Applied mathematical model for a heat transfer

Maysoon M. Aziz
Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Iraq

ABSTRACT

It is implicit for some time ago, that the radiative magneto hydrodynamic stands for an interacting between a radiation field and magneto hydrodynamic (MHD) field that is concerned with interacted electrical conducting fluids and electromagnetic field. An accurate solution to the heat transfer equation is obtained by calculating the radiant heat emission in the boundary layer MHD because of the eccentric rotations of the pore disk and the fluid for long time. It is found that the asymptotic solution is present in both the cases of suction and blowing states, whereas no such solution exists for the blowing state in the absence of radiative emission of heat. The heat transfer rate at the disk has been determined and the condition is gotten for the heat to flow from the liquid to the disk.

Keywords: Heat equation, Modeling, Ordinary and partial differential equations, Suction, Blowing

Corresponding Author:

Maysoon M. Aziz
Department of Mathematics
College of Computer Sciences and Mathematics
University of Mosul, Iraq
E-mail: aziz_maysoon@uomosul.edu.iq

1. Introduction

The investigation of a MHD layer with radiative heat transfer stands for a great importance in numerous astrophysical and aeronautical problems. Additionally, they also server to enhance our broad under-standing of motions involving plasmas and conducting fluid generally Recently Neild and Bejan [1], Chatti and et al [2], Khan and et al [3], Lee [4], Guenther and Lee [5], Khan and Elkamel [6] studied the boundary Layer because of no coaxial rotations of pores disc and fluid for long time, but they neglected the effects of radiative heat transfer which plays an important role in analyzing heat transfer characteristics under high temperature conditions. Numerical analysis of 3D flow among parallel rotating plates for a common axis had accomplished by Hud and et al [7], by Lai and Rajagopal [8]. Coirier [9] had been firstly studied a flow of Newtonian fluid caused by non-coaxial fluid owing to non-coaxial rotation of fluid and a disk for long time, he takes a rotation for both cases the dissimilar and identical angular velocity. Erdogan [10] studied the same flow in the case of pore disk when the rotation with both cases the same and different angular velocities, respectively. Numerical analysis of 3D flow among parallel rotating plates regarding a common axis has performed by Siddiqui and et al [11] and Maqbool and et al [12]. Impulsively started unsteady flow produced by eccentric rotation of a disk and fluid for long time has been contributed by Pop [13]. Kaloni and Siddiqui [14] investigated the time dependent Navier-stokes equations for a viscous fluid flow among eccentric rotating disks. Dupont and Crochet [15] and Guo and et al [16] extended the analysis done for the case of magnet-hydrodynamic effect. Wazwaz [17] obtained the analytical solution of the unsteady flow induced by non-coaxial rotations of a pore disk and fluid for long time. Bali's and et al [18-22] give the Numerical studies of heat transfer between two eccentric rotating disks. Heat transfer phenomenon due to free convective have ample application in many engineering practices. The objective of this communication is to study the radiative heat emission influence on the flow of heat from the liquid to the disk or from the disk to the liquid[23-36]. This paper is organized as follow, the next sections significance of the problem, section three problem description, section four the mathematical analysis and solution of the problem, exact solution has been established and description of parameters and discussion followed by conclusion in section 5.

2. Significance of the problem

This research we expect is important in applied mathematics and useful for fieldwork in engineering.

3. Problem Formulation

We consider a system of the Cartesian coordinate in which the z-axis has been taken normally to the plan z=0 of the pore disc. It is assumed that the disc and the fluid have been rotating with uniform angular velocity Ω regarding the axes with distance L in the plane x=0 between them, so that the boundary conditions are

$$\begin{cases} u = -\Omega y, v = x\Omega, w = -w_0 \text{ at } z = 0, \\ u = -\Omega y + \Omega L, v = x\Omega, w = w_0 \text{ as } z \to \infty \end{cases} \quad \cdots(1)$$

Where the components of the fluid velocity are w,v,u . The fluid is assumed electrically conducting with a transverse applied magnetic field B_0 in the direction of z-axis. Reynolds number magnetic R_m has been assumed so small that the induced magnetic field is negligible as compared to the applied magnetic field. Following the velocity profits in the magnetohydrodynamic (MHD) boundary layer caused by the rotation of disc are caused by the rotation of disc given by:

$$u = -\Omega y + f(z),\ v = \Omega x + g(z),\ w = -w_0 \quad \cdots(2)$$

where,

$$f = \Omega L(1-e^{-\alpha\mathfrak{z}} \cos\beta\mathfrak{z})$$

$$g = \Omega L\, e^{-\alpha\mathfrak{z}} \sin\beta\mathfrak{z}$$

$$\mathfrak{z} = \left(\frac{\Omega}{2v}\right)^{\frac{1}{2}} z,\ \alpha = \sqrt{2s} + [\,(s^4 + N + 1)^{\frac{1}{2}} + s^2 + N\,]^{\frac{1}{2}}$$

$$\beta = [\,(s^4 + N + 1)^{\frac{1}{2}} - (s^2 + N)\,]^{\frac{1}{2}}$$

$$S = w_0(2\Omega v)^{-\frac{1}{2}},\ N = \frac{\mu\beta_0^2}{(\rho\Omega)}$$

Here,

S: is suction parameter

N: is magnetic parameter

The heat transfer equation to be solved is of the form.

$$PC_p w \frac{dT}{dz} = k \frac{d^2T}{dz^2} - \frac{dq_R}{dz} + pv[(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2] \quad \cdots(3)$$

where

C_p stands for the specific heat at a constant pressure

T denotes absolute temperature

K denotes a coefficient of thermal condition

q_R denotes the flux of radiative heat

V denotes the kinematic coefficient of viscosity.

4. Solution of the problem

Based on optically thin approximation, the mean free path of radiation is much larger than the characteristic length of flow field. The situation is sometimes expressed as that of radiation with negligible self-absorption so that the plank mean absorption coefficient $k_p = \frac{1}{p} L_k$ is very small, where L_R is the characteristic length of mean free path of radiation. Each element of the fluid exchanges radiation straightforwardly with the bounding surfaces and so there is no rad-dative interaction between various fluid elements. Sparrow, and Cess [24] have shown that the equation of radiative heat transfer for a non-grey gas near equilibrium under optically thin limit assumes a simple from:

$\frac{dq_R}{dz} = 4\text{p}k_p\sigma T^4$ ···(4)

Where

σ : is the Stefan – Boltzmann constant k_p: is the Planck mean absorption coefficient

For the case of extremely optically thin limit $k_p \to 0$ as $L_R \to \infty$ and thus the flux of radiative heat will be a constant as has been shown.

Since the temperature T_0 at the pore disc is the highest and the temperature T_∞ of the liquid for long time is the lowest, it can be assumed that the liquid at infinity is the lowest, it can be assumed that the liquid at infinity attains an isothermal state of equilibrium whereas the temperature difference in the whole flow filed is small. Under such condition it is possible to linearize the equation (4), and for the present case the best-fit linear approximation of (4) can be expressed in the form:

$\frac{dq_R}{dz} = c_1\text{T} - c_2$ ···(5)

where

$c_1 = \frac{16}{5}\text{P}k_p\sigma T_0^3(1 - \frac{T_\infty}{T_0})^3$

$c_2 = \frac{4}{5}\text{P}k_p\sigma T_0^4(1 - \frac{T_\infty}{T_0})^4$

The liquid at infinity is in an isothermal state of equilibrium. The boundary conditions to be satisfied by the temperature profile of (4) are.

$T_{(0)} = T_0$, $\text{T} \to \frac{1}{5}T_0$ as $z \to \infty$ ···(6)

Which mean that the pore plate is maintained at a constant temperature T_0 and the fluid temperature at infinity has stationary value $\frac{1}{5}T_0$, we introduce the following dimension less parameters:

$\mathfrak{z} = (\frac{\Omega}{2v})^{\frac{1}{2}}\text{z}$, $\theta = \frac{T - T_0}{T_\infty - T_0}$, $p_r = \frac{2v^c p^p}{k}$, $E_c = \frac{\Omega^2 L^2}{2T_0 c_p}$,

where p_r is called a Prandtl number and E_c is called Eckert number. Transforming (3) in to a dimension less form we get

$-\frac{d^2\theta}{d\mathfrak{z}^2} + p_r\text{s}\frac{d\theta}{d\mathfrak{z}} - \text{F}\theta = -\text{F} + \frac{5}{4}P_r E_C(\alpha^2 + \beta^2)e^{-2\propto\mathfrak{z}}$ ···(7)

Where $\text{F} = \frac{(4)\ (\frac{4}{5})^4\sigma k_p p_r T_0^3}{c_p\Omega} > 0$

The boundary conditions (6) assume the form ,

$\theta(0) = 0, \theta(\infty) = 1$ ···(8)

Solution (7) satisfying (8) is given by.

$\theta(\mathfrak{z}) = 1 - e^{-\lambda\mathfrak{z}} + \frac{5E_c P_r(\alpha^2+\beta^2)}{4(4\alpha^2 - 2\alpha P_r S - F)}(e^{-2\alpha\mathfrak{z}} - e^{-\lambda\mathfrak{z}})$ ···(9)

For $4\alpha^2 - 2\alpha P_r S - F \neq 0$ and

$\theta(\mathfrak{z}) = 1 - e^{-\lambda\mathfrak{z}} - \frac{5}{4}[\frac{E_c P_r(\alpha^2+\beta^2)e^{-2\alpha\mathfrak{z}}}{(4\alpha - P_r S)}]$ ···(10)

For $4\alpha^2 - 2\alpha P_r S - F = 0$, where $\lambda = \frac{1}{2}(\sqrt{P_r^2 S^2 + 4F + P_r S}) > 0$.

Since F > 0 & λ > 0 for both cases of suction (s>0) & blowing (s<0), the asymptotic solution (9) and (10) are valid for the whole flow filed, where as in the absence of radiative heat transfer the asymptotic solution does not exist for the case of blowing [11].

In the limit of vanishing radiative effects (F=0), the solution (9) and (10) reduce to

$\theta(\mathfrak{z}) = 1 - e^{-P_r S} + \frac{5}{4}[\frac{E_c P_r(\alpha^2+\beta^2)}{(2\alpha(2\alpha - P_r S)}(e^{-2\alpha\mathfrak{z}} - e^{-P_r S})]$ ···(11)

For $2\alpha - P_r S \neq 0$ and

$\theta(\mathfrak{z}) = 1 - e^{-P_r S} - \frac{5}{4}[\frac{E_c P_r(\alpha^2+\beta^2)}{4\alpha - P_r S}(e^{-2\alpha\mathfrak{z}})]$ ···(12)

For $2\alpha = P_r S$.

The equation (11) and (12) show that the solution is valid only for the suction case (s>0) and no asymptotic solution exists for the blowing case (s<0). At the disc the rate of heat transfer is given by

c_w = -k $(\frac{dT}{dz})$z = 0

$= \frac{4}{5} T_0 k(\frac{\Omega}{2\nu})^{\frac{1}{2}} [\lambda - \frac{5}{4} \frac{E_c P_r(\alpha^2+\beta^2)}{2\alpha+\frac{1}{2}\left(-P_r S+\sqrt{P_r^2 s^2+4F}\right)}]$

The heat will flow from the disc to the liquid, if the following condition is satisfied:

$\lambda < \frac{5}{4} [\frac{E_c P_r(\alpha^2+\beta^2)}{2\alpha+\frac{1}{2}\left(-P_r S+\sqrt{P_r^2 s^2+4F}\right)}]$

or otherwise, there will be flow of heat from the liquid to the disc.

5. Conclusion and recommendation

1-This research is about model in applied mathematics for an interact with fluid flow, heat transfer and radiative optical thin limit, it includes a solution of heat transfer equation and radiative heat emission in magneto hydrodynamic (MHD)
boundary layer with eccentric rotations of pores and disk and fluid for long time.

2- It is found that the asymptotic solution exists in both the cases of blowing and suction whereas no such solution exists for the case of blowing in the absence of radiative heat emission.

3- The heat transfer rate at the disc is determined, and the condition is obtained for the flowing of heat from the liquid to the disc.

4- The equations (11) and (12) show that the solution is valid only for the suction case of (s>0), and the asymptotic solution does not exist for the blowing case, (s < 0).

5- This research is an important and wide subject especially for engineering students in undergraduate and postgraduate in most of the Universities. Also the important of the researches from the above material and its equations (ordinary differential and partial differential) and their solutions especially for the faculty interest in applied mathematics

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