + (\bar{\theta}\bar{\theta})\mathcal{L}_{\theta\theta} + (\theta\theta)\mathcal{L}_{\bar{\theta}\bar{\theta}}\right], \tag{3.40}$$

where the three $\mathcal{L}$ terms can be any functions of superfields where each term is itself a superfield, and where the indices indicate the highest power of θ in the component field expansion.

Renormalisation puts further restrictions on what terms $\mathcal{L}$ can contain. It has been shown, see for instance [79], that at most third powers of scalar fields Φ can be used. Thus the most general Lagrangian density $\mathcal{L}_S$ we can construct with scalar superfields [11] is

$$\mathcal{L}_S = \Phi_i^\dagger\Phi_i + (\bar{\theta}\bar{\theta})W(\Phi) + (\theta\theta)W(\Phi^\dagger), \tag{3.41}$$

[9] It is differentiable and it vanishes at the surface of R.

[10] For computational details see e.g. Sections 6.8, 7.1 and 7.2 of [77].

[11] Note that the so-called kinetic term $\Phi_i^\dagger\Phi_i$ is a vector field created by the combination of two scalar fields.

with $i \in \{1, \ldots, n\}$, where n is the number of scalar superfields in the model, and where the function

$$W(\Phi) = g_i \Phi_i + m_{ij} \Phi_i \Phi_j + \lambda_{ijk} \Phi_i \Phi_j \Phi_k, \tag{3.42}$$

is the *superpotential*. The superpotential is in turn a scalar superfield since it is a sum and product of scalars. Since superfields commute, m_{ij} and λ_{ijk} are symmetric under an exchange of indices.

3.3.5 Supergauge Transformations

If we are to construct a gauge theory, similar to that of the SM, out of the superfields in the Lagrangian (3.41) we need to define a supersymmetric generalisation of gauge transformations, *supergauge transformations*, for both scalar and vector superfields in such a way that the Lagrangian is invariant, or can be made invariant, under the transformations.

We start by defining the local gauge transformation of a multiplet $\Phi(x)$ of scalar (left-handed) superfields — or scalar supermultiplet — transforming under a gauge group G that may in general be non-abelian, as

$$\Phi(x) \to e^{-iqt^a \Lambda^a(x)} \Phi(x), \tag{3.43}$$

or in terms of the component superfields of Φ,

$$\Phi_i(x) \to (e^{-iqt^a \Lambda^a(x)})_{ij} \Phi_j(x) = U_{ij}(x) \Phi_j(x). \tag{3.44}$$

The matrices t^a are the generators of the representation R of G that satisfy the algebra

$$[t^a, t^b] = i f^{abc} t^c, \tag{3.45}$$

where f^{abc} are the structure constants of G. Furthermore, q is the charge of the multiplet $\Phi(x)$ under the gauge transformation, and is proportional to the strength of the gauge coupling to that supermultiplet. If the Φ_i are to remain scalar superfields after the gauge transformation we must require

$$D_A \Lambda^a(x) = 0, \tag{3.46}$$

thus the Λ^a must also be scalar (left-handed) superfields.

For the superpotential $W(\Phi)$ to be invariant under (3.43) it must obey certain restrictions:

$$g_i = 0 \quad \text{if} \quad g_i U_{ir} \neq g_r, \tag{3.47}$$

$$m_{ij} = 0 \quad \text{if} \quad m_{ij} U_{ir} U_{js} \neq m_{rs}, \tag{3.48}$$

$$\lambda_{ijk} = 0 \quad \text{if} \quad \lambda_{ijk} U_{ir} U_{js} U_{kt} \neq \lambda_{rst}. \tag{3.49}$$

Kinetic terms of the form $\Phi^\dagger \Phi$ can be made invariant by, as in the SM gauge theories, introducing a compensating vector field. We rewrite the kinetic term as an interaction term between the scalar supermultiplet and a multiplet of vector superfields V,

$$\mathcal{L}_I = \Phi^\dagger e^{qt^a V^a} \Phi. \tag{3.50}$$

This term is invariant if the result V' of the supergauge transformation of the vector supermultiplet satisfies

$$e^{qt^a(V^a)'} = e^{-iqt^a\Lambda^{a\dagger}} e^{qt^aV^a} e^{iqt^a\Lambda^a}. \tag{3.51}$$

One can show that for infinitesimal transformation parameters Λ^a this has the solution

$$\begin{aligned}(V^a)' &= V^a + i(\Lambda^a - \Lambda^{a\dagger}) - \frac{1}{2}qf^{abc}V^b(\Lambda^{c\dagger} + \Lambda^c) \\ &\quad -\frac{i}{12}q^2 f^{abc}f^{cde}V^bV^d(\Lambda^e - \Lambda^{e\dagger}) + \ldots \end{aligned}\tag{3.52}$$

It is easy to check that this new field is a vector superfield as it should be.

In the special case of an abelian gauge transformation, i.e. zero structure constants, the transformation of the vector superfield is particularly simple and we can write

$$V' = V + (\Phi + \Phi^\dagger), \tag{3.53}$$

with a slight redefinition of the transformation parameter $\Phi = i\Lambda$. Inserting the generic expressions for the scalar superfields, (3.22) and (3.24), into (3.32), and with the help of a change of variables

$$\lambda(x) \rightarrow \lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\phi}(x), \tag{3.54}$$

$$d(x) \rightarrow d(x) + \frac{1}{4}\Box f(x), \tag{3.55}$$

one sees that the component fields transform as

$$f(x) \rightarrow f(x) + A(x) + A^*(x), \tag{3.56}$$

$$\phi(x) \rightarrow \phi(x) + \sqrt{2}\psi(x), \tag{3.57}$$

$$m(x) \rightarrow m(x) + F(x), \tag{3.58}$$

$$V_\mu(x) \rightarrow V_\mu(x) + i\partial_\mu(A(x) - A^*(x)), \tag{3.59}$$

$$\lambda(x) \rightarrow \lambda(x), \tag{3.60}$$

$$d(x) \rightarrow d(x). \tag{3.61}$$

The main point to notice is that the component vector field $V_\mu(x)$ transforms as one expects from a gauge vector field in the SM under an abelian gauge transformation, see (2.4) and (2.20). We also see that the choice of variables for the generic vector field $V(x,\theta,\bar{\theta})$ in (3.54) and (3.55) implies that the components $\lambda(x)$ and $d(x)$ are invariant under supergauge transformations.

Our new-found gauge freedom also allows us to pick a particular gauge, the *Wess-Zumino gauge*, to simplify some calculations. In this gauge the components of the scalar field Φ in (3.53) are chosen to be

$$2\mathrm{Re}\,A(x) = -f(x), \tag{3.62}$$

$$F(x) = -m(x), \tag{3.63}$$

$$\sqrt{2}\psi(x) = -\phi(x), \tag{3.64}$$

so that the generic vector superfield can be written

$$\begin{aligned} V_{WZ}(x,\theta,\bar{\theta}) &= (\theta\sigma^\mu\bar{\theta})[V_\mu(x) + i\partial_\mu(A(x) - A^*(x))] \\ &\quad +(\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\lambda(x) + (\theta\theta)(\bar{\theta}\bar{\theta})d(x). \end{aligned} \tag{3.65}$$

Notice that we have only fixed the real part of $A(x)$, so we still have the gauge freedom of SM abelian gauge theories. Unfortunately, the Wess-Zumino gauge is non-supersymmetric in the sense that following a generic supersymmetry transformation the vector field will no longer be in this gauge. The component fields $\phi(x)$ and $m(x)$ have transformations under the supersymmetry operators that are non-zero, even if the fields themselves are zero.

One of the main benefits of the Wess-Zumino gauge is the ease with which we can compute powers of V_{WZ}. Since $\theta^n = 0$ for $n > 2$, we have that

$$V_{WZ}^2(x,\theta,\bar{\theta}) = \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})[V(x) + i\partial(A(x) - A^*(x))]^2, \tag{3.66}$$

$$V_{WZ}^3(x,\theta,\bar{\theta}) = 0, \tag{3.67}$$

thus

$$\exp(V_{WZ}) = 1 + V_{WZ} + \frac{1}{2}V_{WZ}^2. \tag{3.68}$$

Yet again in analogy with the SM we would like to add field strength terms for the vector fields to the Lagrangian (3.41). Unfortunately the field strength is not invariant under non-abelian gauge transformations. Using $V = qt^aV^a$ and $\Lambda = qt^a\Lambda^a$ to simplify notation, and inserting (3.51) into the definition of the field strength (3.33), we can show that it transforms as,[12]

$$\begin{aligned} W_A &\rightarrow -\frac{1}{4}(\bar{D}\bar{D})e^{-i\Lambda}e^{-V}e^{i\Lambda^\dagger}D_Ae^{-i\Lambda^\dagger}e^{V}e^{i\Lambda} \\ &= e^{-i\Lambda}W_Ae^{i\Lambda}. \end{aligned} \tag{3.70}$$

However, the trace of W^AW_A is invariant due to its cyclic property:

$$\begin{aligned} \mathrm{Tr}\,(W^AW_A) &\rightarrow \mathrm{Tr}\,(e^{-i\Lambda}W^Ae^{i\Lambda}e^{-i\Lambda}W_Ae^{i\Lambda}) \\ &= \mathrm{Tr}\,(W^AW_Ae^{i\Lambda}e^{-i\Lambda}) \\ &= \mathrm{Tr}\,(W^AW_A). \end{aligned} \tag{3.71}$$

Thus we have

$$\mathcal{L}_V = \frac{1}{4T(R)}\mathrm{Tr}\,[W^AW_A(\bar{\theta}\bar{\theta}) + \bar{W}_{\dot{A}}\bar{W}^{\dot{A}}(\theta\theta)], \tag{3.72}$$

as the term for a free vector superfield in the Lagrangian. The extra numerical factor $T(R)^{-1}$ appears because of the matrix structure of (3.72), and it is included to take care of

[12] We also find use for repeated applications of (3.46) and from (C.22)

$$\{\bar{D}^{\dot{A}}, D_A\} = \varepsilon^{\dot{A}\dot{B}}\{D_A, \bar{D}_{\dot{B}}\} = -2\sigma^\mu_{A\dot{B}}\varepsilon^{\dot{A}\dot{B}}P_\mu. \tag{3.69}$$

the normalisation of the generators t^a of the representation R, so that the energy densities of the gauge fields are correct.[13]

As a side remark, one would perhaps assume that the term $\bar{W}_{\dot{A}}\bar{W}^{\dot{A}}$ is needed to make the Lagrangian real. However one can prove that the term containing $W^A W_A$ is real and thus that $\bar{W}_{\dot{A}}\bar{W}^{\dot{A}}$ is strictly speaking superfluous in $\mathcal{L}_V$, although this is sometimes overlooked in the literature.

3.3.6 Supersymmetry Breaking

What we have achieved at this point is to give a gauge invariant Lagrangian density for supersymmetry in terms of vector and scalar superfields, consisting of the terms in (3.42), (3.50) and (3.72). In doing so we have created an interaction between scalar field multiplets Φ_i and a gauge field multiplet V.

As was discussed in Section 3.2, supersymmetry predicts equal masses for particles in the same representation, and it predicts that all fermions have bosonic partners in their representation. One should then expect to find bosonic partners to the known fermions with equal mass. This is certainly not the case in nature. Thus we need some sort of mechanism that breaks supersymmetry, to give the needed mass differences. Furthermore, one would wish that this mechanism was not entirely ad hoc, and that it in some way explained why these partners have not yet been found in experiments. On this basis we now go on to discuss how supersymmetry breaking terms can be accommodated in the Lagrangian formulation.

One of the earliest suggested approaches is the so-called *spontaneous breaking* of supersymmetry. The mechanism is a parallel to the spontaneous symmetry breaking of the electroweak symmetry in the SM, as discussed in Section 2.5. It does not actually break supersymmetry in the Lagrangian, but it has a vacuum that is not supersymmetric, thus splitting the masses of the particles within a supermultiplet. This relies on a component field having a non-zero VEV, whilst being an auxiliary field that can be removed by the equations of motion that result from the Lagrangian. There are two ways of bringing this about. The O'Raifeartaigh mechanism [80], or F-term breaking, has $\langle 0|F(x)|0\rangle \neq 0$, where $F(x)$ is the highest order, in θ, component field of a scalar superfield, while the Fayet-Iliopoulos mechanism [81], or D-term breaking, uses $\langle 0|d(x)|0\rangle \neq 0$, with $d(x)$ being the highest-order component field of a vector superfield.

Spontaneous symmetry breaking seems an elegant way of solving the problem of degenerate masses. However it has been shown [82] that for a very general class of spontaneous supersymmetry breaking models,[14] the so-called *supertrace* $\mathrm{STr}\,(\mathcal{M}^2)$ of weighted squared-mass eigenvalues of the mass matrix $\mathcal{M}$, vanishes at tree-level. The supertrace is given by

$$\mathrm{STr}\,(\mathcal{M}^2) = \sum_s (-1)^{2s}(2s+1)\mathrm{Tr}\,(M_s^2), \tag{3.74}$$

[13]The generators are normalised by the relation

$$\mathrm{Tr}\,[t^a t^b] = T(R)\delta^{ab}. \tag{3.73}$$

The factor $T(R)$ is the *Dynkin index* of the representation R.

[14]The exception is models with so-called axial gauge invariance. However, these have problems with renormalisability.

where M_s is the mass matrix of particles with spin s. For the masses of a particular scalar supermultiplet of the Lagrangian we have the following relationship,

$$\mathrm{Tr}\,\left[M_0^2 - 2M_{1/2}^2\right] = 0. \tag{3.75}$$

The consequence is that at least one of the scalars must be lighter than the corresponding fermions at tree-level.[15] Other calculations have shown that this formula does not change substantially when computed to higher orders.[16] This indicates that the boson partners should not all be very far away in mass from the known fermions, and makes models of spontaneous supersymmetry breaking very constrained.

The alternative route is to simply parametrise our ignorance of the real mechanism and break supersymmetry explicitly by the addition of terms to the Lagrangian that are not invariant under supersymmetry transformations, and that can give rise to the needed mass differences. In the Minimal Supersymmetric Standard Model that we will describe in the next Section, supersymmetry is broken in this manner. Such terms are restricted to so-called *soft breaking terms*, meaning that they do not give rise to quadratic divergences, or worse, in quantum corrections to scalar masses. Thus this supersymmetry breaking will not reintroduce the quadratic divergences for the Higgs mass described in Section 2.6.1.

Girardello and Grisaru [84] list all possible soft supersymmetry breaking terms, and show rigorously that they are free from quadratic divergences. Written as superfields they are

$$\begin{aligned}\mathcal{L}_{\mathrm{soft}} &= -\frac{1}{6}a_{ijk}\theta\theta\bar{\theta}\bar{\theta}\Phi_i\Phi_j\Phi_k - \frac{1}{2}b_{ij}\theta\theta\bar{\theta}\bar{\theta}\Phi_i\Phi_j - t_i\theta\theta\bar{\theta}\bar{\theta}\Phi_i \\ &\quad -\frac{1}{4T(R)}M\theta\theta\bar{\theta}\bar{\theta}\mathrm{Tr}\,\left[W^AW_A\right] + \mathrm{h.c.} \\ &\quad -m_{ij}^2\theta\theta\bar{\theta}\bar{\theta}\Phi_i^\dagger\Phi_j.\end{aligned} \tag{3.76}$$

Here the a_{ijk}, b_{ij}, t_i, M and m_{ij}^2 are the parameters of the soft breaking, while Φ_i and W^A are scalar superfields in the model and field strengths for vector fields that belong to the gauge group of the theory. From the $\theta\theta\bar{\theta}\bar{\theta}$-factors we see that it is the lowest order component of the various fields that are projected out. For the scalar superfield Φ_i this is, in our notation, the component field A_i. For the trace of W^AW_A, an explicit calculation in component fields shows that this yields $\lambda^A\lambda_A$, where λ_A is the spin-$\frac{1}{2}$ component field of the vector superfield for which W_A is the field strength. In terms of component fields the soft breaking terms can then be written

$$\mathcal{L}_{\mathrm{soft}} = -\frac{1}{6}a_{ijk}A_iA_jA_k - \frac{1}{2}b_{ij}A_iA_j - t_iA_i - \frac{1}{2}M\lambda^A\lambda_A + \mathrm{c.c.} - m_{ij}^2A_i^*A_j. \tag{3.77}$$

The new parameters introduced into the theory by the addition of these extra terms are not arbitrary. They must be chosen so that we avoid large effects from supersymmetric particles in loops, for instance in giving flavor changing neutral currents or a CP-violating dipole moment for the neutron, that contradict current experimental bounds and SM predictions. Thus it is natural to believe that there must be some organising principle behind

[15] Note that to have an equal number of fermionic and bosonic degrees of freedom there must be two scalars in this sum for every fermion.

[16] As noted in [83] there is an exception here for very large couplings.

these parameters. Models of *supergravity*, a generalisation of (rigid) supersymmetry where the supersymmetry transformations are local, i.e. the parameters of the transformations depend on space-time coordinates, can provide such an organising principle and a connection to a theory of gravity. There are a multitude of other viable models on the market, such as Gauge Mediated Symmetry Breaking (GMSB) [85–87] and Anomaly Mediated Symmetry Breaking (AMSB) [88, 89], and different variations of these. We will not go into details on any of these mechanism, and be satisfied with a parametrization of our ignorance.

3.4 The Lagrangian of the MSSM

In Section 3.1 we commented on our restriction of the discussion to an $N = 1$ supersymmetry algebra. Here, we will further restrict ourselves to constructing a superfield Lagrangian with the minimal field content that still contains all the SM fields. This is the Minimal Supersymmetric Standard Model (MSSM). While non-minimal models may be found to solve some problems that the MSSM cannot, most of the physics inherent to phenomenologically realistic models of supersymmetry are to be found in the MSSM.

3.4.1 Field Content

As we saw in (3.24), a left-handed scalar superfield Φ has one left-handed Weyl spinor $\psi_A(x)$, and two complex scalar fields $A(x)$ and $F(x)$ as component fields. The field $F(x)$ can always be eliminated from the equations of motion resulting from the Lagrangian because it contains no derivatives of $F(x)$. This can be seen in a component field expansion of (3.41), using (3.22) and (3.24). Thus, a left-handed scalar superfield can represent a left-handed fermion with spin-$\frac{1}{2}$, by the spinor ψ_A, and a scalar particle, a boson of spin-0, by the field $A(x)$. Similarly the right-handed superfield $\Phi^\dagger$ contains a right-handed Weyl spinor $\bar{\psi}^{\dot{A}}$ and a scalar field.

We have seen that in the Wess-Zumino gauge (3.65) the vector superfields have three independent component fields, the real vector field $V_\mu(x)$, the Weyl spinor field $\lambda_A(x)$ and the real scalar field $d(x)$. This still leaves open one degree of gauge freedom in fixing the imaginary component of the field $A(x)$ from (3.24). Thus, we can put our vector superfields in the Wess-Zumino gauge and still have the freedom of SM gauge transformations. The component field $V_\mu(x)$ can then describe a spin-1 field with three degrees of freedom, with one degree of freedom set aside for the gauge freedom, and can thus represent a vector boson. The Weyl spinor spin-$\frac{1}{2}$ field $\lambda_A(x)$ is the supersymmetric partner of the gauge boson — the *gaugino* — and the field $d(x)$ is another auxiliary field that can be eliminated. This is again because there are no derivatives of $d(x)$ in the Lagrangian, as can be seen in a component expansion of the terms containing vector fields, $\Phi_i^\dagger e^V \Phi_i$ and $W^A W_A$.

We see that the superfields can provide us with both the boson and fermion fields necessary to reconstruct the SM. Anticipating the gauge structure of the SM with its gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y, \tag{3.78}$$

we know that for each generation of leptons, we need a supermultiplet that transforms as a doublet under $SU(2)_L$. One component should be a left-handed superfield e, μ and τ, representing the left-handed leptons e_L, μ_L, τ_L, and their scalar supersymmetry partners, and the other component a left-handed superfield ν_e, ν_μ, ν_τ, representing the neutrinos ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$ and their supersymmetric partners, the left-handed *sleptons* $\tilde{l}_L$. We write these three doublets as

$$L^l = \begin{pmatrix} \nu_l \\ l \end{pmatrix}, \tag{3.79}$$

where $l = 1, 2, 3$ for the three generations of leptons. We will also need three singlet right-handed superfields $E^{l\dagger}$, to represent the right-handed electrons e_R, μ_R and τ_R and their slepton partners.[17] E^l are then the left-handed superfields containing the anti-particles of the right-handed leptons. All of these supermultiplets are singlets under the colour group $SU(3)_c$.

The quarks are treated in a similar manner, corresponding to their $SU(2)_L$ transformation properties, in doublets of left-handed superfields, one representing up-type quark u_L, c_L or t_L, and one down-type quarks d_L, s_L or b_L. The doublets containing the left-handed quarks and their supersymmetric partners the left-handed *squarks* $\tilde{q}_L$ are written

$$Q^f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix}, \tag{3.80}$$

where $f = 1, 2, 3$ labels the three generations of quarks. The right-handed superfield singlets that contain the right-handed quarks and squarks are $D^{f\dagger}$ and $U^{f\dagger}$. D^f is then the various left-handed superfields containing the anti-particles of the right-handed down-type quarks and similarly U^f for the up-type quarks. We have suppressed the colour index these superfields have as they transform as triplets under $SU(3)_c$. If we require this index, we will write Q_i^f, and so on, where $i = 1, 2, 3$.

To take care of the gauge sector we need vector supermultiplets. From (3.50) we find that we need one vector superfield per generator of the gauge group. For $U(1)_Y$ this vector superfield is called B. For $SU(2)_L$, with three generators, we need three superfields W^i, $i = 1, 2, 3$. Mixing of the component gauge fields, with the gauge symmetry broken by the VEVs of Higgs fields, will result in the massive vector bosons $W^\pm$ and Z, and the massless photon, just as in the SM. However, in addition we will also get the fermionic superpartners of the gauge fields. The colour group $SU(3)_c$ has eight superfields named C^j, $j \in \{1, \ldots, 8\}$, representing the gluons and their supersymmetry partners, the *gluinos* $\tilde{g}$. The field strength superfields of these vector superfields are from (3.33) given by

$$B_A = -\frac{1}{4}\bar{D}\bar{D}D_A B, \tag{3.81}$$

$$W_A = -\frac{1}{4}\bar{D}\bar{D}e^{-W}D_A e^W, \tag{3.82}$$

$$C_A = -\frac{1}{4}\bar{D}\bar{D}e^{-C}D_A e^C, \tag{3.83}$$

[17] We assume here that the neutrinos are massless, so that we do not need their right-handed singlets under $SU(2)_L$. Since there is convincing evidence for non-zero neutrino mass this is just a statement of our ignorance of how the neutrinos are given mass.

where $W = g\sigma^i W^i$ and $C = g_s\lambda^j C^j$ contain the SM coupling constants and generators of the gauge groups.

Finally, we need Higgs fields to provide spontaneous electroweak symmetry breaking, giving mass to the gauge bosons and also to the fermions. For the Higgs mechanism to work these could be separate doublets of scalar superfields transforming under $SU(2)_L$, containing the Higgs particle(s) and their fermionic supersymmetry partners, the *higgsinos*. Alternatively, there is the very intriguing notion that the Higgs fields could be the scalar partners of some of the known SM fermions. In this case we must be able to write down mass terms for the SM fermions in the MSSM superpotential, using only the scalar superfields discussed above.

3.4.2 The MSSM Superpotential

With the field content of the MSSM Lagrangian in hand, we want to know what terms are possible in the MSSM superpotential (3.42), given the restrictions of Eqs. (3.47)–(3.49). Writing the superpotential as a function of the superfields needed to describe the known particles,[18] $W(L^k, Q^f, E^l, U^g, D^h, B, W^i, C^j, H_1, H_2, \ldots)$, where the H_n refer to possible Higgs superdoublets, we first note that there can be no linear (tadpole) term in W. By (3.47) this would require that one of the scalar superfields is a gauge singlet under all the factor groups of the SM gauge group. This is not the case as all quarks and leptons have a non-zero hypercharge, see Table 3.3, and any Higgs doublets have a non-zero weak isospin charge. Such a term could however appear in non-minimal models.

The possible direct mass terms of the superpotential on the form $m_{ij}\Phi_i\Phi_j$ are restricted to the cases where $m_{ij}U_{ir}U_{js} = m_{rs}$ for all gauge transformations. For the $U(1)_Y$ case this condition reduces to

$$Y_i + Y_j = 0, \tag{3.84}$$

where Y_i and Y_j are the hypercharges of the two multiplets. In Table 3.3 we give the hypercharges of the quarks and leptons in the SM, which can be found from Eq. (2.21) and Table 2.1. The only possible combinations that satisfy the hypercharge requirement are particle and anti-particle combinations. Since these are in different handed superfields no bilinear terms with supermultiplets containing SM fermions can appear in the superpotential. The exceptions to this is that a separate Higgs superfield doublet H can couple to one of the superfield doublets containing SM fermions. For instance we can have superpotential terms such as

$$\kappa_{ij} L^{iT} i\sigma^2 H_j. \tag{3.85}$$

Two Higgs superdoublets can also, if they have opposite hypercharge, couple to each other. This last possibility will result in terms of the form

$$\mu_{ij} H_i^T i\sigma^2 H_j. \tag{3.86}$$

[18] Note that we use only left-handed superfields, containing the anti-particles of the right-handed leptons and quarks, in the superpotential.

Particle	Hypercharge
ν_l, l_L	$-\frac{1}{2}$
l_R	-1
u_L, d_L	$\frac{1}{6}$
u_R	$\frac{2}{3}$
d_R	$-\frac{1}{3}$

Table 3.3: Hypercharges of the SM fermions.

The factor $i\sigma^2$ that appears in both these terms is necessary to contract the two doublets to a singlet, invariant under $SU(2)_L$.[19]

We are then left with one last possibility, to introduce mass through Yukawa terms $\lambda_{ijk}\Phi_i\Phi_j\Phi_k$. These must satisfy the restriction (3.47), which for $U(1)_Y$ is equivalent to

$$Y_i + Y_j + Y_k = 0. \tag{3.90}$$

With a careful investigation of all the possible combinations one can see that this is only realised by the three terms

$$\lambda_{ijk}L^i i\sigma^2 L^j E^k, \quad \lambda'_{ijk}L^i i\sigma^2 Q^j D^k, \quad \lambda''_{ijk}D^i D^j U^k. \tag{3.91}$$

These terms also satisfy the corresponding constraints from the other two gauge groups of the SM,[20] They can also potentially generate fermion masses through loop diagrams.

However, these terms are the source of phenomenological problems. Non-zero values of the λ-couplings lead to lepton number violating (the first two terms) and baryon number violating (the last term) interactions. In addition, bilinear terms of the form (3.85) also violate lepton number. Such interactions have strict experimental bounds. Should for example both lepton and baryon number violating terms be present the consequence is rapid proton decay. For these terms to be present, the couplings must be small enough to evade experimental bounds, such as the measured lower limit on the lifetime of the proton. Thus they cannot be used to replace the Higgs mechanism in generating masses for all the SM fermions.[21]

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[19]This is easily seen from from the $SU(2)_L$ transformation of the supermultiplets, e.g.

$$L^{iT} \rightarrow L^{iT} e^{ig\sigma^{kT}\Lambda^k}, \tag{3.87}$$

$$H_j \rightarrow e^{ig\sigma^k\Lambda^k} H_j, \tag{3.88}$$

which implies

$$\begin{aligned} L^{iT} i\sigma^2 H_j &\rightarrow L^{iT} e^{ig\sigma^{kT}\Lambda^k} i\sigma^2 e^{ig\sigma^k\Lambda^k} H_j \\ &= L^{iT} i\sigma^2 H_j, \end{aligned} \tag{3.89}$$

where we have used that from (A.5) $\sigma^2\sigma^k = -\sigma^{kT}\sigma^2$.

[20]Again the terms are all singlets under $SU(2)_L$. Invariance under the colour group $SU(3)_c$ is ensured because the multiplets Q^f belong to a 3 particle representation of $SU(3)_c$, and the D^g and U^h belong to a $\bar{3}$ anti-particle representation. These terms form colour singlets in the same manner as pairs of quark and anti-quark can form colour singlets as mesons, and three anti-quarks can form a colour singlet as an anti-baryon.

[21]Thought the lepton-violating terms can potentially be used to give small masses to the neutrinos [90].

Since there is no apparent *a priori* reason for these couplings to be small, the addition of a extra symmetry called *R-parity* has been suggested [91], with the multiplicatively conserved quantum number R defined as

$$R = (-1)^{2s+3B+L}, \tag{3.92}$$

where s, B and L are the spin, baryon and lepton number of a particle. This gives $R = 1$ for all SM particles and $R = -1$ for the supersymmetric particles, or sparticles. One usually defines the MSSM to conserve this parity, meaning it can only have interactions with a total of $R = 1$, disallowing the terms in (3.85) and (3.91). While this evades experimental bounds, the introduction of an extra symmetry seems somewhat artificial. However, it has three extremely important phenomenological consequences:

- The lightest supersymmetric particle (LSP) is absolutely stable.
- Every other sparticle must decay into the LSP.
- Sparticles will always be produced in pairs in collider experiments.

The first of these implies that the LSP, if it is neutral and only weakly interacting, is a good dark matter candidate, as we discussed in Section 2.6.3. With a neutral LSP, the next two points indicate that experimental production of sparticles should be accompanied by missing energy in the form of pairs of escaping LSPs.

3.4.3 Higgs Superfields

The discussion of the previous Section leaves us with no other choice than to introduce separate Higgs superfield doublets to the Lagrangian. In the SM we needed one such doublet, and since we are working in the MSSM, our model should be made with the least possible redundancy in fields. Here however, one Higgs doublet will not suffice. As in the SM, we need the Higgs field to generate masses for the leptons and quarks through Yukawa terms. With only one Higgs doublet it turns out that we cannot give masses to all the quarks.

Because of the fractional charges of the quarks and the effect this has on the hypercharge, and the differences in hypercharge between the right-handed up-type quarks and the down-type, it is impossible to balance hypercharge combinations that satisfy the restriction (3.90), with only one Higgs doublet. The most field efficient thing we can do is to combine the left-handed quark superdoublets, with the Hermitian conjugate of the right-handed singlets, which are the left-handed scalar superfield singlets. From Table 3.3, we see that combinations of Q and D, and Q and U give a collective hypercharge of $\pm\frac{1}{2}$. It is then sufficient to have two Higgs multiplets H_1 and H_2 with hypercharges $\pm\frac{1}{2}$. The H_2 doublet with hypercharge $-\frac{1}{2}$ can also give masses to the leptons.

The resulting superpotential has the following terms, including the possible bilinear Higgs superfield terms given in (3.86),

$$W(L^k, Q^f, E^l, U^g, D^h, H_1, H_2) = \sum_{k,l} Y_e^{kl} L^{kT} i\sigma^2 H_2 E^l$$

$$+\sum_{f,g,i} Y_u^{fg} Q_i^{fT} i\sigma^2 H_1 U_i^g$$
$$+\sum_{f,h,i} Y_d^{fh} Q_i^{fT} i\sigma^2 H_2 D_i^h$$
$$+\mu H_1^T i\sigma^2 H_2, \tag{3.93}$$

where Y_e^{kl}, Y_d^{fg} and Y_u^{fh} are the Yukawa parameters rewritten to account for one of the indices having been determined by the Higgs superfield used. The resulting 3×3-matrices will be proportional to the mass matrices of the quarks and leptons. It should be pointed out that the quarks in the doublets transforming under $SU(2)_L$ are not the same as the quarks observed in strong interactions under $SU(3)_c$. This is taken care of by the usual Cabbibo-Kobayashi-Maskawa mixing of eigenstates found in the SM, here incorporated in the matrices Y_d^{fg} and Y_u^{fg}.

Another way of showing the necessity of two Higgs superfield doublets, is through anomaly cancellations. In the SM triangle loops with fermions have ultra-violet infinities that cancel against each other. Introducing a Higgs superdoublet also introduces new fermions as the Higgs' supersymmetry partners. These will upset the anomaly cancellation. The solution is to introduce a second doublet, with opposite hypercharge, which is the condition for mutual cancellation of their contributions.

With two Higgs doublets the MSSM Higgs potential, and finding its minimum, becomes more complicated than for the SM Higgs potential. Here, we will only briefly comment on the construction of a spontaneous symmetry-breaking Higgs potential in the MSSM. We have already concluded that the Higgs doublets must have opposite hypercharge, $Y = \pm\frac{1}{2}$. By (2.21) this means that we can write them in terms of their component fields as

$$h_1 = \begin{pmatrix} h_1^+ \\ h_1^0 \end{pmatrix}, \quad h_2 = \begin{pmatrix} h_2^0 \\ h_2^- \end{pmatrix}, \tag{3.94}$$

where we write the scalar component field of the Higgs superfield doublet H as h. As in the SM the $SU(2)_L$ gauge freedom allows us to rotate away one of the component fields. After setting $h_1^+ = 0$ in the potential, one finds that any minimum must have $h_2^- = 0$. This is fortunate since by giving VEVs only to the uncharged Higgs fields we ensure that we do not break electromagnetic symmetry. By setting the VEVs to be $\langle 0|h_1^0|0\rangle = v_1/\sqrt{2}$ and $\langle 0|h_2^0|0\rangle = v_2/\sqrt{2}$, we give masses to the electroweak gauge bosons and through Yukawa terms to the SM fermions, and also to their supersymmetric partners. One can show that these VEVs are related to the Z mass and the electroweak coupling constants, and to the Fermi scale of weak interactions, through

$$v_1^2 + v_2^2 = \frac{2m_Z^2}{g^2 + g'^2} \simeq (246 \text{ GeV})^2. \tag{3.95}$$

However, their absolute value is unknown, and this is parametrised in the MSSM by the ratio

$$\tan\beta = \frac{v_1}{v_2}. \tag{3.96}$$

The two-doublet Higgs model used in the MSSM has a richer phenomenology than the SM Higgs sector. Of the eight total degrees of freedom in the Higgs doublets, three are absorbed into the gauge bosons just as in the SM. The remaining degrees can be shown to give two neutral, CP-even, Higgs states, the lighter h and the heavier H, two charged Higgs $H^{\pm}$ and one CP-odd Higgs A.

Because of the spontaneous symmetry breaking the spin-$\frac{1}{2}$ components of the Higgs superfields will mix with the spin-$\frac{1}{2}$ partners of the electroweak gauge bosons in the vector superfields that have the same quantum numbers. Writing the supersymmetric partners of the Higgs and gauge fields by the corresponding superfield sporting a tilde, the $\tilde{H}_1^0$ and $\tilde{H}_2^0$ neutral higgsinos mix with the $\tilde{B}^0$ and $\tilde{W}^0$ gauginos to form four states $\tilde{\chi}_i^0$ called *neutralinos*. The charged higgsinos $\tilde{H}_1^+$ and $\tilde{H}_2^-$ mix with $\tilde{W}^+$ and $\tilde{W}^-$ to form two charged states, the *charginos* $\tilde{\chi}_j^{\pm}$.

3.4.4 Gauge Terms

Now that we have fixed the superpotential, what remains in the construction of the MSSM Lagrangian is the rather easy job of writing down the Lagrangian terms for the gauge superfields, and the interaction terms coupling the gauge fields to the matter and Higgs fields. The pure gauge terms are given in terms of the field strengths in (3.72), and for the MSSM this is

$$\mathcal{L}_V = \frac{1}{2}\mathrm{Tr}\left[W^A W_A(\bar{\theta}\bar{\theta}) + C^A C_A(\bar{\theta}\bar{\theta})\right] + \frac{1}{4}B^A B_A(\bar{\theta}\bar{\theta}) + \mathrm{h.c.} \tag{3.97}$$

were the field strengths can be taken from (3.81), (3.82) and (3.83).[22]

From (3.50) we can write down the interaction terms for the scalar superfields under the familiar gauge transformations of the SM,

$$\begin{aligned}
\mathcal{L}_I \;=\; & \sum_l L^{l\dagger} e^{(gW^i\sigma^i/2 + g'B/2)} L^l + \sum_l E^{l\dagger} e^{(-g'B)} E^l \\
& + \sum_f Q^{f\dagger} e^{(-g_s C^i\lambda^i/2 + gW^i\sigma^i/2 - g'B/6)} Q^f \\
& + \sum_f U^{f\dagger} e^{(-g_s C^i\lambda^i/2 + 2g'B/3)} U^f + \sum_f D^{f\dagger} e^{(-g_s C_i\lambda_i/2 - g'B/3)} D^f \\
& + H_1^{\dagger} e^{(gW^i\sigma^i/2 - g'B/2)} H_1 + H_2^{\dagger} e^{(gW^i\sigma^i/2 + g'B/2)} H_2,
\end{aligned} \tag{3.100}$$

where we have used the coupling constants and generator representations as given in Eqs. (2.13), (2.14) and (2.36).

3.4.5 Supersymmetry Breaking Revisited

We have now constructed a supersymmetric and gauge invariant Lagrangian for the MSSM. The final piece of the complete model is to add soft supersymmetry breaking

[22] The correct numerical factor can be found from (3.73), leading to, in a natural notation, the following Dynkin indices for the non-abelian groups:

$$T(R_L) = \frac{1}{4}\mathrm{Tr}\left[\sigma^1\sigma^1\right] = \frac{1}{2}, \tag{3.98}$$

$$T(R_c) = \frac{1}{4}\mathrm{Tr}\left[\lambda^1\lambda^1\right] = \frac{1}{2}. \tag{3.99}$$

The Pauli and Gell-Mann matrices, σ and λ, are given in Appendix A.

terms to lift the mass degeneracy between the SM particles and their supersymmetric partners as described in Section 3.3.6.

For the sake of clarity we will write down the soft breaking Lagrangian in terms of the component fields involved, thus avoiding the numerous projections by factors of $\theta\theta\bar{\theta}\bar{\theta}$ in (3.76). What we are then writing down is the $\int d^4\theta$-projection of the supersymmetry breaking parts of the Lagrangian density in terms of superfields. The available terms were given in equation (3.77). Though we break supersymmetry with these terms, we do not wish to break gauge symmetry explicitly. This limits the possible terms to those that mirror the terms already in the Lagrangian. This excludes tadpole terms, and most bilinear terms, with the exception of a pure Higgs term $\mu B H_1^T i\sigma^2 H_2$. Note again the appearance of the factor $i\sigma^2$ to make the term an $SU(2)_L$ singlet. We can have trilinear Yukawa terms of the type $a_{jk}\tilde{A}_j i\sigma^2 H\tilde{A}_k$, were one of the fields is a Higgs doublet and where A_j is an $SU(2)_L$ doublet. The partners of the gauge bosons, the fermionic spin-$\frac{1}{2}$ gauginos of the vector superfields B, W^i and C^j, can generate additional mass from terms of the type $\frac{1}{2}M\tilde{\lambda}^A\tilde{\lambda}_A$, where $\tilde{\lambda}_A$ is the spin-$\frac{1}{2}$ component field of a vector superfield. For each of the scalar superfields we can write down a mass term of the form $m_{ij}^2\tilde{A}_i^*\tilde{A}_j$, where m_{ij}^2 is the corresponding mass matrix and the $\tilde{A}_i$ are the spin-0 component fields of the scalar superfields. The m_{ij}^2 may in general have complex entries and mixing between generations, but must be hermitian to make the Lagrangian real. [23]

The resulting Lagrangian for the soft breaking terms is then

$$\begin{aligned}
\int d^4\theta\,\mathcal{L}_B \;=\; & -\frac{1}{2}\left(\sum_j M_3\tilde{C}^{jA}\tilde{C}^j_A + \sum_i M_2\tilde{W}^{iA}\tilde{W}^i_A + M_1\tilde{B}^A\tilde{B}_A + \text{h.c.}\right) \\
& -\left(\sum_{f,g}(a_U)_{fg}\tilde{Q}^{fT} i\sigma^2\tilde{H}_1\tilde{U}^g + \sum_{f,h}(a_D)_{fh}\tilde{Q}^{fT} i\sigma^2\tilde{H}_2\tilde{D}^h + \text{h.c.}\right) \\
& -\left(\sum_{k,l}(a_E)_{kl}\tilde{L}^{kT} i\sigma^2\tilde{H}_2\tilde{E}^l + \text{h.c.}\right) \\
& -\sum_{k,l}(m_L)^2_{kl}\tilde{L}^{k\dagger}\tilde{L}^l - \sum_{k,l}(m_E)^2_{kl}\tilde{E}^k\tilde{E}^{l\dagger} \\
& -\sum_{f,g}(m_Q)^2_{fg}\tilde{Q}^{f\dagger}\tilde{Q}^g - \sum_{f,g}(m_U)^2_{fg}\tilde{U}^f\tilde{U}^{g\dagger} - \sum_{f,g}(m_D)^2_{fg}\tilde{D}^f\tilde{D}^{g\dagger} \\
& -(m_{h_1})^2 h_1^\dagger h_1 - (m_{h_2})^2 h_2^\dagger h_2 - (\mu B h_1^T i\sigma^2 h_2 + \text{h.c.})
\end{aligned} \tag{3.101}$$

The number of parameters of the MSSM is very large, and the vast majority of these appear in the soft terms. Careful counting shows that in addition to the 19 free parameters of the SM, there are 105 more parameters in the MSSM. Supersymmetry as a more fundamental theory seems at first glance to be rather far fetched. The huge number of parameters also mean that most experimental bounds can be avoided by a tuning of the right parameters, and most experimental discoveries can likewise be explained by some version of the MSSM. Thus the predictive power of the MSSM is in some sense limited.

[23] Note that the right-handed leptons and quarks are in right-handed supermultiplets, which means that the structure of their mass terms is changed by hermitian conjugation.

There are important exceptions to this. As we noted in Section 2.6, if we want to avoid reintroducing the fine-tuning problem for the Higgs mass, the scale of SUSY masses must be close to the electroweak scale, something which also seems necessary for a successful unification of forces at high energies for the simplest GUT models. Also, electroweak baryogenesis requires that some sparticles are light and supersymmetric dark matter seems most natural with candidate sparticles having a mass of the order of $\mathcal{O}(100\text{ GeV})$.

There are also strong hints of organising principles that would reduce the number of free parameters. Most of the soft parameters imply flavour mixing and CP violation that is restricted by experiments, preferring diagonal or near diagonal mass matrices. The apparent unification of coupling constants at high energies opens the possibility of unification of the soft parameters at the same high scales. The simplest such models, such as the popular *minimal supergravity* (mSUGRA) models of supersymmetry assume very few fundamental parameters at GUT scale. In mSUGRA there are only four, a common scalar mass m_0, a common gaugino mass $m_{1/2}$, a common trilinear parameter A_0 and $\tan\beta$ (in addition the sign of the Higgs parameter μ is a free parameter).[24] As one moves down in energy from the GUT scale the masses of the sparticles run differently depending on their couplings to gauge fields. The result is a low energy model with great variation in sparticle masses.

In exchange for providing a SUSY breaking mechanism that supplies universal soft breaking terms at the high scale, these models receive a lot of predictive power. So much that mSUGRA models are now highly constrained by experimental lower bounds on Higgs and sparticle masses, and by requiring consistency with the dark matter density measured by WMAP. If supersymmetry is discovered at the LHC, one of the greatest goals of particle physics will be to determine how the parameters are organised, and what the SUSY breaking mechanism is.

As we commented on in the Introduction, the LHC experiment will constitute an important test of low energy supersymmetry. In the next Chapter we will discuss various approaches to searching for supersymmetry at the LHC, and attempts to measure some of the parameters discussed in this Chapter, by using the cascade decays of sparticles.

[24]There are slight differences in terminology regarding mSUGRA. Some authors use the term Constrained Minimal Supersymmetric Standard Model (CMSSM) for these models, imposing a relation between the soft trilinear and bilinear supersymmetry breaking parameters: $A_0 = m_0 + B_0$, in mSUGRA. This fixes one free parameter in the electroweak symmetry breaking, allowing $\tan\beta$ to be determined from the other parameters in mSUGRA models.

Chapter 4

Supersymmetric Cascade Decays

> 'Though this be madness, yet there is method in 't'
>
> William Shakespeare
> *Hamlet, Prince of Denmark*

In the previous Chapter we saw that the conservation, or near conservation, of R-parity leads to the pair production of supersymmetric particles and their cascade decays into pairs of the lightest supersymmetric particle (LSP). There are strong bounds on a stable charged LSP from dark matter searches in cosmic rays, thus the generic signal of supersymmetry at the LHC is missing energy from escaping LSPs, alongside an abundance of high momentum hadronic jets — the collimated showers of particles that result from the production of quarks or gluons — and leptons from the cascade decays.

In this Chapter we will discuss the search for supersymmetry and the measurement of its parameters, in particular the mass of supersymmetric particles, at the LHC. This discussion aims at placing the four papers included at the end of the thesis in a wider context, and at showing how they are connected in a broad investigation of cascade decays at the LHC.

4.1 Supersymmetry at the LHC

In this Section we start with a short overview of the LHC experiment, and continue with a discussion of the standard Monte Carlo simulation techniques we have used to study the potential of the LHC in searching for supersymmetry. Then we go on to finally look at cascade decays and their importance in measuring the masses of supersymmetric particles.

4.1.1 The Machine and the Detectors

The LHC (Large Hadron Collider), being built at CERN, Geneva, is mainly a proton-proton collider, with a centre of mass energy of 14 TeV [92]. It has also been designed with the ability to collide heavy ions. The LHC machine consists of a series of proton accelerators, feeding a final 26.7 km circumference ring of superconducting magnets cooled down to temperatures of 1.9 K and with vacuum of 10^{-10} Torr, making the LHC colder than outer space, and almost as empty.

The machine is capable of handling two trains consisting of 2800 bunches of 10^{11} protons, with a separation of 25 ns, going in opposite directions, giving a total design luminosity of roughly 10^{34} cm^{-2}s^{-1}. This means that the experiments could potentially record data equivalent to an integrated luminosity of 100 fb^{-1} per year at optimal running. It is however planned that the machine will run at lower luminosity for the first years in a start up phase, roughly estimated at 10 fb^{-1} per year. At the design luminosity the focused beams will provide approximately 20 proton-proton collisions per bunch crossing, making reconstruction of events a very challenging task for the experiments.

In addition to the central machinery of the proton/heavy-ion accelerator, the LHC consists of four major experiments. The large detectors ATLAS (A Toroidal LHC ApparatuS) [93,94] and CMS (Compact Muon Solenoid) [95,96], are designed as general-purpose detectors, while the smaller LHCb (Large Hadron Collider beauty) [97] and ALICE (A Large Ion Collider Experiment) [98,99] are specialised b-physics and heavy-ion detectors, respectively. In addition there are also the smaller TOTEM [100] and LHCf [101] experiments, designed to measure the total cross section and to do so-called forward physics measurements, near the beam axis.

The general-purpose detectors ATLAS and CMS will have the best reach for direct discoveries of supersymmetry at the LHC. They are designed to reconstruct high energy jets and leptons well using several layers of sub-detectors. In particular, they have the ability to perform very accurate measurements of high-energy muons, due to their large size and strong magnetic fields. They have also been built to fully utilise the very high collision rates of the LHC, withstanding the strong radiation from running at the highest design luminosity, and being able to record data at a rate of around 100 collisions per second, triggering on those events that are deemed interesting.

The LHCb detector takes a different approach. Optimised for identifying individual hadrons and reconstructing the decays of B-mesons, it will test the SM predictions for physics with bottom quarks, to look for deviations due to the loop effects of New Physics, including supersymmetry. Although direct identification of supersymmetry will be difficult, this allows for a complementary sensitivity in areas that may be difficult to probe for the general-purpose detectors, such as scenarios with very heavy supersymmetric particles.

The work presented in this thesis has focused on the possibilities of direct searches, mainly by ATLAS and CMS, for new particles produced in proton-proton collisions.

4.1.2 Monte Carlo Simulation

To investigate the LHC potential we depend on computer tools to help simulate the complicated physics involved in hadron collisions. We have mainly used various versions of the `PYTHIA` [102] Monte Carlo event generation program to simulate proton collisions, both for signal supersymmetry events and for possible backgrounds. The `PYTHIA` simulation is based on leading-order matrix elements for hard $2 \to 2$ parton processes involving quarks or gluons as estimated from parton distribution functions, evolved with higher order effects in parton showering and hadronization, and also including an estimation of the underlying event, i.e., the collision of the proton remnants after the hard interaction.

The leading order (LO) approach has its limitation in that higher order QCD effects are predicted to become important at the LHC. The expected increase in cross sections can be compensated for by the use of so-called K-factors, multiplying the LO cross sections by results from next-to-leading order (NLO) calculations, e.g. for MSSM events by `Prospino` [103], but this does not correct for the predicted differences in the kinematical distributions of the produced partons [104]. Another issue with the $2 \to 2$ parton processes is that the addition of parton showers underestimates the production of multiple hard partons which are the result of $2 \to n$ processes. For Papers 1 to 3 these effects are believed to be fairly small, while they are discussed in greater detail in Paper 4 where hadronic jets is the central topic.

Tied in to the simulation of collisions is the simulation of detector response. For the first three papers we base ourselves on the modelling of a generic LHC detector by `AcerDET 1.0` [105]. This gives a fast simulation of a simplified detector with realistic geometrical acceptances and jet reconstruction from calorimeter cells. The smearing of momentum measurements and the identification of b- and τ-jets are parameterised based on early full simulations of the ATLAS detector. While not sufficient for the demanding task of comparison to actual data taken by the experiments, the fast simulation of LHC detectors is a good tool for phenomenological investigations of the LHC discovery reach.

4.1.3 Discovery

There are several candidates for dark matter in the MSSM, but most of the work published has focused on a neutralino LSP. For much of the mSUGRA parameter space the lightest neutralino is the natural candidate since it is the lightest sparticle. The mSUGRA alternatives are the lightest scalar tau $\tilde{\tau}_1$, the stau, or the lightest scalar top quark $\tilde{t}_1$, the stop, which are both unfeasible as dark matter due to their charges.

While the neutralino is the lightest sparticle in these models, it is not directly produced in proton collisions to any significant degree. There will be some slepton Drell–Yan pair production and neutralino–chargino production, but the wast majority of the supersymmetry cross section at the LHC will be the production of strongly interacting particles, squarks and gluinos, if they have kinematically accessible masses.

The implications of this is that if supersymmetry is realised in nature, the LHC is likely to see an excess of events with missing energy from escaping neutralinos, and, as a minimum, two jets from the high momentum quarks produced in the decays of squarks into neutralinos, $\tilde{q} \to \tilde{\chi}_1^0 q$. This is reflected in the planned searches, where the so-called *effective mass* defined by

$$M_{\text{eff}} = \not{E}_T + \sum_{\text{jets}} p_{T,i}, \tag{4.1}$$

in terms of the missing transverse energy $\not{E}_T$ and the sum of the transverse momentum of the jets in an event, is one of the main search channels.[1] Other important channels included one or more leptons. The distribution of effective mass has been found to be a good discriminator between SUSY and the SM [106, 107]. In Fig. 4.1 (left) we show the

[1] Only the missing transverse momentum is measurable since the longitudinal momenta of the colliding partons are unknown.

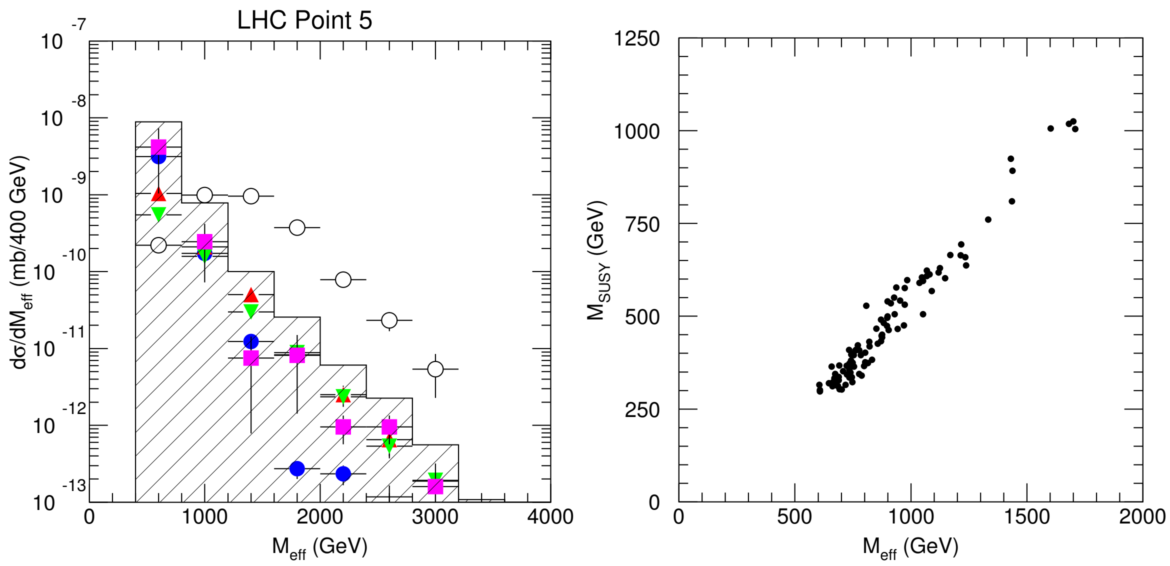

Figure 4.1: The distribution of effective mass for the LHC5 benchmark point (left) and the distribution of SUSY mass scale versus effective mass for mSUGRA scenarios (right). The figures are taken from Chapter 20 of the ATLAS TDR [94].

distribution for the LHC5 mSUGRA benchmark point.[2] The SUSY distribution (open circles), dominates at large effective masses over the SM background, shown here for $t\bar{t}$ (solid circles), W+jet (triangles), Z+jet (downward triangles) and QCD $2 \to 2$ events (squares). If this is the case, the discovery of SUSY at the LHC should be easy, and could be done with a few months of integrated luminosity for sparticle masses below 1 TeV.

The effective mass also gives an indication of the scale of supersymmetry, through the masses of the produced sparticles. The peak of the effective mass distribution is effectively correlated with the SUSY mass scale $M_{\rm SUSY}$, shown in Fig. 4.1 (right) as the minimum of the gluino and down squark masses. For more general MSSM models the correlation weakens, but is still present. In [107] the precision of the mass scale measurement is estimated to be 15% and 40% for mSUGRA and MSSM scenarios, respectively, with one year of running at low luminosity (10 fb^{-1}), and 7% and 20% after one year at high luminosity (100 fb^{-1}).

However, these measurements will require a good understanding of the SM at previously unexplored energies. In particular the NLO effects on the effective mass distribution should be properly accounted for, and experimental effects on the measurement of missing energy and the jet energy scale must be well known. This will certainly take more than the few months required to reach the necessary luminosity.

4.1.4 Invariant Mass Distributions

Following a significant excess in some search channel that is inconsistent with the SM, the season for new particle hunting will be open. The missing LSP of supersymmetry means that the standard method of reconstructing particles from the invariant mass of

[2]See [94] for details.

their decay products, looking for resonance peaks, is not available.[3] The effective mass distribution, and variations of it, also have a limited accuracy in determining individual sparticle masses.

The well studied alternative to this is to look at the endpoints of invariant mass distributions [106, 108–113]. For a number of cascade decay chains the positions of the endpoints of the various possible combinations of SM decay products has been calculated in terms of the sparticle masses involved. In particular the decay chain

$$\tilde{q} \to \tilde{\chi}_2^0 q \to \tilde{\ell} q \ell_n \to \tilde{\chi}_1^0 q \ell_n \ell_f \tag{4.2}$$

has received a lot of attention. Simple kinematical considerations, ignoring the finite width of the sparticles, initial- and final-state radiation, spin effects and any experimental smearing, lead to the following expression for the endpoint of the di-lepton invariant mass distribution for this decay chain, see e.g. [110],

$$(m_{\ell\ell}^{\max})^2 = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\ell}}^2)(m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{\ell}}^2}. \tag{4.3}$$

Tables over the more complicated expressions for other endpoints, including also the possibility of a gluino decaying to the squark of Eq. (4.2) and assuming the experimental indistinguishability of the two leptons, can be found in [112, 113].

Given that the number of linearly independent endpoints is equal to or larger than the number of sparticle masses involved, the masses can in principle be solved for. However, a number of complications can arise. The quadratic structure of the endpoint expressions opens up the possibility of additional false solutions. There is also a strong correlation between the sparticle mass measurements because the endpoints are expressed as mass differences. Thus mass differences will be much better measured through endpoints than absolute masses [112]. The possible appearance of small structures in the distributions near their endpoints, so-called "feet" that can be mistaken for background, result in systematic errors that are difficult to estimate.

The main thrust of Paper 1 is to extend the work on invariant mass distributions by calculating the full shape of the experimentally observable distributions from (4.2), and not just the endpoints. We show that knowing the distribution shape removes the problem of "feet" by predicting their existence, and allows for more sophisticated fits to the endpoints. In a recent conference proceedings contribution we have also showed the successful use of the derived shapes to remove false solutions [114]. Using the full distributions is not without additional complications. Effects from the mis-identification of decay products can be significant in the high multiplicity environment of the LHC colliders, and the cuts used to isolate SUSY events from SM background may affect the distributions, in particular at low invariant masses. However, it should be emphasised that it is not necessary to fit whole distributions to take advantage of knowledge of the theoretical shape.

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[3] Since at least two high momentum particles will be missing, the assignment of the missing momentum is not unique.

4.1.5 Seeing SUSY and Seeing that it is SUSY

While the discovery of significant excesses over SM predictions and the measurement of the masses of new particles, may be attributed to SUSY, the same results may also be interpreted as other New Physics (NP) scenarios. It was shown in [115] that Universal Extra Dimension (UED) scenarios can mimic supersymmetry, where the lightest Kaluza–Klein excitation of the photon takes the role of the LSP, and the conservation of a KK-parity ensures its stability. Other suggestions for SUSY-like NP models are Little Higgs scenarios with so-called T-parity [116]. While the mass spectrum of these models tends to be more degenerate, and the models predict somewhat different cross sections, the existence of this possibility precludes any totally convincing argument for a supersymmetry interpretation. The question is then, how can we identify the New Physics as actually being SUSY?

What is needed is to look for a property peculiar to supersymmetry, and from the discussions of Chapter 3 it should be clear that this is the spin of the particles. If the newfound particles are shown to be the bosonic partners of fermionic SM particles or vice versa, the SUSY interpretation will be greatly strengthened. The measurement of particle spin at the LHC is difficult due to the unknown — on an event by event basis — nature of the parton collision participants and their charges, and to the difficulty of measuring the charges of jets. However, it was shown by Barr in [117] that due to the charge asymmetry in proton–proton collisions, with more positive than negatively charged quarks, the spin of the $\tilde{\chi}_2^0$ in the decay chain (4.2) can be shown to be non-zero by looking at the differences between the invariant mass distributions for ql^- and ql^+ combinations.

In Paper 1 the derivations of the invariant mass distributions was consciously made very generic, in the sense that they can be applied to any decay chain

$$D \to Cc \to Bcb \to Acba \tag{4.4}$$

where the capital letters signify particles in some NP model and the small letters are the SM decay products. We gave specific distributions for the SUSY decay chain (4.2) with spin effects included, but a change to different spin configurations is easily accommodated by the formulae.

4.2 Light Stops

Paper 2 extends the investigation of invariant mass distributions to the more complicated decay chain

$$\tilde{g} \to \tilde{t}_1 t \to \tilde{\chi}_1^0 cWb \to \tilde{\chi}_1^0 c\ell\nu_\ell b, \tag{4.5}$$

where b and c are heavy quarks. The loss of information from the escaping neutrino means that there is no edge structure at the endpoints of the useful invariant mass distributions. The alternative at hand is to make fits to the shapes of the distributions, and for this purpose we derive the shapes of the m_{lc} and m_{bc} distributions.

While the decay (4.5) may occur in mSUGRA models, we focus on more generic MSSM scenarios with a light stop, $m_{\tilde{t}_1} < m_t$, which is the next-to-lightest supersymmetric particle (NLSP). These scenarios can be motived both from cosmological considerations on dark matter and baryogenesis, and from the Higgs mass fine-tuning problem.

4.2.1 Motivation

The issue of dark matter is discussed extensively in Paper 2. It focuses on the predicted tendency for a slight over-density of dark matter in SUSY scenarios when compared to the WMAP measurement given in Eq. (2.54). This is solved by picking scenarios with an efficient annihilation mechanism, increasing the cross section of Eq. (2.55). In particular an NLSP that is close in mass to the LSP can provide a high co-annihilation cross section at freeze-out. Such scenarios with a stop NLSP are called stop-coannihilation scenarios.

As we noted in Section 2.6.4 successful electroweak baryogenesis requires larger CP-violation than what is found in the SM. It also needs the electroweak symmetry breaking to be a strongly first order phase transition to avoid washing out the created baryon asymmetry. This condition can be given as [118],

$$\frac{v(T_c)}{T_c} \gtrsim 1, \tag{4.6}$$

where T_c is the critical temperature of the transition and v is the Higgs vacuum expectation value at this temperature. By studying the Higgs potential at finite temperature one finds that

$$v(T_c) \propto \frac{1}{\mu} \propto \frac{1}{m_h}, \tag{4.7}$$

which necessitates a very light Higgs in the SM, lighter than the LEP bounds [57]. Loop effects in the Higgs potential from bosonic particles with masses around the electroweak scale can help alleviate the situation, but the SM bosons have too weak couplings [119, 120]. In MSSM models the large number of new bosons, and in particular the existence of a stop with a large Yukawa coupling inherited from the heavy top, can allow Higgs masses below 120 GeV [64–71]. This opens up a small window for electroweak baryogenesis above the LEP lower bound of 114.4 GeV [57].

The second prediction of electroweak baryogenesis is that the lightest stop mass must be low, somewhere below the top mass, to increase the Higgs VEV sufficiently. This has implication for the Higgs fine-tuning problem. As we commented on in Section 2.6.1, while supersymmetry removes quadratic corrections to the Higgs mass, its breaking introduces logarithmic ones, whose size depends on the SUSY mass scale. In the MSSM the most important loop contributions to the Higgs mass come from the top and stop because of their large Yukawa couplings, see Eqs. (2.51) and (2.52). The opposite signs of the contributions mean that scenarios with a naturally light stop have reduced fine-tuning.

4.2.2 Searches

With the stop lighter than the top quark its decay channels are limited. The parameter space with a significant decay rate to a chargino $\tilde{t}_1 \to \tilde{\chi}_1^{\pm} b$ is very restricted in scenarios with GUT unification of gaugino masses as these predict a mass for the lightest chargino of about double the LSP mass, which means that the chargino is very likely heavier than the stop.[4] The decay $\tilde{t}_1 \to \tilde{\chi}_1^0 W b$, is likewise restricted because of the W mass, but may

[4]GUT scenarios predict

$$M_2 = (g_2/g_1)^2 M_1 \simeq 2M_1 \tag{4.8}$$

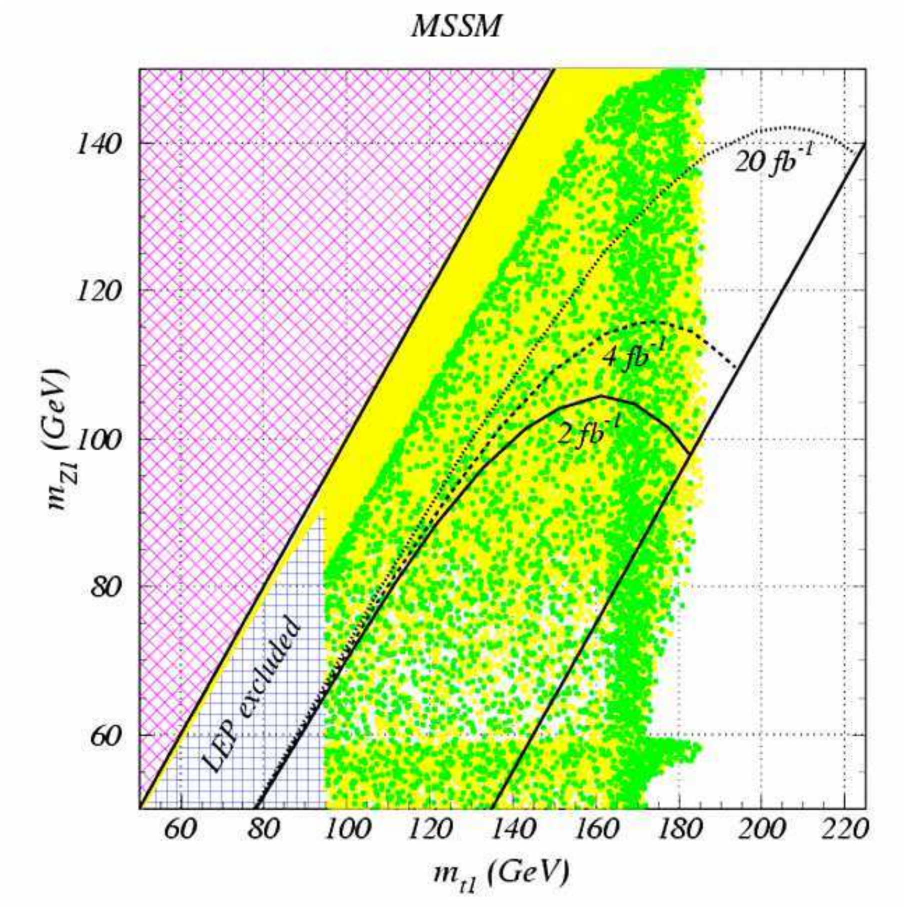

Figure 4.2: Tevatron reach for the $cc\not{E}_T$-channel for various integrated luminosities. The hatched areas are excluded, by a stop LSP (red) or LEP bounds (blue). The yellow and green areas shows the results of a scan over MSSM parameter space, where green represents points found with CDM density within WMAP first-year bounds and yellow points with too low CDM density. The figure is taken from [71].

occur as a four-body decay with an off-shell W. In this case the one-loop decay $\tilde{t}_1 \to \tilde{\chi}_1^0 c$ will be competitive, particularly for the lightest stops in stop-coannihilation scenarios.

Searches for a light stop decaying to a c-quark and the LSP are being performed at the Tevatron with the signature $cc\not{E}_T$ from stop pair-production, using c-jet tagging. Figure 4.2 shows the reach of the Tevatron for total integrated luminosities of 2 fb^{-1}, 4 fb^{-1} and 20 fb^{-1}. While the Tevatron can cover significant parameter space not excluded by LEP, we can see that it has little sensitivity in the stop-coannihilation region where the stop–LSP mass difference is small, due to the soft nature of the c-jets.

At the LHC c-jet tagging abilities will be very limited due to the higher energies and larger jet activity. As a result the $cc\not{E}_T$-channel will be very difficult to use. The main new idea presented in Paper 2 is to instead search for stops produced in the decays of

at the electroweak scale, which means that a mostly wino ($\tilde{W}^{\pm}$) chargino should have double the mass of a mostly bino ($\tilde{B}^0$) neutralino.

gluinos. If lighter than the squarks, with the exception of the stop, gluinos will decay nearly 100% into pairs of top and stop, as in (4.5). Since the gluino is a Majorana particle, it does not distinguish between a stop and anti-top pair, and its charge conjugate, which implies that the production of gluino pairs should lead to a large rate of same-sign top production. In Paper 2 we show that this search channel can be very efficient, and in a later conference proceedings contribution [121] we show that it has a reach up to gluino masses of $m_{\tilde{g}} \lesssim 900$ GeV, largely independent of the stop mass.

Same-sign top production also holds the possibility of demonstrating the Majorana nature of the gluino. If the SM $t\bar{t}$ background can be subtracted, an equal number of same- and opposite-sign top-pair events from the decay (4.5) of gluino pairs shows that the gluino is indeed a Majorana particle. This is the subject of ongoing investigations.

4.3 Hadronic Decay Chains

Most investigation into cascade decays have looked at decay chains with some leptonic content, such as (4.2) and (4.5). The expected large number of jets in SUSY events at the LHC means that purely hadronic decay chains have been thought difficult to reconstruct due to large combinatorial backgrounds from selecting the wrong jets in an event. Yet there are important decay chains such as

$$\tilde{q} \to \tilde{\chi}_1^{\pm} q' \to \tilde{\chi}_1^0 q' W, \tag{4.9}$$

where the leptonic decays of the W makes reconstruction very difficult, that could benefit greatly from the ability to successfully reconstruct the hadronic decays of the W. For scenarios where $m_{\tilde{\chi}_2^0} \simeq m_{\tilde{\chi}_1^{\pm}}$,[5] the left-handed squark typically decays 60% of the time as $\tilde{q}_L \to \tilde{\chi}_1^{\pm} q'$ and 30% as $\tilde{q}_L \to \tilde{\chi}_2^0 q$. This is the case for most of the mSUGRA parameter space, but also in many, more general, MSSM models. Given that the chargino will tend to decay into a W and the LSP if there is enough mass difference between the two, we see that the decay (4.9) will be important in many scenarios.

In mSUGRA scenarios that are consistent with the WMAP results for the CDM density (2.54), the decays of $\tilde{\chi}_2^0$ tend to be dominated by decays into slepton-lepton pairs, as found in (4.2). This is due to the limited allowed range for the scalar mass m_0 in the so-called stau-coannihilation region, where the near degeneracy of the lightest stau and the LSP is the mechanism for keeping the CDM density below the WMAP bound. In Fig. 4.3 we show the branching ratio of $\tilde{\chi}_2^0$ as a function of the gaugino mass $m_{1/2}$ along a line spanning the stau-coannihilation region.

We see that the branching ratio to Z and the lightest Higgs is consistently low. However, this need not be the case for more generic scenarios. By relaxing the mSUGRA unification requirement on the scalar masses for the Higgs doublets, in so-called non universal Higgs mass (NUHM) scenarios, a far larger range of scalar mass values is allowed. This is shown in Fig. 4.3 by the NUHM benchmark points α and β, that are dominated by the Z and Higgs decay of $\tilde{\chi}_2^0$ respectively.

In Paper 4 we study the invariant mass distribution qB for decay chains similar to (4.9), where B is a massive boson, either W, Z or h, that decays hadronically. By using

[5] In the limit where the electroweak masses are a small perturbation on the neutralino and chargino mass matrices both $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^{\pm}$ will be wino-like, with quite degenerate masses given mainly by M_2.

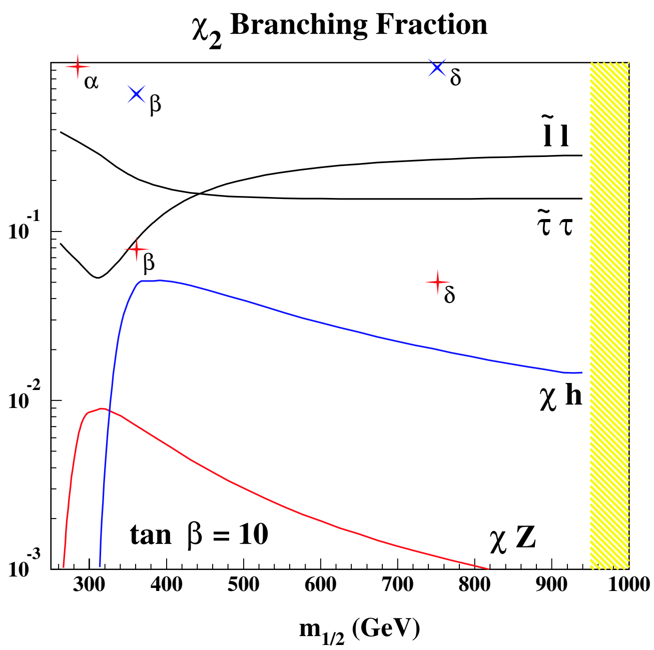

Figure 4.3: Branching ratios for $\tilde{\chi}_2^0$ along the mSUGRA line $m_0 = 13.57 + 0.142\, m_{1/2} + 4.90 \times 10^{-5}\, m_{1/2}^2$, where $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$. Also shown are the branching ratios for the benchmarks α, β and δ, used in Paper 4. The figure and parametrisation are taken from [122].

a sophisticated jet algorithm, the k_T-algorithm, that gives information on the internal structure of jets, we find that for highly boosted bosons, where the jets initiated by the quarks in the boson decay are very collimated and form a single jet, we can successfully identify the correct jet. This allows us to reconstruct the invariant mass distribution, and to extract endpoints that relate the SUSY masses involved.

We also make an extensive investigation into possible background contributions from the production of one or two massive vector bosons with multiple jets, which is not well simulated by the $2 \to 2$ production with added partons showers found in `PYTHIA`. For this we use the `ALPGEN` Monte Carlo event generation program [123–125]. While we find that there can be significant background contribution from such processes for our signal, we are still able to reconstruct endpoints by modelling and removing the background through so-called sideband subtraction, using events with jet-masses away from the boson mass in question.

4.4 Gravitino Dark Matter

In previous sections we have looked at the phenomenology of SUSY scenarios with a neutralino dark matter candidate. Theories of supergravity, such as mSUGRA, imply the existence of a supersymmetric partner of the spin-2 graviton, the gravitational force carrier. This is the *gravitino* $\tilde{G}$, a fermion with spin-$\frac{3}{2}$. Since there is no *a priori* reason for the gravitino to necessarily be heavier than the other sparticles, one should consider the possibility of a gravitino LSP, resulting in Gravitino Dark Matter (GDM) scenarios.

4.4.1 Cosmology

The gravitational coupling to matter is $\propto 1/M_P$, where M_P is the reduced Planck mass, given in terms of Newton's constant G_N as

$$M_P = \frac{1}{8\pi G_N} \simeq 2.4 \cdot 10^{18}\ \text{GeV}. \tag{4.10}$$

Since the gravitational coupling is very weak, the next-to-lightest sparticle (NLSP) can be quite long-lived in GDM scenarios.

In a generic MSSM model almost any sparticle could be the NLSP, but cosmological considerations can limit the field of candidates considerably. If we look at mSUGRA models with gravitino dark matter, the former LSP candidates are now the NLSP candidates. The most stringent constraints on the NLSP come from its decay after the Big Bang Nucleosynthesis (BBN) period in the first three minutes of the universe, where light elements such as Deuterium, ^{3}He, ^{4}He, ^{6}Li and ^{7}Li were formed. The late injection of energy may change the current abundances of these elements significantly. By comparing BBN calculations for a given model with the observed abundances, one finds that the electromagnetic energy released by the decay $\tilde{\chi}_1^0 \to \tilde{G}\gamma$ strongly constrains the parameters of models with a neutralino NLSP. The calculations instead favour a stau NLSP, since the stau decay $\tilde{\tau}_1 \to \tilde{G}\tau$ releases less electromagnetic energy, loosing a significant part of its energy to harmless neutrinos. The effect of the hadronic energy release from tau decays is found to be unimportant for NLSP lifetimes over 10^4 s [126–132].

Recent investigations have shown that a stop NLSP is excluded for strict mSUGRA models, but may be allowed in small sections of the parameter space for NUHM scenarios [133]. Going outside of the mSUGRA NLSP candidates, the decays of sneutrinos release little electromagnetic and hadronic energy and sneutrinos thus seem to be good candidates [128, 134]. For a recent investigation into sneutrino NLSP phenomenology at the LHC, see [135].

In the BBN allowed areas of parameter space for a stau NLSP the gravitino dark matter density produced from the decays of staus after freeze-out is lower than the WMAP limits (2.54). However, the thermal production of out-of-equilibrium gravitinos at the high temperatures of the early universe can account for the missing dark matter [136, 137].

4.4.2 Signatures

In Paper 3 we have studied three GDM benchmarks with a long lived stau NLSP. The long lifetime of the stau —relative to the scale of a detector — means that R-parity conserving GDM scenarios do not have the standard SUSY signature of missing transverse momentum at the LHC. Instead there will be two charged tracks in every event due to the staus. If these have enough momentum to leave the detector they will appear as tracks in the muon system, and will be reconstructed by detector software as muons.

Compared to the highly relativistic muons, a significant number of the staus will have low velocity due to their much larger masses. While this may be a challenge to detector triggers and track reconstruction that assumes $v \simeq c$ for all particles, this opens up some spectacular possibilities. If the stau velocity can be measured through its Time-of-Flight (ToF) in a detector, the stau mass can be very precisely determined from

$$m = \frac{p}{\beta\gamma}, \tag{4.11}$$

where $\beta = v/c$ and $\gamma = (1-\beta^2)^{-1/2}$. Furthermore, requiring two low velocity tracks in the muon system should be a very efficient way of reducing the SM background. Other properties, such as the larger loss of energy to ionisation due to the lower velocity, can also help separate staus from muons.

We show in Paper 3 that we can extract an effectively background-free sample of events with two slow-moving staus in all three benchmarks, and that it may be feasible to measure the stau mass in GDM scenarios down to a precision of 0.1%. Starting from the identified staus we find that we can reconstruct many heavier sparticles from their cascade decays into staus, even for difficult low cross section scenarios with squark and gluino masses above 1.5 TeV.

Current methods for measuring stau velocity at the LHC detectors depend on re-fitting tracks in the muon systems with different assumptions on the velocity, minimising the χ^2 of the fit quality as a function of velocity [138, 139]. We have suggested a complementary method using direct timing information from the muon trigger system, and this is now under full simulation study in ATLAS [140].

4.4.3 Supergravity

Since a fraction of the staus will be produced at low momenta simply by kinematical coincidences, there is a chance that energy loss in detector material will stop some of them, in particular in the calorimeters. When the beam is turned off one could hope to see their decays inside the detectors if a sufficient number is stopped and if their lifetime is of the order of the break in the LHC running. Should the detector material be insufficient to stop a significant number, suggestions have been made for building separate stopping detectors in the experimental caverns [141, 142], or even boring out holes in the cavern walls to extract rock samples with embedded staus [2].

By watching the decays of stopped staus their lifetime can be determined, and if the gravitino mass is not very much lighter than the stau, it could be measured from looking at the tau recoil energy E_τ,

$$m_{\tilde{G}}^2 = m_{\tilde{\tau}_1}^2 + m_\tau^2 - 2m_{\tilde{\tau}_1}E_\tau. \tag{4.12}$$

At the same time one can calculate the lifetime of a stau NLSP from the Feynman rules for the gravitino, given in e.g. [143]. The partial width of the decay $\tilde{\tau}_1 \to \tilde{G}\tau$ can be found to be

$$\Gamma_{\tilde{\tau}_1\to\tilde{G}\tau} = \frac{1}{48\pi}\frac{1}{M_P}\frac{m_{\tilde{\tau}_1}^5}{m_{\tilde{G}}^2}\left(1-\frac{m_{\tilde{G}}^2}{m_{\tilde{\tau}_1}^2}\right)^4, \tag{4.13}$$

where we have used $m_\tau \ll m_{\tilde{\tau}_1}$. Since the stau lifetime is only dependent on the stau and gravitino masses, and the reduced Planck mass, a measurement of both the lifetime and the two masses will amount to a microscopic measurement of the gravitational coupling and through it Newton's constant. If this is found to be consistent with the macroscopic gravity determination of G_N this is clear evidence for the realisation of supergravity in nature.

While speculations into stopping detectors and hole boring may seem a little far fetched, if the gravitino is dark matter and the scale of supersymmetry is anywhere near electroweak energies, such a monumental discovery does lie in our future, if not at the LHC, then at a future International Linear Collider.

Appendix A

Notation

Let me begin with some general notes on the notation used. We consistently work in natural units, setting $\hbar = c = 1$. The index notation uses small Greek letters for Lorentz indices and capital Latin letters, dotted and un-dotted according to representation, for two-component spinor indices. Four-component Dirac spinors are indexed by small Latin letters. Unless otherwise stated we imply a sum over repeated indices. For Lorentz vectors the usual sign-rules for contractions with upper and lower indices apply, using the Minkowski space-time metric tensor $g^{\mu\nu}$ given by

$$g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1), \tag{A.1}$$

to raise or lower indices. For two-component Weyl spinors the indices are handled by the anti-symmetric tensor ϵ_{AB}, where

$$\epsilon_{AB} = -\epsilon^{AB} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{A.2}$$

Gamma matrices

The γ-matrices are matrices that satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{A.3}$$

The γ-matrices have several representations convenient for different purposes. Here we will use the Dirac representation where

$$\gamma_D^0 \equiv \begin{bmatrix} \sigma^0 & 0 \\ 0 & \bar{\sigma}^0 \end{bmatrix}, \quad \gamma_D^i \equiv \begin{bmatrix} 0 & \sigma^i \\ \bar{\sigma}^i & 0 \end{bmatrix}, \tag{A.4}$$

and where the σ^μ-matrices are the Pauli matrices defined by

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{A.5}$$

and where

$$\bar{\sigma}^\mu = -\sigma^\mu. \tag{A.6}$$

The σ-matrices constitute a basis for the vector space of 2×2-matrices. The subscript on the γ-matrices will be dropped when no confusion is likely to occur. To simplify notation we will also use Feynman's notation for contractions with γ-matrices,

$$\not{p} \equiv p_\mu \gamma^\mu. \tag{A.7}$$

A fifth γ-matrix can be defined as the product

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \tag{A.8}$$

and can be shown to have the property

$$(\gamma^5)^\dagger = \gamma^5. \tag{A.9}$$

We also define a commutator between two γ-matrices,

$$\sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]. \tag{A.10}$$

Chirality operators

We define the chirality operators P_L and P_R as

$$P_L = \frac{1}{2}(1 - \gamma_5) \tag{A.11}$$
$$P_R = \frac{1}{2}(1 + \gamma_5). \tag{A.12}$$

One can easily show that these are projection operators, satisfying the projection relations

$$P_L^2 = P_L \tag{A.13}$$
$$P_R^2 = P_R \tag{A.14}$$
$$P_L + P_R = 1, \tag{A.15}$$

which in turn lead to the orthogonality relation

$$P_L P_R = 0, \tag{A.16}$$

and from Eq. (A.9) we have that

$$P_L^\dagger = P_L \tag{A.17}$$
$$P_R^\dagger = P_R. \tag{A.18}$$

SU(2)

The generators J_i of the $SU(2)$ group must satisfy the Lie algebra

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \tag{A.19}$$

where ϵ_{ijk} is a totally anti-symmetric tensor. For a two-dimensional representation this algebra is satisfied by the Pauli matrices σ^i of Eq. (A.5).

SU(3)

The generators λ_i of $SU(3)$ transformations must satisfy the Lie algebra

$$[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c, \tag{A.20}$$

where f_{abc} are the anti-symmetric structure constants for the algebra. For a detailed listing of the f_{abc}, see e.g. Section 36 of [26]. The three-dimensional representations commonly used are the eight Gell-Mann matrices λ_1–λ_8, given by

$$\begin{aligned} \lambda_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \lambda_4 &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}, \quad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \\ \lambda_7 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \end{aligned} \tag{A.21}$$

Spinors

In the four-dimensional representation used here a *Dirac spinor* Ψ_D is a four component spinor with complex entries[1], that can be decomposed into two so-called left-handed and right-handed Weyl spinors, ψ_A and $\bar{\chi}_{\dot{A}}$, as

$$\Psi_D = \begin{pmatrix} \psi_A \\ \bar{\chi}^{\dot{A}} \end{pmatrix}. \tag{A.22}$$

A *Majorana spinor* Ψ_M is a four-component spinor that is its own charge conjugate, i.e. $\Psi_M^c = \Psi_M$, implying that

$$\Psi_M = \begin{pmatrix} \psi_A \\ \bar{\psi}^{\dot{A}} \end{pmatrix}. \tag{A.23}$$

We define the adjoint of a spinor as

$$\bar{\Psi} = \Psi^\dagger \gamma^0. \tag{A.24}$$

[1]Formally a Dirac spinor is an element of the fundamental representation of the complex Clifford algebra, see Eq. (A.3).

Appendix B

Algebra

In the following we give some details on definitions and basic properties of some of the algebraic terms used in the main text. We assume that the reader is familiar with vector spaces and groups.

B.1 Lie Algebras

Lie algebras are defined as a vector spaces L over some field[1] with an added composition rule $\circ$, often called product, defined by the binary operation

$$\circ : L \times L \to L. \tag{B.1}$$

This mapping is required to have the properties that if $u_j, u_k, u_l \in L$, then

1. $u_j \circ u_k \in L$ (closure of algebra),
2. $u_j \circ (u_k + u_l) = u_j \circ u_k + u_j \circ u_l$ and
   $(u_k + u_l) \circ u_j = u_k \circ u_j + u_l \circ u_j$ (bilinearity),
3. $u_j \circ u_k = -u_k \circ u_j$ (antisymmetry),
4. $u_j \circ (u_k \circ u_l) + u_k \circ (u_l \circ u_j) + u_m \circ (u_j \circ u_k) = 0$ (Jacobi relation).

Vector spaces with only the first two properties are called *algebras*.

B.2 Superalgebras

The general definition of a *graded algebra*, called a $\mathbb{Z}_N$ graded algebra, is a vector space L that is the direct sum of N vector spaces L_k:

$$L = \bigoplus_{k=0}^{N-1} L_k, \tag{B.2}$$

[1] Usually taken to be $\mathbb{R}$. If the field is $\mathbb{C}$ we have a complex Lie algebra.

on which a composition rule $\circ$ is defined so that for $u_i \in L_i$ we have

$$u_j \circ u_k \in L_{j+k \bmod N}, \tag{B.3}$$

and where $\circ$ is a bilinear operation.

In supersymmetry we focus on $\mathbb{Z}_2$ graded algebras, so-called *superalgebras*, where

$$L = L_0 \oplus L_1. \tag{B.4}$$

If we additionally demand that the composition rule $\circ$ has similar properties to that in a Lie algebra, i.e. that for $u_i, v_i, w_i \in L_i$,

1. $u_j \circ v_k = -(-1)^{jk} v_k \circ u_j$ (supersymmetrization),
2. $u_j \circ (v_k \circ w_l)(-1)^{jl} + v_k \circ (w_l \circ u_j)(-1)^{kj} + w_l \circ (u_j \circ v_k)(-1)^{lk} = 0$ (generalised Jacobi relation),

then this algebra is a generalisation of a Lie algebra, called a *Lie superalgebra*. Notice that with the above properties the subspace L_0 spans an ordinary Lie algebra, while the vector space L_1 does not, since the properties 1. and 2. in this section are equivalent to 3. and 4. in Section B.1 for elements in L_0.

B.3 The Poincaré Group

Formally the Poincaré group is the group of all isometries of the Minkowski space-time. This means that it governs all transformations of external coordinates that preserve distance. The most general transformation that leaves $(x^\mu - y^\mu)^2$ invariant, where x^μ and y^μ are two points in Minkowski space, is

$$x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu, \tag{B.5}$$

where $\det \Lambda^\mu_\nu = 1$ for transformations that do not contain discrete space or time reflections. The generators of this group are P_μ and $M_{\mu\nu}$, where the P_μ are the generators of translations, and $M_{\mu\nu}$ are the anti-symmetric generators of rotations and relativistic "boosts" (change in velocity). These form the Poincaré algebra where the composition rule is given by the commutators

$$[P_\mu, P_\nu] = 0, \tag{B.6}$$
$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda} P_\mu - g_{\mu\lambda} P_\nu), \tag{B.7}$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(g_{\mu\rho} M_{\nu\sigma} - g_{\mu\sigma} M_{\nu\rho} - g_{\nu\rho} M_{\mu\sigma} + g_{\nu\sigma} M_{\mu\rho}). \tag{B.8}$$

From this it is an easy, but somewhat menial task to show that the Poincaré algebra is indeed a Lie algebra.

We will not go into details on the derivations of these relations from the transformation (B.5), or on the matrix representations of the generators. For the interested reader this can for example be found in Section 1.4 of [144] or Sections 1.1 and 1.2 of [77]. However, we note that from the above the Poincaré algebra is a semi-direct sum of two sub-algebras: the algebra formed by the translation generators P_μ with composition rule (B.6) and the algebra formed by the $M_{\mu\nu}$ and (B.8), the Lorentz algebra.

The representations of the Poincaré algebra can be found from its Casimir operators $P^2 = P_\mu P^\mu$ and $W^2 = W_\mu W^\mu$, where the Pauli-Ljubanski vector W_μ is given by

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} M^{\nu\rho} P^\sigma. \tag{B.9}$$

In the rest frame of the particle this is

$$W_k = \frac{1}{2}\epsilon_{k\nu\rho\sigma} M^{\nu\rho} P^\sigma = \frac{1}{2} m \epsilon_{k\nu\rho 0} M^{\nu\rho} = m S_k, \tag{B.10}$$

where S_k is the spin operator

$$S_i = \frac{1}{2}\epsilon_{ijk} M_{jk}. \tag{B.11}$$

So the particle representations of the Poincaré algebra are characterised by their mass, the eigenvalue of P^2, and their spin, the eigenvalue of S^2.

Appendix C

A (short) Superspace Calculus

Superspace is formally an 8-dimensional manifold that can be constructed as a coset space formed from the supersymmetry group and the Lorentz group, but we will not go into the formal details here.[1] For our uses it suffices to say that the superspace coordinates are given by an 8-vector $z^\pi = (x^\mu, \theta^A, \bar{\theta}^{\dot{A}})$, where x^μ are the ordinary Minkowski 4-vector coordinates, while θ^A and $\bar{\theta}^{\dot{A}}$ represent four anti-commuting Grassmann numbers, θ^1, θ^2, $\bar{\theta}^{\dot{1}}$ and $\bar{\theta}^{\dot{2}}$, in terms of two Weyl spinors. Collectively we can also write these as the four component Majorana spinor $\theta = (\theta_A, \bar{\theta}^{\dot{A}})^T$.

The anti-commutation property of Grassmann numbers amongst themselves gives the following commutation properties for the coordinates of superspace,

$$[x^\mu, x^\nu] = 0, \tag{C.1}$$
$$[x^\mu, \theta_a] = 0, \tag{C.2}$$
$$\{\theta_a, \theta_b\} = 0. \tag{C.3}$$

From this is it easy to see that the square of any Grassmann number vanishes, $(\theta_1)^2 = (\theta_2)^2 = 0$. One should also note the following contraction properties for the Weyl notation:

$$\theta\theta \equiv \theta^A\theta_A = -\theta_A\theta^A = -2\theta^1\theta^2, \tag{C.4}$$
$$\bar{\theta}\bar{\theta} \equiv \bar{\theta}_{\dot{A}}\theta^{\dot{A}} = -\theta^{\dot{A}}\theta_{\dot{A}} = 2\theta_{\dot{1}}\theta_{\dot{2}}. \tag{C.5}$$

where we have used Eq. (A.2) to raise and lower the indices of the Weyl spinors.

C.1 Differentiation and Integration

The operations of differentiation and integration in superspace are defined as normal on the x^μ-coordinates. On the Grassmann numbers differentiation is symbolically defined as

$$\partial_A\theta^B \equiv \frac{\partial}{\partial\theta^A}\theta^B \equiv \delta_A^B, \tag{C.6}$$

with a product rule

$$\begin{aligned}\partial_A(\theta^{B_1}\theta^{B_2}\dots\theta^{B_n}) &= (\partial_A\theta^{B_1})\theta^{B_2}\dots\theta^{B_n} - \theta^{B_1}(\partial_A\theta^{B_2})\theta^{B_3}\dots\theta^{B_n} \\ &\quad + \dots + (-1)^{n-1}\theta^{B_1}\theta^{B_2}\dots\theta^{B_{n-1}}(\partial_A\theta^{B_n}).\end{aligned} \tag{C.7}$$

[1] For a more comprehensive discussion of superspace and its properties, see [75].

Integration over the Grassmann variables is defined by

$$\int d\theta_A \equiv 0, \tag{C.8}$$

$$\int d\theta_A \, \theta_A \equiv 1, \tag{C.9}$$

$$\int d\theta_A \, \{af(\theta_A) + bg(\theta_A)\} \equiv a \int d\theta_A \, f(\theta_A) + b \int d\theta_A \, g(\theta_A) \tag{C.10}$$

where a and b are some complex numbers, and f and g two complex valued functions of the Grassmann number.[2] The function f of a Grassmann number θ_A can be written

$$f(\theta_A) = a_0 + a_1\theta_A, \tag{C.11}$$

because higher powers of θ_A vanish as a result of anti-commutation. The integral of this function is then

$$\int d\theta \, f(\theta_A) = a_1 = \frac{\partial}{\partial\theta_A} f(\theta_A), \tag{C.12}$$

so the operations of integration and differentiation have in fact identical effect, and they both have the property of projecting out the component of the highest power of the Grassmann variable.

Integration over several Grassmann numbers follows directly from the definition given. As with the integration over one Grassmann variable this has interesting projective properties. For superspace we define volume elements by the following relations

$$d^4\theta \equiv d^2\theta d^2\bar{\theta}, \tag{C.13}$$

$$d^2\theta \equiv -\frac{1}{4} d\theta^A d\theta_A, \tag{C.14}$$

$$d^2\bar{\theta} \equiv -\frac{1}{4} d\bar{\theta}_{\dot{A}} d\bar{\theta}^{\dot{A}}. \tag{C.15}$$

As a result we have that

$$\int d^2\theta \, (\theta\theta) = 1, \tag{C.16}$$

$$\int d^2\bar{\theta} \, (\bar{\theta}\bar{\theta}) = 1, \tag{C.17}$$

$$\int d^4\theta \, (\theta\theta\bar{\theta}\bar{\theta}) = 1, \tag{C.18}$$

so that the integral operator $\int d^4\theta$ works to project out the highest order component of a power series expansion in θ and $\bar{\theta}$. All other terms than the one containing $\theta\theta\bar{\theta}\bar{\theta}$ will be zero as a consequence of (C.8). The operators $\int d^2\theta$ and $\int d^2\bar{\theta}$ have similar properties for functions involving only θ or $\bar{\theta}$. The projection operators are useful for constructing supersymmetric Lagrangian densities in superspace.

[2] While the definitions for differentiation and integration have been written down for the left-handed Weyl spinor components here, the corresponding definitions for the right-handed spinor is a trivial change of indices, using Eq. (A.2) to raise or lower indices.

C.2 Covariant Derivatives

In the construction of a supersymmetric Lagrangian it is useful to have a derivative that is invariant under supersymmetry transformations. The ordinary momentum operator $P_\mu = i\partial_\mu$ is invariant, which is easily seen from (3.2). However, we can make a more general covariant derivative. We define, in Weyl spinor notation,

$$D_A \equiv \partial_A + i(\sigma^\mu\bar{\theta})_A\partial_\mu, \tag{C.19}$$

$$\bar{D}_{\dot{A}} \equiv -\bar{\partial}_{\dot{A}} - i(\theta\sigma^\mu)_{\dot{A}}\partial_\mu. \tag{C.20}$$

These covariant derivatives are invariant under supersymmetry transformations and satisfy the following relations

$$\{D_A, D_B\} = \{\bar{D}_{\dot{A}}, \bar{D}_{\dot{B}}\} = 0, \tag{C.21}$$

$$\{D_A, \bar{D}_{\dot{B}}\} = -2\sigma^\mu_{A\dot{B}}P_\mu, \tag{C.22}$$

$$D^3 = \bar{D}^3 = 0, \tag{C.23}$$

$$D^A(\bar{D}\bar{D})D_A = \bar{D}_{\dot{A}}(DD)\bar{D}^{\dot{A}}. \tag{C.24}$$

For detailed proofs see [77].

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