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A DISCRETE-TIME SLIDING MODE CONTROLLER FOR THE FERMENTATION PROCESS

Summary

This paper proposes a discrete-time sliding mode controller for the fermentation process, which is adequately approximated by the first-order plus dead time model. The contribution of the paper is an investigation into the suitable approximation of the time delay for discrete-time sliding mode controller. The dead time is considered in two ways by the first-order Taylor series approximation and the first-order Pade approximation. In the first case, the discrete-time sliding mode controller is based on the integral compensation of an output error, and in the second case, the stable system centre method is used. Numerical simulation examples of the yeast fermentation process are given to show the effectiveness of these two methods.

Key words: *discrete-time controller, first-order Pade approximation, first-order Taylor series approximation, fermentation process, sliding mode control*

1. Introduction

The fermentation process (FP) has a wide range of applications, from the production and storage of food [1] to the production of pharmaceutical products and wastewater treatment. The FP control design is challenging. The main obstacle to precise control is the presence of microorganisms [2], as well as the dynamics of FP, which is often incomprehensible, strongly non-linear, and non-stationary [1]. The model parameters vary over a prolonged period due to metabolic variations and physiological modifications [3]. On the other hand, problems arise due to the inefficiencies of cheap and unreliable sensors used for on-line measurement of model parameters [4].

The complexity and indeterminacy of FP usually do not allow conventional controllers to provide the desired performance control systems and to meet technological conditions [5]. These problems were solved by using modern control methods, such as open-loop control [6, 7], adaptive control [8-10], model predictive control [11, 12], fuzzy control [13], artificial neural networks [14, 15], probing control [16], statistical process control [17], and others. However, control algorithms most commonly proposed are not very robust or very complex and difficult to implement in practice [18].

On the other hand, in recent years, a lot of attention has been paid to sliding mode control (SMC) design [1, 3, 5, 19-30]. The main advantage of sliding mode is its invariance to parameter uncertainty and external disturbances [5, 21, 23, 26, 27, 29, 30]. The smoothness of the input variable is necessary for FP. Chattering phenomena in sliding mode are undesirable for the precise control of FP [5, 30]. However, it should be noted that some attempts have been made to solve the problems of the SMC design of FP [23-26]. Various techniques are proposed to reduce chattering phenomena, such as the introduction of a boundary layer around the sliding surface [23], filtering the control signal [24, 25], and realizing the sliding mode to an auxiliary input variable [26].

All previous papers consider SMC in the continuous domain [1, 3, 5, 19-30]. This paper proposes SMC in a discrete-time domain. The development of this controller significantly simplifies the application of the sliding control theory to chemical processes. As a result, the robustness of the system can be guaranteed from the beginning of the process, and the reaching phase is eliminated. The obtained SMC has many advantages, such as fast response, invariance to plant parameter deviations, and ability to compensate for external disturbances.

2. Model of the fermentation process

The first order plus dead time (FOPDT) model is a simple approximation of the dynamic response of a process for the purpose of control design [31]. Industry reports that the identification of process parameters is still responsible for high costs [32]. Despite significant efforts to improve identification methods and to capture the dominant process parameters in mathematical modelling, the process dynamics, which is not yet modelled, causes errors in process parameters [32]. Therefore, approximations such as FOPDT are useful to allow control design methods for non-control expert operators, such as those widely presented in [33].

An efficient method for the modelling of FP is the use of empirical models, which use low order linear models with dead time [28]. In most cases, FOPDT models are adequate for the analysis and design of chemical process control [21, 27, 28]. They can be represented by the following continuous domain transfer function [21, 27-30].

$$G(s)=\frac{x(s)}{u(s)}=\frac{K e^{-\theta s}}{\tau s+1} \tag{1}$$

where

$x(s)$ – Laplace transform of system state,

$u(s)$ – Laplace transform of control,

K – process gain,

$\theta > 0$ – process dead time,

$\tau > 0$ – process time constant.

By approximating the non-linear dynamics with FOPDT models, the modelling error is introduced, which contributes to the degradation of the controller performance [34]. Conventional controllers (PID, Lead-Lag or Smith Predictors) are sometimes not sufficient to compensate for these effects [29]. SMC can be designed to control non-linear systems assuming that the robustness of the controller will compensate for the modelling errors arising from the linearization of the non-linear process model [27]. The inevitable problem is the choice of an adequate SMC method that maximally compensates for system modelling errors.

Control systems with dead time are difficult to analyse and simulate. The main reason is that the closed-loop control system with a dead time system is an infinite dimension, i.e. the closed-loop has an infinite number of poles [35]. It is also difficult to determine all the poles of the system. Some control analysis methods are designed for process models that do not contain dead time, i.e., the transfer function models must be the ratios of polynomials in the numerator and the denominator. For this requirement to be met, the dead time in a transfer function model must be replaced by an approximation.

The dead time term ($e^{-\theta s}$) can be approximated in two different ways. A first-order Taylor series approximation transforms the dead time term in [21, 28, 30]:

$$e^{-\theta s} \equiv \frac{1}{\theta s+1} \tag{2}$$

The dead time term can also be written as a first-order Pade approximation [28]:

$$e^{-\theta s} \equiv \frac{1-0.5\theta s}{1+0.5\theta s} \tag{3}$$

A comparison among the dead time term, the first-order Taylor series approximation, and the first-order Pade approximation shows that the first-order Pade approximation works very well between 0 and 1 but beyond that, the approximation breaks down. On the other hand, the Taylor series approximation improves as θ increases. In [36], it is shown that the first-order Taylor series approximation or the first-order Pade approximation can be considered as good approximations for the dead time term.

The expression in the denominator of the relations (2) and (3) introduces a negative pole in the left half-plane and thus has effects on the characteristic polynomial of a system. The numerator of the relation (3) introduces a positive zero in the right half-plane, which must be taken into account in the discrete-time sliding mode controller design phase. The Pade approximation often gives a better approximation of the function than the Taylor series approximation even in a situation where the Taylor series does not converge [37].

3. Design of discrete-time sliding mode controller

In the design of the discrete-time sliding mode controller, two approximations are used: the first-order Taylor series and the first order Pade.

3.1 Discrete-time sliding mode controller based on the first-order Taylor series approximation

In this section, a discrete-time sliding mode (DTSM) controller is developed based on the first-order Taylor series approximation. Substituting the relation (2) into the relation (1) yields:

$$G(s) = \frac{K}{(\tau s+1)(\theta s+1)} = \frac{K}{\tau\theta s^2+(\tau+\theta)s+1} = \frac{b}{s^2+a_2 s+a_1} \tag{4}$$

where

$$a_1 = \frac{1}{\tau\theta} \quad a_2 = \frac{\tau+\theta}{\tau\theta} \quad b = \frac{K}{\tau\theta}$$

For the system (4), the traditional sliding mode is not used, because it has two motion phases: the reaching phase, in which the system is sensitive to all disturbances, and the sliding mode phase, in which the control system is robust, and its motion dynamics is described by a reduced order differential equation related to the controlled process. SMC based on the integral compensation of an output error is proposed because it has only the sliding mode phase, the control system is robust during the complete motion, but its motion dynamics is described by a full order differential equation [35].

The system (4) using SMC with the integral compensation of an output error can be described as follows:

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m\,\mathbf{x}_m(t) + \mathbf{b}_m\,u(t) + \mathbf{d}\,r(t) \tag{5}$$

where

$$\mathbf{x}_m(t) = \begin{bmatrix} z(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} \quad \mathbf{A}_m = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_1 & -a_2 \end{bmatrix} \quad \mathbf{b}_m = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

This modified model of the system can be realized by a computer (discretely). The sampling period T is selected according to the sampling. An equivalent discrete-time model of the system (5) is [38]:

$$\delta\,\mathbf{x}_m(kT) = \mathbf{A}_\delta\,\mathbf{x}_m(kT) + \mathbf{b}_\delta\,u(kT) + \mathbf{d}_\delta\,r(kT) \tag{6}$$

where

$$\mathbf{A}_\delta = \frac{e^{\mathbf{A_m}T} - \mathbf{I}}{T} \qquad \mathbf{b}_\delta = \frac{1}{T}\int_0^T e^{\mathbf{A_m}T}\,\mathbf{b_m}\,\mathrm{d}\tau_t \qquad \mathbf{d}_\delta = \frac{1}{T}\int_0^T e^{\mathbf{A_m}T}\,\mathbf{d_m}\,\mathrm{d}\tau_t \tag{7}$$

The expression for $\delta\,\mathbf{x}_m(kT)$ represents the first differential (in the next explanation k stands for kT):

$$\delta\,\mathbf{x}_m(kT) = \frac{\mathbf{x}_m(k+1) - \mathbf{x}_m(k)}{T}$$

First, to realize the control using the DTSM controller, the design of the switching hyperplane is performed:

$$g(k) = \mathbf{c_\delta}\,\mathbf{x}_m(k)$$

where the switching matrix $\mathbf{c_\delta}$ is determined by the solution $\mathbf{P}$ of the discrete-time Riccati equation as

$$\mathbf{c_\delta} = \mathbf{b_\delta}\,\mathbf{P} \tag{8}$$

The discrete-time Ricatti equation is given as

$$\mathbf{A}_\delta^T\,\mathbf{P}\,\mathbf{A}_\delta - \mathbf{P} - \mathbf{A}_\delta^T\,\mathbf{P}\,\mathbf{b}_\delta\,(\mathbf{b}_\delta^T\,\mathbf{P}\,\mathbf{b}_\delta + \mathbf{P})^{-1}\,\mathbf{b}_\delta^T\,\mathbf{P}\,\mathbf{A}_\delta + \mathbf{Q} = 0$$

where

$\mathbf{Q}$ - diagonal matrix with positive elements.

The control of the DTSM controller for the system (6) is [34]

$$u(k) = -(\mathbf{c}_\delta\,\mathbf{b}_\delta)^{-1}\,(\mathbf{c}_\delta\,\mathbf{A}_\delta\,\mathbf{x}_m(k) + \mathbf{c}_\delta\,\mathbf{d}_\delta\,r(k) - K_d\,(\mathbf{c}_\delta\,\mathbf{b}_\delta)^{-1}\,\frac{g(k)}{|g(k)|} \tag{9}$$

For the problem of chattering to be avoided, the control $u(k)$ (9) is modified as follows [39]:

$$u(k) = -(\mathbf{c}_\delta\, \mathbf{b}_\delta)^{-1}(\mathbf{c}_\delta\, \mathbf{A}_\delta\, \mathbf{x}_m(k) + \mathbf{c}_\delta\, \mathbf{d}_\delta\, r(k) - K_d\, (\mathbf{c}_\delta\, \mathbf{b}_\delta)^{-1} \frac{g(k)}{|g(k)| + \alpha} \tag{10}$$

where:

$K_d > |\, \mathbf{c}_\delta\, \mathbf{b}_\delta\, d_{\max}\, |,\ d_{\max}$ - the maximum estimated value of the disturbance (11)

$\alpha \in \Re,\ \alpha > 0,\ \alpha$ - arbitrary positive constant for the elimination of chattering (12)

3.2 Discrete-time sliding mode controller based on first-order Pade approximation

In this section, the DTSM controller is developed based on the first-order Pade approximation. Substituting the relation (3) into the relation (1) yields:

$$G(s) = \frac{K(1-0.5\theta s)}{(\tau s+1)(1+0.5\theta s)} = \frac{2K - 2K\theta s}{\tau\theta s^2 + (2\tau+\theta)s + 2} = \frac{d_2 s + d_1}{s^2 + a_2 s + a_1} \tag{13}$$

where:

$$a_1 = \frac{2}{\tau\theta} \qquad a_2 = \frac{2\tau+\theta}{\tau\theta} \qquad d_1 = \frac{2K}{\tau\theta} \qquad d_2 = \frac{-K}{\tau\theta}$$

For the system (13), the SMC based on the integral compensation of an output error is not used because its motion dynamics is described by a differential equation of the same order as the controlled process. In that case, the robustness of the system can be impaired because it is a transfer function with zero [39]. In order to ensure complete robustness with reduced order dynamics, it is proposed to apply the stable system centre method.

The continuous-time state-space model of the transfer function (13) is

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t) \\ y(t) &= \mathbf{d}\,\mathbf{x}(t) \end{aligned} \tag{14}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} d_1 & d_2 \end{bmatrix}$$

This modified model of the plant can be realized by a computer (discretely). The sampling period T is selected according to the sampling theorem. An equivalent discrete-time model of the system (14) is [38]:

$$\begin{aligned} \delta\,\mathbf{x}(kT) &= \mathbf{A}_\delta\, \mathbf{x}(kT) + \mathbf{b}_\delta\, u(kT) \\ y(kT) &= \mathbf{d}_\delta\, \mathbf{x}(kT) \end{aligned} \tag{15}$$

where

$$\mathbf{A}_\delta = \frac{e^{\mathbf{A}T} - \mathbf{I}}{T} \qquad \mathbf{b}_\delta = \frac{1}{T}\int_0^T e^{\mathbf{A}T}\, \mathbf{b}\, \mathrm{d}\tau_t \qquad \mathbf{d}_\delta = \frac{1}{T}\int_0^T e^{\mathbf{A}T}\, \mathbf{d}\, \mathrm{d}\tau_t \tag{16}$$

The expression for $\delta x(kT)$ represents the first differential (in the next explanation k stands for kT):

$$\delta\,\mathbf{x}(kT) = \frac{\mathbf{x}(k+1) - \mathbf{x}(k)}{T}$$

Applying the transformation matrix $\mathbf{P}$ with the condition that $\mathbf{x}(k)=\mathbf{P}\begin{bmatrix} z(k) \\ q(k) \end{bmatrix}$, the system (15) is transformed into a normal canonical form:

$$\begin{aligned} \delta z(k) &= A_{11} z(k) + A_{12} q(k) \\ \delta q(k) &= A_{21} z(k) + A_{22} q(k) + A_{23} u(k) \\ y(k) &= q(k) \end{aligned} \tag{17}$$

Using the stable system centre method, the internal state generator $z(k)$ is generated first. The desired signal $q_d(k)$ is a function defined by multiple sub-functions, where each sub-function applies to a different interval in the domain. It can be presented by a known linear exo-system. Let the characteristic polynomial for this exo-system be:

$$P(z) = z + p_0$$

The stable system centre is an internal state, $z_c(k)$, which asymptotically converges to the ideal internal dynamics

$$\delta z_r(k) = A_{11} z_r(k) + A_{12} q_d(k)$$

The stable system centre for the system (15) can be defined by [40-42]:

$$\delta z_c(k) = A_{11} z_c(k) + s_c(k) \tag{18}$$

The desired asymptotic behaviour of the internal state dynamics (18) is of the first order. It can be defined as

$$\delta z_c(k) + c_0 z_c(k) = 0 \tag{19}$$

Let us define the input to the stable system centre as:

$$s_c(k) = A_{12} q_d(k) \tag{20}$$

and the equation of the internal state generator (19) as

$$\delta z_c(k) + c_0 z_c(k) = -P_0 s_c(k) \tag{21}$$

The value P_0 is computed to be

$$P_0 = c_0 A_{11}^{-1} \quad Z_c = P_0 A_{12} \tag{22}$$

Internal state is generated as

$$z_c(k) = \frac{Z_c}{z + p_0} r(k) \tag{23}$$

The stable system centre method transforms the output error problem into a state error stabilization problem [40-42]. The state errors will be stabilized to zero by SMC. Let state errors be:

$$e_y(k) = r(k) - y(k), \quad e_z(k) = z_c(k) - z(k) \tag{24}$$

The sliding hyperplane $g(k)$ is defined as

$$g(k) = e_y(k) + M\, e_z(k) \tag{25}$$

The value M is selected so that the value of ($A_{11} - A_{12}\, M$) is a modulo <1. The DTSM control for the system (17) is [40-42]:

$$u(k) = A_{23}^{-1}\, M\ (A_{11} e_z(k) + A_{22} e_y(k)) + A_{23}^{-1} \min(\frac{|g(k)|}{T}, \alpha + \beta |g(k)|)\ \mathrm{sgn}(g(k)) \tag{26}$$

where

$$\alpha > 0, \quad 0 \le \beta T < 1 \tag{27}$$

4. Illustrative example

For the DTSM controller synthesis to be verified, computer simulation of the regulation of the yeast fermentation process based on the first-order Taylor series approximation and the first-order Pade approximation is done.

4.1 Yeast fermentation process

With the presence of living organisms, the control of the fermentation process is more complex when a conventional chemical reactor is used. The dynamics of the fermentation process is highly non-linear and difficult to understand. It should be noted that yeasts have their own regulatory mechanism, which means that model parameters may not remain constant for long. The fermenter receives a jet, with unknown temperature, T_i , and unknown glucose concentration (feed substrate), S_f. The temperature of the fermenter, T is controlled by manipulating the jacket flow rate [43, 44]. The input stream is assumed to be equal to the output stream, i.e. the volume and physical parameters, such as density and heat transfer coefficients, are constant.

The reactor is modelled as a continuous mixing tank with a constant flow rate. The process model is described by five non-linear differential equations [43-45]:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \mu_x X - D\, X \tag{28}$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mu_x X}{R_{sx}} - \frac{\mu_p X}{R_{sp}} + D(S_f - S) \tag{29}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mu_p\, X - D\, P \tag{30}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\mu_x X}{\rho\, C_p Y_H} - \frac{K_T\, A_T}{\rho\, C_p V}\,(T - T_j) + D(T_i - T) \tag{31}$$

$$\frac{\mathrm{d}T_j}{\mathrm{d}t} = \frac{F_j}{V_m}(T_{ji} - T_i) + -\frac{K_T\, A_T}{\rho_j\, C_{pj} V_m}(T - T_j) \tag{32}$$

As we are interested in ethanol for glucose consumption, the evolution of CO_2 is not estimated in this model. Table 1 shows the steady state of the continuous yeast fermentation process. The full non-linear model (28-32) is used to determine the characteristic parameters (K, θ, τ) of the FOPDT model.

The mass and energy balance equations (28-32) describing the behaviour of a continuous yeast fermentation process are non-linear, while the most commonly used control strategies are based on linear system theories. Therefore, for the realization of the control system, it is important to make these equations linear. The above model is made linear by the Taylor series approximation [46]. The transfer function is determined using the linear model and steady-state values of parameters (Table 1). The transfer function thus obtained is then converted to the FOPDT model. The Reaction Curve method [47] FOPDT and the FIT 3 [48] are used to obtain the characteristic parameters (K, θ, τ) of the FOPDT model. The FOPDT model is given by

$$G(s) = \frac{P(s)}{X(s)} = \frac{K e^{-\theta s}}{\tau s + 1} \tag{33}$$

Table 1 Steady state values of the process model parameters

V - fermenter volume	1000 l	F_j - flow rate of jacket	18 l/h
V_m - jacket volume	25 l	R_{sx} - growth coefficients for yeast	0.65
X - yeast concentration	1 g/l	R_{sp} - growth coefficients for alcohol	0.45
S - glucose concentration	30 g/l	ρ - density of mixture	1070g/l
S_f - feed substrate concentration	60 g/l	ρ_j - density of cooling water	1000g/l
P - ethanol concentration	15 g/l	C_p - specific heat capacity of mixture	4J/kgK
T - fermenter temperature	27 ^{0}C	Y_H - heat evolved per gram	1K/gr
T_i - inlet temperature	25 ^{0}C	K_T - heat transfer coefficient	3.6e^{5}J/h^{2}K
T_j - jacket temperature	29 ^{0}C	A_T - area of heat transfer	1m^2

4.2 Discrete-time sliding mode controller based on the first-order Taylor series approximation

For the relations obtained for the DTSM controller synthesis to be verified, computer simulation of the regulation of the yeast fermentation process based on the first-order Taylor series approximation is done. By using the relation (2), the FOPTD model (33) takes the following form:

$$G(s) = \frac{0.0048}{s^2 + 0.0594 s + 0.0327} \tag{34}$$

The system (34) using SMC with the integral compensation of an output error is described as follows (5):

$$\dot{\mathbf{x}}_m(t) = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.0327 & -0.0594 \end{bmatrix} \mathbf{x}_m(t) + \begin{bmatrix} 0 \\ 0 \\ 0.0048 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r(t) \tag{35}$$

The discrete model of the system (35) for the sampling period $T = 1ms$, according to (7), has the following form:

$$\delta \mathbf{x}_m(k) = \begin{bmatrix} 0 & -0.9996 & 0.002 \\ 0 & 0.002 & 0.9998 \\ 0 & -0.032994 & -0.059392 \end{bmatrix} \mathbf{x}_m(k) + \begin{bmatrix} 0 \\ 0.002 \\ 0.004792 \end{bmatrix} u(k) + \begin{bmatrix} 0.998 \\ 0.001 \\ 0 \end{bmatrix} r(k)$$

Let the diagonal matrix be:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then the switching matrix $\mathbf{c}_\delta$ (8) is

$$\mathbf{c}_\delta = \begin{bmatrix} -256.37 & 41.32 & 5.34 \end{bmatrix}$$

For the realization of the DTSM control (10), the following values of parameters can be selected (11, 12):

$$k = 40 \quad \alpha = 0.1$$

Based on the selected parameters of the DTSM controller, the simulation results are presented in the form of a diagram of the response (Fig. 1), control (Fig. 3), and switching hyperplane (Fig. 5). The system has good properties of eliminating chattering (Fig. 1).

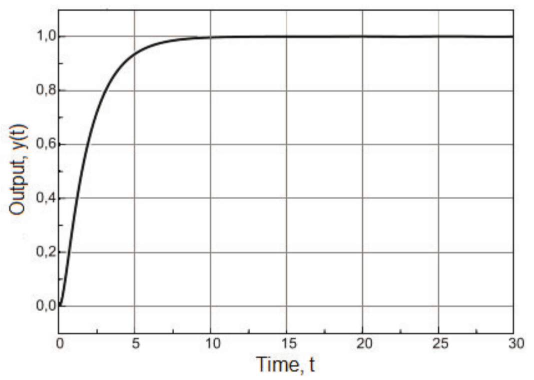

Fig. 1 Step response (Taylor series approximation)

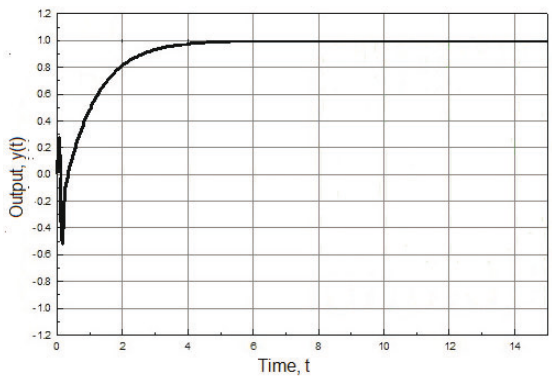

Fig. 2 Step response (Pade approximation)

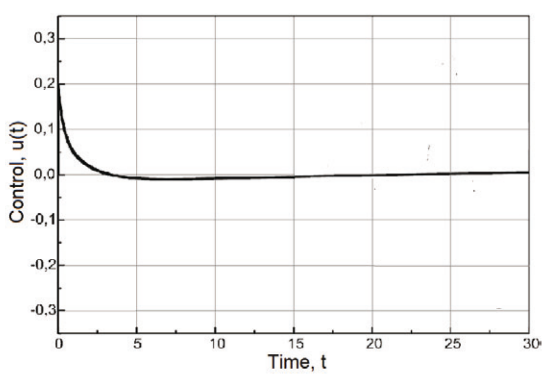

Fig. 3 Control (Taylor series approximation)

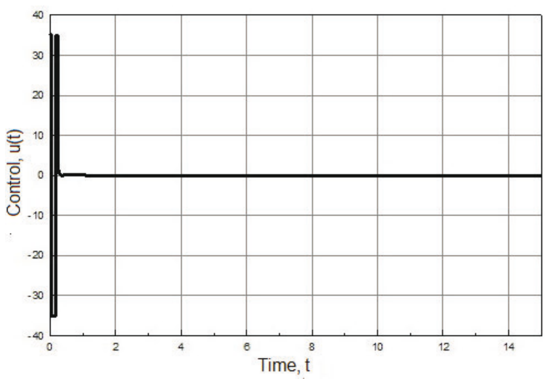

Fig. 4 Control (Pade approximation)

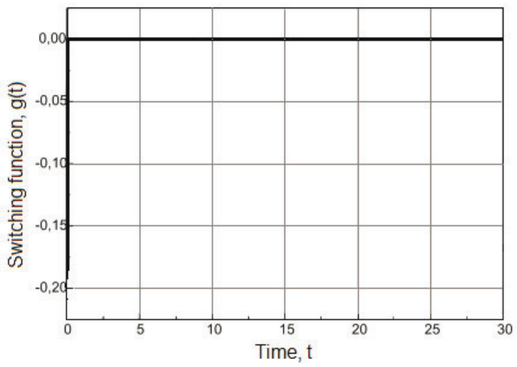

Fig. 5 Switching function (Taylor series approximation)

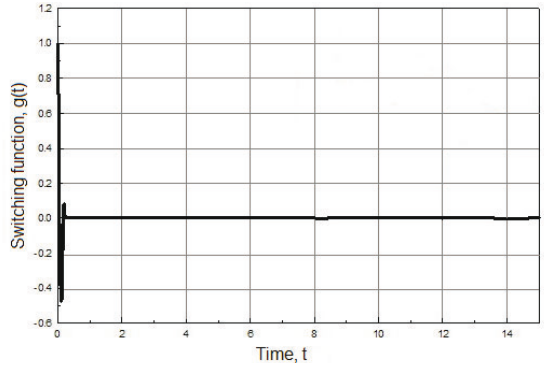

Fig. 6 Switching function (Pade approximation)

4.3 Discrete-time sliding mode controller based on the Pade approximation

For the relations obtained for the DTSM controller synthesis to be verified, computer simulation of the regulation of the yeast fermentation process based on the first-order Pade approximation is done. By using the relation 3, the FOPTD model (33) takes the following form:

$$G(s) = \frac{-0.0048s + 0.0095}{s^2 + 1.1273s + 0.0655} \tag{36}$$

The continuous-time state-space model of the transfer function (36) is:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} 0 & 1 \\ -0.0655 & -1.1273 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0.0095 & -0.0048 \end{bmatrix} \mathbf{x}(t) \end{aligned} \tag{37}$$

Using δ transform for the sampling period $T = 1ms$, the discrete-time model of (37), according to (16), is:

$$\begin{aligned} \delta \mathbf{x}(k) &= \begin{bmatrix} 0 & 0.9996 \\ -0.065492 & -0.004795 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0,0002 \\ 0.9996 \end{bmatrix} u(kT) \\ y(kT) &= \begin{bmatrix} 0.0095 & -0.0048 \end{bmatrix} \mathbf{x}(kT) \end{aligned} \tag{38}$$

Applying the transformation matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} 0.992 & -0.0002 \\ 0..0095 & -0.0048 \end{bmatrix}$$

the system (38) is transformed into a normal canonical form:

$$\begin{aligned} \delta z(k) &= 0.9996\ z(k) + 0.065488 q(k) \\ \delta q(k) &= 0.9996\ z(k) + 0.00499 q(k) + 0.999 u(k) \\ y(k) &= q(k) \end{aligned} \tag{39}$$

The model of the desired signal $r(k) = const$ can be defined by the characteristic polynomial (18), $P(z) = z + p_0$. The stable system centre for the system (39) can be defined as

$$z_c(k) + c_0\, z_c(k) = 0 \text{ with } c_0 = 0.1965 \text{ i.e. } z + p_0 \text{ with } p_0 = -0.9997835$$

Let us define the input to the stable system centre (20), $s_c(k) = -0.065488\,r(k)$, and the equation of the internal state generator (21), $\delta z_c(k) + 0.1965\,z_c(k) = -P_0\,s_c(k)$. The value P_0, (22), is computed to be $P_0 = 0.1965$. The internal state (23) is generated as

$$z_c(k) = \frac{0.0012969}{z + 0.1965}\,r(k)$$

The internal state error dynamics (24) will be substituted by one order selecting sliding hyperplane as (25), where $M = 0.8$. Based on the relation (27), it is chosen that $\alpha = 5$, $\beta = 0$. Then SMC (26) is selected to be

$$u(k) = 0.80049e_z(k) + 0.00399e_y(k) + 1.001\min(1000|g(k)|, 5)\,\mathrm{sgn}(g(k)) \qquad (40)$$

The simulation results are shown in the form of step response (Fig. 2), control (Fig. 4), and switching hyperplane (Fig. 6). Based on the results obtained, it can be concluded that the system is stable. The control (40) is smooth, and the switching function dynamics is without chattering.

Finally, in the case of the first-order Taylor series approximation and the first-order Pade approximation, the realized DTMS controller enabled the system stability and the elimination of chattering. The step response in the case of the first-order Taylor series approximation has a higher response speed and enters the sliding mode faster. The step response in the case of the first-order Pade approximation exhibits non-minimum phase behaviour (undershoots) at the beginning of the process. The control and the switching function have much higher amplitude in the case of the first-order Pade approximation than in the case of the first-order Taylor series approximation. In the former case, it should be noted that the control and the switching function as well as the step response show non-minimum phase behaviour at the beginning of the process.

5. Conclusions

This paper has presented the synthesis of the DTSM controller based on the FOPDT model of the fermentation process. The DTSM controller was obtained by the integral compensation of an output error and the stable system centre method, depending on whether the first-order Taylor series approximation or the first-order Pade approximation was used. These methods are applied to the yeast fermentation process. The examples presented indicate that the DTSM controller performance is stable and quite satisfactory despite nonlinearities over a wide range of operating conditions. The proposed DTSM controller based on the first-order Taylor series approximation can yield better dynamic performance than the DTSM controller based on the first-order Pade approximation.

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