Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA)
Available online at www.inia.es/forestsystems
http://dx.doi.org/10.5424/fs/2012213-02338

Forest Systems 2012 21(3), 383-397
ISSN: 2171-5068
eISSN: 2171-9845

Evaluation of different modeling approaches for total tree-height estimation in Mediterranean Region of Turkey

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Abstract

Efficient management of timber resources and wood utilization practices require accurate and versatile information about important characteristics of forest resources for evaluating the numerous management and utilization alternatives for timber resources. Tree height is considered one of the most useful variables along with stocking and diameter at breast height, in estimating forest stand wood volumes and productivity. Six nonlinear growth functions were fitted to tree height-diameter data of three major tree species in Western Mediterranean Region's forests of Turkey. The generalized regression neural network (GRNN) technique has been applied for tree height prediction, as well, due to its ability to fit complex nonlinear models. The performance of the models was compared and evaluated. Further, equivalence tests of the selected models were conducted. Validation showed the appropriatness of all models to predict tree height. According to the model performance criteria, the six nonlinear growth functions were able to capture the height-diameter relationships and fitted the data almost equally well, while the constructed generalized regression neural network (GRNN) models were found to be superior to all nonlinear regression models, in terms of their predictive ability.

Key words: growth functions; generalized regression neural network models; equivalence testing.

Resumen

Evaluación de diferentes métodos de modelización para la estimación de la altura total del árbol de la Región Mediterránea de Turquía

La gestión eficiente de los recursos forestales y la de utilización de la madera requieren de información precisa y versátil acerca de las características importantes de los recursos forestales para la evaluación de la gestión y de las alternativas de utilización de los recursos forestales. La altura del árbol es considerada como una de las variables más útiles, junto con la densidad, y el diámetro a la altura del pecho, en la estimación de volúmenes de madera y la productividad de masas forestales. Se ajustaron seis modelos de altura total-diámetro y se compararon con el fin de estimar con precisión la altura total del árbol de las tres principales especies de árboles en los bosques de la Región Occidental Mediterráneo de Turquía. La regresión generalizada de redes neuronales (GRNN) se presenta como una técnica alternativa de red neuronal a la técnica de regresión no lineal para estimar la altura total de los árboles debido a su capacidad para adaptarse a modelos complejos no lineales. Se compararon y evaluaron los modelos. Se llevaron a cabo otras pruebas, como la equivalencia de los modelos seleccionados. De acuerdo con los criterios del rendimiento de los modelos, las seis funciones no lineales de crecimiento fueron capaces de capturar las relaciones altura-diámetro y ajustaron los datos casi igual de bien, mientras que las construidas mediante modelos de regresión generalizados de redes neuronales (GRNN) resultaron ser superiores a todos los modelos de regresión no lineal, en términos de su capacidad predictiva.

Palabras clave: funciones de crecimiento; modelos de regresión generalizada de redes neuronales; pruebas de equivalencia.

Introduction

Brutian pine (*Pinus brutia Ten.*), Cedar of Lebanon (*Cedrus libani A. Rich.*), and Cilicica fir (*Abies cilicica Carr.*) are major coniferous tree species in Turkey. There are nearly 6.2 million hectare of Brutian pine (about 5.4 million ha), Cedar of Lebanon (about 0.5 million ha), and Cilicica fir (about 0.3 million ha) forest almost

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Received: 27-02-12. Accepted: 04-06-12.

one-quarter of total forest land in these country (Anonymous, 2006). Turkey has adopted the principles of multipurpose and ecologically based forest management. Therefore, General Directorate of Forests needs to develop and evaluate tree height-diameter equations and growth and yield prediction models for management of these forest resources. Individual tree height and diameter at breast height (dbh) are essential forest inventory measures for estimating timber volume, site index, stand description, damage appraisals, and other important variables in forest growth and yield, succession, and carbon budget models (Parresol, 1992; Peng *et al.*, 2001; Yuancai and Parresol, 2001; Sharma and Zhang, 2004; Castedo-Dorado *et al.*, 2005). Estimating individual tree volume and site index, and describing stand growth dynamics and succession over time, require accurate height-diameter models (Curtis, 1967; Zhang, 1997; Colbert *et al.*, 2002; Peng *et al.*, 2004; Castedo-Dorado *et al.*, 2005). However, the available information about height-diameter relationships concerning the above species is not considered sufficient.

Dbh of a tree can be measured quickly, easily, and accurately, but the measurement of total tree height is relatively complex, time consuming, expensive, and difficult to accurately obtain (Arabatzis and Burkhart, 1992; Zhang, 1997; Fang and Bailey, 1998; Lootens *et al.*, 2007; Sharma and Parton, 2007; Meng *et al.*, 2008). Therefore, the standard sampling procedure for forest inventory is to measure heights of only a few trees and estimate the unmeasured heights from mathematical relationships between height and diameter or between height and diameter and other stand characteristics (Larsen and Hann, 1987; Dolph, 1989).

Development of simple and efficient models that allow forest managers to determine with reliability the height of the trees in a stand from diameter data is a prime objective in forest management. Knowledge of the relation between these variables permits managers to obtain, without investing large amounts of money in height measurement, the input values needed to estimate single tree volume, dominant height of the stands, competition indices for individual tree growth, height/diameter ratio, and structural diversity indexes (Calama and Montero, 2004).

A number of tree height-diameter equations have been developed for various tree species in different countries (Curtis, 1967; Larsen and Hann, 1987; Dolph, 1989; Zhang, 1997; Fang and Bailey, 1998; Huang *et al.*, 2000; Colbert *et al.*, 2002; Soares and Tome, 2002; Zhang *et al.*, 2002; Eerikainen, 2003; Temesgen and Gadow, 2004; Castedo-Durado *et al.*, 2005; Sharma and Parton, 2007). Among a variety of mathematical equations, sigmoid or non-linear growth functions are widely used in developing tree height-diameter equations. Although most of these functions can adequately fit tree height-diameter curves, they may produce large extrapolation errors when applied beyond the range of model development data (Zhang, 1997). Therefore, the models predictive capabilities (accuracy, precision, time dependence, biological realism, and flexibility) should be carefully evaluated and validated before they are used (Zhang, 1997, Yuancai and Parresol, 2001). As an alternative approach to fitting non-linear data, neural network models have gained popularity for their effective manner to manage complex, non-linear systems a feature, which is not the case of statistical regression models where an appropriate non-linear function must first be found. Traditional computing solutions such as regression analysis are based on rules or equations, which define a system and must be explicitly programmed. While this is perfect in situations where the rules are known, many systems exist for which the rules are either not known or difficult to discover and it is these systems to which neural computing techniques can be applied (Swingler, 2001). Regression (Bayesian) networks (often called generalized regression neural networks in the literature) were devised by Specht (Specht, 1991), casting a statistical method of function approximation into a neural network form. It is used for estimation of continuous variables, as in standard regression techniques. GRNN falls into the category of probabilistic neural networks, which means that it is especially advantageous in order to perform predictions and comparisons of system performance in practice. Artificial neural network models (ANNs) have been used widely in environmental sciences including the field of forest modeling. Maier and Dandy (2000) stated a review of neural network (NN) modeling issues and applications for the prediction and forecasting of water resources variables; Liu *et al.* (2003) used neural network models (NNs) in classification of ecological habitats, Corne *et al.* (2004) predicted forest attributes using NNs, Özçelik *et al.* (2008) conducted a comparative study of NNs and standard methods for estimating tree bole volume, Fernández *et al.* (2008) handled ANNs for the prediction of standard particleboard mechanical properties, Esteban *et al.* (2009) utilized ANNs in wood identification, while Esteban *et al.* (2011) employed ANNs for the prediction of plywood bonding quality. It is worth noting that the back-propagation algorithms are the most popular for training feed forward neural networks and are used in most of the published papers about ANNs. However, they have some known disadvantages, such as slow convergence and sensitivity to noise in the training data sets (Diamantopoulou, 2010). The purpose of this study is to step further by introducing the generalized regression neural network (GRNN) modeling architecture as an alternative neural network technique for estimating total tree-height. Additionally, GRNN models performance are tested and finally a comparative analysis is conducted between six standard and widely used nonlinear growth functions and the GRNN models, using sample trees of three major tree species in Western Mediterranean Region's forests of Turkey.

Materials and methods

Data

The data used in this study were obtained for three tree species from even-aged managed stands in Bucak Forest Enterprise-Mediterranean Region of Turkey. All sampled trees were measured for diameter at breast height (dbh) outside bark and total height (h). These trees were felled throughout the clear-cutting areas of Bucak Forest Enterprise, and were subjectively selected to provide representative information for a variety of throughout clear-cutting areas density, height, stand structure, age, and site condition. Namely, the trees were subjectively selected to ensure a representative distribution by diameter and height classes. In each tree, two perpendicular diameters outside-bark (1.3 m above ground level) were measured to 0.1 cm and were then arithmetically averaged. The trees were later felled, leaving stump with an average height of 0.30 m, and total bole length was measured to the nearest 0.05 cm. Summary statistics for the two variables by species are provided in Table 1.

The available tree height-diameter data were split using the three-way data splits method (Fig. 1). The majority (70%) of the data in each diameter class was used for model development while the remaining data (30%) in each diameter class for each species were randomly selected and reserved for model validation (Fig. 1). In order to construct a neural network model, it is very important to have both training and testing data sets as insurance against overfitting (Leahy, 1994). Due to the nature of the GRNN modeling, it is necessary to use both training and test data sets within the development data set, in order to construct the most suitable model for the examined species. For this reason we have used the k-fold cross validation method with k = 10. The k taking the value 10 has been the most common practice (Olson and Delen, 2008). Using this method of data division, all data of the development data set have eventually been used for the construction of the GRNN model. In regression model building, there is little to be gained by separating development data into parts for fitting and testing (Hirsch, 1991). For this reason, in regression model building we used the development data set without any further division. Finally, for both non-linear and generalized neural regression models building, all data of the development data set have eventually been used.

Both model development and validation data sets covered the same ranges of diameter and height (Table 1).

Table 1. Tree summary statistics for model development and validation data sets

[1] Species	Number of trees	dbh (cm)				h (m)			
		Mean	Minimum	Maximum	Standard deviation	Mean	Minimum	Maximum	Standard deviation
Model Development Data									
CL	251	35.46	11.0	75.0	14.38	16.80	5.2	27.2	5.09
CF	264	37.21	16.5	73.0	12.97	17.05	9.0	28.0	4.38
BP	354	38.79	11.0	76.0	15.13	17.92	6.8	26.4	4.73
Model Validation Data									
CL	71	35.31	15.0	66.5	13.15	17.17	9.3	25.2	4.47
CF	65	40.05	16.0	73.0	13.87	18.25	9.0	27.8	4.56
BP	132	42.07	12.0	75.0	16.54	18.64	7.4	25.7	4.72

[1] CL: Cedar of Lebanon (*Cedrus libani A. Rich.*); CF: Cilicica fir (*Abies cilicica Carr.*); BP: Brutian pine (*Pinus brutia Ten.*).

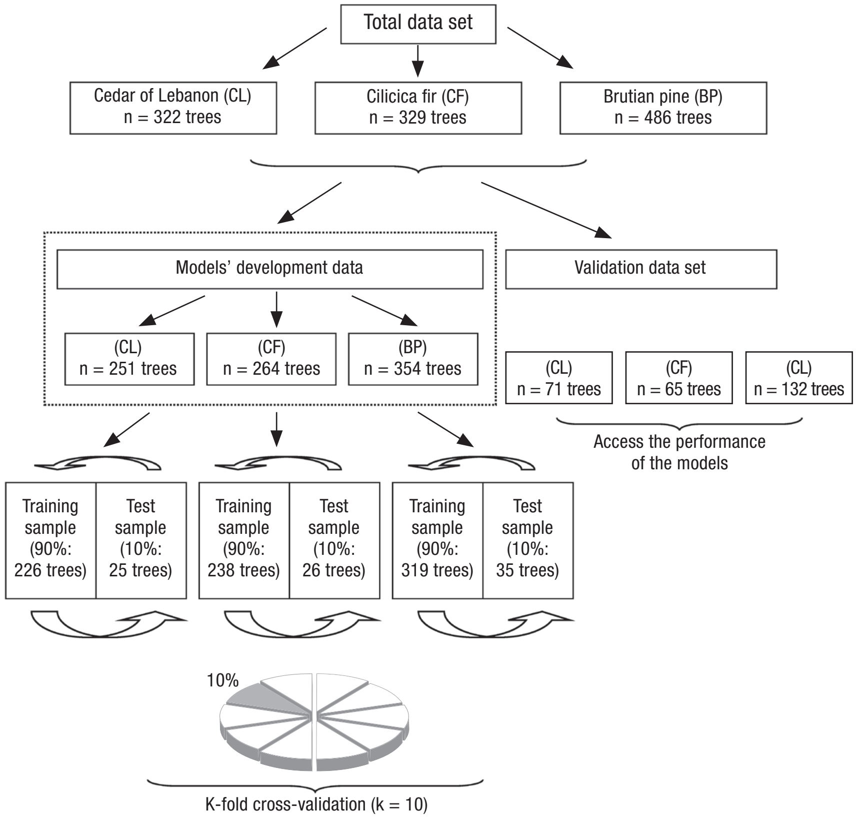


Figure 1. Data division according to the three-way data splits method.

Non-linear regression modeling

Six non-linear growth functions (Table 2) were selected as candidate height-diameter equations. These models based on their appropriate mathematical properties (typical sigmoid shape, number of parameters, flexibility), possible biological interpretation of parameters (upper asymptote, maximum or minimum growth rate), and satisfactory prediction for tree height-diameter relationship in the literature (Arabatzis and Burkhart, 1992; Huang and Titus, 1992; Zeide, 1993; Zhang, 1997; Fang and Bailey, 1998). As indicated by Peng *et al.* (2001); these six nonlinear growth functions have been widely used for two major reasons. First they define sigmoid curves, in which the growth rate increases from minimum value to a maximum at a point of inflection, and then declines towards zero at an upper asymptote. Secondly; they have three parameters (an upper asymptote, a rate parameter, and a shape parameter) that describe various biological processes and behaviors. For example, the Chapman-Richards and Weibull models are well known flexible growth functions with biologically interpretable coefficients (Pienaar and Turnbull, 1973; Fang and Bailey, 1998; Yuancai and Parresol, 2001).

These growth functions of Table 2 were fitted to model development data of height-diameter for each tree species, respectively. Parameter estimations were accomplished using the PROC NLIN procedure in SAS (SAS Institute, 2002). As stated in Fang and Bailey (1998) and Peng *et al.* (2001), we selected to use the Levenberg -Marquardt algorithm because it is considered to be most useful when parameter estimates are highly correlated and represents a combination of the best features of linearization method and the steepest descent method (Fekedulegn *et al.*, 1999). The tree total height versus diameter at breast height scatter plots present typical sigmoidal-concave curves for all tree species (Fig. 2). In order to examine the existence of heteroscedasticity in our data, the White's general test (White, 1980) has been used under the null hypothesis of homoskedasticity. The null hypothesis was rejected for the significance level of $\alpha = 0.05$, concluding that the problem of heteroscedasticity is apparent.

Parresol (1993) indicated that forest modelers often face the problem of heteroscedasticity in their data,

Table 2. Nonlinear height-diameter models selected for comparison using data from Mediterranean Region Forests of Turkey

Model		References
Chapman-Richards	$h = 1.3 + a \cdot \left[1 - \exp\left(-b \cdot dbh\right)\right]^{c}$	Huang and Titus, 1992
Weibull	$h = 1.3 + a \cdot \left[1 - \exp\left(-b \cdot dbh^{c}\right)\right]$	Huang and Titus, 1992
Exponential	$h = 1.3 + a \cdot \exp\left[\frac{b}{\left(dbh + c\right)}\right]$	Ratkowsky, 1990
Modified Logistic	$h = 1.3 + \frac{a}{\left(1 + b^{-1} \cdot dbh^{-c}\right)}$	Huang *et al.*, 2000
Korf/Lundgvist	$h = 1.3 + a \cdot \exp\left[-b \cdot dbh^{-c}\right]$	Zeide, 1989
Gompertz	$h = 1.3 + a \cdot \exp\left[-b \cdot \exp\left(-c \cdot dbh\right)\right]$	Huang and Titus, 1992

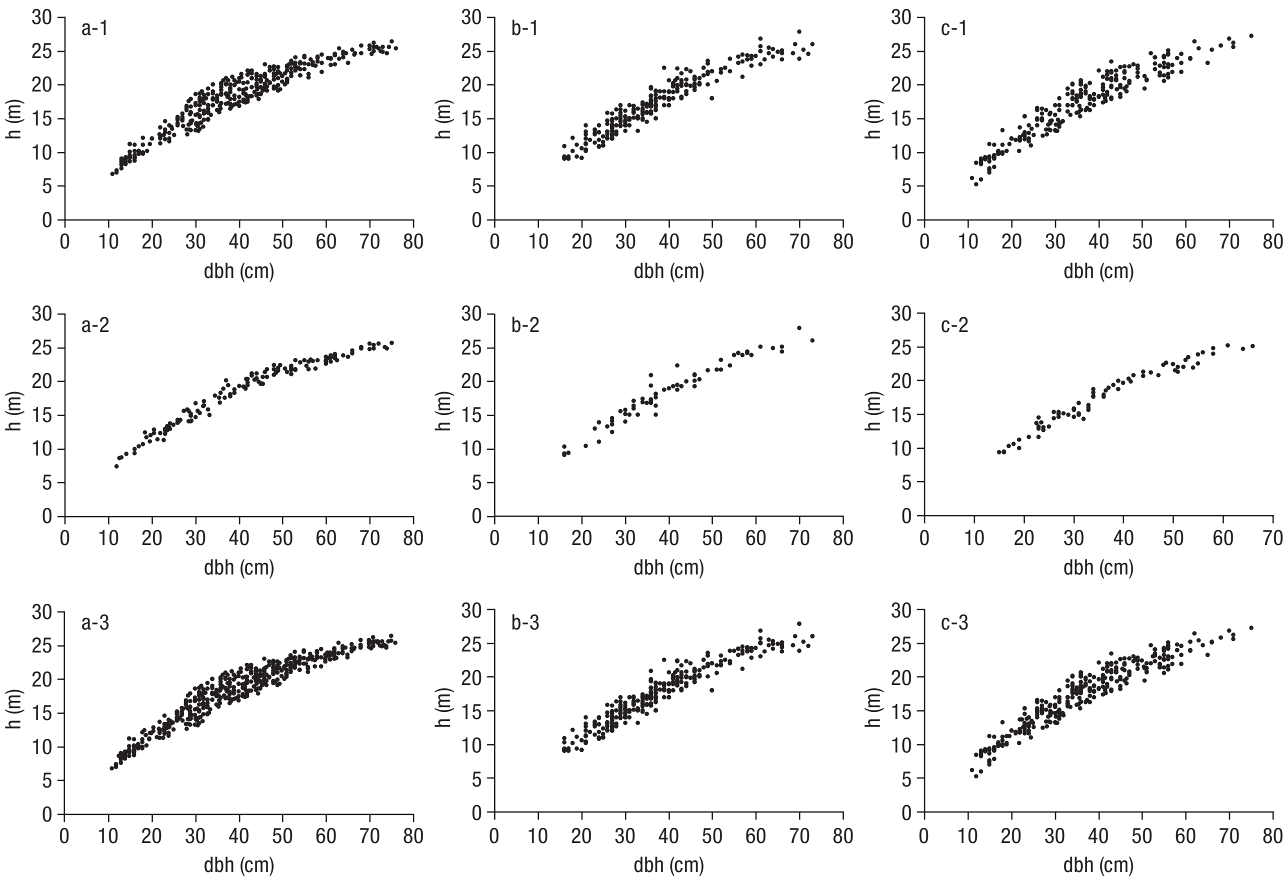


Figure 2. Scatter plot of total height (h) against diameter at breast height (dbh) for a: Cedar of Lebanon trees and for the model development data (a-1), model validation data (a-2) and combined data (a-3), b: Cilicica fir for the (b-1), (b-2) and (b-3) data sets and c: Brutian Pine for the (c-1), (c-2) and (c-3) data sets.

which would lead to non-minimum variance parameter estimates and unreliable prediction intervals. Therefore, weighted least-squares estimation that corrects the estimated standard errors of the least-squares coefficients for heteroscedasticity was used. This offers the possibility of more efficient estimation (Fox, 1991). Exploratory graphing of diameter at breast height against total tree-height showed that there was a strong relationship between the variance of the (h) and the values of (dbh). Using the SPSS program (Norusis, 2000), a wide range (from –4 to 4 by 0.1) of possible power values were examined and this variance was found to be proportional to the (1.1) power of (dbh) for all species.

Generalized regression neural network modeling

A generalized regression neural network (GRNN) is often used for function approximation (Specht, 1991; Bishop, 1995; Patterson, 1996). It does not require an iterative training procedure, and it approximates any arbitrary function between input and output vectors, drawing the function estimate directly from the training data. The regression network uses Bayesian techniques to estimate the expected mean value of the output, given an input case as follows:

$$E\left[y|x\right] = \int_{-\infty}^{+\infty} y f\left(x,y\right) dy \Big/ \int_{-\infty}^{+\infty} f\left(x,y\right) dy \qquad [1]$$

where, y is the output value, which is being estimated; x is the input case; f is the joint probability density function of the inputs and outputs. As the joint probability density function is not known, it must be estimated from a sample of observations x and y. Defining the distance d_i^2 between the training sample (x_i) and the point of prediction (x) as:

$$d_i^2 = (x - x_i)^t (x - x_i) \qquad [2]$$

we obtain the resulting equation [Eq.3] which is directly applicable to problems involving numerical data:

$$y(x) = \left(\sum_{i=1}^{n} y_i \cdot e^{\left(-d_i^2 / 2\sigma^2 \right)} \right) \Big/ \left(\sum_{i=1}^{n} e^{\left(-d_i^2 / 2\sigma^2 \right)} \right) \qquad [3]$$

where y(x) is the output value, which is being estimated for the input case x, n is the number of the observations and σ is the smoothness parameter and its optimal value is subject to a research.

This type of network is kernel-based approximation method cast in the form of neural networks (see appendix). Gaussian Kernel functions are located at each training case. The GRNN copies the training cases introduced to the first layer (input layer) into the network to be used to estimate the response on new points. The output is estimated using a weighted average of the outputs of the training cases, where the weighting is related to the distance of the point from the point being estimated (so that points nearby contribute most heavily to the estimate). The first hidden layer in the GRNN contains the pattern units. A second hidden layer contains units, which help to estimate the weighted average. Each output has a special unit assigned in this layer, which forms the weighted sum for the corresponding output. Therefore, the second hidden layer is considered as the summation layer. To get the weighted average from the weighted sum, the weighted sum must be divided by the sum of the weighting factors. A single special unit in the second layer calculates the latter value. The output layer then performs the actual divisions (using special division units). Hence, the second hidden layer always has exactly one more unit than the output layer. In regression problems, typically, only a single output is estimated, and so the second hidden layer usually has two units.

In order to select an optimal value of σ Specht suggested (Specht, 1991) the holdout method. As a step further, in this paper the k-fold cross-validation method was used (Fig. 1). Following this method, the 10% of the development data set is used as test data set and the remaining 90% of the development data set as the training sample. After the selection of a fixed value of σ, the model was trained using the training data set. Then the model was evaluated using the test data set. This process was repeated for each k = 10 folds (90% for training and 10% for testing) and then many times using different values of the smoothing coefficient. Finally, the best value of σ that should be used was selected as the value that minimized the error that was estimated as the average error rate on these cross-validation examples.

Models performance criteria

Each model was evaluated using the model's correlation coefficient, bias, mean absolute deviation, mean square error and % root mean square error of the dependent variable mean for the regression models which has been used as the output node for the GRNN models. These measures have been computed by the following equations:

Correlation coefficient (r):

$$r = \frac{\sum_{i=1}^{n}\left[(y_i - \bar{y})\cdot(y_{iest} - \bar{y}_{est})\right]}{\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}\cdot\sqrt{\sum_{i=1}^{n}(y_{iest} - \bar{y}_{est})^2}} \quad [4]$$

Bias:

$$Bias = \sum_{i=1}^{n}\left(y_i - y_{iest}\right)\Big/ n \quad [5]$$

Mean absolute deviation (MAD):

$$MAD = \sum_{i=1}^{n}\left|y_i - y_{iest}\right|\Big/ n \quad [6]$$

Mean square error (MSE):

$$MSE = \sum_{i=1}^{n}(y_i - y_{iest})^2\Big/ n - p \quad [7]$$

% Root mean square error (RMSE%):

$$RMSE\% = \left(\sqrt{\frac{\sum_{i=1}^{n}\left(y_i - y_{iest}\right)^2}{n-p}}\Big/ \bar{y}\right)\cdot 100 \quad [8]$$

where y_i are the observed values, y_{iest} are the estimated by the model values, $\bar{y}$ is the mean of the observed values, $\bar{y}_{est}$ is the mean of the estimated values; n is the total number of data used for fitting the model, and p is the number of parameters to be estimated.

The correlation coefficient (r) was selected to measure the linear correlation between the observed and the estimated by the model values. Bias indicates on average under-prediction or over-prediction by the model. The mean absolute deviation was selected for an overall indication of the error variance, while the mean square error and the root mean squared error expressed as percentage of the mean of the observed values (RMSE%), were calculated as the common performance measure as they show the global goodness of fit.

Equivalence testing

Model validation is a central aspect to the responsible application of models and is often realized as tests of how well model predictions match a set of independent observations. Traditional tools are optimized to detect differences rather than similarities. This is because the usual null hypothesis is that there is no difference between means of the populations. Equivalence test looks for similarities while the usual hypothesis test looks for differences. The conclusion of no statistically significant difference of the usual statistical test means that the current evidence is not strong enough to support that the examined treatments lead to different outcomes (Robinson *et al.*, 2005; Özçelik *et al.*, 2010). That is different from saying that the two outcomes are the same. Therefore, in model validation, the null hypothesis should be that the model is not valid (Loehle, 1997; Robinson and Froese, 2004). For this reason, the height-diameter models considered as the best fitted to the observed data, were further validated using the equivalence testing strategy, which is a regression-based methodology validation that it has been described extensively by Robinson *et al.* (2005). In this procedure, the null hypothesis is H_0: a difference, e.g. $\mu \neq \mu_o$ and the alternative hypothesis is H_1: not a difference, e.g. $\mu = \mu_o$. The shifted intercept (b_0) was tested for equality to the mean observation ($\bar{y}$), which is identical to testing that the mean of the observations is equivalent to the mean of the predictions. The slope (b_1) was tested for equality to one. For both tests the desired experimental probability level was $\alpha = 0.05$. The regions of equivalence have been established as: $Eq_0 = \bar{y} \pm 2\%$ for the shifted intercept and $Eq_1 = 1.0 \pm 2\%$ for the slope. In order to construct confidence intervals that do not depend on the assumptions such as the model form is correct, the residuals are independent, normally distributed and have constant variance, a non-parametric bootstrap methodology has been applied (Efron, 1979; Efron and Tibshirani, 1993, Davison and Hinkley, 1997). The number of bootstrap replicates was 1000.

Results

It is apparent from the model performance criteria that each growth function was equally well fitted to the tree height-diameter data of the three species. The r values for all models and species of Table 3 shows strong positive correlation (greater than 0.9929) between the observed and the estimated values for all models (Table 3).

According to the non-linear regression models, the highest r-value of 0.9950 was reached for Brutian pine trees (Table 3). The bias ranged between –0.0059 to 0.0104, and MAD values from 0.3128 to 0.3771, for all species. Differences in bias among the six non-linear models for each species were not significant. All model coefficients were significant at the significance level of 0.05. Comparing the mean square errors of the models, the Korf/Lundqvist, the Gompertz and the modified logistic models had the smallest MSE values for the Cedar of Lebanon, Cilicica fir and Brutian pine trees, respectively. The values of the RMSE% ranged from 7.908% to 7.975% for the Cedar of Lebanon trees, from 6.838% to 6.954% for the Cilicica fir trees and from 6.171% to 6.209% for the Brutian pine trees. Residual analyses showed that there were no detectable trends in the plots of residuals against the predicted tree heights. Although the six growth functions were fitted to the same data sets, they resulted in different asymptote coefficients (coefficient a in Table 3). In general, Gompertz's function yielded the smallest asymptote coefficients for all tree species, and the Chapman-Richards and Weibull models had similar asymptotes (Table 3).

According to the GRNN models of Table 3, the K = 10 folds cross-validation method resulted to smoothing coefficient values equal to 0.0101, 0.0079 and 0.0100 for the Cedar of Lebanon, Cilicica fir and Brutian pine trees, respectively. Examination of the final GRNN models performance criteria suggested that the selected models were successful in describing the relationship between height and diameter at breast height, as indicated by the low values of Bias, MAD, MSE and RMSE% and the high values of r. The results of Table 3 for the development data set, indicate that the GRNN models gave the most accurate estimations for all species. The GRNN model gave 0.78% more accurate estimations than the Korf/Lundqvist non-linear model, 0.85% more accurate estimations than the Gompertz non-linear model and

Table 3. Parameter estimates and performance criteria of the six weighted nonlinear height-diameter models and performance criteria of the generalized regression neural network models that showed the best quality of fit for the three tree species and for the development data sets

[1] Model	a	b	c	r	Bias (m)	MAD (m)	MSE	RMSE %
				***Cedar of Lebanon (CL), n* = 251**				
Chapman-Richards	31.331	2.354	1.145	0.9929	–0.0024	0.3731	0.2359	7.921
Weibull	30.964	2.318	1.090	0.9929	–0.0022	0.3732	0.2361	7.923
Exponential	42.088	–0.434	0.105	0.9930	–0.0029	0.3730	0.2353	7.909
Modified logistic	39.249	2.607	1.249	0.9930	–0.0002	0.3729	0.2354	7.911
Korf/Lundqvist	61.963	0.762	0.537	0.9930	–0.0021	0.3734	0.2352	7.908
Gompertz	28.016	2.214	3.955	0.9929	–0.0059	0.3771	0.2392	7.975
GRNN	–	–	–	0.9942	–0.0002	0.3229	0.1915	7.130
				***Cilicica fir (CF), n* = 264**				
Chapman-Richards	29.400	3.078	1.530	0.9941	0.0015	0.3184	0.1896	6.883
Weibull	28.019	3.238	1.323	0.9940	0.0052	0.3166	0.1887	6.868
Exponential	42.006	–0.416	0.069	0.9939	0.0050	0.3229	0.1921	6.930
Modified logistic	34.884	4.007	1.529	0.9940	0.0057	0.3205	0.1907	6.904
Korf/Lundqvist	48.753	0.533	0.725	0.9939	0.0104	0.3254	0.1935	6.954
Gompertz	27.664	2.586	4.266	0.9941	0.0029	0.3128	0.1871	6.838
GRNN	–	–	–	0.9965	0.0180	0.2795	0.1437	5.990
				***Brutian pine (BP), n* = 354**				
Chapman-Richards	29.128	2.591	1.128	0.9950	0.00007	0.3429	0.1891	6.173
Weibull	28.829	2.575	1.079	0.9950	–0.0012	0.3431	0.1892	6.174
Exponential	38.320	–0.373	0.086	0.9950	0.0037	0.3428	0.1890	6.172
Modified logistic	35.447	3.235	1.273	0.9950	0.0006	0.3428	0.1890	6.171
Korf/Lundqvist	48.349	0.566	0.621	0.9950	0.0061	0.3436	0.1893	6.176
Gompertz	27.051	2.062	3.966	0.9950	–0.0056	0.3486	0.1913	6.209
GRNN	–	–	–	0.9956	0.0026	0.3243	0.1648	5.760

[1] Weighted models.

0.41% more accurate estimations than the modified logistic non-linear model for the Cedar of Lebanon, Cilicica fir and Brutian pine trees, respectively.

In order to perform equivalence testing for the models of Table 3, the coverage probabilities of the equivalence regions for the intercepts and the slopes were calculated. The two one-sided confidence intervals for the intercept and the slope of each model were calculated using a non-parametric bootstrap analysis so as the intervals not depend on assumptions such as the correctness of the model form, the independence of the residuals, etc. The proportion of times that the bootstrap sample intercepts were contained within the region of equivalence Eq_0 was 1.000 for all models and species. The proportion of times that the bootstrap sample slopes were contained within the region of equivalence Eq_1 were more than the desirable level of 0.949 (desired experiment level $\alpha = 0.05$) for all models and species. Therefore, it is concluded that the null hypothesis of dissimilarity is rejected, for all models and species, providing a quantitative confirmation of the models utility in estimating the total tree-height of the three species.

Table 4 presents the empirical two one-sided 97.468% intervals CI_0 and CI_1 corresponding to the joint 95% intervals for the intercept and the slope, respectively and the regions of equivalence Eq_0 and Eq_1 for the intercept and the slope, respectively, for all models and species.

The CI_0 and CI_1 intervals are contained within the Eq_0 and Eq_1 regions of equivalence, for all models and species, strengthening the aspect of suitability of the models (Table 4). Additionally, a graphical examination of the intercepts and slopes distributions of the 1000 bootstrap samples was conducted and gave acceptable shapes for both the intercepts and slopes for all species, while the

Table 4. Empirical two one-sided 97.468% intervals CI_0 and CI_1 corresponding to the joint 95% intervals for the intercept and the slope, respectively and regions of equivalence Eq_0 and Eq_1 for the intercept and the slope, respectively, for the development data sets of all species

Model	CI_0 limits		Eq_0 limits		CI_1 limits		Eq_1 limits	
	lower	upper	lower	upper	lower	upper	lower	upper
				[1] ***Cedar of Lebanon (CL)***				
Chapman-Richards	6.074	6.189	6.010	6.225	0.986	1.015	0.98	1.02
Weibull	6.067	6.191	6.010	6.225	0.985	1.017	0.98	1.02
Exponential	6.069	6.191	6.010	6.225	0.984	1.015	0.98	1.02
Modified logistic	6.073	6.193	6.010	6.225	0.985	1.015	0.98	1.02
Korf/Lundqvist	6.071	6.191	6.010	6.225	0.984	1.014	0.98	1.02
Gompertz	6.078	6.187	6.010	6.225	0.992	1.019	0.98	1.02
GRNN	6.079	6.184	6.010	6.225	0.992	1.019	0.98	1.02
				[2] ***Cilicica fir (CF)***				
Chapman-Richards	6.271	6.379	6.199	6.452	0.981	1.017	0.98	1.02
Weibull	6.277	6.384	6.199	6.452	0.982	1.016	0.98	1.02
Exponential	6.277	6.383	6.199	6.452	0.986	1.013	0.98	1.02
Modified logistic	6.277	6.387	6.199	6.452	0.982	1.017	0.98	1.02
Korf/Lundqvist	6.281	6.391	6.199	6.452	0.981	1.018	0.98	1.02
Gompertz	6.299	6.391	6.199	6.452	0.989	1.020	0.98	1.02
GRNN	6.299	6.391	6.199	6.452	0.989	1.019	0.98	1.02
				[3] ***Brutian pine (BP)***				
Chapman-Richards	7.001	7.091	6.904	7.186	0.992	1.009	0.98	1.02
Weibull	6.998	7.092	6.904	7.186	0.992	1.010	0.98	1.02
Exponential	7.007	7.094	6.904	7.186	0.992	1.008	0.98	1.02
Modified logistic	7.002	7.089	6.904	7.186	0.992	1.009	0.98	1.02
Korf/Lundqvist	7.004	7.097	6.904	7.186	0.991	1.009	0.98	1.02
Gompertz	6.993	7.084	6.904	7.186	0.993	1.010	0.98	1.02
GRNN	7.007	7.088	6.904	7.186	0.994	1.009	0.98	1.02

[1] Regions of equivalence: $Eq_0 \in [6.010, 6.225]$, $Eq_1 \in [0.980, 1.020]$. [2] Regions of equivalence: $Eq_0 \in [6.199, 6.452]$, $Eq_1 \in [0.980, 1.020]$. [3] Regions of equivalence: $Eq_0 \in [6.904, 7.186]$, $Eq_1 \in [0.980, 1.020]$.

bootstrap sample intercepts were contained within the region of equivalence for all models, and the proportion of times that the bootstrap slopes were contained within the region of equivalence was acceptable for all models.

In order to validate all models predictive ability to a new set of data, the performance criteria of all weighted models were calculated for the validation data sets (Table 5). It can be seen that GRNN model produced the lowest values of Bias, MAD, MSE and RMSE%, for all species, while the Korf/Lundqvist non-linear model gave the highest values of MAD, MSE and RMSE% for the Cedar of Lebanon and the Cilicica fir trees. The Gompertz non-linear model gave the highest values of MSE and RMSE% for the Brutian pine trees.

Further examining the predictive ability of the models, the 45-degree line plots were produced for all models and species (Fig. 3). As can be seen (Fig. 3), all models tended to make an angle of 45 degrees with the axes, meaning there was no significant difference between the measured and the predicted values. Since the constructed models never saw the data in the validation set, the good predictions on these data demonstrated the adequacy and the potential of all models to estimate the total tree-height values. All the non-linear growth functions produced similar proximity of points with slopes very close to 45 degrees. It is worth noting that the GRNN models provided the most accurate predictions with the closest to 45 degrees values, for all species.

Discussion

Knowledge of tree heights such as total height is fundamental for developing growth and yield models in forest stands. Based on their appropriate mathematical properties, six non-linear growth functions were evaluated for estimating total tree-height and found to perform equally well, resulting to RMSE% values from 6.17 to 7.98 for all different species. GRNN modeling was used as an alternative approach to fitting non-linear data. During the last ten years or so GRNN modeling approach

Table 5. Performance criteria of all height-diameter models for the three tree species and for the validation data set

[1] Model	r	Bias	MAD	MSE	RMSE (%)
	***Cedar of Lebanon (CL), n* = 71**				
Chapman-Richards	0.997945	0.06794	0.20999	0.07298	4.2498
Weibull	0.997942	0.06851	0.21048	0.07331	4.2594
Exponential	0.997944	0.06681	0.20924	0.07299	4.2501
Modified logistic	0.997942	0.06949	0.21009	0.07359	4.2676
Korf/Lundqvist	0.997915	0.06488	0.21070	0.07425	4.2866
Gompertz	0.997960	0.06936	0.21068	0.07148	4.2060
GRNN	0.997880	0.05236	0.19491	0.06869	4.1231
	***Cilicica fir (CF), n* = 65**				
Chapman-Richards	0.996379	0.11028	0.26871	0.15873	5.4462
Weibull	0.996366	0.11309	0.26871	0.15978	5.4642
Exponential	0.996398	0.11945	0.27248	0.15976	5.4638
Modified logistic	0.996382	0.11673	0.27146	0.15972	5.4632
Korf/Lundqvist	0.996371	0.12254	0.27615	0.16226	5.5064
Gompertz	0.996373	0.11003	0.26552	0.15936	5.4569
GRNN	0.997451	0.09450	0.24222	0.10566	4.4434
	***Brutian pine (BP), n* = 132**				
Chapman-Richards	0.998701	–0.04829	0.19697	0.06613	3.2115
Weibull	0.998698	–0.04979	0.19719	0.06638	3.2176
Exponential	0.998704	–0.04294	0.19730	0.06547	3.1954
Modified logistic	0.998705	–0.04652	0.19751	0.06577	3.2028
Korf/Lundqvist	0.998684	–0.04079	0.20015	0.06620	3.2134
Gompertz	0.998682	–0.05529	0.19923	0.06765	3.2484
CRNN	0.998885	–0.03496	0.17884	0.05578	2.9495

[1] Weighted models.

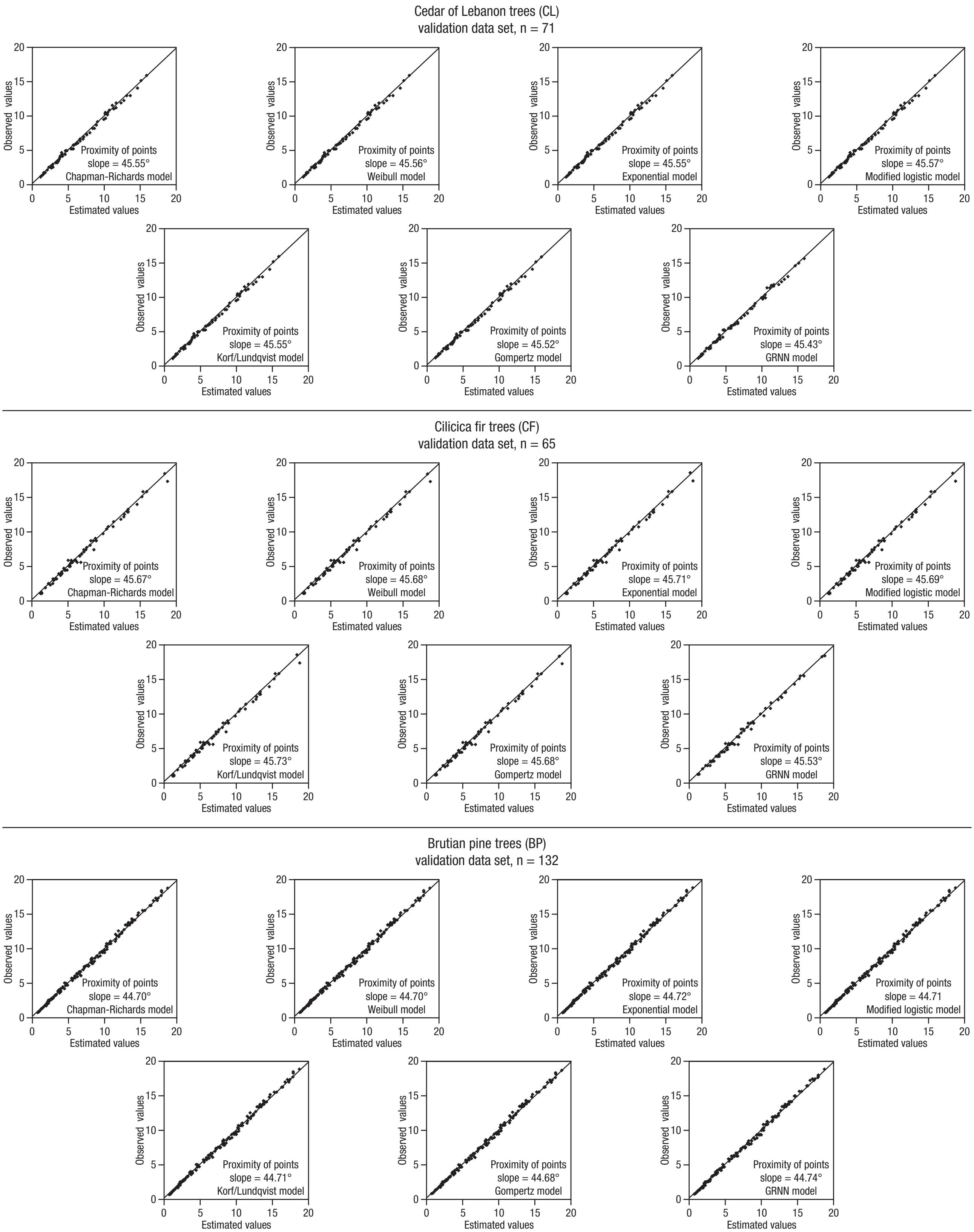


Figure 3. 45-degree line plots for the validation data set for all weighted models and tree species.

has been gained the interest along with an ever increasing array of practical applications in the field of environmental and forest modeling. Specifically, GRNN models have been successfully used for estimating leaf wetness prediction to forecast plant disease (Chtioui *et al.*, 1999), for mapping the biomass of tropical rain forest (Foody *et al.*, 2001), in daily-suspended sediment estimation (Cigizoglou and Apl, 2006), for estimating the transpiration of cherry trees (Li *et al.*, 2009), etc. However, the implementation of the GRNN modeling approach in tree height prediction is limited. According to the authors' knowledge no work has been reported that address the above issue. As a challenge for further research in the topic of GRNN modeling implementation to real data, in this work three different GRNN models were constructed and were found superior to non-linear growth functions, providing the highest estimation ability with RMSE% values from 5.69 to 7.13, for all different species. Although the results referred to the specific three tree species in Bucak Forest Enterprise, this research could be a basis for further research by highlighting the applicability of GRNN's in tree height predictions. In general, GRNN modeling offers significant advantages over the non-linear regression modeling. One of the most useful is their ability to learn from data including an effective manner to manage complex, non-linear systems a feature, which is not the case of statistical regression models where an appropriate nonlinear function must first be found, meaning that they do not require assumptions about the form of a fitting function. The estimation of the joint probability density function is derived from the training data set with no preconceptions about its form. This characteristic makes the system perfectly general. Furthermore, GRNN is consistent (Cigizoglu and Alp, 2006). GRNNs are simple in their application, since the smoothness parameter is the only parameter of the procedure of which the optimal value is subject to a research. However, the σ parameter can influence considerably the performance of the GRNN model. Therefore, considerable effort has to be spent for the proper selection of its value.

The adjustment of the equivalence tests indicated that all models were sufficient for describing the total height-diameter relationship for the species analyzed, providing a quantitative confirmation of the models utility in estimating the total tree-height of the three species.

Finally, the validation of the models predictive ability to a new set of data provided the lowest values of Bias, MAD, MSE and RMSE%, for all species, for the GRNN models, showing the indication that GRNN models are the most reliable height-diameter models for all three species. Rational directions and indications on a more general applicability of the proposed GRNN models could be consider the very good performance of the models and their high accuracy of the tree height predictions using a totally different data set from the same forest (validation data set).

Conclusions

Nonlinear growth functions have been commonly used for modeling tree height-diameter relationship. Development and analysis of six nonlinear height-diameter models fitted to three tree species in Western Mediterranean Region Forests of Turkey show that most concave and sigmoidal functions are able to accurately describe tree height-diameter relationship. Generalized regression neural network models were used as an alternative approach to fitting nonlinear data, due its ability to fit complex nonlinear models while these models do not have to be specified in advance, like other nonlinear modeling techniques such as regression analysis. GRNN model provide the highest estimation ability, in terms of MSE and RMSE%. The equivalence testing which was used in order to further validate the selected models, showed that all models could be used for accurate total height estimation. Validation of all constructed models using independent data sets indicated that non-linear models gave satisfactory results for all species, while GRNN models were found to be superior to all models, in terms of their predictive ability. Furthermore, a GRNN model, based on artificial networks associative ability, is generally well fitted to missing or inaccurate data, once it has been developed. It is worth noting that these types of data are frequently faced in forest-data measurements. Finally, all proposed models could also be utilized to predict total tree-heights, missing tree heights from field diameter measurements, and could significantly help for estimating individual tree total volume.

References

Anonymous. 2006. Forest Resources. Ankara: The General Directorate of Forests.

Arabatzis AA, Burkhart HE. 1992. An evaluation of sampling methods and model forms for estimating height-diameter relationships in loblolly pine plantations. For Sci 38, 192-198.

Bishop CM. 1995. Neural Networks for Pattern Recognition. Oxford University Press. 498 pp.

Calama R, Montero G. 2004. Interregional nonlinear height-diameter model with random coefficient for Stone pine in Spain. Can J For Res - Rev Can Rech For 34, 150-163.

Castedo-Dorado F, Anta MB, Parresol BR, Gonzalez JGA. 2005. A stochastic height-diameter model for maritime pine ecoregions in Galicia (Northwestern Spain). Ann For Sci 62, 455-465.

Chtioui Y, Panigrahi S, Francl L. 1999. A generalized regression neural network and its application for leaf wetness prediction to forecast plant disease. Chemometrics and Intelligent Laboratory Systems 48, 47-58.

Cigizoglu HK, Alp M. 2006. Generalized regression neural network in modeling river sediment yield. Adv Eng Softw 37, 63-68.

Colbert KC, Larsen DR, Lootens JR. 2002. Height-diameter equations for thirteen midwetern bottomland hardwood species. North J Appl For 19, 171-176.

Corne SA, Carver SJ, Kunin WE, Lennon JJ, Van Hees WWS. 2004. Predicting forest attributes in southeast Alaska using artificial neural networks. For Sci 50, 259-276.

Curtis RO. 1967. Height-diameter and height-age euations for second-growth Douglas-fir. For Sci 13, 365-375.

Davison AC, Hinkley DV. 1997. Bootstrap methods and their application. Cambridge Univercity Press, Cambridge. 585 pp.

Diamantopoulou MJ. 2010. Filling gaps in diameter measurements on standing tree boles in the urban forest of Thessaloniki, Greece. Environ Modell Softw 25, 1857-1865.

Dolph KL. 1989. Height-diameter Equations for young-growth red fir in California and southern Oregon. Pacific Soutwest Forest and Range Experiment Station, Reserach Note PSW-408.

Eerikainen K. 2003. Predicting the height-diameter pattern of planted Pinus kesiya stands in Zambia and Zimbabwe. For Ecol Manage 175, 355-366.

Efron B. 1979. Bootstrap methods: another look at the jackknife. The Annals of Statistics 7, 1-26.

Efron B, Tibshirani RJ. 1993. An introduction to the bootstrap. Chapman and Hall, NY. 436 pp.

Esteban LG, Fernández FG, De Palacios P, Romero RM, Cano NN. 2009. Artificial neural networks in wood identification: The case of two Juniperus species from the Canary Islands. IAWA Journal 30, 87-94.

Esteban LG, Fernández FG, De Palacios P. 2011. Prediction of plywood bonding quality using an artificial neural network. Holzforschung 65, 209-214.

Fang Z, Bailey RL. 1998. Height-diameter models for tropical forest on Hainan Island in southern China. For Ecol Manage 110, 315-327.

Fekedulegn D, Siurtain MPM, Colbert JJ. 1999. Parameter estimation of nonlinear growth models in forestry. Silva Fenn 33, 327-336.

Fernández FG, Esteban LG, De Palacios P, Navarro N, Conde M. 2008. Prediction of standard particleboard mechanical properties utilizing an artificial neural network and subsequent comparison with a multivariate regression model. Investigacion Agraria Sistemas y Recursos Forestales 17, 178-187.

Foody G, Cutler M, Mcmorrow J, Pelz D, Tangki H, Boyd D, Douglas I. 2001. Mapping the biomass of Bornean tropical rain forest from remotely sensed data. Global Ecology & Biogeography 10, 379-387.

Fox J. 1991. Regression Diagnostics: An introduction (Quantitative Applications in the Social Sciences). Sage Publications, Newbury Park, CA. 92 pp.

Huang S, Titus SJ. 1992. Comparison of nonlinear height-diameter functions for major Alberta tree species. Can J For Res - Rev Can Rech For 22, 1297-1304.

Huang S, Price D, Titus SJ. 2000. Development of ecoregion-based height-diameter models for white spruce in boreal forests. For Ecol Manage 129, 125-141.

Hirsch R. 1991. Validation samples. Biometrics 47, 1193-1194.

Larsen DR, Hann DW. 1987. Height-diameter equations for seventeen tree species in southwest Oregon. Oregon State University Forest Research Laboratory Corvallis Research Paper: 49.

Leahy K. 1994. The overfitting problem in perspective. AI Expert 9(IO), 35-36.

Li X, Yang P, Ren S. 2009. Study on Transpiration Model for fruit tree based on generalized regression neural network. Proc ICEC Int Conf on «Engineering Computation», Washington, DC, (UDA), doi 10.1109/ICEC.2009.71.

Liu C, Zhang L, DavIs CJ, Solomon DS, Brann TB, Caldwell LE. 2003. Comparison of neural networks and statistical methods in classification of ecological habitats using FIA data. For Sci 49, 619-631.

Loehle C. 1997. A hypothesis testing framework for evaluating ecosystem modl performance. Ecol Model 97, 153-165.

Lootens JR, Larsen DR, Shifley SR. 2007. Height-diameter equations for 12 upland species in the Missouri Ozrak highland. North J Appl For 24, 149-152.

Maier HR, Dandy GC. 2000. Neural networks for the prediction and forecasting of water resources variables: a review of modeling issues and applications. Environ Modell Softw 15, 101-124.

Meng SX, Huang S, Lieffers VJ, Nunifu T, Yang Y. 2008. Wind speed and crown class influence the height-diameter relationship of lodgepole pine: Nonlinear mixed effects modeling. For Ecol Manage 256, 570-577.

Norusis MJ. 2000. SPSS for Windows. SPSS Inc., Chicago, IL.

Olson D, Delen D. 2008. Advanced Data Mining Techniques. Spriger-Verlang. 180 pp.

Özçelik R, Diamantopoulou MJ, Wiant HV, Brooks JR. 2008. Comparative study of standard and modern methods for estimating tree bole volume of three species in Turkey. For Prod J 58, 73-81.

Özçelik R, Brooks JR, Diamantopoulou MJ, Wiant HV. 2010. Estimating Breast Height Diameter and Volume from Stump Diameter for Three Economically Important Species in Turkey. Scand J Forest Res 25, 32-45.

Parresol BR. 1992. Baldcypress height-diameter Equations and their prediction confidence intervals. Can J For Res - Rev Can Rech For 22, 1429-1434.

Parresol BR. 1993. Modeling multiplicative error variance: an example predicting tree diameter from stump dimensions in baldcypress. For Sci 39, 670-679.

Patterson D. 1996. Artificial Neural Networks: Theory and Applications. Prentice Hall, Singapure. 477 pp.

Peng C, Zhang L, Liu J. 2001. Developing and validating nonlinear height-diameter models for major tree species of Ontario's boreal forests. North J Appl For 18, 87-94.

Peng C, Zhang L, Zhou X, Dang Q, Huang S. 2004. Developing and evaluating tree height-diameter models at three geographic scales for black spruce in Ontario. North J Appl For 21, 83-92.

Pienaar LV, Turnbull KJ. 1973. The Chapman-Richards generalization of von Beralanffy's growth model for basal area growth and yield in even-aged stands. For Sci 19, 2-22.

Ratkowsky DA. 1990. Handbook of nonlinear regression models. Marcel Dekker, N.Y. 241 pp.

Robinson AP, Froese RE. 2004. Model validation using equivalence tests. Ecol Model 176, 349-358.

Robinson AP, Duursma RA, Marshall JD. 2005. A regression-based equivalence test for model validation: shifting the burden of proof. Tree Physiol 25, 903-913.

SAS InstItute Inc., 2002. SAS/ETS User's Guide, 9th ver. SAS Institute Inc, Cary, NC.

Sharma M, Zhang SY. 2004. Height-diameter models using stand characteristics for Pinus banksiana and Picea mariana. Scand J Forest Res 19, 442-451.

Sharma M, Patron J. 2007. Height-diameter equations for boreal tree species in Ontario using a mixed-effects modeling approach. For Ecol Manage 249, 187-198.

Soares P, Tome M. 2002. Height-diameter equation for first rotation eucalypt plantations in Portugal. For Ecol Manage 166, 99-109.

Specht DF. 1991. A Generalized Regression Neural Network. IEEE Transactions on Neural Networks 2, 568-576.

Swingler K. 2001. Applying Neural Networks. A practical Guide. Morgan Kaufman Publishers Inc, Great Britain. 303 pp.

Temesgen H, Gadow K. 2004. Generalized height-diameter models: an application for major tree species in complex stands of interior British Colombia. Eur J For Res 123, 45-51.

White H. 1980. A Heteroskedastic consistent covariance matrix and a direct test of heteroskedasticity. Econometrica 48, 817-838.

Yuancai L, Parresol BR. 2001. Remarks on height-diameter modeling. Res. Note. SRS-10. Asheville, NC: U.S. Department of Agriculture, Forest Service, Southern Research Station. 6p.

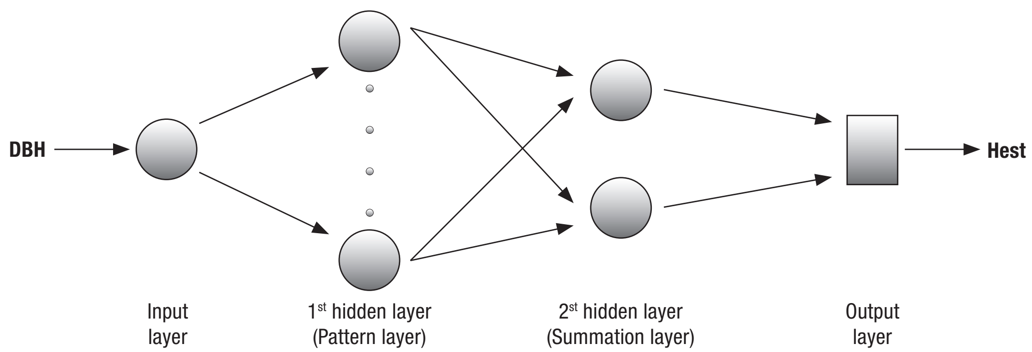
Zeide B. 1989. Accuracy of equations describing diameter growth. Can J For Res - Rev Can Rech For 19, 1283-1286.

Zeide B. 1993. Analysis of growth Equations. For Sci 39, 594-616.

Zhang L. 1997. Cross-validation of non-linear growth functions for modeling tree height-diameter relationship. Annals of Botany 79, 251-257.

Zhang L, Peng C, Huang S, Zhou X. 2002. Evaluation of ecoregion-based tree height-diameter models for jack pine in Ontario. The Forestry Chronicle 78, 530-538.

Appendix. Generalized Regression Neural Network Implementation for total tree height estimation



Input layer: is used to distribute the input diameter values to each node of the pattern layer.

Pattern layer: consists of n nodes, where n is the number of samples within the training set. In this layer the distances (di^2) are calculated (Eq. 2).

Summation layer: consists of two nodes. The input to the first node is the nominator of the Eq. 3, while the input of the second node is the denominator of the Eq. 3.

Output layer: receives two summations from the previous layer and divides them according to Eq. 3, in order to get an estimation of the total tree height.

Numerical example: Considering the above GRNN architecture, suppose we have the data:

(dbhi, h_i)
Tree no 1: (0.20, 9.1)
Tree no 2: (0.40, 18.0)
Tree no 3: (0.60, 24.2)

The algorithm that handles the above data was written by the authors in C^{++}:

```
#include<iostream.h>
#include<math.h>
#define N 3


main(){
        double X[N],U[N],k[N],d[N][N],Y[N],yekt[N],sigma=0.01,t,t1;
        double temp1=0,temp2=0;
        int i,j;
        double z;

        for(i=0;i<N;i++){
                cout<<"Insert tree num"<<i+1<<" diameter"<<"\n";
                cin>>X[i];
                cout<<"Insert tree num"<<i+1<<" total height"<<"\n";
                cin>>Y[i];
                Y[i]/=(pow(X[i],-1.1));
        }
        for(i=0;i<N;i++){
                for(j=0;j<N;j++){
                        temp1+=X[j];
                }
                U[i]=temp1/N;
                temp1=0;
                k[i]=U[i]-X[i];
        }
        for(i=0;i<N;i++){
                for(j=0;j<N;j++){
                        d[i][j]=k[i]*k[j];
                }
        }
        for(i=0;i<N;i++){
                for(j=0;j<N;j++){
                        t=d[i][j];
                        t1=2*sigma*sigma;
                        z=(double)t/t1;
                        d[i][j]=exp(z);
                }
        }
        temp1=0;
        for(i=0;i<N;i++){
                for(j=0;j<N;j++){
                        temp1+=d[i][j]*Y[j];
                        temp2+=d[i][j];
                }
                yekt[i]=temp1/temp2;
                yekt[i]*=pow(X[i],-1.1);
                temp1=0;
                temp2=0;
        }

        cout<<"y-est is:\n";
        cout<<yekt[0]<<»\n»<<yekt[1]<<»\n
```

Results:
$h_{1est} =$ 9.099773 m
$h_{2est} =$ 20.01658 m
$h_{3est} =$ 24.20174 m