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Spontaneous symmetry breaking in three-Higgs-doublet S_3-symmetric models

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Abstract. The talk summarises work done by the authors consisting of a detailed study of the possible vacua in models with three Higgs doublets with S_3 symmetry and without explicit CP violation. Different vacua require special regions of the parameter space which were analysed in our work. We establish the possibility of spontaneous CP violation in this framework and we also show which complex vacua conserve CP. In our work we discussed constraints from vacuum stability. The results presented here are relevant for model building.

1. Introduction

In the Standard Model of Particle Physics (SM) there is one Higgs doublet responsible for spontaneous electroweak symmetry breaking and for the mechanism that gives mass to fermions and to electroweak gauge bosons. The model predicts the existence of one Higgs boson. In 2012 a scalar boson was discovered at the LHC [1, 2] with properties consistent with those predicted by the SM. However there are good motivations to consider models with more than one Higgs doublet such as the possibility of having CP symmetry broken spontaneously [3] or new sources of CP violation. Supersymmetric models require two Higgs doublets. Furthermore, models with two Higgs doublets have a rich phenomenology with many interesting possible manifestations of physics beyond the SM [4, 5]. Extensions of the SM with more than one Higgs doublet are good candidates to explain some of the present flavour anomalies and to solve some of the puzzles left unanswered by the SM.

Models with more than one Higgs doublet can give substantial contributions to flavour changing neutral currents (FCNC). Current experimental bounds require these to be strongly suppressed. One possibility is to completely forbid Higgs mediated FCNC at tree level via a symmetry, as is the case in models with natural flavour conservation (NFC) [6, 7] where only one Higgs doublet is allowed to couple to each charge quark sector. In the case of two Higgs doublets this is achieved by means of a Z_2 symmetry and as a result neither spontaneous nor hard CP violation can occur in the Higgs sector. It is possible to have CP violation in the scalar sector,

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with two Higgs doublets and NFC, with the Z_2 symmetry softly broken in the Higgs potential [8]. Three Higgs doublets and NFC with exact Z_2 symmetries allow for CP to be violated either explicitly [9] or spontaneously [10]. FCNC can also be eliminated at tree level assuming alignment of Yukawa couplings in flavour space [11], in this case complex Yukawa couplings give rise to new sources of CP violation. There have been several attempts at obtaining Yukawa alignment in several extensions of the SM [12, 13, 14, 15]. Another very interesting possibility to avoid the problem of having too large FCNC with two Higgs doublets, is to build models allowing for Higgs mediated FCNC which are under control by means of a symmetry that leads to couplings suppressed by small off-diagonal elements of the quark mixing matrix V_{CKM}. The first models of this type, based on a symmetry, were proposed by Branco, Grimus and Lavoura (BGL) [16]. Implications of BGL-type models have been extensively analysed recently, in the light of the LHC results, for several different implementations [17, 18, 19, 20, 21, 22]. BGL-type models can have FCNC either in the up sector or in the down sector but not in both sectors at the same time. Recently, a generalisation of BGL-type models allowing for FCNC in both sectors was built [23]. BGL models models can also be extended to the case of three doublets [17].

Three-Higgs-doublet models may provide good dark matter candidates [24, 25, 26, 27, 28, 29]. One may also speculate that nature is such that three generations of fermions come with three Higgs doublets, which is, of course, an issue to be settled by experiment. As the number of doublets increases so does the complexity of the scalar potential and the number of free parameters in the theory [30]. Discrete symmetries play an important role in reducing this number and lead at the same time to testable predictions. Symmetries also play an important role in stabilising dark matter [31, 32, 33].

As mentioned above, CP can be spontaneously violated in models with three Higgs doublets with Z_2 symmetries [10]. Such is also the case for an S_3 symmetry [34, 35, 36].

This talk is based on the work done in Ref. [35] where the possible vacuum solutions were analysed and the possibility of having spontaneous CP violation in the context of three Higgs doublets, with an S_3 symmetry, was studied. In this work we concentrated our attention on the scalar potential. Several authors have considered implications of three-Higgs-doublet models with an S_3 symmetry for flavour physics (see, for example: [37, 38, 39, 40, 41, 42, 43, 44]).

2. Vacua of S_3-symmetric three-Higgs-doublet potential

The S_3 group is the permutation group involving three Higgs doublets ϕ_1, ϕ_2 and ϕ_3, identified as the defining representation and constituting a reducible triplet of Higgs doublets. The study of the S_3-symmetric three-Higgs-doublet potential can be done in two different frameworks, either in terms of the defining representation [45, 46], or in terms of the irreducible representations [47]. In what follows, we classify the vacua in terms of constraints on the potential. Vacua can also be classified in terms of their residual symmetries [48].

2.1. The scalar potential

The S_3 symmetric potential has a quadratic and a quartic part, which in terms of the defining representation can be written [45]:

$$V = V_2 + V_4 \tag{1}$$

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \text{h.c.}], \tag{2a}$$

$$\begin{aligned} V_4 = {} & A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \overline{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \text{h.c.}]\} \\ & + \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{h.c.}] + \sum_{i \neq j \neq k \neq i, j<k} \{\frac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \text{h.c.}] \\ & + \frac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \text{h.c.}] + \frac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \text{h.c.}]\}. \end{aligned} \tag{2b}$$

Here all fields appear on an equal footing. This representation is not irreducible, it splits into two irreducible representations consisting of a singlet, h_S and a doublet of S_3 with components h_1 and h_2. The decomposition into these two irreducible representations is given by:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{3}$$

This matrix has a striking similarity with the tribimaximal mixing matrix [49] which is very close to the observed leptonic mixing.

The scalar potential written in term of fields from irreducible representations has the form [47, 41, 50]

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \tag{4a}$$

$$\begin{aligned} V_4 = {} & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2. \end{aligned} \tag{4b}$$

In this form the potential has no symmetry for the interchange of h_1 and h_2 but there is a Z_2 symmetry of the form $h_1 \to -h_1$. There is an equivalent doublet representation which has also been used in the literature [40]:

$$\begin{pmatrix} \hat{\chi}_1 \\ \hat{\chi}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{5}$$

where the symmetry appears as a symmetry for the interchange of the fields $\hat{\chi}_1$ and $\hat{\chi}_2$. Both expressions for the potential describe the same physics and the coefficients in the different frameworks are related through linear equations.

With the special choice of $\lambda_4 = 0$ the potential acquires a continuous $SO(2)$ symmetry defined by:

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \tag{6}$$

Spontaneous breaking of this symmetry leads to a massless scalar which of course must be avoided.

S_3 has three irreducible representations, a doublet, a singlet and a pseudosinglet, h_A. The latter has no direct translation into the initial fields ϕ_1, ϕ_2 and ϕ_3 and transforms under S_3 into

(-h_A). From a group theoretical point of view we can choose to write an S_3 symmetric potential in terms of the doublet and h_A. The new potential becomes:

$$V_2 = \mu_0^2 h_A^\dagger h_A + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \tag{7a}$$

$$\begin{aligned} V_4 = {} & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_A^\dagger h_2)(h_1^\dagger h_2 + h_2^\dagger h_1) - (h_A^\dagger h_1)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_A^\dagger h_A)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_A^\dagger h_1)(h_1^\dagger h_A) + (h_A^\dagger h_2)(h_2^\dagger h_A)] + \lambda_7 [(h_A^\dagger h_1)(h_A^\dagger h_1) + (h_A^\dagger h_2)(h_A^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_A^\dagger h_A)^2, \end{aligned} \tag{7b}$$

which reduces to the same potential we had before with h_1 and h_2 interchanged. At this stage there is no new physics from this choice of representations. However this may change depending on how the couplings to the fermions are introduced.

In order to study the possibility of having spontaneous CP violation we start with a potential with real coefficients. This choice guarantees, without loss of generality, that the potential conserves CP. In this case we are left with ten independent parameters irrespective of the choice of representations. This potential does not fall into a CP conserving potential with irremovable complex parameters [51].

We use the following field notations for the decomposition of the $SU(2)$ Higgs doublets:

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ (\rho_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, 3, \tag{8}$$

$$h_i = \begin{pmatrix} h_i^+ \\ (w_i + \tilde{\eta}_i + i\tilde{\chi}_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, \quad h_S = \begin{pmatrix} h_S^+ \\ (w_S + \tilde{\eta}_S + i\tilde{\chi}_S)/\sqrt{2} \end{pmatrix}. \tag{9}$$

2.2. Real vacuum solutions

Real vacuum solutions do not violate CP spontaneously. It is interesting to understand what are the possible real solutions for the vacuum. In this case one has to solve three minimisation conditions corresponding to the vanishing of the three relevant derivatives of the potential. In the irreducible framework these conditions can be solved in terms of μ_0^2 and μ_1^2 leading to [50]:

$$\mu_0^2 = \frac{1}{2w_S} \left[\lambda_4 (w_2^2 - 3w_1^2) w_2 - (\lambda_5 + \lambda_6 + 2\lambda_7)(w_1^2 + w_2^2) w_S - 2\lambda_8 w_S^3 \right], \tag{10a}$$

$$\mu_1^2 = -\frac{1}{2} \left[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) + 6\lambda_4 w_2 w_S + (\lambda_5 + \lambda_6 + 2\lambda_7) w_S^2 \right], \tag{10b}$$

$$\mu_1^2 = -\frac{1}{2} \left[2(\lambda_1 + \lambda_3)(w_1^2 + w_2^2) - 3\lambda_4 (w_2^2 - w_1^2) \frac{w_S}{w_2} + (\lambda_5 + \lambda_6 + 2\lambda_7) w_S^2 \right]. \tag{10c}$$

The first equation comes from the derivative of the potential with respect to w_S and the second and third from the derivatives with respect to w_1 and w_2. Eqs. (10b) and (10c) were obtained dividing by w_1 and w_2 respectively. Clearly, these two equations are not automatically consistent. There are several possible consistency cases:

- for $w_1 = 0$ the corresponding derivative is zero and there is no clash with the determination of μ_1^2 from Eq. (10c).
- otherwise, $\lambda_4(3w_2^2 - w_1^2)w_S = 0$ is required. This can be achieved in three different ways: $\lambda_4 = 0$ or $w_1 = \pm\sqrt{3}w_2$ or $w_S = 0$.
- for $w_S = 0$ a special condition arises from Eq. (10a): $\lambda_4 w_2 (3w_1^2 - w_2^2) = 0$ so that in addition we must have $\lambda_4 = 0$ or $w_2 = \pm\sqrt{3}w_1$, or $w_2 = 0$.

Table 1. Possible real vacua (partly after Derman and Tsao [46]). This classification uses the notation R-X-y, where R refers to "real". The roman numeral X gives the number of constraints on the parameters of the potential that arise from solving the stationary-point equations. The letter y is used to distinguish different vev's that have the same X, and λ_a is defined in Eq. (11).

Vacuum	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	Comment
R-0	$0,0,0$	$0,0,0$	Not interesting
R-I-1	x,x,x	$0,0,w_S$	$\mu_0^2 = -\lambda_8 w_S^2$
R-I-2a	$x,-x,0$	$w,0,0$	$\mu_1^2 = -(\lambda_1+\lambda_3)\, w_1^2$
R-I-2b	$x,0,-x$	$w,\sqrt{3}w,0$	$\mu_1^2 = -\frac{4}{3}(\lambda_1+\lambda_3)\, w_2^2$
R-I-2c	$0,x,-x$	$w,-\sqrt{3}w,0$	$\mu_1^2 = -\frac{4}{3}(\lambda_1+\lambda_3)\, w_2^2$
R-II-1a	x,x,y	$0,w,w_S$	$\mu_0^2 = \frac{1}{2}\lambda_4\frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -(\lambda_1+\lambda_3)\, w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1b	x,y,x	$w,-w/\sqrt{3},w_S$	$\mu_0^2 = -4\lambda_4\frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -4(\lambda_1+\lambda_3)\, w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1c	y,x,x	$w,w/\sqrt{3},w_S$	$\mu_0^2 = -4\lambda_4\frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -4(\lambda_1+\lambda_3)\, w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-2	$x,x,-2x$	$0,w,0$	$\mu_1^2 = -(\lambda_1+\lambda_3)\, w_2^2$, $\lambda_4 = 0$
R-II-3	$x,y,-x-y$	$w_1,w_2,0$	$\mu_1^2 = -(\lambda_1+\lambda_3)\,(w_1^2+w_2^2), \lambda_4 = 0$
R-III	ρ_1,ρ_2,ρ_3	w_1,w_2,w_S	$\mu_0^2 = -\frac{1}{2}\lambda_a(w_1^2+w_2^2) - \lambda_8 w_S^2$, $\mu_1^2 = -(\lambda_1+\lambda_3)\,(w_1^2+w_2^2) - \frac{1}{2}\lambda_a w_S^2$, $\lambda_4 = 0$

Derman and Tsao [46] analysed spontaneous symmetry breaking with real vacua taking also into account the residual symmetries. Their work was done in the reducible framework where the condition $\lambda_4 = 0$ corresponds to $4A - 2(C + \overline{C} + D) - E_1 + E_2 + E_3 + E_4 = 0$. This condition was obtained before by Derman [45] who considered it very unnatural, since in his context it was not clear that it was associated to an additional symmetry. With $\lambda_4 \neq 0$ there were only three possible real solutions [46]:

- (x, x, x) leaving S_3 unbroken and translating into the doublet-singlet notation as $(0, 0, w_S)$; consistency condition: $w_1 = 0$ (also verifies $w_1 = \pm\sqrt{3}w_2$).
- (x, x, y) leaving a residual S_2 symmetry. In terms of the reducible representation any ordering of the vevs is equivalent, however, in the definition of the doublet of S_3 a special direction is chosen. As a result, different orderings correspond to different translations:
  (x, x, y) translates into $(0, w_2, w_S)$; consistency condition: $w_1 = 0$.
  (x, y, x) translates into $(w_1, -\frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = -\sqrt{3}w_2$.
  (y, x, x) translates into $(w_1, \frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = \sqrt{3}w_2$.
- $(x, y, z) = (x, -x, 0)$ leaving a residual S_2 symmetry. This is the only possible real solution with all three vevs different from each other, unless one imposes $4A - 2(C + \overline{C} + D) - E_1 + E_2 + E_3 + E_4 = 0$ ($\lambda_4 = 0$). The translation into the irreducible representation is now:
  $(x, -x, 0)$ translates into $(w_1 = \sqrt{2}x, 0, 0)$: consistency conditions: $w_S = 0$ together with $w_2 = 0$.
  $(x, 0, -x)$ translates into $(w_1 = \frac{1}{\sqrt{2}}x, w_2 = \frac{\sqrt{3}}{\sqrt{2}}x, 0)$; consistency conditions: $w_S = 0$ together with $w_2 = \sqrt{3}w_1$.
  $(0, x, -x)$ translates into $(w_1 = -\frac{1}{\sqrt{2}}x, w_2 = \frac{\sqrt{3}}{\sqrt{2}}x, 0)$; consistency conditions: $w_S = 0$

together with $w_2 = -\sqrt{3}w_1$.

Table 1 summarises all the possible real solutions together with the constraints imposed on the parameters of the potential. The following abbreviation was introduced:

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7. \tag{11}$$

Table 2. Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3\sin 2\rho_1/\sin 2\rho_2}$, $\psi = \sqrt{[3+3\cos(\rho_2-2\rho_1)]/(2\cos\rho_2)}$. Due to the constraints the vacua labelled with an asterisk (*) are in fact real.

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho} + x, -re^{-i\rho} + x, -re^{i\rho} + re^{-i\rho} + x$
C-IV-c	$\sqrt{1+2\cos^2\sigma_2}\hat{w}_2$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho} + r\sqrt{3(1+2\cos^2\rho)} + x$, $re^{i\rho} - r\sqrt{3(1+2\cos^2\rho)} + x, -2re^{i\rho} + x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1}\xi + x, re^{i\rho_2} - re^{i\rho_1}\xi + x$, $-2re^{i\rho_2} + x$
C-IV-f	$\sqrt{2+\frac{\cos(\sigma_1-2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2}\psi + x$, $re^{i\rho_1} - re^{i\rho_2}\psi + x, -2re^{i\rho_1} + x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

2.3. Complex vacuum solutions

In the discussion of possible complex vacua we now adopt a convention where w_S is real and non-negative and take

$$w_1 = \hat{w}_1 e^{i\sigma_1}, \quad w_2 = \hat{w}_2 e^{i\sigma_2}, \tag{12}$$

with the $\hat{w}_i$ also real and non-negative. With this convention w_S is also denoted by $\hat{w}_S$. A systematic analysis of possible solutions was performed in [35]. The results are summarised in Table 2. The list of the constraints on the potential that are consistent with each solution is not given here, it can be found in Ref. [35].

Several solutions require $\lambda_4 = 0$. This is not a new feature, it also happened in the context of real solutions. For $\lambda_4 = 0$ the potential acquires a continuous $SO(2)$ symmetry which can be

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broken spontaneously by the vacuum solutions, therefore, leading to a massless scalar. Massless scalars are ruled out by experiment. It is possible to avoid this problem by introducing soft breaking terms. The most general form for the V_2 part of the potential with soft breaking terms would be:

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2) + \mu_2^2 (h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2}\nu^2 (h_2^\dagger h_1 - h_1^\dagger h_2) \\ + \mu_3^2 (h_S^\dagger h_1 - h_1^\dagger h_S) + \mu_4^2 (h_S^\dagger h_2 - h_2^\dagger h_S). \tag{13}$$

However, soft breaking terms involving h_S and one h_i are not consistent with $\lambda_4 = 0$.

In Table 3 we collect all possible complex vacuum solutions indicating whether or not they require λ_4 equal to zero and whether or not they allow for spontaneous CP violation. One important conclusion from our analysis is that there are cases where CP can be violated spontaneously, however, no solution requiring $\lambda_4 = 0$ can lead to spontaneous CP violation. In order to confirm that CP could indeed be violated spontaneously we used a powerful tool based on CP-odd Higgs-basis-invariant conditions, verifying that there were indeed conditions that were violated. There are several such conditions which were especially built for the analysis of the Higgs potential [52, 53, 54, 55, 56]. In the next subsection we discuss spontaneous CP violation using a few illustrative examples.

Table 3. Spontaneous CP violation

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

2.4. Spontaneous CP violation

Spontaneous CP violation can only occur if the Lagrangian conserves CP but the vacuum does not. This can only happen when there is no transformation that can be identified with a CP transformation leaving both the Lagrangian and the vacuum invariant. Under a CP transformation a single Higgs doublet Φ transforms into its complex conjugate. In models with several Higgs doublets the most general CP transformation is given by:

$$\Phi_i \xrightarrow{\text{CP}} U_{ij} \Phi_j^*. \tag{14}$$

Here, U is a unitary matrix mixing different Higgs doublets and corresponds to a Higgs basis transformation[6]. Higgs basis transformations do not change the physics.

If all the coefficients of the potential are real the potential conserves CP explicitly and the above equation is verified for U the identity matrix. Checking for explicit CP invariance of a multi-Higgs potential may be a non-trivial task since Higgs basis transformations, in general, can transform couplings that are real in one Higgs basis into couplings that are complex in another basis. For this purpose CP-odd Higgs basis invariants are of great help [53, 54]. Once

[6] This transformation is often referred to as a "generalized" CP transformation, thus suggesting that there is also a "non-generalized" CP transformation. This is, of course, misleading.

it is known that a Lagrangian conserves CP it remains to check whether or not CP is violated spontaneously. It has been shown [57] that in order for the vacuum to conserve CP the following relation has to be obeyed:

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle \tag{15}$$

with U now a unitary matrix corresponding to a symmetry of the Lagrangian. This relation is very powerful and allows to show that vacua that would at first sight violate CP are indeed CP conserving. This can be illustrated with a few examples taken from Table 3. For a full discussion see Ref. [35].

- Let us consider the vacuum identified as C-I-a, given by $\left(x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}\right)$ in the reducible representation. It is not possible to rephase the three Higgs doublets in such a way that the three vevs become real keeping at the same time the potential real. This is a vacuum solution with calculable non-trivial phases, fixed by the symmetry of the potential with no explicit dependence on the parameters of the potential. Such phases are called geometrical phases [57]. It was shown in Ref. [57] that this vacuum does not violate CP since Eq. (15) can be verified for U given by:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{16}$$

  This matrix makes use of the symmetry of the potential for the interchange of ϕ_2 and ϕ_3.
- Another interesting example is the C-III-c vacuum which is of the form $\left(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0\right)$ in the irreducible representation framework. It can also be written, without loss of generality, through an overall phase rotation, in the form $\left(\hat{w}_1 e^{i\sigma}, \hat{w}_2, 0\right)$. At first sight this vacuum looks like a CP violating vacuum, especially taking into consideration the fact that the moduli of w_1 and w_2 are different from each other. However, once again we can use Eq. (15) to show that this vacuum conserves CP. Notice that this solution requires $\lambda_4 = 0$ (see Table 3) and therefore there is an $SO(2)$ symmetry for the fields h_1 and h_2. With this knowledge one can build the necessary matrix U and Eq. (15) becomes:

$$e^{i(\delta_1+\delta_2)} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{w}_1 e^{i\sigma} \\ \hat{w}_2 \\ 0 \end{pmatrix}^* = \begin{pmatrix} \hat{w}_1 e^{i\sigma} \\ \hat{w}_2 \\ 0 \end{pmatrix}, \tag{17}$$

  or

$$e^{i(\delta_1+\delta_2)} \begin{pmatrix} \sin 2\theta & \cos 2\theta & 0 \\ \cos 2\theta & -\sin 2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{w}_1 e^{i\sigma} \\ \hat{w}_2 \\ 0 \end{pmatrix}^* = \begin{pmatrix} \hat{w}_1 e^{i\sigma} \\ \hat{w}_2 \\ 0 \end{pmatrix}. \tag{18}$$

  In this example the matrix U has several components:
  - an $SO(2)$ rotation of h_1 and h_2 by an angle θ, which should be chosen as:

$$\tan 2\theta = \frac{\hat{w}_1^2 - \hat{w}_2^2}{2\hat{w}_1\hat{w}_2\cos\sigma}. \tag{19}$$

  With this choice, the vevs of the new S_3 doublet fields acquire the same modulus and the new vacuum acquires the form $(ae^{i\delta_1}, ae^{i\delta_2}, 0)$,
  - an overall phase rotation of the three Higgs doublets by $\exp[-i(\delta_1+\delta_2)/2]$, so that now the first two vevs acquire symmetric phases: $(ae^{i\delta}, ae^{-i\delta}, 0)$,
  - finally we just need to use the symmetry for the interchange $h'_1 \leftrightarrow h'_2$ in the S_3 doublet representation.

The last example illustrates how powerful the condition given by Eq. (15) can be, but at the same time it shows that, as complexity grows, it may be non-trivial, in cases where such a matrix exists, to build the necessary matrix U. In fact this may require special insight and there is the danger of missing it, in a CP conserving case. In Ref. [58] we propose an alternative simple method, which is very useful in such cases, and allows to detect or eliminate the possibility of having spontaneous CP violation in multi-Higgs models. The three tools, consisting of the use of CP-odd invariant conditions, the relation given by Eq. (15) and the simple method proposed in Ref. [58], combined together, provide a reliable procedure to determine whether or not a given Higgs potential violates CP spontaneously.

3. Conclusions

We have presented here a summary of the work done in Ref. [35]. We have focused on some important features of three-Higgs-doublet models with an S_3 symmetry with emphasis on the discussion of spontaneous CP violation. Some aspects which were dealt with in the paper were not included in this short presentation. We refer the reader to the original work for a more detailed discussion of these aspects and for other topics such as ideas about constraining the potential by the vevs, relations among complex and real vacua, and a discussion on positivity beyond the necessary conditions given by Das and Dey [50] following the approach of Refs. [59, 24] (see also [60]). Models with multi-Higgs doublets such as those discussed in our work are very interesting and can in principle provide answers for several open questions. In particular they can provide viable dark matter candidates. These and other questions such as ways of generating realistic fermion masses and mixing in this context or looking for viable models with spontaneous CP violation are still challenging despite the fact that a lot of work has been already done along these lines. These questions are very timely due to the potential for being tested at the LHC.

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