From Gatekeeper to Gateway: The Role of Quantitative Reasoning

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Abstract: *This article presents Quantitative Reasoning (QR) as a pathway to smooth students' mathematical transition from high school to higher education. An explanation of what QR is, why it is important, and for whom it is most appropriate is presented. A current leader in the QR movement, Ohio has flourishing secondary and postsecondary QR courses, which are described.*

Keywords: *Quantitative reasoning, mathematical pathways, gateway course, Ohio's Learning Standards, Ohio Mathematics Initiative*

Introduction

Background and Context

> Many colleges and universities use mathematics as an admissions gatekeeper. Admissions requirements often explicitly stipulate that a student must have completed Algebra II. Yet requirements that prioritize algebra have little to no relation to students' readiness to succeed in courses such as statistics or quantitative reasoning—which are more relevant to a wide range of credentials and careers. (Dana Center, 2020, p. 12)

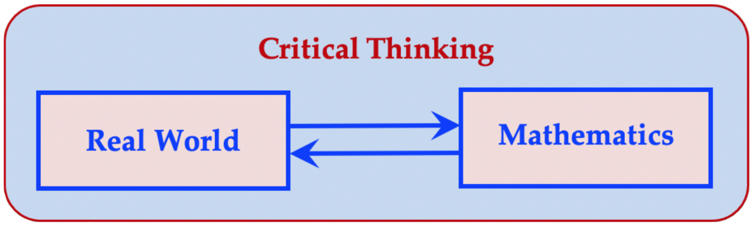
Mathematics, especially in the form of algebraic skills, serves as an educational gatekeeper. In Ohio, due to Senate Bill 311 (2006), beginning with the class of 2014, high school graduation has required four years of mathematics including "Algebra II or its equivalent." Most Ohio school districts have interpreted this law to mean that all students should complete Algebra II. At the national level, the algebra-centric ACT and SAT college admissions tests often serve as gatekeepers. Moreover, most tests used for college mathematics placement focus on algebra skills, using these mechanical procedures as a proxy for mathematical thinking.

But what mathematics do students really need to be successful in their careers and life in general? That is, what mathematics is "necessary to function on the job, in the family of the individual, and in society" (Public Law 105-220, Title II, Section 203, Number 12)? Is mastery of factoring and the quadratic formula needed to be mathematical literate? We argue that—for all people—problem solving, proportional reasoning, probability, statistical thinking, and modeling with mathematics are important and that—for most people—these quantitative competencies are more important "on the job, in the family . . . , and in society" than the automation of by-hand algebraic procedures. To be sure, understanding variables and functions as tools for modeling and some degree of algebraic facility are essential, but knowing six or seven methods to solve a quadratic equation may be excessive emphasis on one topic at the expense of other more important knowledge and skills.

In Praise of Quantitative Reasoning

A relatively new pathway for learning mathematics—called Quantitative Reasoning—focuses on students' ability to think critically about how the world around us connects to mathematics and vice versa. As Figure 1 suggests, students in a Quantitative Reasoning course are challenged to think deeply about real-world contexts and link them to relevant mathematics (cf. Elrod, 2014; Mayes et al., 2014).

Figure 1: *A model of student engagement in Quantitative Reasoning.*



We acknowledge that some innovative Algebra II and College Algebra teachers—and most curriculum standards—integrate genuine applications, authentic data, multiple linked representations, and mathematical modeling into the teaching and learning of algebra. In particular, Ohio's Learning Standards for Algebra 2 (Ohio Department of Education [ODE], 2018) and the Ohio Transfer 36 student learning outcomes for College Algebra (Ohio Department of Higher Education [ODHE], 2016) are examples of such innovation. We applaud these efforts. However,

- Tests and textbooks have not kept pace with such innovative learning standards.
- Many high school teachers and college faculty lack the motivation or preparation to teach algebra as envisioned in these standards.
- Even these innovative standards focus heavily on symbolic representations and by-hand symbol manipulation and give relatively little attention to problem solving, proportional reasoning, probability, statistical thinking, and modeling when compared to a course in Quantitative Reasoning.

QR Tasks: A Nonexample and Three Examples

A Tale of Two Tasks

Rather than dwelling on algebra, with which the reader is likely familiar, we now use a pair of sample tasks to contrast quantitative reasoning versus traditional algebra (Table 1). We follow the tasks by a detailed analysis of their features and classroom implications.

Table 1: *Comparing Tasks.*

Task A: Traditional Algebra	Task B: Quantitative Reasoning
If the sequence 3, 7, 11, 15, ... continues in an arithmetic (additive) pattern, what is the value of the 100th term?	On the first day of April, Brianna has $1.50 in her piggy bank. On April 2nd, she adds 25¢. Brianna keeps adding a quarter dollar each day. Model this situation in at least two ways. Write and answer two questions about the situation.

These two sample tasks illustrate several differences between a traditional algebra pathway and a quantitative reasoning pathway:

1. Task A is a mathematical problem á la Pólya (1957): Understand the problem; devise a plan; carry out the plan; and look back. By constrast, Task B is a modeling task á la Pollak (1966): *Here is a situation—think about it*.
2. Task A has one correct answer. A student response will be either right or wrong. Task B is an open-ended situation. It calls for modeling and problem posing in addition to problem solving. Task B has a low floor and a high ceiling.
3. Task A can be solved using a memorized formula: namely, $a_n = a_1 + (n-1)d$, where n is the term number, a_n is the nth term, a_1 is the first term, and d is the common difference. This formula is highly symbolic and anxiety-inducing for many students. Admittedly, Task A can be approached in many other ways, but it can also be presented as one in a list of similar exercises to be solved quickly using such a formula without connections to underlying concepts. By contrast, although Task B can also be represented as an arithmetic sequence, it more likely would be represented as a linear function or using a spreadsheet, table, or graph. Plus, students are asked to represent Task B in at least two ways.
4. Task A has no context. Many students, and even some teachers, may wonder why we even include arithmetic sequences in the curriculum. Task B has a simple, everyday context. But, to be sure, some students may not know what a "piggy bank" is and may be unfamiliar with other aspects of the situation. An accompanying photo of a piggy bank may help, but this may not be enough for some students.
5. Task A uses highly mathematical language. Task B uses natural language.
6. Task A involves numbers. Task B involves quantities: that is, numbers in context with associated attributes and units of measure. The attributes correspond to variables: in this case, time and money. The respective quantities are presented in multiple formats: (a) Time: "the first day of April" and "April 2nd." These may require some explanation, especially for students for whom English is an additional language. (b) Money: "$1.50" (read "one dollar and fifty cents" by native speakers), "25¢," and "a quarter dollar." (The symbol for cents and the idea of a quarter dollar may be tricky, especially for international students.) Students will need to decide whether to work in dollars or cents, or if using a spreadsheet, which format to select.

Linguistically and culturally, Task B is much more complex than Task A. Moreover, the cognitive demand of Task B is greater than that of Task A. Cognition and cognitive demand refer to "the mental processes and contents of thought involved in attention, perception, memory, reasoning, problem solving, and communication" (National Research Council, 2001, p. 20). Using tasks with higher cognitive demand leads to greater student learning (Stein et al., 1996).

Task B and quantitative reasoning tasks in general are not only more complex and more cognitively demanding than context-free algebra exercises, but they also are messier—they are more difficult to deal with for both teachers and students. We argue that engaging students in doing fewer, richer, messier tasks is worth the effort. Such an approach can transform high school, college, and university mathematics: "Each student should feel empowered and engaged as a mathematical learner, experience success in mathematics, and become fully prepared for the quantitative demands of their future careers and lives" (Dana Center, 2020, p. 3). We see Quantitative Reasoning (QR) as central to this transformation.

Additional QR Tasks

Lest the reader get the wrong impression, Task B is merely a possible jumping off point for a QR course. Quantitative Reasoning engages students in a progression of problems, explorations, and investigations, preparing students ultimately to carry out statistical studies and modeling projects:

Task C: *Is a DoubleStuf Oreo really double stuffed?*

In "Smarter Cookies," Strayer and Edwards (2015) turned this seemingly simple question into a three-phase statistical project for their students. We encourage you to read how they engaged their students in replication, critique, and reformulation and in the process helped them become shrewder consumers of media claims.

Task D: *Luna Lake is stocked with trout. What will happen to the trout population in the long run? Justify your answer using mathematics.* (Alhammouri et al., 2018, p. 418)

In "Tracking Trout," Alhammouri et al. (2018) explained how they used this task to lead ordinary high school students to carry out a full-fledged modeling project. Getting students ready to do such a project did not happen overnight. Developing students' competence and confidence took months of preparation while being "supported by peers, teachers, and technology" (p. 416).

Role of the Teacher

Every mathematics course or curriculum is a sequence of tasks. And for any mathematics class, the teacher's role is critical. Will the tasks be part of a rich and meaningful experience? Or, alternatively, will the teacher's orchestration of the tasks reinforce negative beliefs and attitudes toward mathematics? How will the teacher organize the students into productive cooperative groups? How will the teacher lead whole-class discussions to drive home the key ideas of the task? How will the teacher have the students share their work? How will the teacher press students for explanations and help students link their thinking to other students' thinking?

We do not have the space here to delve into these pedagogical issues. So, we point the reader to resources that do: Adabor and Foley (2010, 2013); Alhammouri and Foley (2019); Alhammouri et al. (2017, 2018); Foley et al. (2017); Stein et al. (2009); Strayer and Edwards (2015).

Mathematical Pathways

In Ohio and other U.S. states, the most common mathematical pathway has been the algebra to calculus pathway. College Algebra—which historically, along with College Trigonometry and Analytic Geometry, was a key component of university preparation for Differential and Integral Calculus—has become the default gateway course in college mathematics. In addition, College Algebra is the most common mathematics course taken for dual credit, that is, taken by students while still in high school to earn college credit (known in Ohio as College Credit Plus).

However, as many careers and everyday life are becoming increasingly quantitative, the mathematics that people use most involves problem solving, proportional reasoning, probability, statistical thinking, and modeling—often accompanied by spreadsheets and other computer software. Although basic algebraic concepts and skills are still needed, the skills taught in College Algebra are not as critical for most careers or for personal financial and medical decisions as the competencies taught in Quantitative Reasoning or Statistics. Further, pushing students into the algebra to calculus pathway creates a hurdle for students in programs that do not actually require the specific content taught in that pathway. To address this academic barrier, rather than placing all students in the traditional algebra to calculus (STEM) pathway, many states, high schools, colleges, and universities have created a variety of mathematical pathways aligned to students' career aspirations. The two most common alternative pathways are the statistics and quantitative reasoning pathways. Other pathways—including discrete mathematics to computer science and data science—are also being considered and developed.

The Statistics Pathway

Over the past quarter century, the statistics pathway has become established and now is flourishing. The growth of Advanced Placement (AP) Statistics since it was first offered in 1997 has been nothing short of phenomenal: by 2017, "the number of students taking the exam grew to 217,000" ("Celebrating 20 years . . . ," 2017). Even though many high schools—especially small or poor high schools—still do not offer AP Statistics (or even AP Calculus for that matter), statistics content has become increasingly integrated into the school mathematics curriculum. At the higher education level, most colleges and universities do offer a freshman-level (gateway) course in Statistics. In addition, the statistics pathway has received both guidance and a boost from the school and college GAISE Reports (American Statistical Association [ASA], 2016; National Council of Teachers of Mathematics & ASA, 2020), which are now both in their second editions. Therefore, compared to quantitative reasoning, the statistics pathway is relatively established. Moreover, Ohio's Learning Standards (ODE, 2018) incorporate the GAISE framework for statistical problem solving, so at least in theory all secondary students in Ohio get a grounding in how to do statistics.

The Quantitative Reasoning Pathway

At the national level, the American Mathematical Association of Two-Year Colleges, the Carnegie Foundation for the Advancement of Teaching, the Dana Center at the University of Texas, the Mathematical Association of America (MAA), the National Numeracy Network, the National Organization for Student Success, and Transforming Postsecondary Education in Mathematics (TPSE Math) have all advocated for multiple pathways—including quantitative reasoning—that align mathematical coursework with students' programs of study at institutions of higher education (IHEs). At the state level, a committee headed by Leitzel (2014) launched ODHE's Ohio Mathematics Initiative (OMI). The OMI's goal is to reform mathematics at the state's 23 community colleges and 13 public universities. The work of the OMI has resulted in expanding gateway courses from just College Algebra to include Introduction to Statistics (Intro to Stats), Quantitative Reasoning, Data Science, Technical Mathematics, Discrete Mathematics, and Mathematics for Elementary School Teachers. A significant feature of the OMI has been to establish Quantitative Reasoning as an alternative to College Algebra across Ohio's 36 IHEs. In addition, the ODHE has created the Ohio Transfer Module (now known as Ohio Transfer 36), which sets guidelines and learning outcomes for general education courses so that students can transfer credits from one institution to another. In particular, Ohio Transfer 36 specifies the content for Quantitative Reasoning and other entry-level mathematics courses needed to obtain state-level approval for the transferability of such courses.

QR Courses at Institutes of Higher Education in Ohio

Building on MAA recommendations (CUPM, 2004), the ODHE created *TMM011–Quantitative Reasoning* to provide IHEs with guidelines for developing college-level QR courses. To be transferable a QR course should "engage students in a meaningful intellectual experience, . . . increase students' quantitative and logical reasoning abilities, . . . improve students' ability to communicate quantitative ideas, . . . encourage students to take other courses in the mathematical sciences . . . [and] strengthen mathematical abilities that students will need in other disciplines" (ODHE, 2015, p. 1). The ODHE has organized the learning outcomes into units on (a) Numeracy, (b) Mathematical Modeling, and (c) Probability and Statistics. The ODHE learning outcomes for this QR course focus on problem solving and conceptual understanding rather than on memorization and procedures and are designed to foster a deep understanding of the applicability of mathematical skills and tools.

According to ODHE's Paula Compton (personal communication, 20 April 2021), 22 of Ohio's 36 public IHEs, including 16 community colleges and 6 universities, have had their QR courses approved at the state level through Ohio Transfer 36. For instance, Ohio University began offering MATH 1060 *Quantitative Reasoning* in Fall Semester 2018 and completed the state approval process effective Summer Semester 2020. Other IHEs offer QR course but have not completed the approval process. For example, Cleveland State University offers two college-credit bearing QR courses: MTH 116 *Foundations of Quantitative Literacy*, which focuses on financial mathematics, statistics, and probability, and MTH 117 *Math Applications in the Real World*, designed to develop QR skills as they apply to personal and social issues. Ironically, these two courses have been offered since before the *TMM011–Quantitative Reasoning* guidelines were developed.

College and universities across the state have heeded OMI's call to offer Quantitative Reasoning courses. Statewide, Quantitative Reasoning has grown in enrollment from 251 students in 2015–2016 to 5,675 students in 2019–2020 (T. Sudkamp, personal communication, 1 February 2021), more than doubling in enrollment each year on average. At Ohio University, for example, enrollment in MATH 1060 *Quantitative Reasoning* has grown from 9 students in Fall 2018 to 126 students in Spring 2021. Based on an analysis of majors and enrollment trends, MATH 1060 enrollment ultimately should stabilize at 800–850 students per academic year.

Increasingly, the state's IHEs are offering QR courses that incoming students can use to satisfy their general education requirement. Therefore, College Algebra no longer needs to serve as a general education quantitative requirement, but rather only to satisfy degree-specific requirements for students with majors that require the mathematical content of College Algebra.

Ohio Course Sequences That Include QR Courses

Meanwhile, at the high school level, the ODE has been rolling out *Mathematical Modeling and Reasoning* (MMR), a QR course for high school seniors. Beginning at just three high schools in the pre-pilot year of 2018–2019, the course is now being offered at scores of high schools across the state. During 2021–2022, the MMR course will be adapted to become an Algebra 2 equivalent (A2E) QR course for high school juniors. This opens up a host of mathematical options in Grades 11–13 for students who have completed Algebra 1 and Geometry—or Mathematics 1 and 2—while in Grades 9 and 10. Table 2 shows some of many possibilities in Grades 11–13 for such students.

Table 2: *Sample Course Sequences for Students Who Have Completed Algebra I and Geometry.*

Grade 11	Grade 12	Grade 13
A2E QR	College Credit Plus QR	Intro to Stats
Alg 2 or Math 3	MMR	QR or Intro to Stats
A2E QR	Precalculus or AP Stats	Numerous gateway options

Final Thoughts

QR lies at the intersection of critical thinking, mathematical competencies, and genuine contexts from disciplines or everyday life. It is ideal for preparing high school and incoming college students for a wide range of majors and careers. A student taking a QR course is challenged to think critically and apply mathematics and statistics to solve problems, interpret data, and draw conclusions within disciplinary and interdisciplinary contexts.

With an increasing number of Ohio IHEs developing and offering QR courses, high school teachers, principals, and counselors can be confident in recommending QR and MMR to their students. These courses build students quantitative understanding and self-confidence and prepare students for a variety of majors and for the financial and medical decisions of adult life. IHE faculty, administrators, and advisors can be self-assured in recommending QR to incoming students, especially undecided and non-STEM majors. Already 22 Ohio IHEs offer QR courses approved through Ohio Transfer 36. As Madison (2019) put it, QR "is no longer an orphan" and is beginning to develop a "supporting family structure" (p. 37). We invite you to join in supporting QR as a gateway and a pathway to student success in mathematics.

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