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A plan for transportation and distribution the products based on multi-objective travelling salesman problem in fuzzy environmental

Wakas S. Khalaf [1], Batool Atiyah Khalaf [2], Noorah Osama Abid [3]
[12]Department of Industrial Management, College of Administration and Economics, University of Baghdad, Iraq
[3]Department of Statistics[3], College of Administration and Economics, University of Baghdad, Iraq

ABSTRACT

Transportation and distribution are the most important elements in the work system for any company, which are of great importance in the success of the chain work. Al-Rabee factory is one of the largest ice cream factories in Iraq and it is considered one of the most productive and diversified factories with products where its products cover most areas of the capital Baghdad, however, it lacks a distribution system based on scientific and mathematical methods to work in the transportation and distribution processes, moreover, these processes need a set of important data that cannot in any way be separated from the reality of fuzziness industrial environment in Iraq, which led to use the fuzzy sets theory to reduce the levels of uncertainty. The decision-maker has several goals that he aspires to accomplish for two stages, so, the decision-maker adopted in his work system on a multi-objective travelling salesman problem. A network of paths for transportation and distribution of the products has been designed based on a multi-objective travelling salesman problem, by building a mathematical model that finds the best paths for each stage, taking into account the goals required by the decision-maker. The results obtained from the use of (Lingo) software showed the importance of these methods in determining the optimal path for the processes of collecting and transporting milk from their collection centers to the Al-Rabee factory as a first stage, as well as transporting the final products and distributing them from the Al-Rabee factory to the shopping centers as a second stage.

Keywords: Transportation and distribution, Travelling Salesman Problem, Goal Programming, Ranking function, Lingo.

Corresponding Author:

Wakas S. Khalaf

Department of Industrial Management, College of Administration and Economics
University of Baghdad, Iraq
E-mail: dr.wakkas1@coadec.uobaghdad.edu.iq

1. Introduction

Business organizations seek to keep going with technological developments in order to meet the great challenges presented by the business environment including intense competition and the short life cycle of products.

Researchers in the field of operations management and marketing have paid great attention to these developments. The topic of transportation and distribution have gained great importance in the academic and applied field, it considers the network that connects all parties that deal with the company to deliver its product to the final consumer.

One of the methods that have received much attention is the travelling salesman problem (TSP).

The TSP method is summarized in finding the optimal path through a number (n) of cities or regions so that the salesman visits each city only once before returning to the city from which he started, consequently the level of ambition the decision-maker will be achieved through the use of this method, the optimal path is determining with the least time, cost, distance or all together.

The decision-maker in Al-Rabee factory has important goals that he wishes to arrive at through goal

programming (GP), which is distinguish and effective instrument technique for finding suitable solutions for multiple goals.
The most important challenge facing the decision maker is choosing the appropriate alternative from among the number of alternatives. The decision-making process with multiple objectives is one of the important issues in the process of transportation and distribution.
The Iraqi environment is characterized by its being fuzzy in different aspects, which requests for use of the fuzzy logic in addressing instances of uncertainty imposed by the fuzzy environment. Using the fuzzy sets theory is for achieving the goals inspired from this study which it is hoped to contribute in providing proposed acceptable solutions to solve the multi-objective travelling salesman problem.
Many studies have proposed different problems on this topic
Changdar *et al.* displayed a multi-objective solid travelling salesman problem in a fuzzy environment. The aim of the study is to obtain a complete tour such that both the total cost and the time are reduced [1].
Yalcın and Erginel proposed a new algorithm based on multi-objective optimisation models to guide the work of compounds used to transfer ceramics and distribution process. This algorithm achieved the best results by making the vehicles behave optimally in the transportation and distribution process, which was conducted in the logistics department of a ceramic manufacturing company-Turkey [2].
Miranda *et al.* proposed the bi-objective isle traveling salesman problem, where a set of rural isles must be served using a single barge following a single path. Two objective functions are aimed to be minimized: maritime and ground transportation costs. Miranda *et al.* suggested mixed integer programming model is solved for a set of real instances from Chile by a weighted sum method [3].
O'Neil and Hoffman used the traveling salesman problem method to find optimal paths in the process of receiving materials from their source and then transporting and delivering them to the sources of their sale and consumption. The graphical method was used to find the optimal paths for the transportation process of a pickup truck. The results obtained indicated that this method led to the optimal transportation plan that a pickup truck could use in the process of receiving and delivering materials [4].
Pavithraa and Ganesanb suggested a simple approach for solving multi objective travelling salesman problem whose decision parameters are expressed as triangular fuzzy number. Pavithraa and Ganesanb applied arithmetic operation and ranking for the parametric form of triangular fuzzy number [5].
Remer and Malikopoulos propused a method to optimize the last-mile delivery path of a truck using coordination with unmanned aerial vehicles (UAVs). Initial, a TSP is formulated to determine the truck's route. Then, a scheduling problem is formulated to find the paths for the UAVs. A genetic algorithm is utilized to solve these problems, and simulated results are presented [6]
Khan *et al.* used the theory of switch operation and switch sequence on the sequence of paths of a TSP. Khan *et al.* amended Artificial Bee Colony (ABC) algorithm to solve multi-objective TSP. The suitability of a solution is found using a rule dependent the supremacy property of a multi-objective optimization problem [7]. Cheikhrouhou and Khoufi aimed to provide a comprehensive evaluation of current studies on the Multiple Traveling Salesman Problem (MTSP). Cheikhrouhou and Khoufi focused on MTSP's new contributions to both classical vehicles/robots and unmanned aerial automobiles. They focused the methods applied to solve the MTSP as well as its application domains. They examined the MTSP variants and propose taxonomy and a categorization of current studies [8]
In this study the effective plan of transportation and distribution processes were designed in the Al-Rabee factory in a fuzzy environment where the multi-objective travelling salesman problem was used to determine the optimal paths for two stages, the first is to collect raw milk from its sources and transport it to the factory, and the second is to transport and distribute the final products to the places where they are consumed with the presence of multiple goals for the decision-maker.

1.1 The distinguishing points of this study from the literature

Al-Rabee factory is considered one of the most famous ice cream factories in Iraq and looks forward to maximizing its profitability and increasing its productivity. However, it is distinguished from the literature in two main axes.

1.1.1 Research problem

1. There is no effective distribution system for its products and the distribution process is random and is not based on a mathematical method.

2. Reliance on traditional methods and personal experience in the process of collecting and transporting milk from their collection centers to the Al-Rabee factory, as well as transporting the final products and distributing them from the Al-Rabee factory to the shopping centers.
3. Weakness in the relationship and communication between suppliers, manufacturers and distributors.

1.1.2 Research objectives

1. Draw an ideal reliable roadmap at the first stage (collect and transport milk from their collection centers to the Al-Rabee factory) and the second stage (transport the final products and distribute them from the Al-Rabee factory to the shopping centers) in a fuzzy environment.
2. The ranking function was applied to defuzzification the objective constraints of the mathematical model.
3. Design an effective processing chain based on efficient scientific mathematical methods.
4. Depend on the multi-objective travelling salesman problem to determine the optimal path with several goals for the decision-maker who wants to achieve them for the first and second stage.

2. Traveling salesman problem (TSP)

2.1 . Concept of traveling salesman problem

The TSP is a problem of combanatorail optimaization where the problem consists of a set of nodes (cities) and the distance, time or cost is known between any two nodes. The salesman starts from the city of origin and visits all cities exactly once and returns to the city from which he started in such a way that the total distance, time or cost is the lowest possible.
Therefore, the goal of the problem is to find a shortest path (tour) possible through the set of cities to be visited so that each visited city only once except for the city of departure, he visits twice [7]

2.2 Type of TSP

In the field of TSP study, two types of methods were addressed. The data matrix of the problem was adopted in determining the type of problem for TSP. The data matrix of the problem is the distance, the value adopted between each two nodes in the problem is called Euclidian, meaning the calculated value. Two types of problem were identified [7]
1. Symmetric.
2. Asymmetric.
According to this concept, the problem can be represented for the species referred to above as follows [5,8]:

1. Problem Symmetric

This type of problem can be represented as follows

$$X(i,j) = X(j,i) \qquad (1)$$

$i = 1,2,\dots m$, $j = 1,2,\dots,n$

2. Problem Asymmetric

$$X\ (i,j) \neq X(j,i) \qquad (2)$$

$i = 1,2,\dots m$ $j = 1,2,\dots,n$

2.3 A mathematical model of TSP with the presence of multi-objective

A mathematical model can be formulated as follows [5,7,8]:

1. If there is one goal, the model is as follows:

$$\left.\begin{aligned}
&Minimize\,Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\\
&\quad S.to.\\
&x_{ij} + x_{ji} \le 1\\
&\textstyle\sum_{i=1}^{m} x_{ij} = 1\\
&\textstyle\sum_{j=1}^{n} x_{ij} = 1\\
&x_{ij} \ge 0
\end{aligned}\right\} \qquad (3)$$

Where:
$i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$, $i \neq j$
$c_{ij} = 0$ *when* $i = j$

Where c_{ij} represents a variable value for a specific unit of measurement from city i to city j and the standard above is similar (Symmetric TSP) if c_{ij}=c_{ji}, n represents the number of values of decision variable parameters, m represents the number of decision variables and n = m, while x_{ij} represents decision variable for two cases:

$$x_{ij} = \begin{pmatrix} 1 & \text{where } i \rightarrow j \text{ is found} \\ 0 & \text{other wise} \end{pmatrix}$$

2. If there are multi-objective, the mathematical model is as follows:

$$\left.\begin{array}{l} \text{Max or Min} \quad Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \\ \text{S. to} \\ \sum_{j=1}^{n} x_{ij} = 1 \\ \sum_{i=1}^{n} x_{ij} = 1 \\ x_{ij} + x_{ji} \leq 1 \qquad \forall\, i,j \\ \textit{Where:} \\ i = 1,2,\ldots,m \;, \quad j = 1,2,\ldots,n \quad i \neq j \\ r = 1,2,\ldots,k \\ x_{ij} \geq 0 \qquad c_{ij} = 0 \;\; all \;\; i = j \end{array}\right\} \tag{4}$$

Since cij represents the value of a variable of a certain unit of measure from the city i to the city j and the above criterion is considered symmetric TSP if it is (c_{ij}= c_{ji}), and (n) represents the number of values of the decision variable parameters, and (m) represents the number of decision variables and always is (n = m), k represents the number of goals in the research, while xij represents the decision variable and as follow:

$$x_{ij} = \begin{pmatrix} 1 & \text{where } i \rightarrow j \text{ is found} \\ 0 & \text{other wise} \end{pmatrix}$$

3. Methods of goal programming (GP)

There are two main ways to solve the GP problems, as follows [9,10]:

Weighted GP: This method depends on determining the numerical weighted weights for each goal. These weights are the coefficients of the deviational variables that exist for each goal; for example, if the company has a problem it wants to solve and the solution of this problem lies in achieving three goals, the decision-maker will give the most important goal a greater weight than the others and so on.

Pre-emptive GP: The weight algorithm is characterized by the difficulty in reaching the best solution for the multiple goals in which the difficulty lies in determining the weights and their importance for the other remaining goals. This is why decision-makers prefer to use the priority method instead, where the most important goal is given the first priority, then followed by the less important goal and so on according to the decision-maker's classification.

The two methods represent and embody the multiple goals in one goal function which is (Min). The determination of the weights and priorities for each goal is not subject to a specific law or a specific formula, but, rather, they are developed by the decision-maker and the desire to give the important goals a higher priority than others.

4. Fuzzy preliminaries

Zadeh advanced the fuzzy theory in 1965. The fuzzy set theory delivered a mathematical model for dealing with imprecise concepts and problems that have many possible solutions. The following definitions of the fuzzy numbers and some basic arithmetic operations on it may be helpful [11].

4.1 . Definition

A fuzzy number $\tilde{A}$ is a Triangular-Fuzzy number represented by (a_1, a, a_2) and it's a membership function $\mu_{\tilde{A}}(x)$ is given as follows in [11,12]:

$$\mu_A(x, a_1, a, a_2) = \left.\begin{array}{lll} 0 & if & x < a_1 \\ \frac{x-a}{a-a_1} & if & a_1 \le x \le a \\ \frac{a_2-x}{a_2-a_1} & if & a \le x \le a_2 \\ 0 & if & x > a_2 \end{array}\right\} \quad (5)$$

The α- cut off the triangular fuzzy number $\tilde{A}= (a_1, a, a_2)$ is the closed interval

4.2. Definition

Let $\tilde{A} = (a_1, \alpha_1, \beta_1)$ and $\tilde{B} = (a_2, \alpha_2, \beta_2)$ be two triangular fuzzy numbers. Then [11]:

$\tilde{A} \oplus \tilde{B} = (a_1, \alpha_1, \beta_1) \oplus (a_2, \alpha_2, \beta_2) = (a_1 + a_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$

$\tilde{A} \ominus \tilde{B} = (a_1, \alpha_1, \beta_1) \ominus (a_2, \alpha_2, \beta_2) = (a_1 - a_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2)$

4.2 Ranking functions

A convenient method for comparing of fuzzy number is by use of ranking function [12,13]. A ranking function $\Re$: F(R) → R, where F(R) (a set of all fuzzy numbers defined on a set of real numbers), charts each fuzzy number into a real number of F(R).

Let ã and $\tilde{b}$ be two fuzzy numbers in F(R), then

(i) $\tilde{a} \ge_{\Re} \tilde{b}$ if and only if $\Re(\tilde{a}) \ge \Re(\tilde{b})$

(ii) $\tilde{a} >_{\Re} \tilde{b}$ if and only if $\Re(\tilde{a}) > R(\tilde{b})$

(iii) $\tilde{a} =_{\Re} \tilde{b}$ if and only if $\Re(\tilde{a}) = \Re(\tilde{b})$

Let $\Re$ be any linear ranking function. Then,

$\tilde{a} \ge_{\Re} \tilde{b}$ if and only if $\tilde{a} - \tilde{b} \ge_{\Re} \tilde{0}$ if and only if $-\tilde{b} \ge_{\Re} - \tilde{a}$.

If $\tilde{a} \ge_{\Re} \tilde{b}$ and $\tilde{c} \ge_{\Re} \tilde{d}$, then $\tilde{a} + \tilde{c} \ge_{\Re} \tilde{b} + \tilde{d}$.

4.2.1. Ranking functions for triangular fuzzy number

A triangular fuzzy number $\tilde{a} = (a_1, a, a_2)$ or (a, α, β), the ranking function is given by

$\Re(\tilde{a}) = \frac{1}{2}\int_0^1 (inf\ a_\alpha + sup\ a_\alpha)\, d_\alpha$, where a_α is α −cut on $\tilde{a}$. This decreases to:

$$\Re(\tilde{a}) = \frac{1}{4}(a_1 + 2a + a_2) or \left\{ a + \frac{(\beta-\alpha)}{4} \right\} \quad (6)$$

Then triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ and

$\tilde{b} = (b, y, \theta)$, have

$\tilde{a} \ge_{\Re} \tilde{b}$ if and only if

$\Re(\tilde{a}) = \left[a + \frac{(\beta-\alpha)}{4}\right] \ge \left[b + \frac{(\theta-y)}{4}\right] = \Re(\tilde{b})$

5. A real practical example

Al-Rabee factory is one of the most famous ice cream factories in Baghdad-Jamila. The factory produces different types of ice cream, the manufacture of ice cream depends mainly on raw milk that the factory buys from some villages and rural areas surrounding Baghdad by a refrigerated truck to collect and transport milk from their collection centers in those areas to the Al-Rabee factory as a first stage, as well as, transport the final products and distribute them from the Al-Rabee factory to the shopping centers as the second stage.

5.1. Mechanism of the first stage

Designing a plan for the process of collecting and transporting raw milk from its sources to the factory in the existence of four objectives and the decision-maker wish to give the important objectives higher priority than others, these goals according to their importance are as follows:

1. The first objective is to obtain milk with a high fat density

The percentage of milk fat density (percentages measured by the quality control department in the factory which is a fuzzy percentage):
The quality control department determined the fuzzy percentage of row milk fat for the four specified centers, which are as follows:
Fudayliah Center =(38%, 48%, 58%), Alkamalia Center =(35%, 45%, 55%), Khan Bani Saad Center = (26%, 36%, 46%), Khalis Center = (25%, 35%, 45%)
The factory aims to obtain row milk with a high density of fat because it contributes to increasing the quality of the final product for all kinds of ice cream.

2. The second objective is to collect and transport milk in the least possible time

The data of the fuzzy time (per minute) for the process of collecting and transporting milk from the areas where raw milk is available, as well as the time required to go from one area to another, shown in Table 2.

Table 2. The fuzzy time (per minute) between the factory (Jamila) and the milk collection centers and between the same areas

	Jamila	Fudayliah	Kamalia	Khan Bani Saad	Khalis
Jamila	(0,0,0)	(25,35,45)	(35,45,55)	(40,45,50)	(60,70,80)
Fudayliah	(25,35,45)	(0,0,0)	(15,20,25)	(22,25,33)	(65,70,75)
Kamalia	(35,45,55)	(15,20,25)	(0,0,0)	(20,25,30)	(60,70,80)
Khan Bani Saad	(40,45,50)	(22,25,33)	(20,25,30)	(0,0,0)	(50,60,70)
Khalis	(60,70,80)	(65,70,75)	(60,70,80)	(50,60,70)	(0,0,0)

3. The third objective is to collect and transporting milk with the lowest possible fuel cost

The data of the fuzzy cost of fuel consumed (in dinars) for trucks used to collect and transport raw milk from milk collection centers to the factory, as well as the fuzzy cost of fuel consumed for the moving between one center and another, as shown in Table 3.

Table 3. The fuzzy cost of fuel consumed for trucks used in the process of collecting and transporting milk (in thousands of Iraqi dinars)

	Jamila	Fudayliah	Kamalia	Khan Bani Saad	Khalis
Jamila	(0,0,0)	(8,10,12)	(9,11,13)	(11,13,15)	(15,17,19)
Fudayliah	(8,10,12)	(0,0,0)	(3,5,7)	(5,7,9)	(16,18,20)
Kamalia	(9,11,13)	(3,5,7)	(0,0,0)	(4,6,8)	(15,17,19)
Khan Bani Saad	(11,13,15)	(5,7,9)	(4,6,8)	(0,0,0)	(16,18,20)
Khalis	(15,17,19)	(16,18,20)	(15,17,19)	(16,18,20)	(0,0,0)

4. The fourth objective is the quantity of demand (in tons) of raw milk is not less than 25 tons/day

The data of fuzzy demand quantity of raw milk from collection centers (determined by the decision maker in the factory) and these quantities were determined according to the fat percentage in it and in each center as follows:
Fudayliah center = (7,9,11) tons of raw milk, Kamalia center = (5,6,7) tons of raw milk, Khan Bani Saad center = (3,4,5) tons of raw milk, Khalis center = (2,4,6) tons of raw milk

5.1. Building the fuzzy mathematical model for the first stage

The fuzzy mathematical model is built for the first stage, which is to collect and transport raw milk from its sources (collection centers) to the factory according to the general formula of the fuzzy multi-objective travelling salesman problem method, as follows:

5.1.1. Variables of the fuzzy mathematical model

The decision variables and their fuzzy parameters are shown in Table 4 as follows:

Table 4. the matrix of decision variables and their fuzzy parameters

	x_0	x_1	x_2	x_3	x_4
x_0	0	$\tilde{c}_{01}x_{01}$	$\tilde{c}_{02}x_{02}$	$\tilde{c}_{03}x_{03}$	$\tilde{c}_{04}x_{04}$
x_1	$\tilde{c}_{10}x_{10}$	0	$\tilde{c}_{12}x_{12}$	$\tilde{c}_{13}x_{13}$	$\tilde{c}_{14}x_{14}$
x_2	$\tilde{c}_{20}x_{20}$	$\tilde{c}_{21}x_{21}$	0	$\tilde{c}_{23}x_{23}$	$\tilde{c}_{24}x_{24}$
x_3	$\tilde{c}_{30}x_{30}$	$\tilde{c}_{31}x_{31}$	$\tilde{c}_{32}x_{32}$	0	$\tilde{c}_{34}x_{34}$
x_4	$\tilde{c}_{40}x_{40}$	$\tilde{c}_{41}x_{41}$	$\tilde{c}_{42}x_{42}$	$\tilde{c}_{43}x_{43}$	0

Whereas:
x_0: a decision variable indicating the location of the factory (Jamila)
x_1: a decision variable indicating the Fudayliah milk collection center
x_2: a decision variable indicating the Kamalia milk collection center
x_3: a decision variable indicating the Khan Bani Saad milk collection center
x_4: a decision variable indicating the Khalis milk collection center

The other decision variables are as follows:
x_{ij}: The decision variable from node (i) to node (j), for example: such as:
x_{01}: The decision variable from the factory in (Jamila) to the milk collection center in Fudayliah, and so on for other variables from the factory in (Jamila) to other milk collection centers.
$\tilde{c}_{ij}$: The fuzzy parameter of the decision variable and it is either a distance, time, fuel cost...etc, between node (i) and node (j)

5.1.2. The objective constraints of the fuzzy mathematical model

1. The first objective constraint is to obtain raw milk with a high percentage of fat density, as the decision-maker wants the fuzzy percentage of fat in raw milk to be not less than (0.38, 0.48, 0.58).

$$\begin{aligned}\text{Max } \tilde{Z}_1 = {} & (38\%, 48\%, 58\%)x_{01} + (35\%, 45\%, 55\%)x_{02} + (26\%, 36\%, 46\%)x_{03} \\ & + (25\%, 35\%, 45\%)x_{04} \geq (38\%, 48\%, 58\%)\end{aligned}$$

2. The second objective constraint is to reduce the fuzzy time to move from the factory to the raw milk collection centers and to move between the centers themselves, and the decision-maker wishes that the fuzzy total time for the process of collecting and transporting raw milk from its sources to the factory is not more than (170,180,190) minutes/day.

$$\begin{aligned}\text{Min } \tilde{Z}_2 = {} & (25,35,45)x_{01} + (35,45,55)x_{02} + (40,45,50)x_{03} + (60,70,80)x_{04} + (25,35,45)x_{10} \\ & + (15,20,25)x_{12} + (22,25,33)x_{13} + (65,70,75)x_{14} + (35,45,55)x_{20} \\ & + (15,20,25)x_{21} + (20,25,30)x_{23} + (60,70,80)x_{24} + (40,45,50)x_{30} \\ & + (22,25,33)x_{31} + (20,25,30)x_{32} + (50,60,70)x_{34} + (60,70,80)x_{40} \\ & + (65,70,75)x_{41} + (60,70,80)x_{42} + (50,60,70)x_{43} \leq (170,180,190)\end{aligned}$$

3. The third objective is to reduce the fuzzy of fuel consumed for trucks during the process of moving from the factory to the raw milk collection centers, and moving between the centers themselves and back to the factory, and the decision-maker wishes that the fuzzy of fuel consumed should not exceed (180,200,220) in liters/day.

$$\text{Min } \tilde{Z}_3 = (8,10,12)x_{01} + (9,11,13)x_{02} + (11,13,15)x_{03} + (15,17,19)x_{04} + (8,10,12)x_{10} + (3,5,7)x_{12} + (5,7,9)x_{13} + (16,18,20)x_{14} + (9,11,13)x_{20} + (3,5,7)x_{21} + (4,6,8)x_{23} + (15,17,19)x_{24} + (11,13,15)x_{30} + (5,7,9)x_{31} + (4,6,8)x_{32} + (16,18,20)x_{34} + (15,17,19)x_{40} + (16,18,20)x_{41} + (15,17,19)x_{42} + (16,18,20)x_{43} \leq (48,50,52)$$

4. The forth objective constraint is to maximize the fuzzy quantity of demand of raw milk (in tons) and the decision-maker wishes to maximized this quantity to be within (17,19,21) tons / day of raw milk.

$$\text{Max } \tilde{Z}_4 = (7,9,11)x_{01} + (5,6,7)x_{02} + (3,4,5)x_{03} + (2,4,6)x_{04} \geq (17,19,21)$$

5.2. The objective constraints of the mathematical model after defuzzification

After using the ranking function as in (6) to eliminate fuzziness of the objective constraints of the mathematical model, the objective constraints are as follow:

$$\text{Max } Z_1 = 48\%x_{01} + 45\%x_{02} + 36\%x_{03} + 35\%x_{04} \geq 48\%$$

$$\text{Min } Z_2 = 35x_{01} + 45x_{02} + 45x_{03} + 70x_{04} + 35x_{10} + 20x_{12} + 25x_{13} + 70x_{14} + 45x_{20} + 20x_{21} + 25x_{23} + 70x_{24} + 45x_{30} + 25x_{31} + 25x_{32} + 60x_{34} + 70x_{40} + 70x_{41} + 70x_{42} + 60x_{43} \leq 180$$

$$\text{Min } Z_3 = 10x_{01} + 11x_{02} + 13x_{03} + 17x_{04} + 10x_{10} + 5x_{12} + 7x_{13} + 18x_{14} + 11x_{20} + 5x_{21} + 6x_{23} + 17x_{24} + 13x_{30} + 7x_{31} + 6x_{32} + 18x_{34} + 17x_{40} + 18x_{41} + 17x_{42} + 18x_{43} \leq 50$$

$$\text{Max } Z_4 = 9x_{01} + 6x_{02} + 4x_{03} + 4x_{04} \geq 19$$

The system constraints are as follows:

Accordingly,

$x_{01} + x_{02} + x_{03} + x_{04} = 1,\quad x_{10} + x_{20} + x_{30} + x_{04} = 1$

$x_{10} + x_{12} + x_{13} + x_{14} = 1,\quad x_{01} + x_{21} + x_{31} + x_{41} = 1$

$x_{20} + x_{21} + x_{23} + x_{24} = 1,\quad x_{02} + x_{12} + x_{13} + x_{14} = 1$

$x_{30} + x_{31} + x_{32} + x_{34} = 1,\quad x_{03} + x_{13} + x_{23} + x_{24} = 1$

$x_{40} + x_{41} + x_{42} + x_{43} = 1,\quad x_{04} + x_{41} + x_{42} + x_{43} = 1$

$x_{01} + x_{10} \leq 1,\quad x_{02} + x_{20} \leq 1,\quad x_{03} + x_{30} \leq 1,$

$x_{04} + x_{40} \leq 1,\quad x_{12} + x_{21} \leq 1,\quad x_{13} + x_{31} \leq 1,$

$x_{14} + x_{41} \leq 1,\quad x_{23} + x_{32} \leq 1,\quad x_{24} + x_{42} \leq 1,$

$x_{34} + x_{43} \leq 1,$

$x_{ij}\ (0,1) \geq 0$

5.3. Discuss the results of the first stage

The most important results obtained by applying the Lingo software are as follows, for more details see appendix A:

Obtaining the best path to collect and transport raw milk from its sources (collection centers) to the factory according to the general formula of the multi-objective travelling salesman problem method, as the following:

$$X_{01} \longrightarrow X_{13} \longrightarrow X_{32} \longrightarrow X_{24} \longrightarrow X_{40} \longrightarrow X_{01}$$

Jamila → Fudayliah → Khan Bani Saad → Kamalia → Khalis → Jamila

Through the results of the lingo software solution, it was found that the first goal was achieved, where the value of p_1^- is zero, which means obtaining raw milk with the highest percentage of fat density from Fudayliah milk collection center.

As for the second goal, it was not achieved, as the p_2^+ value is 25. This means that the time to move from the Al-Rabee factory to the raw milk collection centers and to move between the same centers is 25 minutes

greater than the time available, while the third goal has been achieved where the value of p_3^+ is zero, means that the fuel consumed for trucks during the process of moving from the factory to the raw milk collection centers, and moving between the centers themselves and back to the factory not exceed 60 in litres/day.
Finally, the last goal was not achieved, as the p_4^- is equal to 2, and it means that the amount of raw milk that Fudayliah center is supplied as the center that was chosen in the first goal to obtain raw milk with the highest fat percentage is less than the amount of raw milk required by Al-Rabee factory by 2.

5.4. Mechanism of the second stage

The process of transporting and distributing the final products is a very important process due to its impact on all elements of the process, starting from the milk collection process then the production process that takes place inside the factory to the process of transporting and distributing the final products.
The distribution process is carried out by a group of individuals working in the organization through whom the process of transporting the final products through the refrigerated truck of the Al-Rabee factory and then distributing them to the shopping centers located on both sides of Karkh and Rusafa. The decision-maker wants to achieve two objectives during the process of transferring and distributing the final product to the shopping centers, which are as follows:

1. The first objective is to reduce the fuzzy transportation cost of truck during the process of transporting and distributing the main products from the factory to the shopping centers and then return to the factory.

The data of the fuzzy transportation cost of truck (in dollar) to transport the main products from the factory and distribute them to the points of sale, as well as the fuzzy transportation cost of truck for the process of moving among sales centers, as in Table 5.

Table 5. the fuzzy transportation cost of truck (in dollar) for the process of transporting products from the Al Rabee factory and distributing them to shopping centers as well as the fuzzy transportation cost of truck for the process of moving among sales centers

	Jamila	Hayi-awr	Waziriya	Karrada	Adhamiya	Mansour	Palestine Street
Jamila	(0,0,0)	(2,3,4)	(5,7,9)	(7,9,11)	(6,8,10)	(8,10,12)	(5,7,9)
Hayi-awr	(2,3,4)	(0,0,0)	(6,7,8)	(8,10,12)	(6,8,10)	(9,11,13)	(4,5,6)
Waziriya	(5,7,9)	(6,7,8)	(0,0,0)	(7,9,11)	(4,5,6)	(7,9,11)	(5,7,9)
Karrada	(7,9,11)	(8,10,12)	(7,9,11)	(0,0,0)	(8,10,12)	(8,10,12)	(6,8,10)
Adhamiya	(6,8,10)	(6,8,10)	(4,5,6)	(8,10,12)	(0,0,0)	(7,9,11)	(5,7,9)
Mansour	(8,10,12)	(9,11,13)	(7,9,11)	(8,10,12)	(7,9,11)	(0,0,0)	(8,10,12)
Palestine Street	(5,7,9)	(4,5,6)	(5,7,9)	(6,8,10)	(5,7,9)	(8,10,12)	(0,0,0)

2. The second objective is to reduce the fuzzy total time for the transportation and distribution of the main products from the factory to the shopping centers.

The data of the fuzzy time for the process of transporting and distributing the main products from the factory to the shopping centers and among the centers themselves, as shown in Table 6:

Table 6: The fuzzy time (in minutes) for the process of transporting the main products from the Al-Rabee factory and distributing them to shopping centers.

	Jamila	Hayi-awr	Waziriya	Karrada	Adhamiya	Mansour	Palestine Street
Jamila	(0,0,0)	(10,15,20)	(15,20,25)	(30,40,50)	(25,35,45)	(50,60,70)	(25,35,45)
Hayi-awr	(10,15,20)	(0,0,0)	(22,27,32)	(35,40,45)	(28,33,38)	(45,55,65)	(20,25,30)
Waziriya	(15,20,25)	(22,27,32)	(0,0,0)	(30,40,50)	(10,15,20)	,50,60)(40	(15,20,25)
Karrada	(30,40,50)	(35,40,45)	(30,40,50)	(0,0,0)	(50,60,70)	(35,40,45)	(40,50,60)
Adhamiya	(25,35,45)	(28,33,38)	(10,15,20)	(50,60,70)	(0,0,0)	(60,70,80)	(35,45,55)

	Jamila	Hayi-awr	Waziriya	Karrada	Adhamiya	Mansour	Palestine Street
Mansour	(50,60,70)	(45,55,65)	,50,60)(40	(35,40,45)	(60,70,80)	(0,0,0)	(30,40,50)
Palestine Street	(25,35,45)	(20,25,30)	(15,20,25)	(40,50,60)	(35,45,55)	(30,40,50)	(0,0,0)

5.5. Building the fuzzy mathematical model for the second stage

The fuzzy mathematical model is built for the second stage, which is the transportation of the main products from the Al-Rabee factory and their distribution to the shopping centers according to the general formula of the multi-objective travelling salesman problem method, as follows:

5.5.1. Variables of the fuzzy mathematical model

The decision variables and their fuzzy parameters are shown in Table 7 as follows:

Table 7. The matrix the decision variables and their fuzzy parameters

	x_0	x_1	x_2	x_3	x_4	x_5	x_5
x_0	0	$\tilde{c}_{01}x_{01}$	$\tilde{c}_{02}x_{02}$	$\tilde{c}_{03}x_{03}$	$\tilde{c}_{04}x_{04}$	$\tilde{c}_{05}x_{05}$	$\tilde{c}_{06}x_{06}$
x_1	$\tilde{c}_{10}x_{10}$	0	$\tilde{c}_{12}x_{12}$	$\tilde{c}_{13}x_{13}$	$\tilde{c}_{14}x_{14}$	$\tilde{c}_{15}x_{15}$	$\tilde{c}_{16}x_{16}$
x_2	$\tilde{c}_{20}x_{20}$	$\tilde{c}_{21}x_{21}$	0	$\tilde{c}_{23}x_{23}$	$\tilde{c}_{24}x_{24}$	$\tilde{c}_{25}x_{25}$	$\tilde{c}_{26}x_{26}$
x_3	$\tilde{c}_{30}x_{30}$	$\tilde{c}_{31}x_{31}$	$\tilde{c}_{32}x_{32}$	0	$\tilde{c}_{34}x_{34}$	$\tilde{c}_{35}x_{35}$	$\tilde{c}_{36}x_{36}$
x_4	$\tilde{c}_{40}x_{40}$	$\tilde{c}_{41}x_{41}$	$\tilde{c}_{42}x_{42}$	$\tilde{c}_{43}x_{43}$	0	$\tilde{c}_{45}x_{45}$	$\tilde{c}_{46}x_{46}$
x_5	$\tilde{c}_{50}x_{40}$	$\tilde{c}_{51}x_{41}$	$\tilde{c}_{52}x_{42}$	$\tilde{c}_{53}x_{53}$	$\tilde{c}_{54}x_{54}$	0	$\tilde{c}_{56}x_{56}$
x_6	$\tilde{c}_{60}x_{60}$	$\tilde{c}_{61}x_{61}$	$\tilde{c}_{62}x_{62}$	$\tilde{c}_{63}x_{63}$	$\tilde{c}_{64}x_{64}$	$\tilde{c}_{65}x_{65}$	0

Whereas:
x_0: Decision variable indicating the location of the factory in Jamila
x_1: Decision variable indicating the location to the Hayi-awr shopping center
x_2: Decision variable indicating the location to the Waziriya shopping center
x_3: Decision variable indicating the location to the Karrada shopping center
x_4: Decision variable indicating the location to the Adhamiya shopping center
x_5: Decision variable indicating the location to the Mansour Shopping Center
x_6: Decision variable indicating the location to the Palestine Street

The remaining decision variables are defined as follows:
x_{ij}: Refers to the decision variable from node (i) to node (j), for example: such as:
x_{01}: Refers to the decision variable from the Al-Rabee factory to Hayi-awr Shopping Center and so on for other variables from the Al-Rabee factory to other shopping centers.
c_{ij}: Refers to the coefficient of the decision variable and it is either a distance, time, fuel cost ... etc, between node (i) and node (j)

5.5.2. The objective constraints of the fuzzy mathematical model

The decision-maker desires to achieve the two objectives, in addition to clarifying the contribution of each decision variable to achieving the specified levels of the objectives, these two objectives are arranged according to their importance to the decision-maker and as follows:

1. The first objective constraint is to reduce fuzzy transportation cost of truck during the process of moving from the factory to the shopping centers and then return to the factory. The decision-maker wishes that fuzzy transportation cost of truck does not exceed (55,60,65) dollar/day.

$$\text{Min } Z_1 = (2,3,4)\text{X}_{01} + (5,7,9)\text{X}_{02} + (7,9,11)\text{X}_{03} + (6,8,10)\text{X}_{04} + (8,10,12)\text{X}_{05} + (5,7,9)\text{X}_{06} + (2,3,4)\text{X}_{10} + (5,7,9)\text{X}_{12} + (8,10,12)\text{X}_{13} + (7,9,11)\text{X}_{14} + (9,11,13)\text{X}_{15} + (4,5,6)\text{X}_{16} + (5,7,9)\text{X}_{20} + (6,7,8)\text{X}_{21} + (7,9,11)\text{X}_{23} + (4,5,6)\text{X}_{24} + (7,9,11)\text{X}_{25} + (5,7,9)\text{X}_{26} + (7,9,11)\text{X}_{30} + (8,10,12)\text{X}_{31} + (7,9,11)\text{X}_{32} + (8,10,12)\text{X}_{34} + (8,10,12)\text{X}_{35} + (6,8,10)\text{X}_{36} + (6,8,10)\text{X}_{40} + (7,9,11)\text{X}_{41} +$$

$(4,5,6)X_{42} + (8,10,12)X_{43} + (7,9,11)X_{45} + (5,7,9)X_{46} + (8,10,12)X_{50} + (9,11,13)X_{51} + (7,9,11)X_{52} + (8,10,12)X_{53} + (7,9,11)X_{54} + (8,10,12)X_{56} + (5,7,9)X_{60} + (4,5,6)X_{61} + (5,7,9)X_{62} + (6,8,10)X_{63} + (5,7,9)X_{64} + (8,10,12)X_{65} \leq (55,60,65)$

2. The second objective constraint is to reduce the fuzzy time to move from the factory to the shopping centers and then return to the factory where the decision-maker wishes that the fuzzy total time for the distribution of products to shopping centers does not exceed (220,225,230) minutes/day.

$$\text{Min } Z_2 = (10,15,20)X_{01} + (15,20,25)X_{02} + (30,40,50)X_{03} + (25,35,45)X_{04} + (50,60,70)X_{05} + (25,35,45)X_{06} + (10,15,20)X_{10} + (22,27,32)X_{12} + (35,40,45)X_{13} + (28,33,38)X_{14} + (45,55,65)X_{15} + (20,25,30)X_{16} + (15,20,25)X_{20} + (22,27,32)X_{21} + (30,40,50)X_{23} + (10,15,20)X_{24} + (40,50,60)X_{25} + (15,20,25)X_{26} + (30,40,50)X_{30} + (35,40,45)X_{31} + (30,40,50)X_{32} + (50,60,70)X_{34} + (35,40,45)X_{35} + (40,50,60)X_{36} + (25,35,45)X_{40} + (28,33,38)X_{41} + (10,15,20)X_{42} + (50,60,70)X_{43} + (60,70,80)X_{45} + (35,45,55)X_{46} + (50,60,70)X_{50} + (45,55,65)X_{51} + (40,50,60)X_{52} + (35,40,45)X_{53} + (60,70,80)X_{54} + (30,40,50)X_{56} + (25,35,45)X_{60} + (20,25,30)X_{61} + (15,20,25)X_{62} + (40,50,60)X_{63} + (35,45,55)X_{64} + (30,40,50)X_{65} \leq (220,225,230)$$

5.6. The objective constraints of the mathematical model after defuzzification

After using the ranking function as in (2) to eliminate fuzziness of the objective constraints of the mathematical model, the objective constraints are as follow:

$$\text{Min } Z_1 = 3X_{01} + 7X_{02} + 9X_{03} + 8X_{04} + 10X_{05} + 7X_{06} + 3X_{10} + 7X_{12} + 10X_{13} + 9X_{14} + 11X_{15} + 5X_{16} + 7X_{20} + 7X_{21} + 9X_{23} + 5X_{24} + 9X_{25} + 7X_{26} + 9X_{30} + 10X_{31} + 9X_{32} + 10X_{34} + 10X_{35} + 8X_{36} + 8X_{40} + 9X_{41} + 5X_{42} + 10X_{43} + 9X_{45} + 7X_{46} + 10X_{50} + 11X_{51} + 9X_{52} + 10X_{53} + 9X_{54} + 10X_{56} + 7X_{60} + 5X_{61} + 7X_{62} + 8X_{63} + 7X_{64} + 10X_{65} \leq 60$$

$$\text{Min } Z_2 = 15X_{01} + 20X_{02} + 40X_{03} + 35X_{04} + 60X_{05} + 35X_{06} + 15X_{10} + 27X_{12} + 40X_{13} + 33X_{14} + 55X_{15} + 25X_{16} + 20X_{20} + 27X_{21} + 40X_{23} + 15X_{24} + 50X_{25} + 20X_{26} + 40X_{30} + 40X_{31} + 40X_{32} + 60X_{34} + 40X_{35} + 50X_{36} + 35X_{40} + 33X_{41} + 15X_{42} + 60X_{43} + 70X_{45} + 45X_{46} + 60X_{50} + 55X_{51} + 50X_{52} + 40X_{53} + 70X_{54} + 40X_{56} + 35X_{60} + 25X_{61} + 20X_{62} + 50X_{63} + 45X_{64} + 40X_{65} \leq 225$$

System constraints: They are as follows:

S.to

$$X_{01} + X_{02} + X_{03} + X_{04} + X_{05} + X_{06} = 1$$
$$X_{10} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} = 1$$
$$X_{20} + X_{21} + X_{23} + X_{24} + X_{25} + X_{26} = 1$$
$$X_{30} + X_{31} + X_{32} + X_{34} + X_{35} + X_{36} = 1$$
$$X_{40} + X_{41} + X_{42} + X_{43} + X_{45} + X_{46} = 1$$
$$X_{50} + X_{51} + X_{52} + X_{53} + X_{54} + X_{56} = 1$$
$$X_{60} + X_{61} + X_{62} + X_{63} + X_{64} + X_{65} = 1$$
$$X_{10} + X_{20} + X_{30} + X_{40} + X_{50} + X_{60} = 1$$
$$X_{01} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} = 1$$
$$X_{02} + X_{12} + X_{32} + X_{42} + X_{52} + X_{62} = 1$$
$$X_{03} + X_{13} + X_{23} + X_{43} + X_{53} + X_{63} = 1$$
$$X_{04} + X_{14} + X_{24} + X_{34} + X_{54} + X_{64} = 1$$
$$X_{05} + X_{15} + X_{25} + X_{35} + X_{45} + X_{65} = 1$$
$$X_{06} + X_{16} + X_{26} + X_{36} + X_{46} + X_{56} = 1$$

$X_{01} + X_{10} \leq 1,\quad X_{02} + X_{20} \leq 1,\quad X_{03} + X_{30} \leq 1,$
$X_{04} + X_{40} \leq 1,\quad X_{05} + X_{50} \leq 1,\quad X_{06} + X_{60} \leq 1,$
$X_{12} + X_{21} \leq 1,\quad X_{13} + X_{31} \leq 1,\quad X_{14} + X_{41} \leq 1,$
$X_{15} + X_{51} \leq 1,\quad X_{16} + X_{61} \leq 1,\quad X_{23} + X_{32} \leq 1,$
$X_{24} + X_{42} \leq 1,\quad X_{25} + X_{52} \leq 1,\quad X_{26} + X_{62} \leq 1,$
$X_{34} + X_{43} \leq 1,\quad X_{35} + X_{53} \leq 1,\quad X_{64} + X_{46} \leq 1,$

$X_{45} + X_{54} \leq 1, \qquad X_{46} + X_{64} \leq 1, \qquad X_{56} + X_{65} \leq 1,$
$(0,X_{ij},1)$

5.6.1. Discussing the results of the second stage

The most important results obtained are as follows, for more details see Appendix B:
The best path of transporting and distributing the main products from the factory to the shopping centers and then return to the factory as the follow:

$$X_{01} \longrightarrow X_{12} \longrightarrow X_{23} \longrightarrow X_{35} \longrightarrow X_{54} \longrightarrow X_{46} \longrightarrow X_{60}$$

Jamila→ Hayi-awr→ Waziriya→ Karrada→ Mansour → Adhamiya→ Palestine Street→ Jamila

Through the results of the lingo software solution, it was found that the first goal was achieved, where the value of p_1^+ is zero, which means the transportation cost of the truck during the process of transporting and distributing the main products from the factory to the shopping centers and then return to the factory is not exceed 60 dollar /day
As for the second goal was achieved also, where the value of p_2^+ is zero. This means the transportation time of the truck during the process of transporting and distributing the main products from the factory to the shopping centers and then return to the factory is not exceeding the available time for the factory.

6. Conclusions

1. In this study an efficient plan of transportation and distribution processes were designed in the Al-Rabee factory in a fuzzy environment where the fuzzy sets theory was applied which it helped to contribute and provide proposed acceptable solutions for the decision-maker.
2. The ranking function equation was used to eliminate fuzziness of the objective constraints of the mathematical model.
3. In this study an efficient plan of transportation and distribution processes were designed in the Al-Rabee factory in a fuzzy environment where the multi-objective travelling salesman problem was used to determine the best paths for two stages through adopting two important scientific methods: The traveling salesman problem and the goal programming method, where a mathematical model was built and solved through the Lingo software which achieved the best results within multiple goals for the decision-maker.
4. The multi-objective travelling salesman problem was applied to determine the best paths for two stages through adopting two important scientific methods: The traveling salesman problem and the goal programming method, where a mathematical model was built and solved through the Lingo software which achieved the best results within multiple goals for the decision-maker.
5. Through the results of the first stage, it was found that the path (Jamila → Fudayliah → Khan Bani Saad→ Kamalia → Khalis → Jamila) is the efficient path for the process of collecting and transporting milk from its collection centers to Al-Rabee factory.
6. Through the results of the second stage, it was found that the path (Jamila→ Hayi-awr→ Waziriya→ Karrada→ Mansour → Adhamiya→ Palestine Street→ Jamila) is the best path for the process of transporting and distributing the main products from the Al-Rabee factory to the shopping centers and then return to the factory.

   Using these paths would give an efficient and convincing solution to the decision-maker within the limited resources of the factory.

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Appendix A

```
min=P1minus ;

0.48*X01 + 0.45*X02 + 0.36*X03 + 0.35*X04 +P1minus - P1plus  =0.48;

35*X01 + 45*X02 + 45*X03 + 70*X04 + 35*X10 + 20*X12 + 25*X13 + 70*X14 + 45*X20 + 20*X21 + 25*X23 +
70*X24 + 45*X30 + 25*X31 + 25*X32 + 60*X34
+ 70*X40 + 70*X41 + 70*X42 + 60*X43+  P2minus -P2plus =200;

10*X01 + 11*X02 + 13*X03 + 17*X04 + 10*X10 + 5*X12 + 7*X13 + 18*X14 + 11*X20 + 5*X21 + 6*X23 + 17*X24
+ 13*X30 + 7*X31 + 6*X32 + 18*X34
+ 17*X40 + 18*X41 + 17*X42 + 18*X43+  P3minus - P3plus =60;

18*X01 + 25*X02 + 22*X03 + 30*X04+  P4minus -P4plus =20;

X01 + X02 + X03 + X04 = 1;
X10 + X12 + X13 + X14 = 1;
X20 + X21 + X23 + X24 = 1;
X30 + X31 + X32 + X34 = 1;
X40 + X41 + X42 + X43 = 1;
X10 + X20 + X30 + X40 = 1;
X01 + X21 + X31 + X41 = 1;
X02 + X12 + X32 + x42 = 1;
X03 + X13 + X23 + X43 = 1;
X04 + X14 + X24 + X34 = 1;
X01 + X10 <= 1;
X02 + X20 <= 1;
X03 + X30 <= 1;
X04 + X40 <= 1;
X12 + X21 <= 1;
X13 + X31 <= 1;
X14 + X41 <= 1;
X23 + X32 <= 1;
X24 + X42 <= 1;
X34 + X43 <= 1;
@bnd( 0,X01,1);@bnd( 0,X02 ,1);@bnd( 0,X03,1);@bnd( 0,X04,1);
@bnd( 0,X10,1);@bnd( 0,X12,1);@bnd( 0,X13,1);@bnd( 0,X14,1);
@bnd( 0,X20,1);@bnd( 0,X21,1);@bnd( 0,X23,1);@bnd( 0,X24,1);
@bnd( 0,X30,1);@bnd( 0,X31,1);@bnd( 0,X32,1);@bnd( 0,X34,1);
@bnd( 0,X40,1);@bnd( 0,X41,1);@bnd( 0,X43,1);@bnd( 0,X43,1);
```

```
Global optimal solution found.
Objective value:                    0.000000
Infeasibilities:                    0.000000
Total solver iterations:                  13
Elapsed runtime seconds:                0.06

Model Class:                              LP

Total variables:            28
Nonlinear variables:         0
Integer variables:           0

Total constraints:          25
Nonlinear constraints:       0

Total nonzeros:            117
Nonlinear nonzeros:          0
```

Variable	Value	Reduced Cost
P1MINUS	0.000000	0.000000
X01	1.000000	0.000000
X02	0.000000	0.000000
X03	0.000000	0.9000000E-01
X04	0.000000	0.1000000
P1PLUS	0.000000	1.000000
X10	0.000000	0.3000000E-01
X12	0.000000	0.000000
X13	1.000000	0.000000
X14	0.000000	0.000000
X20	0.000000	0.000000
X21	0.000000	0.000000
X23	0.000000	0.000000
X24	1.000000	0.000000
X30	0.000000	0.000000
X31	0.000000	0.000000
X32	1.000000	0.000000
X34	0.000000	0.000000
X40	1.000000	0.000000
X41	0.000000	0.000000
X42	0.000000	0.000000
X43	0.000000	0.000000
P2MINUS	0.000000	0.000000
P2PLUS	25.00000	0.000000
P3MINUS	3.000000	0.000000
P3PLUS	0.000000	0.000000
P4MINUS	2.000000	0.000000
P4PLUS	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	-1.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.4500000
7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	0.000000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	0.000000	0.3000000E-01
17	1.000000	0.000000
18	1.000000	0.000000
19	0.000000	0.000000
20	1.000000	0.000000
21	0.000000	0.000000
22	1.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	1.000000	0.000000

Appendix B

```
min= P1plus;

3*X01 + 7*X02 + 9*X03 + 8*X04 + 10*X05 + 7*X06 + 3*X10 + 7*X12 + 10*X13 + 9*X14 + 11*X15 + 5*X16 + 7*X20 + 7*X21 + 9*X23 + 5*X24
+ 9*X25 + 7*X26 + 9*X30 + 10*X31 + 9*X32 + 10*X34 + 10*X35 + 8*X36 + 8*X40 + 9*X41 + 5*X42 + 10*X43 + 9*X45 + 7*X46 + 10*X50 + 11*X51
+ 9*X52 + 10*X53 + 9*X54 + 10*X56 + 7*X60 + 5*X61 + 7*X62 + 8*X63 + 7*X64 + 10*X65 + P2minus - P2plus <=60;


15*X01 + 20*X02 + 40*X03 + 35*X04 + 60*X05 + 35*X06 + 15*X10 + 27*X12 + 40*X13 + 33*X14 + 55*X15 + 25*X16 + 20*X20 + 27*X21 + 40*X23 + 15*X24
+ 50*X25 + 20*X26 + 40*X30 + 40*X31 + 40*X32 + 60*X34 + 40*X35 + 50*X36 + 35*X40 + 33*X41 + 15*X42 + 60*X43 + 70*X45 + 45*X46 + 60*X50 + 55*X51
+ 50*X52 + 40*X53 + 70*X54 + 40*X56 + 35*X60 + 25*X61 + 20*X62 + 50*X63 + 45*X64 + 40*X65 + P1minus - P1plus <= 225;


X01 + X02 + X03 + X04 + X05 + X06 = 1;
X10 + X12 + X13 + X14 + X15 + X16 = 1;
X20 + X21 + X23 + X24 + X25 + X26 = 1;
X30 + X31 + X32 + X34 + X35 + X36 = 1;
X40 + X41 + X42 + X43 + X45 + X46 = 1;
X50 + X51 + X52 + X53 + X54 + X56 = 1;
X60 + X61 + X62 + X63 + X64 + X65 = 1;
X10 + X20 + X30 + X40 + X50 + X60 = 1;
X01 + X21 + X31 + X41 + X51 + X61 = 1;
X02 + X12 + X32 + x42 + X52 + x53 = 1;
X03 + X13 + X23 + X43 + X53 + X63 = 1;
X04 + X14 + X24 + X34 + X54 + X64 = 1;
X05 + X15 + X25 + X35 + X45 + X65 = 1;
X06 + X16 + X26 + X36 + X46 + X56 = 1;
X01 + X10 <= 1;
X02 + X20 <= 1;
X03 + X30 <= 1;
X04 + X40 <= 1;
X05 + X50 <= 1;
X06 + X60 <=1;
X12 + X21 <= 1;
X13 + X31 <= 1;
X14 + X41 <= 1;
X15 + X51 <= 1;
X16 + X61 <= 1;
X23 + X32 <= 1;
X24 + X42 <= 1;
X25 + X52 <= 1;
X26 + X62 <= 1;
X34 + X43 <= 1;
X35 + X53 <= 1;
X36 + X63 <= 1;
X45 + X54 <= 1;
X46 + X64 <= 1;
X56 + X65 <= 1;
@bnd( 0,X01,1);@bnd( 0,X02 ,1);@bnd( 0,X03,1);@bnd( 0,X04,1);@bnd( 0,X05,1);@bnd( 0,X06 ,1);
@bnd( 0,X10,1);@bnd( 0,X12,1);@bnd( 0,X13,1);@bnd( 0,X14,1);@bnd( 0,X15,1);@bnd( 0,X16 ,1);
@bnd( 0,X20,1);@bnd( 0,X21,1);@bnd( 0,X23,1);@bnd( 0,X24,1);@bnd( 0,X25,1);@bnd( 0,X26 ,1);
@bnd( 0,X30,1);@bnd( 0,X31,1);@bnd( 0,X32,1);@bnd( 0,X34,1);@bnd( 0,X25,1);@bnd( 0,X36 ,1);
```

@bnd(0,X40,1);@bnd(0,X41,1);@bnd(0,X43,1);@bnd(0,X43,1);@bnd(0,X25,1);@bnd(0,X46 ,1);
@bnd(0,X50,1);@bnd(0,X51 ,1);@bnd(0,X52,1);@bnd(0,X53,1);@bnd(0,X54,1);@bnd(0,X56 ,1);
@bnd(0,X60,1);@bnd(0,X61 ,1);@bnd(0,X62,1);@bnd(0,X63,1);@bnd(0,X64,1);@bnd(0,X65 ,1);

Variable	Value	Reduced Cost
P1PLUS	0.000000	1.000000
X01	1.000000	0.000000
X02	0.000000	0.000000
X03	0.000000	0.000000
X04	0.000000	0.000000
X05	0.000000	0.000000
X06	0.000000	0.000000
X10	0.000000	0.000000
X12	1.000000	0.000000
X13	0.000000	0.000000
X14	0.000000	0.000000
X15	0.000000	0.000000
X16	0.000000	0.000000
X20	0.000000	0.000000
X21	0.000000	0.000000
X23	1.000000	0.000000
X24	0.000000	0.000000
X25	0.000000	0.000000
X26	0.000000	0.000000
X30	0.000000	0.000000
X31	0.000000	0.000000
X32	0.000000	0.000000
X34	0.000000	0.000000
X35	1.000000	0.000000
X36	0.000000	0.000000
X40	0.000000	0.000000
X41	0.000000	0.000000
X42	1.000000	0.000000
X43	0.000000	0.000000
X45	0.000000	0.000000
X46	1.000000	0.000000
X50	0.000000	0.000000
X51	0.000000	0.000000
X52	0.000000	0.000000
X53	0.000000	0.000000
X54	1.000000	0.000000
X56	0.000000	0.000000
X60	0.000000	0.000000
X61	0.000000	0.000000
X62	0.000000	0.000000
X63	0.000000	0.000000
X64	0.000000	0.000000
X65	0.000000	0.000000
P1MINUS	0.000000	0.000000
P2MINUS	0.000000	0.000000
P2PLUS	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000

7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	0.000000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	0.000000
18	0.000000	0.000000
19	1.000000	0.000000
20	0.9253731	0.000000
21	0.7462687E-01	0.000000
22	1.000000	0.000000
23	1.000000	0.000000
24	0.9253731	0.000000
25	1.000000	0.000000
26	1.000000	0.000000
27	1.000000	0.000000
28	0.7462687E-01	0.000000
29	0.000000	0.000000
30	0.7462687E-01	0.000000
31	1.000000	0.000000
32	1.000000	0.000000
33	1.000000	0.000000
34	0.7462687E-01	0.000000
35	1.000000	0.000000
36	0.9253731	0.000000
37	0.9253731	0.000000
38	0.000000	0.000000