Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA)
http://dx.doi.org/10.5424/fs/2014233-04937

Forest Systems 2014 23(3): 494-499
ISSN: 2171-5068
eISSN: 2171-9845

OPEN ACCESS

Resource Communication. Modeling dominant height growth including site attributes in the GADA approach for *Quercus faginea* Lam. in Spain

Eduardo Lopez-Senespleda[1,2]*, Andres Bravo-Oviedo[1,2], Rafael Alonso[1] and Gregorio Montero[1,2]

[1] Forest Research Centre (INIA-CIFOR). Ctra. A Coruña, km 7,5. 8040 Madrid, Spain. [2] Sustainable Forest Management Research Institute. Universidad de Valladolid-INIA. Spain

Abstract

Aim of the study: To develop a site index model for *Quercus faginea* Lam. stands.

Area of study: Spain

Material and methods: Data from 81 growth series collected in plots where *Q. faginea* was the main species were used for modelling. Different generalized algebraic difference equations (GADA) were fitted from traditionally used models. Richards model was selected and used to expand the parameters with environmental variables.

Research highlights: Winter rainfall (WR), annual potential evapotranspiration (PET) and pH were introduced increasing the prediction ability of the GADA. It is strongly recommended to apply the model with ages lower than 80 years because the lack of data above that age makes bias increase and efficiency decrease.

Key words: site index; Lusitanian oak; environmental variables.

Introduction

Lusitanian oak (*Q. faginea* Lam.) is a semi-deciduous species widely extended in the Iberian Peninsula with a fragmented distribution (Costa *et al.*, 2005). Despite their multiple benefits, its stands have been reduced to the current distribution in favour of the holm oak (*Quercus ilex* L.), because of its better ability to sprout from both root and stump and the sweetness of its acorns.

Dominant tree height at a given age is commonly used as a measure of site quality, which in turn expresses the potential productivity of a site for a particular tree species (Clutter, 1983; Ortega & Montero, 1988). There are many studies about growth equations for Mediterranean species like cork oak (Sánchez González *et al.*, 2005; Sánchez González *et al.*, 2010), stone pine (Calama & Montero, 2004), pyrenean oak (Adame *et al.*, 2006), holm oak (Gea-Izquierdo, *et al.*, 2008), juniper (Alonso Ponce, 2008) and Mediterranean maritime pine (Bravo-Oviedo *et al.*, 2007; Bravo-Oviedo *et al.*, 2008).

Nevertheless, there are few studies about site index for *Q. faginea* (San Miguel, 1986; López-Senespleda & Sánchez-Palomares, 2007), thus the aim of this work was to build a height-age site index model for *Q. faginea* stands in Spain, based on stem analysis covering the main range of its distribution area. The functions tested in this study were Richards, Gompertz, Levakovic III & Hossfeld IV (Richards, 1959; Kiviste *et al.*, 2002), using the Generalized Algebraic Difference Approach (GADA) (Cieszewski & Bailey, 2000) including environmental variables in the model.

Data

Data used in this work comprise dominant height growth series from 159 stem analysis collected in 91 plots of 500 m^2 where *Q. faginea* was the main species. Plots were sampled in different physiographic, edaphic and climatic conditions within the natural extent of Lusitanian oak stands in Spain. In each plot, one or two dominant trees were selected, cut down and then sectioned in 1-m length logs above the diameter at breast height (DBH) level. Disks were collected from

* Corresponding author: elopez@inia.es
Received: 02-09-13. Accepted: 25-04-14.

each log, from the bottom to the tree top. The TSAP© software (Rinn, 2005) was used combined with a linear positioning digitizer tablet (LINTAB) to measure the tree-rings and age for each disk. For the true height estimation at a given age, the Carmean's algorithm was used (Carmean, 1972). Finally 81 growth series were used to build the site index models with a mean age of 60 years (range 27-120 years), and a mean height of 7.9 m (range 4.9-14.8 m).

Modeling approach

To apply the GADA approach (Cieszewski and Bailey, 2000), the growth functions must be expressed via dynamic equations obtained through the generalization of the main parameters. The methodology could be summarized by the following steps (Krumland and Eng, 2005): i) Select a suitable base equation; ii) Identify all the parameters that potentially change for different levels of site productivity and reformulate the base equation by replacing these parameters as function of X; iii) Solve the resulting GADA formulated model for X and find a specific solution in terms of initial conditions of height and age (H_0, t_0). The GADA approach has desirable attributes when modeling dominant height equations, *i.e.,* polymorphism, inflection point, asymptote, biologically acceptable behavior and base-age invariance (Goelz and Burk, 1992; Álvarez González *et al*., 2004). In order to fit the models, the dummy variable approach was apply (Cieszewski *et al*., 2000).

The candidate growth functions considered as base equations were: Richards, Gompertz, Levakovic III and Hossfeld IV (Richards, 1959; Kiviste *et al*., 2002). Six different assumptions derived from Cieszewski (2004) and Cieszewski & Strub (2008) were used to obtain 24 dynamic equations. The candidate functions were fitted by nonlinear regression using the PROC MODEL (SAS, 2004).

Model selection

The evaluation of the models was based on qualitative and quantitative examinations of the residuals of the fitted models and the plotted curves. The first step was to evaluate the statistics of the fitted models (Amaro *et al*., 1998): the bias or the deviation of the model with respect to the observed values, the mean square error (MSE), the adjusted coefficient of determination (R^2_{adj}) and Akaike's information criterion (AIC) (Akaike, 1974).

The second step was focused on the graphical analyses. The biological sense of the model was evaluated through the shape of the fitted curves and the value of their asymptotes. Once the analysis of the logical and biological consistency of the parameters estimation was made, the selected functions were subjected to Jackknife cross-validation (Davidson and Hincley, 1997).

In order to set the age of reference, the relative error in height and the number of observations were represented by age classes. The site index predictions with age were plotted to characterize their consistency also (Álvarez González *et al*., 2004).

Parameter expansion

Once a GADA model was selected, the expansion of the parameters was performed. In order to select the environmental variables to be used, a Principal Component Analysis (PCA) was made and the variables which accounted for the maximum amount of variance were chosen and combined in different growth theory assumptions (Bravo-Oviedo *et al*., 2008). The model selection was carried out with the same criteria than in the previous section. Finally, the model with the expanded parameters and the GADA model where compared to analyze their behavior.

Results and discussion

All the models were fitted with a second order continuous autoregressive error structure (Gregoire *et al*., 1995; Zimmerman and Núñez-Antón, 2001; Diéguez Aranda *et al*., 2005), which seems to be more appropriate to account for autocorrelation. Only 6 of the 24 tested equations met the convergence criterion (Table 1).

All the models showed an appropriate graphic performance except models M5 and M6 which had the poorest fitting values. The other models exhibited very small differences among them.

The asymptotes of the M1 model are 7.5-20.6 m with t_0 = 50 years and H_0 = 4-12 m as initial conditions. In the M2 model, the asymptotes are 7.7-19.7 m, and in the M3 model are 9.2-27.5 using the same initial

Table 1. Dynamic equations and solutions for X_0 used in the four models or base equations

Model or base equation	Solution for X with initial values for (t_0, H_0)	Dynamic equation	Code
Richards $y = A(1 - e^{-b \cdot t})^c$	$X_0 = \frac{1}{2}\left[-Z_0 + \sqrt{Z_0^2 - 4 \cdot Ln(\varphi_0)}\right]$ $Z_0 = c_1 \cdot Ln(\varphi_0) - Ln(H_0)$ $\varphi_0 = (1 - e^{b \cdot t_0})$	$H = e^{X_0} \cdot \varphi^{(c_1 + 1/X_0)}$ $\varphi = (1 - e^{-b \cdot t})$	M1
Levakovic III $y = A\left(\frac{t^2}{b + t^2}\right)^c$	$X_0 = \frac{1}{2}\left[-Z_0 + \sqrt{Z_0^2 - 4 \cdot Ln(\varphi_0)}\right]$ $Z_0 = c_1 \cdot Ln(\varphi_0) - Ln(H_0)$ $\varphi_0 = \left(\frac{t_0^2}{b + t_0^2}\right)$	$H = e^{X_0} \cdot \varphi^{(c_1 + 1/X_0)}$ $\varphi = \left(\frac{t^2}{b + t^2}\right)$	M2
Hossfeld IV $y = a\frac{t^c}{b + t^c}$	$X_0 = \frac{1}{2 \cdot t_0^c}\left[-Z_0 + \sqrt{Z_0^2 + 4 \cdot t_0^c H_0}\right]$ $Z_0 = -H_0 \cdot (t_0^c + b_1)$	$H = \frac{X_0 \cdot t^c}{b_1 + 1/X_0 + t^c}$	M3
	$X_0 = \frac{1}{2 \cdot H_0}\left[-Z_0 + \sqrt{Z_0^2 + 4 \cdot t_0^c H_0}\right]$ $Z_0 = t_0^c \cdot (H_0 - a_1)$	$H = \frac{(a_1 + 1/X_0) \cdot t^c}{X_0 + t^c}$	M4
Gompertz $y = e^{a - (b/t^c)}$	$X_0 = \frac{1}{2}\left[-Z_0 + \sqrt{Z_0^2 + 4 \cdot e^{-c \cdot t_0}}\right]$ $Z_0 = -b_1 \cdot e^{-c \cdot t_0} - Ln(H_0)$	$H = e^{[X_0 - (b_1 + 1/X_0)]e^{-c \cdot t}}$	M5
	$X_0 = \frac{1}{2 \cdot e^{-c \cdot t_0}}\left[-Z_0 + \sqrt{Z_0^2 + 4 \cdot e^{-c \cdot t_0}}\right]$ $Z_0 = Ln(H_0) - a_1$	$H = e^{(a_1 + 1/X_0) - X_0 \cdot e^{-c \cdot t}}$	M6

conditions in both models. The M4 model has a unique asymptote of 22.9 m. Finally, model M1 was selected because of its better goodness of fit statistics and biological behavior (Fig. 1a). The age of reference chose is consistent with the results plotted in Fig. 1e. Finally, the site index shows an acceptable stability which increase with age (Fig. 1b).

The environmental variables extracted from the PCA were pH, fine earth fraction (FE in %), winter rainfall (WR in mm) and annual potential evapotranspiration (Thornthwaite, 1948) (PET in mm). Different structures combining the environmental variables were tested to expand the parameters in the M1 model. The values of the fitted parameters, the coefficient of determination, MSE, AIC and bias obtained for each of the four structures are summarized in Table 2.

The inclusion of climatic and edaphic variables into the model corrected the bias showed by the model approach without environmental variables and increase the model efficiency. The model selected was CM1.4 whose final equations are:

$$\varphi_0 = (1 - e^{-0.003357 \cdot pH\, t_0}) \quad [1]$$

$$Z_0 = 1.687275 \cdot \sqrt{\frac{WR}{(PET + 1)}} \cdot Ln\,(\varphi_0) - Ln(H_0) \quad [2]$$

$$X_0 = 0.5 \cdot [-Z_0 + \sqrt{Z_0^2 - 4 \cdot Ln(\varphi_0)}] \quad [3]$$

$$H = e^{X_0} \cdot (1 - e^{-0.003357 \cdot pH \cdot t})^{(1.687275 \cdot \sqrt{\frac{WR}{(PET+1)}} + 1/X_0)} \quad [4]$$

H and t means height and age respectively and the 0 subscript means initial conditions.

In order to assess the influence of the environmental variables over the model, the curves were plotted with one free variable while the other two remains fixed (mean value). For pH values of 8 and 5 the

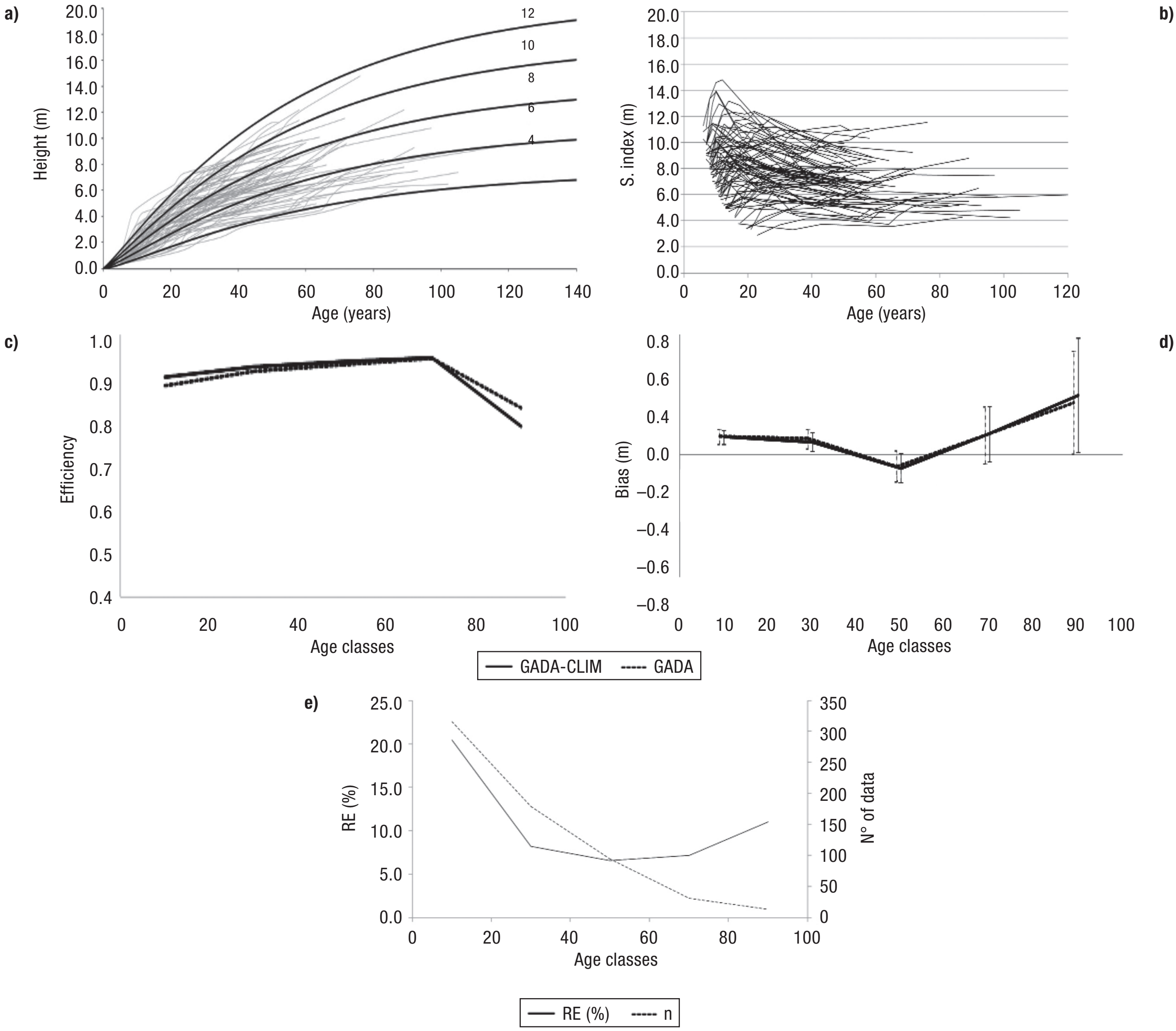

Figure 1. (a) GADA M1 (Richards base model) potential growth curves (81 series) for site indexes 4, 6, 8, 10 and 12 m at a reference age of 50 years. Grey thin lines correspond to the growth series used to fit the model. (b) Site index predictions vs age. (c) and (d) shows the model efficiency and the bias classified in age intervals. Dashed line correspond to M1 model, solid line correspond to CM1.4 model. (e) Relative error (%) in height prediction (solid line) and number of observations n (dotted line).

asymptotes of the model varies between 11.9-16.3 m respectively. If PET varies between 550-750 mm, asymptotes varies between 13.4-12.7 m respectively, and finally for WR values of 75 and 275 mm, asymptotes varies between 11.5-13.8 m.

The inclusion of environmental variables into the model allows comparing variations in growth pattern between different sites with different climatic conditions or comparing the growth pattern of the same site under different future climatic conditions. The differences between two sites with the same height at the same age could follow different growth patterns as a result of the climate conditions, that is, the intraregional variations that existed during the observed growing period (Bravo-Oviedo *et al.*, 2010).

Bias and efficiency were analyzed with data distributed in age classes. The bias increases and the efficiency decreases (Figs. 1c and 1d) when ages over 80 years are used mainly due to lack of sufficient data in older trees. Thus, it is strongly recommen-

Table 2. Parameter estimates and statistics for the model structures analyzed

GADA model selected								
Model	a_1/b	c/c_1	R^2_{adj}	**MSE**	**AIC**	**Mean bias**	**LCI**	**UCI**
M1	0.019451**	0.811387**	0.9664	0.2487	–970.9	0.0531395	0.08866842	0.1241973

Model with expanded parameters									
Structure used expand parameters	b'	c'	R^2_{adj}	**MSE**	**AIC**	**Mean bias**	**LCI**	**UCI**	**Code**
$\delta = b' \cdot \text{pH}$ $\gamma' = c' \cdot \sqrt{WR+1}$	0.003309**	0.06470**	0.9696	0.2251	–1,034.3	0.0814848	0.0476437	0.1153259	CM1.1
$\delta = b' \cdot \text{pH}$ $\gamma' = c' \sqrt{PET+1}$	0.003165**	0.032821**	0.9669	0.2450	–980.3	0.0831750	0.0478369	0.1185132	CM1.2
$\delta = b' \cdot \text{pH}$ $\gamma' = c' \cdot \frac{FE}{\sqrt{PET+1}}$	0.002684**	0.292371**	0.9548	0.3347	–633.2	0.0763393	0.0347718	0.1179069	CM1.3
$\delta = b' \cdot \text{pH}$ $\gamma' = c' \cdot \sqrt{\frac{WR}{PET+1}}$	0.003357**	1.687275**	0.9699	0.2228	–1,040.7	0.0815500	0.0478870	0.1152130	CM1.4

** $p < 0.001$; LCI: lower confidence interval of bias. UCI: upper confidence interval of bias.

ded to use the model for ages below 80 years, which on the other hand includes the rotation period in Lusitanian oak coppices forests (Serrada *et al.*, 2008).

Acknowledgements

The authors thank the reviewer who contributes to improve this communication. We also want to express our thanks for their comments to R. Calama and R. Ruiz-Peinado.

References

Adame P, Cañellas I, Roig S, Río M, 2006. Modelling dominant height growth and site index curves for rebollo oak (*Quercus pyrenaica* Willd.). Ann For Sci 63: 929-940.

Akaike H, 1974. A new look at the statistical model identification. IEEE Transactions on Automatic Control 19: 716-723.

Alonso Ponce R, 2008. Autoecología paramétrica de *Juniperus thurifera* L. en Castilla y León. Doctoral thesis. Universidad Politécnica, Madrid, Spain.

Álvarez González JG, Barrio Anta M, Diéguez Aranda U, Rojo Alboreca A, 2004. Metodología para la construcción de curvas de calidad de estación. Cuad Soc Esp Cien For 18: 303-309.

Amaro A, Reed AD, Tomé M, 1998. Modeling dominant height growth: eucaliptus plantations in Portugal. Forest Sci 44: 37-46.

Bravo-Oviedo A, Gallardo-Andrés C, Río M, Montero G, 2010. Regional changes of *Pinus pinaster* site index in Spain using a climate-based dominant height model. Can J Forest Res 40: 2036-2048.

Bravo-Oviedo A, Río M, Montero G, 2007. Geographic variation and parameter assessment in generalized algebraic difference site index modelling. Forest Ecol Manag 247: 107-119.

Bravo-Oviedo A, Tomé M, Bravo F, Montero G, Río M, 2008. Dominant height growth equations including site attributes in the generalized algebraic difference approach. Can J Forest Res 38: 2348-2358.

Calama R, Montero G, 2004. Interregional nonlinear height-diameter model with random coefficients for stone pine in Spain. Can J Forest Res 34: 150-163.

Carmean WH, 1972. Site index curves for upland oaks in the central Status. Forest Sci 18: 109-120.

Cieszewski CJ, 2004. GADA derivation of dynamic site equations with polymorphism and variable asymptotes from Richards, Weibull and other exponential functions. PMRC Technical Report 2004.

Cieszewski CJ, Bailey RL, 2000. Generalized algebraic difference approach: theory based derivation of dynamic site equations with polymorphism and variable asymptotes. Forest Sci 46: 116-126.

Cieszewski CJ, Harrison M, Martin S.W, 2000. Practical methods for estimating non-biased parameters in self-referencing growth and yield models. PMRC Technical Report 2000. p: 11.

Cieszewski CJ, Strub M, 2008. Generalized algebraic difference approach derivation of dynamic site equations with polymorphism and variable asymptotes from exponential and logarithmic functions. Forest Sci 54: 303-315.

Clutter JL, 1983. Timber management: a quantitative approach. J Wiley, New York, USA.

Costa M, Morla C, Sainz H (eds), 2005. Los bosques Ibéricos: una interpretación geobotánica. Ed Planeta, Madrid, Spain.

Davidson AC, Hincley DV, 1997. Bootstrap methods and their application. Cambridge series in Statistical and Probabilistic Mathematics, Cambridge University Press. p: 582.

Diéguez Aranda U, Álvarez González JG, Barrio Anta M, Rojo Alboreca A, 2005. Site quality equations for *Pinus sylvestris* L. plantations in Galicia (northwestern Spain). Ann For Sci 62: 143-152.

Gea-Izquierdo G, Cañellas I, Montero G, 2008. Site Index in agroforestry systems: Age-dependent and age-independent dynamic diameter growth models for *Quercus ilex* L. in Iberian open oak woodlands. Can J Forest Res 138: 101-113.

Goelz JCG, Burk TE, 1992. Development of a well-behaved site index equation: jack pine in north central Ontario. Can J Forest Res 22: 776-784.

Gregoire T, Schanbenberger O, Barrett JP, 1995. Linear modelling of irregularly spaced, unbalanced, longitudinal data from permanent-plot measurements. Can J Forest Res 25: 137-156.

Kiviste A, Álvarez González JG, Rojo Alboreca A, Ruiz González D, 2002. Funciones de crecimiento de aplicación en el ámbito forestal. INIA, Madrid, Spain.

Krumland B, Eng H, 2005. Site index systems for mayor young-growth forest and woodland species in northern California. The Resources Agency Dpt Forestry & Fire Protection. p: 219.

López-Senespleda E, Sánchez-Palomares O, 2007. Modelo de calidad de estación y crecimiento en altura dominante para *Quercus faginea* Lam. en España. Cuad Soc Esp Cien For 23: 199-205.

Ortega A, Montero G, 1988. Evaluación de la calidad de las estaciones forestales. Revisión bibliográfica. Ecología. 2: 155-184.

Richards FJ, 1959. A flexible growth function for empirical use. J Exp Bot 10: 290-300.

Rinn F, 2005. TSAPWin Professional. RINNTECH, Heidelberg, Germany.

San Miguel A, 1986. Ecología, tipología, valoración y alternativas silvopascícolas de los quejigares (*Quercus faginea* Lamk.) de Guadalajara. Doctoral thesis. Universidad Politécnica, Madrid, Spain.

Sánchez-González M, Stiti B, Chaar H, Cañellas I, 2010. Dynamic dominant height growth model for Spanish and Tunisian cork oak (*Quercus suber* L.) forests. Forest Systems 19: 285-298.

Sánchez-González M, Tomé M, Montero G, 2005. Modelling height and diameter growth of dominant cork oak trees in Spain. Ann For Sci 62: 633-643.

SAS I.I, 2004. SAS/STAT® 9.1 User's Guide, SAS Inst Inc, Cary, NC.

Serrada R, Montero G, Reque J (eds), 2008. Compendio de Selvicultura aplicada en España, Madrid, Spain.

Thornthwaite CW, 1948. An approach to a rational classification of climate. Geogr Rev 38: 55-94.

Zimmerman DL, Núñez-Antón V, 2001. Parametric modelling of growth curve data: an overview. Test 10: 1-73.