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Representation of Nonlinear Pseudo-Random Generators Using State-Space Equations

Raghad K. Salih
University of Technology, Iraq, Raghad.k.Salih@uotechnology.edu.iq

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Raghad K. Salih

University of Technology, Applied Science Department, Iraq.

Raghad.k.Salih@uotechnology.edu.iq

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تمثيل المولدات الشبه-عشوائية اللاخطية باستخدام معادلات فضاء الحالة

ملخص

فكرة البحث تمثيل للمولدات الشبه عشوائية اللاخطية باستخدام معادلات فضاء الحالة . يعتمد هذا التمثيل على المصفوفات حيث يتم توليد المتتابعة باستخدام المصفوفات و ليس كما هو متعارف عليه عند توليد متتابعات المولدات باستخدام مسجلات الإزاحة. كما تم تقديم خوارزميات لانواع مختلفة من المولدات الشبه عشوائية اللاخطية و استخراج متتابعاتها. علاوة على ذلك تم اعطاء مثالين لتوضيح هذا التمثيل.

Abstract

The idea of research is a representation of the nonlinear pseudo-random generators using state-space equations that is not based on the usual description as shift register synthesis but in terms of matrices. Different types of nonlinear pseudo-random generators with their algorithms have been applied in order to investigate the output pseudo-random sequences. Moreover, two examples are given for conciliated the results of this representation.

1. INTRODUCTION

Pseudo-random generators (PRG) are used as spectrum modulations for direct sequence spread spectrum design for digital communication system, in wireless technique and as a key in encryption to produce the ciphertext in cipher systems. The sequence appears random in nature but in reality, it is deterministic and available to the privileged users [1].

The state space equations SSE has emerged in the last fifty years in the field of control theory. This method uses vector and matrices for system representation, so it permits a simple notation that is easily accepted and processed by digital computer [2]. In this work the nonlinear PRGs were viewed using SSEs.

2. Pseudo-Random Generators PRG

2.1 Nonlinear Feedback Shift Register (NLFSR) Generators [3,4]

A NLFSR of length n is commonly used for producing PR-sequence. It is made up of two parts: shift registers (SR) and a feedback function. The SR is a storage element of a sequence of (n) bits. These (n) binary storage are called the stages of the SR. The algorithm is shown below.

NLFSR Algorithm

Step 1

Input :-

(1) The length (n) of the NLFSR.

(2) The initial state of the NLFSR as $[s_0\ s_1\ .\ .\ .\ s_{n-1}]$.

(3) The nonlinear feedback function $f\ (s_k, s_{k+1},\dots,s_{k+n-1})$.

Step 2

Set $k = 0$

Step 3

Shift the bits in the register by one position to the left and calculate the feedback bit s_{k+n} from the nonlinear function $f\ (s_k, s_{k+1},\dots,s_{k+n-1})$.

Step 4

Set a new state $[s_{k+1}\ s_{k+2}\ .\ .\ .\ s_{k+n}]$ of NLFSR.

Step 5

If the new state is equal to the initial state then :

a) Stop

b) Print the output s_k , $k = 0,1,2,\dots$ of NLFSR.

c) Print the period (k+1) of the sequence of NLFSR.

else

a) k = k+1

b) go to (step 3).

Example

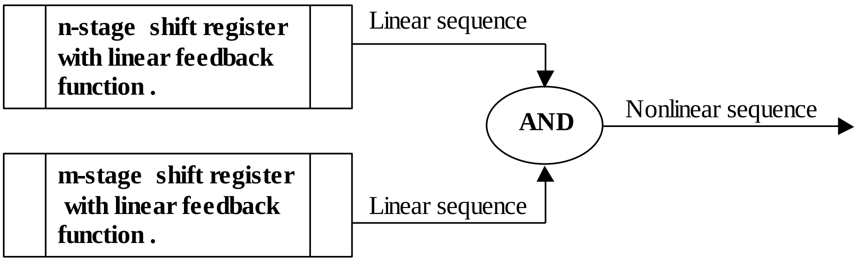
Consider the following NLF function (3-stage):

$f(s_k, s_{k+1}, s_{k+2}) = s_k + s_{k+1} + s_{k+1}.s_{k+2} + 1$

with initial state 101. The output sequence can be generated by applying NLFSR algorithm the produced sequence is : 10100011 .

2.2 Hadmard Generator HG [5,6]

A nonlinear generator consists of two LFSRs, one with (n-stage) and the other with (m-stage) where the gcd (m,n) =1 and each of which produces a sequence with maximal period. The two LFSR's are combined with nonlinear function "AND" to produce a nonlinear sequence with period $((2^n-1)\times(2^m-1))$ as illustrated in figure (1).



Figure(1) Hadmard generator.

Hadmard Algorithm

Step 1:

Input:

(1) The (n,m) stages of two LFSR's.

(2) The initial states of them .

(3) The coefficients of the linear feedback functions of them .

Step 2:

Use (step 1) and call LFSR algorithm to find their sequences.

Step 3:

Combine the two linear sequences in (step 2) with "AND" function to produce the Hadmard sequence .

3. State - Space Equations SSE [7,8,9,10]

The SSE of the linear system is:

$$\left.\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}\right\} \qquad \ldots(1)$$

Where A is ($n\times n$) matrix, B is the ($n\times m$) input matrix, C is the $p\times n$ output matrix and D is the $p\times m$ transmission matrix.

A mathematical model of a nonlinear system was described using SSE as follows:

$$\left.\begin{aligned} x_{k+1} &= f(x_k, u_k) \\ y_k &= h(x_k, u_k) \end{aligned}\right\} \quad , \quad k = 0,1,2,\ldots \qquad \ldots(2)$$

where x_k is the state of the system, u_k is the input of the system and y_k is the output of the system.

4. SSE of Non-Linear PRG:

If we have NLFSR n-stage with nonlinear feedback function $f(s_k, s_{k+1}, \ldots, s_{k+n-1})$, where $s_{k+n} = f(s_k, s_{k+1}, \ldots, s_{k+n-1})$, k = 0,1,2,... ; then the nonlinear SSE can be derived using Eq.(2) as:

$$x(k+1) = f(x(k)) \qquad \ldots(3)$$

$$y(k) = h(x(k)) \quad , \quad k = 0,1,2,\ldots \qquad \ldots(4)$$

where ,

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} = \begin{bmatrix} s_k \\ s_{k+1} \\ \vdots \\ s_{k+n-1} \end{bmatrix}$$

therefore,

$$x(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} x_2(k) \\ x_3(k) \\ \vdots \\ s_{k+n} = f(s_k, s_{k+1}, \ldots, s_{k+n-1}) = f(x_1(k), x_2(k), \ldots, x_n(k)) \end{bmatrix}$$

$$= f(x(k)) \qquad \ldots(5)$$

while the output equation of the nonlinear state space model is

$$y(k) = s_k = x_1(k) = h(x(k)) \quad , \quad k = 0,1,2,\ldots. \qquad \ldots(6)$$

When the nonlinear PRG is constructed from a nonlinear combination of two or more LFSR's, the state space model can be derived as follows

Let $x_1, x_2, \ldots, x_j$ be j LFSR's , j >1 with $n_1, n_2, \ldots, n_j$ stages and $f_1(x), f_2(x), \ldots, f_j(x)$ characteristic polynomials respectively , where :

$$\left.\begin{array}{l} f_1(x)=c_0+c_1x+c_2x^2+\cdots+c_{n_1-1}x^{n_1-1}+x^{n_1} \\ f_2(x)=c_0+c_1x+c_2x^2+\cdots+c_{n_2-1}x^{n_2-1}+x^{n_2} \\ \vdots \\ f_j(x)=c_0+c_1x+c_2x^2+\cdots+c_{n_j-1}x^{n_j-1}+x^{n_j} \end{array}\right\} \quad \ldots(7)$$

The linear recursion relations of the above LFSR's are :

$$\left.\begin{array}{l} s_1(k+n_1)=\sum_{i=0}^{n_1-1}c_i s_1(k+i) \\ s_2(k+n_2)=\sum_{i=0}^{n_2-1}c_i s_2(k+i) \\ \vdots \\ s_j(k+n_j)=\sum_{i=0}^{n_j-1}c_i s_j(k+i) \end{array}\right\} \quad \ldots(8)$$

where $k=0,1,2,\ldots$.

The state variables are:

$$\left.\begin{array}{l} x_1(k)=s_1(k) \\ x_2(k)=s_1(k+1) \\ \vdots \\ x_{n_1}(k)=s_1(k+n_1-1) \\ x_{n_1+1}(k)=s_2(k) \\ x_{n_1+2}(k)=s_2(k+1) \\ \vdots \\ x_{n_1+n_2}(k)=s_2(k+n_2-1) \\ \vdots \\ x_{n_1+n_2+\cdots+n_{j-1}+1}(k)=s_j(k) \\ \vdots \\ x_{n_1+n_2+\cdots+n_j-1}(k)=s_j(k+n_j-2) \\ x_{n_1+n_2+\cdots+n_j}(k)=s_j(k+n_j-1) \end{array}\right\} \quad \ldots(9)$$

From Eq.(8) and Eq.(9) the SSE is

$$x(k+1)=A\,x(k) \quad \ldots(10)$$

where x (k) , $k\geq 0$ is the state vector :

$$x(k)=\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n_1}(k) \\ x_{n_1+1}(k) \\ \vdots \\ x_{n_1+n_2}(k) \\ \vdots \\ x_{n_1+n_2+\cdots+n_j-1}(k) \\ x_{n_1+n_2+\cdots+n_j}(k) \end{bmatrix}_{(n_1+n_2+\cdots+n_j)\times 1}$$

and A is $(n_1+n_2+\cdots+n_j)\times(n_1+n_2+\cdots+n_j))$ matrix

$$A=\begin{bmatrix} 0 & 1 & 0 & \ldots & 0 & & & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ldots & 0 & & & 0 & 0 & & 0 \\ \vdots & & & & \vdots & & & \vdots & & \ddots & \vdots \\ c_0 & c_1 & c_2 & \ldots & c_{n_1-1} & & & 0 & 0 & \cdots & 0 \\ & & & & & \ddots & & & & & \\ & & & & & & \ddots & & & & \\ 0 & 0 & \cdots & 0 & & & & 0 & 1 & 0 \ldots & 0 \\ 0 & 0 & & 0 & & \cdots & & 0 & 0 & 1 \ldots & 0 \\ \vdots & & \ddots & \vdots & & & & \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 & & & & c_0 & c_1 & c_2 \ldots & c_{n_j-1} \end{bmatrix}_{(n_1+n_2+\cdots+n_j)\times(n_1+n_2+\cdots+n_j)}$$

The nonlinear output equation of SSE can be obtained by using Eq.(2)

$$y(k)=h(x(k))=x_1(k)\,x_{n_1+1}(k)\,x_{n_1+n_2+1}(k)\ldots x_{n_1+n_2+\cdots+n_{j-1}+1}(k) \quad , \; k=0,1,2,\ldots \quad \ldots(11)$$

5. Test Examples

Example (1)

Retrieval the example in section (2.1). To represent the NLFSR by using SSE, use Eq.(5) and Eq.(6):

$$x(k+1)=\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix}=\begin{bmatrix} x_2(k) \\ x_3(k) \\ x_1(k)+x_2(k)+x_2(k)x_3(k)+1 \end{bmatrix}=$$

f (x(k))

$$y(k)=x_1(k)=h(x(k)).$$

Example (2) :

Consider HG with two LFSRs

$f_1(x)=x^3+x+1$ with initial state 100 ,

and $f_2(x)=x^2+x+1$ with initial state 10 .

By applying Hadmard algorithm, the following nonlinear sequence with period $(2^3-1)\times(2^2-1)$ is obtained :

$Seq\,(Hadmard\,)=100101100000101001001$

Use Eq.(10) and Eq.(11) to represent HG by using SSE

$$x(k+1)=A\,x(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \end{bmatrix}$$

and the output is $y(k)=x_1(k)x_4(k)=h(x(k))$.

Conclusion

State space models have been derived to represent PRG's. From solving some test examples, the following points are included:

1- State space model represent nonlinear PRG's in a simplified mathematical way using matrices of first-order difference equations.

2- State space model of PRG gives rapid generation because its simple logic where it is computed easily in digital computer.

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