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Probabilistic sensitivity studies on modal properties and response of occupied pedestrian bridge

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The enthusiastic behaviour of people on structures can cause synchronized rhythmic movements which may lead to resonance between forcing frequency and one or more natural frequencies of the structure. Such a human-structure synchronization results in strong vibrations that affect structural stability and create discomfort among people. The Indian codes and standards do not consider Crowd-Structure Interaction (CSI), resulting in an unreliable vibration assessment of structures. The present work investigates the effect of CSI on the dynamic behaviour of pedestrian bridges in the vertical direction. The system is modelled as a two degree of freedom mass spring damper system and the modal properties are determined. The modal properties and response of the system are studied for varying crowd parameters such as size and location. A probabilistic sensitivity analysis is carried out to understand the sensitivity of modal properties and response of the CSI system to the parameters of crowd and empty structure, using a variance based method. Monte Carlo simulation is used to simulate samples of the random parameters. Crowd activity frequency and generated load factor are observed to be the most sensitive parameters affecting the vibration of the structure.

KEYWORDS: Crowd load; vibrations; crowd-structure interaction; modal properties; sensitivity; Monte Carlo simulation.

Vibration of pedestrian structures when subjected to dynamic crowd loads is of a serious concern for both safety and comfort of the occupants of the structure. The incidents of large vibrations on newly built structures like on London Millennium bridge in 2000 are being reported. Current available guidelines (ISO, Canadian etc.) have settled on adopting conservative load models without studying the behaviour of crowd-structure interaction. This leads to an over or under estimation of the actual vibration response of the structure. In Indian scenario, the value of live/imposed load is produced based on the intended use of the building including the load due to impact and vibration[1]. Here crowd load is often characterized as this live load where the presence of crowd is treated as an addition of weight on the structure. Similar approach is considered in the design of bridges as well where the vehicular load is specified in kg/m^2 for various class of vehicles[2]. In areas where crowd loads are likely to occur, such as bridges near towns, which are either centers of pilgrimage or where large congregational fairs are held, an increased intensity of footway loading in kg/m^2 is allowed. These load intensity based approaches do not yield realistic response of structures when occupied with crowd. There is a need for the development of improved models for the analysis of crowd occupied structures.

Dougill et al.[3], Jones et al.[4] and Vasilatou et al.[5] have modelled existing structures with the presence of crowd as equivalent mass-spring damper models to predict structural response. Jimenez-Alonso et al.[6] proposed a biomechanical crowd-structure interaction model which can estimate the energy exchange between pedestrians and footbridge. Shahabpoor et al.[7] have proposed a method for serviceability assessment by modelling the Crowd-Structure Interaction (CSI) system considering the inter- and intra-subject variability of the crowd.

The model was based on actual vibrations perceived by an individual and the occupied structure's modal parameters were predicted.

The present study deals with determination of the behaviour of crowd occupied structure i.e. the modal properties and peak acceleration for given properties of empty structure and crowd. It is obvious that the activity, level of synchronization and other characteristics of crowd vary and the parameters considered cannot be assigned with deterministic values. This uncertain nature of the inputs resulting in uncertainty of outputs brings in the necessity of a probabilistic sensitivity analysis of the CSI system.

Uncertainty is associated with every physical phenomenon. Any variation of inputs from the exact value may lead to incorrect estimation of the output parameters. Small errors in certain input parameters may result in larger error in the output if the output is sensitive to those parameters. On the contrary, error in certain parameters may not affect the output at all. Consideration of uncertainties involved leads to a more realistic assessment of quantities of interest. Sensitivity analysis helps in identifying the sensitivity of the quantities of interest towards other parameters.

In this study, sensitivity analysis is performed to find out the extent to which the properties of crowd and empty structure affect the behaviour of occupied structure. A simple probabilistic model is developed and a global response sensitivity analysis is carried out. Monte Carlo Simulation (MCS) is adopted to simulate the samples of parameters. The sensitivity of modal properties and peak acceleration of the CSI system to the parameters of crowd and empty structure is found using a variance based method. This study helps in identifying the most sensitive parameters so that care could be taken while measuring and processing them.

CSI system

The pedestrian structure considered for this study is a footbridge with Pratt type steel truss and a concrete deck represented in Fig. 1. The bridge is designed to be 38.5 m long, 2 m wide and 4.85 m high. Analysis is carried out to determine the modal properties and response of the CSI system, for the dominant vertical mode, considering the interaction between the footbridge and its occupants. In order to obtain these, a two Degree of Freedom (DoF) system is developed where first DoF represents the vertical motion of the structure and the second DoF represents the vertical motion of crowd. The force exerted by the crowd on the structure depends on the crowd activity. This force and the reaction developed by the structure constitute an internal force pair that affects the response of the system. In order to quantify this internal force, Generated Load Factor (GLF) and crowd activity frequency (f_a) are considered[4,8]. The vertical frequency ranges for different activities are specified in literature[9].

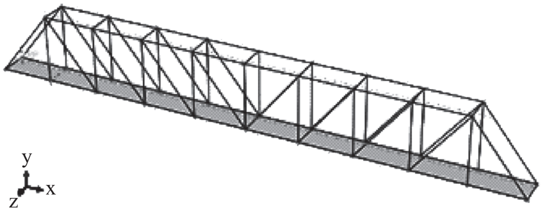


Fig. 1 Numerical model of footbridge (ABAQUS CAE)

The inputs for the analysis are the properties of the empty structure and crowd characteristics whereas the outputs from the analysis are the modal properties and the response of the CSI system. The modal properties of the empty bridge are found numerically. ABAQUS CAE is adopted in the present study to obtain natural frequency and unity normalized mode shape ordinates corresponding to the vibration mode of interest. The first vertical mode is considered for the present study, as this has more probability to cause resonance being at the lower frequency. Modal mass (m_{es}) and modal stiffness (k_{es}) associated with this mode are calculated as 33928.16 kg and 4.501 Hz, respectively. Damping of 5% is assumed. Unity normalized mode shape ordinates for the first vertical mode at various locations along the bridge are extracted and a best fit polynomial equation given below is obtained.

$$\varnothing_i = \gamma_1 . x_i^4 + \gamma_2 . x_i^3 + \gamma_3 . x_i^2 + \gamma_4 . x_i + \gamma_5 \qquad (1)$$

where, γ_1 = 0.000003011, γ_2 = -0.00023186, γ_3 = 0.002937, γ_4 = 0.05875 and γ_5 = 0.008043. A comparison between numerically obtained mode shape and the polynomial curve fit is given in Fig. 2. It is seen that the error is negligible and this polynomial equation is used for further analysis.

The properties assumed for an average person are tabulated in Table 1. The modal properties for an equivalent single DoF crowd model viz. mass, stiffness and damping are derived from the properties of each person. The unity normalized mode shape ordinate of the empty structure at the location of the individuals are used to account for the location of individuals on the structure[10].

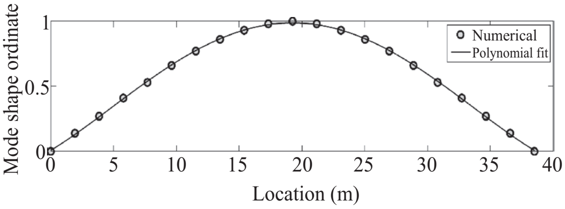


Fig. 2 Numerically obtained mode shape (dominant) and best fitted curve using polynomial equation

TABLE 1
MODAL PROPERTIES OF AN AVERAGE PERSON

Parameter	Unit	Value
Modal mass (m_i)	kg	70
Stiffness (k_i)	N/m	23360
Damping (c_i)	Ns/m	770

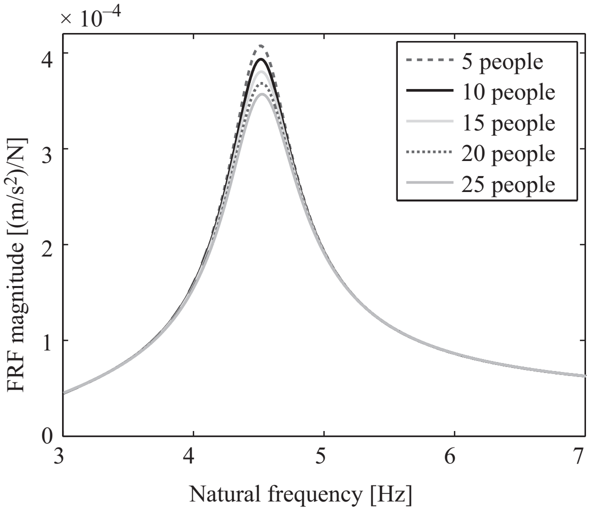
Analysis of the CSI system

Frequency Response Function (FRF) analysis is adopted in this study to obtain the outputs. Natural frequency and damping ratio are obtained by peak picking and half power bandwidth method, respectively. The physical acceleration experienced by the crowd at specific locations is obtained by multiplying the acceleration with the mode shape ordinate at that location.

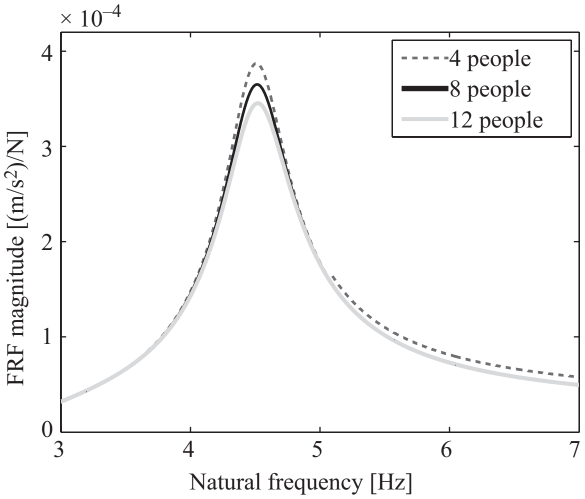
The FRF curves obtained for crowd walking along the bridge and in a circle at mid-span for different crowd size are shown in Fig. 3. It is observed that the peak of the FRF curve shifts towards right with increase in crowd size. This indicates the increase in natural frequency of the CSI system with increase in crowd size. It can be further seen that the FRF curves becomes more flat with increase in crowd size, indicating the increase in damping ratio of the CSI system with increase in crowd size.

Parametric studies

The response of the CSI system for varying crowd size, location and activity is analysed. Figure 4 shows the trend of change in natural frequency, damping ratio and peak acceleration with varying crowd size. As the crowd size increases, the natural frequency, damping ratio and peak acceleration of the system increase. The trend in variation of natural frequency, damping ratio and peak acceleration agrees with other studies reported in literature[10,11].



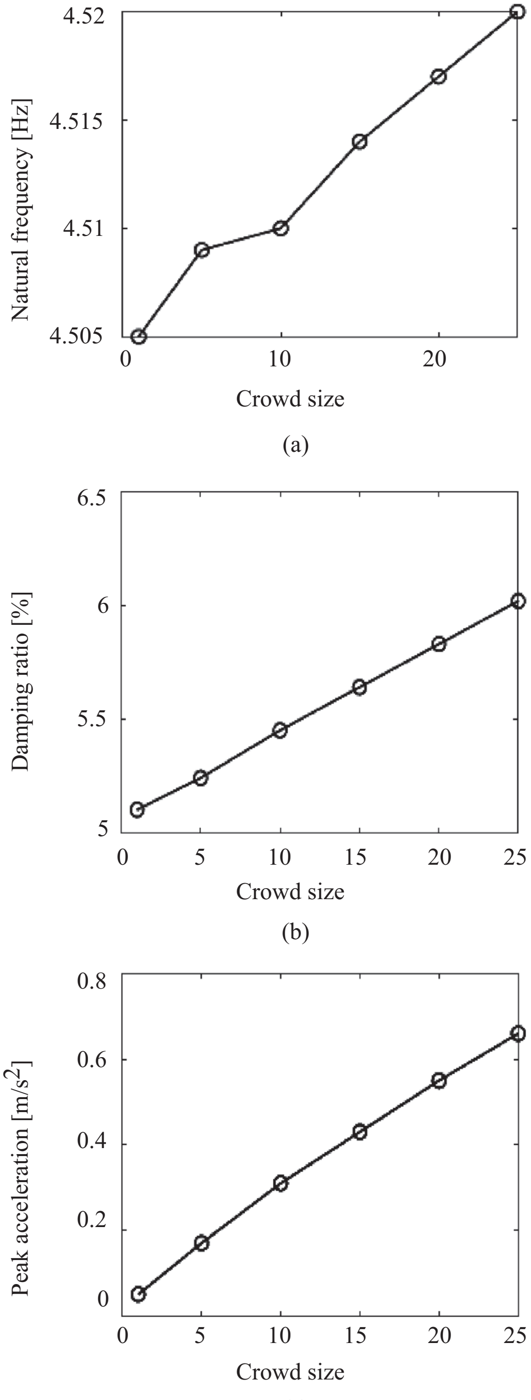
(a)



(b)

Fig. 3 FRF curves of CSI system for crowd walking (a) along the structure (b) in a circle at mid-span

The variation of natural frequency, damping ratio and peak acceleration with varying crowd location is plotted in Fig. 5 for different crowd size. All parameters considered here are found to have maximum value when the crowd is located at mid-span. These values are minimum when crowd is located at the supports. A significant difference in the natural frequency and damping ratio is observed with variation in crowd size at the mid-span when compared to other locations. This is because the mode shape of the vibration mode adopted here has the maximum ordinate at its mid-span. Therefore, the mid-span is more sensitive to the crowd forces when compared to other locations due to maximum interaction between crowd and structure at that location. A significant difference is observed in peak acceleration for different crowd size at the same crowd location. Therefore, the trend in variation of peak acceleration for various crowd locations of the occupied structure is affected by the trend in change of modal properties of the occupied structure.



(a)

(b)

(c)

Fig. 4 Parametric studies on the effect of crowd size on (a) natural frequency; (b) damping ratio and (c) peak acceleration

Global response sensitivity analysis

A global response sensitivity analysis is carried out to have a better understanding of the parameters involved in the dynamic response of the CSI system. Monte Carlo simulation is adopted to simulate the samples of various parameters. The sensitivity of modal properties and response of the CSI system to the parameters of crowd and empty structure is found using a variance based method. The rate of variation in the output parameters for given variation in input parameters is observed to determine the sensitivity of output parameters towards the inputs. The probability distributions are chosen appropriately for the input parameters. The steps involved in the sensitivity analysis are as follows:

1. Assume probability distributions to the input parameters, X_i.
2. Generate samples of Random Variables (RVs) that follow the assumed distribution. MATLAB random variable generator 'randn', produces a sequence of standard normally distributed random numbers. Apply suitable transformation to obtain RVs of required distribution. Transformation of a RV from standard normal distribution (X_N) to normal distribution (X) of given mean (μ) and standard deviation (σ) is given by

$$X_i = \mu_i + (\sigma_i \times X_{Ni}) \tag{2}$$

3. Obtain the samples of output parameters (Y_i) for the performance function in two ways:
    i. By varying all the input RVs simultaneously according to the given distribution.
    ii. By keeping all the input RVs except one at mean, ie. varying only one RV according to the given distribution at a time. Repeat this for all the input RVs.
4. Plot the distribution curves of the output parameters obtained in the second way.
5. Determine the coefficient of variation of output parameters in both the ways. Calculate the coefficient of sensitivity of each output parameter

for each input parameter using

$$CoS_i = \left(\frac{CoV_i}{CoV}\right)^2 \times 100 \tag{3}$$

where CoS_i is the coefficient of sensitivity for i^{th} input parameter, CoV is the coefficient of variation of the output parameter obtained by varying all input parameters estimated by

$$CoV = \frac{\sigma}{\mu} \tag{4}$$

and CoV_i is the coefficient of variation of the output parameter obtained by varying only i^{th} input parameter at a time, estimated by

$$CoV_i = \frac{\sigma_i}{\mu_i} \tag{5}$$

TABLE 2
PROBABILITY DISTRIBUTION PARAMETERS OF INPUT QUANTITIES USED IN SENSITIVITY STUDIES

Parameter	Units	RV	Distribution	μ_i	σ_i
m_{es}	kg	X_1	Normal	33930	1696.5
f_{es}	Hz	X_2	Normal	4.501	0.225
ζ_{es}	—	X_3	Normal	0.05	2.5×10^{-3}
m_i	kg	X_4	Normal	70	3.5
k_i	N/m	X_5	Normal	23360	1168
c_i	Ns/m	X_6	Normal	770	38.5
GLF	—	X_7	Normal	0.188	9.4×10^{-3}
f_a	Hz	X_8	Normal	2	0.1

Here, all the input quantities are assumed to be Gaussian and independent. The distribution parameters are listed in Table 2 and the distribution plots are shown in Fig. 6. The distribution of output parameters such as peak acceleration, damping ratio and natural frequency are determined according to the above mentioned steps. CoS_i is found for all parameters for a sample size of 50000 with a crowd of 20 people walking along the structure and walking in a circle at the mid-span.

RESULTS AND DISCUSSIONS

The cumulative distribution plots of natural frequency, damping ratio and peak acceleration for crowd walking in a tight circle at mid-span obtained by varying each input parameter and keeping the rest at their mean are shown in Fig. 7. The CoS_i of various parameters for the cases of crowd walking in a tight circle at the mid-span of the structure and walking along the structure is determined and is listed in Table 3.

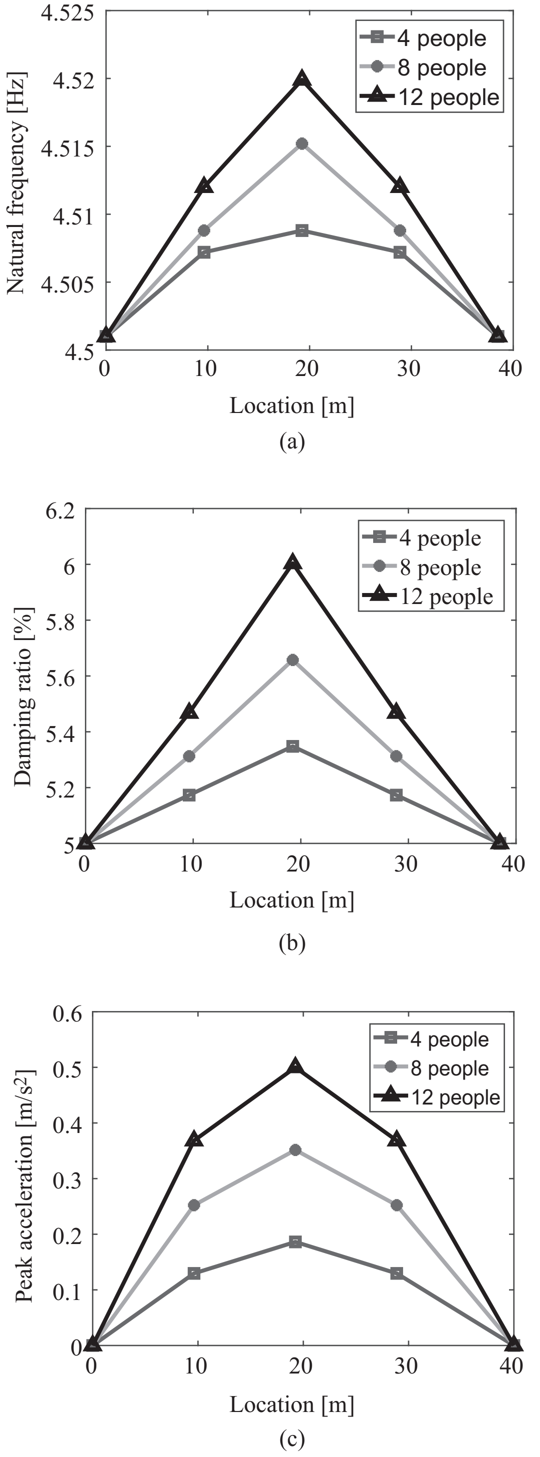


Fig. 5 Parametric studies on the effect of crowd location on (a) natural frequency; (b) damping ratio and (c) peak acceleration

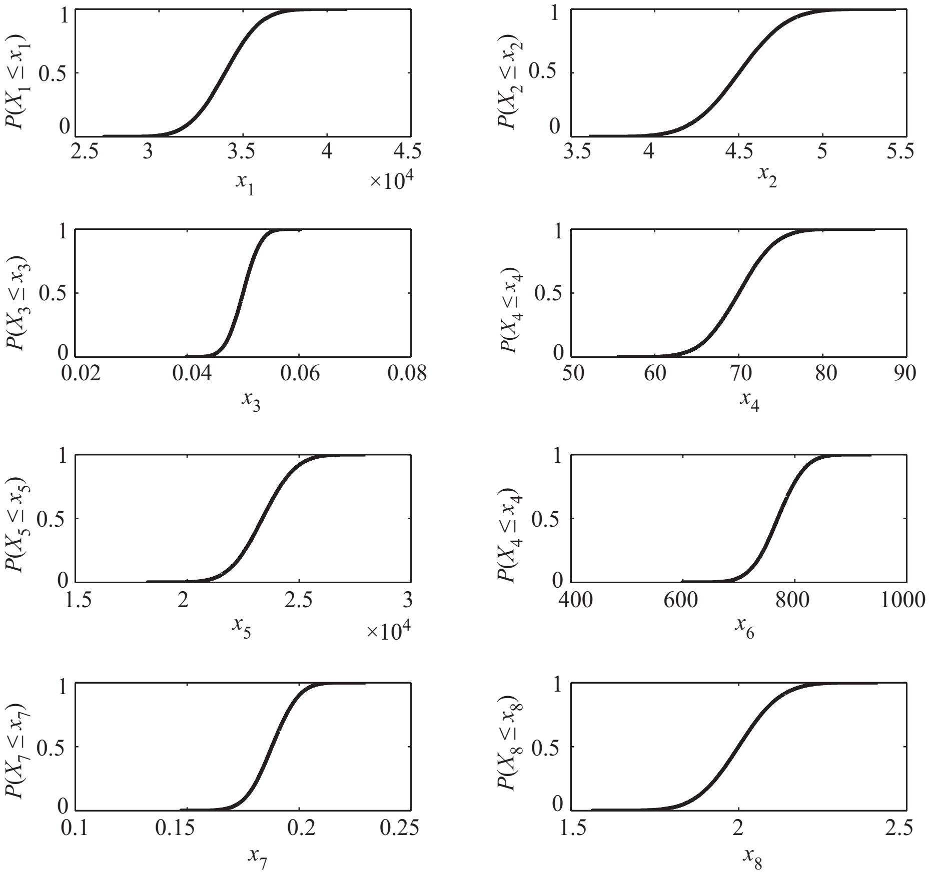


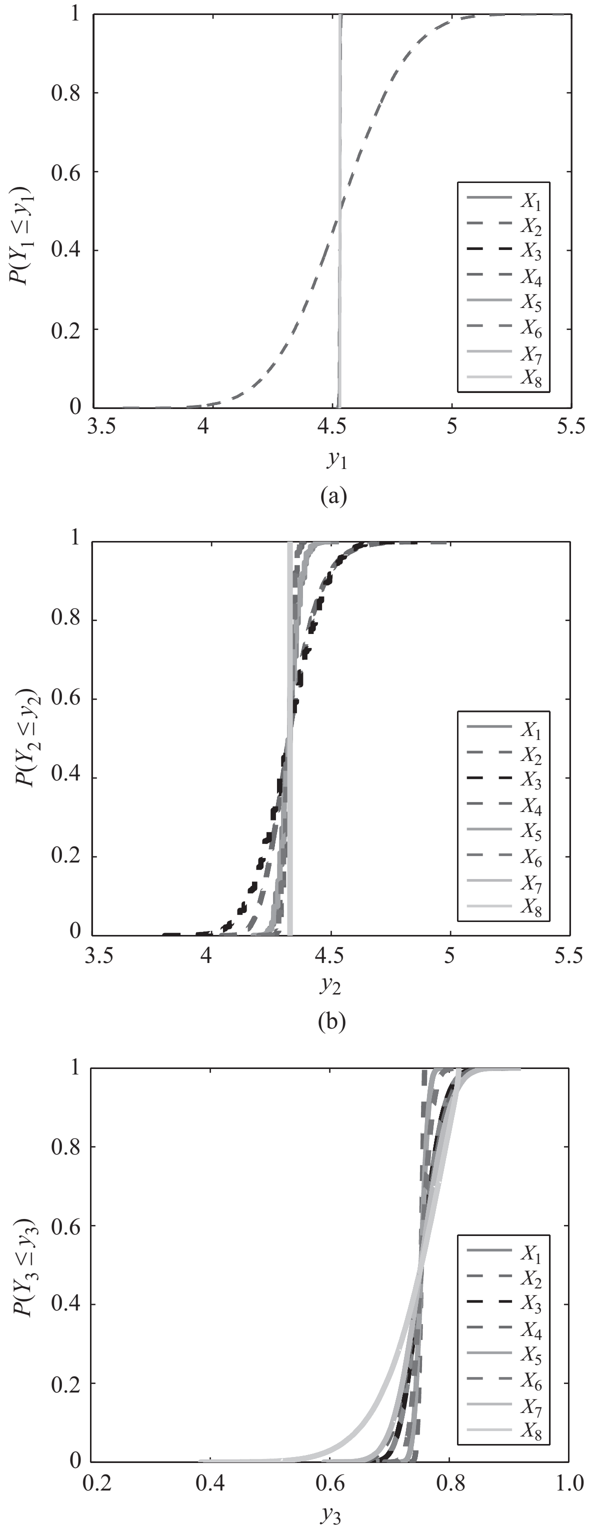
Fig. 6 Probability distribution plots of input quantities used in sensitivity studies

TABLE 3

SENSITIVITY FOR CROWD WALKING IN A TIGHT CIRCLE AT THE MID-SPAN (SCENARIO 1) AND CROWD WALKING ALONG THE STRUCTURE (SCENARIO 2)

Parameter	RV	Sensitivity (%) for scenario 1			Sensitivity (%) for scenario 2		
		Y_1	Y_2	Y_3	Y_1	Y_2	Y_3
m_{es}	X_1	0.0044	5.1651	10.0098	0.0014	2.1269	12.1960
f_{es}	X2	99.9163	35.0422	0.1889	99.9353	13.4548	1.9744
ζ_{es}	X_3	0.0014	50.8696	10.0927	0.0012	81.6901	11.8881
m_i	X_4	0.0117	1.2923	14.2684	0.0033	0.3855	12.3558
k_i	X_5	0.0053	4.9472	1.1573	0.0011	2.0486	2.1523
c_i	X_6	0.0129	1.7726	2.8370	0.0022	0.6262	2.0987
GLF	X_7	0.0000	0.0000	17.1048	0.0000	0.0000	15.6394
f_a	X_8	0.0000	0.0000	44.8332	0.0000	0.0000	42.1183

The parameter that largely affects the natural frequency of the CSI system is the natural frequency of the empty structure with a sensitivity of 99.92% for crowd walking in a circle at mid-span and 99.94% for crowd walking along the structure. The damping ratio and natural frequency of the empty structure are observed to affect the damping ratio of the CSI system for crowd walking at the mid-span. The *CoS* value obtained are 50.87% and 35.04% respectively. It is noticed that for crowd walking along the structure,



(a)

(b)

(c)

Fig. 7 Distribution of (a) natural frequency (Y_1); (b) damping ratio (Y_2) and (c) peak acceleration (Y_3) for variation in each input parameter

the damping ratio of the system is significantly affected by the damping ratio of the empty structure with a sensitivity of 81.69%. The natural frequency of the empty structure does not have much effect on the damping ratio of the CSI system with sensitivity being 13.45%. In other words, for crowd walking at mid-span, both damping ratio and natural frequency of the empty structure are found to be very significant parameters whereas for crowd walking along the span, damping ratio of the empty structure is more significant compared to natural frequency of the empty structure. Therefore, it is inferred that the location of the crowd is a significant factor that influences the damping ratio of the CSI system.

The crowd parameters such as activity frequency and GLF show maximum effect on the peak acceleration of the system. The activity frequency exhibited sensitivity coefficients of 44.83% and 42.11% for crowd at mid-span and along the structure respectively. This means crowd activity and GLFs are the most sensitive parameters affecting the peak acceleration of the CSI system. As the comfort level of crowd is directly dependent on the peak acceleration of the system, it is concluded that crowd load is a significant factor that needs consideration in the vibration serviceability assessment of pedestrian structures.

CONCLUSIONS

The behaviour of pedestrian bridges when occupied with crowd is studied. It is observed that the properties of occupied structure largely depend on the properties of the empty structure. The natural frequency, damping ratio and peak acceleration of the system increase with increase in crowd size. It is also seen that the crowd size and location impact the response of the system. Therefore, these parameters are to be carefully studied during vibration serviceability assessment of pedestrian structures. Based on the sensitivity analysis carried out, the most sensitive parameter for the natural frequency of the CSI system is the natural frequency of the empty structure. The same is found to be independent of crowd location. However, the sensitivity of damping ratio of the system is found to be dependent on the location of the crowd. Both natural frequency and damping ratio of the empty structure are found to impact the damping ratio of the CSI system when the crowd is walking at the mid-span. Only damping ratio of the empty structure is observed to affect when crowd is walking along the span. The peak acceleration is largely affected by the crowd activity frequency and GLF, which proves that crowd load is a significant factor to be considered in the vibration assessment of pedestrian structures. Ignoring the presence of crowd on such structures may lead to incorrect estimation of structural response, resulting in accidents.

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