INQUIRE BASED LEARNING IN AN ALGEBRA CLASS

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Contents

1. Introduction

I analyzed the class I was assigned to teach for the Spring semester 2017, MAT220-Algebra - which I had already directed during the Spring 2017, under the magnifying glass of the scholarship of teaching and learning. As many advanced - post-calculus - mathematics courses, the teaching method adopted at the UiB, and in many other both national and international institutions, is to use frontal lectures with the traditional "definition-statement-proof" pattern. The shortcomings of this teaching method are well known both to educators and researchers in the field (see for example the incipit of [5]). A possible better approach is the inquiry based learning (IBL) developed from the Moore method in the 1900. In it, the role of the teacher is to be a facilitator rather than a "sage on stage", and the students learn from mutual discussions and group works. However, while ample literature covers the implementation of IBL in math classes from secondary school to Calculus, its use in more advanced courses in general, and in abstract algebra classes in particular, is more recent - starting with the pioneering paper by Leron– Dubinski [7], and going to the more recent description of a teaching experiment by Capaldi [1].

In the remainder of this first section, I will describe the content of the Algebra class taught at the UiB, and I will reflect on the pedagogical challenges its learning implies. In Section 2, I will review some literature on IBL, with particular emphasis to its application to high-level math classes. Then, in Section 3, I will describe my pedagogical experiment in the teaching of the MAT-220 class using IBL methods. The outcome of the experiment, and my personal conclusion will be reported in the last section of this elaborate.

1.1. **MAT220-Algebra at the UiB.** Abstract algebra is one of the branches of pure mathematics. It concerns the study of algebraic structures - roughly speaking sets provided of some operations and satisfying determined axioms. It is among the most abstract part of mathematics, but it influences also our everyday life. Below, you can find a list of questions whose answer lies in the study of this subject, for a more detailed descriprion of what Abstract Algebra is, please see [4].

(i). Why $(-1) \times (-1) = 1$?
(ii). How many prime numbers are there?
(iii). We know that there is a formula which allows us to solve a second degree equation as like as, e.g., $x^2 - 7x + 1 = 0$. Is there any such formula for a higher degree equation as, e.g., $x^7 + x^2 - 2 = 0$?
(iv). Is it possible to trisect an angle, duplicate a cube - that is, given a cube, one would like to construct a second cube whose volume is exactly twice that of the first object - or square a circle - i.e. given a circle one wants to construct a square whose area is the same as the area of the circle - using just a straight rule, and a compass?
(v). Which regular polygon with n sides is it possible to construct with a straight rule, and a compass?
(vi). Find the whole solutions of $x^2 + y^2 = z^2$.
(vii). Given three whole numbers: e, d, and n. Is there a fast way to compute the remainder of the division of e^d by n.
(viii). Given three whole numbers: e, r, and n. Is there a fast way to find a whole number d such that the remainder of the division of e^d by n is equal to r?

Unfortunately, almost none of these applications can be addressed by a one-semester undergraduate course, since they require a much deeper baggage of knowledge. As a consequence, the syllabus concerns the axiomatic definitions of algebraic structures such as *groups*, *rings*, and *fields*, and the meticulous proof of their most important proprieties; which, apparently, are of no use outside the academic world. Thus, it is particularly difficult to engage the attention of the whole class - which usually is an in-homogeneous mixture of math, computer science, hard sciences, and math-education majors.

Beside its many applications, one of the most important aspects of an upper-level mathematics class such as Algebra is that it helps the students to develop and hone their problem-solving strategy. In fact, the ability of thinking as mathematicians, and acquiring mathematical approach to problem solving, are among the most important skills that the students should retain after the successful completion of the class. This is also one of the factor that makes abstract algebra courses normally very challenging to students. In fact, as observed by Capaldi [1, pg. 12]:

> Suddenly, students are expected to go beyond simple imitation and memorization, which often carried them through calculus and other lower-level mathematics courses. At the same time, they are introduced to new concept that are presented in an abstract manner, by describing their properties and what those properties imply. These new expectation and levels of abstractness can result in a challenging transition [...].

It does not help the overall situation a consisten discrepancy between what is taught in class and what is object of assessment. In fact, the class is taught under the "black-board lecture style" - typical in the teaching of pure mathematics (see, [5]) - in which, the axiomatic definitions, and the main theorems of the theory are presented. Oppositely, in the five hours written examination, the students are asked to use the knowledge they acquired in class to perform computations, and write short proofs concerning algebraic structures. Brutally simplifying: there is a juxtaposition between theoretical, and practical aspects of this field.

Summarizing, the MAT220 class at the UiB presents the following issues:

- it is objectively a very challenging class, and the average of grades is usually really low. For these reason, it is considered by the students a very hard course, and many of them delay in placing it in their study plan. This is particularly bad because, for this choice, they will be prevented some paths, if they wished to pursue an academic, or research based, career;
- what is taught in the lectured is pretty different from what is evaluated in the final exams;
- classes are usually not homogeneous, with students having many different interests and priorities. It is really hard to engage them.

When I taught this field for the first time, in the Spring 2016, I had a firsthand experience of this problems. So, after witnessing these pedagogical challenges, I wanted to see, if with a different teaching approach, many of the aforementioned obstacles might be overcome.

2. Theoretical Framework

Some of the difficulties observed while I was teaching MAT220 at the UiB during the Spring 2016, can be found mentioned also in the existing literature. I already quoted Capaldi thoughts on this matter, and she is not alone in underlining the obstacles students encounters when approaching to this subject. For example, Dubinski–Leron begin their paper [7] in the following strong and traumatic manner.

> **Statement.** *The teaching of abstract algebra is a disaster, and this remains true almost independently of the quality of the lectures.*
>
> We agree.
> And we think there's a fairly wide consensus on this among experienced abstract algebra instructors, and an even wider one among experienced students.

The same authors, in the book [3] claim that there are some cognitive obstacles to the students understanding of concepts as cosets and quotients. In [2], Cullinane argument that these difficulties are common to many post-calculus classes where

> to be successful, students must adapt quickly, and develop new perspective on the very nature of mathematical activities in which they will engage.[...][The reason for this is that a] not-so-subtle shift- from merely implementing given algorithms to creating solution strategies from the ground up- occurs.

In addition, to this "shift in problem solving expectations", the material of the lectures in post-calculus classes become suddenly more abstract, a greater precision and formality is required when solutions to problems are offered, and the role of the textbooks change. In fact, while on basic math classes the textbook can be viewed as a box of construction blocks - it contains all the tools and the instructions to build what is pictured on the box - in post-calculus courses, the book is more like a toolbox with precise instructions for every tool, but with no guidance on how to use these instruments to reach one's goal. As a consequence, the students are asked to "become active readers of their book".

Other difficulties students normally encounter in abstract algebra courses - but, that go, in part, beyond this writing - are also described in [6] and [8].

After to have clear this situation. A natural question was born in my mind: what can be done to ease this transition? In literature, one can find many proposed solutions.

In [5] a revolutionary method for teaching abstract algebra, adopted at the Department of Mathematics of the London School of Economics is described. The classes were organized in seminar in which the student themselves had to present the material either from reading textbooks or research articles. The role of the teacher was just the one of coordinator of the seminar (drawing the schedule, and assigning some reading material). The result of this teaching method were extraordinary.

> Our experience has shown that most students love this method of teaching and that they derive a great sense of satisfaction from their work. They tend to achieve better exam results than students who are taught in the traditional way. Furthermore, a higher percentage of graduates among those who have been exposed to seminars in algebra become interested in taking a higher degree in mathematics (usually specializing in algebra).

Another advantage of this method is that any student can work at her or his pace. Unfortunately, by the author own admission, such a method can be effective only in small classes. Consequently, since I expected to have at least 40 students enrolled, I decided to discard this approach.

In [8], a didactic experiment in which two students are lead to formulate their own definition of group - by starting from observing the symmetries of a triangle - is described. Again, this proposed methods is really intriguing but, by the authors' own admission the pace of the class is much slower. Since among my duties as an instructor there is to provide mathematics majors with the tools they will need in next courses, I had to discard also this method. In addition, I have doubts that it could have worked in a large class, in which each student should have come up with their own definition. Even if, naturally, this second obstacle could have been avoided by letting the students work in groups.

In [7], Dubinski–Leron suggest to use computer algebra system as ITSEL to help the student deal with the abstraction proper of Abstract Algebra courses, and to favor in-class discussions. Again the logistic of this project was not easily implementable in the classrooms at the UiB where not only there is not a sufficient number of computer stations available, but also they are bereft of power outlets.

The method I decided to implement, due to the similarities between my class situation - even if I had at least double the number of students - and what described there, is that employed by Capaldi in [1].

> For most class periods, I put students into five groups of four students each.[...] While groups were working, I wandered around the room to check in on their progress or answer clarification questions. A day in this course always incorporated collaborative work. Once every group finished, a representative would present their results.

The group work in Capaldi's class was based on worksheet written with the purpose of leading the students through the labyrinth of the theory, both using the discovery method of [8] and allowing for plenty of time for practice with exam-like exercises. Capaldi declares that the whole experiment was a success: she managed to cover much the same material as in the previous year, when she had used the traditional lecture method, and the average of the student in the IBL class were significantly higher. She got some student complain, but mostly were about the fairness of her grading system in general, and the use of participation points in particular. I have to remark here that the grading system used in the US and Canada is really favorable to IBL style classes. In fact, the final grade is usually a weighted average of the work of the whole semester, and is not assigned just by considering the final exam. This ensure that the students are kept under pressure the whole term, it is less likely they get lost. The use of a similar method in an European framework will, without doubt, advantage the more motivated, and mature students. But, it could leave behind those that for personal reasons are not full-time in school.

3. Execution

3.1. **The Class.** Unfortunately, the Department of Mathematics at UiB does not gather/did no give me access to data concerning the composition of my class (e.g. major, year of study, gender, and nationality of each student). Thus, all the data exposed here as to be consider as "not-official". I see from my class-roster that I had 57 students enrolled; of these, about 15 were female, and at least 5 where Erasmus students from France and Germany - where abstract algebra is studied also in secondary schools. 44 of them decided to attempt the exam in the Spring semester, and 5 in the Fall (these latter could have also have already attempted the previous time, but to know this for certain I would have question the administration of the Department of Mathematics, and in doing so breaching the strict student privacy rules of the UiB). These data are consistent with what I had witnessed in the Spring 2016, where I had more than 80 students enrolled, of these 40 attempted the exam in the first semester and 7 in the Fall. However, I have to underline that prior to Spring 2016 the number of pupils in MAT220 was far lower - usually about 30 student added the class to their study plan, and no more than 20 took the final exams. This drastic raise is partly a consequence of the oil-crisis: MAT220 is required in order to be able to teach mathematics in secondary schools, and many oil engineers were given the option of taking this class, together with few others, in order to get a teaching qualification.

Attendance to the lecture was not required, and about 50% of students showed up to lectures. This is well in the average with other mathematics class of the same level taught at the UiB. An higher percentage of women (almost all of them) attended regularly the lectures. Among the participants, almost one third were mathematics major while the remaining two thirds were divided among IT majors, and students enrolled in teachers programs. So, I will assume that this distribution was the effective composition of my course.

3.2. **The Method.** Instead of using the frontal lecture method, I decided to experiment a student centered class. I asked to students to read the book at home by themselves. Instead of lecturing on the theory, for every class, I prepared some handouts in which the learning outcome for the current lecture were explained. In addition, on every handout the students would find a list of questions on which they were supposed to work in small groups. I gave them some time to spend on each point. Afterward, a chosen volunteer representative from a group explained what they had came out as relative answer, and their reasoning. At this point, I would ask to the other groups whether they had anything to add, and, if I deemed it necessary, I would also add some my own comments to their answers. The questions in the handouts were of varying difficulty, and ranged from straightforward computations to more theoretical questions. At the end of these handout sheets, the student would find the reading assignment for the next lecture. After a few weeks, since I noticed that the pace of the class was too slow for some of the students, I started to add to the handouts also a section with more difficult questions that would not have been object of assessment. Clearly, I did that with the goal of keeping everyone engaged. Some examples of such handouts can be found in the appendix of this document.

Similarly to what Capaldi did, I explained, during the first day of course, what the teaching methods would have been, and what were the motivations - and the objectives - behind such revolution. I think that some of the students in the teaching program were already familiar with this teaching style, but many others were quite surprised. I will speak more on students' reactions in the following section.

I made also use of slides in which the main definitions and the most important results were shown. So, the student would have had them handy, in case they did not remember fluently the content of their reading assignment. Especially at the beginning, with the aim to break the ice, I also prepared some pop quizzes related to the handout in which the students could test their knowledge, and maintain the anonymity with the use of clickers, such as Kahot.

Since one of the big risk of this form of teaching is that students do not keep up with the pace of the lectures, I composed some mandatory assignments which had to be turned in in order to be admitted to the final exam. You can also find examples of these assignment in the appendices.

Finally, I asked the student for an anonymous feedback at the end of each lecture: they had to turn in a minute answering the following three questions.

(i). What was the most important topic today?
(ii). What did you understand most?
(iii). What did you understand least?

Some of these minutes can be found in the appendices.

I conclude this paragraph by mentioning briefly what I could not implement - due to the Norwegian higher education system, and other practical/logistical obstacles - who could have been of great help.

- I could not test the students during the semester on their reading assignment. Having a weakly pop-quiz on them would have assured that the material would have been read.
- I could not require mandatory participation to the class nor assign "participation points" toward the final grade as Capaldi did in her experiment.
- I could not make the mandatory assignments count towards the final grade.

4. Outcome and Conclusions

It is difficult to write an impartial paragraph of conclusions because there was not any external person overseeing my teaching. I requested for the class to be evaluated, but at this moment, I have not received the students evaluations - and, honestly, at this point, I am pretty sure I will never receive them. So, I have no idea whether the students liked this teaching method, or whether they would have preferred the usual frontal lecture one. My personal impression is that this drastic change could have been too much of a shock, especially within a Norwegian classroom, where interaction with the audience is not usually sought for. In addition, at the beginning of the semester a student (enrolled in the Bachelor in Mathematics) complained directly to me because to his eyes it looked as I was not explaining the *pensum*. I had to sit with him and showing as many questions asked in the handouts lead to a better explanation of the definitions, and results of the book.

Attendance was not great (around 50%), but not different from what I had previously experienced in this, and other classes at UiB. I am not entirely satisfied in class participation: some groups were fare more active then others, and when I tried to get answer from the shyest part of the class, by addressing them directly, I rarely managed anything.

One of the best ways to see whether this class was, at least practically, a success is to compare the final grades students received with those of the previous academic year, in which I had taught it with a more traditional method. In Table 1, you can see all these data, grouped for semester and year. Each year summary can be seen in Figure 1.

Semester	**Total**	**A**		**B**		**C**		**D**		**E**		**F**	
		♯	%	♯	%	♯	%	♯	%	♯	%	♯	%
Spring 2016	40	2	5	4	10	12	30	9	22.5	4	10	9	22.5
Fall 2016	7	1	14.3	1	14.3	1	14.3	0	0	1	14.3	3	42.8
Tot. 2016	47	3	6.4	5	10.6	13	27.7	9	19.1	5	10.6	12	25.6
Spring 2017	44	3	6.8	3	6.8	16	36.4	10	22.7	5	11.4	7	18.9
Fall 2017	5	1	20	1	20	1	20	0	0	1	20	1	20
Tot. 2017	49	4	8.1	4	8.1	17	34.7	10	20.4	6	12.3	8	16.4

TABLE 1. Grades for MAT220 in 2016 and 2017

One can see that, respect to the last time I taught the same class in 2016, I managed to maintain more or less the same ratios of A's and B's (slightly more A's and slightly less B's). But, far less people failed, and I had a bigger number of C's. This is consistent with the feeling I had while I was lecturing: my impression was that this teaching method was indifferent for the strongest students, but it could help the average, and weaker students. It is important to remark here that given the small cohort, and the short time series of data accessible to me, I cannot honestly tell - under a more strictly statistical point of view - whether there was an actual improvement, or whether the perceived changes were just outliers.

I was not able to cover as much material as before, but if this material was understood better, then it was certainly a worth effort. However this could have hindered the mathematics majors in their next endeavors since they need all the *pensum* in order to avoid problems with the Commutative Algebra class, of which MAT220 is a prerequisite.

Due to my experience in the flipped class environment, I decided to make some changes to my regular teaching, even when I use frontal lectures. For example, if the room where lectures are held allows it to me, I am going to use slides which contains the theoretical material, and I will write on the black board only computations and proofs. I would also try to involve students in my reasoning, and sometimes I will ask them to come to the black board (but, I have yet to find one brave enough).

In conclusion, I think this was a positive experience even if it had some dark sides. In fact, if I have to go back, I perhaps would have tried to flip the class only partially. For example, reserving only one day a week to a flipped class environment in which exercises, and exams could be discussed, instead of a complete revolution of my teaching method. As the Romans would say *in media res stat virus* (the virtue is in the middle).

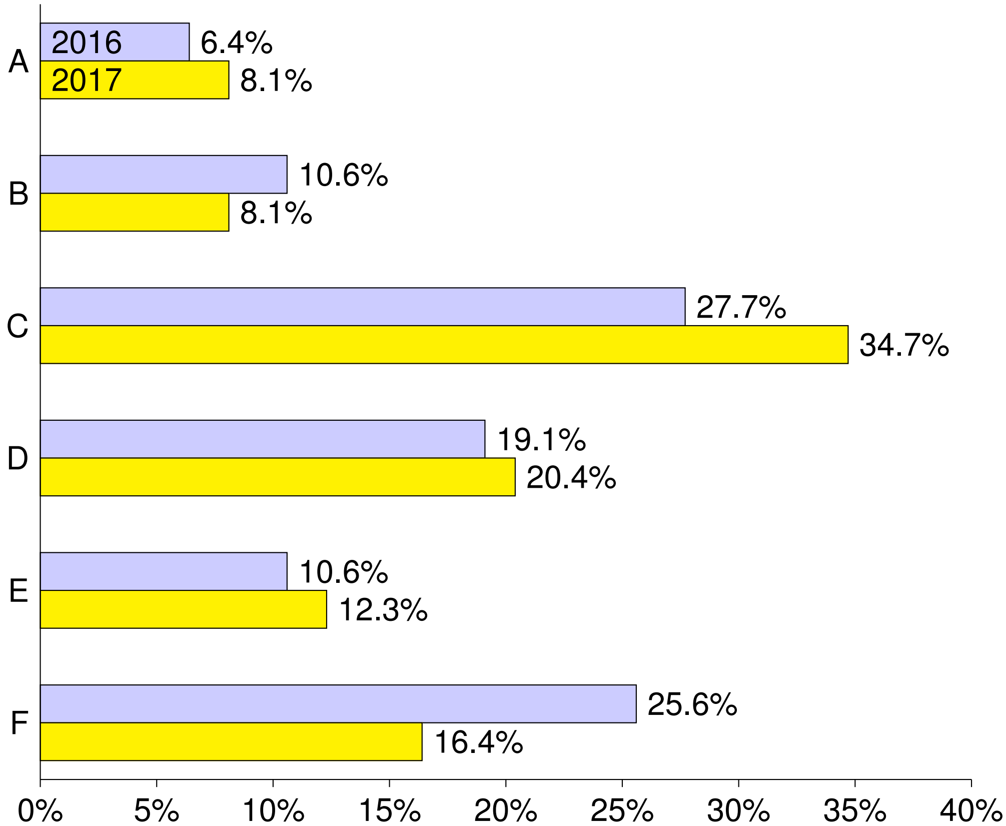

FIGURE 1. Grades distribution in 2016 and 2017

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Appendix A. Examples of Handouts

MAT220 - 1. Review of basic concepts

Sofia Tirabassi

Learning Outcomes:
At the end of today lecture you will remember the basic concept of set, function, cartesiant product of sets... which you encountered in earlier classes and which will be used through the whole semester.
Instructions: Hello and Welcome to MAT220! Gather in group (preferabily with people you do not know) and answer together the following questions.

1. Write the definition of set.
2. What is the cartesian product $A \times B$ of two sets?
3. Write the definition of function between two sets.
4. When is a function injective (1-1) surjective (onto) or bijective?
5. Given two functions $f : A \to B$ and $g : B \to C$, what is the composition $g \circ f$.
6. Let T be the set $\{1, 2, 3\}$. How many bijective functions $f : T \to T$ are there? List them using the notation just explained!
7. If T is a set of n elements, how many bijective functions $f : T \to T$ are there?
8. Is the composition of two bijective functions still bijective? Argument your answer.
9. Compose the following functions $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$. Meaning write down $f \circ g$ and $g \circ f$.
10. Find the inverse of $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$
11. Given three (bijective) functions, f, g and $h : T \to T$, what can you say of the two functions $(f \circ g) \circ h$ and $f \circ (g \circ h)$?
12. Given two (bijective) functions, f, $g : T \to T$, what can you say of the two functions $f \circ g$ and $g \circ f$?

13. If I say “binary operation” what do you think of? Try to give a definition and make examples. Find a binary operation on S_3 the set of bijective functions $T \to T$.

Reading assignment for next time: Read section 4 of the textbook. In particular know the content of Definition 4.1. Read the paragraph on multiplication tables (from pg. 43 til the end of the section).

MAT220 - 12. Group Actions and Burnside's Formula

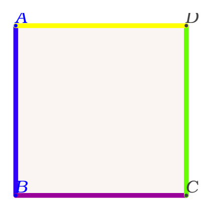
Sofia Tirabassi

Learning Outcomes:

1. You will learn to apply Burnisde formula's to various problems
2. You will learn how to compute the quotiens of various groups.
3. You will learn the proof and applications of the fundameteal theorem of isomorphism.

Instuctions: Gather in groups and answer together the following questions. Choose a spokeman for the group.

1. Show that $\sigma \cdot i = \sigma(i)$ define an action of S_n on $X := \{1, 2, \ldots, n\}$.
2. Consider the following square whose side are colored in 4 differents colors, chosen out of k. Let X be the set of coloring and let D_4 act on X via the corresponding symmetry of the square.

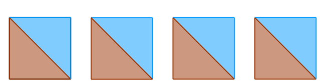


   (a) For any permutation σ in D_4, find X_σ.

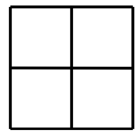
   (b) Suppose $k = 4$ how many differents orbits are there?

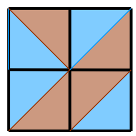
   (c) And for any k?

3. (**This was a past exam question!!!**) We place four identical square pieces



in the following figure:



Example: 

Let X be a set of all possible patterns that we can make in this way. The group of symmetries of the big square is $D_4 = \{e, \rho, \rho^2, \rho^3, \tau_1, \tau_2, \mu_1, \mu_2\}$ where ρ is the rotation on 90^{o}, τ_1 and τ_2 are the reflections with respect to the vertical and, respectively, the horizontal axes of symmetry, and μ_1, μ_2 are the reflections with respect to the diagonals. The group D_4 acts on the set X.

(a) Draw all elements of the orbit of the example given at the beginning of the Problem. Draw the elements of the fixed point set X_ρ.

(b) Explain why $|X| = 256$, $|X_{\rho^2}| = 16$ and $|X_{\mu_1}| = 16$.

(c) Calculate the number of orbits of the action of D_4 on the set X.

(d) Decide whether some of the patterns in X has a symmetry group that is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

4. Let X be the 720 different markings of faces of a cube using from one to six dots. Let G be ther group of 24 rotations of the cube. Compute the number of the orbits of X.

5. Suppose that the face of a cube are colored chosing form k different colors allowing repetitions. Let X be the number of colorig. Let G be the group of the 24 rotations of the cube, and suppose that G acts on X. Count the number of orbits of the actions. **Hint:** The 24 rotations of the cube consist of the identity, of 9 rotations which leaves a pair of opposite faces fixed, 8 that leaves a pair of opposite vertex inveriants, and 6 that leaves a pair of opposite edges invariant.

Reading Assignment for next time: Sections 18, 19.

Appendix B. Examples of Exercise sheets

MAT220 - First Mandatory Exercise Sheet

Sofia Tirabassi

1. Let $S = \mathbb{R}\setminus\{-1\}$ with the following operation $a * b = a + b + ab$
   (a) is $*$ a binary operation on S?
   (b) is it associative?
   (c) is there an identity element?
   (d) is there an inverse for every element?
   (e) is $(S, *)$ a group? Is it abelian?
   (f) find the solution of the equation $3 * x * 2 = 4$.

2. For everyone of the following groups find their order and the order of each one of their elements
$$\mathbb{Z}_4, \quad \mathbb{Z}_4 \times \mathbb{Z}_2, \quad S_3, \quad D_4, \quad \mathbb{Z}.$$

3. Prove that every nonabelian group G has order at least 6. (**Hint:** Suppose by contradiction that the group is not abelian, then there are two element a, b with $ab \neq ba$. Show that all the elements of $S = \{e, a, b, ab, ba\}$ are distinct and deduce that the order of G is at least 5. Show that either $a^2 \notin S$ or $a^2 = e$, if the latter holds then $aba \notin S$. Conclude.)

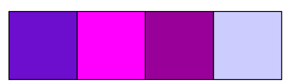
4. Given a group G and one of its elements a we introduce the set
$$C(a) := \{g \in G \mid ga = ag\}.$$
Show that $C(a)$ is a subgroup of G.

MAT220 - Fourth Mandatory Exercise Sheet

Sofia Tirabassi

Instuctions: Solve the following exercises. Your answer should be precises with ALL the details included. Every point in every question is worth 10pts

1. **(This was part of an exam question)** Suppose that A_4 acts on a strip of 4 square tiles by exchanging them. Suppose that the tiles are colored with k-colors allowing repetitions. How many orbits of the action are there?



2. **(This question could have been an exam question)** Let $R = (R, +_R, 0_R, \cdot_R, 1_R)$ be a commutative ring with unit. Consider the set $S := R \times R$ and endow it with the following two operations.

   1. $+_S$ is the usual component wise addition, that is $(a_1, b_1) +_S (a_2, b_2) := (a_1 +_R a_2, b_1 +_R b_2)$
   2. $(a_1, b_1) \cdot_S (a_2, b_2) = (a_1 \cdot_R a_2, a_1 \cdot_R b_2 +_R a_2 \cdot_R b_1)$

   (a) Show that $\cdot_S$ is associative.

   (b) Show that S with the two given operations is a ring.

   (c) Is it a commutative ring?

   (d) Let $(0, b) \in S$. Compute $(0, b)^2$. Is S an integrity domain?

   (e) Let $R = \mathbb{Z}_2$. Write down the table of addition and multiplication for S.

3. Find the characteristic of $\mathbb{Z}_4 \times \mathbb{Z}_6$.

4. If $f : \mathbb{Z} \to \mathbb{Z}$ is a morphism of rings with unity (i.e. a morphism of rings st $f(1) = 1$), then f is the identity.

Appendix C. Examples of Minutes

MAT220 -Minute questionnarie

Sofia Tirabassi

Return this (with answer) at the end of every lecture.

1. What was the most important thing today?
2. What did you understand best?
3. What did you understand least?

Remember that the learning lies
in the group disgussions. You have to
give more time to disgussion. ~~to~~

I think much confusion lies in
the question on what ~~[illegible]~~ ruels fro
Known arithmetics apply and what d
not apply to algebra.

I also think that you started off
way too advansed. in the last se

Mind also that many people di

[illegible]

MAT220 -Minute questionnarie

Sofia Tirabassi

Return this (with answer) at the end of every lecture.

1. What was the most important thing today?
2. What did you understand best?
3. What did you understand least?

1) Definition of group.

2) Working out if something is a group.

3) ~~Keeping up with order of elements~~
~~Sometimes, But I think I understood it ok~~
~~And~~ differentiating between the terms

- Associative

and

- Commutative

I mix them sometimes....

MAT220 -Minute questionnarie

Sofia Tirabassi

Return this (with answer) at the end of every lecture.

1. What was the most important thing today?
2. What did you understand best?
3. What did you understand least?

(1) Group theory - stuff

(2) Binary operations

(3) How to check if a group i
abelian.

MAT220 -Minute questionnarie

Sofia Tirabassi

Return this (with answer) at the end of every lecture.

1. What was the most important thing today? Understanding the natu
2. What did you understand best? The nature of identity e
3. What did you understand least? Proving that an operati
is associative.
Proving not is easier

I really like the kahoot session.
It does test our understanding wh
at the same time giving the theory

MAT220 -Minute questionnarie

Sofia Tirabassi

Return this (with answer) at the end of every lecture.

1. What was the most important thing today? Groups. Order matters
2. What did you understand best? Assosativity
3. What did you understand least? Some Kanoot questions

Appendix D. Final Exam

ENGLISH

UNIVERSITETET I BERGEN
Det matematisk-naturvitenskapelige fakultet

Eksamen i emnet MAT220/MAT644 - Algebra

Thursday June 1st 2017, 09-14

Allow: Allowed help resources: Calculator in accordance with the Faculty regulations.
The exam set has 2 pages. All the sub-questions carry the same weight.

All answers must be explained.

Oppgave 1

Let G be a group with operation $\star$. In this exercise you will prove Caley's theorem.

(a) Consider the symmetric group over G, S_G. Describe its elements and its operation.

(b) Given an element $g \in G$, consider the function $\lambda_g : G \to G$ defined by $\lambda_g(h) = g \star h$. Show that $\lambda_g \in S_G$.

(c) Consider the function $f : G \to S_G$ which to any g assign the element λ_g defined in (b). Suppose that $G = \mathbb{Z}_3$, compute $f(1)$ and $f(2)$.

(d) Let again G be any group, show that the function f defined in (c) is a group homomorphism.

(e) Compute the kernel of f and deduce that f is injective.

Oppgave 2

Let a group G act on a set X.

(a) Take an element x in X and g an element of G. Give the definitions of the set of fixed points X_g, and of the orbit Gx.

(b) Let $G_x := \{g \in G \mid g \cdot x = x\}$. Show that it is a subgroup of G.

Let G be the group of all the isometry of the plane. Take x and y two distinct point in the plane.

(c) Describe the elements of G_x. Find the number of elements in $G_x \cap G_y$.

(c) Take τ a non-trivial glide reflection and let$H = \langle \tau \rangle$ be the subgroup generated by τ. Describe the orbit of Hx.

Oppgave 3

Determine wether the following statements are true or fale.

(a) Consider the set $\mathbb{R}^*$ with the following binary operation $a \star b = |a|b$. The operation $\star$ does not have an identity element.

(b) Given a ring R and two of its ideals I_1 and I_2, then their intersection $I_1 \cap I_2$ is an ideal.

Oppgave 4 – Ringer

Let k be the field $\mathbb{Z}_2$ and denote its operations by $+$ and $\cdot$ respectively. Now consider the set

$$R := k \times k$$

with the following operations $\oplus$ and $\odot$. The operation $\oplus$ is just the usual componentwise addition in the cartesian product, that is

$$(x_1, x_2) \oplus (y_1, y_2) := (x_1 + y_1, x_2 + y_2),$$

while $\otimes$ is given by

$$(x_1, x_2) \odot (y_1, y_2) := (x_1 \cdot y_1, x_1 y_2 + x_2 y_2).$$

(a) Write the multiplication table for $\odot$

(b) Show that R with the aformetioned operation is a commutative ring with unity.

(c) Explain why R is not a field.

(d) Let $I = \{(0, x) \mid x \in k\}$. Show that is an ideal of R.

(e) Explain why the quotient R/I is a field.

Oppgave 5 – Kroppsutvidelser

Let $f(x) = x^3 + x + 1 \in \mathbb{Z}_5[x]$.

(a) Explain why $F = \mathbb{Z}_5[x]/\langle f \rangle$ is a field.

(b) We have that $F = \mathbb{Z}_5(\alpha)$ for $\alpha = x + \langle f \rangle$ in F. Use α to write a basis for F over $\mathbb{Z}_5$. Find the number of elements in F.

(c) Show that $\alpha^4 + \alpha = \alpha^2 + 1$. Express α^5 in the basis found in (b).

(d) Calculate $(x^3 + x + 1) : (x + 4)$ in $\mathbb{Z}_5[x]$ with remainder. Express $(\alpha + 4)^{-1}$ in the basis in (b).

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