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Comparative Analytical Study on Crack Width of Reinforced Concrete Structures

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Abstract

This paper presents a comparative study for the cracking limit state according to design codes. It aims mainly to connect research findings with design code equations. Appropriate recommendations are reached and the various factors and parameters influencing crack width investigated. The most appropriate equation for crack width calculation can be found. This is done by creating an analytical and numerical program studied various factors and parameters affecting on the crack width. The Analytical study includes some variables affecting the crack width such as steel stress, concrete cover, flexural reinforcement ratio and rebar arrangement. A 3-D finite element analysis by ABAQUS were used to model and idealize the problem. The numerical results were compared with the analytical results. It was concluded that some codes did not take into account the impact of some major variables and cases on the crack width. Also, it was found that some codes are not clear in the region concerning the position of the crack width calculation and the values obtained for the crack width. For calculating crack width values, JSCE (2007) equation is the most appropriate equation as it takes into account the main parameters that affect crack width.

Keywords: Flexural Cracks; Crack Width; Serviceability; Reinforced Concrete; Codes Provisions.

1. Introduction

Crack formation and crack control were some of the most important considerations in designing systems of reinforced concrete structures. The basic concepts of cracking in reinforced concrete members were fairly well understood, and the influence of crack size and distribution was generally accepted. However, it still a complex problem because of the different characteristic of each investigator specimens and the indirect effect of many parameters therefore some conclusions were unclear as to whether these parameters were important. Cracks of large widths lead to corrosion, degradation of the surface and thus harm the overall quality of life-being of the structure. Excessive crack width would reduce the structure's existence by allowing for faster penetration of high humidity corrosive factors, repeated moisture absorption, steam, salt water and chemical-related gases, in ordered to achieve reinforcement. Moreover, structural performance, including rigidity, energy absorption, capacity and ductility, was impaired by cracks in reinforced concrete structures. For reinforced concrete flexural members' usability, therefore, prediction and the cracking control and cracking widths is important. Designers were able to used guidelines in

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different design codes to monitor the crack width on the tension surface of the flexural member. These guidelines were based on certain analytical solutions to crack width that was developed by various researchers. In the workshop experiments with rectangular reinforced concrete beams, two criteria, a concrete cover and an average effective concrete area surrounding each bar were studied by Albandar and Mills [1]. Frosch [2] have had studied the development and used of the crack width equation for the measurement of large coverings and presents a new approached formulation physically dependent phenomena. Gilbert and Nejadi [3] studied the cracking caused by shrinkage in restrained reinforced concrete members both experimentally and analytically.

The effect of thick concrete covers on the maximum flexural crack width was studied by Makhlouf and Malhas [4]. Soltani et al. [5] had focused on the significant variation in the cracking performance among high resistance steel reinforcement and conventional steel reinforcement in concrete. Rasmussen et al. [6] had analyzed the crack pattern in bending members in experimental findings by Sherwood [7] and identified them with two existing models; primary bending cracks and secondary local cracks. At the Center for Infrastructure Engineering and Safety (CIES), Hussain et al. [8] performed a long-term experiment to bring the impact of long-term effects, such as concrete creep and shrinkage, on concrete cracking into focus. Creazza and Russo [9], Chiu et al. [10] and Hamrat et al. [11] explored the bending crack creation of beams of high-strength reinforced concrete (HSRC). Allam et al. [12] proposed five rectangular reinforced concrete models to theoretically analyze the provisions of codes within certain equations that some researchers had found. Chowdhury and Loo [13] had developed a new method to estimate average crack widths for beams with reinforced or partially pre-stressed concrete beams. The calculation and comparison of the maximum bending crack width for different structural members, based on experimental data tension on both sides by axial force, Nam et al. [14] have had been studied. A critical review of the literature related to design of reinforced concrete structures in the cracking limit state was provided by Basteskår et al. [15]. Bakis et al. [16] studied the width of flexural cracks in a concrete member internally reinforced with fiber reinforced polymer (FRP). Flexural cracks of a reinforced concrete beam blended with fly ash as supplementary material had been studied by Oktaviani et al. [17]. Khorasani et al. [18] studied flexural cracking behavior of concrete beams reinforced with GFRP bars.

This paper present a comparative study for the cracking limit state according to some design codes. Analytical and numerical program studied various factors and parameters affecting on the crack width.

2. Research Methodology

The following figure show the research methodology steps: (1) studying theoretical the cracking limit state according to some famous codes in this field. (2) Develop numerical software for model and idealize the problem and verify it results with experimental results. (3) Comparing the analytical results of codes with experimental results of some previous researcher. (4) Studying analytically the effect of the various variables or parameter on the cracking behavior of some suggested reinforced concrete beams models. (5) Comparing the analytical results of the suggested beams models with the numerical results. (6) Final conclusion and recommendation for research.

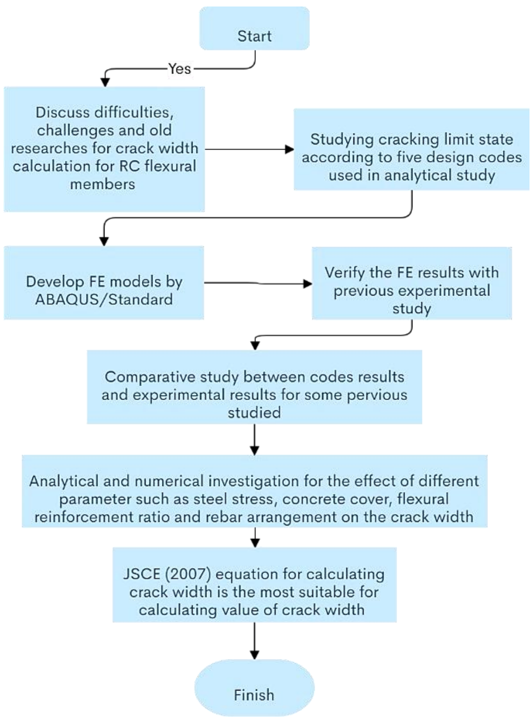


Figure 1. Methodology of the research

3. Codes Provisions for Crack Width Calculations

3.1. Egyptian Code (ECP 203-2018)

The Egyptian code ECP 203-2007 [19] gives the crack width by the following equation:

$$w_k = \beta . S_{rm} . \varepsilon_{sm} \tag{1}$$

Where w_k = crack width; β = the factor connects the average width of the crack and its design value: β = 1.7 for loading-induced cracks and for cracks caused by the deformation limitation in a segment that is greater than 800mm wide or deeper (whichever is smaller), and β = 1.3 for cracks caused by the deformation limitation in a segment that is less than 300mm wide or deeper (whichever is smaller).

The Egyptian code give a crack spacing depend on strain in steel reinforcement and other factors by the Equation 2:

$$S_{rm} = \left(50 + 0.25 k_1 . k_2 . \frac{\emptyset}{\rho r}\right) \tag{2}$$

Where k_1 = Coefficient representing the bonding impact of steel and concrete between cracks: k_1 = 0.8 for deformed bars, 1.6 for smooth bars and $k_1 = k.k_1$ for member subjected to imposed deformation; k_2 = coefficient representing the effect on the cross-sectional strain distribution: k_2 = 0.5 for the pure bending member and k_2 = 1.0 for the axial tension section; Ø = diameter of bar in mm; ρr = ratio of effective tension reinforcement: $\rho r = \frac{As}{Acef}$ where A_{cef} = area of effective tension concrete. The Egyptian code give a coefficient that reflect the effect of steel strain on the crack width by the following equation:

$$\varepsilon_{sm} = \frac{fs}{Es}\left(1 - \beta_1 . \beta_2 . \left(\frac{fsr}{fs}\right)^2\right) \tag{3}$$

Where f_s = longitudinal steel stress in tension zone based on a continuously loaded cracked component study; E_s = the steel reinforcement elasticity modulus; f_{sr} = stress in tension zone longitudinal steel, based on the study of the cracked component resulting in major cracking load; β_1 = coefficient reflecting the bonded reinforcement properties: β_1 = 0.8 for bars which are deformed and β_1 =0.5 for bars which are smooth; β_2 = coefficient which takes the loading duration into consideration: β_2 = 1.0 for short term loading and β_2 = 0.5 for long term loading or cyclic loading.

3.2. Eurocode 2 (EN 1992-1-1: 2004)

The Eurocode EC2 [20] gives the crack width by the Equation 4:

$$w_k = S_{r,max} . (\varepsilon_{sm} - \varepsilon_{cm}) \leq w_{kmax} \tag{4}$$

Where w_k = crack width.

The maximum crack spacing may be calculated from the expression;

$$S_{r,max} = k_3 . c + k_1 . k_2 . k_4 . \left(\frac{\emptyset}{\rho_{p,eff}}\right) \tag{5}$$

Where c = the cover to the longitudinal reinforcement; k_1 = coefficient reflecting the bonded reinforcement properties: k_1 = 0.8 for bars which are deformed and k_1 =1.6 for bars which are smooth; k_2 = coefficient representing the effect on the cross-sectional strain distribution: k_2 = 0.5 for the pure bending member and k_2 =1.0 for the axial tension section; k_3 = 3.4; k_4 = 0.425; Ø and $\rho_{p,eff}$ are similar to the items of the equation given by the ECP [19].

$\varepsilon_{sm} - \varepsilon_{cm}$ may be calculated from the following expression;

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma s - kt\left(\frac{f_{cteff}}{\rho_{peff}}\right)(1 + \alpha_e \rho_{peff})}{Es} \geq 0.6 \left(\frac{\sigma s}{Es}\right) \tag{6}$$

Where ε_{sm} is mean strain under relevant combination of loads and allowing for effects, such as tension stiffening or shrinkage; ε_{cm} is the average strain in the solid concrete between the cracks; σs = the stress in tension reinforcement; f_{cteff} = the main value of the concrete tensile strength when the cracks can be expected first; α_e = the ratio E_s/E_{cm}; K_t factor depend on the duration of the load: ; K_t = 0.6 for loading on short term and K_t = 0.4 for loading on long term.

3.3. ACI 318-14 [21] (ACI 224R-01 [22])

The equation that were considered to best predict the probable maximum bottom and side crack width are:

$$w_b = 0.091 \sqrt[3]{t_b . A}\ \beta\ (f_s - 5).10^{-3} \tag{7}$$

$$w_s = \frac{0.091 \sqrt[3]{t_b . A}}{1 + {t_s}/{h_1}} (f_s - 5).10^{-3} \tag{8}$$

Where w_b = the maximum crack width at the bottom tension fiber of the beam, in; w_s = the maximum crack width at reinforcement level, in; f_s = reinforcing steel stress, ksi; A = symmetric concrete area, divided by a number of bars with steel reinforcement, in^2: $A = \frac{t.b_w}{n}$ where t = twice the distance from centroid of the flexural tension reinforcement to the extreme tension fiber and b_w = the efficient tension area width. t_b = bottom cover to center of bar, in; t_s = side cover to center of bar, in; β = ratio of distance between neutral axis and tension face to distance between neutral axis and reinforcing steel regarding 1.2 in beams and 1.35 for slabs.

Simplification of Equation 7 yielded the following equation:

$$w = 0.076\, \beta\, f_s \sqrt[3]{d_c A}\ .10^{-3} \tag{9}$$

Where d_c = distance from the maximum tension fiber to the nearest bar, cover thickness, in.

3.4. JSCE Guidelines for Concrete (2007)

The crack width, w, can be determined according to JSCE [23] by the next expression;

$$w = 1.1 k_1 k_2 k_3 \{4c + 0.7(c_s - \emptyset)\}[\frac{\sigma_{se}}{E_S} + \varepsilon'_{csd}] \tag{10}$$

Where w = crack width; k_1 = a constant to consider the effect of surface reinforcement geometry on crack width: k_1 = 1.00 for bars deformed and k_1 = 1.30 for bars smooth and prestressing steel; k_2 = a coefficient to identify the impact of the material goodness on crack width: $k_2 = 15/(f'_c + 20) + 0.7$ where f'_c = concrete resistance compressive (N/mm^2); k_3 = a constant that takes the effect of multiple tensile reinforcement layers on crack width into account; $k_3 = 5(n+2)/7n+8$ where n = number of the layers of tensile reinforcement; c = concrete cover (mm); c_s = tensile reinforcement center to center gap (mm); $\emptyset$ = diameter of tensile reinforcement; ε'_{csd} = compressive strain for evaluation of increment of crack width due to shrinkage and creep of concrete: $\varepsilon'_{csd} = 150\times10^{-6}$ general cases and $\varepsilon'_{csd} = 100\times10^{-6}$ for high-strength concrete; σ_{se} = increased stress reinforcement by the state with a concrete stress of zero.

3.5. British Standard (BS 8110-1: 1997) and Singapore Standard (CP65-1999)

The widths of flexural cracks at a particular point on the surface of a member depend primarily on three factors [24-27]:

- The distance from point considered to reinforce bars perpendicular to the cracks;
- The neutral axis distance to the point considered;
- The average surface strain at the given point.

The following equation gives a relationship between the crack width and these three major variables that results in accurate acceptability under most common design conditions but the formula in the member dominantly exposed to axial tension should be applied with caution.

$$\text{Design surface crack width} = \frac{3a_{cr}\varepsilon_m}{1+2(\frac{a_{cr}-c_{min}}{h-x})} \tag{11}$$

Where c_{min} = minimum cover to the tension steel; h = member's total depth; x = neutral axis depth; ε_m = average strain at the point of consideration for cracking. a_{cr} = gap from the point to the next longitudinal bar surface.

The c_{min} and h values are derived from the member's section. The assessment is explained for the other variables.

- Evaluation of a_{cr}

$$a_{cr} = \sqrt{(\frac{s}{2})^2 + (d_c)^2} - \frac{d_b}{2} \tag{12}$$

Where d_b = longitudinal bar diameter; d_c = effective cover = $c_{min} + d_b/2$; s = longitudinal bars spacing center-to-centre.; d_b, d_c and s values are derived from the member section.

- Evaluation of ε_m

$$\varepsilon_m = \varepsilon_1 - \frac{b(h-x)(a-x)}{3E_sA_s(d-x)} \tag{13}$$

Where a = distance from the face of the compression to the level at which the width of the crack is determined; a = h, when measuring the width of the crack at a soffit; b = width of the rectangular zone; d = effective depth of the longitudinal reinforcement; A_s = reinforcement area; E_s = elasticity modulus for steel; ε_1 = at the chosen point, strain based on a cracked sectional analysis: $\varepsilon_1 = \varepsilon_s (a - x)/(d - x)$ where ε_s = longitudinal reinforcing strain.

4. Finite Element Analysis

The non-linear study of tension concrete contributions after the bending cracking of the reinforced concrete beams was carried out using a Finite Element Model (FEM). The goal was to compare the FEM findings with those obtained from the building code equations. The current research is carried out by a commercial FEM software, ABAQUS Program Version 6.14. Concrete was modeled using 3-dimensional, 8-node solid elements; C3D8, with three degrees of freedom for each node; translations u, v, and w in the three orthogonal directions; x, y and z, respectively. Reinforcement was modeled on a double nodded linear truss (T3D2) element. The embedded method used in assembling the model, as the reinforcement components are embedded into the host element (concrete components) and the interaction of the model between the concrete and reinforcement is constructed by this method. Concrete damaged plasticity model (CDP) approach was chosen to represent full inelastic compression and tension behaviors of concrete including damages properties.

4.1. Material Modeling

The material modeling was as follows:

- The complete stress-strain relationship of concrete under compression used in this research study is derived from Hsu and Hsu (1994) [28], which is described by Wahalathantri et al. (2011) [29]. Typical compressive stress-strain relationship with properties and terms of damage taking according to ABAQUS Manual [30] as shown in Figure 2 The user needs to insert stresses (σ_c), inelastic strains (ε_c^{in}) correspond to stress levels, damage characteristics (d_c) with inelastic strains in tabular form to describe the stress-strain relation of the concrete.
- To model the complete tensile behavior of reinforced concrete in ABAQUS, a stress-strain post failure relationship for concrete subjected to tension is used to take into consideration tension stiffening as shown in Figure 2 The user needs to enter a young module (E_o), stress (σ_t), cracking stress values (ε_t^{ck}) and the value of damage parameters (d_t) for the relevant concrete grade to develop this model.
- Steel reinforcement is known as a linearly elastic plastic material as shown in Figure 3 for finite element modeling. Elastic modules were assumed to be 200×10^3 MPa and 0.3, respectively, for the poison ratio for steel reinforcement. Steel plate used in ABAQUS has been assumed to be a linear elastic material with the same elastic modulus and Poisson's ratio of the steel reinforcement. A steel plate's thickness as 50 mm has been taken into account as a support and load plates.

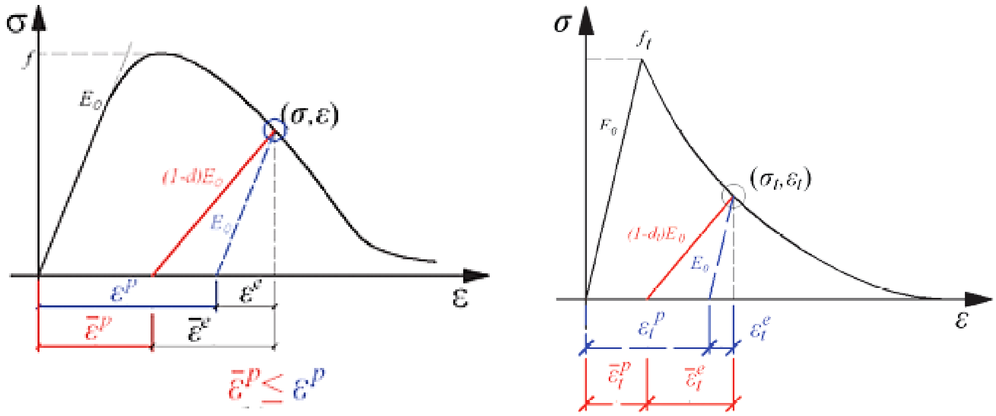


Figure 2. Stress-strain relationship of concrete under uniaxial compression and tension [30]

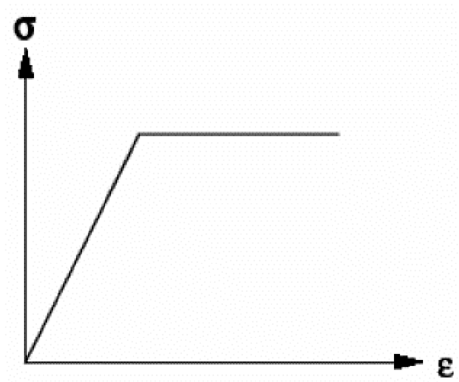


Figure 3. Idealizations of the steel stress-strain curve

4.2. Crack Width in ABAQUS

ABAQUS is used usually to analyze structures such as beams and show the distributions of stresses and strains. The crack can be graphically represented in a concrete damage plasticity model by adding an effective crack directions and the direction of the main plastic strain, as it is normal for a crack plane. Loading stage can contain the cracking pattern, but the crack width is not unfortunately determined. Since crack width is the key parameter in the control of cracking. Concrete properties are a weakness in tension so that any step of loading leads to stress in concrete tension or compression, for tension leads to cracking [31]. This strain in node for concrete element is same crack width generation from plastic strain are shown in Figure 4 and can estimation by Equation 14.

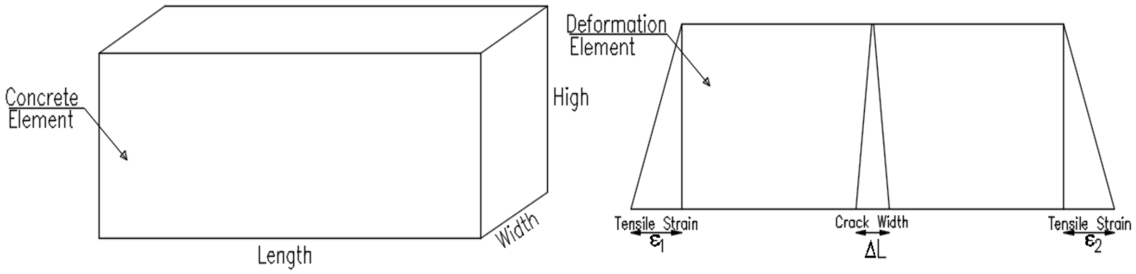


Figure 4. Method calculated of crack width [31]

$$w_c = (\varepsilon_1 + \varepsilon_2) \times l_e \tag{14}$$

Where, w_c = crack width in concrete element, (ε_1, ε_2) = tensile plastic strain in concrete element, l_e = element length (mesh size in ABAQUS).

4.3. Verification of ABAQUS Results

First of all, the modeling for FEM was checked against the results of one of Makhlouf and Malhas [4] beams, two beams tested by Albandar [32] and one of Gilbert and Nejadi [3] beams.

4.3.1. Makhlouf and Malhas [4]

Test results for three beams reinforced by high-strength deformed bars tested by Makhlouf and Malhas [4] have been recorded by Vincenzo and Giuseppe [33], and by Allam et al. (2012) [12]. The specimens consisted of three beams; each beam differed in amount of longitudinal reinforcement to measure the size of surface crack widths under working load when applying a 50 mm thick concrete cover to different steel stress levels. All beams were of rectangular cross section with the same dimension. All bottom longitudinal reinforcement was provided by M14 and M20 bars, while M10 bars were used for all top longitudinal reinforcement. Shear reinforcement, where provided, was in the form of closed stirrups constructed from D8 bars. Cross-section details and the beam profiles are shown in Figure 5. Table 1 provides additional relevant details and shows the steel stress–crack width relationship obtained experimentally together with that obtained from the FEM analysis for the beam G2. Results indicates that the finite element model matches well with the experimental results.

Table 1. Cross section detail, material characteristics and comparison of determined crack width numerical values with measured experimental values for Makhlouf and Malhas [4] test specimen (G2).

Beam number	d (mm)	f_y (MPa)	fc` (MPa)	f_s (MPa)	w_k (EXP.) (mm)	Mesh size (mm)	Plastic strain	w_k (NUM.) (mm)	w_k(EXP./NUM.) (%)
G2-1	336	430	40	154	0.0900	50	0.002636	0.1318	0.6831
G2-2	336	430	40	184	0.1500	50	0.003358	0.1679	0.8932
G2-3	336	430	40	230	0.2200	50	0.004392	0.2196	1.0019
G2-4	336	430	40	288	0.2700	50	0.005476	0.2738	0.9862

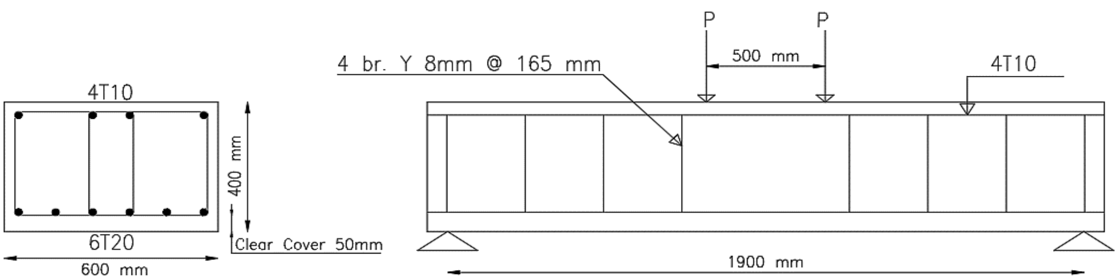


Figure 5. Cross sections and elevation details of beams tested by Makhlouf and Malhas [4, 12]

4.3.2. Albandar [32]

Tested finds of the 9 beams with high strength and deformed bars tested by Albandar (1973) [32] were stated by Albandar and Mills [1]. Experimental program include 3×3 simply supported full-scale beams in which the side covers were varied in the ratio 3 to 1 and the effective area while all other parameters (except bar diameter) were held constant. The basis of the programme was to investigate the effect of varying a single parameter within a series of three beams, each one cast and tested under uniform conditions. Cross-section details and the beam profiles are shown in Figure 6. Table 2 provides additional relevant details and shows the steel stress–crack width relationship obtained experimentally together with that obtained from the FEM analysis for the beam S1 and S4. Results indicates that the finite element model matches well with the experimental results.

Table 2. Cross section detail, material characteristics and comparison of determined crack width numerical values with measured experimental values for beams tested by Albandar [32]

Beam number	d (mm)	s (mm)	f_{cu} (MPa)	f_y (MPa)	f_s (MPa)	w_k (EXP.) (mm)	Mesh size (mm)	Plastic strain	w_k (NUM.) (mm)	w_k(EXP./NUM.) (%)
S1	379	178.00	47.1	410	288	0.3200	50	0.005592	0.2796	1.1443
S4	379	64.67	45.7	410	288	0.1700	50	0.003972	0.1986	0.8561

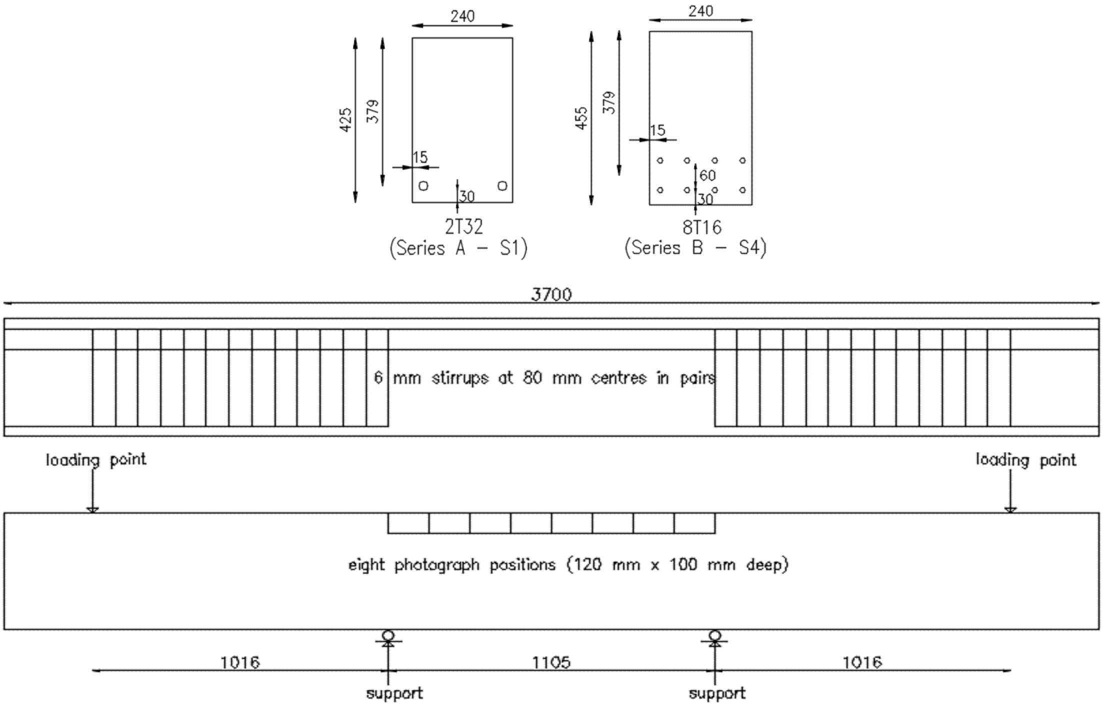


Figure 6. Cross sections and elevation details of beams tested by Albandar [32] (adapted from Albandar and Mills [1]) (all dimension in mm)

4.3.3. Gilbert and Nejadi [3]

In six beams with high strength deformed bars, Gilbert and Nejadi [3] document results of the tests. The specimens consisted of six beams; each beam differed in tensile steel area, bar diameter, bar spacing, tensile steel stress and concrete cover. After almost 400 days of load, cracks at the various steel stress level were reported over the longer term. Cross-section data and models of beams are shown in Figure 7. Table 5 provides additional relevant details and shows the steel stress–crack width relationship obtained experimentally together with that obtained from the FEM analysis for the beam B1-b. Results indicates that the finite element model matches well with the experimental results.

Table 3. Cross section detail, material characteristics and comparison of determined crack width numerical values with measured experimental values for beams tested by Gilbert and Nejadi [3]

Beam number	d (mm)	s (mm)	fc` (MPa)	f_s (MPa)	w_k (EXP.) (mm)	Mesh size (mm)	Plastic strain	w_k (NUM.) (mm)	w_k(EXP./NUM.) (%)
B1-b	300	150	24.8	154	0.13	50	0.002448	0.1224	1.0622

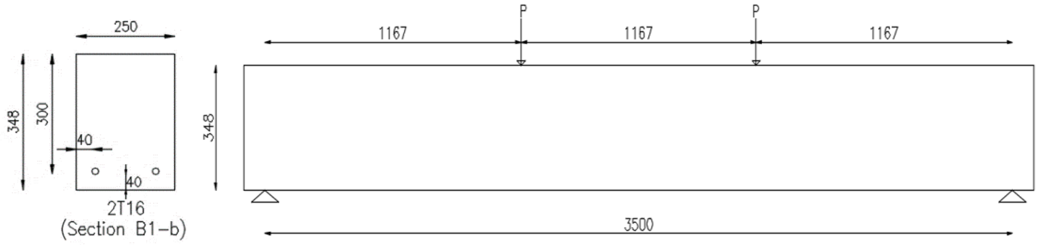


Figure 7. Cross sections and elevation details of beams tested by Gilbert and Nejadi [3] (all dimension in mm)

5. Application of Some Code Equations with Published Available Experimental Data

To get links to code equation precision for predicting crack width in reinforced concrete members, a comparison is carried out with the experimental results reported by Gilbert and Nejadi [3] and Makhlouf and Malhas [4].

5.1. Makhlouf and Malhas [4]

Cross-section details and the beam profiles are shown in Figure 5.

Table 4. Cross section details and material characteristics of the checked beams by Makhlouf and Malhas [4]

Beam number	b (mm)	h (mm)	d (mm)	Bottom steel	Top steel	Stirrups	f_y (MPa)	fc` (MPa)
G1	600	400	338	4Φ14+2Φ20	4Φ10	4 br. Φ8mm@165mm	430	40
G2	600	400	336	6Φ20	4Φ10	4 br. Φ8mm@165mm	430	40
G3	600	400	336	9Φ20	4Φ10	4 br. Φ8mm@165mm	430	40

Table 5. Application of some code equations with published available experimental data presented by Makhlouf and Malhas [4]

Beam number	w_k(EXP.) (mm)	w_k (ECP) (mm)	w_k (EC2) (mm)	w_k (ACI) (mm)	w_k (JSCE) (mm)	w_k (BS OR SS CP) (mm)
G1-1	0.08	–	0.1379	0.1817	0.0772	0.1062
G1-2	0.11	–	0.1657	0.2183	0.1182	0.1408
G1-3	0.19	0.1151	0.2058	0.2712	0.1775	0.1907
G1-4	0.31	0.2346	0.2573	0.3390	0.2534	0.2548
G2-1	0.09	0.1060	0.1426	0.2134	0.1624	0.1585
G2-2	0.15	0.1628	0.1704	0.2549	0.2076	0.1976
G2-3	0.22	0.2416	0.2300	0.3187	0.2770	0.2574
G2-4	0.27	0.3333	0.3195	0.3990	0.3644	0.3329
G3-1	0.14	0.1200	0.1278	0.1861	0.1769	0.1595
G3-2	0.18	0.1585	0.1692	0.2243	0.2193	0.1974
G3-3	0.26	0.2112	0.2294	0.2797	0.2810	0.2522
G3-4	0.34	0.2753	0.3056	0.3500	0.3590	0.3218

The following are shown in Table 5:

- Egyptian code [19] indicates an underestimated crack width values in G1 where the beam is reinforced with a limited reinforcement ratio (μ = 0.62 %). At steel stress f_s = 134 MPa and f_s = 161 MPa ($f_s = 0.31 f_y$ and $f_s = 0.37\ f_y$), The Egyptian code [19] indicates that the beam is considered to be uncracked. The crack value specified in the Egyptian code [19] well correlated the experimental values for beam with reinforcement ratio (μ = 0.95%) in G2 at all stages of reinforcement bar stress. For beam with higher reinforcement ratio (μ = 1.42%) in G3, the Egyptian code [19] is less than the crack width measured in experiments;
- The Eurocode 2 [20] equation gives crack width values greater than the experimental values, with the exception of the heavy reinforcement ratio (μ = 1.42%) values. Eurocode 2 [20] predictions for beams with reinforcement ratio (μ= 0.62% and μ = 0.95%) give overestimated values of the crack width because Eurocode 2 [20] restricts the level of the ($\varepsilon_{sm}-\varepsilon_{cm}$) not to be less than 0.6 f_s/E_s. As the reinforcement ratio increased in G3 the concrete contribution in tension zone increased. The value of ($\varepsilon_{sm}-\varepsilon_{cm}$) bigger than the value of 0.6 f_s/E_s and the value of crack width specified in the Eurocode 2 [20] well correlated the experimental values;
- Generally, the ACI 318-14 [21] (ACI 224R-01[22]) equation substantially overestimated crack width values except for high stage of steel stress f_s = 227 MPa and f_s = 284 MPa ($f_s = 0.53 f_y$ and $f_s = 0.66\ f_y$) and at high values of reinforcement ratio (μ = 1.42%);
- JSCE (2007) [23] give a crack width values well correlated the experimental values in G1 and G2, except at level of steel stress f_s = 250 MPa ($f_s = 0.58 f_y$) and low reinforcement ratio (μ = 0.62%). For G2 where the

reinforcement ratio (μ = 0.95%), JSCE (2007) [23] appear overestimated values for the crack width. This may be a result of use big bar diameter (Φ = 20mm) which is main factor in the equation of crack width. However, the bar diameter in G3 is the same but the value of bar spacing is smaller;

- The width of cracks values expected by BS 8110-1997 [24, 25] or SS CP 65-1999 [26, 27] equations was greater than test values for beams in G1 and G2, with the exception of elevated steel stress stages f_s = 250 MPa (f_s = 0.58f_y) and low reinforcement ratio (μ = 0.62%). For G3, the width of the crack values expected by BS 8110-1997 [24, 25] or SS CP 65-1999 [26, 27] equations well correlated the experimental values.

5.2. Gilbert and Nejadi [3]

Cross-section data and models of beams are shown in Figure 7.

Table 6. Cross-Section Details and material properties of beams tested by Gilbert and Nejadi [3]

Beam number	b (mm)	h (mm)	d (mm)	Bottom steel	c_b (mm)	c_s (mm)	s (mm)	fc` (MPa)
B1	250	348	300	2Φ16	40	40	150	24.8
B2	250	333	300	2Φ16	25	25	180	24.8
B3	250	333	300	3Φ16	25	25	90	24.8

Table 7. Application of some code equations with published available experimental data presented by Gilbert and Nejadi [3]

Beam number	w_{max} (EXP.) (mm)	w_{avge} (EXP.) (mm)	w_k (ECP) (mm)	w_k (EC2) (mm)	w_k (ACI) (mm)	w_k (JSCE) (mm)	w_k (BS OR SS CP) (mm)
B1-a	0.3800	0.2800	0.2659	0.2354	0.2521	0.2434	0.1942
B1-b	0.1800	0.1300	0.1345	0.1355	0.1718	0.1382	0.1116
B2-a	0.3600	0.2000	0.2135	0.2853	0.1846	0.2145	0.1762
B2-b	0.1800	0.1100	0.1144	0.1617	0.1256	0.1253	0.1022
B3-a	0.2800	0.1700	0.1745	0.1527	0.1541	0.1635	0.1266
B3-b	0.1300	0.0900	0.0889	0.0771	0.0926	0.0894	0.0665

The following are shown in Table 7:

- Generally, codes give a crack width values greatly underestimated the values of the maximum crack width. However the crack width values, as some code indicates, were well associated with average crack width test values;
- The Egyptian code [19] and JSCE (2007) [23] the crack width values corresponded well to the average crack width experimental values of all steel stress levels and differed the value of the concrete covering;
- The Eurocode 2 [20] equation give crack width values smaller than experimental values of average crack width, except in beam B2. Eurocode 2 [20] predictions for beam B2 give values of the crack width bigger than values predicted for beam B1. However, the concrete cover in beam B1 (c = 40mm) bigger than its value in beam B2 (c = 25mm). Eurocode 2 [20] limits the value of spacing between reinforcement bar (spacing ≤ 5(c +Φ/2)), if the spacing between reinforcement bar exceeds 5(c +Φ/2), maximum crack spacing equation change (s_{rmax} = 1.3(h−x)). Maximum crack spacing predicted from previous equation give a value bigger than the value predicted by usual equation for calculating crack spacing ($s_{rmax}= k_3.c + k_1.k_2.k_4.(Ø/\rho_{p,eff})$) so crack width values of beam B2 bigger than crack width value of beam B1.
- Generally, the ACI 318-14 [21] (ACI 224R-01[22]) equation give crack width values less than average crack width experimental values exception of the low degree of steel stress f_s = 154 MPa and f_s = 128 MPa (M_s/M_u = 30%).
- Generally, the BS 8110-1997 [24, 25] or SS CP 65-1999 [26, 27] equation underestimated the values of the crack width.
- The findings for the B1-a and B2-a beams as well as the B1-b and B2-b beams indicate the effect of the concrete cover on the width of the crack. With the increasing concrete cover in beam B1 the width of the crack increased. The maximum crack width can be affected by 5.56% according to experimental findings, as the concrete cover rises between 25 to 40 mm on high steel stress levels f_s= 226 MPa (Ms/Mu = 45%) and 0.00% at low level of steel stress f_s= 154 MPa (Ms/Mu = 45%). This is compared to 40.00%, 18.18% increase observed from experimental average crack width, 24.54%, 17.57% increase calculated by ECP 203-2018 [19], 17.49%, 16.20% decrease calculated by EC2 (2004) [20], 36.57%, 36.78% increase calculated by ACI 318-14 [21, 22], 13.47%, 10.30% increase calculated by JSCE (2007) [23] and 10.22%, 9.20% increase calculated by BS 8110-1997 [24, 25] or SS CP 65-1999 [26, 27].

6. Analytical and Numerical Comparison between Code Equations for Predicting the Crack Width

According to the previous equations for calculating the crack width, it is possible to conclude the factors affecting on the crack width are:

(1) Steel stress (f_s);

(2) Concrete cover for longitudinal steel at tension zone (c);

(3) Spacing between bars in tension zone (s);

(4) Diameter of bars in tension zone (Ø);

(5) Area of longitudinal tension steel (A_s);

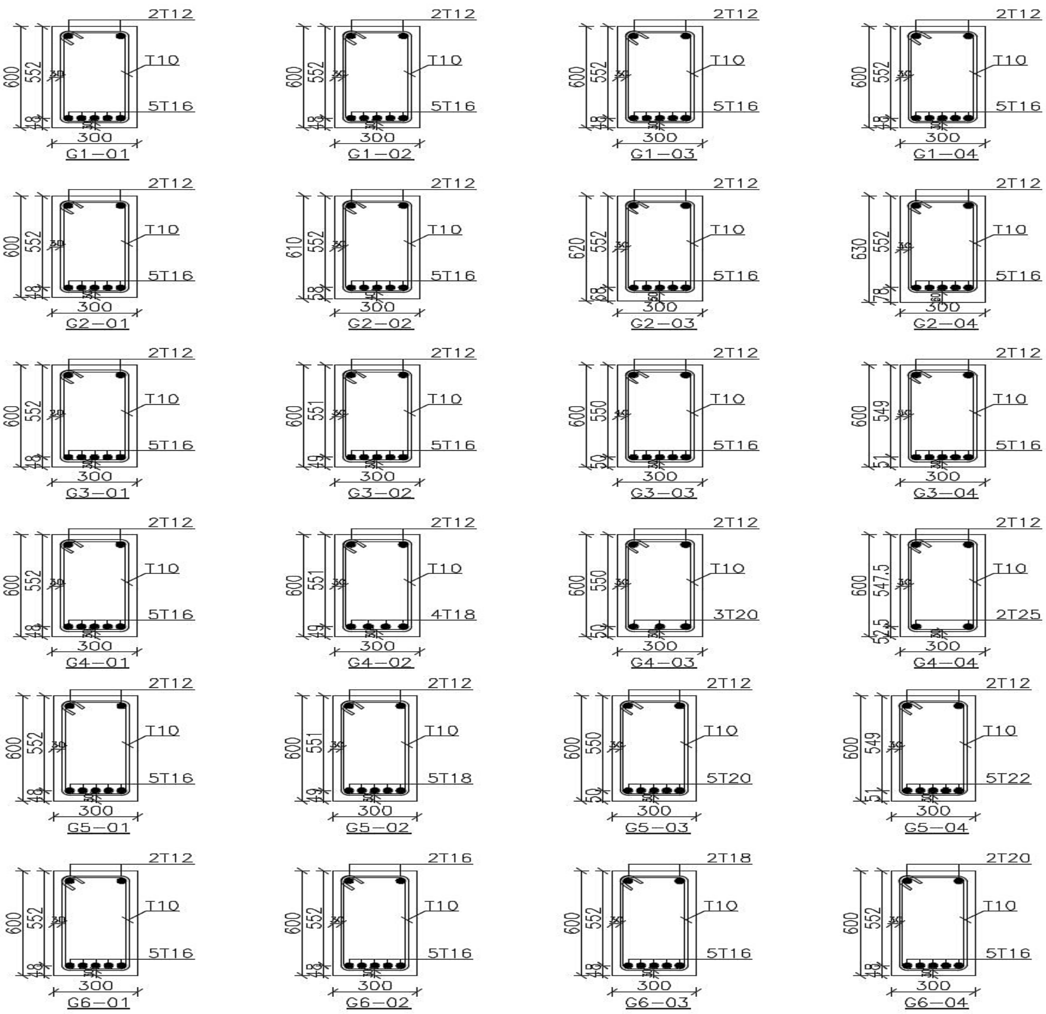
(6) Area of longitudinal compression steel ($A_s^{\backslash}$);

(7) Surface geometry of reinforcement bar (smooth, deformed) which effect on the bond between steel and concrete;

(8) Compressive strength for concrete;

(9) Number of tension reinforcement layers.

In this section construction codes and ABAQUS are applied to reinforced concrete modules with different parameter values. Calculations were conducted on nine groups of reinforced concrete beams. Group 1, included four specimens that had four steel stress, 160, 180, 200, and 220 Mpa. Group 2, included four specimens that had four concrete cover, 30, 40, 50 and 60 mm. Group 3, included four specimens that had four reinforcement bar spacing, 41, 46, 51 and 56 mm. Group 4, included four specimens that had four reinforcement bar diameter, 16, 18, 20 and 25 mm. Group 5, included four specimens that had four area of longitudinal tension steel, 1005.31, 1272.35, 1570.80 and 1900.66 mm^2. Group 6, included four specimens that had four area of longitudinal compression steel, 226.19, 402.12, 508.94 and 628.32 mm^2. Group 7, included two specimens that had two type of reinforcement bar, smooth and deformed. Group 8, included four specimens that had four compressive strength, 30, 40, 50 and 70 Mpa. Group 9, included three specimens that had three number of reinforcement layer, 1, 2 and 3 layer. Figure 8 show the concrete dimensions and reinforcement arrangement of the studied specimens in all groups and Table 8 provides Cross-section details and material properties of these beams.



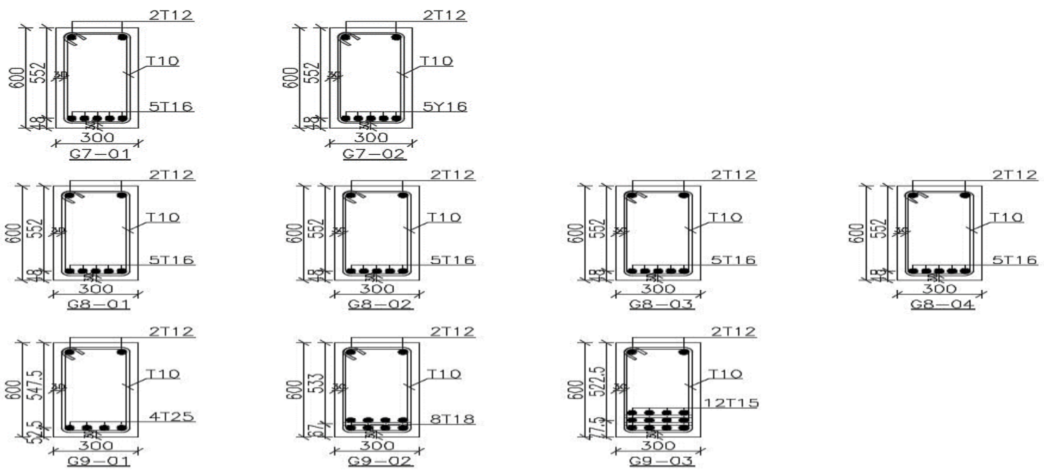


Figure 8. Concrete dimensions and reinforcement arrangement of the studied specimens in all groups (all dimension in mm)

Table 8. Cross-section details and material properties of beams in all groups

Beam number	b (mm)	h (mm)	d (mm)	c (mm)	s (mm)	Φ (mm)	A_s (mm^2)	f_s (MPa)	f_{cu} (MPa)
G1-01	300	600	552	30	51	16	1005.31	160	30
G1-02	300	600	552	30	51	16	1005.31	180	30
G1-03	300	600	552	30	51	16	1005.31	200	30
G1-04	300	600	552	30	51	16	1005.31	220	30
G2-01	300	600	552	30	51	16	1005.31	220	30
G2-02	300	610	552	40	51	16	1005.31	220	30
G2-03	300	620	552	50	51	16	1005.31	220	30
G2-04	300	630	552	60	51	16	1005.31	220	30
G3-01	300	600	552	30	56	16	1005.31	220	30
G3-02	300	600	552	30	51	16	1005.31	220	30
G3-03	300	600	552	30	46	16	1005.31	220	30
G3-04	300	600	552	30	41	16	1005.31	220	30
G4-01	300	600	552.0	30	51.00	16	1005.31	220	30
G4-02	300	600	551.0	30	76.33	18	1017.88	220	30
G4-03	300	600	550.0	30	100.00	20	942.48	220	30
G4-04	300	600	547.5	30	195.00	25	981.75	220	30
G5-01	300	600	552	30	51.0	16	1005.31	220	30
G5-02	300	600	551	30	50.5	18	1272.35	175	30
G5-03	300	600	550	30	50.0	20	1570.80	145	30
G5-04	300	600	549	30	49.5	22	1900.66	120	30
G6-01	300	600	552	30	51	16	1005.31	220	30
G6-02	300	600	552	30	51	16	1005.31	220	30
G6-03	300	600	552	30	51	16	1005.31	220	30
G6-04	300	600	552	30	51	16	1005.31	220	30
G7-01	300	600	552	30	51.00	16	1005.31	140	30
G7-02	300	600	552	30	51.00	16	1005.31	140	30
G8-01	300	600	552	30	51.0	16	1005.31	220	30
G8-02	300	600	552	30	51.0	16	1005.31	220	35
G8-03	300	600	552	30	51.0	16	1005.31	220	40
G8-04	300	600	552	30	51.0	16	1005.31	220	45
G9-01	300	600	547.5	30	65.00	25	1963.50	220	30
G9-02	300	600	533.0	30	67.33	18	2035.75	220	30
G9-03	300	600	522.5	30	68.33	15	2120.58	220	30

c = Clear concrete cover; s = Reinforcement bar spacing; Φ = Reinforcement bar diameter.

Table 9 shows the values of the crack width for the studied beams in all groups. Table 10 shows statistical analysis for calculated crack width values according to five codes versus numerical crack width values.

Table 9. Values of the crack width for the studied beams in all groups

Beam number	w_k (ECP) (mm)	w_k (EC2) (mm)	w_k (ACI) (mm)	w_k (JSCE) (mm)	w_k (BS OR SS CP) (mm)	w_k (NUM.) (mm)
G1-01	0.1147	0.1375	0.1286	0.1288	0.0864	0.1118
G1-02	0.1364	0.1609	0.1446	0.1499	0.1016	0.1293
G1-03	0.1574	0.1842	0.1607	0.1711	0.1167	0.1469
G1-04	0.1779	0.2075	0.1768	0.1922	0.1318	0.1649
G2-01	0.1779	0.2075	0.1768	0.1922	0.1318	0.1649
G2-02	0.1960	0.2451	0.2050	0.2309	0.1607	0.1734
G2-03	0.2136	0.2775	0.2329	0.2685	0.1911	0.1925
G2-04	0.2308	0.3059	0.2606	0.3050	0.2229	0.2193
G3-01	0.1779	0.2075	0.1768	0.1958	0.1346	0.1772
G3-02	0.1779	0.2075	0.1768	0.1922	0.1318	0.1648
G3-03	0.1779	0.2075	0.1768	0.1885	0.1292	0.1529
G3-04	0.1779	0.2075	0.1768	0.1849	0.1268	0.1450
G4-01	0.1779	0.2075	0.1768	0.1922	0.1318	0.1648
G4-02	0.1912	0.2188	0.1936	0.2034	0.1419	0.1699
G4-03	0.2110	0.2358	0.2162	0.2193	0.1602	0.1808
G4-04	0.2471	0.2659	0.2574	0.2872	0.2298	0.1808
G5-01	0.1779	0.2075	0.1768	0.1922	0.1318	0.1648
G5-02	0.1352	0.1576	0.1443	0.1585	0.1067	0.1348
G5-03	0.1062	0.1234	0.1207	0.1342	0.0885	0.1095
G5-04	0.0856	0.0989	0.1030	0.1161	0.0749	0.0945
G6-01	0.1779	0.2075	0.1768	0.1922	0.1318	0.1648
G6-02	0.1778	0.2076	0.1768	0.1921	0.1315	0.1637
G6-03	0.1778	0.2076	0.1764	0.1921	0.1313	0.1535
G6-04	0.1778	0.2076	0.1764	0.1921	0.1312	0.1492
G7-01	0.0920	0.1142	0.1125	0.1077	0.0713	0.0949
G7-02	0.1616	0.1618	0.1125	0.1400	0.0713	0.0949
G8-01	0.1779	0.2075	0.1768	0.1922	0.1318	0.1656
G8-02	0.1743	0.2025	0.1766	0.1798	0.1316	0.1639
G8-03	0.1707	0.1977	0.1764	0.1687	0.1314	0.1621
G8-04	0.1671	0.1932	0.1764	0.1587	0.1312	0.1605
G9-01	0.1812	0.2141	0.2076	0.2301	0.1611	0.1450
G9-02	0.1710	0.1928	0.1729	0.2167	0.1716	0.1512
G9-03	0.1652	0.1812	0.1563	0.2090	0.1793	0.1588

Table 10. Statistical analysis for calculated crack width values according to five codes versus numerical crack width values

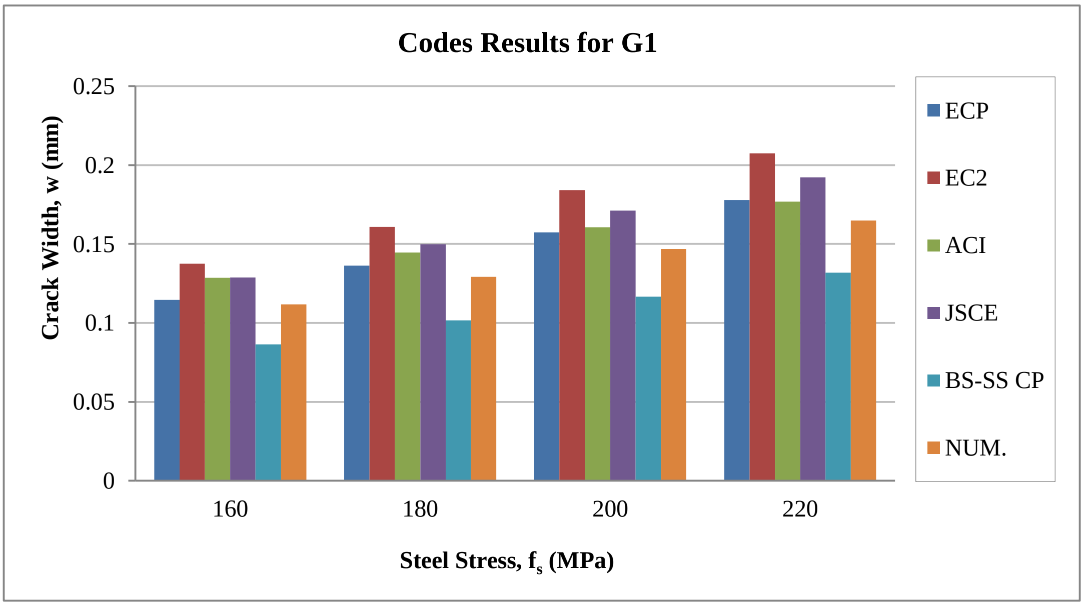
Codes	Min $wk_{(CAL.)/(NUM.)}$	Max $wk_{(CAL.)/(NUM.)}$	Avg. $wk_{(CAL.)/(NUM.)}$	Stdev.
ECP	0.9058	1.7028	1.1095	0.1357
EC2	1.0466	1.7050	1.2915	0.1234
ACI	0.9843	1.4317	1.1348	0.0937
JSCE	0.9888	1.5885	1.2357	0.1372
BS OR SS CP	0.7513	1.3667	0.9188	0.1654

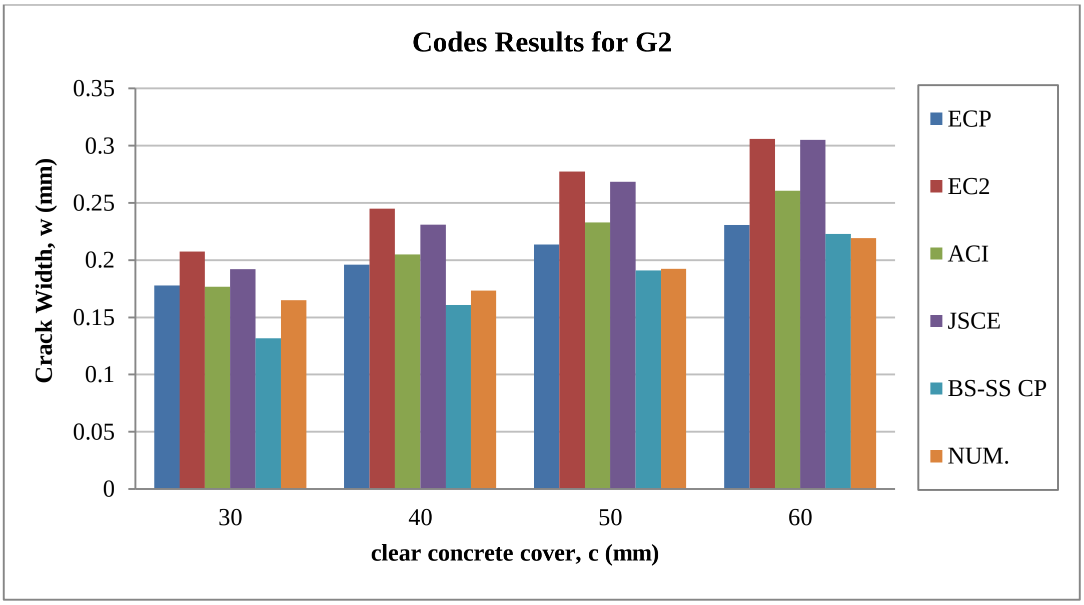
Figures 9 to 11 show comparison between codes and numerical results for beams studied at the bottom tension fibers in all groups. Table 11 provides a summary for the degree of influence of each variable on the width of the crack for nine reinforced concrete groups according to ABAQUS and all codes involved in this research.

From the analytical and numerical results, the following comparative points are discussed:

- The results of Group 1 clearly shows that the width of the crack increased with increasing the steel stress. The results demonstrates that the width of the crack can be varied by 12-55% as the steel stress vary, respectively, from 160 MPa to 180, 200 and 220 MPa for the same section.

- The results of Group 2 clearly shows that the width of the crack increased with increasing the concrete cover. The results demonstrates that the width of the crack can be varied by 5-70% as the concrete cover vary, respectively, from 30 mm to 40, 50 and 60 mm for the same bar spacing.
- The results of Group 3 clearly shows that the width of the crack increased with increasing the reinforcement bar spacing in tension zone. The results demonstrates that the width of the crack can be varied by 2-22% as the reinforcement bar spacing vary, respectively, from 41 mm to 46, 51 and 56 mm for the same concrete cover.
- The results of Group 4 clearly shows that the width of the crack increased with increasing the reinforcement bar diameter in tension zone. The results demonstrates that the width of the crack can be varied by 3-75% as the reinforcement bar diameter vary, respectively, from 16 mm to 18, 20 and 25 mm for the same steel stress.
- The results of Group 5 clearly shows that the width of the crack decreased with increasing the area of longitudinal tension reinforcement. The results demonstrates that the width of the crack can be varied by 17-52% as the area of longitudinal tension reinforcement vary, respectively, from 1005.31 mm^2 to 1272.35, 1570.8 and 1900.66 mm^2 for the same applied load (constant bending moment).
- The results of Group 6 clearly shows that the width of the crack decreased with increasing the longitudinal compression reinforcement area. The results demonstrates that the width of the crack can be varied by 0 - 9% as the area of longitudinal tension reinforcement vary, respectively, from 226.19 mm^2 to 402.12, 508.94 and 628.32 mm^2 for the same applied load (constant bending moment).
- The results of Group 7 clearly shows that the width of the crack increased with the changing reinforcement surface type from deformed to smooth. The results demonstrates that the width of the crack can be varied by 30-75% as the reinforcement surface type vary, respectively, from deformed to smooth for the same steel stress.
- The results of Group 8 clearly shows that the width of the crack decreased with increasing the concrete compressive strength. The results demonstrates that the width of the crack can be varied by 0.5-18% as the concrete compressive strength vary, respectively, from 30 MPa to 35, 40 and 45 MPa for the same section.
- The results of Group 9 according to ECP 203-2018 [19], EC2 (2004) [20], ACI 318-14 [21] (ACI 224R-01 [22]) and JSCE (2007) [23] clearly shows that the width of the crack decreased with increasing the number of tensile reinforcement layers increases. The results demonstrates that the width of the crack can be varied by 5-25% as the number of tensile reinforcement layers vary, respectively, from 1 layer to 2 and 3 layer for the same reinforcement ratio. This is compared to 4-12% increase calculated by BS 8110-1997 [24, 25] or SS CP 65-1999 [26, 27] and NUMERICAL.
- ACI 318-14 [21] (ACI 224R-01 [22]) code give the lowest value of the standard deviation for statistical analysis of calculated crack width values versus numerical crack width values.
- JSCE (2007) [23], ECP 203-18 [19] and EC2 (2004) [20] give the second lowest value of the standard deviation for statistical analysis of calculated crack width values versus numerical crack width values.
- BS 8110-1997 [24, 25] or SS CP 65-1999 [26, 27] give the highest value of the standard deviation for statistical analysis of calculated crack width values versus numerical crack width values.





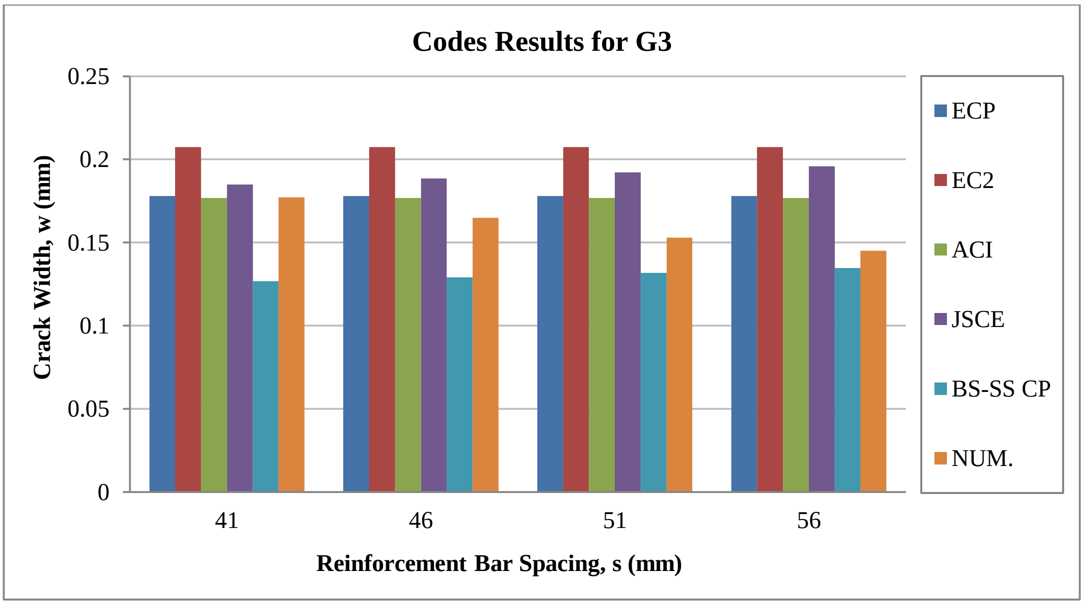
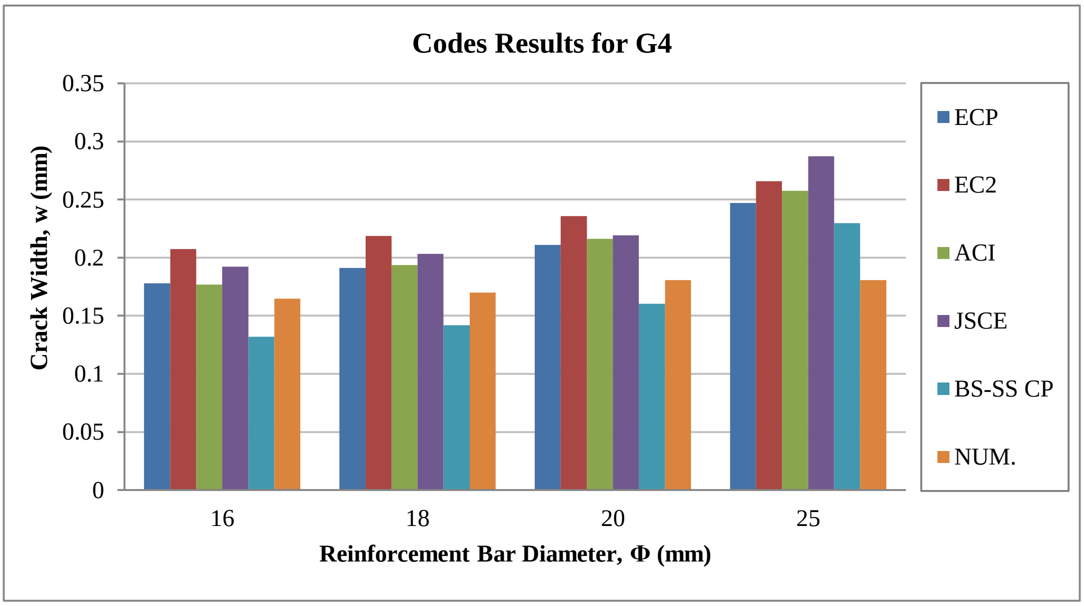


Figure 9. Comparison numerical results with code results for beams studied in G1, G2 and G3



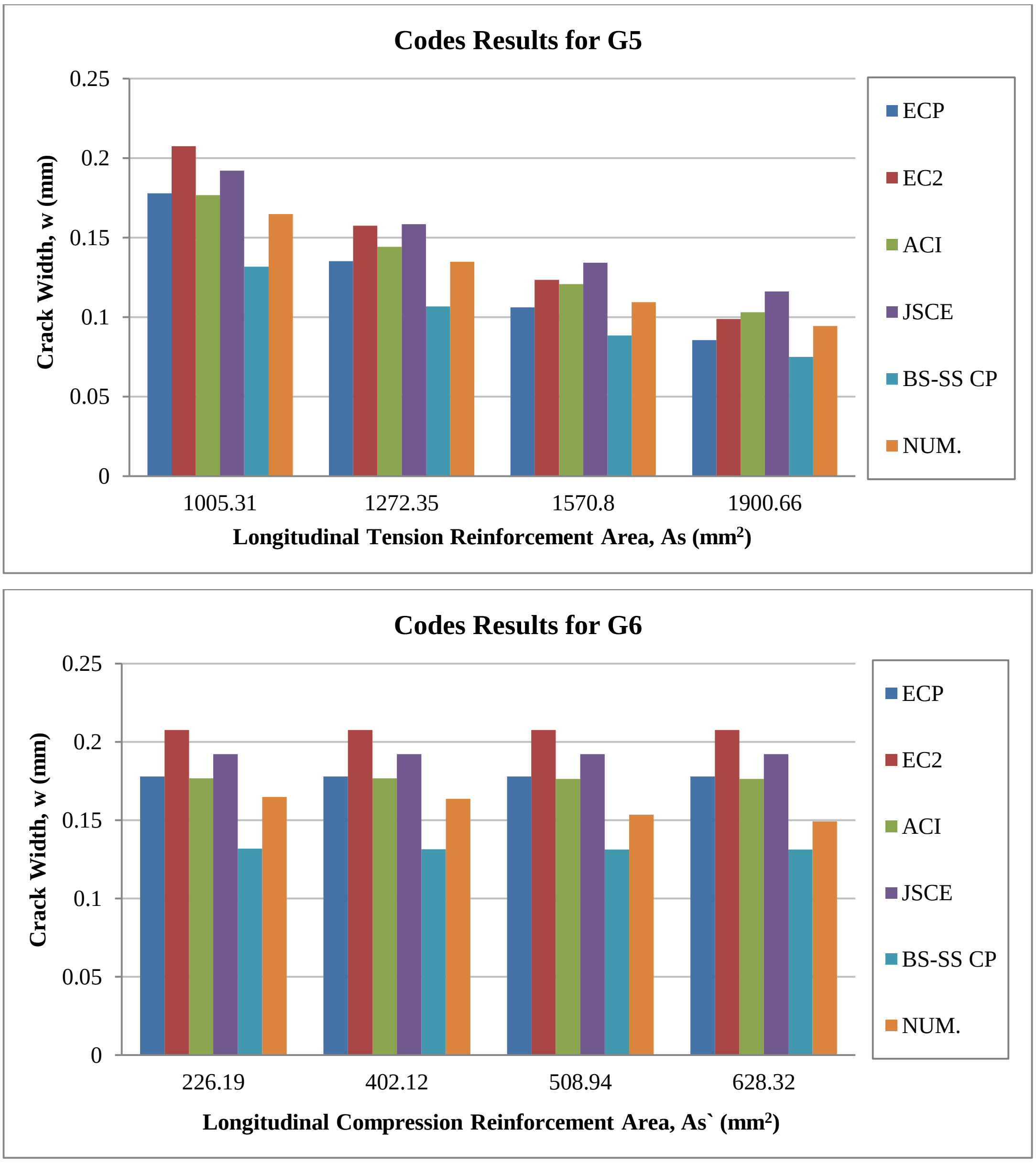
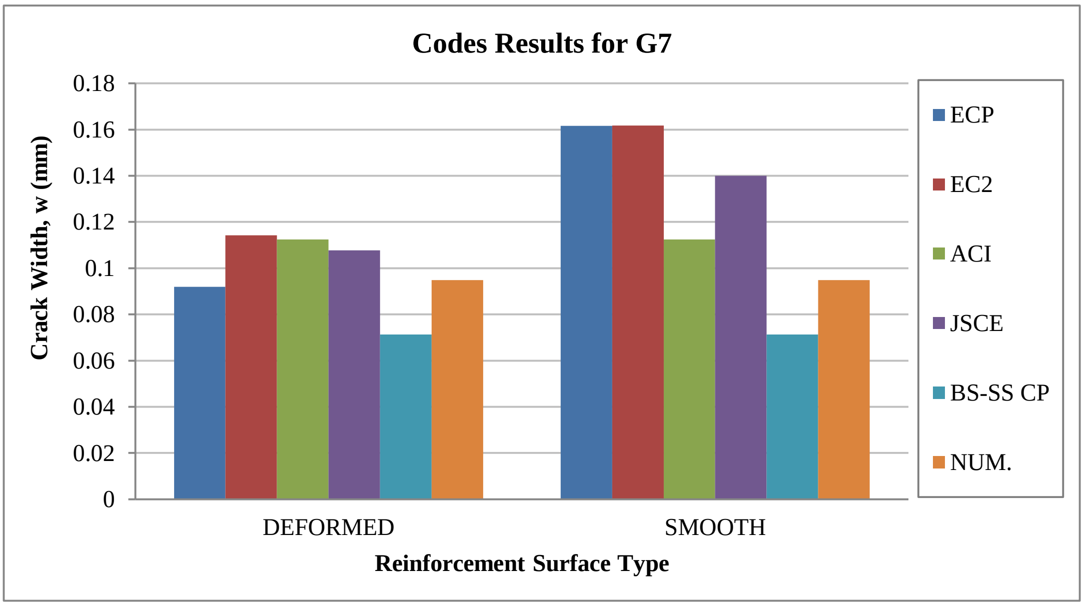
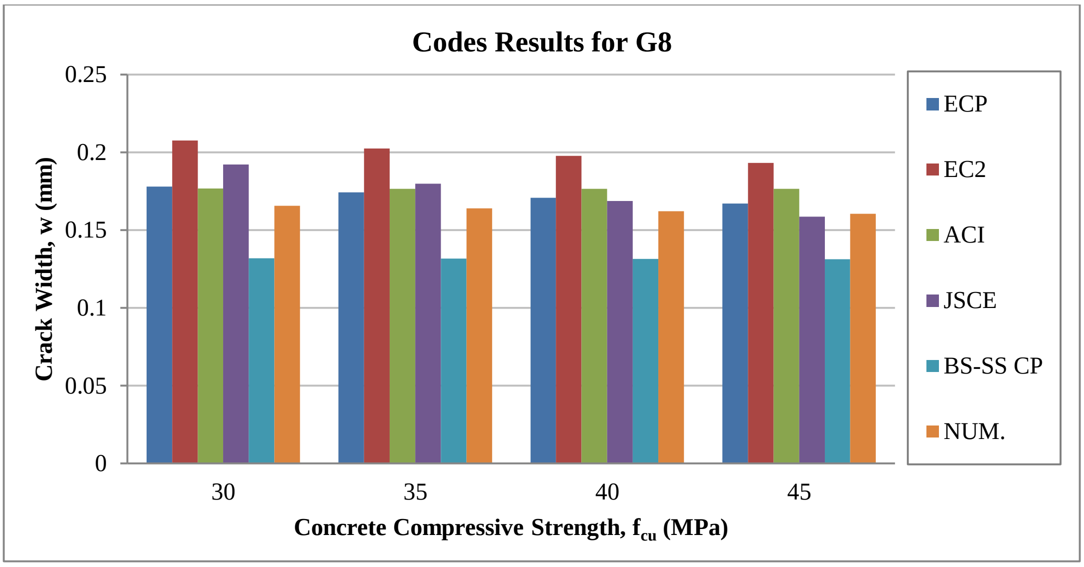


Figure 10. Comparison numerical results with code results for beams studied in G4, G5 and G6





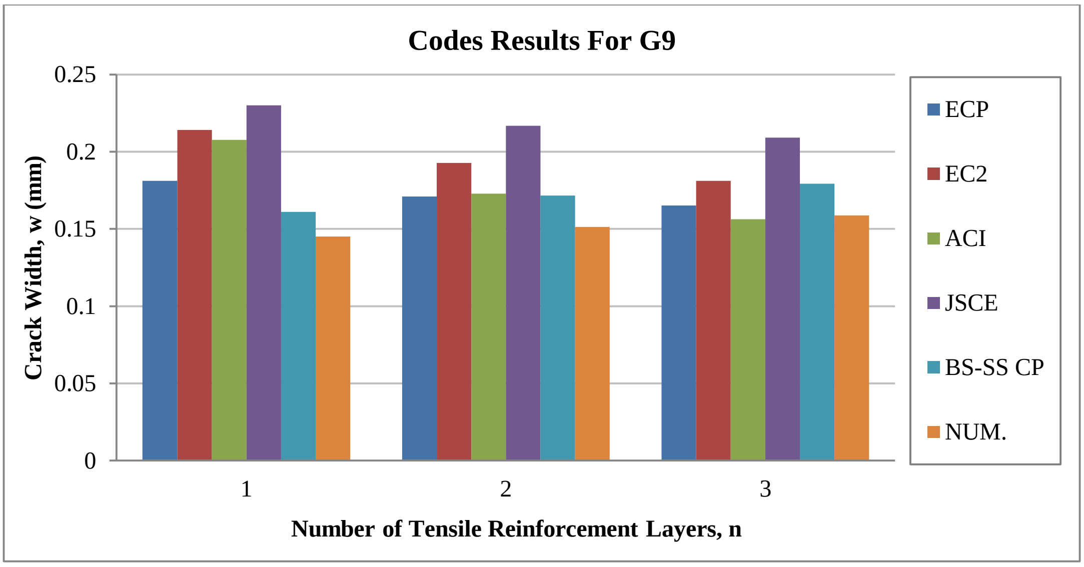


Figure 11. Comparison numerical results with code results for beams studied in G7, G8 and G9

Table 11. The degree of influence of each variable on the width of the crack for nine reinforced concrete groups according to ABAQUS and all codes involved in this research

Group	Parameter	Parameter variation		References	Influence percentage			
					Increment		Decrement	
		From	To		From	To	From	To
G1	f_s	180 MPa	220 MPa	All codes and ABAQUS	12%	55%	-----------------	
G2	C_c	40 mm	60 mm	All codes and ABAQUS	5%	70%	-----------------	
G3	S	41 mm	56 mm	JSCE, BS and ABAQUS	2%	22%	-----------------	
				ECP, EC2 and ACI	Didn't considered in codes equations		Didn't considered in codes equations	
G4	Ø	16 mm	25 mm	All codes and ABAQUS	3%	75%	-----------------	
G5	A_s	1005 mm^2	1900 mm^2	All codes and ABAQUS	-----------------		17%	52%
G6	A_s`	226 mm^2	628 mm^2	All codes and ABAQUS	-----------------		0%	9%
G7	Surface type	Def.	Smo.	ECP, EC2 and JSCE	30%	75%	-----------------	
				ACI, BS and ABAQUS	Didn't considered in ABAQUS & codes equations		Didn't considered in ABAQUS & codes equations	
G8	f_{cu}	30 MPa	45 MPa	All codes and ABAQUS	-----------------		0.5%	18%
G9	Num. of R.F.T layers	1 layer	3 layers	ECP, EC2, ACI and JSCE	-----------------		5%	25%
				BS and ABAQUS	4%	12%	-----------------	

7. Conclusions and Recommendations

The following conclusions can be arranged from the comparative analysis of multiple building codes approach, analytical and numerical research provided by this study through the analysis of the significant factors that can influence on the width of the crack, such as concrete cover, steel stress, reinforcement ratio, bar surface and reinforcement arrangement:

- The magnitude of the tensile stress in the reinforcing steel is the major factor influencing bending crack width under any given load. Controlling steel strain is the most efficient way of reducing crack widths in structures. The concrete covering over the reinforcing steel, bar size, surface geometry of reinforcement bar and steel distribution in the tension zone are also important factors which affect crack widths.
- ECP 203-2018, EC2 (2004) and ACI 318-14 (ACI 224R-01) doesn't take the effect of reinforcement bar spacing (side concrete cover) on the crack width into consideration so that the values of crack widths expected for beams models in Group 3 is constant according to the results of these codes. This is compared to 7, 14 and 18% decreased with decreasing the reinforcement bar spacing in tension zone in ABAQUS.
- ACI 318-14 (ACI 224R-01), BS 8110-1997 or SS CP 65-1999 and ABAQUS doesn't take the effect of reinforcement bar surface on the crack width into consideration so that the values of crack widths expected for beams models in Group 7 is constant according to the results of these references.
- ACI 318-14 (ACI 224R-01), JSCE (2007), BS 8110-1997 or SS CP 65-1999 and ABAQUS doesn't take the effect of loading duration on the crack width into consideration.
- The inadequate consistency of the references in the region concerning the position of the crack width calculation is at the bottom of the beam or at the reinforcement level except ACI 318-14 (ACI 224R-01), BS 8110-1997 or SS CP 65-1999 and ABAQUS.
- The inadequate consistency of the references in the region concerning the values obtained for the crack width is that maximum crack width or average crack width of the cracks generated in the constant moment region except ACI 318-14 (ACI 224R-01) and ABAQUS.
- Any statistical analyses of the code results with experimental or numerical results give deceptive indications without avoiding the pervious defects of each code. Specifically, the crack width values calculated according to each code, is it an average value or a maximum value.
- JSCE (2007) equation is the most suitable for calculating value of crack width because it take into consideration the most important factor which effected on the crack width.
- The numerical outputs in terms of load–crack width are in good agreement with the experimental results found in literature review so ABAQUS is a good software in our numerical study on crack width and the parameter that affecting on it.

8. Declarations

8.1. Author Contributions

Conceptualization, H.M. and K.F.; methodology, H.M. and K.F.; software, H.M., K.F. and A.A.; validation, H.M. and K.F.; formal analysis, H.M. and K.F.; investigation, H.M., K.F. and A.A.; resources, H.M., K.F. and A.A.; data curation, H.M., K.F. and A.A.; writing—original draft preparation, H.M., K.F. and A.A.; writing—review and editing, H.M., K.F. and A.A.; visualization, H.M. and K.F.; supervision, H.M. and K.F.; project administration, H.M. and K.F. All authors have read and agreed to the published version of the manuscript.

8.2. Data Availability Statement

The data presented in this study are available in article.

8.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

8.4. Conflicts of Interest

The authors declare no conflict of interest.

9. References

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