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Kinetic Equation for an Electron Gas (Non-Neutral Plasma) in Strong Fields and Inhomogenities

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Abstract.

The first two equations of the BBGKY-hierarchy are discussed and solved in order to derive a kinetic equation for an electron gas (non-neutral plasma) where strong electric and magnetic fields as well as inhomogenities are taken into account on scales relevant for collisions between particles. The gyrotropic assumption is not made. The magnetic field and the inhomogenities are shown to have special effects on the collision terms. A strong magnetic field approximation is then made in order to simplify the collision term, and a new, proper collision term has been found when a strong magnetic field is present.

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I. Introduction.

It is possible to generate a pure electron gas on a neutral background of He , say, in a cylindrical tube with an axial magnetic field and reflecting ends, Malmberg & de Grassie (1975) and in particular measure the diffusion of the electrons across the magnetic field towards the walls, de Grassie, Malmberg & Douglas (1976). Taking into account classical collisions between electrons and neutrals a theoretical interpretation of the diffusion that fits well with some measurements has been obtained, Douglas & O'Neil (1976), though other processes may be important, de Grassie et al. and de Grassie and Malmberg (1977).

This paper is a study of how the electron - electron collisions may be taken into account. In the parameter range used till now such collisions can be shown to be less important than electron - neutral collisions. However, lowering the neutral gas pressure (density) by several orders of magnitude may drastically alter this picture and the effects of electron - electron collisions are of interest. The question then is if these collisions can be described by ordinary collision terms, for instance by Boltzmann or Landau collision terms. Table I shows how some typical plasma parameters vary with electron temperature from 1eV and downwards for two values of the magnetic field holding the electron density fixed. We note that this fixed electron density $2 \cdot 10^6/cm^3$, the temperature 1eV and a magnetic field strength of order 100 G are typical in the works of Malmberg & de Grassie, de Grassie et al. and Douglas & O'Neil. The lowering

of the electron temperature so that eventually the electron gas may liquefy and crystallize is of interest, Malmberg and O'Neil (1977). Down to temperatures as low as 10^{-3}eV the electron gas behaves "classically", i.e. $\lambda_L < n^{-1/3} < \lambda_D$. Since screening effects for a pure electron gas are much the same as for a neutral plasma, Davidson (1971), typical collisions occur over the range between λ_L and λ_D, and the table shows that quantum mechanical effects are negligible in this process even down to 10^{-4}eV. A striking feature from the table is that the Larmor radius r_e always is much less than the Debye length. Consequently ordinary Boltzmann or Landau collision terms are inadequate to describe the collisions since they do not include the effect of the gyration of electrons in collisions. Indeed, the table shows that for a very cool electron gas all interactions are influenced by gyrations. This motivates a study of the effects of a magnetic field on the collision integral for a gas of charged particles. We are interested in deriving a collision integral that holds for every strength of the magnetic field or every value of r_e relative to λ_L and λ_D. If possible we should like to simplify the collision integral so to be tempting for further studies. This might be obtained starting with existing collision integrals, for instance Rostoker (1960) Haggerty & de Sobrino (1964), Schram (1969), Montgomery, Turner & Joyce (1974) and Montgomery, Joyce & Turner (1974). However for different reasons we make our own derivation from the beginning: We do not want to make the usual gyrotropic assumption and we want to consider from the beginning the effects of electric fields as well as inhomogenities on the collision scales. For the non-neutral plasma these effects may be important.

We start to derive from the BBGKY-equations an equation which is a "generalized" Boltzmann equation. This equation takes into account the effects of strong electric and magnetic fields as well as a class of strong inhomogenities. The equation is derived so to be valid in both the "initial" and "kinetic" stages of Bogoliubov (1962). From this equation we derive in section III a corresponding equation making the (usual) assumption about weak interactions. The collision integral consists of two parts: One "velocity space collision integral" and one "gradient driven collision integral". The first is a generalization of the collision term of Montgomery et al. and reduces to Landau's collision integral in appropriate limits. The latter represents an effect that may correspond to terms derived for other models by Wu and others (see Wu 1966) by a method different from ours. Both parts simplify somewhat when in "first order" we may neglect all inhomogenities in the distribution function over the collision range. Assuming this we study the velocity space collision integral further in section IV. Emphasis is there laid on the strong magnetic field effects. Finally we propose a (new) velocity space collision integral that is simpler than the derived form in section III. It has all the properties of a "proper" collision integral.

The gradient driven collision integral is studied a bit further in Appendix C for the zero magnetic field case.

II. Derivation of a kinetic equation.

The starting equations for the one component gas are taken as the first two equations of the BBGKY-hierarchy for the one particle distribution function $f(\underline{r}_i,\underline{c}_i,t) = f(i,t)$, $i = 1,2,\ldots$ and the two particle correlation function $g(\underline{r}_i,\underline{c}_i,\underline{r}_j,\underline{c}_j,t) = g(i,j,t)$, $i \neq j = 1,2,\ldots$. When terms arising from "third" particles are ignored except contributions that can be absorbed in the electric field terms, we have

$$\frac{\partial f}{\partial t} + \underline{c}_1\cdot\frac{\partial f}{\partial \underline{r}_1} + \frac{e}{m}\,(\underline{E}_1+\underline{c}_1\times\underline{B}_1)\cdot\frac{\partial f}{\partial \underline{c}_1} = \frac{1}{m}\int d\underline{c}_2 d\underline{r}_2\, \frac{\partial \varphi_{12}}{\partial \underline{r}_1}(|\underline{r}_1-\underline{r}_2|)\cdot\frac{\partial g}{\partial \underline{c}_1}(1,2,t) \tag{1}$$

$$\frac{\partial g}{\partial t} + \underline{c}_1\cdot\frac{\partial g}{\partial \underline{r}_1} + \underline{c}_2\cdot\frac{\partial g}{\partial \underline{r}_2} + \frac{e}{m}(\underline{E}_1+\underline{c}_1\times\underline{B}_1)\cdot\frac{\partial g}{\partial \underline{c}_1} + \frac{e}{m}(\underline{E}_2+\underline{c}_2\times\underline{B}_2)\cdot\frac{\partial g}{\partial \underline{c}_2} - \frac{1}{m}\frac{\partial \varphi_{12}}{\partial \underline{r}_1}(|\underline{r}_1-\underline{r}_2|)\cdot\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right)g =$$

$$= \frac{1}{m}\frac{\partial \varphi_{12}}{\partial \underline{r}_1}(|\underline{r}_1-\underline{r}_2|)\cdot\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right)f(1,t)f(2,t) \tag{2}$$

Because of this restriction these equations may lead to generalizations of the usual Boltzmann- and Landau equations, not to a generalization of the Balescu-Lenard equation. The subscripts on $\underline{E}$ and $\underline{B}$ indicate the coordinates they are evaluated at. e designates the charge on the particles that takes on a negative value for the electrons. $\varphi_{12}(|\underline{r}_1-\underline{r}_2|)$ is the (Coulomb) interaction potential between (like) particles 1 and 2 .

To circumvent the great difficulties in attacking the equations in general we first observe the following:

(i) For space and time coordinates that are relevant for a collision the right hand side of Eq. (1) is small (compared to terms with $\underline{E}_1$ and $\underline{B}_1$ and inhomogenities that may be large).

(ii) Only a g from Eq. (2) describing the evolution of g in a collision is necessary on the right hand side of Eq. (1).

Therefore we may proceed as follows to derive a kinetic equation that takes collisions into account: First we solve

$$\frac{\partial f}{\partial t} + \underline{c}_1 \cdot \frac{\partial f}{\partial \underline{r}_1} + \frac{e}{m}(\underline{E}_1 + \underline{c}_1 \times \underline{B}_1) \cdot \frac{\partial f}{\partial \underline{c}_1} = 0 \tag{3}$$

and substitute this result for f into the right hand side of Eq. (2). We then solve Eq. (2) for g and substitute that result into the right hand side of Eq. (1), which then constitutes the desired (kinetic) equation. Following this procedure it is clear that we do not make explicitly the Bogoliubov functional assumption but rather derive that relationship for this special case. However, following this prescription we meet serious difficulties at the first step: to solve Eq. (3) which is the Vlasov equation. $\underline{E}$ and $\underline{B}$ are given from the Maxwell equations and therefore are functionals of f by space charges and currents. To circumvent this difficulty we here assume space charges and currents are nearly uniform and stationary over scales for a collision, i.e. over distances less then λ_D and times less then $1/\omega_P$ where ω_P is the plasma frequency. Assuming this we may solve Eq. (3) with $\underline{E}$ and $\underline{B}$ uniform and stationary. Formally we may

put the solution of Eq.(3) in the form

$$f(\underline{r}_1,\underline{c}_1,t) = S_{-t}(1)f(\underline{r}_1,\underline{c}_1,t = 0) \tag{4}$$

where $S_{-t}(i)$ is a "streaming" operator similar to those introduced by Bogoliubov: It has the property to transform particle i's position and velocity coordinates backward a time $-t$ according to the equations of motion

$$\begin{aligned} \frac{d\underline{r}_i}{dt} &= \underline{c}_i \\ \frac{d\underline{c}_i}{dt} &= \frac{e}{m}(\underline{E} + \underline{c}_i \times \underline{B}) \end{aligned} \tag{5}$$

(Since we here operate on scales relevant for a collision, we leave out the subscripts on $\underline{E}$ and $\underline{B}$). Substituting then Eq.(4) into Eq.(2) and solving for g, assuming that $g = 0$ at time $t = 0$, then gives, cf. Appendix A,

$$g(1,2,t) = S_{-t}(1,2)f(1,t=0)f(2,t=0) - S_{-t}(1)S_{-t}(2)f(1,t=0)f(2,t=0) \tag{6}$$

Here $S_{-t}(1,2)$ is another streaming operator that transforms particle 1 and 2's position and velocity coordinates backward a time $-t$ according to the equations of motion in a collision:

$$\begin{aligned} \frac{d\underline{r}_i}{dt} &= \underline{c}_i \\ \frac{d\underline{c}_i}{dt} &= \frac{e}{m}(\underline{E} + \underline{c}_i \times \underline{B}) - \frac{1}{m}\frac{\partial\varphi_{ij}}{\partial\underline{r}_i}(|\underline{r}_i - \underline{r}_j|) \quad , \; i \neq j \; , \; i,j = 1,2 \end{aligned} \tag{7}$$

Thus the two particle correlation builds up from $t = 0$ according to the departure of particle motion in a collision from particle motion due to "external" fields only. Substituting now Eq. (6) back into Eq. (1) gives the following equation

$$\frac{\partial f}{\partial t} + \underline{c}_1 \cdot \frac{\partial f}{\partial \underline{r}_1} + \frac{e}{m}(\underline{E}_1 + \underline{c}_1 \times \underline{B}) \cdot \frac{\partial f}{\partial \underline{c}_1} = \frac{1}{m} \int d\underline{r}_2 d\underline{c}_2 \frac{\partial \varphi_{12}}{d\underline{r}_1}(|\underline{r}_1 - \underline{r}_2|) \cdot \frac{\partial}{\partial \underline{c}_1}\Big(S_{-t}(1,2) - S_{-t}(1)S_{-t}(2)\Big) f(1,t=0) f(2,t=0) \qquad (8)$$

Here $\underline{E}_1$ and $\underline{B}_1$ may be non-uniform and non-stationary on scales longer than the collision scales. The right hand side, whose form was deduced on the assumption that f obeys Eq. (3), takes care of the collision effects on the evolution of f. In Eq. (8) the time runs from $t = 0$, and for $0 \leq t \lesssim \tau_c$, where τ_c is a typical time for a collision, the equation describes the building up of what may be a (collisional) kinetic equation, i.e. for $t \gg \tau_c$ the collisional effects on the right hand side of Eq. (8) may approach an almost time constant level and we are in the kinetic stage. This would be the case, for instance, when neglecting the effects of external fields and inhomogenities on the collision scales. Thus Eq. (8) is more general than a traditional kinetic equation. The effect of a $g(t=0) \neq 0$ could easily be added, too.

III. Weak interaction approximation.

We now make the usual assumption that weak interactions

between charged particles are the dominant ones. We find an approximate solution of Eq. (7) in the following way: The position coordinates in the interaction force term are taken as given by the operator $S_{-t}(i)$, i.e., instead of Eq. (7) we solve:

$$\frac{d\underline{r}_i}{dt} = \underline{c}_i$$

$$\frac{d\underline{c}_i}{dt} = \frac{e}{m}(\underline{E}+\underline{c}_i\times\underline{B}) - \frac{1}{m} S_t(i)S_t(j) \frac{\partial\varphi_{ij}}{\partial\underline{r}_i} (|\underline{r}_i(0) - \underline{r}_j(0)|) \tag{7'}$$

Therefore we must solve Eq. (5) first: Locally, let the z-axis be directed along the magnetic field and x- and y-axes perpendicular to it. We then get

$$S_t(i)\underline{c}_i = \begin{cases} \left(c_{ix} - \frac{E_y}{B}\right)\cos \Omega t + \left(c_{iy} + \frac{E_x}{B}\right)\sin \Omega t + \frac{E_y}{B} \\ -\left(c_{ix} - \frac{E_y}{B}\right)\sin \Omega t + \left(c_{iy} + \frac{E_x}{B}\right)\cos \Omega t - \frac{E_x}{B} \\ c_{iz} + \frac{e}{m} tE_z \end{cases} \tag{9}$$

Here $\Omega = eB/m$ and we have $\underline{c}_i(t=0) = \underline{c}_i$. Integrating Eq. (9) gives

$$S_t(i)\underline{r}_i = \underline{r}_i + \begin{cases} \frac{1}{\Omega}\left(c_{ix} - \frac{E_y}{B}\right)\sin \Omega t - \frac{1}{\Omega}\left(c_{iy} + \frac{E_x}{B}\right)(\cos \Omega t-1) + t\,\frac{E_y}{B} \\ \frac{1}{\Omega}\left(c_{ix} - \frac{E_y}{B}\right)(\cos \Omega t-1) + \frac{1}{\Omega}\left(c_{iy} + \frac{E_x}{B}\right)\sin \Omega t - t\,\frac{E_x}{B} \\ c_{iz}t + \frac{1}{2}t^2 \frac{e}{m} E_z \end{cases} \tag{10}$$

We here have $\underline{r}_i(t=0) = \underline{r}_i$.

From Eq.(10) we then derive

$$S_t(i)S_t(j)(\underline{r}_i-\underline{r}_j) = \underline{r}_i - \underline{r}_j + \begin{cases} \frac{1}{\Omega} c_{ijx}\sin \Omega t - \frac{1}{\Omega} c_{ijy}(\cos \Omega t-1) \\ \frac{1}{\Omega} c_{ijx}(\cos \Omega t-1) + \frac{1}{\Omega} c_{ijy}\sin \Omega t \\ c_{ijz}t \end{cases} \tag{11}$$

where $\underline{c}_{ij} = \underline{c}_i - \underline{c}_j$ is the relative velocity.

We are now ready to solve Eq.(7'). For shorthand we set

$$\underline{\omega}_{12}(t) = S_t(1)S_t(2)\,\frac{\partial\varphi_{12}}{\partial\underline{r}_1}\,(|\underline{r}_1(0) - \underline{r}_2(0)|) \tag{12}$$

and then have from the last part of Eq.(7') (with $i=1$, $j=2$)

$$S_t(1,2)\underline{c}_1 = S_t(1)\underline{c}_1 + \Delta\underline{c}_1(t) \tag{13}$$

where

$$\Delta\underline{c}_1(t) = \int_0^t d\tau\, \underline{\omega}_{12}(\tau)\cdot \begin{pmatrix} -\cos \Omega(t-\tau) & \sin \Omega(t-\tau) & 0 \\ -\sin \Omega(t-\tau) & -\cos \Omega(t-\tau) & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{14}$$

Integrating Eq.(13) we solve the first part of Eq.(7')

$$S_t(1,2)\underline{r}_1 = S_t(1)\underline{r}_1 + \Delta\underline{r}_1(t) \tag{15}$$

where

$$\Delta \underline{r}_1(t) = \int_0^t d\tau \int_0^\tau d\tau' \, \underline{\omega}_{12}(\tau') \cdot \begin{pmatrix} -\cos\Omega(\tau-\tau') & \sin\Omega(\tau-\tau') & 0 \\ -\sin\Omega(\tau-\tau') & -\cos\Omega(\tau-\tau') & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{16}$$

Letting $1 \longleftrightarrow 2$ in Eqs. (13) and (15) we get the solution for particle "2" . Substituting these results back into Eq. (6) we have

$$g(1,2,t) = f\left(S_{-t}(1)\underline{r}_1 + \Delta\underline{r}_1(-t),\ S_{-t}(1)\underline{c}_1 + \Delta\underline{c}_1(-t),\ t = 0\right) \times$$

$$\times\ f\left(S_{-t}(2)\underline{r}_2 + \Delta\underline{r}_2(-t),\ S_{-t}(2)\underline{c}_2 + \Delta\underline{c}_2(-t),\ t = 0\right) -$$

$$- f\left(S_{-t}(1)\underline{r}_1,\ S_{-t}(1)\underline{c}_1,\ t = 0\right) f\left(S_{-t}(2)\underline{r}_2, S_{-t}(2)\underline{c}_2, t=0\right)$$

Due to the weak interaction $\Delta\underline{c}_1(-t)$ may be considered small for every t . $\Delta\underline{r}_1(-t)$ on the other hand may grow with t . Assuming for a moment that t is finite, we formally make a series expansion in the small terms, retaining only first order terms. The result is

$$g(1,2,t) = \left\{\Delta\underline{c}_1(-t)\cdot\left(\frac{\partial}{\partial(S_{-t}(1)\underline{c}_1)} - \frac{\partial}{\partial(S_{-t}(2)\underline{c}_2)}\right) + \right.$$

$$\left. + \Delta\underline{r}_1(-t)\cdot\left(\frac{\partial}{\partial(S_{-t}(1)\underline{r}_1)} - \frac{\partial}{\partial(S_{-t}(2)\underline{r}_2)}\right)\right\} S_{-t}(1)S_{-t}(2)f(1,t=0)f(2,t=0) \tag{17}$$

Use has been made of the property

$$\Delta\underline{r}_1(t) = -\Delta\underline{r}_2(t)$$

$$\Delta\underline{c}_1(t) = -\Delta\underline{c}_2(t)$$

Eq.(17) may be transformed further, cf. Appendix B, so that (explisitly) time growing terms cancel out. The result is then substituted into Eq.(1) giving the new equation

$$\frac{\partial f}{\partial t} + \underline{c}_1 \cdot \frac{\partial f}{\partial \underline{r}_1} + \frac{e}{m}(\underline{E}_1 + \underline{c}_1 \times \underline{B}_1)\cdot\frac{\partial f}{\partial \underline{c}_1} = C^V_{ee} + C^P_{ee} \tag{18}$$

where

$$C^V_{ee} = \frac{1}{m}\frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{x}d\underline{c}_2 \frac{\partial \varphi_{12}}{\partial \underline{x}} \int_0^t d\tau S_{-\tau}(1)S_{-\tau}(2)\left(\frac{1}{m}\frac{\partial \varphi_{12}}{\partial \underline{x}}(|\underline{x}|)\right)\cdot$$

$$\cdot \frac{\partial}{\partial \tau}\begin{pmatrix} \frac{1}{\Omega}\sin\Omega\tau & \frac{1}{\Omega}(\cos\Omega\tau - 1) & 0 \\ -\frac{1}{\Omega}(\cos\Omega\tau-1) & \frac{1}{\Omega}\sin\Omega\tau & 0 \\ 0 & 0 & \tau \end{pmatrix}\cdot\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right)f(1,t)f(2,t) \tag{19}$$

$$C^P_{ee} = \frac{1}{m}\frac{\partial}{\partial \underline{c}_1}\cdot \int d\underline{x}d\underline{c}_2 \frac{\partial \varphi_{12}}{\partial \underline{x}} \int_0^t d\tau\ S_{-\tau}(1)S_{-\tau}(2)\left(\frac{1}{m}\frac{\partial \varphi_{12}}{\partial \underline{x}}(|\underline{x}|)\right)\cdot$$

$$\cdot\begin{pmatrix} \frac{1}{\Omega}\sin\Omega\tau & \frac{1}{\Omega}(\cos\Omega\tau - 1) & 0 \\ -\frac{1}{\Omega}(\cos\Omega\tau-1) & \frac{1}{\Omega}\sin\Omega\tau & 0 \\ 0 & 0 & \tau \end{pmatrix}\cdot\left(\frac{\partial}{\partial \underline{r}_1} - \frac{\partial}{\partial \underline{r}_2}\right)f(1,t)f(2,t) \tag{20}$$

Here $\underline{x} = \underline{r}_1 - \underline{r}_2$. $S_{-\tau}(1)$ and $S_{-\tau}(2)$ operate only on the terms inside the parentheses immediately following them.

Eq.(18) is a weak interaction approximation to Eq.(8). It shows more explisitly how the collisional effects build up from $t = 0$.

This buildup is given by the τ-integrals as $t \to \infty$ in the upper integration limits. C_{ee}^{V}, a velocity space collision integral, describes the diffusion of particles in velocity space due to collisions. It corresponds to traditional collision terms. In addition to this, C_{ee}^{P}, a gradient driven collision integral, shows that inhomogenities may have a collisional effect too.

We observe that C_{ee}^{P} is roughly of order $\tau_c c_m/L$ as compared to C_{ee}^{V}. This may be of order one for strong inhomogenities in f. (Such inhomogenities must be limited to be consistent with the assumption of (nearly) uniform and stationary fields on the collision scales). When $\tau_c c_m/L \ll 1$ we may simplify C_{ee}^{V} and C_{ee}^{P} somewhat. Making a series expansion of $f(2)$:

$$f(\underline{r}_2,\underline{c}_2,t) = f(\underline{r}_1-\underline{x},\underline{c}_2,t) = f(\underline{r}_1,\underline{c}_2,t)-\underline{x}\cdot\frac{\partial f}{\partial \underline{r}_1}(\underline{r}_1,\underline{c}_2,t) + \frac{\underline{x}\underline{x}}{2}:\frac{\partial^2 f}{\partial \underline{r}_1 \partial \underline{r}_1} +.$$

where the terms after the first are relatively small on collision scales, we can do the approximations

$$\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right) f(1,t)f(2,t) \approx \left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right) f(\underline{r}_1,\underline{c}_1,t)f(\underline{r}_1,\underline{c}_2,t)$$

and

$$\left(\frac{\partial}{\partial \underline{r}_1} - \frac{\partial}{\partial \underline{r}_2}\right) f(1,t)f(2,t) \approx f(\underline{r}_1,\underline{c}_2,t)^2 \frac{\partial}{\partial \underline{r}_1}\left(\frac{f(\underline{r}_1,\underline{c}_1,t)}{f(\underline{r}_1,\underline{c}_2,t)}\right).$$

Then in Eqs.(19) and (20) we can decouple the terms following the matrixes from the $\underline{x}$-integration. It should be noted that even this simplification applies to situations which may be far outside the range of the usual classical assumption of weak inhomogenities, i.e. $L >> \lambda_m$, where λ_m denotes the "effective" mean free path of particles.

Assuming $\tau_c c_m/L << 1$ we shall consider two cases: When $\underline{B} \to \underline{0}$ we expect that C^V_{ee} gives the collision term of Landau . In Appendix C we discuss both C^V_{ee} and C^P_{ee} in this limit case. In the next section we consider a simplification of C^V_{ee} when the magnetic field effects are important.

IV. Magnetic field effects on the collision term.

In this section we study C^V_{ee} when $\tau_c c_m/L << 1$ and the decoupling of terms applies as noted at the end of section III. First let us substitute the Maxwellian

$$f_M = n \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m}{2kT}(\underline{c} - \underline{U})^2\right]$$

into Eq.(19) and see if C^V_{ee} then vanishes. We get

$$C^V_{ee} =$$

$$= \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{x} d\underline{c}_2 \frac{\partial \varphi_{12}}{\partial \underline{x}} \int_0^t \frac{\partial}{\partial \tau}\left[S_{-\tau}(1)S_{-\tau}(2)\varphi_{12}(|\underline{x}|)\right] d\tau \left(-\frac{m}{kT}\right) f_M(1) f_M(2)$$

$$= \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{x} d\underline{c}_2 \frac{\partial \varphi_{12}}{\partial \underline{x}} \Big(S_{-t}(1)S_{-t}(2)\varphi_{12}(|\underline{x}|) - \varphi_{12}(|\underline{x}|)\Big)\left(-\frac{m}{kT}\right) f_M(1) f_M(2)$$

$$= \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{x} d\underline{c}_2 \frac{\partial \varphi_{12}}{\partial \underline{x}} S_{-t}(1)S_{-t}(2)\varphi_{12}(|\underline{x}|)\left(-\frac{m}{kT}\right) f_M(1)\, f_M(2)$$

In the last transformation we used the above mentioned decoupling. We observe from this that $C_{ee}^V \to 0$ as $t \to \infty$. This again reflects that only when $t \to \infty$ in the upper τ-integral limit does Eq.(19) correspond to a traditional collision term.

We now write C_{ee}^V as follows when $t \to \infty$ in the upper limit of the τ-integral :

$$C_{ee}^V = \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{c}_2 \, \underset{\sim}{\Phi} \, (\underline{c}_1,\underline{c}_2,\underline{B})\cdot\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right) f(\underline{r}_1,\underline{c}_1,t) f(\underline{r}_1,\underline{c}_2,t) \tag{21}$$

where

$$\underset{\sim}{\Phi} \, (\underline{c}_1,\underline{c}_2,\underline{B}) = $$
$$= \int d\underline{x} \, \frac{\partial \varphi_{12}}{\partial \underline{x}}(|\underline{x}|) \int_0^\infty d\tau \left(S_{-\tau}(1) S_{-\tau}(2) \, \frac{\partial \varphi_{12}}{\partial \underline{x}}(|\underline{x}|)\right) \begin{pmatrix} \cos \Omega\tau & -\sin \Omega\tau & 0 \\ \sin \Omega\tau & \cos \Omega\tau & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{22}$$

Making a Fourier transformation we get

$$\underset{\sim}{\Phi} = (2\pi)^3 \int d\underline{k} \, \underline{k} \, \underline{k} \, (\varphi(k))^2 \int_0^\infty d\tau \, \exp\left\{ i\underline{k}\cdot \left[\underline{c}_{12z}\tau + \right.\right.$$
$$\left.\left. + \frac{1}{\Omega}\left(\sin \Omega\tau \underline{c}_{12\perp} - \frac{\underline{c}_{12\perp}\times \underline{B}}{B}(1-\cos \Omega\tau)\right)\right] - \varepsilon\tau \right\} \begin{pmatrix} \cos \Omega\tau & -\sin \Omega\tau & 0 \\ \sin \Omega\tau & \cos \Omega\tau & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{23}$$

where

$$\varphi(k) = \left(\frac{1}{2\pi}\right)^3 \int d\underline{x} \, \varphi_{12} \, (|\underline{x}|) \, e^{-i\underline{k}\cdot\underline{x}} = \frac{e^2}{2\pi^2 k^2} \tag{24}$$

is the transform of the Coulomb-potential and $\varepsilon \to 0^+$ in the $\underline{k}$-integration. $\underline{c}_{12\perp}$ is the vector component of $\underline{c}_{12}$ transverse $\underline{B}$. Eq. (23) has a rather complicated form. However, as will be shown, collisions where the magnetic field effects are important may be more easily tractable. We proceed as follows. First we make an expansion of the exponential function in Eq. (23) retaining only the first few terms:

$$\exp\left\{i\underline{k}\cdot\left[\underline{c}_{12z}\tau + \frac{1}{\Omega}\left(\sin\Omega\tau\,\underline{c}_{12\perp} - \frac{\underline{c}_{12\perp}\times\underline{B}}{B}(1-\cos\Omega\tau)\right)\right] - \varepsilon\tau\right\} \approx$$

$$\approx e^{i\underline{k}\cdot\underline{c}_{12z}\tau-\varepsilon\tau}\left[1 + \frac{1}{\Omega}\,i\underline{k}\cdot\left(\sin\Omega\tau\,\underline{c}_{12\perp} - \frac{\underline{c}_{12\perp}\times\underline{B}}{B}(1-\cos\Omega\tau)\right) + \right.$$

$$\left. + \frac{1}{2}\left(\frac{1}{\Omega}\,i\underline{k}\cdot\left(\sin\Omega\tau\,\underline{c}_{12\perp} - \frac{\underline{c}_{12\perp}\times\underline{B}}{B}(1-\cos\Omega\tau)\right)\right)^2\right] \qquad (25)$$

This expansion we multiply with the matrix of Eq. (23). However, in this product we are not keeping all terms:

$$\exp\left\{i\underline{k}\cdot\left[\underline{c}_{12z}\tau + \frac{1}{\Omega}\left(\sin\Omega\tau\,\underline{c}_{12\perp} - \frac{\underline{c}_{12\perp}\times\underline{B}}{B}(1-\cos\Omega\tau)\right)\right] - \varepsilon\tau\right\}\cdot \begin{pmatrix} \cos\Omega\tau & -\sin\Omega\tau & 0 \\ \sin\Omega\tau & \cos\Omega\tau & 0 \\ 0 & 0 & 1 \end{pmatrix} \approx$$

$$\approx e^{i\underline{k}\cdot\underline{c}_{12z}\tau-\varepsilon\tau}\left[1+\frac{i\underline{k}}{\Omega}\cdot\left(\sin\Omega\tau\;\underline{c}_{12\perp}-\frac{\underline{c}_{12\perp}\times\underline{B}}{B}(1-\cos\Omega\tau)\right)\right]\begin{pmatrix}\cos\Omega\tau & -\sin\Omega\tau & 0\\ \sin\Omega\tau & \cos\Omega\tau & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$-\tfrac{1}{2}\left(\frac{\underline{k}}{\Omega}\cdot\left(\sin\Omega\tau\;\underline{c}_{12\perp}-\frac{\underline{c}_{12\perp}\times\underline{B}}{B}(1-\cos\Omega\tau)\right)\right)^2\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix} \tag{26}$$

The calculation of $\underset{\sim}{\Phi}$ using this approximate form is rather long and we only state the procedure: First all the τ-integrals are evaluated. Then the $\underline{k}$-integration is performed in cylindrical coordinates, $\underline{k}=(k_\perp\cos\varphi,\ k_\perp\sin\varphi,\ k_z)$. We get

$$\underset{\sim}{\Phi}=(2\pi)^3\pi\int dk_\perp k_\perp^3\left[p_1\begin{pmatrix}1 & 0 & -\frac{c_{12x}}{c_{12z}}\\ 0 & 1 & -\frac{c_{12y}}{c_{12z}}\\ -\frac{c_{12x}}{c_{12z}} & -\frac{c_{12y}}{c_{12z}} & \frac{c_{12\perp}^2}{c_{12z}^2}\end{pmatrix}+p_2\begin{pmatrix}0 & -1 & \frac{c_{12y}}{c_{12z}}\\ 1 & 0 & -\frac{c_{12x}}{c_{12z}}\\ -\frac{c_{12y}}{c_{12z}} & \frac{c_{12x}}{c_{12z}} & 0\end{pmatrix}\right.$$

(27)

where

$$p_1 = \frac{\pi}{|c_{12z}|}\left[\varphi\left(\left(k_\perp^2 + \frac{\Omega^2}{c_{12z}^2}\right)^{\frac{1}{2}}\right)\right]^2 \tag{28}$$

$$p_2 = P\int_{-\infty}^{\infty} \varphi(k)^2 \frac{dk_z}{k_z c_{12z}+\Omega} \tag{29}$$

Here P means the principal value of the integral. Before discussing the $k_\perp$-integration in Eq. (27) we show that C_{ee}^V from Eq. (21) with $\underset{\sim}{\Phi}$ from Eqs. (27)-(29) conserves particle number, momentum and kinetic energy and also, when acting alone, drives any distribution to a Maxwellian. To show this we first observe that $\underset{\sim}{\Phi}$ from Eqs. (27)-(29) has the following properties:

A) $\underset{\sim}{\Phi}(\underline{c}_1,\underline{c}_2) = \underset{\sim}{\Phi}(\underline{c}_2,\underline{c}_1)$

B) $\underset{\sim}{\Phi}(\underline{c}_1,\underline{c}_2)\cdot\underline{c}_{12} = \underline{c}_{12}\cdot\underset{\sim}{\Phi}(\underline{c}_1,\underline{c}_2) = \underline{0}$

C) $\underline{V}\cdot\underset{\sim}{\Phi}(\underline{c}_1,\underline{c}_2)\cdot\underline{V} \geq 0$ where the sign of equality holds if and only if $\underline{V} \parallel \underline{c}_{12}$. $(\underline{V} \neq \underline{0})$

Properties A) and B) are easily derived. Indeed, the properties hold for each of the two matrix-parts of $\underset{\sim}{\Phi}$ separately. Property C) follows from

$$\underline{V}\cdot\underset{\sim}{\Phi}\cdot\underline{V} = (2\pi)^3\pi\int_0^\infty dk_\perp k_\perp^3\, p_1\left[\left(V_x - \frac{c_{12x}}{c_{12z}}V_z\right)^2 + \left(V_y - \frac{c_{12y}}{c_{12z}}V_z\right)^2\right] \geq 0 .$$

Observe that the second matrix part of $\underset{\sim}{\Phi}$ does not contribute here at all.

Now, particle conservation

$$\int C_{ee}^V\, d\underline{c}_1 = 0$$

follows directly from the form of C_{ee}^{V} . Momentum conservation tion

$$\int m\underline{c}_1\, C_{ee}^{V}\, d\underline{c}_1 = \underline{0}$$

follows when using property A above. Kinetic energy conservation

$$\int \tfrac{1}{2}mc_1^2\, C_{ee}^{V}\, d\underline{c}_1 = 0$$

follows using property B . The Maxwellization property follows by an H-theorem showing that

$$\int (1 + \ln f)C_{ee}^{V}\, d\underline{c}_1 \leq 0$$

which readily follows using property C .

Thus C_{ee}^{V} from Eq. (21) with $\underset{\sim}{\Phi}$ from Eqs. (27)-(29) formally is a proper (velocity space) collision integral.

We now discuss the remaining $k_{\perp}$-integration in Eq. (27). Setting for the moment $\frac{1}{L}$ and $\frac{1}{\ell}$ as lower and upper integration limits, direct integration gives

$$\int_{\frac{1}{L}}^{\frac{1}{\ell}} dk_{\perp} k_{\perp}^3\, p_1 = \left(\frac{e^2}{2\pi^2}\right)^2 \frac{\pi}{|c_{12z}|}\, \kappa_1(\ell, L) \tag{30}$$

and

$$\int_{\frac{1}{L}}^{\frac{1}{\ell}} dk_{\perp} k_{\perp}^3\, p_2 = \left(\frac{e^2}{2\pi^2}\right)^2 \frac{\pi}{|c_{12z}|}\, \kappa_2(\ell, L) \tag{31}$$

where

$$\kappa_1(\ell, L) = \tfrac{1}{2}\ln\left[\frac{1 + \dfrac{r_e^2}{\ell^2}\dfrac{c_{12z}^2}{c_{\perp}^2}}{1 + \dfrac{r_e^2}{L^2}\dfrac{c_{12z}^2}{c_{\perp}^2}}\right] - \tfrac{1}{2}\frac{c_{12z}^2}{c_{\perp}^2}\,\frac{\dfrac{r_e^2}{\ell^2} - \dfrac{r_e^2}{L^2}}{\left(1 + \dfrac{r_e^2}{\ell^2}\dfrac{c_{12z}^2}{c_{\perp}^2}\right)\left(1 + \dfrac{r_e^2}{L^2}\dfrac{c_{12z}^2}{c_{\perp}^2}\right)} \tag{32}$$

$$\kappa_2(\ell,L) = -\tfrac{1}{2}\ \mathrm{sgn}\ e\ \frac{|c_{12z}|}{c_\perp}\ \frac{\left(\frac{r_e}{\ell} - \frac{r_e}{L}\right)\left(1 - \frac{r_e^2}{\ell L}\frac{c_{12z}^2}{c_\perp^2}\right)}{\left(1 + \frac{r_e^2}{L^2}\frac{c_{12z}^2}{c_\perp^2}\right)\left(1 + \frac{r_e^2}{\ell^2}\frac{c_{12z}^2}{c_\perp^2}\right)} +$$

$$+ \ \mathrm{sgn}\ e \left(\mathrm{Arctg}\left(\frac{r_e}{\ell}\frac{|c_{12z}|}{c_\perp}\right) - \mathrm{Arctg}\left(\frac{r_e}{L}\frac{|c_{12z}|}{c_\perp}\right)\right) \tag{33}$$

Here $r_e = \frac{c_\perp}{|\Omega|}$ is the Larmor radius ($c_\perp = (\frac{\pi kT}{m})^{\frac{1}{2}}$ is a characteristic relative particle speed transverse the magnetic field). sgn e is the sign of the electric charge. We observe that κ_1 is convergent when $L \to \infty$ and divergent when $\ell \to 0$. Thus a lower cut-off in the $k_\perp$-integration is not necessary for convergence. κ_2 on the other hand is finite in both limits $L \to \infty$ and $\ell \to 0$. However, the choice we do of the $k_\perp$ interval $\left[\frac{1}{L}, \frac{1}{\ell}\right]$ should be consistent with the approximation Eq. (25) which seems to be valid only when

$$\frac{k_\perp c_\perp}{|\Omega|} \lesssim 1 \ , \quad \text{i.e.} \quad k_\perp \lesssim \frac{|\Omega|}{c_\perp} = \frac{1}{r_e} \tag{34}$$

This part of the $k_\perp$-interaction band takes into account the collisions which are (strongly gyrating) not-winded and only partly the winded collisions (characterized by $k_\perp \gtrsim \frac{1}{r_e}$). Our κ_1 and κ_2 may nevertheless show what happens in the transition regime from not-winded to winded collisions. We consider this point first: Setting $L = \lambda_D$ in Eqs. (32) and (33), thereby taking account of the screening effect, despite that this is not necessary for convergence, we study κ_1 and κ_2 as functions of ℓ. When the magnetic field is

strong so that $r_e << \lambda_D$ $\kappa_1(\ell)$ in particular simplifies to

$$\kappa_1(\ell) \approx \tfrac{1}{2} \ln\left[1 + \frac{r_e^2}{\ell^2}\frac{c_{12z}^2}{c_\perp^2}\right] - \tfrac{1}{2}\frac{c_{12z}^2}{c_\perp^2} \frac{\frac{r_e^2}{\ell^2}}{\left(1 + \frac{r_e^2}{\ell^2}\frac{c_{12z}^2}{c_\perp^2}\right)} \tag{35}$$

When ℓ varies so that $r_e << \ell$ $(< L = \lambda_D)$ $\kappa_1(\ell)$ is small. Then only not-winded collisions are taken into account in the $k_\perp$-integration. As $\ell \to r_e$ (while $c_{12z}^2/c_\perp^2 \approx 1$) $\kappa_1(\ell)$ grows strongly. This shows that when collisions become winded they are much more effective than the not-winded ones. Due to the condition Eq.(34) ℓ should not pass below r_e in Eq.(35). However, it is likely that Eq.(35) shows some of the features of the actual behaviour when $\ell < r_e$: We get from Eq. (35) that $\kappa_1(\ell) \to \ln\left(\frac{r_e}{\ell}\frac{|c_{12z}|}{c_\perp} e^{-\frac{1}{2}}\right)$ when ℓ decreases below r_e . If we have $\lambda_L << r_e << \lambda_D$ and set ℓ equal to λ_L this logaritmic term may correspond to the logaritm in the collision term set up by Schram (1969) and by Montgomery, Joyce and Turner (1974). We shall later represent the winded collisions with the collision integral they derived for this strong magnetic field case, i.e. a modified Landau collision term with cut-off's in k at $\frac{1}{r_e}$ and $\frac{1}{\lambda_L}$ taking account of the collision range where the particles are almost straight-lined.

Returning to the not-winded collisions which our expansion procedure are best suited for the condition Eq.(34) must be considered together with $k_\perp$-values consistent with the weak inter-

action approximation. We take such collisions to be represented by

$$\frac{1}{\lambda_D} \leq k_\perp \leq \frac{1}{\lambda_L} \tag{36}$$

This choice is in accordance with the k-values used in traditional kinetic theory of weakly Coulomb-interacting particles. The conditions Eqs.(34) and (36) now determine L and ℓ. Discussing this we separate between the following three regimes of the magnetic field strength (expressed through r_e) :

$$\begin{gathered} r_e < \lambda_L \\ \lambda_L \leq r_e \leq \lambda_D \\ r_e > \lambda_D \end{gathered} \tag{37}$$

In the first case it is consistent to use $\frac{1}{\lambda_D}$ and $\frac{1}{\lambda_L}$ as lower and upper limits, i.e. all (weak) interaction is taken into account. All this interaction is of the not-winded type. In the second case we take $\frac{1}{\lambda_D}$ and $\frac{1}{r_e}$ as lower and upper integration limits. The left out interaction $k_\perp$-band from $\frac{1}{r_e}$ to $\frac{1}{\lambda_L}$ then corresponds to the winded collisions discussed above whose collision integral contribution we represent as stated there. In the third case there is no overlap of conditions Eqs.(34) and (36). In this case Eqs.(27) - (29) are inadequate. The gyration motion is so small that the whole collision integral may be represented by an ordinary Landau collision integral. With this in mind we get from Eqs.(32) and (33)

$$
\kappa_1 = \begin{cases}
\frac{1}{2}\ln\left[\dfrac{1+\dfrac{c_{12z}^2}{c_\perp^2}}{1+\dfrac{r_e^2}{\lambda_D^2}\dfrac{c_{12z}^2}{c_\perp^2}}\right] - \frac{1}{2}\dfrac{c_{12z}^2}{c_\perp^2}\,\dfrac{1-\dfrac{r_e^2}{\lambda_D^2}}{\left(1+\dfrac{c_{12z}^2}{c_\perp^2}\right)\left(1+\dfrac{r_e^2}{\lambda_D^2}\dfrac{c_{12z}^2}{c_\perp^2}\right)}\,, & \lambda_L < r_e < \lambda_D\,, \\[3ex]
\frac{1}{2}\ln\left[\dfrac{1+\dfrac{r_e^2}{\lambda_L^2}\dfrac{c_{12z}^2}{c_\perp^2}}{1+\dfrac{r_e^2}{\lambda_D^2}\dfrac{c_{12z}^2}{c_\perp^2}}\right] - \frac{1}{2}\dfrac{c_{12z}^2}{c_\perp^2}\,\dfrac{\dfrac{r_e^2}{\lambda_L^2}-\dfrac{r_e^2}{\lambda_D^2}}{\left(1+\dfrac{r_e^2}{\lambda_L^2}\dfrac{c_{12z}^2}{c_\perp^2}\right)\left(1+\dfrac{r_e^2}{\lambda_D^2}\dfrac{c_{12z}^2}{c_\perp^2}\right)}\,, & r_e \leq \lambda_L\,,
\end{cases}
\tag{38}
$$

and

$$
\kappa_2 = \begin{cases}
-\frac{1}{2}\operatorname{sgn}e\,\dfrac{|c_{12z}|}{c_\perp}\,\dfrac{\left(1-\dfrac{r_e}{\lambda_D}\right)\left(1-\dfrac{r_e}{\lambda_D}\dfrac{c_{12z}^2}{c_\perp^2}\right)}{\left(1+\dfrac{r_e^2}{\lambda_D^2}\dfrac{c_{12z}^2}{c_\perp^2}\right)\left(1+\dfrac{c_{12z}^2}{c_\perp^2}\right)} + \\
\quad + \operatorname{sgn}e\left(\operatorname{Arctg}\left(\dfrac{|c_{12z}|}{c_\perp}\right) - \operatorname{Arctg}\left(\dfrac{r_e}{\lambda_D}\dfrac{|c_{12z}|}{c_\perp}\right)\right)\,, & \lambda_L < r_e < \lambda_D\,, \\[3ex]
-\frac{1}{2}\operatorname{sgn}e\,\dfrac{|c_{12z}|}{c_\perp}\,\dfrac{\left(\dfrac{r_e}{\lambda_L}-\dfrac{r_e}{\lambda_D}\right)\left(1-\dfrac{r_e^2}{\lambda_L\lambda_D}\dfrac{c_{12z}^2}{c_\perp^2}\right)}{\left(1+\dfrac{r_e^2}{\lambda_D^2}\dfrac{c_{12z}^2}{c_\perp^2}\right)\left(1+\dfrac{r_e^2}{\lambda_L^2}\dfrac{c_{12z}^2}{c_\perp^2}\right)} + \\
\quad + \operatorname{sgn}e\left(\operatorname{Arctg}\left(\dfrac{r_e}{\lambda_L}\dfrac{|c_{12z}|}{c_\perp}\right) - \operatorname{Arctg}\left(\dfrac{r_e}{\lambda_D}\dfrac{|c_{12z}|}{c_\perp}\right)\right)\,, & r_e \leq \lambda_L\,.
\end{cases}
\tag{39}
$$

In table II and III we estimate κ_1 and $|\kappa_2|$ from Eqs.(38) and (39) numerically as functions of the magnetic field strength through r_e. For definiteness we have put $\lambda_D = 10^4 \lambda_L$, we have set $r_e = \lambda_L x$ and keep $c_{12z}^2/c_\perp^2$ fixed in three steps: As 10, 1 and 1/10. Commenting mainly on the values for κ_1 we observe the sensibility on the ratio $|c_{12z}|/c_\perp$. Thus when $|c_{12z}|$ increases κ_1 grows. Indeed we have

$$\kappa_1 \to \begin{cases} \ln \dfrac{\lambda_D}{r_e}, & \lambda_L < r_e < \lambda_D \\ \ln \dfrac{\lambda_D}{\lambda_L}, & r_e \leq \lambda_L \quad \text{as} \quad \dfrac{|c_{12z}|}{c_\perp} \to \infty \end{cases} \tag{40}$$

and also

$$\kappa_2 \to 0 \quad \text{as} \quad \frac{|c_{12z}|}{c_\perp} \to \infty \ .$$

This behaviour may be attributed to the fewer gyrations during the interactions, which make the collisions more effective. However, the collisions occur over shorter times when $|c_{12z}|$ increases tending to lower their effect. This we see from the factor $\frac{1}{|c_{12z}|}$ of Eqs.(30) and (31). Table II shows further that for each value of $|c_{12z}|/c_\perp$ κ_1 has a very characteristic variation with x : When x decreases from $x = 1$, κ_1 drops quite abruptly. Since Eqs.(38)-(39) then describe the whole weak interaction this shows how destructive the effect of an extremely strong magnetic field is on the collision frequency, say. When x increases from $x = 1$, κ_1 has a plateau over a wide range (Indeed, this plateau would continue infinitely if we did not cut off the $k_\perp$-integration at the lower limit,

$1/\lambda_D$, i.e. if we let $\frac{1}{\lambda_D} \to 0$.) This plateau means the following: Since the cut-offs in this case are at $\frac{1}{\lambda_D}$ and $\frac{1}{r_e}$, increasing x means that a more narrow $k_\perp$-band is taken into account at the remote collision-side of the band. However, this narrowing of the $k_\perp$-band is exactly counterbalanced by the less gyration motion of particles and gives the plateau in the κ_1-values. This plateau finally falls off slowly towards zero as $r_e \to \lambda_D$.

We notice that the drop of κ_1 when r_e becomes less than λ_L is sensitive to the particular choice of the cut-off $\frac{1}{\lambda_L}$. For such values of r_e κ_1 and κ_2 include all (weak) interactions. The effect of stronger interactions taking account of the departure of particle trajectories from them given by Eq.(10) may better estimate the collisional contribution in this regime. For Coulomb-interacting particles in traditional kinetic theory, using the Boltzmann collision term, Chapman and Cowling (1970), the effect of close encounters (i.e. when the impact parameter is less than the Landau length) falls off rapidly with decreasing impact parameter. Strongly gyrating particles along field lines a distance apart less than λ_L and having r_e's less than λ_L may have an effect on the collision term much less than this: Bound to the magnetic field lines as the particles are, all such collisions may be almost one-dimensional. Thus with λ_L neatly chosen κ_1 and κ_2 from Eqs.(38) and (39) may adequately describe the whole collisional contribution when $r_e < \lambda_L$.

On the basis of the foregoing derivations and discussion we propose the following (velocity space) collision integral:

$$C^V_{ee} = \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1}\cdot \int d\underline{c}_2\, \underset{\sim}{\Phi}\cdot \left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right) f(\underline{r}_1,\underline{c}_1,t)f(\underline{r}_1,\underline{c}_2,t)$$

where, in tensor form,

$$\underset{\sim}{\Phi} = \begin{cases} = 2\pi e^4 \ln\left(\frac{\lambda_D}{\lambda_L}\right) \dfrac{\underset{\sim}{I}c^2_{12} - \underline{c}_{12}\underline{c}_{12}}{c^3_{12}}\,, & r_e > \lambda_D \\[2ex] = 2\pi e^4 \left\{ \ln\left(\frac{r_e}{\lambda_L}\right) \dfrac{\underset{\sim}{I}\, c^2_{12} - \underline{c}_{12}\underline{c}_{12}}{c^3_{12}} + \right. \\ \quad + \kappa_1 \dfrac{1}{|c_{12\|}|}\left(\dfrac{B^2c^2_{12\|}\underset{\sim}{I} + \underline{B}\underline{B}(c^2_{12\perp} - c^2_{12\|}) - Bc_{12\|}(\underline{B}\,\underline{c}_{12\perp} + \underline{c}_{12\perp}\underline{B})}{B^2\, c^2_{12\|}}\right) + \\ \quad \left. + \kappa_2 \dfrac{1}{|c_{12\|}|}\left((\underline{c}_{12\perp}\times\frac{\underline{B}}{B})(P\frac{\underline{B}}{c_{12\|}B} - \frac{\underline{c}_{12\perp}}{c^2_{12\perp}}) - (P\frac{\underline{B}}{Bc_{12\|}} - \frac{\underline{c}_{12\perp}}{c^2_{12\perp}})(\underline{c}_{12\perp}\times\frac{\underline{B}}{B})\right)\right\} \\ & \lambda_L < r_e \le \lambda_D \\[2ex] = 2\pi e^4 \left\{ \kappa_1 \dfrac{1}{|c_{12\|}|}\left(\dfrac{B^2c^2_{12\|}\underset{\sim}{I} + \underline{B}\underline{B}(c^2_{12\perp} - c^2_{12\|}) - Bc_{12\|}(\underline{B}\underline{c}_{12\perp} + \underline{c}_{12\perp}\underline{B})}{B^2c^2_{12\|}}\right) + \right. \\ \quad \left. + \kappa_2 \dfrac{1}{|c_{12\|}|}\left((\underline{c}_{12\perp}\times \frac{\underline{B}}{B})(P\frac{\underline{B}}{c_{12\|}B} - \frac{\underline{c}_{12\perp}}{c^2_{12\perp}}) - (P\frac{\underline{B}}{Bc_{12\|}} - \frac{\underline{c}_{12\perp}}{c^2_{12\perp}})(\underline{c}_{12\perp}\times\frac{\underline{B}}{B})\right)\right\} \\ & r_e \le \lambda_L\,. \end{cases} \tag{41}$$

κ_1 and κ_2 in the appropriate ranges are given from Eqs. (38) and (39) ($c_{12z} = c_{12\|}$). P denotes that the principal value should be taken in the $c_{12\|}$-integration: We easily derive from Eqs. (38) and (39) that $\kappa_1 = O(c^4_{12\|})$ and $\kappa_2 = O(|c_{12\|}|)$ when $c_{12\|}$ is small. For the part involving κ_2 in Eq. (41) this may

introduce a divergence in the $c_{12\|}$-integration of the collision integral. Taking the principal value eliminates such a divergence. The effect is nearly the same as introducing a cut-off for small $|c_{12\|}|$ for this part of the collision term. On the other hand the interesting relaxation part of the collision term involving κ_1 is sufficiently regular for small $c_{12\|}$ that a similar cut-off is unnecessary.

We observe that the collision integral varies continuously with r_e : When $r_e > \lambda_D$ we have the usual Landau collision term. When r_e decreases below λ_D the ordinary Landau collision term continuously transforms into the modified Landau term, representing the winded collision, and in addition a new collision term shows up when $r_e \leq \lambda_D$ taking care of the not-winded collisions (we observe that both κ_1 and $\kappa_2 \to 0$ as $r_e \to \lambda_D$ in appropriate ranges). As $r_e \to \lambda_L$ the winded collision integral dies away. When r_e decreases below λ_L the collision integral stems from not-winded (weak) collisions and ultimately dies away as $r_e \to 0$. We note when $\lambda_L << r_e << \lambda_D$ the accordance with the result of Schram and Montgomery, Joyce and Turner: From the numerical values of table II and III it then follows that the modified Landau part dominates over the not-winded collision part of the collision integral.

We end with pointing at a feature the collision term expressions in each interval of r_e has in common: As $|c_{12\|}| \to \infty$ we get that $\underset{\sim}{\Phi}$ in each of the three ranges tends to the same asymptotic expression:

$$\underset{\sim}{\Phi} \sim 2\pi e^4 \ln \frac{\lambda_D}{\lambda_L} \frac{\underset{\sim}{I} - \frac{\underset{\sim}{BB}}{B^2}}{|c_{12\|}|} \quad \text{for large} \quad |c_{12\|}| \,.$$

Use has been made of Eq. (40) in the last two ranges of Eq. (41).

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Appendix A.

From Eqs. (2) and (4) we get

$$g(1,2,t) = S_{-t}(1,2)g(1,2,t=0) + \int_0^t d\tau\, S_{-\tau}(1,2)\left[\frac{1}{m}\frac{\partial\varphi_{12}}{\partial \underline{r}_1}\cdot\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right)f(1,t-\tau)f(2,t-\tau)\right] \tag{A1}$$

where $g(t=0) \to 0$ (sufficiently fast) as $|\underline{r}_1 - \underline{r}_2| \to \infty$, otherwise quite arbitrary. We now show that the τ-integrand can be written as a derivative with respect to τ : We have

$$\frac{\partial}{\partial\tau}\left[S_{-\tau}(1,2)f(1,t-\tau)f(2,t-\tau)\right] = \left(\frac{\partial S_{-\tau}\underline{r}_1}{\partial\tau}\cdot\frac{\partial}{\partial(S_{-\tau}\underline{r}_1)} f(S_{-\tau}\underline{r}_1, S_{-\tau}\underline{c}_1, t-\tau) + \frac{\partial S_{-\tau}\underline{c}_1}{\partial\tau}\cdot\frac{\partial}{\partial(S_{-\tau}\underline{c}_1)} f(S_{-\tau}\underline{r}_1, S_{-\tau}\underline{c}_1, t-\tau) + \frac{\partial}{\partial\tau} f(S_{-\tau}\underline{r}_1, S_{-\tau}\underline{c}_1, t-\tau)\right) f(S_{-\tau}\underline{r}_2, S_{-\tau}\underline{c}_2, t-\tau) + (1 \longleftrightarrow 2) \tag{A2}$$

Since

$$\frac{\partial}{\partial\tau} S_{-\tau}\underline{r}_i = - S_{-\tau}\underline{c}_i$$

$$\frac{\partial}{\partial\tau}(S_{-\tau}\underline{c}_i) = - S_{-\tau}\dot{\underline{c}}_i = S_{-\tau}\left(\frac{1}{m}\frac{\partial\varphi_{ij}}{\partial\underline{r}_i} - \frac{e}{m}(\underline{E} + \underline{c}_i \times \underline{B})\right)$$

we further get

$$\frac{\partial}{\partial\tau}\left[S_{-\tau}(1,2)f(1,t-\tau)f(2,t-\tau)\right] = S_{-\tau}\left[\left(-\underline{c}_1\cdot\frac{\partial}{\partial\underline{r}_1}f(\underline{r}_1,\underline{c}_1,t-\tau)+\left(\frac{1}{m}\frac{\partial\varphi_{12}}{\partial\underline{r}_1}-\right.\right.\right.$$

$$\left.\left.-\frac{e}{m}(\underline{E}+\underline{c}_1\times\underline{B})\right)\cdot\frac{\partial}{\partial\underline{c}_1}f(\underline{r}_1,\underline{c}_1,t-\tau)-\frac{\partial}{\partial(t-\tau)}f(\underline{r}_1,\underline{c}_1,t-\tau)\right)f(\underline{r}_2,\underline{c}_2,t-\tau)+$$

$$\left.+(1\longleftrightarrow 2)\right] = S_{-\tau}\left(\frac{1}{m}\frac{\partial\varphi_{12}}{\partial\underline{r}_1}\cdot\frac{\partial}{\partial\underline{c}_1}f(\underline{r}_1,\underline{c}_1,t-\tau)f(\underline{r}_2,\underline{c}_2,t-\tau)+\right.$$

$$\left.+\frac{1}{m}\frac{\partial\varphi_{12}}{\partial\underline{r}_2}\cdot\frac{\partial}{\partial\underline{c}_2}f(\underline{r}_2,\underline{c}_2,t-\tau)f(\underline{r}_1,\underline{c}_1,t-\tau)\right) = \tag{A3}$$

$$= S_{-\tau}\left(\frac{1}{m}\frac{\partial\varphi_{12}}{\partial\underline{r}_1}\cdot\left(\frac{\partial}{\partial\underline{c}_1}-\frac{\partial}{\partial\underline{c}_2}\right)f(1,t-\tau)f(2,t-\tau)\right)$$

Here use has been made of Eq. (3) at time $t - \tau$. The last expression is the integrand of Eq. (A1) and we then get

$$g(1,2,t) = S_{-t}(1,2)g(1,2,t=0) + S_{-t}(1,2)f(1,t=0)f(2,t=0) -$$

$$- S_{-t}(1)S_{-t}(2)f(1,t=0)f(2,t=0) .$$

Appendix B.

Transforming Eq. (17) we first observe that

$$\frac{\partial}{\partial(S_{-t}\underline{c}_1)}f(S_{-t}\underline{r}_1,S_{-t}\underline{c}_1,t=0) = S_{-t}\frac{\partial}{\partial\underline{c}_1}f(\underline{r}_1,\underline{c}_1,t=0) =$$

$$= S_{-t}\frac{\partial}{\partial\underline{c}_1}f(S_t\underline{r}_1,S_t\underline{c}_1,t) = S_{-t}\left(\frac{\partial S_t\underline{r}_1}{\partial\underline{c}_1}\cdot\frac{\partial}{\partial(S_t\underline{r}_1)}+\right.$$

$$\left.+\frac{\partial S_t\underline{c}_1}{\partial\underline{c}_1}\cdot\frac{\partial}{\partial(S_t\underline{c}_1)}\right)f(S_t\underline{r}_1,S_t\underline{c}_1,t) .$$

From Eqs. (9) and (10) we have

$$\frac{\partial}{\partial \underline{c}_1} (S_t \underline{c}_1) \equiv \underset{\sim}{A}(t) = \begin{pmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{B1}$$

$$\frac{\partial}{\partial \underline{c}_1} (S_t \underline{r}_1) \equiv \underset{\sim}{B}(t) = \begin{pmatrix} \frac{1}{\Omega} \sin \Omega t & \frac{1}{\Omega}(\cos \Omega t - 1) & 0 \\ -\frac{1}{\Omega}(\cos \Omega t - 1) & \frac{1}{\Omega} \sin \Omega t & 0 \\ 0 & 0 & t \end{pmatrix} \tag{B2}$$

Thus we have

$$\frac{\partial}{\partial (S_{-t} \underline{c}_1)} f(S_{-t}\underline{r}_1, S_{-t}\underline{c}_1, t = 0) = \left(\underset{\sim}{B}(t) \cdot \frac{\partial}{\partial \underline{r}_1} + \underset{\sim}{A}(t) \cdot \frac{\partial}{\partial \underline{c}_1}\right) f(\underline{r}_1, \underline{c}_1, t) \tag{B3}$$

We also get

$$\frac{\partial}{\partial (S_{-t} \underline{r}_1)} f(S_{-t}\underline{r}_1, S_{-t}\underline{c}_1, t = 0) = \frac{\partial}{\partial \underline{r}_1} f(\underline{r}_1, \underline{c}_1, t) \tag{B4}$$

Then $g(1,2,t)$ from Eq. (17) may be written as

$$g(1,2,t) = \Delta \underline{c}_1(-t) \cdot \underset{\sim}{A}(t) \cdot \left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2} \right) f(\underline{r}_1, \underline{c}_1, t) f(\underline{r}_2, \underline{c}_2, t) +$$

$$+ \left(\Delta \underline{r}_1(-t) + \Delta \underline{c}_1(-t) \cdot \underset{\sim}{B}(t) \right) \cdot \left(\frac{\partial}{\partial \underline{r}_1} - \frac{\partial}{\partial \underline{r}_2} \right) f(\underline{r}_1, \underline{c}_1, t) f(\underline{r}_2, \underline{c}_2, t) \tag{B5}$$

Transforming further we observe from Eqs. (14) and (B1) that

$$\Delta\underline{c}_1(-t)\cdot\underset{\sim}{A}(t) = \int_0^t d\tau\ \underline{\omega}_{12}(-\tau)\cdot\begin{pmatrix} \cos\Omega\tau & -\sin\Omega\tau & 0 \\ \sin\Omega\tau & \cos\Omega\tau & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{B6}$$

Also, Eq. (16) may be transformed as follows

$$\Delta\underline{r}_1(-t) = \int_0^t d\tau\ \underline{\omega}_{12}(-\tau)\cdot\left[\begin{pmatrix} -\frac{1}{\Omega}\sin\Omega(t-\tau) & \frac{1}{\Omega}\cos\Omega(t-\tau) & 0 \\ -\frac{1}{\Omega}\cos\Omega(t-\tau) & -\frac{1}{\Omega}\sin\Omega(t-\tau) & 0 \\ 0 & 0 & -t \end{pmatrix} - \right.$$

$$\left. - \begin{pmatrix} 0 & \frac{1}{\Omega} & 0 \\ -\frac{1}{\Omega} & 0 & 0 \\ 0 & 0 & -\tau \end{pmatrix}\right] \tag{B7}$$

and from Eqs. (14) and (B2) we get

$$\Delta\underline{c}_1(-t)\cdot\underset{\sim}{B}(t) = \int_0^t d\tau\ \underline{\omega}_{12}(-\tau)\cdot\left[\begin{pmatrix} \frac{1}{\Omega}\sin\Omega\tau & \frac{1}{\Omega}\cos\Omega\tau & 0 \\ -\frac{1}{\Omega}\cos\Omega\tau & \frac{1}{\Omega}\sin\Omega\tau & 0 \\ 0 & 0 & t \end{pmatrix} + \right.$$

$$\left. + \begin{pmatrix} \frac{1}{\Omega}\sin\Omega(t-\tau) & -\frac{1}{\Omega}\cos\Omega(t-\tau) & 0 \\ \frac{1}{\Omega}\cos\Omega(t-\tau) & \frac{1}{\Omega}\sin\Omega(t-\tau) & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] \tag{B8}$$

Then

$$\Delta \underline{r}_1(-t) + \Delta \underline{c}_1(-t)\cdot \underset{\sim}{B}(t) = \int_0^t d\tau\ \underline{\omega}_{12}(-\tau)\cdot \begin{pmatrix} \frac{1}{\Omega}\sin\Omega\tau & \frac{1}{\Omega}(\cos\Omega\tau-1) & 0 \\ -\frac{1}{\Omega}(\cos\Omega\tau-1) & \frac{1}{\Omega}\sin\Omega\tau & 0 \\ 0 & 0 & \tau \end{pmatrix} \tag{B9}$$

We note that the time growing parts in Eqs. (B7) and (B8) exactly cancelled out setting up Eq. (B9).

Substituting Eqs. (B6) and (B9) back into Eq. (B5) give with an obvious transformation the form of $g(1,2,t)$ used in Eqs. (19) and (20).

Appendix C.

Letting $B \to 0$ when also $\tau_c c_m/L \ll 1$ and $t \approx \infty$ in the upper τ-integration limit we have from Eqs. (19) and (20):

$$C^V_{ee} = \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1}\cdot\int d\underline{x}d\underline{c}_2\ \frac{\partial\varphi_{12}}{\partial \underline{x}}\int_0^\infty d\tau\ \frac{\partial\varphi_{12}}{\partial \underline{x}'}\ [\underline{x}' = \underline{x} - \underline{c}_{12}\tau]\cdot$$

$$\cdot\left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right) f(\underline{r}_1,\underline{c}_1,t) f(\underline{r}_1,\underline{c}_2,t) \tag{C1}$$

and

$$C^P_{ee} = \frac{1}{m^2}\frac{\partial}{\partial \underline{c}_1}\cdot\int d\underline{x}d\underline{c}_2\ \frac{\partial\varphi_{12}}{\partial \underline{x}}\int_0^\infty d\tau\ \tau\ \frac{\partial\varphi_{12}}{\partial \underline{x}'}\ [\underline{x}' = \underline{x} - \underline{c}_{12}\tau]\cdot$$

$$\cdot\ f(\underline{r}_1,\underline{c}_2,t)^2\ \frac{\partial}{\partial \underline{r}_1}\left(\frac{f(\underline{r}_1,\underline{c}_1,t)}{f(\underline{r}_1,\underline{c}_2,t)}\right) \tag{C2}$$

Using in C_{ee}^{V} the Coulomb potential and taking Fourier-transforms of functions of $\underline{x}$ we get the usual logarithmic-divergent integral in k which is made finite by cut-offs at $\frac{1}{\lambda_D}$ and $\frac{1}{\lambda_L}$ (lower and upper limits in k-integration). Following the same procedure with C_{ee}^{P} reveals a divergent integral in k due to the lower k-integration limit. This divergens is stronger than logarithmic and making a cut-off gives a result that varies quite strongly with variations in the cut-off. Thereby it seems that C_{ee}^{P} is more sensible than C_{ee}^{V} from the assumption we did at the outset to limit the discussion of collisions to two-particle collisions. Therefore "collective" collisions with the screening mechanism should be included from the beginning. We may artificially circumvent this if we substitute for the Coulomb potential between two particles a potential that partly incorporates the effect of other particles. We here substitute for the Coulomb potential the following potential:

$$\varphi(r) = \frac{e^2}{r}\left(e^{-\frac{r}{\lambda_D}} - e^{-\frac{r}{\lambda_L}}\right) \tag{C3}$$

For $r \ll \lambda_L$ $\varphi(r)$ tends to $e^2(\frac{1}{\lambda_L} - \frac{1}{\lambda_D})$, for $\lambda_L \ll r \ll \lambda_D$ $\varphi(r)$ behaves as $\frac{e^2}{r}$ and for $r \gg \lambda_D$ we have $\varphi(r) \approx \frac{e^2}{r} e^{-\frac{r}{\lambda_D}}$. To get a measure of how "good" such a substitution is we evaluate at first C_{ee}^{V} . We have from Eq. (C3)

$$\varphi(k) = \frac{e^2}{2\pi^2} (k_L^2 - k_D^2) \frac{1}{(k^2+k_D^2)(k^2+k_L^2)} \tag{C4}$$

where $k_D = 1/\lambda_D$ and $k_L = 1/\lambda_L$. After some algebra we get (without doing any cut-offs)

$$C_{ee}^V = \frac{1}{m^2} 2\pi e^4 \left(\frac{\lambda_D^2 + \lambda_L^2}{\lambda_D^2 - \lambda_L^2} \ln \frac{\lambda_D}{\lambda_L} - 1\right) \frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{c}_2 \frac{\partial^2}{\partial \underline{c}_1 \partial \underline{c}_1} (|\underline{c}_1 - \underline{c}_2|) \cdot \left(\frac{\partial}{\partial \underline{c}_1} - \frac{\partial}{\partial \underline{c}_2}\right) f(1,t) f(2,t) \quad \text{(C5)}$$

Thus when $\lambda_D >> \lambda_L$, which is the case for classical plasmas, we only have a small lowering of the collision frequency as compared to the one from Landau's equation. Making the same substitution for the Coulomb potential in Eq. (C2) we get (without cut-offs)

$$C_{ee}^P = \frac{1}{m^2} 2\pi e^4 \frac{(\lambda_D - \lambda_L)^2}{\lambda_D + \lambda_L} \frac{\partial}{\partial \underline{c}_1} \cdot \int d\underline{c}_2 \frac{\partial^2}{\partial \underline{c}_1 \partial \underline{c}_1} (\ln |\underline{c}_1 - \underline{c}_2|) \cdot f(\underline{r}_1, \underline{c}_2, t)^2 \frac{\partial}{\partial \underline{r}_1} \left(\frac{f(\underline{r}_1, \underline{c}_1, t)}{f(\underline{r}_1, \underline{c}_2, t)}\right) \quad \text{(C6)}$$

We observe that $C_{ee}^P / C_{ee}^V \approx \lambda_D / L \cdot \ln(\lambda_D / \lambda_L)$. For very weak inhomogenities, for instance when L is much larger than the (effective) mean free path, as in classical kinetic-transport theories, C_{ee}^P is vanishing small to all (usual) relevant orders of approximations. However, for stronger inhomogenities C_{ee}^P must also be counted for.

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emperature	1eV	10^{-1}eV	10^{-2}eV	10^{-3}eV	10^{-4}eV
$\frac{4}{3}\pi n\lambda_D^3$	$1,2\cdot 10^6$	$3,9\cdot 10^4$	$1,2\cdot 10^3$	$3,9\cdot 10$	$1,2$
λ_L	$1,44\cdot 10^{-7}$	$1,44\cdot 10^{-6}$	$1,44\cdot 10^{-5}$	$1,44\cdot 10^{-4}$	$1,44\cdot 10^{-3}$
$n^{-1/3}$	$8\cdot 10^{-3}$	$8\cdot 10^{-3}$	$8\cdot 10^{-3}$	$8\cdot 10^{-3}$	$8\cdot 10^{-3}$
λ_D	$5,3\cdot 10^{-1}$	$1,7\cdot 10^{-1}$	$5,3\cdot 10^{-2}$	$1,7\cdot 10^{-2}$	$5,3\cdot 10^{-3}$
r_e / r_e	$2\cdot 10^{-2}$ / $5\cdot 10^{-3}$	$8\cdot 10^{-3}$ / $1,5\cdot 10^{-3}$	$2\cdot 10^{-3}$ / $5\cdot 10^{-4}$	$8\cdot 10^{-4}$ / $1,5\cdot 10^{-4}$	$2\cdot 10^{-4}$ / $5\cdot 10^{-5}$
$\lambda\!\!\!^{-}$	$2,8\cdot 10^{-8}$	$8,7\cdot 10^{-8}$	$2,8\cdot 10^{-7}$	$8,7\cdot 10^{-7}$	$2,8\cdot 10^{-6}$
kT	$1,6\cdot 10^{-12}$	$1,6\cdot 10^{-13}$	$1,6\cdot 10^{-14}$	$1,6\cdot 10^{-15}$	$1,6\cdot 10^{-16}$
$\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$	$9,2\cdot 10^{-23}$	$9,2\cdot 10^{-23}$	$9,2\cdot 10^{-23}$	$9,2\cdot 10^{-23}$	$9,2\cdot 10^{-23}$

Table I : Variation with temperature of the plasma parameter, classical distance of closest approach (Landau length) (λ_L), Debye length (λ_D), electron gyroradius (r_e), electron de Broglie wavelength ($\lambda\!\!\!^{-}$), kinetic energy and the Fermi level for electrons for $n = 2\cdot 10^6/cm^3$ and magnetic field $B = 10^2$ Gauss and $5\cdot 10^2$ Gauss. All lengths in cm. kT and Fermi level in erg.

x	κ_1		
	$c_{12z}^2/c_\perp^2 = 10$	$c_{12z}^2/c_\perp^2 = 1$	$c_{12z}^2/c_\perp^2 = \frac{1}{10}$
0,2	0,025	$4\cdot 10^{-4}$	$4\cdot 10^{-6}$
0,4	0,170	$5\cdot 10^{-3}$	$7\cdot 10^{-5}$
0,6	0,372	0,021	$3\cdot 10^{-4}$
0,8	0,568	0,052	10^{-3}
1	0,744	0,097	$2,2\cdot 10^{-3}$
10	0,744	0,097	$2,2\cdot 10^{-3}$
10^2	0,744	0,097	$2,2\cdot 10^{-3}$
10^3	0,744	0,097	$2,2\cdot 10^{-3}$
$2;10^3$	0,719	0,096	$2,2\cdot 10^{-3}$
$4\cdot 10^3$	0,574	0,091	$2,2\cdot 10^{-3}$
$6\cdot 10^3$	0,373	0,075	$1,9\cdot 10^{-3}$
$8\cdot 10^3$	0,176	0,044	$1,3\cdot 10^{-3}$
10^4	0	0	0

Table II : Some numerical values for κ_1 for different magnetic field strengths through x and for three different values of $c_{12z}^2/c_\perp^2$. x is given from $r_e = \lambda_L x$ and $\lambda_D = 10^4 \lambda_L$.

x	$\lvert\kappa_2\rvert$		
	$c^2_{12z}/c^2_{\perp}=10$	$c^2_{12z}/c^2_{\perp}=1$	$c^2_{12z}/c^2_{\perp}=\frac{1}{10}$
0,2	0,338	0,101	0,032
0,4	0,659	0,208	0,064
0,6	0,879	0,320	0,096
0,8	1,023	0,431	0,129
1	1,121	0,535	0,163
10	1,119	0,535	0,162
10^2	1,105	0,530	0,161
10^3	0,958	0,485	0,147
$2\cdot 10^3$	0,783	0,434	0,131
$4\cdot 10^3$	0,462	0,327	0,099
$6\cdot 10^3$	0,241	0,216	0,067
$8\cdot 10^3$	0,097	0,105	0,034
10^4	0	0	0

Table III : Some numerical values of $\lvert\kappa_2\rvert$ for the same parameter values as in table II.