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Comparison between VG-levy and Kernel function estimation with application

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ABSTRACT

In this article, we present the variance Gamma Levy model that was obtained from the Brownian motion Gamma with their parameter estimation methods (Maximum Likelihood (MLE) and method of moments (MME)). Then, we compare them with Kernel density function depending on MASE.
Our application concerned with the Apple company that is listed on the Nasdaq, their data are suitable for the VG-Levy model and achieved the proper conditions of Levy of stability and independence, which means that Apple company was efficient in providing the information to investors.

Keywords: MLE, MME, MSE,Kernel function, VG-levy model

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1. Introduction

Random processes describe synonym of events defined at time intervals that can be used in stock markets and exchange rates. There are many types of random processes, including Brownian motions, Poisson processes, Gaussian processes, as well as Levy processes that refers to a continuous function from the right and marked on and satisfies the conditions of Independent and stationary increments.
The Levy process consists of two types, the first is Jump Diffusion models and the second type is Infinite activity models whose main movement is by means of oscillations [1].
In this article, we will study one of the models of the second type of Levy's processes, which is the Variance Gamma Levy Model (VG-Levy) for logarithmic returns data, and then estimating the parameters of the model based on the two methods of estimating (MME and MLE), and then modeling the processes of risk assets in finance and comparing the VG-levy model with the estimations of kernel density function using the rule of thumb method for selecting the smoothing parameter.
The reason for using Levy models in finance is because of their ability to explain stock returns such as heavy spikes and fluctuations. Loregian, Merenri, Roji in (2011) stated that the one of the advantages of the VG-Levy model is kurtosis and skewnwss, as well as having pure jump, i.e. empty of large jumps, so it is not possible to depend on the type of models with long periods as these include periods of reaction in the markets [2]. Hurst (1997) estimated the model for the Dow-Jones markets for the period from 1973 to 1993 after he removed the large negative increases on 19/10/1987 due to the bankruptcy of the stock markets on that day. While Rathgeber (2012) estimated the pricing assets for long periods after being classified or divided into used sectors (NAICS) system [3].

2. Variance Gamma Levy process

The variance gamma process is Levy process; it is a process of independent and stationary increments (Michael 2007) [4]. A variance gamma is constructed from Brownian motion gamma process, suppose that $X=\{X_t\}_{t\geq 0}$ be Arithmetic Brownian motion with drift θ and variance σ^2 and $\{X_t^G\}_{t\geq 0}$ be a gamma process with shape parameter 1/v and rate parameter 1/v. Then a variance gamma (VG) process $\{X_t^{VG}\}_{t\geq 0}$ is defined by [5]:

$$X_t^{VG} = \theta X_t^G + \sigma w_{X_t^G} \qquad \dots (1)$$

and the probability density function of VG processes with parameters $(\theta t, \sigma^2 t \frac{v}{t})$

$$fX_t^{VG}(x) = \frac{2exp\left[\frac{\theta}{\sigma^2}x\right]}{v^{t/v}\,\Gamma\left(\frac{t}{v}\right)\sqrt{2\pi\sigma^2}}\left(\frac{x^2}{\frac{2\sigma^2}{v}+\theta}\right)^{\frac{t}{2v}-\frac{1}{4}} k_{\frac{t}{v}-\frac{1}{2}}\left(\frac{\sqrt{x^2\left(\frac{2\sigma^2}{v}+\theta^2\right)}}{\sigma^2}\right) \quad \dots (2)$$

with x ,σ, v, θ ∈ R and σ > 0 , v > 0
$k_{\cdot}(.)$ is a modified Bessel function of the second kind, σ: is the scale or variability parameter, θ is skewness parameter, v: shape parameter (or kurtosis parameter).

The properties of VG-Le'vy process is as follows:
1. By Levy khintchine formula (for the purpose of associated distribution to the process) for Variance gamma distribution the characteristic function $\emptyset_{X_t}(u)$ of X_t^{VG} is given by [5]:

$$\emptyset_{X_t}(u) = \left(1 - iu\theta v + \frac{\sigma^2 u^2 v}{2}\right)^{\frac{-t}{v}}$$

2. VG–Levy processes is an infinitely divisible (independent and identically distribution increments over intervals of equal length) this is clear from the linearity of the log of the characteristic function in the time variable.
3. Infinite activity
4. Finite variation (it can be written as the difference of two increasing process, the first of specify for the price increases, while the second explains the price decreases). In the case of the gamma process the two increasing processes that are differenced to obtain the VG process are themselves gamma process) [6].
It means that the VG process is free of jumps, as it is difficult to estimate VG processes for long periods, as during this period there is a reaction or a high rise in prices with a difference in the stages of the market due to the change in the behavior of stocks.
6. Four moments of the VG-Levy process can be calculated by using moment generating function $\emptyset X(u) = Ee^{iux}$, so the moments for increments X_t is [7]:

$$M_1 = \theta_t \quad \dots (3)$$
$$M_2 = (\theta_t^2 v_t + \sigma_t^2) \quad \dots (4)$$
$$M_3 = 2\theta_t^3 v_t^2 + 3\sigma_t^2 \theta_t v_t \quad \dots (5)$$
$$M_{4=} 3\sigma_t^4 v_t + 12\sigma_t^2\theta_t^2 v_t^2 + \sigma_t\theta_t^4 v_t^3 + 3\sigma_t^4 + 6\sigma_t^2\theta_t^2 v_t + 3\theta_t^4 v_t^2 \quad \dots (6)$$

7. Statistical properties of the VG-levy

$$\mathrm{E}(X_t^{VG}) = \theta_t \quad \dots (7)$$
$$\mathrm{Var}(X_t^{VG}) = (\theta_t^2 v_t + \sigma_t^2) \quad \dots (8)$$
$$\mathrm{Skew}(X_t^{VG}) = \frac{2\theta_t^3 v_t^2 + 3\sigma_t^2\theta_t v_t}{(\theta_t^2 v_t + \sigma_t^2)^{\frac{3}{2}}} \quad \dots (9)$$
$$\mathrm{kurt}(X_t^{VG}) = \frac{(3\sigma_t^4 v_t + 12\sigma_t^2\theta_t^2 v_t^2 + \sigma_t\theta_t^4 v_t^3)}{(\theta_t^2 v_t + \sigma_t^2)^2} \quad \dots (10)$$

Seneta (2004) and Finlay and Seneta (2006) assume that we face a symmetric case of the empirical distribution of log returns and we can approximate $\theta \approx 0$ this restriction implies $\theta^2 = \theta^3 = \theta^4 = 0$ Therefore the statistic properties write to (7-10) as following [7, 8]:

$$\mathrm{E}(X_t^{VG}) = \theta_t \quad \dots (11)$$
$$\mathrm{Var}(X_t^{VG}) = \sigma_t^2 \quad \dots (12)$$
$$\text{Skewness} \quad (X_t^{VG}) = \frac{3\theta_t v_t}{\sigma_t} \quad \dots (13)$$
$$\text{Kurtoses} \quad (X_t^{VG}) = 3(1 + v_t) \quad \dots (14)$$

8. The risk asset or stock price be given by

$$S_t = S_0 e^{X_t^{VG}} \quad \dots (15)$$

Where: S_t denote to the price of risk asset in time

3. Methods of estimation of VG-Levy model

3.1. Method of moment (MOM)

The (MOM) approach is used to determine the parameters of VG-Levy process by means of the empirical moments M_i^E $(M_2^E: variance, M_3^E: skewness, M_4^E: kurtosis) empirical$ and equations (12-14), we get the following four estimators after matching the moments of the population parameters with their estimates [9].

So for equation (12): $\hat{\sigma}_t^2 = M_2^E$

$$\therefore \hat{\sigma}_t = \sqrt{M_2^E} \quad \dots (16)$$

and for equation (13) : $M_3^E = \frac{3\hat{\theta}_t \hat{v}_t}{\hat{\sigma}_t}$

$$\therefore \hat{\theta}_t = \frac{M_3^E \hat{\sigma}_t}{3\hat{v}_t} \quad \dots (17)$$

And for equation (14): $M_4^E = 3(1 + \hat{v}_t)$

$$\therefore \hat{v}_t = \frac{M_4^E}{3} - 1 \quad \dots (18)$$

3.2. Method of maximum likelihood estimation (MLE)

The MLE procedure is based on maximization of the log likelihood function, which in the variance gamma model is[1]

$$L(\theta, \sigma, v) = \frac{n}{2}\log\left(\frac{2}{\pi}\right) + \sum_{t=1}^{n} \frac{x\theta}{\sigma^2} - \sum_{t=1}^{n} \log(\Gamma(v)\sigma) + \sum_{t=1}^{n} \log\left(k_{v-0.5}\left(\frac{\sqrt{2\sigma^2 + \theta^2} + |x|}{\sigma^2}\right)\right) + \sum_{t=1}^{n}\left(v - \frac{1}{2}\right)\left[\log\left(|x| - \frac{1}{2}\log\left(2\sigma^2 + \theta^2\right)\right)\right] \quad \dots (19)$$

As observed in [6], the direct optimization of log likelihood is computationally expensive.
Because there are three parameters even in the case of symmetry (θ = 0) and this is due to the presence of the BASEL function, this leads to a clear difficulty in finding the estimators in addition to defining the BASEL function depending on the parameter v.
So the Nelder Mead algorithm was used. In addition, the MLE results are strongly influenced by the initial values of the parameters. In order to overcome this problem, the initial values of the MLE method were chosen based on the estimators of the MOM [11-14].

4. Kernel density estimator

Kernel function density estimators were used for logarithm data of experimental returns, depending on the rule of thumb method to select the smoothing parameter.
For the purpose of comparing, it with the estimators of VG-levy at the MOM and ML estimators, suppose that X_i represent to the logarithm of the experimental returns (actual) and that the estimator of the probability density function at point x are as follows [10]:

$$\hat{f}(X) = \frac{1}{nh}\sum_{i=1}^{n} k\left(\frac{X - X_i}{h}\right) \quad \dots (20)$$

Where n represents to the sample size, h refers to the smoothing parameter.so one of the methods that can be used to select it named the rule of thumb or Silverman's method $h = 1.06\,\sigma\, n^{\frac{-1}{5}}$, where σ :denotes to the standard deviation.

5. Practical application

For analyzing the VG-levy process we use logarithm of the daily return of the Apple INC (Nasdag) stocks and of the index over the period from [18/4/2018. To 18/4/2019] as follows:

$$X_t = \log \frac{X_t}{X_{t-1}} \qquad \dots (21)$$

The chi-square test was used to know whether Apple's stock dividend data is distributed in the distribution of Variance Gamma or not, and through the results it was found that there is acceptance of the null hypothesis, which is that the data is distributed in distribution of Variance Gamma.
Then, as a result of matching the data to the distribution of VG, fulfilling the conditions of VG-Levy is achieved through the independence and stability. From figure (1) we can see fulfillment of the characteristic (1) of the Levy process.
In addition, the Augmented Dickey-Fuller (ADF) test was used to verify the stability of the data on the assumption that the null hypothesis (is the instability of the stock returns data for Apple Inc. for the period studied).
For alternative hypothesis (stability of Apple's stock returns data for the period studied), there was a rejection of the null hypothesis and the acceptance of the alternative. Then the runs test was used also to test the independence of the returns on the assumption of null hypothesis (it is the independence of returns) versus the alternative hypothesis (that stock returns are not independent), we found the acceptance of the null hypothesis and that the Apple Inc. share earnings data are independent.

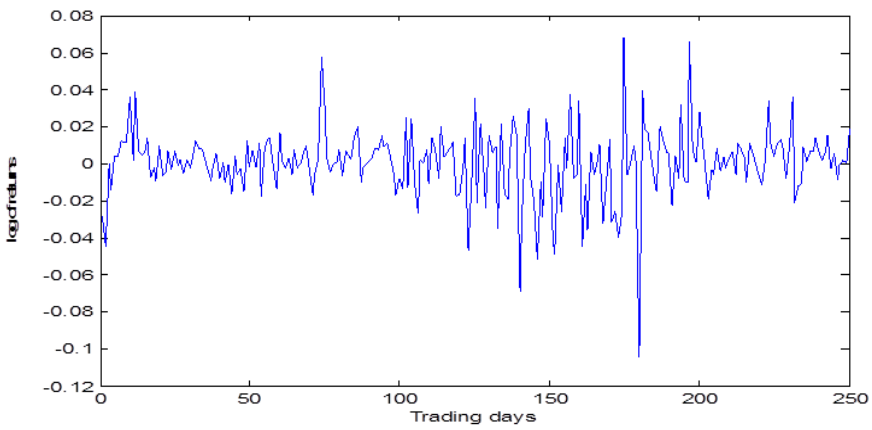

Figure 1. The independence and stability of Apple's revenue algorithm

t is noticed through the aforementioned figure that the independence of returns indicates the efficiency of stock pricing for Apple in providing the information to investors, and that the stability of stock prices is reflected in reducing the volatility of investment returns and hence the associated risks.
It is known that the goal of building the VG-Levy Random Process Model is to reveal the known statistical features of stock returns in order to make inference about the parameters control the model based on the process observations.
Therefore, the parameters of the VG model were estimated based on the maximum potential and momentum methods, as shown in Table 1.

Table 1. The moment and potential estimators of the variance gamma model

Method	θ	σ	V
MOM	-0.0027	0.0189	1.703
MLE	-0.0022	0.0394	0.4827

From the estimators of σ, we notice a remarkable relative discrepancy that reflects on the varying levels of risk associated with investing stocks and the extent of the matter related to the time period for research and the nature of the data used in the analysis at the level of the company's daily closing prices.
Also, for Skewness parameter θ (that represent the internal risk) the results indicate a negative Skewness in the curve of returns dividend, that indicates to the fluctuation in prices with a positive deviation.
However, if the kurtosis parameter V (represent by external risk) is low, this will indicate to less risk for investors than in the case where an investment with high kurtosis values.
It was found that the Skewness parameter (internal risk) is a result of fluctuations and reflects the risks. As for the kurtosis parameter V (external risk), we found that Apple had less internal risks than external risks, and the price fluctuations represented by the value of σ are related to the value of internal risk θ, meaning that the increase in volatility leads to high kurtosis and thus a higher risk for investors.

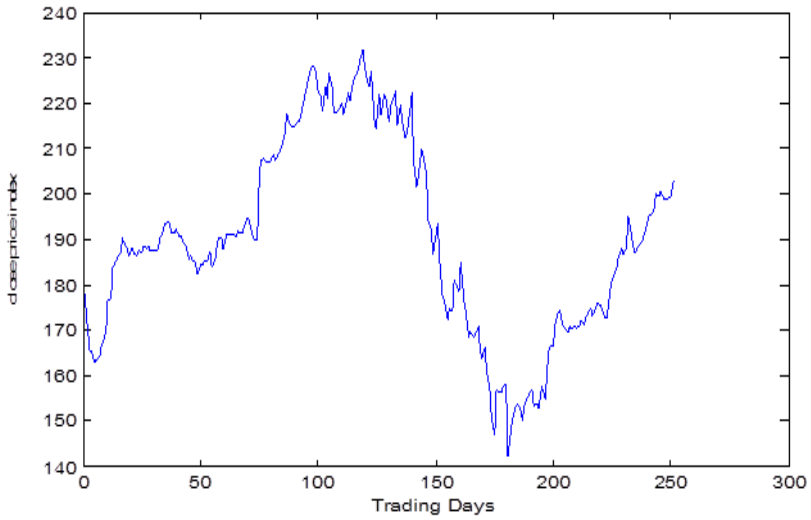

Figure 2. Apple Inc. stock prices

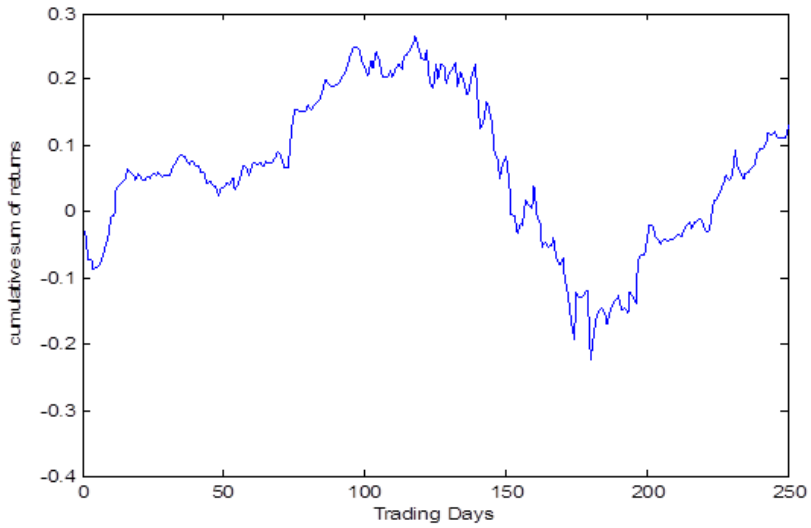

Figure 3. The cumulative sum of returns

Figure 3 shows the development of stock prices and earnings during the aforementioned period from periods of rise (increase in returns) and decline (decrease in returns) and that through the figure it can be seen that the path was in a clear fluctuation and this is evident through the values σ, v
For the purpose of comparing the logarithm of returns with the estimators of MOM and MLE in the VG model based on the following figures and the mean square error (MSE):

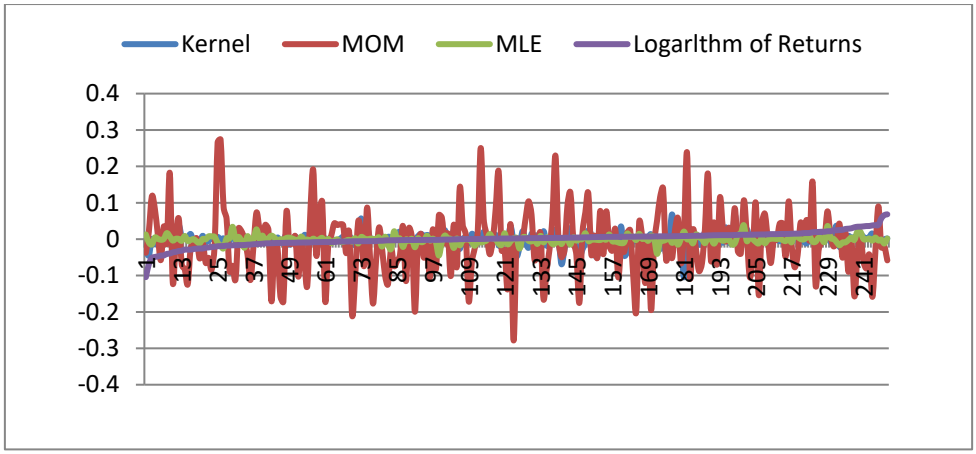

Figure 4. Comparison of the logarithm of returns with the estimators of MOM and MLE in the VG model and kernel.

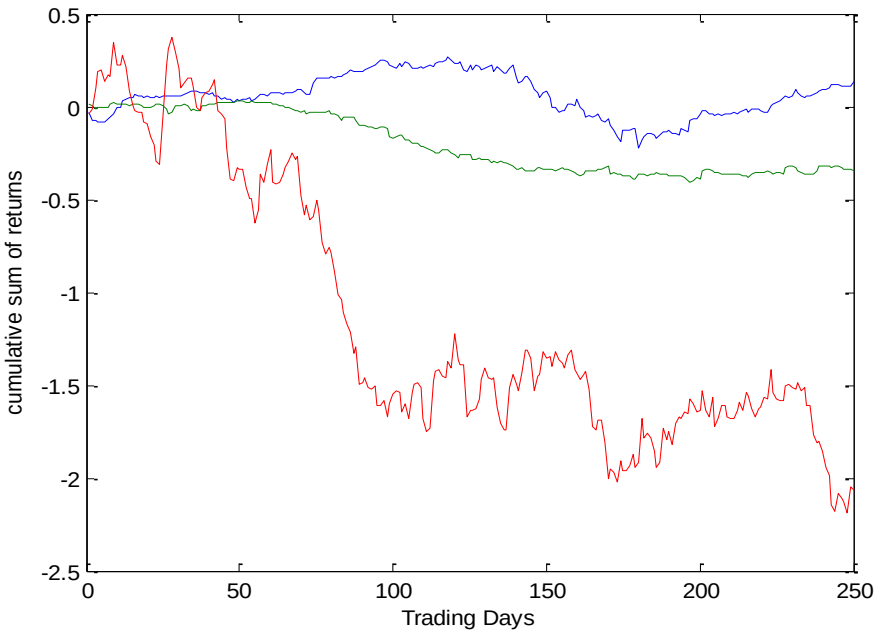

Figure 5. A comparison the cumulative sum of returns with the estimators of MME, MLE when using the VG model

From Table 2 and Figures 4-5, we compare MOM,MLE and Kernel as estimators of the VG-levy model using the mean square error(MSE) criterion for the empirical data (log of returns).

$$MSE = \frac{\sum_{i=1}^{n}\left(X_i - \hat{X}_i\right)^2}{n}$$

Table 2. MSE for MOM, MLE and Kernel estimators

Method	MSE
MOM	0.0073
MLE	0.000473
Kernel	0.000689

From the results we find that the best fit is achieved by the MLE method followed by the Kernel.

6. Conclusions

The aim of studying the price fluctuations through the parameters of the model and thus the possibility of knowing the trends of stock prices in the financial markets and the consequent risks associated with investing in them in a manner consistent with the investor's preferences regarding bearing a certain degree of risk in the sense of a pre-preparedness for the potential sacrifice of the investor's capital with the limits of risk resulting from those fluctuations in prices for the returns resulting from price movements in those markets. So, it was found that the VG-levy model with MLE is the best.

References

[1] M. Jumma," Methods of estimating model parameters levy processes and employment in returns stock prices north Bank and United, Master thesis, Baghdad University, 2016.

[2] A. Loregian, L. Mercuri, and E. Rroji, "Approximation of the variance gamma model with a finite mixture of normals", Statistics & Probability Letters, vol.82, no. 2, pp.217-224, 2012.

[3] S.R Hurst and E. Platen,"The Marginal Distribution of Returns and Volatility in L1 " Statistical Procedures and Related Topics, IMS Lecture Notes –Monograph series, vol. 31, pp.301-314, 1997

[4] M.C. Fu,"Variance Gamma and Monte Carlo", Advances in Mathematical Finance, part of Book Series Applied and Numerical Harmonic Analysis, publisher Name Birkhauser Boston, 2007.

[5] W. Lydia, "American Monte Carlo Option Pricing Under Pure Jump Levy Models"thesis presented partial fulfillment of the requirements for degree of Master of science in the faculty of science, university Stellenbosch, 2012.

[6] D.B. Madan, P. P. Carr, and E. C. Chang, "The variance gamma process and option pricing", Review *of Finance*, vol.2, no. 1 (1998): 79-105.

[7] S. Eugene, "Fitting the variance Gamma model of financial data", Journal of Applied probability, vol. 41, pp. 177-187, 2004.

[8] R. Finaly, and S. Eugene, stationary -increment student and Variance Gamma processes, Journal of Applied probability, vol.43, pp 441-453, 2006.

[9] A. Rathgebr, J. Stadler and S. Stock, "Modelling Share Returns an Empirical Study on The Variance Gamma Model", Journal of Economics and finance, vol.40, pp 653-682, 2013.

[10] D. H Rashed. M. Y. Hmood,"Comparing nonparametric density estimators using different canonical kernel functions, 2nd scientific conference of the college of Computers Sciences & Mathematics (SCOMS), Iraq, pp172-181, 2009.

[11] D.B. Madan, P. P. Carr, and E. C. Chang, "The variance gamma process and option pricing", Review *of Finance*, vol.2, no. 1, 79-105, 1998.

[12] D.B. Madan and S. Eugene,"Characteristic Function Estimation Using Maximum Likelihood on Transformed Variables" Econometrics Discussion papers, Department of Econometrics, University of Sydney, 1987.

[13] M. Y. Hmood and M. Jumma,"Use of Le'vy's Model to Simulate Stock Dividends Actual Estimations of Some Iraqi Banks", International Journal of Psychosocial Rehabilitation, vol.24, no.9, pp.83-97, 2020.

[14] M. Y. Hmood and M. Jumma,"Using the Le'vy Model to estimate the stock returns for some Iraqi banks", Journal of Economics and Administrative Sciences, vol.22, no.94, pp.460- 479, 2016.