# AALBORG UNIVERSITY

# An Environment for Graphical Models

Part II

## A Guide to CoCo

Jens Henrik Badsberg

Jens Henrik Badsberg (2001). A Guide to CoCo,
*Journal of Statistical Software*, Volume 6, issue 4:
http://www.jstatsoft.org/v06/

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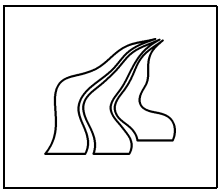

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## Part II:

## A Guide to **CoCo**

Jens Henrik Badsberg

March 1995

Second edition

Jens Henrik Badsberg (2001). A Guide to CoCo, *Journal of Statistical Software*, Volume 6, issue 4: http://www.jstatsoft.org/v06/

Part II of a thesis submitted to the Faculty of Technology and Science at Aalborg University for the degree of Doctor of Philosophy.

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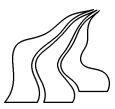

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# Preface

CoCo is a program for estimation, test and model search among hierarchical interaction models for large complete contingency tables. The name CoCo is derivated of "Co"mplete "Co"ntingency tables, since the initial program could only handle complete tables, but the program has been enhanced to handle incomplete tables.

CoCo works especially efficiently on *graphical models*, and some of the commands are designed to handle graphical models.

Graphical models are log-linear interaction models for contingency tables that can be represented by a simple undirected graph with as many vertices as the table has dimension. Further all these models can be given an interpretation in terms of conditional independence and the interpretation can be read directly off the graph in the form of a Markov property. The class of graphical model is a proper subclass of the *hierarchical models*, but the class strictly contains the *decomposable models*, e.g., Haberman (1974).

See Darroch, Lauritzen & Speed (1980) for how graphical models are defined by the close connection between the theory of Markov fields and that of log-linear interaction models for contingency tables.

Besides incomplete tables and tables with incomplete observations can be handled and exact tests between any two nested decomposable models computed.

CoCo is a program designed to perform estimation and tests in large contingency tables. By using graph-theoretical results (Rose, Tarjan & Lueker 1976, Tarjan & Yannakakis 1984, Tarjan 1985, Leimer 1993) the hierarchical log-linear interaction models are decomposed. See also chapter 3 of Part I. The IPS-algorithm is not used on the full table, but only on the non-decomposable irreducible components (chapter 2 of Part I). Furthermore, the optimized version of the IPS-algorithm of Jiroušek (1991) is used on these non-decomposable atoms.

If one model is tested against another and the two models have common decompositions, then the test can be partitioned in tests on smaller tables (Goodman 1971). Collapsibility (Asmussen & Edwards 1983) of models and tests is used. If two large sparse models (many factors but few interactions) have no common decompositions, they can be tested against each other by computing the deviance for each model, not by summing over all cells in the full table, but by summing over only cells in sufficient marginal tables (chapter 2 of Part I).

Tests between models with an unlimited number of factors can then be performed on a PC, if the largest clique in the fill-in graphs of non-decomposable atoms of graphs for the models has no more than 14 binary vertices (24 on workstations).

The likelihood ratio test statistic, Pearson's $\chi^2$ test statistic and the power divergence statistics can be computed. In sparse tables an adjusted number of degrees of freedom is computed. Exact tests between any two nested decomposable models can be performed.

Tables with "structural zeros" (Incomplete tables) can be declared, and a modified version of the IPS-algorithm is used on these.

Commands for interactive model search among graphical models are implemented. A semi-automatic model search is possible by backward elimination and forward selection. The model search from Edwards & Havránek (1985) is included in the program.

Besides functions for tests and model search CoCo also has some procedures for model control. The test of one decomposable model against another decomposable model can be factorized into a sequence of tests with one edge (Sundberg 1975, Frydenberg & Lauritzen 1989), and the test between any two submodels can be factorized in tests with one interaction. Functions for producing these factorizations are available in CoCo. CoCo also has a function for computing a wide range of measures of associations on 2-dimensional tables.

Observed counts, estimated counts and probabilities and residual (absolute, adjusted, standard, Freeman-Tukey, etc.) can be listed, plotted pairwise, printed in tables and given an univariate description with mean, variance, median, range, etc.

Finally, to make the use of CoCo somewhat easier and more flexible some data selection is possible and cases with missing values (incomplete observations) can be excluded at various levels in CoCo or the EM-algorithm applied. CoCo can exclude all observations with any variable marked as unobserved, when reading the observations, exclude observations with variables unobserved in a given set, after reading the observations, or exclude observations with relevant variables for a given test marked as unobserved, when performing the test.

CoCo can be loaded into New S (S-Plus) and XLISP-STAT. Functions for returning statistics and residuals from CoCo, e.g., high-resolution plotting are provided.

The program is originally designed to test methods described in the dissertation Badsberg (1986).

### Size of Tables

The range of the sizes of the tables CoCo can handle can be classified as follows:

**Tiny tables:** 2 and 3 dimensional tables;

In these tables a wide range of measures of associations on the 2-dimensional tables given other variables can be computed.

**Small tables:** tables with up to between 7 and 10 variables;

On small tables the global model search procedure of Edwards & Havránek (1985) is useful, and will terminate after an acceptable computing time. Also exact tests by Monte Carlo simulation and the EM-algorithm are useful on these tables.

Before any computation all the marginal tables are found to reduce the computing time (unless some cases have unobserved variables).

**Medium tables:** tables with up to 20 variables;

Any test between two hierarchical models can be performed. The procedures for backward elimination and forward selection of edges in graphical models are useful.

In these tables only sufficient tables of observed counts and tables of estimated probabilities for few tables can be stored internally in the computer.

**Large tables:** tables with more than 20 variables;

We do not say that all models on large tables are large, e.g., a model with only main effects is called a *small* model. If all the tables of the sufficient marginals of a model cannot fit in the computer memory at one time together with the state spaces of all the non-decomposable components of the model, then the model is *large*. A model is *very large*, if any of the sufficient marginal tables cannot fit in the computer memory. We cannot handle very large models, if we cannot at least store the state spaces of the non-decomposable components of the model one by one. Such models are called *huge* models. Thus, in very large models the largest clique of the fill-in graphs of non-decomposable atoms of the graph of the very large model cannot have more than 14 binary vertices (24 on workstations).

One model can be tested against another by using partitioning and collapsibility of tests, or by computing the deviance for each model by summing over only non-zero cells in the sufficient marginal tables as described in chapter 2 of Part I. These tests can be performed between any two large (or very large) models, but if the parts of the test involve tests on large tables, then only the deviance can be computed, and the adjusted degrees of freedom cannot be computed, if the parts involve large models.

The procedures for backward elimination and forward selection of edges in graphical models are still useful on large tables. (Various runs of model selection by respectively forward selection and backward elimination has been performed on the 121 dimensional table with 10.000 cases of Wedelin (1993).)

In large tables the observations are placed as a case list on an internal file.

This crude classification of tables follows the classification of datasets by Huber (1994): Tiny datasets are suitable for black-boards, a small set can be printed on a few pages, a medium set fits on a floppy disk, a large dataset requires a tape, and a huge dataset requires many tapes.

### Other programs

Other programs for analysis of log-linear interaction models for contingency tables are DIGRAM (Kreiner 1989) and MIM (Edwards 1989). CoCo finds closed form expres-

sion for estimates in decomposable models, handles incomplete tables and tables with incomplete observations, computes exact tests between any two nested decomposable models, handles larger tables and has commands for semi-automatic and automatic search. DIGRAM handles recursive graphical models on contingency tables. MIM also handles continuous variables in mixed interaction models, CG-distributions. Both DIGRAM and MIM runs only on Personal Computers, and are on these machines able to present graphical models as graphs.

## Using this Tutorial

This guide is the second volume of the thesis "A Environment for Graphical Models". Volume 3, "Xlisp+CoCo — A Tool for Graphical Models", of the thesis is a guide to CoCo within XLISP-STAT.

Chapter 1 of this guide is a Tutorial Introduction to CoCo. A semi-automatic model search with exact tests is performed on the data from Reiniš, Pokorný, Bašiká, Tišerová, Goričan, Horáková, Stuchliková, Havránek & Hrabovský (1981), and some tables are printed, plotted and described. This part of the guide is most useful for users, who are familiar with analysis of discrete data, and who want a quick introduction to CoCo.

Chapters 3 to 12 make up the User Manual. Commands are described in detail, arranged according to the step in the analysis to which they belong. Some examples are given. Some of the used methods (algorithms) are described.

The appendix contains parts of a Reference Manual: a Quick Reference Card and an Installation Guide.

The program CoCo is not only useful for working statisticians and other scientists analyzing discrete data by contingency tables, but also in courses teaching analysis of discrete data and graphical models. The guide is probably useful in such courses, but the guide to CoCo is not a text-book on contingency table, and should only be used in courses assisted by text-books as, e.g., Lauritzen (1982).

## Availability

The source code in C, executable for Sun 4 (Sparc) and executable of the standalone version of CoCo for PC's and Macintosh for this version of CoCo is available free of charge for non-commercial use.

The source code may only be read and edited for the purpose of porting CoCo to other machines. No new features may be added to CoCo and no parts of the program may be included in other systems or new interface-procedures to New S, S-Plus, XLISP-STAT or any other extendable system may be made without the written permission from the author.

The source code in C and executable for Sun 4 (Sparc), both with Lisp-code for the graphics, and executable of the standalone version of CoCo for PC's and Macintosh can be obtained by anonymous *ftp* over internet from *ftp.iesd.auc.dk*, or by *WWW* from *http://www.iesd.auc.dk/pub/packages/CoCo*. Or CoCo is available on disks (3.5" or 5.25" High density). You should, however, by prepared to bear the costs of copying, e.g., by supplying a disk or tape and a stamped mailing envelope. This guide and the

guide to CoCo is also available in TeX and postscript code from the ftp site or printed from Aalborg University.

## Disclaimer

CoCo is an experimental program. It has not been extensively tested. The author of CoCo, the University of Aalborg and any other part assisting the author of CoCo in distributing CoCo take no responsibility for losses or damages resulting directly or indirectly from the use of this program.

CoCo is an evolving system. Over the time new features will be introduced, and existing features that do not work may be changed. Every effort will be made to keep CoCo consistent with the information in this guide, but if this is not possible, the help information in CoCo and XLISP-STAT should give accurate information about the current use of a command.

## Acknowledgments

Many thanks go to Professor Steffen Lilholt Lauritzen who inspired the creation of CoCo. Also thanks to David Edwards and Svend Kreiner for useful comments and suggestions to new features in CoCo. Thanks to Flemming Skjøth, Søren Højsgaard, Bo Thiesson and Jørgen Greve, especially Flemming Skjøth, who in their study patiently used and tested CoCo, and found numerous errors.

Thanks to my sister Annette Badsberg for correcting the English language of (earlier versions of) this guide.

Aalborg, Danmark, March 15th 1995,

Jens Henrik Badsberg

# Contents

# Part I

# Tutorial

# Chapter 1

# Introduction to CoCo

## 1.1 Starting and Finishing

**DOS:** On a PC: Install CoCo as described in Appendix A, make sure that your `PATH` is pointing to the directory with CoCo, and just type `COCO`.

**Macintosh:** On Macs: If not installed: Install CoCo as described in Appendix A. Click the `CoCo` folder.

**Unix:** On UNIX-Workstations: If not installed: Install CoCo as described in Appendix A, make sure that the directory with the shell script `CoCo` is in your `PATH`. Type `CoCo` (or `fep CoCo`).

CoCo will then start with something like:

```
CoCo     -     A program for estimation, test and model search
in very large 'Co'mplete and 'InCo'mplete 'Co'ntingency tables.
Version 1.3   Friday March 17 12:00:00 MET 1995
Compiled with gcc, a C compiler for Sun4
Copyright (c) 1991, by Jens Henrik Badsberg

>
```

> is the normal prompt from CoCo. If CoCo is expecting integer or real numbers, file-names, factor-names, sets, generating classes, data, etc. the prompt is changed. Give the expected data to CoCo, or try some numbers, names or `;`.

CoCo is ended with `quit`.

## 1.2 An Example

The data of table 1.1 from Reiniš et al. (1981) is also used in Edwards & Havránek (1985).

| | | | B | No | | Yes | |
|---|---|---|---|---|---|---|---|
| | | | A | No | Yes | No | Yes |
| F | E | D | C | | | | |
| Negative | <3 | <140 | No | 44 | 40 | 112 | 67 |
| | | | Yes | 129 | 145 | 12 | 23 |
| | | >140 | No | 35 | 12 | 80 | 33 |
| | | | Yes | 109 | 67 | 7 | 9 |
| | >3 | <140 | No | 23 | 32 | 70 | 66 |
| | | | Yes | 50 | 80 | 7 | 13 |
| | | >140 | No | 24 | 25 | 73 | 57 |
| | | | Yes | 51 | 63 | 7 | 16 |
| Positive | <3 | <140 | No | 5 | 7 | 21 | 9 |
| | | | Yes | 9 | 17 | 1 | 4 |
| | | >140 | No | 4 | 3 | 11 | 8 |
| | | | Yes | 14 | 17 | 5 | 2 |
| | >3 | <140 | No | 7 | 3 | 14 | 14 |
| | | | Yes | 9 | 16 | 2 | 3 |
| | | >140 | No | 4 | 0 | 13 | 11 |
| | | | Yes | 5 | 14 | 4 | 4 |

A, smoking; B, strenuous mental work; C, strenuous physical work;
D, systolic blood pressure; E, ratio of $\alpha$ to $\beta$ lipoproteins;
F, family anamnesis of coronary heart disease.

Table 1.1: *Risk factors for coronary heart disease.*

Start CoCo and give the command `SET KEYBOARD On;` to indicate that you want to read data from the keyboard. Then type `READ DATA;`. CoCo will respond with the prompt `DATA->` indicating that CoCo is expecting `data`. Declare the table with `Factors A2/B2/C2/D2/E2/F2//` and give the keyword `Table` to tell CoCo that the data is to be entered in table-form. Then type the 64 cell-counts and end with `/`.

```
>keyboard;
 Keyboard set ON
>read data;
 DATA->Factors A 2 / B 2 / C 2 / D 2 / E 2 / F 2 //
 DATA->Table
 DATA-> 44 40 112 67  129 145  12 23   35 12  80 33  109  67   7  9
 DATA-> 23 32  70 66   50  80   7 13   24 25  73 57   51  63   7 16
 DATA->  5  7  21  9    9  17   1  4    4  3  11  8   14  17   5  2
 DATA->  7  3  14 14    9  16   2  3    4  0  13 11    5  14   4  4/
  64 cells with  1841 cases read.
 Finding all marginals.
>
```

The table can also be entered as a list of observations. See chapter 3 for details on

this and for details on the declaration of factors.

The declaration of factors and the table can be placed on a file, e.g., reinis81.dat:

```
Factors A 2 / B 2 / C 2 / D 2/ E 2 / F 2 //
Table
 44  40  112  67
129 145   12  23
 35  12   80  33
109  67    7   9
 23  32   70  66
 50  80    7  13
 24  25   73  57
 51  63    7  16
  5   7   21   9
  9  17    1   4
  4   3   11   8
 14  17    5   2
  7   3   14  14
  9  16    2   3
  4   0   13  11
  5  14    4   4
```

This file is then declared to CoCo by `SET INPUTFILE DATA reinis81.dat;` and then read by `READ DATA;` without arguments:

```
 CoCo     -     A program for estimation, test and model search
 in very large 'Co'mplete and 'InCo'mplete 'Co'ntingency tables.
 Version 1.3   Friday March 17 12:00:00 MET 1995
 Compiled with gcc, a C compiler for Sun4
 Copyright (c) 1991, by Jens Henrik Badsberg

>set inputfile data reinis81.dat;
>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
>
```

The declaration of factors and the table or list of cases can be placed on separate files. See chapter 3. Chapter 3 also describes how to read incomplete tables, group levels, make data selection and handle missing values in lists of observations.

After reading the data any marginal table of observations can be printed with the command `PRINT OBSERVED` ⟨*set*⟩`;`, where ⟨*set*⟩ is a subset of the declared factors. The asterisk `*` is an abbreviation of the set containing all factors, so in the following `PRINT OBSERVED ABCDEF.;` is equivalent to `PRINT OBSERVED *;`:

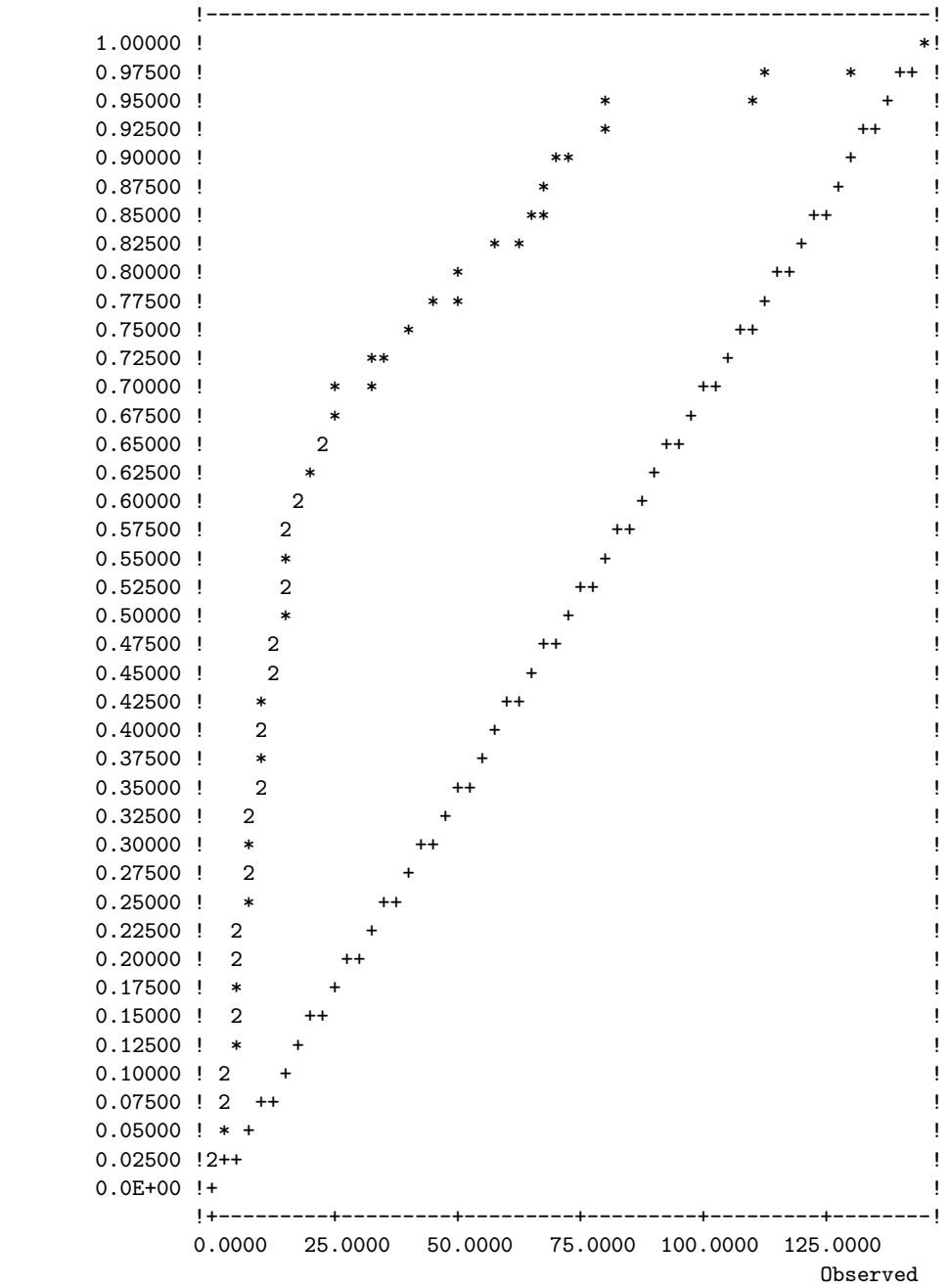
```
>print observed ABCDEF.;

     [ABCDEF]

          C                   1                   2
          B         1         2         1         2
          A     1   2     1   2     1   2     1   2
  F  E  D

  1  1  1     44  40  112  67   129 145   12  23
        2     35  12   80  33   109  67    7   9

     2  1     23  32   70  66    50  80    7  13
        2     24  25   73  57    51  63    7  16


  2  1  1      5   7   21   9     9  17    1   4
        2      4   3   11   8    14  17    5   2

     2  1      7   3   14  14     9  16    2   3
        2      4   0   13  11     5  14    4   4

>print observed AB.;

     [AB]

    A    1   2
  B

  1    522 541
  2    439 339

>
```

`DESCRIBE OBSERVED` ⟨*set*⟩`;`, where ⟨*set*⟩ is a subset of the declared factors, will give a univariate description of the values in the marginal table. The `SET PAGE FORMATS` ⟨*line-length*⟩ ⟨*page-length*⟩`;` is used to make the plot fit into this guide.

```
# Version 1.3   Friday March 17 12:00:00 MET 1995
# Compiled with gcc, a C compiler for Sun4
# Compile-time: Mar 17 1995, 13:00:00.
# Copyright (c) 1991, by Jens Henrik Badsberg
# Licensed to ...

 Diary set ON

>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:           0.083secs.
```

```
>#
># Change the size of a page to fit into this guide:
>#
>set page formats 72 52;
 TIME:            0.000secs.

>#
># Describe the observed counts in the saturated table
># (print also a uniform plot):
>#
>describe table uniform observed * ;

 DESCRIBE TABLE:         Observed

 NUMBER OF CELLS IN TABLE:            64
 NUMBER OF INVALID CELLS:              0
 NUMBER OF CELLS TO DESCRIBE:         64

 SORTED LIST

    0.0000    1.0000    2.0000    2.0000    3.0000    3.0000    3.0000
    4.0000    4.0000    4.0000    4.0000    4.0000    5.0000    5.0000
    5.0000    7.0000    7.0000    7.0000    7.0000    7.0000    8.0000
    9.0000    9.0000    9.0000    9.0000   11.0000   11.0000   12.0000
   12.0000   13.0000   13.0000   14.0000   14.0000   14.0000   14.0000
   16.0000   16.0000   17.0000   17.0000   21.0000   23.0000   23.0000
   24.0000   25.0000   32.0000   33.0000   35.0000   40.0000   44.0000
   50.0000   51.0000   57.0000   63.0000   66.0000   67.0000   67.0000
   70.0000   73.0000   80.0000   80.0000  109.0000  112.0000  129.0000
  145.0000

 STATISTICS

 NUMBER OF VALUES:   64

 25%:   RANK:    16 VALUE:     7.0000    RANK:    17 VALUE:     7.0000
 50%:   RANK:    32 VALUE:    14.0000    RANK:    33 VALUE:    14.0000
 75%:   RANK:    48 VALUE:    40.0000    RANK:    49 VALUE:    44.0000

 MINIMUM:     0.0000    MAXIMUM:   145.0000    RANGE:     145.0000
 MAX RUN:          5    MODE:        4.0000    # < Eps.:          1

 SUM  (X)   :        1841.0000    MEAN:                 28.7656
 SUM  (1/X) :                -    HARMONIC MEAN:              -
 PROD (X)   :           0.0000    GEOMETRIC MEAN:        0.0000
 SUM (X-MEAN)^2 :    7.096E+04    VARIANCE:           1108.6794
 SUM (X-MEAN)^3 :    3.956E+06    SKEWNESS:              1.6744
 SUM (X-MEAN)^4 :    4.113E+08    KURTOSIS:              5.2282
```

```
UNIFORM PLOT

Unit horizontal: - =     2.5000
```



```
HISTOGRAM

Unit horizontal: * =          1
Unit vertical:   ! =    10.0000

   0.0000 ->  !  ************
  10.0000 ->  !  ************************
  20.0000 ->  !  ********
  30.0000 ->  !  ***
  40.0000 ->  !  ***
  50.0000 ->  !  **
  60.0000 ->  !  **
  70.0000 ->  !  *****
  80.0000 ->  !  **
  90.0000 ->  !
 100.0000 ->  !
 110.0000 ->  !  **
 120.0000 ->  !
 130.0000 ->  !  *
 140.0000 ->  !
 150.0000 ->  !  *
 160.0000

COUNTS
```

| Cell count | Number of cells with count | Cumm. number of cells | % of total number of cells | Cumm. % |
|---|---|---|---|---|
| 0 | 1 | 1 | 0.0156 | 0.0156 |
| 1 | 1 | 2 | 0.0156 | 0.0312 |
| 2 | 2 | 4 | 0.0312 | 0.0625 |
| 3 | 3 | 7 | 0.0469 | 0.1094 |
| 4 | 5 | 12 | 0.0781 | 0.1875 |
| 5 | 3 | 15 | 0.0469 | 0.2344 |
| 7 | 5 | 20 | 0.0781 | 0.3125 |
| 8 | 1 | 21 | 0.0156 | 0.3281 |
| 9 | 4 | 25 | 0.0625 | 0.3906 |
| 11 | 2 | 27 | 0.0312 | 0.4219 |
| . | . | . | . | . |
| . | . | . | . | . |
| 67 | 2 | 56 | 0.0312 | 0.8750 |
| 70 | 1 | 57 | 0.0156 | 0.8906 |
| 73 | 1 | 58 | 0.0156 | 0.9062 |
| 80 | 2 | 60 | 0.0312 | 0.9375 |
| 109 | 1 | 61 | 0.0156 | 0.9531 |
| 112 | 1 | 62 | 0.0156 | 0.9688 |
| 129 | 1 | 63 | 0.0156 | 0.9844 |
| 145 | 1 | 64 | 0.0156 | 1.0000 |

```
 TIME:            0.126secs.
```

The hierarchical log-linear interaction model described by the generating class ⟨*gc*⟩ is read by `READ MODEL` ⟨*gc*⟩`;`; for example,

```
>read model AB,BC.;
```

Observed counts, estimated counts, probabilities and residuals (absolute, adjusted, standard, Freeman-Tukey, etc.) can be printed in tables or described with the commands, e.g., `PRINT TABLE Probabilities` ⟨*set*⟩`;` and `DESCRIBE TABLE Adjusted Residuals` ⟨*set*⟩`;` or the values can be listed or plotted against each other with `LIST` ⟨*set*⟩`;` and `PLOT` ⟨*x-value-name*⟩ ⟨*y-value-name*⟩ ⟨*set*⟩`;`. See chapter 4 for details.

```
>#
># Read model ACE,ADE,BC,F.
>#
>read model ACE,ADE,BC,F. ;
 TIME:            0.017secs.

#
# Change the page-size
#
>set page formats 72 36 ;
 TIME:            0.000secs.

>#
># Describe the adjusted residuals in the marginal table BCDEF.:
>#
>describe table  rankit  adjusted BCDEF. ;

 DESCRIBE TABLE:         Adjusted

 NUMBER OF CELLS IN TABLE:           32
 NUMBER OF INVALID CELLS:             0
 NUMBER OF CELLS TO DESCRIBE:        32

 SORTED LIST

   -3.4465   -2.2064   -2.1418   -0.9380   -0.8524   -0.7695   -0.6893
   -0.4999   -0.4912   -0.4444   -0.4315   -0.2857   -0.2795   -0.2417
   -0.1614   -0.1219   -0.0463    0.1210    0.1490    0.2556    0.3147
    0.7317    0.8584    0.9000    0.9719    1.1956    1.2852    1.3426
    1.4399    1.7381    1.9406    4.1246

 STATISTICS

 NUMBER OF VALUES:   32

 25%:   RANK:     8 VALUE:    -0.4999    RANK:     9 VALUE:    -0.4912
 50%:   RANK:    16 VALUE:    -0.1219    RANK:    17 VALUE:    -0.0463
 75%:   RANK:    24 VALUE:     0.9000    RANK:    25 VALUE:     0.9719
```

```
MINIMUM:    -3.4465    MAXIMUM:     4.1246    RANGE:       7.5711
MAX RUN:          1    MODE:             -    # < Eps.:         0

SUM  (X)   :           3.3213    MEAN:                  0.1038
SUM  (1/X) :         -30.8929    HARMONIC MEAN:        -1.0358
PROD (X)   :          -0.0000    GEOMETRIC MEAN:       -0.6163
SUM (X-MEAN)^2 :      58.7837    VARIANCE:              1.8370
SUM (X-MEAN)^3 :      12.1395    SKEWNESS:              0.1524
SUM (X-MEAN)^4 :     506.5294    KURTOSIS:              4.6907

RANKIT PLOT

Unit horizontal: - =     0.1250
Unit vertical:   ! =     0.2000

         !-------------------------------------------------------------!
 2.20000 !                                                    ++        !
 2.00000 !                                                  ++         *!
 1.80000 !                                                ++            !
 1.60000 !                                             * ++             !
 1.40000 !                                           *+++               !
 1.20000 !                                         *++                  !
 1.00000 !                                       ***                    !
 0.80000 !                                     ***                      !
 0.60000 !                                   2+                         !
 0.40000 !                                * ++                          !
 0.20000 !                              2*++                            !
 0.0E+00 !                            ***                               !
-0.20000 !                           2*                                 !
-0.40000 !                         +**                                  !
-0.60000 !                       ++2                                    !
-0.80000 !                     +++ *                                    !
-1.00000 !                  +++ **                                      !
-1.20000 !                ++   *                                        !
-1.40000 !            * ++                                             !
-1.60000 !           ***                                               !
-1.80000 !         ++                                                  !
-2.00000 !*      ++                                                     !
-2.20000 !    +++                                                       !
         !+---------+---------+---------+---------+---------+---------+-!
        -3.5000   -2.2500   -1.0000    0.2500    1.5000    2.7500
                                                              Adjusted

TIME:            0.104secs.
```

(A histogram is omitted from the above output.)

```
>#
># Make a plot of the adjusted residuals against observed counts
>#
>plot  observed  adjusted * ;

  PLOT OF:  Adjusted BY Observed
  PLOT

 Unit horizontal: - =     2.5000
 Unit vertical:   ! =     0.2500


Adjusted
            !------------------------------------------------------------!
     2.5000 !  *                                                         !
     2.2500 !                                                            !
     2.0000 !                                                            !
     1.7500 !  2                                                         !
     1.5000 !      **        *                                           !
     1.2500 !                                            *               !
     1.0000 !   2                                                        !
     0.7500 ! * *  3 *    *                                      *       !
     0.5000 ! 2  ***                                      *              !
     0.2500 ! ** *    **                  *                             *!
     0.0000 !     *        *   * 2  *    *                              !
    -0.2500 !  *2  *                   **                               !
    -0.5000 !    **       *                 2                           !
    -0.7500 ! *2 2     *              * *                                 !
    -1.0000 !* *      *                                                  !
    -1.2500 !     * *                                                   !
    -1.5000 !                                                           !
    -1.7500 !   *                                                       !
    -2.0000 !*                                                          !
    -2.2500 !    *                                                      !
            !+---------+---------+---------+---------+---------+--------!
            0.0000   25.0000   50.0000   75.0000  100.0000  125.0000
                                                        Observed

             EXCLUDED:         0

 TIME:            0.066secs.
```

The likelihood ratio test statistic -2log(Q), the $\chi^2$ test statistic and the power divergence $2nI^{\lambda}$ for one model under another model is computed by the command `TEST;`. The power $\langle\lambda\rangle$ in computing the power divergence is by default set to 2/3. CoCo has to know two models to perform the test: the **current** model and a **base** model.

The model first read is both **base** and **current** and is given the model number 1. The first model stays **base** until another model is declared as **base**. Additional read models are given an increasing number. The last read model is the **current** model. The **current** model can be declared as **base** with the command `BASE;`. Model number

⟨*a*⟩ is named **base** or **current** with the commands `MAKE BASE` ⟨*a*⟩`;` and `MAKE CURRENT` ⟨*a*⟩`;` respectively.

In the example, the model {{ACE},{ADE},{BC},{F}} can be tested against {{ABCDEF}} by:

```
>read model *;
>base;
>read model ACE,ADE,BC,F.;
>test;
 Test of [[F][BC][ADE][ACE]]
 against [[ABCDEF]]

            Statistic       Asymptotic       Adjusted
 -2log(Q) =   62.0779   P =     0.0994  /      0.0994
 Power    =   59.7593   P =     0.1396  /      0.1396
 X^2      =   59.9956   P =     0.1350  /      0.1350
 DF.      =                        49  /          49
>
```

The first column of figures is the test statistic, the second is the asymptotic $p$-values based on an unadjusted number of degrees of freedom and the third is the $p$-values based on an adjusted number of degrees of freedom.

With `SET EXACT TEST On;` exact test is turned on. The exact test for one decomposable model against another decomposable model containing the model is then computed by `EXACT TEST;`:

```
>set exact test on;
 EXACT TEST  SET  ON
>exact test;
 Test of [[F][BC][ADE][ACE]]
 against [[ABCDEF]]

            Statistic       Asymptotic       Adjusted          Exact
 -2log(Q) =   62.0779   P =     0.0994  /      0.0994  /      0.1550
 Power    =   59.7593   P =     0.1396  /      0.1396  /      0.1630
 X^2      =   59.9956   P =     0.1350  /      0.1350  /      0.1570
 DF       =                        49  /          49
>
```

The following is an example of a model-search with the commands `BACKWARD;` and `FORWARD;`. Comments are inserted in the source-file with the character #. The following is a copy of the diary, edited a little: some blank lines removed, echo of comments removed, page-breaks removed and inserted:

```
# Version 1.3   Friday March 17 12:00:00 MET 1995
# Compiled with gcc, a C compiler for Sun4
# Compile-time: Mar 17 1995, 13:00:00.
# Copyright (c) 1991, by Jens Henrik Badsberg
# Licensed to ...
```

```
>#
># Set diary on:
>#
>set diary on;
 Diary set ON

>#
># Keep the log:
>#
>set keep log;
 Keep Log set ON

>#
># Set timer on:
>#
>set timer on;
 Timer set ON
 TIME:            0.002secs.

>#
># Set the input-file:
>#
>set input data reinis81.dat ;
 TIME:            0.003secs.

>#
># Read data
>#
>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:            0.025secs.

>#
># Read the saturated model:
>#
>read model *;
 TIME:            0.001secs.

>#
># Name this model "current" and then the "current" model "base"
>#
>current;
 TIME:            0.001secs.

>base;
 TIME:            0.001secs.
```

```
>#
># Choose exact test and
># compute also exact test P-value for Pearson's chi-spuare:
>#
>set exact test all;
 Exact test set ON
 Only Exact Deviance set OFF
 TIME:            0.001secs.

>#
># Let CoCo decide on the number of tables to generate:
>#
>set number of tables null;
 TIME:            0.001secs.

>#
># Ignore non-decomposable models:
>#
>set decomposable mode on;
 Decomposable mode set ON
 TIME:            0.001secs.

>#
># Do not compute the adjusted number of degrees of freedom:
>#
>set adjusted df off;
 Adjusted df set OFF
 TIME:            0.001secs.

>#
># Do not compute power divergence
>#
>set power lambda null;
 Power Divergence not printed
 TIME:            0.002secs.

>#
># Change the size of a page to fit into this guide:
>#
>set page formats 72 256;
 TIME:            0.002secs.

>#
># Change the test formats:
>#
>set test formats 8 3 6 4;
 TIME:            0.000secs.
```

```
>#
># Do a recursive backward-elimination,
># where tests are done against the previously accepted model (follow)
># and edge exclusions once rejected remain rejected (coherent).
># Print only the sorted list of tests (only sorted):
>#

>only  sorted  recursive  coherent  follow  backward;

 Sorted list

   Edge   DF -2log(Q)      P      X^2      P       Models

   [AC]   16   42.804 0.0005   41.617 0.0007  R [[ABCDEF]] / [[ABDEF][BCDEF]]
 Exact (   1000 )     0.0000          0.0000
   [BC]   16  684.989 0.0000  631.304 0.0000  R [[ABCDEF]] / [[ABDEF][ACDEF]]
 Exact (   1000 )     0.0000          0.0000
   [AE]   16   40.024 0.0010   38.762 0.0015  R [[ABCDEF]] / [[ABCDF][BCDEF]]
 Exact (   1000 )     0.0030          0.0020
   [DE]   16   31.059 0.0132   30.222 0.0168  R [[ABCDEF]] / [[ABCDF][ABCEF]]
 Exact (   1000 )     0.0310          0.0230
   [AD]   16   28.724 0.0257   27.465 0.0363  R [[ABCDEF]] / [[ABCEF][BCDEF]]
 Exact (   1000 )     0.0420          0.0380
   [AB]   16   22.652 0.1232   21.213 0.1702  R [[ABCDEF]] / [[ACDEF][BCDEF]]
 Exact (   1000 )     0.1740          0.1770
   [CF]   16   22.153 0.1381   20.746 0.1881  R [[ABCDEF]] / [[ABCDE][ABDEF]]
 Exact (    100 )     0.1800          0.1400
   [BF]   16   22.788 0.1193   23.124 0.1102  R [[ABCDEF]] / [[ABCDE][ACDEF]]
 Exact (   1000 )     0.1890          0.1100
   [AF]   16   21.305 0.1668   19.710 0.2331  R [[ABCDEF]] / [[ABCDE][BCDEF]]
 Exact (    100 )     0.2700          0.2600
   [CE]   16   18.629 0.2878   17.517 0.3526  R [[ABCDEF]] / [[ABCDF][ABDEF]]
 Exact (    100 )     0.3700          0.3800
   [DF]   16   18.345 0.3035   17.196 0.3728  R [[ABCDEF]] / [[ABCDE][ABCEF]]
 Exact (    100 )     0.3700          0.3700
   [EF]   16   18.316 0.3052   17.820 0.3341  R [[ABCDEF]] / [[ABCDE][ABCDF]]
 Exact (    200 )     0.4000          0.3450
   [BE]   16   17.226 0.3709   16.113 0.4454  R [[ABCDEF]] / [[ABCDF][ACDEF]]
 Exact (    100 )     0.4600          0.4600
   [CD]   16   14.808 0.5389   13.427 0.6422  R [[ABCDEF]] / [[ABCEF][ABDEF]]
 Exact (     20 )     0.6000          0.6500
   [BD]   16   12.226 0.7293   10.985 0.8109  R [[ABCDEF]] / [[ABCEF][ACDEF]]
 Exact (     20 )     0.8500          0.9500

 Edge selected:     [BD]
 Rejected edges:    [[AD][DE][AE][BC][AC]]
 Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
 Model:             [[ABCEF][ACDEF]]
 Edges eligible:    [[AB][AF][BE][BF][CD][CE][CF][DF][EF]]
```

```
  [AF]  [[ABCE][ACDE][BCEF][CDEF]] is not decomposable
  [CE]  [[ABCF][ABEF][ACDF][ADEF]] is not decomposable
  [CF]  [[ABCE][ABEF][ACDE][ADEF]] is not decomposable
  [EF]  [[ABCE][ABCF][ACDE][ACDF]] is not decomposable

Sorted list

  Edge   DF -2log(Q)  P      X^2      P       Models

  [AB]    8   15.574 0.0484   15.364 0.0519  R [[ABCEF]] / [[ACEF][BCEF]]
Exact (   1000 )     0.0530          0.0560
  [BF]    8   15.744 0.0457   16.733 0.0327  R [[ABCEF]] / [[ABCE][ACEF]]
Exact (    200 )     0.0850          0.0450
  [DF]    8   11.302 0.1845   11.478 0.1754  R [[ACDEF]] / [[ACDE][ACEF]]
Exact (    200 )     0.1750          0.1550
  [BE]    8   11.364 0.1812   11.268 0.1863  R [[ABCEF]] / [[ABCF][ACEF]]
Exact (    200 )     0.1850          0.1850
  [CD]    8    7.149 0.5214    7.067 0.5301  R [[ACDEF]] / [[ACEF][ADEF]]
Exact (    100 )     0.5600          0.5700

Edge selected:     [CD]
Rejected edges:    [[AD][DE][AE][BC][AC]]
Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
Model:             [[ABCEF][ADEF]]
Edges eligible:    [[EF][DF][CF][CE][BF][BE][AF][AB]]

  [EF]  [[ABCE][ABCF][ADE][ADF]] is not decomposable
  [AF]  [[ABCE][ADE][BCEF][DEF]] is not decomposable

Sorted list

  Edge   DF -2log(Q)  P      X^2      P       Models

  [AB]    8   15.574 0.0484   15.364 0.0519  R [[ABCEF]] / [[ACEF][BCEF]]
Exact (   1000 )     0.0530          0.0560
  [BF]    8   15.744 0.0457   16.733 0.0327  R [[ABCEF]] / [[ABCE][ACEF]]
Exact (    200 )     0.0850          0.0450
  [CF]    8   12.237 0.1403   11.687 0.1651  R [[ABCEF]] / [[ABCE][ABEF]]
Exact (    100 )     0.1300          0.1300
  [BE]    8   11.364 0.1812   11.268 0.1863  R [[ABCEF]] / [[ABCF][ACEF]]
Exact (    200 )     0.1850          0.1850
  [DF]    4    6.368 0.1733    6.583 0.1596  R [[ADEF]] / [[ADE][AEF]]
Exact (    200 )     0.1950          0.1750
  [CE]    8   12.735 0.1207   12.666 0.1233  R [[ABCEF]] / [[ABCF][ABEF]]
Exact (    200 )     0.2000          0.1950

Edge selected:     [CE]
Rejected edges:    [[AD][DE][AE][BC][AC]]
Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
Model:             [[ABCF][ABEF][ADEF]]
Edges eligible:    [[AB][AF][BE][BF][CF][DF][EF]]
```

```
  [AB]  [[ACF][ADEF][BCF][BEF]] is not decomposable
  [AF]  [[ABC][ABE][ADE][BCF][BEF][DEF]] is not decomposable
  [BF]  [[ABC][ABE][ACF][ADEF]] is not decomposable
  [EF]  [[ABCF][ABE][ADE][ADF]] is not decomposable

Sorted list

  Edge   DF -2log(Q)  P      X^2      P       Models

  [BE]    4   21.159 0.0003   21.168 0.0003  R [[ABEF]] / [[ABF][AEF]]
Exact (   1000 )     0.0000          0.0000
  [CF]    4    7.018 0.1349    7.459 0.1135  R [[ABCF]] / [[ABC][ABF]]
Exact (   1000 )     0.1320          0.0930
  [DF]    4    6.368 0.1733    6.583 0.1596  R [[ADEF]] / [[ADE][AEF]]
Exact (    200 )     0.1950          0.1750

Edge selected:     [DF]
Rejected edges:    [[BE][AD][DE][AE][BC][AC]]
Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
Model:             [[ABCF][ABEF][ADE]]
Edges eligible:    [[EF][CF][BF][AF][AB]]

  [BF]  [[ABC][ABE][ACF][ADE][AEF]] is not decomposable
  [AF]  [[ABC][ABE][ADE][BCF][BEF]] is not decomposable
  [AB]  [[ACF][ADE][AEF][BCF][BEF]] is not decomposable

Sorted list

  Edge   DF -2log(Q)  P      X^2      P       Models

  [CF]    4    7.018 0.1349    7.459 0.1135  R [[ABCF]] / [[ABC][ABF]]
Exact (   1000 )     0.1320          0.0930
  [EF]    4    3.951 0.4127    4.075 0.3959  R [[ABEF]] / [[ABE][ABF]]
Exact (    100 )     0.3900          0.3800

Edge selected:     [EF]
Rejected edges:    [[BE][AD][DE][AE][BC][AC]]
Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
Model:             [[ABCF][ABE][ADE]]
Edges eligible:    [[AB][AF][BF][CF]]

  [AB]  [[ACF][ADE][BCF][BE]] is not decomposable

Sorted list

  Edge   DF -2log(Q)  P      X^2      P       Models

  [BF]    4   11.151 0.0249   13.302 0.0099  R [[ABCF]] / [[ABC][ACF]]
Exact (   1000 )     0.0270          0.0060
  [AF]    4    7.386 0.1169    7.336 0.1192  R [[ABCF]] / [[ABC][BCF]]
Exact (    200 )     0.1200          0.1200
```

```
   [CF]    4    7.018 0.1349    7.459 0.1135  R [[ABCF]] / [[ABC][ABF]]
 Exact (   1000 )     0.1320          0.0930

 Edge selected:     [CF]
 Rejected edges:    [[BF][BE][AD][DE][AE][BC][AC]]
 Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
 Model:             [[ABC][ABE][ABF][ADE]]
 Edges eligible:    [[AF][AB]]

   [AB]  [[AC][ADE][AF][BC][BE][BF]] is not decomposable

 Sorted list

   Edge   DF -2log(Q)  P      X^2      P       Models

   [AF]    2    2.658 0.2647    2.648 0.2661  R [[ABF]] / [[AB][BF]]
 Exact (    100 )     0.2300          0.2300

 Edge selected:     [AF]
 Rejected edges:    [[BF][BE][AD][DE][AE][BC][AC]]
 Accepted edges:    [[BD][CD][BE][EF][DF][CE][AF][BF][CF][AB]]
 Model:             [[ABC][ABE][ADE][BF]]
 Edges eligible:    [[AB]]

   [AB]  [[AC][ADE][BC][BE][BF]] is not decomposable
 TIME:           12.096secs.

>#
># Print all models, the sequence of accepted models:
>#
>print all models;
 Model no.   8 [[ABC][ABE][ADE][BF]]
 Model no.   7 [[ABC][ABE][ABF][ADE]]
 Model no.   6 [[ABCF][ABE][ADE]]
 Model no.   5 [[ABCF][ABEF][ADE]]
 Model no.   4 [[ABCF][ABEF][ADEF]]
 Model no.   3 [[ABCEF][ADEF]]
 Model no.   2 [[ABCEF][ACDEF]]
 Model no.   1 [[ABCDEF]] /BASE/    /CURRENT/
 TIME:            0.005secs.

>#
># Name the last accepted model "current"
>#
>current;
 TIME:            0.001secs.
```

```
>#
># Name "current" model "base"
>#
>base;
 TIME:            0.000secs.

>#
># Print all models:
>#
>print all models;
 Model no.   8 [[ABC][ABE][ADE][BF]] /BASE/    /CURRENT/
 Model no.   7 [[ABC][ABE][ABF][ADE]]
 Model no.   6 [[ABCF][ABE][ADE]]
 Model no.   5 [[ABCF][ABEF][ADE]]
 Model no.   4 [[ABCF][ABEF][ADEF]]
 Model no.   3 [[ABCEF][ADEF]]
 Model no.   2 [[ABCEF][ACDEF]]
 Model no.   1 [[ABCDEF]]
 TIME:            0.005secs.

>#
># Can an edge rejected by 'coherence' now be rejected:
>#
>backward;

   Edge   DF -2log(Q)  P      X^2      P       Models

   [AB]  [[AC][ADE][BC][BE][BF]] is not decomposable
   [AC]    2   23.805 0.0000   23.929 0.0000    [[ABC]] / [[AB][BC]]
 Exact (   1000 )     0.0000          0.0000
   [AD]    2   16.542 0.0003   16.427 0.0003    [[ADE]] / [[AE][DE]]
 Exact (   1000 )     0.0000          0.0000
   [AE]  [[ABC][AD][BE][BF][DE]] is not decomposable
   [BC]    2  682.296 0.0000  631.416 0.0000    [[ABC]] / [[AB][AC]]
 Exact (   1000 )     0.0000          0.0000
   [BE]    2   20.929 0.0000   20.930 0.0000    [[ABE]] / [[AB][AE]]
 Exact (   1000 )     0.0000          0.0000
   [BF]    1    4.732 0.0296    4.772 0.0289    [[BF]] / [[B][F]]
 Exact (   1000 )     0.0300          0.0300
   [DE]    2   18.319 0.0001   18.298 0.0001    [[ADE]] / [[AD][AE]]
 Exact (   1000 )     0.0000          0.0000
 TIME:            1.561secs.

>#
># Did accept wrong edges in the start:
>#
>forward;
```

```
   Edge   DF -2log(Q)  P      X^2      P       Models

   [AF]    2    2.658 0.2647    2.648 0.2661  R [[ABF]] / [[AB][BF]]
 Exact (    100 )     0.2300          0.2300
   [BD]    4    2.504 0.6439    2.502 0.6443    [[ABDE]] / [[ABE][ADE]]
 Exact (     20 )     0.7000          0.7000
   [CD]  [[ABC][ABE][ACD][ADE][BF]] is not decomposable
   [CE]    4    7.516 0.1110    7.602 0.1073    [[ABCE]] / [[ABC][ABE]]
 Exact (   1000 )     0.1010          0.0940
   [CF]    2    2.290 0.3182    2.426 0.2974    [[BCF]] / [[BC][BF]]
 Exact (    100 )     0.3300          0.3100
   [DF]  [[ABC][ABE][ADE][BF][DF]] is not decomposable
   [EF]    2    2.405 0.3004    2.427 0.2972    [[BEF]] / [[BE][BF]]
 Exact (    100 )     0.2700          0.2700
 TIME:            0.523secs.

>#
># Set DecomposableMode off to compute
># asymptotic P-values for non-decomposable models:
>#
>set decomposable mode off;
 Decomposable mode set OFF
 TIME:            0.001secs.

>#
># Compute asymptotic P-values
>#
>backward;

   Edge   DF -2log(Q)  P      X^2      P       Models

   [AB]    3    6.177 0.1033    6.162 0.1040    [[ABE][ABC]] / [[BE][BC]
                                                  [AE][AC]]
   [AC]    2   23.805 0.0000   23.929 0.0000  R [[ABC]] / [[AB][BC]]
 Exact (   1000 )     0.0000          0.0000
   [AD]    2   16.542 0.0003   16.427 0.0003  R [[ADE]] / [[AE][DE]]
 Exact (   1000 )     0.0000          0.0000
   [AE]    3   26.077 0.0000   25.991 0.0000    [[ADE][ABE]] / [[DE][BE]
                                                  [AD][AB]]
   [BC]    2  682.296 0.0000  631.416 0.0000  R [[ABC]] / [[AB][AC]]
 Exact (   1000 )     0.0000          0.0000
   [BE]    2   20.929 0.0000   20.930 0.0000  R [[ABE]] / [[AB][AE]]
 Exact (   1000 )     0.0000          0.0000
   [BF]    1    4.732 0.0296    4.772 0.0289  R [[BF]] / [[B][F]]
 Exact (   1000 )     0.0300          0.0300
   [DE]    2   18.319 0.0001   18.298 0.0001  R [[ADE]] / [[AD][AE]]
 Exact (   1000 )     0.0000          0.0000
 TIME:            0.076secs.

>forward;
```

```
   Edge   DF -2log(Q)  P      X^2      P       Models

   [AF]    2    2.658 0.2647    2.648 0.2661  R [[ABF]] / [[AB][BF]]
 Exact (    100 )     0.2300          0.2300
   [BD]    4    2.504 0.6439    2.502 0.6443  R [[ABDE]] / [[ABE][ADE]]
 Exact (     20 )     0.7000          0.7000
   [CD]    2    1.084 0.5817    1.084 0.5817    [[ABC][ABE][ACD][ADE]]
                                                 / [[ABC][ABE][ADE]]
   [CE]    4    7.516 0.1110    7.602 0.1073  R [[ABCE]] / [[ABC][ABE]]
 Exact (   1000 )     0.1010          0.0940
   [CF]    2    2.290 0.3182    2.426 0.2974  R [[BCF]] / [[BC][BF]]
 Exact (    100 )     0.3300          0.3100
   [DF]    1    1.054 0.3045    1.059 0.3035    [[ABE][ADE][BF][DF]]
                                                 / [[ABE][ADE][BF]]
   [EF]    2    2.405 0.3004    2.427 0.2972  R [[BEF]] / [[BE][BF]]
 Exact (    100 )     0.2700          0.2700
 TIME:            0.072secs.

>#
># Try to drop the significant interaction ABC:
>#
>drop interaction ABC.;
   9: [[BF][BC][AC][ABE][ADE]]
 Model is not graphical
 Cliques:[[ABC][ABE][ADE][BF]]
 2-Section Graph is decomposable
 Test of [[BF][BC][AC][ABE][ADE]]
 against [[ABC][ABE][ADE][BF]]

          DF -2log(Q)  P      X^2      P       Models

           1    5.986 0.0144    5.951 0.0147    [[ABC]] / [[BC][AC][AB]]

 TIME:            0.026secs.

>#
># Drop the edge AB, and accept a non-decomposable model:
>#
>drop edge AB.;
  10: [[AC][ADE][BC][BE][BF]]
 Model is graphical
 Graph is not decomposable
 Generating class for Fill In: [[ACE][ADE][BCE][BF]]
 Test of [[AC][ADE][BC][BE][BF]]
 against [[ABC][ABE][ADE][BF]]

          DF -2log(Q)  P      X^2      P       Models

           3    6.177 0.1033    6.162 0.1040    [[ABE][ABC]] / [[BE][BC]
                                                  [AE][AC]]
 TIME:            0.032secs.
```

```
>#
># Name the last accepted model "current"
>#
>current;
 TIME:            0.001secs.

>#
># Name "current" model "base"
>#
>base;
 TIME:            0.001secs.

>#
># Print all models:
>#
>print all models;
 Model no.  10 [[AC][ADE][BC][BE][BF]] /BASE/    /CURRENT/
 Model no.   9 [[BF][BC][AC][ABE][ADE]]
 Model no.   8 [[ABC][ABE][ADE][BF]]
 Model no.   7 [[ABC][ABE][ABF][ADE]]
 Model no.   6 [[ABCF][ABE][ADE]]
 Model no.   5 [[ABCF][ABEF][ADE]]
 Model no.   4 [[ABCF][ABEF][ADEF]]
 Model no.   3 [[ABCEF][ADEF]]
 Model no.   2 [[ABCEF][ACDEF]]
 Model no.   1 [[ABCDEF]]
 TIME:            0.006secs.

>#
># No edges can be removed from this model:
>#
>backward;

   Edge   DF -2log(Q)  P      X^2      P       Models

   [AC]    1   30.292 0.0000   30.211 0.0000    [[BE][BC][AE][AC]]
                                                 / [[BE][BC][AE]]
   [AD]    2   16.542 0.0003   16.427 0.0003  R [[ADE]] / [[AE][DE]]
 Exact (   1000 )     0.0000          0.0000
   [AE]    2   25.887 0.0000   25.828 0.0000    [[AC][ADE][BC][BE]]
                                                 / [[AC][AD][BC][BE][DE]]
   [BC]    1  688.782 0.0000  638.896 0.0000    [[BE][BC][AE][AC]]
                                                 / [[BE][AE][AC]]
   [BE]    1   20.740 0.0000   20.767 0.0000    [[BE][BC][AE][AC]]
                                                 / [[BC][AE][AC]]
   [BF]    1    4.732 0.0296    4.772 0.0289    [[BF]] / [[B][F]]
 Exact (   1000 )     0.0300          0.0300
   [DE]    2   18.319 0.0001   18.298 0.0001  R [[ADE]] / [[AD][AE]]
 Exact (   1000 )     0.0000          0.0000
 TIME:            0.135secs.
```

```
>#
># No edges should be added to the found model:
>#
>forward;

   Edge   DF -2log(Q)  P      X^2      P       Models

   [AB]    3    6.177 0.1033    6.162 0.1040    [[ABE][ABC]] / [[BE][BC]
                                                 [AE][AC]]
   [AF]    1    1.388 0.2388    1.389 0.2385    [[AC][AE][AF][BC][BE][BF]]
                                                 / [[AC][AE][BC][BE][BF]]
   [BD]    2    1.397 0.4974    1.396 0.4976    [[AC][ADE][BC][BDE]]
                                                 / [[AC][ADE][BC][BE]]
   [CD]    2    1.082 0.5823    1.082 0.5823    [[ACD][ADE][BC][BE]]
                                                 / [[AC][ADE][BC][BE]]
   [CE]    3    7.365 0.0611    7.456 0.0587    [[BCE][ACE]] / [[BE][BC]
                                                 [AE][AC]]
   [CF]    2    2.290 0.3182    2.426 0.2974    [[BCF]] / [[BC][BF]]
 Exact (    100 )     0.3300          0.3100
   [DF]    1    1.057 0.3038    1.062 0.3027    [[DF][BF][BE][BC]
                                                 [ADE][AC]] / [[BF][BE]
                                                 [BC][ADE][AC]]
   [EF]    2    2.405 0.3004    2.427 0.2972    [[BEF]] / [[BE][BF]]
 Exact (    100 )     0.2700          0.2700
 TIME:            0.214secs.

>#
># Print all models:
>#
>print all models;
 Model no.  10 [[AC][ADE][BC][BE][BF]] /BASE/    /CURRENT/
 Model no.   9 [[BF][BC][AC][ABE][ADE]]
 Model no.   8 [[ABC][ABE][ADE][BF]]
 Model no.   7 [[ABC][ABE][ABF][ADE]]
 Model no.   6 [[ABCF][ABE][ADE]]
 Model no.   5 [[ABCF][ABEF][ADE]]
 Model no.   4 [[ABCF][ABEF][ADEF]]
 Model no.   3 [[ABCEF][ADEF]]
 Model no.   2 [[ABCEF][ACDEF]]
 Model no.   1 [[ABCDEF]]
 TIME:            0.007secs.

>#
># Exit CoCo
>#
>quit;
 TIME:            0.002secs.
 TOTAL TIME:     15.017secs.
```

The model {{ADE},{AC},{BC},{BE},{BF}} found by coherent backward elimination with exact and local tests on the data from Reiniš et al. (1981) adds the

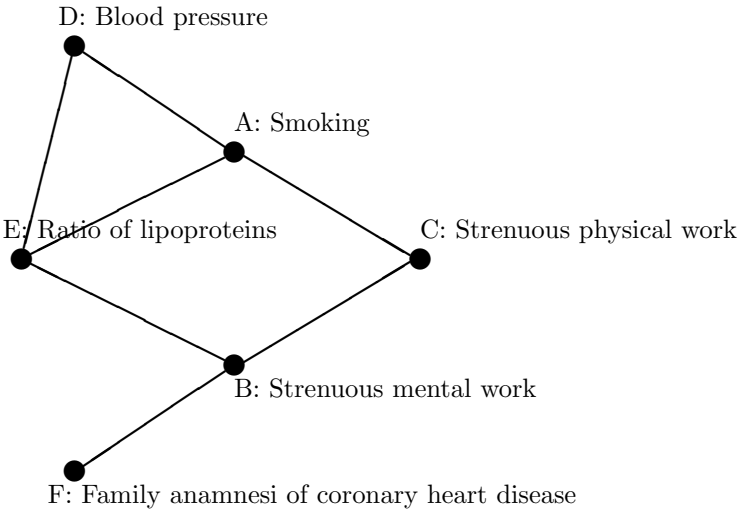



Figure 1.1: A final model from *recursive coherent backward elimination* with *local and exact tests* on data from Reinis et al. 81. The TeX-code of the figure is written by Xlisp+CoCo.

edge $< BF >$ to the first of the two models {{ADE},{AC},{BC},{BE},{F}} and {{ACE},{ADE},{BC},{F}} found in Edwards & Havránek (1985). Note also that we in the search observed that the interaction $< ABC >$ is significant.

The model {{ACE},{ADE},{BC},{F}} from Edwards & Havránek (1985) will in the next part of the guide often be used as an example. This model is selected mainly because it is decomposable.

The log-file after the run of CoCo to make output in this chapter looks like:

```
#
# Name the diary-file:
#
set output diary diary.2 ;
#
# Set diary on:
#
set diary on;
#
# Keep the log:
#
set keep log;
```

```
#
# Set timer on:
#
set timer on;
#
# Set the input-file:
#
set input data reinis81.dat ;
#
# Read data
#
read data;
#Data: Factors A2/B2/C2/D2/E2/F2//
#Data:
#Data: Table
#Data:  44  40  112  67
#Data: 129 145   12  23
#Data:  35  12   80  33
#Data: 109  67    7   9
#Data:  23  32   70  66
#Data:  50  80    7  13
#Data:  24  25   73  57
#Data:  51  63    7  16
#Data:   5   7   21   9
#Data:   9  17    1   4
#Data:   4   3   11   8
#Data:  14  17    5   2
#Data:   7   3   14  14
#Data:   9  16    2   3
#Data:   4   0   13  11
#Data:   5  14    4   4
#
# Change the print-width used when printing tables:
#
set table formats 5 2 2 2;
#
# Print counts in the full table:
#
print observed ABCDEF.;
#
# Print counts in the marginal table AB:
#
print observed AB.;
#
# Print the total count:
#
print observed .;
#
# Change the size of a page to fit into this guide:
#
set page formats 72 52;
```

```
>#
># Describe the observed counts in the saturated table
># (print also a uniform plot):
>#
>describe table uniform observed * ;
#
# Read model ACE,ADE,BC,F.
#
read model ACE,ADE,BC,F.;
#
# Change the page-size
#
set page formats 72 36;
#
# Describe the adjusted residuals in the marginal table BCDEF.
# Print a 'rankit'-plot:
#
describe table rankit adjusted BCDEF.;
#
# Make a plot of the adjusted residuals against observed counts
#
plot observed adjusted *;
#
# Read the saturated model:
#
read model *;
#
# Name this model "current" and then the "current" model "base"
#
current;
base;
#
# Choose exact test and
# compute also exact test P-value for Pearson's chi-spuare:
#
set exact test all;
#
# Let CoCo decide on the number of tables to generate:
#
set number of tables null;
#
# Ignore non-decomposable models:
#
set decomposable mode on;
#
# Do not compute the adjusted number of degrees of freedom:
#
set adjusted df off;
```

```
#
# Do not compute power divergence
#
set power lambda null;
#
# Change the size of a page to fit into this guide:
#
set page formats 72 256;
#
# Change the test formats:
#
set test formats 8 3 6 4;
#
# Do a recursive backward-elimination,
# where tests are done against the previously accepted model (follow)
# and edge exclusions once rejected remain rejected (coherent).
# Print only the sorted list of tests (only sorted):
#
only sorted recursive coherent follow backward;
#
# Print all models, the sequence of accepted models:
#
print all models;
#
# Name the last accepted model "current"
#
current;
#
# Name "current" model "base"
#
base;
#
# Print all models:
#
print all models;
#
# Can an edge rejected by 'coherence' now be rejected:
#
backward;
#
# Did accept wrong edges in the start:
#
forward;
#
# Set DecomposableMode off to compute
# asymptotic P-values for non-decomposable models:
#
set decomposable mode off;
```

```
#
# Compute asymptotic P-values
#
backward;
#
# Compute asymptotic P-values
#
forward;
#
# Try to drop the significant interaction ABC:
#
drop interaction ABC.;
#
# Drop the edge AB, and accept a non-decomposable model:
#
drop edge AB.;
#
# Name the last accepted model "current"
#
current;
#
# Name "current" model "base"
#
base;
#
# Print all models:
#
print all models;
#
# No edges can be removed from this model:
#
backward;
#
# No edges should be added to the found model:
#
forward;
#
# Print all models:
#
print all models;
#
# Exit CoCo
#
quit;
```

It might be read as input to CoCo by `coco < log` or the command `SOURCE log;` in CoCo.

# Part II

# User's Manual

# Chapter 2

# Commands

Command-names in this guide are written in upper-case, like in the command `HELP;`. Arguments to CoCo-commands are capitalized, if they are keywords, like in the command `SET DIARY [ On | Off ];` or, if they are numbers, names, sets, models, etc. surrounded by ⟨ and ⟩, like in `MENU NUMBER` ⟨*a*⟩`;`.

Keywords and arguments surrounded by [ and ] are optional. If keywords are surrounded by [ and ] and separated by `|` either one of the keywords can be given. If keywords or arguments are surrounded by { and } and separated by `|` one of the keywords or arguments must be given.

When reading commands, CoCo does not distinguish between lower and upper-case letters. But note that CoCo distinguishes between lower and upper-case letters when reading factor names. Several commands separated by `;` can be entered on the same line. A command consisting of more than one word can be divided between two lines with `&`.

The current version of CoCo only uses the first 4 characters of each word of a command name to recognize the command. Commands do not have to be ended by the character `;`. EndOfLine, "Return", works as well.

## 2.1 Online HELP

`HELP [` ⟨*command-name*⟩ `|` ⟨*abbreviation*⟩ `];`
`MENU NUMBER` ⟨*a*⟩`;`
`NEXT MENU;`
`CURRENT MENU;`
`PREVIOUS MENU;`

An introduction to CoCo is given by the command `HELP;` without any arguments. The command `HELP;` can be given a command-name as an argument: The command `HELP` ⟨*command-name*⟩`;` will print the full command-name with arguments, and a description of the command with name ⟨*command-name*⟩ is printed (when this description is entered in the `help-file`). `HELP help;` will give a description of how to use "help".

If only the beginning of a command-name is given to the help-command, `HELP` ⟨*start of command-name*⟩`;`, then CoCo will show expected continuation of the beginning of command-names, e.g., `HELP read;` or `HELP set;`, and some documentation for that beginning might be printed.

`HELP` ⟨*value-name*⟩`;` will give documentation for the table-value ⟨*value-name*⟩. `HELP` ⟨*keyword*⟩`;` is used to print documentation for special topics like limits, overlays, data, etc., See `HELP help;`.

`MENU NUMBER 0;` will print a quick-reference-card. With `MENU NUMBER` ⟨*a*⟩`;`, `NEXT MENU;`, `CURRENT MENU;`, `PREVIOUS MENU;` it is possible to step through the quick-reference-card.

If a command is removed from CoCo, then `HELP` ⟨*command-name*⟩`;` will not be able to reach help information on how the command is replaced. See `HELP help;` or `HELP removed;`.

## 2.2 Comments

If `#` is the first character of a command, then characters from `#` to `;` or end of line are skipped.

## 2.3 Make, Delete and Print Alias

`PRINT ALIAS` { ⟨*command-name*⟩ `|` ⟨*abbreviation*⟩ }`;`
`MAKE ALIAS` ⟨*command-name*⟩ ⟨*abbreviation*⟩`;`
`DELETE ALIAS` ⟨*abbreviation*⟩`;`

Abbreviations for commands with long names can be seen with the command `PRINT ALIAS` ⟨*command-name*⟩`;`. The command `HELP` ⟨*abbreviation*⟩`;` will print the full command-name with arguments for the command with abbreviation ⟨*abbreviation*⟩. New abbreviations can be declared with the command `MAKE ALIAS` ⟨*command-name*⟩ ⟨*abbreviation*⟩`;`. An abbreviation is deleted with `DELETE ALIAS` ⟨*abbreviation*⟩`;`.

The author uses the following abbreviations:

```
make alias quit                  end ;
make alias end                   e ;
make alias set inputfile data    sid ;
make alias set outputfile diary  sod ;
make alias set outputfile report sor ;
make alias read data             rd ;
make alias read specification    rs ;
make alias read observations     ro ;
make alias read model            rm ;
make alias print formula         pf ;
make alias print all models      pam ;
```

These abbreviations can be placed on the file, e.g., `my.alias`, and read by the command `SOURCE` ⟨*file-name*⟩`;`: `SOURCE my.alias;`. See also the next section about

the local parser. The file `.cocorc` (`COCO.SRC` under DOS) is read, when CoCo is started. See also the chapter 11 about source files.

## 2.4 Reading a Local Parser

```
READ LOCAL PARSER;
```

If you are running a public CoCo on workstations and declaring new abbreviations, a local version of the parser-table is generated in the current directory. Next time you start CoCo up in that directory, the local version of the parser table with your abbreviations can be read in with `READ LOCAL PARSER;`.

The following two commands are of no interest to the normal user:

```
INIT PARSER;
NO ACTION;
```

`INIT PARSER;` is used by the author to create a new parser-table from the file `INIT.TAB`. With the command `NO ACTION;` CoCo enters a mode, where commands are read, but no action is performed. Exit this mode by `END;`.

## 2.5 Sets and Models

A set is entered as a list of factor names enclosed in sharp parentheses; e.g., the set with the three variables `A`, `B` and `C` is entered as

```
[ABC]
```

A model (generating class, e.g., in the command `READ MODEL` ⟨*gc*⟩`;`) is entered as a list of sets enclosed in sharp parentheses; for example, the model with the two generators <AB> and <BC> is entered as

```
[[AB][BC]]
```

A shorter form can be used. Comma (`,`) ends sets whereas period (`.`) ends both sets and generating classes so the set above set with the three variables `A`, `B` and `C` can be also be entered as

```
ABC.
```

and the above model can be entered as

```
AB,BC.
```

EndOfLine, “Return”, ends sets, generating classes and sets of generating classes unless the EndOfLine is presided by the character `&`.

The asterisk `*` is an abbreviation for the set containing all declared factors and for the saturated model. Period (`.`, with no factors preceding) is an abbreviation for the model with only main-effects. The uniform model (model with no effects) can be entered as `;` or `[[]]`.

### 2.5.1 Variable names of length longer than one character

If variable names longer than one character are used, then the first character of each name is `:`, and sets are entered as, e.g.,

```
[:Sex:Smoking:Drinking]
```

or

```
:Sex:Smoking:Drinking.
```

and models can be entered as, e.g.,

```
[[:Sex:Smoking][:Smoking:Drinking]]
```

or

```
:Sex:Smoking,:Smoking:Drinking.
```

## 2.6 Pausing and Front End Processor: History Mechanism

No history-mechanism is implemented in CoCo. On Unix: Use '`fep`' (If not available on your system, get your system administrator to install it), or run CoCo as a sub-process in '`emacs`'. On DOS: use the F3-key.

`SET PAUSE [ On | Off ];`
`SET PAGING LENGTH` ⟨*number of lines*⟩`;`

At some commands, and especially on some fast machines, it has become a problem to reach to read output from CoCo before it scrolls out of the top of the screen.

With `Pausing` set on by `SET PAUSE [ On | Off ];` (Default: Off) the output is paused for each written 22 lines (after last NewPage), if EndOfLine is found after the total command is read by CoCo. E.g., if `Pausing` is on, then CoCo will pause for each printed 22 lines after `STATUS;`. If `Pausing` is on, then CoCo will not pause during the status command, if `STATUS;;` or `STATUS; READ MODEL A,B.;` is entered. The next 22 lines are printed by pressing "Return". If a `;` is entered when pausing, then the command will finish without pausing. The number of lines to print between pausing are set by `SET PAGING LENGTH` ⟨*number of lines*⟩`;` (Default: 22).

Note that not only printing but also computation in CoCo are paused. It is not recommended to pipe the output from CoCo into pagers like '`more`' or '`less`' since you will not see entered characters to CoCo before you have typed 'return'. Use '`cmdtool`', run CoCo as a sub-process in '`emacs`', use Control-S and Control-Q or set the diary on, and use some '`pager`' on the diary.

A log of the CoCo-session is kept. Unless you use the command `SET KEEP LOG On;` or change the `LogFileName`, the `LogFile` is removed on Unix when CoCo terminates normally. See chapter 11 about the `LogFile` and use the command `STATUS Files;` to see default file-names.

## 2.7 Print-formats, Tests, Diary, Source, Sink, Log, etc.

See chapter 11: Options.

## 2.8 Interrupts

On Unix a Control-C will terminate the IPS-algorithm and the computation of an exact test. Control-\ or two close Control-C's will terminate the EM-algorithm, factorization of a test, a `BACKWARD;`-command , a `FORWARD;`-command or the computation of a dual in the EH-procedure. See section 11.7: "Interrupts" in chapter 11: "Miscellaneous Options for ..." for how to set interrupts off.

# Chapter 3

# Reading Data

The command `READ DATA;` is used to read *observations* and *specifications* of factors from input or from a file.

## 3.1 Keyboard and Files

`SET KEYBOARD [ On | Off ];`

`SET INPUTFILE DATA` ⟨*file-name*⟩`;`
`SET INPUTFILE SPECIFICATION` ⟨*file-name*⟩`;`
`SET INPUTFILE OBSERVATIONS` ⟨*file-name*⟩`;`

`READ DATA [` ⟨*specification*⟩ ⟨*observations*⟩ `];`

`READ SPECIFICATION [` ⟨*specification*⟩ `];`
`READ FACTORS [` ⟨*factor-list*⟩ `];`
`READ NAMES [` ⟨*name-list*⟩ `];`

`READ OBSERVATIONS [` ⟨*observations*⟩ `];`
`READ TABLE [` ⟨*table*⟩ `];`
`READ LIST [` ⟨*list of cases*⟩ `];`

The data consists of two parts: specification and observations. The specification is the definition of variables and table. The observations are the cell counts in the table. Data can either be read from keyboard or file. This is controlled by the command `SET KEYBOARD [ On | Off ];`. The following sections will describe how the specification and observations have to be formatted.

When reading from file, the file with data (specification and observations) is named (and reset) with the command `SET INPUTFILE DATA` ⟨*file-name*⟩`;`. If specification and observations are not in the same file, the two files are named and reset with `SET INPUTFILE SPECIFICATION` ⟨*file-name*⟩`;` and `SET INPUTFILE OBSERVATIONS` ⟨*file-name*⟩`;`.

After naming files, the specification and observations are read with `READ DATA;` without arguments. If data are read from key-board, the specification and observations are expected as arguments to `READ DATA [` ⟨*specification*⟩ ⟨*observations*⟩ `];`.

If data selection is to be done, it has to be declared between reading specification and observations, so `READ DATA [` ⟨*specification*⟩ ⟨*observations*⟩ `];` has to be divided in to the two actions `READ SPECIFICATION [` ⟨*specification*⟩ `];` and `READ OBSERVATIONS [` ⟨*observations*⟩ `];` See section 3.8 on data selection.

The command `READ LIST [` ⟨*list of cases*⟩ `];` is the same command as `READ OBSERVATIONS [` ⟨*observations*⟩ `];` except that the key-word `LIST;` is not expected. Analogous to `READ NAMES [` ⟨*name-list*⟩ `];`, `READ FACTORS [` ⟨*factor-list*⟩ `];` and `READ TABLE [` ⟨*table*⟩ `];`. On the key-words `Names`, `List`, `Factors` and `Table`, see the following sections.

Specifications and observations are defaults read from the file `COCO.DAT` in the home directory for CoCo.

```
SET DATASTRUCTURE { All | Necessary | File | Large };
SET LARGE [ On | Off ];
SET SORTED [ On | Off ];
```

These commands control the selected data structure, see the chapter 11 “Options”.

## 3.2 Specification

The specification declares the number of factors, order of factors and name and number of levels (non-missing and missing levels) for each factor. Factor-names in CoCo consist either all of a single character, or names of any length can be used. If factor-names consist of only a single character, then the name can be any character except slash (`/`). E.g., `a .. z`, `A .. Z`, `0 .. 9`, (and `?`, `!`, `@`, `$`, `#`, `%`, ∧, `_`, `-`, +, `=`, `|`, \, ∼, ‘, ’ and `"`). (Some of the special characters may be reserved in later versions of CoCo.) Note that CoCo distinguishes between lower-case and upper-case letters when reading factor names. The characters `#`, `*`, `,`, `.`, `;`, `:`, `/`, and parentheses (`(`, `)`, `[`, `]`, <, >, { and } ) can be used as factor names, but should not, since it would cause trouble when reading sets and models.

If names consisting of more than one character are to be used, then all names must be declared with `:` as the first character.

The specification starts either with the key-word `Factors` or with the key-word `Names`:

### 3.2.1 Factor by Factor: Factors

If `Factors` is found, a list of items separated by `/` and ended by `//` is expected. The number of items defines the number of factors, and the order of the items defines the order of factors used when reading data. Each item consists of a name (a character), number of non-missing levels (an integer) and an optional number of levels to be interpreted as missing values (see section 3.9 Missing Values: Incomplete observations); for example,

```
Factors A 2 / B 2 / C 2 1 //
```

or

```
Factors :Sex 2 / :Smoking 2 / :Drinking 2 1 //
```

### 3.2.2 Attributes one by one: Names

These specification could also be entered as

```
Names A B C / 2 2 2 / 0 0 1
```

and

```
Names :Sex :Smoking :Drinking / 2 2 2 / 0 0 1
```

respectively.

After the key-word `Names` a list of characters giving names and order of factors is expected. Then a list of integers giving number of non-missing levels for each factor is expected. An optional list of number of levels for each factor to be marked as missing can be entered. If this list is not entered, end the list of numbers of non-missing levels with `//`; for example

```
Names A B C / 2 2 2 //
```

Factor-levels are integers from 1 up to the total number of levels (number of non-missing levels plus the number of levels marked as missing).

New users are often confused by the above two ways of declaring the variables. The form `Factors` is useful to specify the factors one by one. `Names` is useful if the data is entered as a list. The names, number of levels and number of levels to marked as missing can then be written as headings to the columns of levels for each factor. The form `Names` is also used when defining a table from S-Plus or XLISP-STAT, see chapter 12.

Use `STATUS Specification;` to see the entered specification.

## 3.3 Ordinal Variables

`SET ORDINAL` ⟨*set*⟩`;`

With this command the factors of the subset ⟨*set*⟩ of the factors are declared to be ordinal. Then when two factors are tested conditionally independent Goodman and Kruskal's Gamma coefficient is also computed. The argument ⟨*set*⟩ has to be a set with the names of the ordinal factors.

## 3.4 Observations

Observations can either be entered as a table or as a list. The table is read cell by cell, in a standard cell order. A list is a list of cases, each case consisting of the levels of the factors in the same order as the factors are defined or some values to be recoded by the command `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;`. Characters (including `.` and `,` when no cutpoints for the variable reading is declared) not a digit are interpreted as separators between integers. (The characters `.` and `*` are also abbreviations for the lowest level marked as missing.) The integers must be greater than or equal to 1 and less than 30 (if not `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;` is used, see section 3.7). The 6 cases in the $2 \times 2 \times 3$ table 3.1 (factors declared with one of the first two specifications shown in the previous section) can be entered as

| | A = 1 | A = 2 | A = 1 | A = 2 |
|---|---|---|---|---|
| | B = 1 | B = 1 | B = 2 | B = 2 |
| C = 1 | 1 | 0 | 0 | 0 |
| C = 2 | 0 | 0 | 1 | 1 |
| C = 3 | 1 | 2 | 0 | 0 |

Table 3.1: *A simple data set.*

### 3.4.1 Table

```
Table
1 0 0 0
0 0 1 1
1 2 0 0  /
```

or as

### 3.4.2 List

```
List
1 1 1
2 2 2
1 1 3
2 1 .
2 1 *
1 2 2    /
```

When reading from key-board the list or table has to be ended with a slash. The characters `.` and `*` are abbreviations for the lowest level marked as missing. Thus, at the factor C `3`, `.` and `*` are all codes for the level marked as missing.

If `List` is replaced by `Accumulated-list`, then an accumulated list is expected: Each identification of a cell has to be preceded by the number of cases in that cell.

### 3.4.3 Accumulated List

```
Accumulated-list
1  1 1 1
1  2 2 2
1  1 1 3
2  2 1 *
1  1 2 2    /
```

The accumulated list for data entered by `List` or `Table` can be printed by `CASE LIST *;`, see section 4.4.

## 3.5 Replacing the Observations with a Random Data Set

```
SUBSTITUTE DATA WITH GENERATED TABLE;
```

This command will replace the table of observed counts with a random table of counts, where the sufficient marginals for the **current** model are unchanged. The **current** model has to be decomposable. The table is generated as when computing an exact $p$-value.

## 3.6 Initial values to the IPS-algorithm

`READ Q-TABLE` ⟨*set*⟩ ⟨*table*⟩`;`
`READ Q-LIST` ⟨*set*⟩ ⟨*list of cells*⟩`;`
`CLEAN DATA;`

Initial values (integers) to the IPS-algorithm are entered by `READ Q-TABLE` ⟨*set*⟩ ⟨*table*⟩`;`[1]. Zero, `0`, is entered for cells to be zero by structure. Cells to be zero by structure can also be entered by the command `READ Q-LIST` ⟨*set*⟩ ⟨*list of cells*⟩`;`. By cells to be zero by structure incomplete tables can be handled.

### 3.6.1 Structural Zeros = Incomplete Tables

If the cells (B,C) = (2,1), (B,C) = (1,2) and (B,C) = (2,3) in the previously used example are structurally zero, this can be specified by using the character `-` for structural zero cells when entering the observations as a table:

```
Table
1 0 - -
- - 1 1
1 2 - -  /
```

Or after reading the observations without specifying structural zero cells, the structural zero cells can be specified by the `READ Q-TABLE` ⟨*set*⟩ ⟨*table*⟩`;`-command:

```
>READ Q-TABLE ABC.  1 1 0 0   0 0 1 1   1 1 0 0  /
```

`0` or `-` is entered for structural zero cells, `1` for non-structural zero cells.
Since a cell in this table is structural zero if, and only if, the corresponding cell in the BC-marginal table is zero, this table can be specified by:

```
>READ Q-TABLE BC.  1 0   0 1   1 0  /
```

If more than one marginal table for structural zero cells is specified, then a cell in the full table is set to structural zero by CoCo, if, and only if, the corresponding cell in one of the marginal tables is zero. Thus

[1] At the time of writing this guide, the table entered by `READ Q-TABLE` ⟨*set*⟩ ⟨*table*⟩`;` should be a table of integers. This should not be much of a restriction, since the table can just be multiplied with a factor.

```
>READ Q-TABLE AB.  1 0   1 1 /
>READ Q-TABLE BC.  1 0   0 1   1 0  /
```

is equivalent to:

```
>READ Q-TABLE ABC.  1 0 0 0   0 0 1 1   1 0 0 0  /
```

or since all the counts in the non-zero cells in the example are 1:

```
Table
1 - - -
- - 1 1
1 - - -  /
```

If a Q-table can be factorized into a product of Q-tables, it should be, since it saves space and computing time.

If the ⟨*a*⟩-marginal Q-table exists, when `READ Q-TABLE` ⟨*a*⟩ ⟨*table*⟩`;` is used, then the existing Q-table is overwritten. If the specified table contains or is a marginal table in an existing Q-table, a warning message is printed. The used Q-table will still be the product of all defined Q-tables.

The cell (A,B,C) = (2,1,3) is structural zero, but the observed count in this cell was `2`. Such illegal observations are removed with the command `CLEAN DATA;`. (If illegal observations are not removed, computed statistics, probabilities, expected counts, residuals, etc. will be incorrect. A simple test is that `READ MODEL *; PRINT TABLE Probabilities .;` should return the value `1`)

The `PRINT TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;` and `DESCRIBE TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;` commands are described in later chapters. In tables with structural zeros the following is useful:

`PRINT TABLE Complete Observed` ⟨*a*⟩`;` will print a table with the observed count in the cell, if the cell is not structurally zero, else `-`.

`PRINT TABLE Zero` ⟨*a*⟩`;` will print a table with `0` in the cell, if the cell is structurally zero, else `1`.

`PRINT TABLE Errors` ⟨*a*⟩`;` will print a table with the observed counts in the cell, if the cell is structurally zero, else `-`.

`DESCRIBE TABLE Errors` ⟨*a*⟩`;` will give a description of the observed counts in cells with structural zeroes.

The command `READ Q-LIST` ⟨*set*⟩ ⟨*list of cells*⟩`;` declares individual cells in the ⟨*set*⟩-marginal table to be structural zero, i.e., adds structurally zero cells to the ⟨*set*⟩-marginal Q-table. Thus,

```
>READ Q-TABLE AB.  1 0   1 1 /
>READ Q-TABLE BC.  1 0   0 1   1 0  /
```

is equivalent to:

```
>READ Q-TABLE BC.  1 0   0 1   1 0  /
>READ Q-LIST  AB.  2 1 /
```

or

```
>READ Q-LIST  ABC.
 Data->2 1 1
 Data->1 2 1
 Data->2 2 1
 Data->1 1 2
 Data->2 1 2
 Data->2 1 3
 Data->1 2 3
 Data->2 2 3
 Data->/
```

If the ⟨*a*⟩-marginal Q-table exists when `READ Q-LIST` ⟨*a*⟩ ⟨*list of cells*⟩`;` is used then the ⟨*list of cells*⟩ is added to the existing Q-table. If the specified table contains or is a marginal table in an existing Q-table, a warning message is printed.

Commands `Q-list` ⟨*a*⟩ ⟨*table*⟩`;` and `Q-list` ⟨*a*⟩ ⟨*list of cells*⟩`;` can be placed on the file with the list of observations, and is read when the command `READ DATA;` or `READ OBSERVATIONS;` is used, see the beginning of this chapter.

`DISPOSE OF Q-TABLE` ⟨*set*⟩`;`
`DISPOSE OF ALL Q-TABLES;`

These two commands dispose of the specified ⟨*set*⟩-marginal Q-table and all specifications of structural zeros respectively.

Since the estimated probabilities depend on the read Q-tables, the estimated probabilities and tests are disposed of when reading Q-tables. Models are not disposed of, but when the estimated probabilities are used after the reading of the Q-table, the probabilities are re-estimated by the IPS-algorithm. The list of tests is disposed of when reading Q-tables, see `SET REUSE OF TESTS [ On | Off ];`) in later sections (section 6.6 and chapter 11).

## 3.7 Grouping of Factor Levels - Cutpoints and Redefine Factors

`REDEFINE FACTOR` ⟨*factor-name*⟩ ⟨*# of observed levels*⟩ ⟨*# of missing levels*⟩`;`
`CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;`

If a factor has too many levels (or is continuous) the number of levels can be reduced with the command `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;` between reading specifications and observations. Note that the tests in CoCo do not take into account an ordering or scaling on the levels.

Observations have to be entered as a list, if cutpoints are to be used. Before using `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;` the factor with name ⟨*factor-name*⟩ has to be

declared with number of levels equal to number of groups that `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;` result in. The number of cutpoints must be the number of resulting levels minus one. The first group of the resulting factor will consist of values less than or equal to the first cutpoint, values greater than the first cutpoint and less than or equal to the second cutpoint will be grouped in the second group, etc.

Commands `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;` can be placed on the file with the list of observations, and is read when the command `READ DATA;` or `READ OBSERVATIONS;` is used, see the beginning of this chapter. If two `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;`-commands for the same ⟨*factor-name*⟩ are found on the observation-file, the first of them is used and the others ignored.

If the specification and the list of observations are written on files, it is annoying to have to edit the file with specification to experiment with different numbers of cutpoints for a specific factor. The specification has to be edited, since the number of cutpoints must be equal to the number of declared levels minus one. But the declared number of levels can be changed by the command `REDEFINE FACTOR` ⟨*factor-name*⟩ ⟨*# of observed levels*⟩ ⟨*# of missing levels*⟩`;` after reading the specification, but before using the `CUTPOINTS` ⟨*factor-name*⟩ ⟨*cutpoints*⟩`;`-command and reading data. The number of non-missing levels is set to ⟨*# of observed levels*⟩ and the number of levels marked as missing is set to ⟨*# of missing levels*⟩ for factor ⟨*factor-name*⟩. The total number of levels is set to ⟨*# of observed levels*⟩ + ⟨*# of missing levels*⟩.

Consider the following artificial data-file "liver", where cutpoints for the continuous variable Bilirubin are placed on the data-file:

```
# Jaundice:  No: 1,  Mild: 2,  Moderate: 3,  Severe: 4,  Not coded: 5;
# Bilirubin: in umol/l:
#               0: < 1 umol/l,
#          900.00: > 1000 umol/l
#          999.99: No measurement
#               *: Missing on form
Factors J 4 1 / B 6 1 //
Cutpoints B 0 50.00 100.00 200.00 899.99 900.00
#                1:  < 1
#                2:    1    -   50.00 umol/l
#                3:   50.01 -  100.00 umol/l
#                4:  100.01 -  200.00 umol/l
#                5:  200.01 - 1000.00 umol/l
#                6:         > 1000.00 umol/l = code ‘900.00’
#                7:  Missing: Code ‘*’ or ‘999.99’
List
4 123.12       1  27.36       3  49.59       1  15.39       3  53.01
2  22.23       1  49.99       1  50.01       1  50.00       1   8.00
1  12.00       1  73.00       5   7.00       *   8.00       1   9.00
1   6.00       4 108.00       1   0.00       2 900.00       1 999.99
3 999.99       *      *       4      *       /
```

Comments are inserted in the data-file by '#' as the first character at lines before the key-words `Names`, `List`, `Factors` and `Table`.

The commands

```
set output diary diary.l1;
set input data liver;
read data;
print observed *;
quit;
```

will then result in

```
# Version 1.3   Friday March 17 12:00:00 MET 1995
# Compiled with gcc, a C compiler for Sun4
# Compile-time: Mar 17 1995, 13:00:00.
# Copyright (c) 1991, by Jens Henrik Badsberg

 Diary set ON

>set input data liver;
>read data;
 Finding all marginals.
   23 cases read.

>print observed *;

     [JB]

    J          1         2         3         4         5
  B

  1            1         0         0         0         0
  2            8         1         1         0         2
  3            2         0         1         0         0
  4            0         0         0         2         0
  5            0         0         0         0         0
  6            0         1         0         0         0
  7            1         0         1         1         1

>quit;
```

The variables Jaundice and Bilirubin are both ordinal. Without changing the file "liver" they can be redefined by the following commands:

```
set output diary diary.l2;
set input data liver;
read specification;
redefine factor J 2 1;
cutpoints J 1 4;
# 1: 1,                2: 2, 3 and 4,          3 (Missing): 5 (Missing) ;
redefine factor B 2 1;
cutpoints B 50.00 900.00;
# 1: 0 - 50.00 umol/l,  2: 50.01 - Max umol/l,   3: Missing ;
read observations;
print observed *;
quit;
```

These commands will result in:

```
# Version 1.3   Friday March 17 12:00:00 MET 1995
# Compiled with gcc, a C compiler for Sun4
# Compile-time: Mar 17 1995, 13:00:00.
# Copyright (c) 1991, by Jens Henrik Badsberg

 Diary set ON

>set input data liver;

>read specification;

>redefine factor J
 Levels(J)->    2
 Missing(J)->   1

>cutpoints J
 Cutpoint(J,  1)->      1.000
 Cutpoint(J,  2)->      4.000

>#  1: 1,                 2: 2, 3 and 4,           3 (Missing): 5 (Missing) ;

>redefine factor B
 Levels(B)->    2
 Missing(B)->   1

>cutpoints B
 Cutpoint(B,  1)->     50.000
 Cutpoint(B,  2)->    900.000

>#  1: 0 - 50.00 umol/l,  2: 50.01 - Max umol/l,   3: Missing ;

>read observations;
 Ignoring cutpoint for factor 'B' on datafile
 Finding all marginals.
   23 cases read.

>print observed *;

     [JB]

    J          1         2         3
  B

  1            9         2         2
  2            2         4         0
  3            1         2         1

>quit;
```

## 3.8 Data Selection

SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;
OR SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;
REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;
OR REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;

### 3.8.1 Select or Reject Cases During the Reading of Data

With the commands SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩; and OR SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩; between READ SPECIFICATION [ ⟨*specification*⟩ ]; and READ OBSERVATIONS [ ⟨*observations*⟩ ]; only cases in particular cells in one or more marginal tables are read. For example, only cases with (A=1 and B=2) or (C=1) or (D=2) are read by

```
READ SPECIFICATION;
SELECT CASES     AB. 1 2;
OR SELECT CASES  C.  1;
OR SELECT CASES  D.  2;
READ OBSERVATIONS;
```

The command SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩; disposes of previous select-statements. Use the command SELECT CASES .; to select all cases not rejected.

Cases in particular cells can be skipped with REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩; and OR REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;. For example,

```
READ SPECIFICATION;
SELECT CASES     AB. 1 2;
OR SELECT CASES  C.  1;
REJECT CASES     CD. 1 1;
OR REJECT CASES  D.  2;
READ OBSERVATIONS;
```

Cases with

```
( (A=1 and B=2) or (C=1) )
  and not ( (C=1 and D=1) or (D=2) )
```

are read.

As with the command SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩; the command REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩; disposes of previous reject-statements. All cases selected are read after the command REJECT CASES .;.

Note that selection will result in zero cells in the read data, if factors done selection on is read. The selection is done on the result of REDEFINE FACTOR ⟨*factor-name*⟩ ⟨*# of observed levels*⟩ ⟨*# of missing levels*⟩;- and CUTPOINTS ⟨*factor-name*⟩ ⟨*cutpoints*⟩;-commands.

(Cases with a particular level at a non-binary factor can be excluded without making zero cells by reading a data-list with the factor twice, doing selection on one copy of the factor, and grouping the other copy of the factor by the CUTPOINTS ⟨*factor-name*⟩ ⟨*cutpoints*⟩;-commands.)

### 3.8.2 Read only a Subset of Variables

`SET READ { All | Subset` ⟨*set*⟩ `};`

`SET READ Subset` ⟨*a*⟩`;` causes only a subset of the declared factors to be read. The command must be used after reading specification and before data selection and before reading observations. Data selection can still be performed at all declared factors.

`SET READ All;` will cancel the effect of `SET READ Subset` ⟨*a*⟩`;`.

## 3.9 Missing Values: Incomplete Observations

One may want to exclude cases with particular levels at some factors in tests. If e.g., a factor A on the form has the 3 levels: 1) `Yes`, 2) `No` and 3) `Not able to decide Yes or No` and a fourth level is added during transformation of data from forms to data in computer: 4) `No information about factor A`. Then one may want to mark level 3) and 4) as missing values. The levels to be marked have to be the levels with the highest numerical value. The factor A is declared together with a factor C with a total of 4, levels one of which is to be marked as missing and two binary factors B and D as

```
Factors A 2 2 / B 2 / C 3 1 / D 2 //
```

or

```
Names
A B C D /
2 2 3 2 /
2 0 .   /
```

When declaring factors, the characters `*` and `.` are abbreviations of `1`. When reading data in the form `List`, the characters are abbreviations of the lowest level marked as missing. For example, at the factor A declared in this section `.` is equal to `3` and at the factor C equal to `4` in the following list:

```
List
. 2 . 2
/
```

Missing values can in CoCo be handled in the following three ways:

- Skip all cases with missing values.
  This is selected by the command `SKIP MISSING;`. Cases with missing values among the subset of variables to be read (set by `SET READ Subset` ⟨*set*⟩`;`) are then skipped during the reading of cases.

- Use all cases without missing values for relevant factors.
  After the command `EXCLUDE MISSING On;` all cases without missing values for relevant factors are used in each test. This is turned off again by the command `EXCLUDE MISSING Off;` and all cases are used. The command `EXCLUDE`

`MISSING In` ⟨*set*⟩`;` will exclude the cases with variables having levels marked as missing for any of the variables in the set ⟨*set*⟩ of factors. The command is used after entering the data.

- Use the EM-algorithm to estimate the values of unobserved factors.
  This is selected by the command `EM ON;` after entering the data. The current implementation is tested on Fuchs (1982) for situations, where factors at random are unobserved, and on Dawid & Skene (1979) with a latent variable, i.e., a variable unobserved for all cases.

### 3.9.1 Skip Missing: Read only Cases with Complete Information

```
SKIP MISSING;
```

If the command `SKIP MISSING;` is used between reading specification and data, only cases with complete information in the subset to be read are read. Cases with variables with values marked as missing among the subset variables to be read are skipped. (The subset of factors to read is controlled by the command `SET READ` { `All | Subset;`⟨*set*⟩ }.)

### 3.9.2 Exclude Missing On/Off: Use Maximal Number of Cases with Complete Information in each Test

```
EXCLUDE MISSING [ On | Off ];
```

`EXCLUDE MISSING [ On | Off ];` and `EXCLUDE MISSING In` ⟨*set*⟩`;` is used after reading the data without using the command `SKIP MISSING;`. All cases are read into the program (if not, some cases are selected or rejected).

After the command `EXCLUDE MISSING On;` only cases with complete information about all used factors are considered. `ExcludeMissing` may be turned off again with `EXCLUDE MISSING Off;`.

Consider the tables onto which the tests in the commands `TEST;`, `FIND LOG(L);`, `FIND DEVIANCE;`, `EXACT TEST;`, `TEST ONE EDGE;`, `PARTITIONING TEST;`, `GENERATE DECOMPOSABLE MODEL;`, `GENERATE GRAPHICAL MODEL;`, `DROP EDGES` ⟨*edge-list*⟩`;`, `ADD EDGES` ⟨*edge-list*⟩`;`, `DROP INTERACTIONS` ⟨*gc*⟩`;`, `ADD INTERACTIONS` ⟨*gc*⟩`;`, `DROP FACTOR` ⟨*factor*⟩`;`, `MEET OF MODELS;`, `JOIN OF MODELS;`, `REDUCE GENERATOR` ⟨*set*⟩`;`, `REMOVE GENERATOR` ⟨*set*⟩`;`, `REMOVE TOTAL INTERACTION` ⟨*set*⟩`;`, `BACKWARD [ Edges | Interactions ];`, `FORWARD [ Edges | Interactions ];`, `FACTORIZE ONE EDGE;`, `FACTORIZE ONE INTERACTION;` and the tests in the EH-procedure are collapsible. Then if `ExcludeMissing` is on, then all cases with complete information in these tables are included in the tests. Hence the number of observations entering in to tests may be varying from test to test.

### 3.9.3 Exclude Missing in "Subset": Use only Cases with Complete Information for a Specific Subset of Factors.

```
EXCLUDE MISSING In ⟨set⟩;
```

Cases with complete information in a particular subset of the factors may be considered with the command `EXCLUDE MISSING In` ⟨*set*⟩`;`. One can then print, describe, etc. marginal tables of the ⟨*set*⟩-marginal table. The command finds the ⟨*set*⟩-marginal table of observed counts with cases with complete information for the factors ⟨*set*⟩. The status of `EXCLUDE MISSING [ On | Off ];` is unchanged, (`ExcludeMissing` has to be on before use of `EXCLUDE MISSING In` ⟨*set*⟩`;`). The considered models have to have generating classes with union of factors a subset of the set ⟨*set*⟩ used in `EXCLUDE MISSING In` ⟨*set*⟩`;`. Commands resulting in a test will cancel the `EXCLUDE MISSING In` ⟨*set*⟩`;`-command.

### 3.9.4 EM-algorithm

```
EM ON;

SET EM INITIAL { Uniform | First | Last | Mean | Random | Input };
SET EM EPSILON ⟨epsilon⟩;
SET EM ITERATIONS ⟨max⟩;

SET DATASTRUCTURE { All | Necessary | File | Large };
```

Given the data and a model, the probability of each level for a missing factor at a case can be estimated by the EM-algorithm. CoCo is told to use the EM-algorithm by `EM ON;`.

If `Report` is on, the likelihood ratio test statistics are reported for each step in the EM-algorithm. If `Trace` is on, the probability for each cell for each case is reported for each step in the EM-algorithm.

The data structure `Necessary` should be selected just after reading the specifications. The data structure `File` can be used, but since the propagation methods described in Lauritzen (1991), are not implemented in CoCo, it will be very slow. If the data structure `File` is selected, missing values in tables, that are too large to be stored in the memory can be handled, if the sufficient marginals for the model can be stored in the memory. But it will take time.

Initial values for the EM-algorithm are set by `SET EM INITIAL { Uniform | First | Last | Mean | Random | Input };`. `Uniform` means a uniform distribution. `First`, `Last` and `Mean` sets initially the variables with values marked as missing to lowest observable level, highest observable level and rounded mean-value of observed factors for the case respectively. `Mean` is useful to control the value of latent variates. With `Input`, the initial values can be given as input to CoCo. By `Input` the initial value of a factor with a value marked as missing is set to a level equal to the coded level for this factor of the case minus the number of non-missing, un-marked, levels for this factor. `Random` sets missing levels to a random observable level. The same random-generator as used in `ExactTest` is used. `SET SEED` ⟨*seed*⟩`;` sets the initial value ⟨*seed*⟩ for the random number generator. `SET SEED 'Random;` sets a random initial value (computed from CPU-time and/or real time).

Cycles in the EM-algorithm is repeated until the difference between values of the likelihood ratio test-statistics -2log(Q), the deviance, for each cycle is less than the ⟨$\epsilon$⟩

set by `SET EM EPSILON` ⟨$\epsilon$⟩`;`. The command `SET EM ITERATIONS` ⟨$max$⟩`;` controls the maximum number of cycles in the EM-algorithm.

The computing-time is proportional to the sum over (different) cases of the product of number of levels at the unobserved factors at the case (times some polynomium in the dimension). Not too many cases should have more than a single missing value: If a latent variable is used there should not be many other missing values, and if cases have more than a single missing value (around 5), it should be at a limited number of cases. The computing time depends on the product over missing factors of observable level for the case with the largest product of number of observable levels for factors marked as missing. `ReportOn` is recommended to get the likelihood ratio test statistic reported for each step in the EM-algorithm. The commands for semi-automatic and automatic search are over-night commands, when the EM-algorithm is used.

The EM-algorithm in CoCo is tested on data from Fuchs (1982) for an example with sporadic missing observations, and on Dawid & Skene (1979) for an example with latent class variables.

# Chapter 4

# Description of Data and Fitted Values

The two first sections of this chapter will describe how table-values are computed. These values can be printed in tables, described, plotted, listed or exported to files by the commands of the following sections. The values can be marginal counts, probabilities, expected values and residuals as expected minus observed counts, standardized, adjusted, etc.

## 4.1 Notation and Computation of Marginal Values

Let $\Delta$ denote declared variables. For each variable $\delta \in \Delta$ the set $\mathcal{I}_\delta = \{1, 2, ..., |\mathcal{I}_\delta|\}$ is the set of unmarked levels $i_\delta$ for the variable. A cell in the table is then an element $i = (i_\delta)_{\delta\in\Delta}$ in the product $I$ of level sets:

$$i = i_\Delta = (i_\delta, \delta \in \Delta) \in \mathcal{I} = \mathcal{I}_\Delta = \times_{\delta\in\Delta}\mathcal{I}_\delta.$$

The number of cells in the table is $|\mathcal{I}_\Delta| = \prod_{\delta\in\Delta} |\mathcal{I}_\delta|$. The counts $(n(i))_{i\in\mathcal{I}}$ are the contingency table. The number $|\Delta|$ of variables is the dimension of the table.

For a subset $a$ of the declared variables $\Delta$, the $a$-marginal table is obtained by only classifying the observations according to variables in $a$. The marginal table on $a$ has then cells

$$i_a = (i_\delta, \delta \in a) \in \mathcal{I}_\mathrm{a} = \times_{\delta\in\mathrm{a}}\mathcal{I}_\delta$$

and marginal counts

$$n(i_a) = \textstyle\sum_{j:j_a=i_a} n(j).$$

The number of cells in the $a$-marginal table is $|\mathcal{I}_\mathrm{a}| = \prod_{\delta\in\mathrm{a}} |\mathcal{I}_\delta|$.

If the table is incomplete then a table $(q(i))_{i\in\mathcal{I}}$ with $q(i) = 0$ for cells that are structurally zero is defined. The Q-table might be defined as a product of $a$-marginal tables $(q(i_a))_{i_a\in\mathcal{I}_\mathrm{a}}$, $a \in \mathcal{A}$:

Figure 4.1: Collaps of $M_A$ onto $B$ containing $a \cap A$.

$$q(i) = \prod_{a \in \mathcal{A}} q_a(i_a) = \begin{cases} 0 & \text{if } \exists\ a \in \mathcal{A} : \mathrm{q_a(i_a)} = 0, \\ 1 & \text{if } \forall\ a \in \mathcal{A} : \mathrm{q_a(i_a)} = 1, \\ \prod_{a \in \mathcal{A}} q_a(i_a) & \text{otherwise.} \end{cases}$$

If some levels for the factor $\delta$ are marked as missing, then let $\mathcal{I}_\delta = \{1, 2, ..., |\mathcal{I}_\delta|\}$ denote the unmarked levels and let $\mathcal{I}^*_\delta = \{1, 2, ..., |\mathcal{I}^*_\delta|\}$ denote the total set of levels. $\mathcal{I}^*_\delta \setminus \mathcal{I}_\delta = \{|\mathcal{I}_\delta| + 1, |\mathcal{I}_\delta| + 2, ..., |\mathcal{I}^*_\delta|\}$ is the set of marked levels. The counts $(n(i))_{i \in \mathcal{I}^*}, \mathcal{I}^* = \mathcal{I}^*_\Delta = \times_{\delta \in \Delta} \mathcal{I}^*_\delta$, is the total table. The counts $n(i), i \in \mathcal{I} = \mathcal{I}_\Delta = \times_{\delta \in \Delta} \mathcal{I}_\delta$, is the table with only cases with complete information. The cells $i \in \mathcal{I}^* \setminus \mathcal{I}$ have at least one level marked as missing.

Let $\hat{p}(i_A)$ be the maximum likelihood estimate on the table $(n(i_A))_{i_A \in \mathcal{I}_A}$ for a model $M_A$, where the union $A$ of variables in the model is a subset of the declared variables $\Delta$. $\hat{p}(i)$ is the maximum likelihood estimate for the same model, but on the full table $(n(i))_{i \in \mathcal{I}}$ with no effect of variables in $\Delta \setminus A$, corresponding to the distribution on $\Delta \setminus A$ being uniform. Then

$$\hat{p}(i) = \hat{p}(i_A)/|\mathcal{I}_{\mathrm{A^C}}|.$$

The $a$-marginal estimated probability:

$$\hat{p}(i_a) = \textstyle\sum_{j : j_a = i_a} \hat{p}(j).$$

For any subset $a$ of the declared variables $\Delta$ we have

$$\hat{p}(i_{A \cup a}) \quad = \hat{p}(i_A)/|\mathcal{I}_{\mathrm{a} \cap \mathrm{A^C}}|$$

and

$$\hat{p}(i) = \hat{p}(i_{A \cup a})/|\mathcal{I}_{(\mathrm{A} \cup \mathrm{a})^\mathrm{C}}| = \hat{\mathrm{p}}(\mathrm{i}_{\mathrm{A} \cup \mathrm{a}})/|\mathcal{I}_{\mathrm{A^C} \cap \mathrm{a^C}}|.$$

For a subset $a$ of the declared variables $\Delta$ let $B$ be the smallest set of variables onto which the model $M_A$ is *collapsible* (Asmussen & Edwards 1983), and such that $a \cap A \subset B \subset A$. Recall that $M_A$ is collapsible onto $B$ if

$$\hat{p}_B(i_B) = \hat{p}_A(i_B).$$

$M_A$ is collapsible onto $B$ if and only if in the 2-section graph for the model the boundary $\partial(c_i)$ of every connected component $c_i$ of the complement $B^C$ of $B$ in $\Delta$ is contained in a generator of $M_A$ (Asmussen & Edwards 1983). See the chapter 5: "Models" about 2-section graphs.

Then the estimated probabilities in the $a$-marginal are computed as:

$$\begin{aligned}
\hat{p}_A(i_a) &= \sum_{j:j_a=i_a} \hat{p}_A(j) \\
&= \sum_{j_{B\cup a}:j_a=i_a} \sum_{k_{A\cup a}:k_{B\cup a}=j_{B\cup a}} \sum_{l:l_{A\cup a}=k_{A\cup a}} \hat{p}_A(l) \\
&= \sum_{j_{B\cup a}:j_a=i_a} \hat{p}_A(j_B)/|\mathcal{I}_{\mathrm{a\cap A^C}}| \\
&= \sum_{j_{B\cup a}:j_a=i_a} \hat{p}_B(j_{a\cup B}).
\end{aligned}$$

The factor $c(i)$ used in computing the adjusted residual (Haberman 1978) for a given decomposable model is found by:

$$c(i) = n\hat{p}_B(i_B)\{1 - n\hat{p}_B(i_B)(-\frac{1}{n} - \sum_{a\subset\Delta} \frac{\nu(a)}{n(i_a)})\},$$

where $\nu(a), a \subset \Delta$ is the index or adjusted replication number for subsets of sets in the generating class for the model. The index $\nu(a)$ is the power to which the marginal counts $n(i_a)$ are raised in the closed form expression for the maximum likelihood estimate of the probability in a decomposable model. See decomposition of models in chapter 5.

## 4.2 Values

The *values* for the `PRINT TABLE ...;`, `DESCRIBE TABLE ...;`, `PLOT ...;`, `RETURN VECTOR ...;`, `RETURN MATRIX ...;` and `LIST ...;` statements are then computed as follows:

If $a$ is a subset of $B$, the values are computed by summing over cells in the $B$-marginal table. If the set $a$ contains factors not in $B$, the values are computed by summing over cells in the $(B \cup a)$-marginal table, the table determined by the union of the set $a$ and $B$ where no effect of the extra factors is assumed when estimating probabilities.

If the optional key-word `Complete` is given, the value is only computed for non structural zero cells in the $a$-marginal table. Values for the structural zero cells are excluded from plots and the univariate descriptions, and in tables and lists the character `-` is printed for structural zero cells.

If the optional key-word `Log` is given, the values are log-transformed before returned (printed, described, plotted or listed), but not before summing over cells in the $(B\cup a)$-marginal table.

If the optional key-word `Random` is given, then the values are computed in a random table with sufficient marginals as in the **current** model (or in the **base** model, if keyword `Base` is given).

See Aickin (1983) pp. 152 for F-res, R-F, G-res and R-G, Bishop, Fienberg & Holland (1975) pp. 136 for Standardized, -2log(Q) and Freeman-Tukey, Haberman (1974) pp. 138 for Adjusted, Standardized, -2log(Q), Freeman-Tukey and 2(/n-/m), Haberman (1978) pp. 272 for Adjusted and Standardized and Read & Cressie (1988) for Power.

| ⟨*value-name*⟩ | := |
|---|---|
| `[ Base \|` | Compute the values under the **base** model |
| `Current ]` | Compute the values under the **current** model, default |
| `[ Complete ]` | Compute only values with $q(i_a) \neq 0$, See definition of value `Zero` |
| `[ Log ]` | Natural logarithm of the values |
| `[ Random ]` | Compute the values in a random table |
| `{ Observed` | $n(i_a) = \sum_{j:j_a=i_a} n(j)$ |
| `\| Probabilities` | $\hat{p}_A(i_a) = \sum_{j:j_a=i_a} \hat{p}_A(j) = \sum_{j_{B\cup a}:j_a=i_a} \hat{p}_B(j_{a\cup B})$ |
| `\| Expected` | $m(i_a) = n\hat{p}_A(i_a)$ |
| `\| Unadjusted` | $m(i_a) - n(i_a)$ |
| `\| Adjusted` | $\sum_{j_{B\cup a}:j_a=i_a} \frac{n(j_{B\cup a})-n\hat{p}_B(j_{B\cup a})}{\sqrt{c(j)}}$ |
| `\| Standardized` | $\sum_{j_{B\cup a}:j_a=i_a} \frac{n(j_{B\cup a})-n\hat{p}_B(j_{B\cup a})}{\sqrt{n\hat{p}_B(j_{B\cup a})}}$ |
| `\| -2log(Q)` | $2\sum_{j_{B\cup a}:j_a=i_a} n(j_{B\cup a})\ ln(\frac{n(j_{B\cup a})}{n\hat{p}_B(j_{B\cup a})})$ |
| `\| F-res` | $\sum_{j_{B\cup a}:j_a=i_a} \sqrt{n}\frac{n(j_{B\cup a})/n-\hat{p}_B(j_{B\cup a})}{\sqrt{(n(j_{B\cup a})/n)(1-n(j_{B\cup a})/n)}}$ |
| `\| R-F` | $\sum_{j_{B\cup a}:j_a=i_a} \sqrt{n}\frac{n(j_{B\cup a})/n-\hat{p}_B(j_{B\cup a})}{\sqrt{\hat{p}_B(j_{B\cup a})(1-\hat{p}_B(j_{B\cup a}))}}$ |
| `\| G-res` | $\sum_{j_{B\cup a}:j_a=i_a} \sqrt{n}\frac{\hat{p}_B(j_{B\cup a})-\lvert\mathcal{I}_{\mathrm{a\cup B}}\rvert^{-1}}{\sqrt{\hat{p}_B(j_{B\cup a})(1-\hat{p}_B(j_{B\cup a}))}}$ |

| | |
|---|---|
| `\| R-G` | $\sum_{j_{B\cup a}:j_a=i_a} \sqrt{n}\frac{\hat{p}_B(j_{B\cup a})-\lvert\mathcal{I}_{\mathrm{a\cup B}}\rvert^{-1}}{\sqrt{\lvert\mathcal{I}_{\mathrm{a\cup B}}\rvert^{-1}(1-\lvert\mathcal{I}_{\mathrm{a\cup B}}\rvert^{-1})}}$ |
| `\| Freeman-Tukey` | $\sum_{j_{B\cup a}:j_a=i_a}\{\sqrt{n(j_{B\cup a})}+\sqrt{n(j_{B\cup a})+1}-\sqrt{4n\hat{p}_B(j_{B\cup a})-1}\,\}$ |
| `\| 2(/n-/m)` | $\sum_{j_{B\cup a}:j_a=i_a}\{\sqrt{n(j_{B\cup a})}-\sqrt{n\hat{p}_B(j_{B\cup a})}\,\}$ |
| `\| Power` | $\frac{2}{\lambda(\lambda+1)}\sum_{j_{B\cup a}:j_a=i_a} n(j_{B\cup a})\{(\frac{n(j_{B\cup a})}{n\hat{p}_B(j_{B\cup a})})^\lambda-1\}$ |
| `\| Index` | $\mathrm{Index}(i_a)=\mathrm{Index}(i,a)=1+\sum_{\delta\in a}\{(i_\delta-1)\prod_{\gamma\prec\delta,\gamma\in a}\lvert\mathcal{I}_\gamma\rvert\}$ |
| `\| Zero` | $q(i_a)=\begin{cases}1 & \text{if } \exists j_\Delta: j_a=i_a \ \wedge\ q(j)\neq 0\\ 0 & \text{otherwise}\end{cases}$ |
| `\| Error }` | $n(i_a)$ for $q(i_a)=0$ |

The power $\lambda$ is by default set to 2/3. It is changed by the statement `SET POWER LAMBDA { Null | ⟨lambda⟩ };`.

## 4.3 Printing Tables

`PRINT TABLE` ⟨*value-name*⟩ ⟨*a*⟩ `[ /` ⟨*b*⟩ `];`

By the command `PRINT TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;` the marginal table determined by the set ⟨*a*⟩ and with the values ⟨*value-name*⟩ for the (by default) current model is printed. (The commands `PRINT TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;`, `DESCRIBE TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;`, `RETURN VECTOR` ⟨*value-name*⟩ ⟨*a*⟩`;`, `RETURN MATRIX` ⟨*value-names;*⟩ ⟨*a*⟩`;`, `PLOT` ⟨*x-value-name*⟩ ⟨*y-value-name*⟩ ⟨*a*⟩`;`, and `LIST` ⟨*a*⟩`;` can only be used if a **current** model exists. See the next chapter, use `PRINT OBSERVED` ⟨*a*⟩`;` or type `READ MODEL *;`. The command `PRINT OBSERVED` ⟨*a*⟩`;` prints a table of marginal observed counts without expecting a **current** model.)

`PRINT TABLE`

| | |
|---|---|
| `[ Current` | Use values from **current** model |
| `\| Base ]` | Use values from **base** model |
| ⟨*value-name*⟩ | Name of value to print in table |
| ⟨*a*⟩ | Print values in the *a*-marginal table |

The format of the numbers printed in the table is controlled by the command `SET TABLE FORMATS` ⟨*width*⟩ ⟨*dec-prob.*⟩ ⟨*dec-expt.*⟩ ⟨*dec-diff.*⟩`;`. The layout of the table is controlled by the command `SET PAGE FORMATS` ⟨*line-length*⟩ ⟨*page-length*⟩`;`. See chapter 11.

Examples of `PRINT TABLE ...;`-statements

```
print observed *;
print table expected .;
```

```
print table probabilities ABCDE.;

print table random observed *;
print table random base expected ABCD.;

print table unadjusted ABCD.;
print table f-res *;
print table adjusted ABCDE.;

print table log -2log(Q) *;
print table log power    *;
print table log standard *;

print table complete observed ABCDEF.;
print table error ABCD.;
print table zero  AB.;
```

For examples of output, see chapter 1.

The cells in the printed table are ordered according to the order of the factors in $\langle a \rangle$. The first factor met in $\langle a \rangle$ is alternating at the fastest speed.

With `PRINT STANDARD TABLE` $\langle$*value-name*$\rangle$ $\langle a \rangle$`;` the cells are ordered according to the order of the factors in the specification of the factors when declaring the table.

### Row-, Column-, Total-percents or percents of any other marginal:

When $\langle$*value-name*$\rangle$ is `Observed` and the optional set $\langle b \rangle$ is given then the counts in the $\langle a \rangle$-marginal table divided by the counts in the corresponding cell in the $\langle b \rangle$-marginal table times 100 are printed.

Then Row-, Column-, Total-percents or percents of any other marginals table are printed.

E.g.

```
print table observed AB ;
print table observed AB / B ;
print table observed ABC / A ;
print table observed ABC / BC ;
```

To print Total-percents, let the $\langle b \rangle$ be the empty set:

```
print table observed AB / ;
```

### Missing values

If `ExcludeMissing` is `On` and the argument $\langle a \rangle$ of the command `PRINT TABLE` $\langle$*value-name*$\rangle$ $\langle a \rangle$`;` is not a subset of the factors with complete information, then an action similar to `EXCLUDE MISSING In` $\langle a \cup \Delta_{\mathcal{B}} \cup \Delta_{\mathcal{C}} \rangle$`;` is performed before printing the table where $\Delta_{\mathcal{B}}$ is the factors of the **base** model and $\Delta_{\mathcal{C}}$ is the factor of the **current** model. Analogously for other commands of this chapter.

Consider using the command `EXCLUDE MISSING In` $\langle a \rangle$`;` before the command `PRINT TABLE` $\langle$*value-name*$\rangle$ $\langle a \rangle$`;`.

## 4.4 Printing Sparse Tables

`CASE LIST [` ⟨*a*⟩ `];`

With `CASE LIST` ⟨*a*⟩`;` the observed counts in the marginal table determined by the set ⟨*a*⟩ is printed. For each cell with non-zero count, the cell count and an identification of the cell is printed. The list printed by `CASE LIST *;` (when the EM-algorithm is not used) can be read as an accumulated list by the keyword `Accumulated-list`, see section 3.4.3. See also section 4.7.

## 4.5 Describing Tables

`DESCRIBE TABLE [ Uniform ] [ Normal ] [ Rankit ]` ⟨*value-name*⟩ ⟨*a*⟩`;`

The `DESCRIBE TABLE ...;` procedure gives a detailed univariate description of values in marginal tables. It gives statistics, a histogram, uniform and quantile-quantile plot and if short, a list of sorted values, and for observed counts, a table with extreme cell counts.

| | |
|---|---|
| `DESCRIBE TABLE` | |
| `[ Normal ]` | Print `Normal` (``'probit''``) plot |
| `[ Rankit ]` | Print `Rankit` plot |
| `[ Uniform ]` | Print `Uniform` plot |
| `[ Current` | Use values from **current** model |
| `\| Base ]` | Use values from **base** model |
| ⟨*value-name*⟩ | Name of value to be described |
| ⟨*a*⟩ | Describe values in the *a*-marginal table |

With `DESCRIBE TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;` the marginal table determined by the set ⟨*a*⟩ and with values computed as in the previous section is described.

If a sorted list of the values can fit onto two pages, it is printed. The size of a page is controlled by ⟨*page-length*⟩ and ⟨*line-length*⟩ in the statement `SET PAGE FORMATS` ⟨*line-length*⟩ ⟨*page-length*⟩`;`.

The mean (arithmetic, harmonic and geometric), variance, skewness, kurtosis, median, mode, minimum, maximum, and range are computed. (The command `SET PRINT FORMATS` ⟨*format-width*⟩ ⟨*decimals*⟩`;` can be used to change the formats for the printed statistics).

A histogram and a `Uniform` plot are printed. With the key-words `Uniform`, `Rankit` and `Normal` immediately after TABLE in the `DESCRIBE TABLE ...;` statement you can choose between different plots.

The plots are made to fit a page. Let $(x^{(1)}, x^{(2)}, ..., x^{(n)})$ be the sorted list of values to be described. In the uniform plot $x^{(i)}$ is then plotted against $i/n$, where $i$ is the rank of $x^{(i)}$. `Normal` and `Rankit` are normal probability plots, quantile-quantile plots. The empirical quantiles are plotted against the quantiles of a standard normal distribution. In the rankit plot $x^{(i)}$ is plotted against $\Phi^{-1}((3i-1)/(3n+1))$, and in the normal plot $x^{(i)}$ is plotted against $\Phi^{-1}((i-1/2)/n)$. $\Phi^{-1}$ is the inverse of the standard normal distribution function. See Haberman (1978) pp. 20 for rankit-plots.

If ⟨*value-name*⟩ is `Observed` and no log-transformation is used, a table showing the number of cells with extreme cell-counts is printed.

With `DESCRIBE OBSERVED` ⟨*a*⟩`;` the observed counts in the marginal table determined by the set ⟨*a*⟩ are described without expecting a **current** model.

Examples of `DESCRIBE TABLE ...;`-statements:

```
describe observed *;
describe table expected *;

describe table probabilities ABCDE.;
describe table unadjusted *;
describe table g-res    *;

describe table random current observed ABEF;
describe table random current observed *;
describe table random base expected ABEF;

describe table log observed *;
describe table log 2(/n-/m) *;
describe table log freeman-tukey *;

describe table rankit         2(/n-/m) *;
describe table uniform normal standard *;
describe table uniform rankit adjusted *;
describe table rankit normal  adjusted *;
describe table rankit normal uniform adjusted *;

describe table complete log observed *;
describe table errors *;
```

See chapter 1 for examples of output.

## 4.6 Plots

`PLOT` ⟨*x-value-name*⟩ ⟨*y-value-name*⟩ ⟨*a*⟩`;`

A scatter-plot of one value against another value in the same marginal table is made by the `PLOT ...;`-statement:

| | |
|---|---|
| `PLOT` | |
| `[ Current` | Use values from **current** model |
| `\| Base ]` | Use values from **base** model |
| ⟨*value-name*⟩ | Name of x-value |
| `[ Current` | Use values from **current** model |
| `\| Base ]` | Use values from **base** model |
| ⟨*value-name*⟩ | Name of y-value |
| ⟨*a*⟩ | Plot the values in the *a*-marginal table |

The command `PLOT` ⟨*x-value-name*⟩ ⟨*y-value-name*⟩ ⟨*a*⟩`;` plots the values ⟨*x-value-name*⟩ in the *a*-marginal table against the values ⟨*y-value-name*⟩ in the same

table. The values are by default computed in the **current** model. Values for the **current** model can be plotted against values for the **base** model with the key-words `Current` and `Base`. With the keyword `Complete` only values for non structural zero cells is plotted, and with `Log` ⟨*value-name*⟩ the value is log-transformed.

A value can be plotted against the number of the cell in the $a$-marginal table in a standard order by letting ⟨*x-value-name*⟩ or ⟨*y-value-name*⟩ be `Index`. This may be useful to find cells with large residuals, patterns in the residuals or changes in residuals between models by making the same plot for more models.

The plots are made to fit a page. The size of a page is controlled by ⟨*page-length*⟩ and ⟨*line-length*⟩ in the statement `SET PAGE FORMATS` ⟨*line-length*⟩ ⟨*page-length*⟩`;`.

Examples of `PLOT ...;`-statements:

```
plot observed expected *;
plot observed unadjusted *;
plot observed adjusted *;
plot observed standard *;
plot observed -2log(Q) *;
plot observed freeman-tukey *;
plot observed 2(/n-/m) *;
plot observed power *;

plot f-res r-f *;
plot f-res g-res *;

plot log observed log expected *;

plot index observed *;
plot index adjusted ABCD.

plot base adjusted current adjusted *;
plot base log expected current adjusted *;

plot index complete observed *;
plot index error *;
```

See chapter 1 for an example of output.

See chapter 12: "CoCo in other systems" for how to return values from CoCo for high-resolution graphics in S-Plus and XLISP-STAT.

## 4.7 Lists

`LIST` ⟨$a$⟩`;`
`CASE LIST;`

`LIST` ⟨$a$⟩`;` lists observed counts, expected counts, estimated probabilities and residuals (unadjusted, adjusted, standardized etc.) in the marginal table determined by the set ⟨$a$⟩ with values computed by summing over cells in the table determined by the union of the set ⟨$a$⟩ and the set $\mathcal{B}$ containing the intersection between $a$ and the model set and on which the model is collapsible.

The list can be used for, e.g., further computation, tabulation, high-resolution-graphics etc.

The used print format in the list is controlled by the command `SET TABLE FORMATS` ⟨*width*⟩ ⟨*dec-prob.*⟩ ⟨*dec-expt.*⟩ ⟨*dec-diff.*⟩`;`.

The cells in the printed list are ordered according to the order of the factors in ⟨*a*⟩, the first factor met in ⟨*a*⟩ is alternating at the fastest speed.

Example:

```
>read model *;
>read model ACE,ADE,BC,F.;
>test;
 Test of [[F][BC][ADE][ACE]]
 against [[ABCDEF]]

            Statistic       Asymptotic       Adjusted
 -2log(Q) =   62.0779   P =     0.0994  /      0.0994
 Power    =   59.7593   P =     0.1396  /      0.1396
 X^2      =   59.9956   P =     0.1350  /      0.1350
 DF.      =                        49  /          49

>list AB.;

 A B  Observed  Probabi.  Residual     F-res     Res-F     G-res     Res-G

 1 1       522  0.283706    0.3036   -0.3053   -0.2734    1.7525    4.3730
 2 1       541  0.293697   -0.3036    0.4024    0.4458   -0.5794    5.6692
 1 2       439  0.238292   -0.3036   -0.9062   -0.7652  -15.2189   -1.5189
 2 2       339  0.184304    0.3036    0.4927    0.5854  -18.1416   -8.5233


 A B  Expected  Adjusted  Standard  -2log(q)   Freeman  2(/n-/m)     Power

 1 1    522.30   -0.8228   -0.2969    0.3237   -0.2798   -0.3138    0.5583
 2 1    540.70    1.3433    0.4825    1.7218    0.4930    0.4591    1.4953
 1 2    438.70   -2.2441   -0.8091    2.7436   -0.8250   -0.8778    2.4241
 2 2    339.30    1.6995    0.6149    1.1986    0.6080    0.5656    1.4852
```

In incomplete tables the observed count for structural zero cells is marked by the asterisk *.

If `EM` is on, then `CASE LIST;` will for each cell $i$ with non-zero count in the full table print the number $n(i)$ of cases. If the cell $i$ is not marked as missing (none of the levels for the cell are marked as missing), $i \in \mathcal{I}$, then the estimated probability $\hat{p}(i)$ in the cell and an identification of the cell $i$ are printed, else if the cell $i$ has levels marked as missing, $i \in \mathcal{I}^* \setminus \mathcal{I}$, then for each cell $j \in \mathcal{I} : \mathrm{j_b} = \mathrm{i_b}$ in the $(\Delta \setminus b)$-marginal table determined by the factors $b$ with unmarked levels, $i_\delta \in \mathcal{I}_\delta$ for $\delta \in b$, in the cell $i$, the estimated probability $\hat{p}(j)$ of the cell $j$, the sum $\hat{p}(i_b)$ of these probabilities and an identification of the cell $j$ is printed. The levels $j_\delta$ for factors $\delta \in \Delta \setminus b$ with marked levels in the cell $i$ to describe are in the identification marked by the asterisk `*`.

The following example is on data from the Glostrup Study, see section 6.1.3.

```
# Version 1.3   Friday March 17 12:00:00 MET 1995
# Compiled with gcc, a C compiler for Sun4
# Compile-time: Mar 17 1995, 13:00:00.
# Copyright (c) 1991, by Jens Henrik Badsberg
# Licensed to ...

>set input data ex7a.dat;
>read specification;
>set read subset ABC. ;
>set datastructure necessary;
 Datastructure selected:   NECESSARY
>read observations;
 1160 cases read.
>read model *;
>set em on;
>case list;

      Count        P(j)     P(i{b})   A   B   C
        129                0.148546   1   1   1
          8                0.009212   2   1   1
         46    0.148546    0.157759   1*  1   1
         46    0.009212    0.157759   2*  1   1
         46 5.35230E-17    0.157759   3*  1   1
         11                0.017385   1   2   1
          1                0.001580   2   2   1
         10    0.017385    0.018966   1*  2   1
         10    0.001580    0.018966   2*  2   1
         10 1.73891E-11    0.018966   3*  2   1
        388                0.454953   1   1   2
        203                0.239503   2   1   2
         15                0.017471   3   1   2
        212    0.454953    0.711927   1*  1   2
        212    0.239503    0.711927   2*  1   2
        212    0.017471    0.711927   3*  1   2
         55                0.067956   1   2   2
         27                0.033578   2   2   2
          8                0.009815   3   2   2
         38    0.067956    0.111349   1*  2   2
         38    0.033578    0.111349   2*  2   2
         38    0.009815    0.111349   3*  2   2
          3    0.454953    0.522909   1   1*  2
          3    0.067956    0.522909   1   2*  2
          3    0.239503    0.273081   2   1*  2
          3    0.033578    0.273081   2   2*  2
          3    0.454953    0.823276   1*  1*  2
          3    0.239503    0.823276   2*  1*  2
          3    0.017471    0.823276   3*  1*  2
          3    0.067956    0.823276   1*  2*  2
          3    0.033578    0.823276   2*  2*  2
          3    0.009815    0.823276   3*  2*  2
>quit;
```

## 4.8 Exporting Table Values

`RETURN VECTOR` ⟨*value-name*⟩ ⟨*a*⟩`;`
`RETURN MATRIX` ⟨*list of value-names;*⟩ ⟨*a*⟩`;`

The command `RETURN VECTOR` ⟨*value-name*⟩ ⟨*a*⟩`;` returns a vector with the values ⟨*value-name*⟩ in the *a*-marginal table. Similar to `PRINT TABLE` ⟨*value-name*⟩ ⟨*a*⟩`;` but without headings. The print format is controlled by `SET PRINT FORMATS` ⟨*format-width*⟩ ⟨*decimals*⟩`;`.

The cells in the printed vector are ordered according to the order of the factors in ⟨*a*⟩. The first factor met in ⟨*a*⟩ is alternating at the fastest speed.

If `RETURN STANDARD VECTOR` ⟨*value-name*⟩ ⟨*a*⟩`;` is used, then the values will be ordered according to the order of the factors in specification of the factors when declaring data.

If `Dump` is on (See chapter 11: "Options"), the vector is written to the `DumpFile` instead of the standard output. `Dump` is by default off. `Dump` is set on or off by the command `SET DUMP [ On | Off ];`. The default name on the `DumpFile` is `/tmp/CoCo.dump.no.pid-number` under Unix and `COCO???.DMP` in the current directory under DOS. Another `DumpFile` can be named by `SET OUTPUTFILE DUMP` ⟨*file-name*⟩`;`. Note that this command rewrites the file ⟨*file-name*⟩ and that under DOS the file `COCO???.DMP` is rewritten every time CoCo is invoked. If the `DumpFile` is not renamed, the `DumpFile` is removed, when CoCo terminates normally, unless the command `SET KEEP DUMP [ On | Off ];` is used.

The command `RETURN MATRIX` ⟨*value-names;*⟩ ⟨*a*⟩`;` writes row by row to a matrix where the columns contain the values ⟨*value-names*⟩ for the *a*-marginal table. If `Dump` is on, the matrix is written to the `DumpFile` instead of the standard output. Note that the list of ⟨*value-names;*⟩ is terminated by `;` or an EndOfLine: "Return".

The cells in the printed matrix are ordered according to the order of the factors in ⟨*a*⟩. The first factor met in ⟨*a*⟩ is alternating at the fastest speed.

# Chapter 5

# Models

## 5.1 Reading Models

`READ MODEL` ⟨*gc*⟩`;`
`READ N-INTERACTIONS` ⟨*order*⟩ ⟨*set*⟩`;`
`COLLAPSE MODEL` ⟨*set*⟩`;`
`NORMAL TO DUAL;`
`DUAL TO NORMAL;`

A model is read with the command `READ MODEL` ⟨*gc*⟩`;`, where ⟨*gc*⟩ is the *generating class* for the model, for example,

```
READ MODEL AB,BC.;
```

The model with only *main effects* is read by `READ MODEL .;` and the *saturated model* by `READ MODEL *;`. The command `READ N-INTERACTIONS` ⟨*a*⟩ ⟨*set*⟩`;` will read a model with only and all ⟨*a*⟩'th-order interactions among factors in ⟨*set*⟩. A generator (clique) with $n$ factors is a $(n-1)$'th-order interaction. The model last read is the **current** model.

New models can be generated from the **current** model with the commands `DROP EDGES` ⟨*edge-list*⟩`;`, `ADD EDGES` ⟨*edge-list*⟩`;`, `DROP INTERACTIONS` ⟨*gc*⟩`;`, `ADD INTERACTIONS` ⟨*gc*⟩`;`, etc., see chapter 7: "Editing Models with Tests". With `Only` before these model-editing commands, no test is produced when generating the new model.

With `READ MODEL` ⟨*gc*⟩`;` models are entered by their generating classes, and CoCo writes models by their generating classes, the maximal interaction terms in the log-linear interaction models. The *2-section graph* for the model with generating class $\mathcal{C}$ is the graph with *vertices* $e \in a$ for some $a \in \mathcal{C}$ and *edges* $\{e_1, e_2\} \subset a$ for some $a \in \mathcal{C}$. The 2-section graph is a simple undirected graph, where the edges only contains two vertices. The model is *graphical* if, and only if, the cliques in the 2-section graph for the model are the generating class $\mathcal{C}$ for the model. The *cliques* are the maximal complete subsets of the graph. A subset of the graph is *complete* if all pairs of vertices in the subset are mutual adjacent. Two vertices are *adjacent* if there is an edge between them.

The command `DROP EDGES` ⟨$e_1 e_2$⟩`;` will add the statement, that ⟨$e_1$⟩ and ⟨$e_2$⟩ are *conditionally independent* given the rest variables in the **current** (graphical) model. See Lauritzen (1982) or Whittaker (1990).

The command `COLLAPSE MODEL` ⟨*set*⟩`;` will collapse the **current** model onto the smallest set containing ⟨*set*⟩ and onto which the model is collapsible.

The command `DUAL TO NORMAL;` will generate a model, where the generating class for the **current** model is the *dual representation* of the generated model. The dual representation is the minimal interaction-terms set to zero. Together with `MEET OF MODELS;` this can be used to do `DROP INTERACTIONS` ⟨*gc*⟩`;` on several models, see chapter 7: "Editing Models with Tests". Analogously with `NORMAL TO DUAL;`.

See chapter 10: "Search" for classes of models.

## 5.2 The Model List: Shifting Base and Current

```
BASE;
CURRENT;
MAKE BASE ⟨a⟩;
MAKE CURRENT ⟨a⟩;
```

In tests the model called **current** is tested against **base**.

The model first read is both **base** and **current**, and is given the model number 1. The first model stays **base** until another model is declared as **base**. Additional read models are given an increasing number. The model last read is the **current** model. The **current** model can be declared as **base** with the command `BASE;`. Model number ⟨$a$⟩ is named **base** or **current** with the commands `MAKE BASE` ⟨$a$⟩`;` and `MAKE CURRENT` ⟨$a$⟩`;` respectively. The last model generated as a result of the procedures `DROP EDGES` ⟨*edge-list*⟩`;`, `ADD EDGES` ⟨*edge-list*⟩`;`, `DROP INTERACTIONS` ⟨*gc*⟩`;`, etc. is the **last** model and can be named **current** with the command `CURRENT;`.

## 5.3 Printing, Describing and Disposing of Models

```
PRINT MODEL [ Base | Current | Last | Number ⟨a⟩ | All models
    | Interval of models ⟨a⟩ ⟨b⟩ ];
```

These six commands respectively prints the **current** model, the **base** model, the **last** model, model number ⟨$a$⟩, all models, and finally models with numbers between ⟨$a$⟩ and ⟨$b$⟩ The generating classes for the models are printed. If a list of models is printed the **current** and **base** model are marked.

A more detailed description of the models is obtained by replacing PRINT with DESCRIBE.

```
DESCRIBE MODEL [ Base | Current | Last | Number ⟨a⟩ | All models
    | Interval of models ⟨a⟩ ⟨b⟩ ];
```

The command `DESCRIBE CURRENT;` gives a very detailed (data-independent) description of the **current** model with *adjacency matrix*, an *expression* for the maximum

likelihood estimate of cell-probabilities, the *number of cells* in sufficient marginal tables and intersections of sufficient marginal tables and for each edge, the *number of cliques the edge is in*. (If a model is decomposable, and an edge is only in one clique, the model resulting from removing the edge, is decomposable.) See the next section for examples.

Models are disposed of with the commands `DISPOSE OF ...;`. As with `PRINT ...;` and `DESCRIBE ...;`, six `DISPOSE OF ...;`-commands are available:

```
DISPOSE OF MODEL [ Base | Current | Last | Number ⟨a⟩ | All models
    | Interval of models ⟨a⟩ ⟨b⟩ ];
```

## 5.4 Formulas

```
PRINT FORMULA;
PRINT VERTEX ORDERING;
DISPOSE OF FORMULA;
```

Two subgraphs of a graph are a *decomposition* of a graph with respect to a subset $a$ of the vertices if the graph is the union of two subgraphs and the intersection $a$ between the two subgraphs is complete. The graph is *decomposed* in to the two subgraphs. The subgraphs might be decomposed in to further subgraphs. If the graph and subgraphs can be decomposed recursively until the subgraphs are complete, the graph is *decomposable*.

The maximum likelihood estimate for the cell-probabilities can be expressed as

$$\hat{p}_{\Delta}(i) = c \prod_{k=1..u} n(i_{a_k})^{\nu(a_k)}$$

if the model is decomposable. The constant $c$ is equal to $1/|\mathcal{I}_{A^C}|$ where $A$ is the union of variables in the model. See Lauritzen (1982) and chapter 2 of Part I.

If the model can be decomposed, but the model not is decomposable, then the maximum likelihood estimate for the cell-probabilities is found by

$$\hat{p}_{\Delta}(i) = c \prod_{k=1..u} n(i_{a_k})^{\nu(a_k)} \prod_{l=1..v} \hat{p}_{\mathcal{B}_l}(i_{b_l})$$

where $\mathcal{B}_l$ is the generating classes for *irreducible components*, *atoms*, that cannot be decomposed further. The irreducible components $a_k$ and $B_l$ are vertex-sets for the subgraphs, into which the graph is decomposed. The maximum likelihood estimates $\hat{p}_{\mathcal{B}_l}$ for the cell-probabilities on the non-decomposable irreducible components $B_l$ are found by the IPS-algorithm, the *iterative proportional fitting*, *iterative proportional scaling* or *Deming-Staphan algorithm*, see, e.g., Darroch & Ratcliff (1972), Fienberg (1970), Haberman (1972), Jiroušek (1991) and chapter 2 of Part I.

The formula and vertex ordering is printed with `PRINT FORMULA;` and `PRINT VERTEX ORDERING;` respectively.

The following continuous the "Introduction", the final model found, but with the interaction $< ABC >$:

```
>read model ADE,ABE,ABC,BF.;
 TIME:            0.002secs.

>set page formats 70 65;
 TIME:            0.001secs.

>print vertex ordering;
  V   Order(V)   C(V)    Complete(V)
  A:         6   []       TRUE
  B:         5   [A]      TRUE
  C:         4   [AB]     TRUE
  D:         2   [AE]     TRUE
  E:         3   [AB]     TRUE
  F:         1   [B]      TRUE
 TIME:            0.007secs.

>describe current;
   2: [[ADE][ABE][ABC][BF]]
 Model is graphical
 Graph is decomposable
 Dual rep:     [[AF][CF][BD][CD][DF][EF][CE]]

   Adjacency matrix

    A B C D E F
  A   * * * *     [BCDE]
  B *   *   * *   [ACEF]
  C * *           [AB]
  D *       *     [AE]
  E * *   *       [ABD]
  F   *           [B]

  #Cliques   Edges
         1:  [DE][BF][BE][BC][AD][AC]
         2:  [AE][AB]

      #Cells      Expression
           1 /    N ( I []       ) ^  -1 *
           2 /    N ( I [B]      ) ^  -1 *
           4 /    N ( I [BF]     ) ^   1 *
           4 /    N ( I [AE]     ) ^  -1 *
           4 /    N ( I [AB]     ) ^  -1 *
           8 /    N ( I [ADE]    ) ^   1 *
           8 /    N ( I [ABE]    ) ^   1 *
           8 /    N ( I [ABC]    ) ^   1 *
                  1.0000000

 Log(L):                   -
 Dimension:               17

 TIME:            0.016secs.
```

```
>set power lambda null;
 Power Divergence not printed
 TIME:            0.001secs.

>set adjusted df off;
 Adjusted df set OFF
 TIME:            0.000secs.

>drop interaction ABC.;
   3: [[ABE][ADE][BF][BC][AC]]
 Model is not graphical
 Cliques:[[ABC][ABE][ADE][BF]]
 2-Section Graph is decomposable
 Test of [[ABE][ADE][BF][BC][AC]]
 against [[ABCDEF]]

          DF  -2log(Q)       P       X^2       P        Models

          47   58.0908 0.12885   57.0962 0.14863    [[ABCDEF]] / [[AC][BC]
                                                      [BF][ADE][ABE]]
 TIME:            0.033secs.

>current;
 TIME:            0.002secs.

>print formular;
 P [[ABE][ADE][BF][BC][AC]] ( I [ABCDEF] ) =
   N ( I [B]      ) ^  -1 *
   N ( I [BF]     ) ^   1 *
   N ( I [AE]     ) ^  -1 *
   N ( I [AB]     ) ^  -1 *
   N ( I [ADE]    ) ^   1 *
   N ( I [ABE]    ) ^   1 *
   P [[AB][BC][AC]] ( I [ABC] ) *
   1.0000000
 TIME:            0.005secs.

>drop edge AB.;
   4: [[AC][ADE][BC][BE][BF]]
 Model is graphical
 Graph is not decomposable
 Generating class for Fill In: [[ACE][ADE][BCE][BF]]
 Test of [[AC][ADE][BC][BE][BF]]
 against [[ABCDEF]]

          DF  -2log(Q)       P       X^2       P        Models

          49   58.2814 0.17094   57.5445 0.18832    [[ABCDEF]] / [[BF][BE]
                                                      [BC][ADE][AC]]
 TIME:            0.029secs.
```

```
>current;
 TIME:            0.001secs.

>print formular;
 P [[AC][ADE][BC][BE][BF]] ( I [ABCDEF] ) =
   N ( I [B]      ) ^  -1 *
   N ( I [BF]     ) ^   1 *
   N ( I [AE]     ) ^  -1 *
   N ( I [ADE]    ) ^   1 *
   P [[BE][BC][AE][AC]] ( I [ABCE] ) *
   1.0000000
 TIME:            0.004secs.

>quit;
 TIME:            0.000secs.
 TOTAL TIME:      0.187secs.
```

The model {{ADE},{ABE},{ABC},{BF}} is the resulting decomposable model from the coherent backward elimination with exact tests on data from Reiniš et al. (1981). The closed form expression for the maximum likelihood estimate for the cell-probabilities is

$$\hat{p}(i_\Delta) = \frac{n(i_{ABC}) \cdot n(i_{ABE}) \cdot n(i_{ADE}) \cdot n(i_{BF})}{n(i_{AB}) \cdot n(i_{AE}) \cdot n(i_B) \cdot n}.$$

`PRINT VERTEX ORDERING;` shows that a perfect vertex elimination ordering is F, D, E, C, B and A. If the vertices are removed from the graph in that order, then the boundary $C(V)$ around the removed vertex in the rest of the graph will be complete. $Complete(V)$ is `TRUE` if the boundary is complete.

Removal of the edges DE, BF, BE, BC, AD and AC will result in a decomposable model, since they are only in one clique. Removal of AE or AB will result in a non-decomposable model. See the output from `DESCRIBE CURRENT;`.

The model {{ADE},{ABE},{AC},{BC},{BF}} generated by removing the interaction $< ABC >$ is not graphical, but the 2-section graph is decomposable. The model is not graphical since the clique $< ABC >$ in the 2-section graph is not in the generating class for the model. See the result from `DROP INTERACTIONS ABC.;`. The expression for the maximum likelihood estimate for the cell-probabilities is

$$\hat{p}(i_\Delta) = \frac{\hat{p}_{\{\{AB\}\{BC\}\{AC\}\}}(i_{ABC}) \cdot n(i_{ABE}) \cdot n(i_{ADE}) \cdot n(i_{BF})}{n(i_{AB}) \cdot n(i_{AE}) \cdot n(i_B)}.$$

The model {{ADE},{AC},{BC},{BE},{BF}} generated by removing the edge $< AB >$ is graphical, but not decomposable. The graph contains the 4-cycle $< AC >< CB >< BE >< EA >$. See the output from `DROP EDGES AB.;`. The expression for the maximum likelihood estimate for the cell-probabilities is

$$\hat{p}(i_\Delta) = \frac{\hat{p}_{\{\{BE\}\{BC\}\{AE\}\{AC\}\}}(i_{ABCE}) \cdot n(i_{ADE}) \cdot n(i_{BF})}{n(i_{AE}) \cdot n(i_B)}.$$

## 5.5 Is Graphical?, Is Decomposable?, Is Submodel? and Is In One Clique?

`IS GRAPHICAL [` $\langle gc \rangle$ `];`
`IS DECOMPOSABLE [` $\langle gc \rangle$ `];`
`IS SUBMODEL OF [` $\langle gc_1 \rangle$ `] [` $\langle gc_2 \rangle$ `];`
`IS IN ONE CLIQUE` $\langle factor_1 \rangle$ $\langle factor_2 \rangle$ `[` $\langle gc \rangle$ `];`

`IS GRAPHICAL;` without any arguments will return 'True', if the **current** model is graphical, else 'False'. `IS GRAPHICAL` $\langle gc \rangle$`;` with the argument $\langle gc \rangle$ will return 'True', if the model $\langle gc \rangle$ is graphical, else 'False'. Analogously with `IS DECOMPOSABLE;` and `IS DECOMPOSABLE` $\langle gc \rangle$`;`. 'True' is returned, if the model is decomposable.

`IS SUBMODEL OF;` without any arguments will return 'True', if the **current** model is a submodel of the **base** model. `IS SUBMODEL OF` $\langle gc \rangle$`;` with the argument $\langle gc \rangle$ will return 'True', if the model $\langle gc \rangle$ is a submodel of the **current** model. `IS SUBMODEL OF` $\langle gc_1 \rangle$ $\langle gc_2 \rangle$`;` with the arguments $\langle gc_1 \rangle$ and $\langle gc_2 \rangle$ will return 'True', if the model $\langle gc_1 \rangle$ is a submodel of the model $\langle gc_2 \rangle$.

The command `IS IN ONE CLIQUE` $\langle factor_1 \rangle$ $\langle factor_2 \rangle$`;` with the two arguments $\langle factor_1 \rangle$ and $\langle factor_2 \rangle$ will return 'True', if the edge $\langle factor_1 factor_2 \rangle$ is an edge of the **current** model. `IS IN ONE CLIQUE` $\langle factor_1 \rangle$ $\langle factor_2 \rangle$ $\langle gc \rangle$`;` with the arguments $\langle factor_1 \rangle$, $\langle factor_2 \rangle$ and $\langle gc \rangle$ will return 'True', if the edge $\langle factor_1 factor_2 \rangle$ is an edge of the model with generating class $\langle gc \rangle$.

## 5.6 Common Decompositions

`PRINT COMMON DECOMPOSITIONS;`
`DECOMPOSE MODELS` $\langle set \rangle$`;`

The command `PRINT COMMON DECOMPOSITIONS;` prints common decompositions for the **current** and **base** models. If the **current** and the **base** models are both decomposable with respect to $\langle a \rangle$ they are decomposed with the command, `DECOMPOSE MODELS` $\langle a \rangle$`;`. This will result in 4 new models, which are added to the model-list. See also section 6.5.1: "Partitioning and Common Decompositions".

# Chapter 6

# Tests

## 6.1 The Main Test Procedure

This section will describe how test-statistics and degrees of freedom are computed by the command `TEST;`, by the model-editing commands described in chapter 7, and by procedures for model-selection in CoCo. The likelihood ratio test-statistic -2log(Q), i.e., the deviance, Pearson's chi-square and the power divergence are by these commands computed as follows:

### 6.1.1 -2log(Q), Pearson's $\chi^2$ and Power Divergence

`TEST;`

If the generating class $\mathcal{C}$ for the **current** model is a sub-generating class of the generating class $\mathcal{B}$ for the **base** model, then the *deviance*, *Pearson's chi-square* $\chi^2$ and the *power divergence* $2nI^\lambda$ for the hypothesis that $p \in P_\mathcal{C}$ under $p \in P_\mathcal{B}$ is computed as

$$
\begin{aligned}
-2\log Q &= 2\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_b(i_\Lambda)}{\hat{p}_c(i_\Lambda)},\\
\chi^2 &= \sum_{i_\Lambda \in I_\Lambda} \frac{(n\hat{p}_b(i_\Lambda) - n\hat{p}_c(i_\Lambda))^2}{n\hat{p}_c(i_\Lambda)},\\
2nI^\lambda &= \frac{2}{\lambda(\lambda+1)}\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\{(\frac{\hat{p}_b(i_\Lambda)}{\hat{p}_c(i_\Lambda)})^\lambda - 1\},
\end{aligned}
$$

by the command `TEST;`, where $\hat{p}_c$ and $\hat{p}_b$ are the maximum likelihood estimates of $p$ under $P_\mathcal{C}$ and $P_\mathcal{B}$, respectively, and $\Lambda$ is the union of variables in $\mathcal{C}$ and $\mathcal{B}$. For each cell in the table spanned by factors in both models, a term for the statistics is computed and the terms are summarized.

If the table spanned by the factors of the test is to big to fit into the memory (after performing common decompositions) then Pearson's chi-square $\chi^2$ and the power

divergence $2nI^{\lambda}$ is not be computed, and the deviance is computed by the method described in section 6.4.

In this example one of the final models from Edwards & Havránek (1985), and analyzed in the introduction, is tested against the saturated model:

```
>read model *;
>read model ACE,ADE,BC,F.;
>test;
 Test of [[F][BC][ADE][ACE]]
 against [[ABCDEF]]

            Statistic       Asymptotic
 -2log(Q) =   62.0779   P =     0.0994
 Power    =   59.7593   P =     0.1396
 X^2      =   59.9956   P =     0.1350
 DF.      =                        49
>
```

The command `TEST;` performs a test of **current** model against the **base** model.

The power $\langle\lambda\rangle$ is by default set to 2/3. It is changed by the statement `SET POWER LAMBDA` $\langle\lambda\rangle$`;`. If $\lambda = 1$ is entered the power divergence is not printed, if the test-output is in table-format (See the `FACTORIZE ...;`- and `BACKWARD -;`statements).

### 6.1.2 Goodman and Kruskal's Gamma

`SET ORDINAL` $\langle set\rangle$`;`

If two factors are tested conditionally independent, e.g., by the command `TEST;`, by the model-editing commands described in chapter 7, or by procedures for model-selection, then Goodman and Kruskal's Gamma coefficient is computed besides the deviance, Pearson's chi-square $\chi^2$ and the power divergence $2nI^{\lambda}$. Factors are declared to be ordinal by the command `SET ORDINAL` $\langle set\rangle$`;`.

### 6.1.3 Dimension and Degrees of Freedom

The degrees of freedom $D.F.$ for the test of $\mathcal{C}$ under $\mathcal{B}$ is computed as the difference between the dimensions $\dim(P_{\mathcal{B}})$ and $\dim(P_{\mathcal{C}})$ for the two models:

$$D.F. = \dim(P_{\mathcal{B}}) - \dim(P_{\mathcal{C}}).$$

The dimension for a model with generating class $\mathcal{A}$ is computed by:

(1) If $\mathcal{A} = \{a\}$ is complete, i.e., the generating class contains only one generator

$$\dim(P_{\mathcal{A}}) = \textstyle\prod_{\delta\in a} |\mathcal{I}_\delta| - 1.$$

(2) If $\mathcal{A}$ is the union of two generating classes $\mathcal{A}_1$ and $\mathcal{A}_2$

$$\dim(P_{\mathcal{A}}) = \dim(P_{\mathcal{A}_1}) + \dim(P_{\mathcal{A}_2}) - \dim(P_{\mathcal{A}_1\wedge\mathcal{A}_2}),$$

where $\mathcal{A}_1 \wedge \mathcal{A}_2$ is the meet $\{a_1 \cap a_2 \mid a_1 \in \mathcal{A}_1, a_2 \in \mathcal{A}_2\}$ between the two generating classes (Lauritzen 1982).

### 6.1.4 Adjusted D.F.

For $\mathcal{A} = \{a\}$ complete, denote the number of cells with zero count in the $a$-marginal table $(m(i_a))_{i_a \in I_a}$ by

$$\mathcal{Z}(\mathcal{A}, m) = \mathcal{Z}(\{a\}, m) = \#\text{ of elements in } \{(m(i_a))_{i_a \in I_a} |\ m(i_a) = 0\}.$$

For $\mathcal{A}$ a union of two generating classes $\mathcal{A}_1$ and $\mathcal{A}_2$, set

$$\mathcal{Z}(\mathcal{A}, m) = \mathcal{Z}(\mathcal{A}_1, m) + \mathcal{Z}(\mathcal{A}_2, m) - \mathcal{Z}(\mathcal{A}_1 \wedge \mathcal{A}_2, m).$$

For a decomposable model the number $\mathcal{Z}(\mathcal{A}, n)$ is the number of zeros in sufficient marginal plus the index $\nu(a)$ times the number of zeros in intersecting $a$-marginal tables.

Then the adjustment of the degrees of freedom is computed as

$$\mathcal{Z}(\mathcal{B}, \hat{m}_\mathcal{C}) - \mathcal{Z}(\mathcal{C}, n)$$

and adjusted degrees of freedom

$$D.F_{adj} = \dim(P_\mathcal{B}) - \dim(P_\mathcal{C}) - (\mathcal{Z}(\mathcal{B}, \hat{m}_\mathcal{C}) - \mathcal{Z}(\mathcal{C}, n)),$$

where $\hat{m}_\mathcal{C}$ is the expected counts in the **current** model $\mathcal{C}$.

$\mathcal{Z}(\mathcal{C}, n)$ is (approximately) the number of parameters not estimable in the **current** model $\mathcal{C}$. $\mathcal{Z}(\mathcal{B}, \hat{m}_\mathcal{C})$ is the number of parameters used in the **base** model to describe the not-estimable effects in the **current** model. So $\mathcal{Z}(\mathcal{B}, \hat{m}_\mathcal{C}) - \mathcal{Z}(\mathcal{C}, n)$ is a false reduction in the number of parameters.

The adjusted D.F. is probably only correct for decomposable models on connected tables. But in decomposable models exact tests are available. In the non-decomposable models, in which no exact test is available, the adjustment is not correct. The adjusted D.F. computed by the above algorithm can be negative.

The following example is used to explain how output with adjusted degrees of freedom is to be read:

```
>set input data /user/jhb/Examples.CoCo/ksl.d;
 TIME:            0.003secs.
>read data;
 1082 cases read.
 TIME:            0.981secs.
>set power lambda null;
 Power Divergence not printed
 TIME:            0.002secs.

>read model *;
 TIME:            0.002secs.

>drop edge SW.;
   2: [[FKHBSAGY][FKHBAWGY]]
 Model is graphical
 Model is decomposable
 Test of [[FKHBSAGY][FKHBAWGY]]
 against [[FKHBSAWGY]]
```

```
          DF   #0  -2log(Q)       P       X^2       P       Models

          40   88   47.1253 0.20390   38.6293 0.53186   [[FKHBSAWGY]]
                                                          / [[FKHBAWGY]
                                                          [FKHBSAGY]]
 TIME:            0.048secs.

>print all models;
 Model no.   2 [[FKHBSAGY][FKHBAWGY]]
 Model no.   1 [[FKHBSAWGY]] /BASE/    /CURRENT/
 TIME:            0.001secs.

>current;
 TIME:            0.001secs.

>test;
 Test of [[FKHBAWGY][FKHBSAGY]]
 against [[FKHBSAWGY]] Re-use

            Statistic       Asymptotic       Adjusted
 -2log(Q) =   47.1253   P =     1.0000  /      0.2039
 Power    =   38.6293   P =     1.0000  /      0.5319
 X^2      =   38.6293   P =     1.0000  /      0.5319
 DF.      =                       128  /          40
 TIME:            0.009secs.

>find -2log(Q);
 Test of [[FKHBSAGY][FKHBAWGY]]
 against [[FKHBSAWGY]]
 -2log(Q) = 2 * ( -5548.1011 - -5571.6637 ) =    47.1253
 DF.      =              511 -        383 =          128   P =     1.0000
 Adjustment:             230 -        142 =           88
                 (       258)                        40   P =     0.2039
 TIME:            0.020secs.

>quit;
 TIME:            0.000secs.
 TOTAL TIME:      1.144secs.
```

The data for this example and the example of section 6.4 originated in an epidemiological panel study in County of Copenhagen (the so-called Glostrup Study).

In this example the statement `DROP EDGES ...;` was used to generate a model. See the next chapter about `DROP ...;`-statements. From the saturated model a model without the edge `SW` is generated and a test of the generated model against the saturated model is performed. The first test-output is given in the `ShortTestOutput`-format. After adjustment, the degree of freedom is equal to 40. The adjustment (labeled by `#0`) is 88. In the normal test-output (generated by the `TEST;`-statement) both the $p$-values for the unadjusted and adjusted degrees of freedom is printed (second and third column).

If the `FIND DEVIANCE;`-statement is used for the test instead of the `TEST;`-statement, then the number of zeros $\mathcal{Z}(\mathcal{B}, \hat{m}_{\mathcal{C}})$ and $\mathcal{Z}(\mathcal{C}, n)$ are printed. In this case 230 and 142 respectively. Also the observed number of zeros $\mathcal{Z}(\mathcal{B}, n)$ in the sufficient marginal tables (minus number of zeros in intersections of sufficient marginal tables, plus ...) for the **base** model is printed. In this case 258. (If `Trace` is on, the cumulated number of zeros for each sufficient marginal table is printed when computing $\mathcal{Z}(\mathcal{B}, \hat{m}_{\mathcal{C}})$ and $\mathcal{Z}(\mathcal{C}, n)$.)

Since all cells in the sufficient marginal tables have to be visited in order to compute the adjustment, the computing-time used to compute the adjustment is comparable to the time used to compute, e.g., the deviance. So, if you are not using the adjusted degrees of freedom anyway, the computing of it may be turned off with the `SET ADJUSTED DF [ On | Off ];`-statement.

The current implementation of the algorithm for finding the adjustment of the degrees of freedom requires space for large models. If CoCo is unable to compute the adjustment for a test, and the program is running under Unix, increase the value given by the `N` option when starting CoCo (and the `P` option if the models are non-decomposable), see section 11.5.5.

## 6.2 Exact Tests

`EXACT TEST;`

The distribution of the likelihood ratio statistic is poorly approximated by the Chi Square distribution. But exact $p$-values can be computed to any accuracy by Monte Carlo approximation. Random tables with margins equal to observed sufficient marginal tables for the hypotheses are generated, and it is counted how large a proportion of the generated tables have a test statistic larger than the observed.

The exact test for the removal of one edge in the saturated model is computed by the method in Kreiner (1987).

If two decomposable models differ with one edge, the test can be collapsed to a test of removal of one edge in a saturated model. The algorithm by Patefield (1981) is used to generate the individual tables.

The Monte Carlo algorithm of Patefield (1981) can also be extended to give approximation to any degree of accuracy of exact $p$-values for tests between any two nested decomposable models (Lauritzen 1993). The sequence of edges given by `FACTORIZE ONE EDGE;` is used to generate the sufficient marginal tables for the **base** model from the observed sufficient marginal tables for the **current** model.

An exact test between a decomposable **base** and decomposable **current** model is done by the command `EXACT TEST;`.

Exact tests between models generated by the commands `EXACT TEST;`, `FACTORIZE ONE EDGE;`, the model-editing commands, commands for stepwise model-selection and the EH-procedure are controlled by the following commands:

`SET EXACT TEST [ On | All | Deviance | Off ];`
`SET ASYMPTOTIC` ⟨*p-value*⟩`;`
`SET NUMBER OF TABLES { Null |` ⟨*number*⟩ `};`

```
SET LIST OF NUMBERS OF TABLES ⟨list⟩;
SET EXACT TEST FOR TOTAL TEST [ On | Off ];
SET EXACT TEST FOR PARTS OF TEST [ On | Off ];
SET EXACT TEST FOR UNPARTED TEST [ On | Off ];
SET EXACT EPSILON ⟨epsilon⟩;
SET SEED { ⟨seed⟩ | Random };
SET FAST [ On | Off ];
```

The command `SET EXACT TEST [ On | All | Deviance | Off ];` with no arguments sets the computation of `ExactTest` on or off (Default: Off). With `SET EXACT TEST Deviance;` it is possible to compute exact tests for only the deviance, i.e., the likelihood ratio test-statistics -2log(Q). (This does not save much computing-time, but exact $p$-values for the power-divergence $2nI^{\lambda}$ and Pearson's chi-square $\chi^2$ are typically less interesting).

Only if the asymptotic $p$-value is less than the ⟨*p-value*⟩ set by `SET ASYMPTOTIC` ⟨*p-value*⟩`;`, exact tests are computed (Default: 1, i.e., exact tests for all tests if `ExactTest` is on).

`SET NUMBER OF TABLES { Null |` ⟨*number*⟩ `};` sets the number of tables to generate in each exact test (Default: 1000). If the number of tables is set to 0 by `SET NUMBER OF TABLES Null;` or `SET NUMBER OF TABLES 0;` then first 20 tables are generated. Let $m$ be the number of these generated tables with deviance greater than or equal to the observed deviance. If $m$ is equal 0, 1 or 2 then the exact test will be based on 1000 additional random tables, if $m$ is equal 3, 4 or 5 then the exact test will be based on 500 additional random tables, if $m$ is equal 6, 7 or 8 then the exact test will be based on 100 additional random tables and if $m$ is greater than 8 then the exact test will be based on 20 additional random tables.

This variable number of tables to generate can be changed by `SET LIST OF NUMBERS OF TABLES` ⟨*initial number*⟩ ⟨*list of pairs*⟩`;`. The first argument is the initial number of tables to generate. The second argument is a list of pairs, where the first item of each pair is the upper limit of $m$ and the second item is the number of tables to generate. The list is terminated by a pair, where the first item is greater than or equal to the initial number of generated tables. E.g., the default list is entered by `SET LIST OF NUMBERS OF TABLES 20, 2, 1000, 5, 500, 8, 100, 20, 20;`.

Please disregard the following options before you is familiar with exact tests in CoCo:

If `ExactTest` and `Partitioning` are on, then exact tests are computed for decomposable models, that differ with one edge. If `Partitioning` is on and the decomposable models differ with more than one edge, then the computation of exact tests for each part of the test is controlled by `SET EXACT TEST FOR PARTS OF TEST [ On | Off ];` (Default: On), and the computation of exact tests for the total test is controlled by `SET EXACT TEST FOR TOTAL TEST [ On | Off ];` (Default: On). If `Partitioning` is off and the decomposable models differ with more than one edge, then the exact test is only computed if `SET EXACT TEST FOR UNPARTED TEST [ On | Off ];` is on. (Default: On).

If `Fast` is on: `SET FAST [ On | Off ];` (Default: Off), then for each sufficient marginal table of counts generated (each edge in the list of edges given by `FACTORIZE`

`ONE EDGE;`), a term to the deviance is computed, and for each generation of all sufficient marginals for the **base** model, the terms for the deviances are added and compared with the observed deviance, else for each generation of all sufficient marginals for the **base** model, the deviance between the **base** model and the **current** model is computed. This is the fastest for decomposable models which only differ with few edges, but if the models differ with many edges, the computing-time is the smallest, if fast is off. (In later versions of CoCo, the author might have decided what to do, and the commands `SET EXACT TEST [ All | Deviance ];` and `SET FAST [ On | Off ];` are removed.)

`SET SEED` ⟨*seed*⟩`;` sets the initial value ⟨*seed*⟩ for the random number generator. `SET SEED Random;` sets a random initial value (computed from CPU-time, process-id and/or real time).

The ⟨*epsilon*⟩ set by `SET EXACT EPSILON` ⟨*epsilon*⟩`;` is added to test-statistics for the generated table when deciding whether the generate statistics is greater than or equal to the observed statistics.

In the following example a exact test for A being conditionally independent of D given B, C, E and F and an exact test for one of the final models against the saturated is computed on the data from Edwards & Havránek (1985):

```
>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:            0.025secs.

>set exact on;
 Exact test set ON
 TIME:            0.002secs.

>set power lambda null;
 Power Divergence not printed
 TIME:            0.002secs.

>read model *;
 TIME:            0.001secs.

>drop edge AD.
   2: [[ABCEF][BCDEF]]
 Model is graphical
 Model is decomposable
 Test of [[ABCEF][BCDEF]]
 against [[ABCDEF]]
          DF   #0  -2log(Q)       P       X^2       P       Models

          16    0   28.7241 0.02566   27.4652 0.03630   [[ABCDEF]] / [[BCDEF]
                                                          [ABCEF]]
 Exact (   1000 )           0.04000                 -

 TIME:            0.904secs.
```

```
>read model ACE,ADE,BC,F.;
 TIME:            0.002secs.

>exact test;
 Test of [[F][BC][ADE][ACE]]
 against [[ABCDEF]]

            Statistic       Asymptotic       Adjusted          Exact
 -2log(Q) =   62.0779   P =     0.0994  /      0.0994  /      0.1500
 Power    =   59.9956   P =     0.1350  /      0.1350
 X^2      =   59.9956   P =     0.1350  /      0.1350
 DF.      =                        49  /          49
 TIME:            0.912secs.

>quit;
 TIME:            0.000secs.
 TOTAL TIME:      4.889secs.
```

## 6.3 Measures of Associations

The following command will compute lots of statistics for two factors independent etc. given other factors:

`SLICE` ⟨*factor-name-a*⟩ `,` ⟨*factor-name-b*⟩ `[ /` ⟨*set*⟩ `];`

For each 'slice' (2 dimensional table of ⟨*factor-name-a*⟩ by ⟨*factor-name-b*⟩) given configurations of the factors ⟨*set*⟩ the following measures of associations, statistics, is computed and printed:

For each configurations of the factors in the set ⟨*set*⟩ the 2 dimensional table of ⟨*factor-name-a*⟩ by ⟨*factor-name-b*⟩ is printed with margins, and, on each 'slice' (2 dimensional table), the following statistics is computed and printed:

Fisher's exact test, Pearson $\chi^2$ test, $G^2$: likelihood ratio test, Continuity-adjusted $\chi^2$, Yates corrected $\chi^2$, McNemar's test of symmetry, $\kappa$: Kappa, Cramer's V, Phi and maximum of Phi, Contingency Coefficient C and maximum of Contingency Coefficient C, Cross-product ratio alpha and logarithm of Cross-product ratio alpha, Mantel-Haenszel $\chi^2$, Yule's Q, Yule's Y, Cochran-Armitage Trend Test with Goodness of fit: linear trend, Pearson (product-moment) correlation coefficient, Spearman rank correlation coefficient, $\tau_b$: Kendall's Tau b, $\tau_c$: Stuart's Tau c, Somers' D, Goodman and Kruskal's $\tau$, $\gamma$: Gamma, $\lambda_{asym}$: Optimal prediction lambda, $\lambda$: Symmetric optimal prediction lambda, $\lambda^\star$: Modified asymmetric optimal prediction lambda, Uncertainty coefficient $U_{asym}$, Symmetric uncertainty coefficient $U$.

Agresti (1990) and Appendix A.5 of the manual for the BMDP statistical computer package (Dixon 1983) are good sources for references and for formulas for computing the above measures of associations with standard errors.

## 6.4 Computation of log(L) in Large Models

```
FIND LOG(L);
FIND DEVIANCE;
```

Since the maximum likelihood estimate can be expressed as

$$\hat{p}_\Lambda(i) = c \prod_{k=1..u} n(i_{a_k})^{\nu(a_k)} \prod_{l=1..v} \hat{p}_{\mathcal{B}_l}(i_{b_l})$$

the $\log(L)$ can be computed as:

$$\begin{aligned}
\log L &= \sum_{i\in I} \log(p_\Lambda(i)^{n(i)}) \\
&= \sum_{i\in I} n(i) \log\{c \prod_{k=1..u} n(i_{a_k})^{\nu(a_k)} \prod_{l=1..v} \hat{p}_{\mathcal{B}_l}(i_{b_l})\} \\
&= n\log(c) \\
&\quad + \sum_{k=1..u} \nu(a_k) \sum_{i_{a_k}\in I_{a_k}} n(i_{a_k}) \log(n(i_{a_k})) \\
&\quad + \sum_{l=1..v} \sum_{i_{b_l}\in I_{b_l}} n(i_{b_l}) \log(\hat{p}_{\mathcal{B}_l}(i_{b_l})).
\end{aligned}$$

Hence we only need to sum over cells in sufficient marginal tables.

`FIND LOG(L);` computes log(likelihood) by the method, i.e., by only summing up over cells in necessary marginal tables. This is effective in simple models of large dimensions. Differences between log(likelihood)-values for **base** and **current** the deviance, are printed with `FIND DEVIANCE;`, but be careful with errors caused by rounding off.

This method is also used to compute the deviance by any test-procedure of CoCo, if the saturated table is to big to fit into the memory (after performing common decompositions).

This method should only be used if the two models to be tested against each other have no common decomposition. If the deviance is wanted for the test of a large **current** against **base**, then print first common decompositions with the command `PRINT COMMON DECOMPOSITIONS;` (see the next section). If the two models have a common decomposition with respect to the set $a$, they are decomposed into 4 new models $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{C}_1$ and $\mathcal{C}_2$ with `DECOMPOSE MODELS` $\langle a \rangle$`;`. The 4 models can be seen with `PRINT MODEL All;`. The test statistic can then be computed as the sum of the statistic for the test of $\mathcal{C}_1$ against $\mathcal{B}_1$ and of the statistic for the test of $\mathcal{C}_2$ against $\mathcal{B}_2$. The test-procedures of CoCo performs possible common decompositions (as the command `PARTITIONING TEST;`) before computing the test-statistics except when using the command `TEST;`.

Example of computation of log(L) for test between two 23 dimensional models:

```
>set input data /user/jhb/Examples.CoCo/ex7a.dat;
 TIME:            0.002secs.
```

```
>read specification;
 TIME:            0.004secs.

>skip missing;
 TIME:            0.001secs.

>read observations;
  710 cases read.
 TIME:            0.721secs.

>set large;
 Large set ON
 TIME:            0.002secs.

>read model ABI,ACEHILP,CEHILOP,EILPQ,CEHIW,EHJW,HLMO, &
 + ->       DHMO,LPQR,EHVW,LNO,UVW,GIK.;
 TIME:            0.004secs.

>find log(l);
 Log(L):            -7202.4580
 Dimension:               1003
 TIME:            0.206secs.

>read model ABI,ACEHILP,CEHILOP,EILPQ,CEHIW,HLMO, &
 + ->       DHMO,LPQR,EHVW,EHJ,LNO,UVW,GIK.;
 TIME:            0.003secs.

>find log(l);
 Log(L):            -7211.0598
 Dimension:                991
 TIME:            0.037secs.

>print common decompositions;
 Decomposition of [[ABI][ACEHILP][CEHILOP][EILPQ][CEHIW][HLMO][DHMO][LPQR]
                   [EHVW][EHJ][LNO][UVW][GIK]]
 and              [[ABI][ACEHILP][CEHILOP][EILPQ][CEHIW][EHJW][HLMO][DHMO]
                   [LPQR][EHVW][LNO][UVW][GIK]]
  Decompositions:
  [ABCEGHIJKLMNOPQRUVW]    ( 19) / [HMO]( 3) / [DHMO]          ( 4)
  [ABCDEGHIJKLMNOPQUVW]    ( 19) / [LPQ]( 3) / [LPQR]          ( 4)
  [ABCDEGHIKLMNOPQRW]      ( 17) / [EHW]( 3) / [EHJUVW]        ( 6)
  [ABCDEGHIJKLMOPQRUVW]    ( 19) / [LO] ( 2) / [LNO]           ( 3)
  [ABCDEGHIJKLMNOPQRVW]    ( 19) / [VW] ( 2) / [UVW]           ( 3)
  [ABCDEHIJLMNOPQRUVW]     ( 18) / [I]  ( 1) / [GIK]           ( 3)
  Selected:
  [ABCDEGHIKLMNOPQRW]      ( 17) / [EHW]( 3) / [EHJUVW]        ( 6)
 TIME:            0.040secs.

>set adjusted off;
 Adjusted df set OFF
 TIME:            0.002secs.
```

```
>find -2log q;
 Test of [[ABI][ACEHILP][CEHILOP][EILPQ][CEHIW][HLMO][DHMO][LPQR][EHVW]
          [EHJ][LNO][UVW][GIK]]
 against [[ABI][ACEHILP][CEHILOP][EILPQ][CEHIW][EHJW][HLMO][DHMO][LPQR]
          [EHVW][LNO][UVW][GIK]]
 -2log(Q) = 2 * ( -7202.4580 - -7211.0598 ) =    17.2037
 DF.      =             1003 -        991 =           12   P =     0.1417
 TIME:            0.006secs.

>set adjusted on;
 Adjusted df set ON
 TIME:            0.001secs.

>find -2log q;
 Test of [[ABI][ACEHILP][CEHILOP][EILPQ][CEHIW][HLMO][DHMO][LPQR][EHVW]
          [EHJ][LNO][UVW][GIK]]
 against [[ABI][ACEHILP][CEHILOP][EILPQ][CEHIW][EHJW][HLMO][DHMO][LPQR]
          [EHVW][LNO][UVW][GIK]]
 -2log(Q) = 2 * ( -7202.4580 - -7211.0598 ) =    17.2037
 DF.      =             1003 -        991 =           12   P =     0.1417
 Adjustment:             755 -        755 =            0
                 (       757)                         12   P =     0.1417
 TIME:            0.851secs.

>keep log;
 Keep Log set ON
 TIME:            0.002secs.

>quit;
 TIME:            0.001secs.
 TOTAL TIME:      1.947secs.
```

The statement `SET LARGE [ On | Off ];` is used to tell CoCo to dispose of marginal tables immediately after use, that is, to only store marginal tables for one term in the expression for the maximum likelihood estimate at time. Note; the time used for computing the adjusted degrees of freedom is larger than the time used to compute the test statistics.

Since the two models of the above example have common decompositions we should first have decomposed the two models. For each possible common decomposition of the two models the set of vertices for the two parts of the graphs and the intersection between those two vertex-sets are given in the output from `PRINT COMMON DECOMPOSITIONS;`. The number of vertices in each set are also printed.

## 6.5 Miscellaneous Commands for Model Control

```
PARTITIONING TEST;
TEST ONE EDGE;
FACTORIZE ONE EDGE [ ⟨set⟩ ];
FACTORIZE ONE INTERACTION [ ⟨set⟩ ];
```

### 6.5.1 Partitioning and Common Decompositions

If $\mathcal{B}_1$, $\mathcal{B}_2$ is decomposition of $\mathcal{B}$ with respect to $b$, $\mathcal{C}_1$, $\mathcal{C}_2$ is decomposition of $\mathcal{C}$ with respect to $c$ and $b = c = a$, $\cup_{d\in\mathcal{B}_1} d = \cup_{d\in\mathcal{C}_1} d = \Lambda_1$ and $\cup_{d\in\mathcal{B}_2} d = \cup_{d\in\mathcal{C}_2} d = \Lambda_2$ then $\mathcal{B}$ and $\mathcal{C}$ are said to have a common decomposition, and if $\mathcal{C}$ is a sub-generating class of $\mathcal{B}$, the test statistic can be partitioned as:

$$\begin{aligned}
-2\log Q &= 2\sum_{i_\Lambda\in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_{\mathcal{B}}(i_\Lambda)}{\hat{p}_{\mathcal{C}}(i_\Lambda)} \\
&= 2\sum_{i_\Lambda\in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_{\mathcal{B}_1}(i_\Lambda)\hat{p}_{\mathcal{B}_2}(i_\Lambda)/(n(i_a)/n)}{\hat{p}_{\mathcal{C}_1}(i_\Lambda)\hat{p}_{\mathcal{C}_2}(i_\Lambda)/(n(i_a)/n)} \\
&= 2\sum_{i_{\Lambda_1}\in I_{\Lambda_1}} n(i_{\Lambda_1})\{\log\frac{\hat{p}_{\mathcal{B}_1}(i_{\Lambda_1})}{\hat{p}_{\mathcal{C}_1}(i_{\Lambda_1})}\} + 2\sum_{i_{\Lambda_2}\in I_{\Lambda_2}} n(i_{\Lambda_2})\{\log\frac{\hat{p}_{\mathcal{B}_2}(i_{\Lambda_2})}{\hat{p}_{\mathcal{C}_2}(i_{\Lambda_2})}\} \\
&= -2\log Q_1 \;+\; -2\log Q_2.
\end{aligned}$$

If $\mathcal{B}_2 = \mathcal{C}_2$ then $-2\log Q_2 = 0$ and the test is collapsed onto $\Lambda_1$. This is used in `TEST ONE EDGE;`, see the next section.

If **base** and **current** have common decompositions, then they are printed with the command `PRINT COMMON DECOMPOSITIONS;`. If the two models **base** $\mathcal{B}$ and **current** $\mathcal{C}$ are decomposable with respect to the set $a$, they are decomposed into 4 new models $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{C}_1$ and $\mathcal{C}_2$ with `DECOMPOSE MODELS` $\langle a\rangle$`;`. The 4 models can be seen with `PRINT MODEL All;`.

The command `PARTITIONING TEST;` performs recursive and automatic common decompositions on **base** and **current**. Tests are performed. If tests are *collapsible*, it is stated.

If cases with incomplete information (Missing Values) are read and `EXCLUDE MISSING On;` has been used, each test performed as a result of `PARTITIONING TEST;` is performed with all the cases with complete information in the marginal table on which each test is collapsible.

With the command `SET PARTITIONING On;` before `BACKWARD;`, `FORWARD;`, `FACTORIZE ONE EDGE [` $\langle set\rangle$ `];`, etc. each test caused by the command is partitioned. This is turned off again with `SET PARTITIONING Off;`. If `ExactTest` and `Partitioning` is on, then exact test is computed for decomposable models, that differ with one edge. If `Partitioning` is on and the decomposable models differ with more than one edge, then the computation of exact test for each part of the test is controlled by `SET EXACT TEST FOR PARTS OF TEST [ On | Off ];` (Default: On), and the computation of exact test for the total test is controlled by `SET EXACT TEST`

`FOR TOTAL TEST [ On | Off ];` (Default: On). If `Partitioning` is off and the decomposable models differ with more than one edge, then exact test is only computed if `SET EXACT TEST FOR UNPARTED TEST [ On | Off ];` is on. (Default: On).

Example of the use of the commands `PRINT COMMON DECOMPOSITIONS;` and `DECOMPOSE MODELS` $\langle a \rangle$`;` on the data from Edwards & Havránek (1985), see Introduction:

```
>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:            0.026secs.

>read model *;
 TIME:            0.002secs.

>read model ACDE,ABCF.;
 TIME:            0.001secs.

>read model ADE,ACE,ABC,ABF.;
 TIME:            0.001secs.

>make base 2;
 TIME:            0.001secs.

>print common decompositions;
 Decomposition of [[ADE][ACE][ABC][ABF]]
 and              [[ACDE][ABCF]]
  Decompositions:
  [ABCF]  (  4) / [AC]    (  2) / [ACDE]  (  4)
  Selected:
  [ABCF]  (  4) / [AC]    (  2) / [ACDE]  (  4)
 TIME:            0.006secs.

>decompose models AC.;
 Partitioning of [[ADE][ACE][ABC][ABF]]
 and [[ACDE][ABCF]]
 TIME:            0.004secs.

>print all models;
 Model no.   7 [[ACDE]]
 Model no.   6 [[ABCF]]
 Model no.   5 [[ACE][ADE]]
 Model no.   4 [[ABF][ABC]]
 Model no.   3 [[ADE][ACE][ABC][ABF]] /CURRENT/
 Model no.   2 [[ACDE][ABCF]] /BASE/
 Model no.   1 [[ABCDEF]]
 TIME:            0.005secs.
```

```
>read model ACE,ADE,BC,F.;
 TIME:            0.002secs.

>print common decompositions;
 Decomposition of [[ACE][ADE][BC][F]]
 and              [[ACDE][ABCF]]
  Decompositions:
  [ABCF]  (  4) / [AC]    (  2) / [ACDE]  (  4)
  Selected:
  [ABCF]  (  4) / [AC]    (  2) / [ACDE]  (  4)
 TIME:            0.006secs.

>set power lambda null;
 Power Divergence not printed
 TIME:            0.002secs.

>partitioning;
 Test of [[ACE][ADE][BC][F]]
 against [[ACDE][ABCF]]

        DF   #0  -2log(Q)       P       X^2       P       Models

         9    0   20.3954 0.01559   20.0744 0.01738   [[ABCF]] / [[AC][BC][F]]
   +     4    0    2.2147 0.69633    2.2138 0.69651   [[ACDE]] / [[ACE][ADE]]
   =    13    0   22.6101 0.04625   22.2881 0.05069   [[ACDE][ABCF]]
                                                        / [[ADE][ACE][F][BC]]

 TIME:            0.021secs.

>make base 3;
 TIME:            0.001secs.

>print common decompositions;
 Decomposition of [[ACE][ADE][BC][F]]
 and              [[ADE][ACE][ABC][ABF]]
  Decompositions:
  [ABCF]  (  4) / [AC]    (  2) / [ACDE]  (  4)
  [ABCEF] (  5) / [AE]    (  2) / [ADE]   (  3)
  Selected:
  [ABCF]  (  4) / [AC]    (  2) / [ACDE]  (  4)
 TIME:            0.007secs.

>set short off;
 Short test output set OFF
 TIME:            0.002secs.
```

```
>partitioning;
 Test of [[ACE][ADE][BC][F]]
 against [[ADE][ACE][ABC][ABF]]
 Partition of [ABCDEF] in [ABCF],[ACDE] by [AC]
   Test on [ABCF]
   Test of [[AC][BC][F]]
   against [[ABC][ABF]]

              Statistic       Asymptotic       Adjusted
   -2log(Q) =   13.3773   P =     0.0199  /      0.0199
   Power    =   13.1500   P =     0.0218  /      0.0218
   X^2      =   13.1500   P =     0.0218  /      0.0218
   DF.      =                         5  /           5
   Collaps.
   Models  [[ADE][ACE]]
   and     [[ACE][ADE]]
   on [ACDE] identical.
 TIME:            0.021secs.

>quit;
 TIME:            0.001secs.
 TOTAL TIME:      0.174secs.
```

In the output from `PRINT COMMON DECOMPOSITIONS;` all common decompositions are printed. For each decomposition a line with the 3 sets $\Lambda_1$, $a$ and $\Lambda_2$ is printed. The cardinality of the sets is noted. The decomposition where the largest of the two cardinalities of $\Lambda_1$ and $\Lambda_2$ respectively is minimum is selected.

Note in the short test output from `PARTITIONING TEST;`: The characters + and = are used to express that the parts of the test adds up to the total test.

### 6.5.2 Testing One Edge

If two decomposable models (**base** and **current**) differ with exactly one edge $\{\alpha, \beta\}$, then both models are collapsible onto the clique $a$ containing the edge $\{\alpha, \beta\}$ in the **base** model. The $a$-marginal table of maximum likelihood estimates is determined by

$$\hat{p}_{\mathcal{B}}(i_a) = n(i_a)/n$$

and

$$\hat{p}_{\mathcal{C}}(i_a) = \frac{n(i_{a\setminus\{\alpha\}})n(i_{a\setminus\{\beta\}})}{n \cdot n(i_{a\setminus\{\alpha,\beta\}})}.$$

E.g., the deviance can be found as

$$-2\log(Q) = 2\sum_{i_a \in I_a} n(i_a) \log \frac{n(i_a)n(i_{a\setminus\{\alpha,\beta\}})}{n(i_{a\setminus\{\alpha\}})n(i_{a\setminus\{\beta\}})}$$

by only summing over cells in the $a$-marginal table.

The command `TEST ONE EDGE;` finds the test statistics by summing only over cells in the table on which the test is collapsible, if the two models are decomposable and differ with exactly one edge.

### 6.5.3 Factorization of Test into Tests of One Edge Missing

If $\mathcal{C}$ is a decomposable submodel of a decomposable model $\mathcal{B}$ and the two models differ by $k$ edges, then there exist sequences $\mathcal{B} = \mathcal{A}_0$, $\mathcal{A}_1$, ... , $\mathcal{A}_\mathrm{k} = \mathcal{C}$ of models such that for $i = 1, \ldots, k$ $\mathcal{A}_{\mathrm{i}-1}$ and $\mathcal{A}_\mathrm{i}$ differ with exactly one edge, and $\mathcal{A}_\mathrm{i}$ is decomposable. The likelihood ratio statistics can then be computed by:

$$\begin{aligned}
-2\log Q &= 2\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_\mathcal{B}(i_\Lambda)}{\hat{p}_\mathcal{C}(i_\Lambda)} \\
&= 2\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_\mathcal{B}(i_\Lambda)}{\hat{p}_{\mathcal{A}_1}(i_\Lambda)}\frac{\hat{p}_{\mathcal{A}_1}(i_\Lambda)}{\hat{p}_{\mathcal{A}_2}(i_\Lambda)}\cdots\frac{\hat{p}_{\mathcal{A}_{\mathrm{k}-2}}(i_\Lambda)}{\hat{p}_{\mathcal{A}_{\mathrm{k}-1}}(i_\Lambda)}\frac{\hat{p}_{\mathcal{A}_{\mathrm{k}-1}}(i_\Lambda)}{\hat{p}_\mathcal{C}(i_\Lambda)} \\
&= 2\sum_{l=1\ldots k}\sum_{i_{\Lambda_l}\in I_{\Lambda_l}} n(i_{\Lambda_l})\log\frac{\hat{p}_{\mathcal{A}_{1-1}}(i_{\Lambda_l})}{\hat{p}_{\mathcal{A}_1}(i_{\Lambda_l})} \\
&= \sum_{l=1\ldots k} -2\log Q_l.
\end{aligned}$$

The likelihood ratio for the test between the two decomposable models is by the command `FACTORIZE ONE EDGE [` $\langle set \rangle$ `];` factorized into a product of likelihood ratios between decomposable models that differ with exactly one edge. The sequence of models is not unique. Choose between different orders of edges with `SET ALGORITHM { A | B | C };` before `FACTORIZE ONE EDGE;`. Algorithm A and B add edges to the **current** model by testing for decomposability by the algorithm TestForZeroFillIn of Tarjan & Yannakakis (1984). Algorithm B tries the edges in the opposite order as algorithm A. Algorithm C removes edges from the **base** model by looking for edges only in one generator. If the optional argument $\langle set \rangle$ is given, then the edges are tried according to the order of the factors in $\langle set \rangle$.

The following example shows a factorization of the test of one of the final models from Edwards & Havránek (1985) against the saturated model:

```
>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:            0.024secs.

>read model *;
 TIME:            0.001secs.

>read model ACE,ADE,BC,F.;
 TIME:            0.001secs.

>print all models;
 Model no.   2 [[ACE][ADE][BC][F]] /CURRENT/
 Model no.   1 [[ABCDEF]] /BASE/
 TIME:            0.002secs.
```

```
>test;
 Test of [[F][BC][ADE][ACE]]
 against [[ABCDEF]]

            Statistic       Asymptotic       Adjusted
 -2log(Q) =   62.0779   P =     0.0994  /      0.0994
 Power    =   59.7593   P =     0.1396  /      0.1396
 X^2      =   59.9956   P =     0.1350  /      0.1350
 DF.      =                       49  /          49
 TIME:            0.019secs.

>set power lambda null;
 Power Divergence not printed
 TIME:            0.002secs.

>set adjusted df off;
 Adjusted df set OFF
 TIME:            0.002secs.

>set test formats 8 3 6 4;
 TIME:            0.002secs.

>set factorize c;
 TIME:            0.001secs.

>factorize one edge FEDCBA. ;
 Test of [[ACE][ADE][BC][F]]
 against [[ABCDEF]]

  Edge   DF -2log(Q)      P      X^2      P      Models

  [EF]   16   18.316 0.3052   17.820 0.3341   [[ABCDEF]] / [[ABCDE][ABCDF]]
  [DF]    8    9.199 0.3252    9.086 0.3346   [[ABCDF]] / [[ABCD][ABCF]]
  [CD]    8    4.893 0.7705    4.800 0.7801   [[ABCDE]] / [[ABCE][ABDE]]
  [CF]    4    7.018 0.1349    7.459 0.1135   [[ABCF]] / [[ABC][ABF]]
  [BD]    4    2.504 0.6439    2.502 0.6443   [[ABDE]] / [[ABE][ADE]]
  [BE]    4    6.771 0.1485    6.944 0.1389   [[ABCE]] / [[ABC][ACE]]
  [BF]    2    6.321 0.0424    6.349 0.0418   [[ABF]] / [[AB][AF]]
  [AB]    2    5.988 0.0501    5.953 0.0510   [[ABC]] / [[AC][BC]]
  [AF]    1    1.069 0.3012    1.070 0.3010   [[AF]] / [[A][F]]
  =      49   62.078 0.0994   61.983 0.1008   [[ABCDEF]] / [[F][ADE][ACE][BC]]
 TIME:            0.040secs.

>set exact test all;
 Exact test set ON
 Only Exact Deviance set OFF
 TIME:            0.001secs.
```

```
>set number of tables null;
 TIME:            0.001secs.

>set factorize a;
 TIME:            0.002secs.

>dispose tests;
 TIME:            0.001secs.

>factorize one edge * ;
 Test of [[ACE][ADE][BC][F]]
 against [[ABCDEF]]

   Edge   DF -2log(Q)      P      X^2      P      Models

   [AB]    2    5.988 0.0501    5.953 0.0510   [[ABC]] / [[AC][BC]]
 Exact (   1000 )     0.0430          0.0460
   [AF]    1    1.069 0.3012    1.070 0.3010   [[AF]] / [[A][F]]
 Exact (    100 )     0.3500          0.3500
   [BE]    4    6.771 0.1485    6.944 0.1389   [[ABCE]] / [[ACE][ABC]]
 Exact (    200 )     0.1650          0.1550
   [BD]    4    2.504 0.6439    2.502 0.6443   [[ABDE]] / [[ADE][ABE]]
 Exact (     20 )     0.6000          0.6000
   [BF]    2    6.321 0.0424    6.349 0.0418   [[ABF]] / [[AB][AF]]
 Exact (   1000 )     0.0430          0.0400
   [CD]    8    4.893 0.7705    4.800 0.7801   [[ABCDE]] / [[ABCE][ABDE]]
 Exact (     20 )     0.6500          0.6500
   [CF]    4    7.018 0.1349    7.459 0.1135   [[ABCF]] / [[ABC][ABF]]
 Exact (    100 )     0.2400          0.1900
   [DF]    8    9.199 0.3252    9.086 0.3346   [[ABCDF]] / [[ABCD][ABCF]]
 Exact (    100 )     0.3100          0.3300
   [EF]   16   18.316 0.3052   17.820 0.3341   [[ABCDEF]] / [[ABCDF][ABCDE]]
 Exact (    100 )     0.3600          0.3500
   =      49   62.078 0.0994   61.983 0.1008   [[ABCDEF]]
                                                  / [[ADE][ACE][F][BC]]
 Exact (    200 )     0.1750          0.1200

 TIME:            5.645secs.

>quit;
 TIME:            0.000secs.
 TOTAL TIME:      5.581secs.
```

Note that the edge $< BF >$ is significant.

### 6.5.4 Factorization of Test into Tests of One Missing Interaction

If $\mathcal{C}$ is a submodel of $\mathcal{B}$ then sequences $\mathcal{B} = \mathcal{A}_0, \mathcal{A}_1, \ldots, \mathcal{A}_k = \mathcal{C}$ of models exist which for $i = 1, \ldots, k$ $\mathcal{A}_{i-1}$ and $\mathcal{A}_i$ differ with exactly one interaction-term. The likelihood ratio statistics can then be computed by:

$$\begin{aligned}
-2\log Q &= 2\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_{\mathcal{B}}(i_\Lambda)}{\hat{p}_{\mathcal{C}}(i_\Lambda)} \\
&= 2\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_{\mathcal{B}}(i_\Lambda)}{\hat{p}_{\mathcal{A}_1}(i_\Lambda)}\frac{\hat{p}_{\mathcal{A}_1}(i_\Lambda)}{\hat{p}_{\mathcal{A}_2}(i_\Lambda)}\cdots\frac{\hat{p}_{\mathcal{A}_{k-2}}(i_\Lambda)}{\hat{p}_{\mathcal{A}_{k-1}}(i_\Lambda)}\frac{\hat{p}_{\mathcal{A}_{k-1}}(i_\Lambda)}{\hat{p}_{\mathcal{C}}(i_\Lambda)} \\
&= 2\sum_{l=1\ldots k}\sum_{i_\Lambda \in I_\Lambda} n(i_\Lambda)\log\frac{\hat{p}_{\mathcal{A}_{l-1}}(i_\Lambda)}{\hat{p}_{\mathcal{A}_l}(i_\Lambda)} \\
&= \sum_{l=1\ldots k} -2\log Q_l.
\end{aligned}$$

Interactions not in the **current** model but in **base** are added to the current model one at a time and for each interaction the generated model is tested against the previous model with the command `FACTORIZE ONE INTERACTION [` $\langle set \rangle$ `];`. If not the optional argument $\langle set \rangle$ is given, then the interactions are added according to cardinality. If $\langle set \rangle$ is given, then first the first factor in $\langle set \rangle$ is added, then the second factor in $\langle set \rangle$ is added, then the interaction among the first two factors in $\langle set \rangle$ is tried, then the interactions among the first three factors in $\langle set \rangle$ are tried according to cardinality, etc.

If all factors are binary, the test of **current** against **base** is factorized into tests with one degree of freedom.

Since most of the models are not decomposable, the factorization takes time.

The following example shows a factorization of the test of one of the final models from Edwards & Havránek (1985) against some model:

```
>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:            0.026secs.

>read model ABF,ABC,ACDE.;
 TIME:            0.003secs.

>read model ACE,ADE,BC,F.;
 TIME:            0.002secs.

>print all models;
 Model no.   2 [[ACE][ADE][BC][F]] /CURRENT/
 Model no.   1 [[ABF][ABC][ACDE]] /BASE/
 TIME:            0.003secs.
```

```
>test;
 Test of [[F][BC][ADE][ACE]]
 against [[ACDE][ABC][ABF]]

            Statistic       Asymptotic       Adjusted
 -2log(Q) =   15.5920   P =     0.0754  /      0.0754
 Power    =   15.4412   P =     0.0790  /      0.0790
 X^2      =   15.3837   P =     0.0804  /      0.0804
 DF.      =                        9  /           9
 TIME:            0.028secs.

>set power lambda null;
 Power Divergence not printed
 TIME:            0.001secs.

>set adjusted df off;
 Adjusted df set OFF
 TIME:            0.001secs.

>factorize one interaction * ;
 Test of [[ACE][ADE][BC][F]]
 against [[ABF][ABC][ACDE]]

  Edge       DF  -2log(Q)       P       X^2       P       Models

  [AB]        1  1.31E-03 0.97113  1.28E-03 0.97146   [[BC][AC][AB]]
                                                        / [[AC][BC]]
  [AF]        1    1.0687 0.30125    1.0697 0.30102   [[AF]] / [[F][A]]
  [BF]        1    5.0955 0.02399    5.1413 0.02336   [[AB][AF][BF]]
                                                        / [[AF][AB]]
  [CD]        1    1.3559 0.24425    1.3549 0.24443   [[CD][ACE][ADE]]
                                                        / [[ADE][ACE]]
  [ABC]       1    5.9864 0.01442    5.9506 0.01471   [[ABC]] / [[AB][AC][BC]]
  [ABF]       1    1.2255 0.26829    1.2216 0.26905   [[ABF]] / [[AB][BF][AF]]
  [ACD]       1    0.0777 0.78050    0.0776 0.78057   [[ACD][ACE][ADE]]
                                                        / [[ADE][ACE][CD]]
  [CDE]       1    0.4997 0.47961    0.4995 0.47972   [[CDE][ACD][ACE][ADE]]
                                                        / [[ADE][ACE][ACD]]
  [ACDE]      1    0.2816 0.59566    0.2817 0.59560   [[ACDE]] / [[ADE][ACE]
                                                        [ACD][CDE]]
  =           9   15.5922 0.07541   15.5980 0.07528   [[ACDE][ABF][ABC]]
                                                         / [[ACE][ADE][F][BC]]

 TIME:            0.148secs.

>quit;
 TIME:            0.001secs.
 TOTAL TIME:      0.255secs.
```

Note that the edge $< BF >$ and interaction $< ABC >$ are significant. Most is the deviance is due to those two interactions.

## 6.6 Reuse of Tests

```
SET REUSE OF TESTS [ On | Off ];
SHOW TESTS;
DISPOSE OF TESTS;
```

All performed tests are stored and re-used, i.e., each performed test is pushed into a structure, and when a test is to be performed this structure is searched for a test between the two models of the test. This feature is turned on and off with `SET REUSE OF TESTS [ On | Off ];`. The total list of performed tests is printed or disposed of with `SHOW TESTS;` and `DISPOSE OF TESTS;` respectively.

The list of tests is disposed of when the data is changed by reading observations or Q-tables.

Also the exact $p$-values are re-used. Exact tests are reused if the current number of tables (`SET NUMBER OF TABLES { Null |` ⟨*number*⟩ `};`) is equal to the number of tables generated to compute the stored test or if the current number of tables is equal to 0. You may force CoCo to compute new exact $p$-values for the same test without disposing of the list of tests by changing the number of generated tables (e.g., from 1000 to 999).

Stored values of -2Log(Q) may depend on factorization and partitioning.

Note that, if a test is computed by factorization or partitioning, then the sum of $\chi^2$-statistics is only approximately the $\chi^2$-statistic for the full test. Analogously with power divergence.

If a test is reused it is annotated with `Reuse` or an `r` in `ShortTestOutput`. (Tests in a sorted list of tests are always marked by `r`.)

# Chapter 7

# Editing Models with Tests

`[ Only ] GENERATE DECOMPOSABLE MODEL ;`
`[ Only ] GENERATE GRAPHICAL MODEL ;`
`[ Only ] DROP EDGES` ⟨*edge-list*⟩`;`
`[ Only ] ADD EDGES` ⟨*edge-list*⟩`;`
`[ Only ] DROP INTERACTIONS` ⟨*gc*⟩`;`
`[ Only ] ADD INTERACTIONS` ⟨*gc*⟩`;`
`[ Only ] MEET OF MODELS ;`
`[ Only ] JOIN OF MODELS ;`

The commands in this chapter "edit" the **current** model. If not, the keyword `Only` is used, a test is computed.

The computed test-statistics will depend on the use of `SET ORDINAL` ⟨*set*⟩`;` and `SET ADJUSTED DF [ On | Off ];` and on the value set by `SET POWER LAMBDA { Null |` ⟨*lambda*⟩ `};`. See the following chapter.

Exact tests might be selected by `SET EXACT TEST [ On | All | Deviance | Off ];`. If `ExactTest` is selected, then the commands `SET ASYMPTOTIC` ⟨*p-value*⟩`;`, `SET NUMBER OF TABLES { Null |` ⟨*number*⟩ `};`, `SET LIST OF NUMBERS OF TABLES` ⟨*list*⟩`;`, `SET SEED {` ⟨*seed*⟩ `| Random };` `SET EXACT EPSILON` ⟨$\epsilon$⟩`;` and `SET FAST [ On | Off ];` should be noted. See the section 6.2 about exact tests in chapter 6: "Tests".

Partitioning of tests is controlled by `SET PARTITIONING [ On | Off ];`. The commands `SET EXACT TEST FOR TOTAL TEST [ On | Off ];`, `SET EXACT TEST FOR PARTS OF TEST [ On | Off ];` and `SET EXACT TEST FOR UNPARTED TEST [ On | Off ];` will control for which parts of tests to compute exact tests.

`Reuse` of test is controlled by `SET REUSE OF TESTS [ On | Off ];` (and `SET NUMBER OF TABLES { Null |` ⟨*number*⟩ `};`). The output might depend on the options `TestFormats`, `ShortTestOutput`, `PageFormats`, `Report`, `Trace`, `Timer`, `Diary`, `Log`, etc.

Finally the convergence-criterion for the IPS- and EM-algorithm, choice of how to handle missing values and data structure will have effect on the computing time.

## 7.1 Generate Graphical Model

With `GENERATE GRAPHICAL MODEL;` the 2-section graph for the **current** model is made. The generated model is the model with the generating class equal to the cliques in the 2-section graph for the **current** model, the maximal interaction terms are the cliques in the 2-section graph for the **current** model. The **current** model stays **current**. The generated model is added to the model list. It can be disposed of with `DISPOSE OF MODEL Last;` or it can be named **current** with the command `CURRENT;`. If **base** and **current** are the same models then the **current** model is tested against the generated model, else the generated model is tested against **base** model (if not, the key-word `Only` is used). The command `BASE;` names the **current** model **base**.

## 7.2 Add Fill-In: Generate Decomposable Model

A FILL-IN is made (adding extra edges) which is minimal in the sense that no subset of the fill-in will make the graph decomposable. The minimal FILL-IN is added to the 2-section graph for the **current** model. The generated model is the model with the generating class equal to the cliques in that decomposable 2-section graph. (Note: The added FILL-IN is not unique (e.g., there are two ways to add an edge to a graph with only four vertices and consisting of a four-cycle in order that the graph becomes decomposable) and is not minimum: there may be another FILL-IN (not a subset of the selected) with fewer edges.) If **base** is equal to **current**, then the **current** model is tested against the generated model, else the generated model is tested against **base** model (if not the key-word `Only` is used).

## 7.3 Drop Edges

With `DROP EDGES` ⟨*edge-list*⟩`;` the edges ⟨*edge-list*⟩ in the 2-section graph for the **current** model are removed and the generated model is tested against the **base** model. Note that the generated model is graphical. Use `DROP INTERACTIONS` ⟨*gc*⟩`;` if a first-order interaction is to be removed from a hierarchical model. The **current** model stays **current**. The generated model is added to the model list. It can be disposed of with `DISPOSE OF MODEL Last;` or it can be named **current** with the command `CURRENT;`. The command `BASE;` names the **current** model **base**. The following commands in this chapter treat **base**, **current** and **last** model in the same way.

## 7.4 Add Edges

The command `ADD EDGES` ⟨*edge-list*⟩`;` adds the edges ⟨*edge-list*⟩ to the 2-section graph for the **current** model and tests the resulting graphical model against the **base** model. If **base** and **current** are the same models then the **current** model is tested against the generated model, else the generated model is tested against **base** model (if not the key-word `Only` is used).

## 7.5 Drop Interaction

With the command `DROP INTERACTIONS` $\langle gc \rangle$`;` all interaction terms which contain a set in $\langle gc \rangle$ are set equal to 0.

In CoCo this is computed as follows: The command `DROP INTERACTIONS` $\langle gc \rangle$`;` adds the sets $\langle gc \rangle$ to the dual representation for the **current** model. Supersets of other sets in the resulting set of sets are removed and normal representation for that dual representation is found. The generated model is tested against the **base** model.

## 7.6 Add Interaction

The command `ADD INTERACTIONS` $\langle gc \rangle$`;` adds the sets $\langle gc \rangle$ to the generating class for the **current** model. Subsets of other sets in the resulting set of sets are removed and the generated model is tested against the **base** model. If **base** and **current** are the same models then the **current** model is tested against the generated model, else the generated model is tested against **base** model.

## 7.7 Meet of Models

The meet of the two models **base** and **current** is formed by the command `MEET OF MODELS;`. The meet is the largest model contained in both **base** and **current** model. The resulting model will contain the maximal generators that are a subset of at least one generator in both **base** and **current**. For each generator in **base** and for each generator in **current**, the intersection of the two generators is formed, and the resulting model is the generating class with the maximal of these intersections. The generation of the model is symmetric in **base** and **current**.

The resulting model is tested against the **current** model.

In the following example all interactions in a final model from Edwards & Havránek (1985), see the Introduction, are reduced to first-order-interactions:

```
>set page formats 78 256;
 TIME:            0.001secs.

>set power lambda null;
 Power Divergence not printed
 TIME:            0.001secs.

>set adjusted df off;
 Adjusted df set OFF
 TIME:            0.001secs.

>read data;
  64 cells with  1841 cases read.
 Finding all marginals.
 TIME:            0.024secs.
```

```
>read n-interactions 1 ABCDEF.;
   1: [[EF][DF][DE][CF][CE][CD][BF][BE][BD][BC][AF][AE][AD][AC][AB]]
 Model is not graphical
 Cliques:[[ABCDEF]]
 2-Section Model is decomposable
 TIME:            0.011secs.

>read model ACE,ADE,BC,F.;
 TIME:            0.001secs.

>meet of models;
   3: [[F][BC][AD][DE][AC][AE][CE]]
 Model is not graphical
 Cliques:[[ACE][ADE][BC][F]]
 2-Section Model is decomposable
 Test of [[F][BC][AD][DE][AC][AE][CE]]
 against [[ACE][ADE][BC][F]]

          DF  -2log(Q)       P       X^2       P        Models

           1    0.0105 0.91851    0.0105 0.91851    [[ACE]] / [[AC][AE][CE]]
   +       1    2.8340 0.09229    2.8324 0.09238    [[ADE]] / [[AE][DE][AD]]
   =       2    2.8445 0.24117    2.8428 0.24137    [[ADE][ACE]]
                                                      / [[AD][DE][CE][AE][AC]]
 TIME:            0.044secs.
```

The command `NORMAL TO DUAL;` will generate a model, where the generating class for the generated model is the dual representation of the **current** model. Together with `MEET OF MODELS;` this can be used to do `DROP INTERACTIONS` $\langle gc \rangle$`;` on several models.

## 7.8 Join of Models

The join of the two models **base** and **current** is formed by the command `JOIN OF MODELS;`. The join is the smallest model containing both **base** and **current** model. The resulting model will contain the maximal generators in both **base** and **current**. The generation of the model is symmetric in **base** and **current**.

The **current** model is tested against the resulting model.

Continuing the example from the previous section:

```
>join of models;
   4: [[AB][AF][BD][BE][BF][CD][CF][DF][EF][BC][ADE][ACE]]
 Model is not graphical
 Cliques:[[ABCDEF]]
 2-Section Model is decomposable
 Test of [[ACE][ADE][BC][F]]
 against [[AB][AF][BD][BE][BF][CD][CF][DF][EF][BC][ADE][ACE]]
```

```
          DF  -2log(Q)       P       X^2       P       Models

           9   17.5851 0.03991   18.2781 0.03176   [[ACE][ADE][BC][EF][DF][CF]
                                                     [CD][BF][BE][BD][AF][AB]]
                                                     / [[F][BC][ADE][ACE]]
 TIME:            0.074secs.

>print all models;
 Model no.   4 [[AB][AF][BD][BE][BF][CD][CF][DF][EF][BC][ADE][ACE]]
 Model no.   3 [[F][BC][AD][DE][AC][AE][CE]]
 Model no.   2 [[ACE][ADE][BC][F]] /CURRENT/
 Model no.   1 [[EF][DF][DE][CF][CE][CD][BF][BE][BD][BC][AF][AE][AD][AC][AB]]
               /BASE/
 TIME:            0.003secs.
```

## 7.9 Miscellaneous Deprecated Commands

`[ Only ]` `DROP FACTOR` ⟨*factor*⟩`;`
`[ Only ]` `REDUCE GENERATOR` ⟨*set*⟩`;`
`[ Only ]` `REMOVE GENERATOR` ⟨*set*⟩`;`
`[ Only ]` `REMOVE TOTAL INTERACTION` ⟨*set*⟩`;`

**Drop Factor:** `DROP FACTOR` ⟨*factor*⟩`;` drops the factor ⟨*factor*⟩ in the **current** model. The generated model is tested against **base** model. The same effect can be achieved by `DROP INTERACTIONS` ⟨*factor*⟩`;`.

**Reduce Generator:** With `REDUCE GENERATOR` ⟨*set*⟩`;` the generating class for the **current** model is searched for the set ⟨*set*⟩. If the set is found, the set is substituted with all proper subsets of the set. Subsets of other sets in the resulting set of sets are removed and the generated model is tested against the **base** model.

**Remove Generator:** `REMOVE GENERATOR` ⟨*set*⟩`;` searches the generating class for the **current** model for the set ⟨*set*⟩. If the set is found, it is removed from the generating class and the resulting model is tested against the **base** model.

**Remove Total Interaction:** With `REMOVE TOTAL INTERACTION` ⟨*set*⟩`;` all interactions between factors in ⟨*set*⟩ are removed in the **current** model. The resulting model is tested against the **base** model.

## 7.10 Edit the Models without Test: Only ...

With `Only` before one of the `Drop`, `Reduce`, `Remove` or `Add`-commands, then the test is skipped. `Only` is a button. It is effective until after the next model-editing command or until after the next `BACKWARD;` or `FORWARD;`-command.

# Chapter 8

# Controlling the Computed Test Statistics in the Model Selection Procedures

The commands in this chapter will control the criterion for judging the suitability of models in model selection in the CoCo, i.e., the commands will set options for how to compute test statistics, select test statistics and set limits for when to accept or reject a model in the model selection by the commands `BACKWARD;` and `FORWARD;` and in the EH-procedure. Also a command for restricting the model class of the selection procedures to the sub-class of decomposable models is described in this chapter.

Besides by the options set by commands in this chapter the model selection may depend on choice of how to handle missing values, options for controlling the IPS-algorithm, options for controlling output as options for print formats, timer and diary, etc.

Options controlling the computed test statistics and options controlling exact tests will also effect the values computed by the commands `TEST;`, `EXACT TEST;`, etc. and the model-editing commands.

## 8.1 Decomposable Mode(ls)

```
SET DECOMPOSABLE MODE [ On | Off ];
```

This option will restrict both the stepwise forward selection, the backward elimination and the global EH search procedure to decomposable models. I.e., the search procedures will not try to visit non-decomposable models, but edges (or interactions) can still by removed by other commands so that a non-decomposable model is generated.

It is attractive to restrict model selection to the class of decomposable models for a variety of reasons. Firstly, computation efficiency: the ability to use explicit formulae for the maximum likelihood estimates can reduce computational time substantially. Secondly, exact conditional tests for nested decomposable models can be calculated.

A backward elimination by the command `BACKWARD;` with `DecomposableMode` set `On` will give the model selection proposed in Wermuth (1976).

## 8.2 Computed Test-Statistics and Choosing Tests and Significance Level

This section will describe options for how to compute test statistics, select test statistics and set limits for when to accept or to reject a model in the model selection by the commands `BACKWARD;` and `FORWARD;` and in the EH-procedure. Also a short description of the *Information Criterion* is given.

### 8.2.1 Computing Test-Statistics

`SET POWER LAMBDA { Null | ⟨lambda⟩ };`
`SET ADJUSTED DF [ On | Off ];`
`SET REUSE OF TESTS [ On | Off ];`

The computed test statistics in all the test and model search procedures depend on values set by these commands. The power $\lambda$ of the power divergence is set by the command `SET POWER LAMBDA { Null | ⟨lambda⟩ };`. If the command `SET POWER LAMBDA Null;` is used or the argument $\langle\lambda\rangle$ is set to 0 or 1, then the power divergence $2nI^{\lambda}$ is not computed and not printed for computed tests.

The adjusted number of degrees of freedom is only computed, if `AdjustedDF` is set `On` by `SET ADJUSTED DF [ On | Off ];`, `On` by default.

If reuse of tests is on, set by the command `SET REUSE OF TESTS [ On | Off ];`, then changing options by the commands `SET ADJUSTED DF [ On | Off ];` and `SET POWER LAMBDA { Null | ⟨lambda⟩ };` will not result in changes in the used test statistics for already computed tests. See 6.6 and use `SHOW TESTS;` or `DISPOSE OF TESTS;`.

### 8.2.2 Selecting Test Statistic and Significance Level

`SET ORDINAL ⟨set⟩;`
`SET TEST { Lr | Chi | Power };`
`SET { Aic | Bic | Ic Kappa ⟨kappa⟩ | Ic On | Ic Off };`

In the model selection models are accepted or rejected according to the limits set by the commands `SET ACCEPTANCE ⟨alpha⟩;` and `SET REJECTION ⟨alpha⟩;`.

The test statistic to compute the $p$-value or IC-value from is selected according to the option set by the command `SET TEST { Lr | Chi | Power };`, unless two ordinal variables are tested conditionally independent. Goodman and Kruskal's Gamma coefficient is used when two ordinal variables are tested conditionally independent, also when an information criterion is selected. Ordinal variables are declared by the command `SET ORDINAL ⟨set⟩;`, see also section 3.3.

If the Gamma coefficient or the information criterion is computed, then this value is used to determine whether to accept or reject models. Thus these two values cannot be computed and printed without using the value in the model selection.

If both the Gamma coefficient and a information criterion are computed for a model in a model selection, then the Gamma coefficient is used to decide whether to accept or to reject the model.

### Selection by Information Criteria

The Information Criteria $IC^{\kappa}_{\mathcal{A}}$ (Sakamoto & Akaike 1978) for model $\mathcal{A}$ is computed as

$$\begin{aligned} IC^{\kappa}_{\mathcal{A}} &= D_{\mathcal{A}} + \kappa \cdot \dim(\mathcal{A}), \\ AIC_{\mathcal{A}} &= D_{\mathcal{A}} + 2 \cdot \dim(\mathcal{A}) \quad \text{and} \\ BIC_{\mathcal{A}} &= D_{\mathcal{A}} + \log(n) \cdot \dim(\mathcal{A}), \end{aligned}$$

where $D_{\mathcal{A}}$ is the deviance of $\mathcal{A}$ relative to the saturated model, the likelihood ratio test statistic for $\mathcal{A}$ tested under the saturated model.

The change of Information Criteria is then in backward elimination and forward selection computed by

$$\begin{aligned} \Delta(IC)^{\kappa}_{\mathcal{AB}} = IC^{\kappa}_{\mathcal{B}} - IC^{\kappa}_{\mathcal{A}} &= \Delta(D)_{\mathcal{AB}} - \kappa \cdot D.F._{\mathcal{AB}}, \\ \Delta(AIC)_{\mathcal{AB}} &= \Delta(D)_{\mathcal{AB}} - 2 \cdot D.F._{\mathcal{AB}} \quad \text{and} \\ \Delta(BIC)_{\mathcal{AB}} &= \Delta(D)_{\mathcal{AB}} - \log(n) \cdot D.F._{\mathcal{AB}}, \end{aligned}$$

where $\Delta(D)_{\mathcal{AB}} = D_{\mathcal{B}} - D_{\mathcal{A}}$ is the difference between the deviances for the two models considered and $D.F._{\mathcal{AB}}$ is the adjusted or unadjusted number of degrees of freedom for the test of $\mathcal{B}$ under $\mathcal{A}$. An adjusted number of degrees of freedom is used, if it is set on by the `SET ADJUSTED DF;`-command. If $\mathcal{B}$ is a submodel of $\mathcal{A}$ then $\Delta(D)_{\mathcal{AB}}$ and $D.F._{\mathcal{AB}}$ is positive. In a model search the $IC$ value is minimized.

The command `SET IC Kappa` ⟨*kappa*⟩`;` changes the constant $\kappa$ used in computation of the information criterion and sets the computation of this criterion on. The constant is to 2 if the command `SET Aic;` is used, and is set to log(total number of cases) if the command `SET Bic;` is used. When `ExcludeMissing` is `On`, the number of used cases may differ for the different parts of a partitioned test, and the value `BIC` can not be computed.

Note that the commands `SET Aic;`, `SET Bic;`, `SET IC Kappa` ⟨*integer*⟩`;` or `SET IC On;` besides choosing an information criterion also sets both the acceptance and the rejection limit to 0.

### Asymptotic or Exact $p$-values

```
SET EXACT TEST [ On | All | Deviance | Off ];
```

Exact $p$-values are used according to whether exact test are chosen by the command `SET EXACT TEST;`. See the next section about exact tests.

### Acceptance and Rejection of Models

```
SET ACCEPTANCE ⟨alpha⟩;
SET REJECTION ⟨alpha⟩;
SET COMPONENTS ⟨p-value⟩;
```

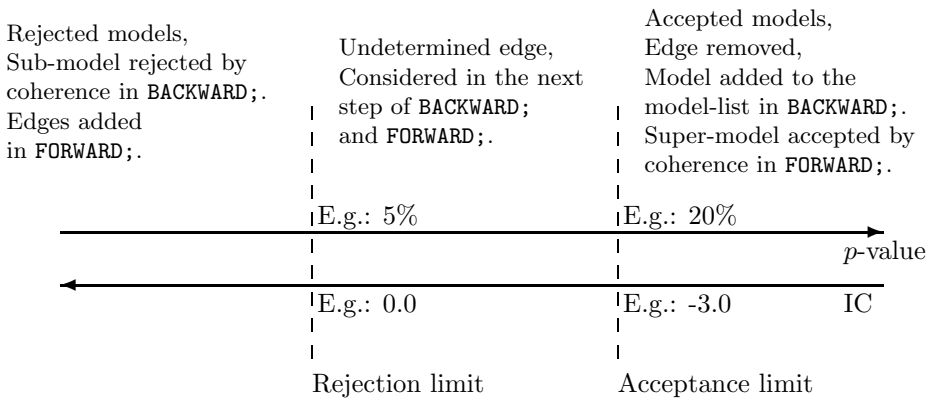



Figure 8.1: Significance levels for stepwise model selection and the EH-procedure.

`SET SEPARATORS` ⟨*p-value*⟩`;`

A hypothesis is accepted, if the $p$-value is greater than a given limit. Thus to be able to classify models by the same expressions regardless of whether $p$-values or an information criterion is used, minus the change in the information criterion is used to classify models.

If the $p$-value (or minus the IC-value) for a model in the stepwise edge or interaction selection or in the EH-procedure is greater than the value set by the command `SET ACCEPTANCE` ⟨*alpha*⟩`;`, then the model is accepted (else the model is rejected in the EH-procedure). In backward elimination edges (or interaction-terms) with a $p$-value greater than this limit are eligible for removal. The recursive backward elimination continues only as long as there are edges (or interaction-terms) with a $p$-value greater than this acceptance limit. Only if the $p$-value for the edge to remove is greater than this acceptance limit a new model is inserted in the model list.

Models with a $p$-value (or minus the IC-value) lower than or equal to the value set by the command `SET REJECTION` ⟨*alpha*⟩`;` are rejected, and sub-models can be rejected by *Coherence* in backward elimination.

Consider in each step of a forward selection all visited edges (or interaction-terms) not in the current model and with a $p$-value lower than or equal to the value set by `SET REJECTION` ⟨*alpha*⟩`;`, and consider the model with these edges added to the current model. This model will be the result of the forward selection step, and such a model is inserted in the model list for each step of the forward selection.

Models with a $p$-value (or minus the IC-value) greater than or equal to the value set by the command `SET ACCEPTANCE` ⟨*alpha*⟩`;` are accepted, and models containing accepted models can be accepted by *Coherence* in forward selection.

When the commands `SET Aic;`, `SET Bic;`, `SET IC Kappa` ⟨*integer*⟩`;` or `SET IC On;` are used, then both the acceptance and the rejection limit are set to 0. If the acceptance limit is set to a value lower than the rejection limit, then the rejection limit is also set to the entered value. Analogously, if the rejection limit is set to a

value greater than the acceptance limit, then the acceptance limit is set to the entered value.

#### Model control

Some further model checking is possible:

If the two models to be tested against each other have common decompositions, then the test can be partitioned. The command `SET PARTITIONING On;` of the last section in this chapter will control this partitioning of tests. The deviance for the total test is then the sum of the deviances for each part of the test. If a limit ⟨*components-limit*⟩ is set by `SET COMPONENTS` ⟨*components-limit*⟩`;`, default 1.0, then tests with a $p$-value lower than ⟨*components-limit*⟩ for any part of the test are rejected in the backward elimination and forward selection.

If two variables are accepted conditionally independent in a graph, then the two variables should by conditionally independent given any separator of the two variables in the graph. If a limit ⟨*separators-limit*⟩ is set by `SET SEPARATORS` ⟨*separators-limit*⟩`;`, default 1.0, then models with a $p$-value lower than ⟨*separators-limit*⟩ for one separator are rejected in the stepwise model selection (when this model control is requested by the keyword `Separators` of the command for backward elimination of forward selection).

## 8.3 Exact Tests in Tests and Model Selection

```
SET EXACT TEST [ On | All | Deviance | Off ];
SET ASYMPTOTIC ⟨p-value⟩;
SET NUMBER OF TABLES { Null | ⟨number⟩ };
SET LIST OF NUMBERS OF TABLES ⟨integer-list⟩;

SET EXACT TEST FOR TOTAL TEST [ On | Off ];
SET EXACT TEST FOR PARTS OF TEST [ On | Off ];
SET EXACT TEST FOR UNPARTED TEST [ On | Off ];

SET SEED { ⟨seed⟩ | Random };
SET EXACT EPSILON ⟨ϵ⟩;
SET FAST [ On | Off ];
```

If exact $p$-values are computed in the model selection, then these $p$-values are used. Exact test is set on by the command `SET EXACT TEST On;`. Exact tests are only computed, if the asymptotic $p$-value is smaller than the value set by `SET ASYMPTOTIC` ⟨*p-value*⟩`;`, 1 by default. The number of tables to generate in the Monte Carlo simulation of the exact $p$-value is set by the command `SET NUMBER OF TABLES;`. A variating number of tables is selected by the command `SET NUMBER OF TABLES Variating;`, and the number of tables to be generated is then set by `SET LIST OF NUMBER OF TABLES;`. See chapter 6 for details.

Note that if the power divergence $2nI^{\lambda}$ or Pearson's chi-square $\chi^2$ is selected by the command `SET TEST { Lr | Chi | Power };`, and exact $p$-values are only computed for the likelihood ratio test-statistics because of the use of the command `SET EXACT`

`TEST Deviance;`, then the asymptotic $p$-value will be used in the model selection procedures.

Please disregard the following options before you is familiar with exact tests in CoCo: The options `SET EXACT TEST FOR TOTAL TEST;`, `SET EXACT TEST FOR PARTS;` and `SET EXACT TEST FOR UNPARTED;` will control computing of exact tests for respectively the total of a test, that can be partitioned, the parts of a partitioned test and for tests not partitioned. The command `SET PARTITIONING;` of the following section will control partitioning of tests, that is, partitioning of the two models by common decompositions and thus collapsibility of tests.

The command `SET SEED` { ⟨*seed*⟩ | `Random` }`;` will set the seed for the random number generator used when generating the exact test. The command `SET SEED Random;` will set this seed to some random value depending on the process number of the CoCo session and the total used computing time. The value set by `SET EXACT EPSILON` ⟨$\epsilon$⟩`;` is to do with round of errors, and one of two algorithms to compute the exact $p$-value by is selected by `SET FAST;`. See chapter 6 for further details.

## 8.4 Partitioning

```
SET PARTITIONING [ On | Off ];
SET ALGORITHM { A | B | C | D | ... };
```

The command `SET PARTITIONING;` of the following section will control partitioning of tests, that is, partitioning of the two models by common decompositions. The tests by the command `TEST;`, the model editing commands and in model selection are then collapsed to the smallest tables on which the tests can be performed. When `ExcludeMissing` is `On`, the number of used cases may differ for the different parts of a partitioned test.

The command `SET ALGORITHM;` will set an option to choose between different algorithms for factorization, computation of the Gamma coefficient, exact $p$-value, fill-in, etc. See chapter 6 and the online help information of CoCo for further details.

# Chapter 9

# Stepwise Model-selection: Backward and Forward

This chapter is on stepwise model selection in CoCo.

The incremental search in CoCo is performed by *forward inclusion* (Dempster 1972) and *backward elimination* (Wermuth 1976) of edges (or interaction-terms). In the backward elimination of edges in graphical models edges are sequentially eliminated from the independence graph. In the forward selection the edges are sequentially added to the independence graph.

Wermuth (1976) considers stepwise edge elimination on decomposable models. Two resent books Christensen (1990) and Whittaker (1990) each contain a chapter on model selection. In Whittaker (1990) with the title "Graphical Models" the model selection is of course treated in relation to graphical models, but also in Christensen (1990) an excellent discussion of model search on graphical model is given. Edwards (1993) considers some computational aspects of both the stepwise edge selection and the EH-procedure, see the next chapter.

In CoCo the backward elimination and the forward selection can be started from any initial model. The backward elimination procedure is not restricted to work on edges, but can also perform stepwise elimination of interaction-terms. Analogously with the forward selection. Steps of the backward elimination and the forward selection can be mixed together in any order, and between each step any set of edges or interaction-terms can be added or eliminated.

The backward elimination from the saturated model has the advantage that the tests are not performed under models later to be found rejected. In the forward selection from the uniform model the initial tests will be performed under models likely to be rejected. But in the forward selection the initial tests often will be collapsible to small tables, and much larger tables can be handled. The marginal associations tested in the first step of a forward selection from the uniform model might also be of some interest.

The resulting models from each cycle of the backward elimination and the resulting model from the forward selection are added to the linked list of models in CoCo.

## 9.1 Selecting Test Statistic and Significance Level

The previous chapter will describe how to set the options for the computed test statistics, select the test statistic and how to set the level of significance.

## 9.2 Fixing of Edges and Interactions

`FIX Edges` { ⟨*edges*⟩ | ⟨*gc*⟩ };
`AND FIX Edges` { ⟨*edges*⟩ | ⟨*gc*⟩ };

The command `FIX Edges` ⟨*gc*⟩; will fix the edges and interaction-terms given by the argument of the command in the models considered in the stepwise model selection commands.

In a backward elimination of interactions, the interaction-terms a subset of a generator in the generating class set by `FIX Edges` ⟨*gc*⟩; are not eligible for removal. Also in a forward selection adding interaction terms containing a fixed generator to the model may not be considered.

In backward elimination of edges on graphical models, the edges (i.e., first-order interactions) given by `FIX Edges` ⟨*edges*⟩; are not tested for removal, and are never added in forward selection of edges. Thus in a forward selection of edges on graphical models an edge can be fixed without being containing a fixed generator.

Edges (and interactions) in the `FixEdgeList` can still be added or removed by the model-editing commands, i.e., by the `Drop-`, `Reduce-`, `Remove-` and `Add`-commands.

The `FixEdgeList` set by `FIX Edges` ⟨*edges*⟩; and `AND FIX Edges` ⟨*edges*⟩; is independent of the `FixIn` and `FixOut` used in the EH-procedure described in the following chapter[1].

Edges (interactions) can be added to the `FixEdgeList` by `AND FIX Edges` ⟨*edges*⟩;. Following use of the `FIX Edges` ⟨*edges*⟩; will clear and replace the `FixEdgeList`.

## 9.3 Backward Elimination

```
[ Only ] [ Reversed ] [ Sorted ] [ Separators ] [ Recursive ]
    [ Headlong ] [ Coherent ] [ Follow ] [ All ] BACKWARD
    [ Edges | Interactions ];
```

The simplest form

```
BACKWARD;
```

[1] One could consider removing the command `FIX Edges` ⟨*edges*⟩; and then also set the fix for stepwise backward elimination and forward selection with the commands `FIX In` ⟨*edges*⟩; and `FIX Out` ⟨*edges*⟩; of the EH-procedure. This would also allow for different fixing for respectively backward elimination and forward selection. But for comparability with former versions of CoCo, separate fixes for the stepwise model selection and the global EH-procedure respectively have been kept.

of the backward elimination command will visit all edges in the **current** model (if it is graphical), and for each visited edge produce a test. (See section 9.3.3 about choosing between backward elimination of edges and backward elimination of interaction-terms.) With the command

```
Recursive BACKWARD;
```

all edges in the **current** model are visited and the least significant edge is removed. This step of the backward elimination is then repeated on the resulting model: All edges are visited in the model, and the least significant edge is removed. This is repeated until all edges are significant in the resulting model. When the keyword `Recursive` is given, the resulting accepted model from each step is added to the model list in CoCo.

### 9.3.1 Controlling the Output

By default a test is printed for each edge (or interaction-term) visited, and a report of rejected, eligible edges, the model resulting of the step of the recursive backward elimination, etc. is given between steps of the backward elimination.

#### Sorted

```
Sorted Recursive BACKWARD;
```

Also a sorted list of the tests is printed for each step of the backward elimination. The list is sorted according to the selected $p$-value (statistics, if `IC` is selected). The $p$-values are ordered in increasing order (`IC` decreasing). The least significant edge (or interaction-term) is then the last edge in the sorted list.

#### Reversed

```
Reversed Sorted Recursive BACKWARD;
```

If this keyword is used, the sorted list of tests is reversed.

#### Only

```
Only Sorted BACKWARD;
```

Only the sorted list of tests is printed. This has the advantage of saving paper, if only the sorted list is wanted. But when running interactively, especially if exact $p$-values are computed and on large tables, there will be a lot of computation without any results shown, since the whole list of tests has to be computed for each step of the backward elimination before any output is printed. The keyword can of course be combined with the keyword `Reversed`.

```
Only BACKWARD;
```

If the keyword `Only` is given without the keyword `Sorted` being given, then all printing from the backward elimination will be eliminated. The only result is then the models being inserted into the model list.

#### Short

```
Short BACKWARD;
```

By this keyword only a short report of the elimination is given: For each edge (or interaction-term) removed the corresponding test is printed.

If `Dump` is set `On` and the keyword `Short` is given then for each completed step of the backward elimination, i.e., each removal of an edge or interaction, the `DumpFile` is rewound and a report of rejected, accepted, eligible edges and the model resulting of the step is printed on the `DumpFile`. In the following step of the backward elimination each tested edge and the selected test statistic for the test of the edge is printed on the `DumpFile`. This may be useful for monitoring a model selection on a large table and for recovering a terminated model selection.

### 9.3.2 Recursive Search

#### Recursive

If the keyword `Recursive` is given then the backward elimination is done recursively until no more edges (or interaction-terms) can be removed according to the selected significance level.

#### Coherent

```
Recursive Coherent BACKWARD;
```

Once an edge (or interaction-term) in the backward elimination process is rejected, it is no more tested for removal.

#### Headlong

```
Headlong Recursive BACKWARD;
```

or

```
Headlong Recursive Coherent BACKWARD;
```

This keyword gives a recursive backward elimination strategy, where the first edge (or interaction) found to be non-significant is eliminated. In each step of this backward elimination, edges with $p$-value less than a given limit (e.g. 1% or 5%) are rejected, and, when the principle of weak rejection, *coherence*, is applied, they are not considered in sub-sequential steps. The first edge found in each step with $p$-value greater than a second limit (e.g. 20%) is removed. All eligible edges not visited in the current step are eligible in the next step together with the edges visited in the current step with a found $p$-value between the two limits. If not the keyword `Coherent` is used, then of course all edges of the model are eligible in the next step.

This gives a faster elimination than a backward elimination where all eligible edges in each step have to be visited in order to find the least significant edge.

If the $p$-value (or minus the IC-value) for a model in this stepwise edge or interaction selection is greater than the value set by `SET ACCEPTANCE` ⟨*alpha*⟩`;`, then the model is accepted. I.e., in the backward elimination edges (or interaction-terms) are

eligible for removal if the $p$-values of the edges are greater than this limit. The recursive backward elimination continues only as long as there are edges (or interaction-terms) with a $p$-value greater than this acceptance limit. Only if the $p$-value for the edge to be removed is greater than this acceptance limit a new model is inserted in the model list.

Edges with a $p$-value (or minus the IC-value) less than or equal to the value set by the command `SET REJECTION` ⟨*alpha*⟩`;` are rejected. Sub-models can be rejected by *Coherence* in the backward elimination.

Not to favour the removal of edges with, e.g., a low lexicographical order, the edges in this headlong backward elimination are visited in a random order. Hence several calls to the procedure might give different results, and a sample of models with all edges significant to a selected level of significance can be generated. The command `SET SEED` { ⟨*seed*⟩ `| Random` }`;` will set the seed for the used random number generator.

#### Follow

```
Recursive Follow BACKWARD;
```

Tests are, if this keyword is given, performed against the previously accepted model. In the first step of the `Recursive Follow BACKWARD;` command the tests will be performed against the **current** model. In the next step, the tests are performed against the model accepted in the first step, etc. If the keyword `Follow` not is given, then all the tests will be performed against the **base** model.

#### Remove all non-significant edges or interactions in one step

```
All BACKWARD;
```

By default only the least significant edge (or interaction-term) is removed. All non significant edges are removed by one step of the backward elimination by the keyword `All`, i.e., all non-significant edges in the **current** model is removed from the model.

### 9.3.3 Graphical or Non-Graphical Models

#### Edges

```
BACKWARD Edges;
```

With the command `BACKWARD Edges;` the 2-section graph for the **current** model is created. In turn, each edge in the 2-section graph is then dropped, and the model with the generating class described by the cliques in the resulting graph is tested against the base model.

By `BACKWARD Edges;` all pairs of factors with an edge in the graph for the **current** model are tested conditionally independent given the remaining factors of the model.

#### Interactions

```
BACKWARD Interactions;
```

`BACKWARD Interactions;` will for each maximal interaction-term in the **current** model remove the interaction-term, and then test the resulting model against the **base** model. This is computed by in turn adding the interaction-terms for the **current** model to the dual representation for the **current** model.

With `BACKWARD Interactions;` and a **current** model with only *n-th order effects* (`READ N-INTERACTIONS` ⟨*n*⟩ `*;` or `READ N-INTERACTIONS` ⟨*n*⟩ ⟨*set*⟩`;`) all *n-th order partial associations* are tested.

For sparse contingency tables, Whittaker (1990) advocates the use of first-order log-linear models, i.e., hierarchical log-linear models with at most two-factor interactions. The model with all two-factor interactions can be read by the command `READ N-INTERACTIONS 1 *;`, and then the backward elimination from this model can be performed by the command `Recursive BACKWARD Interactions;`.

By default the command `BACKWARD;` is the same as the command `BACKWARD Edges;`, if the **current** model is graphical, else `BACKWARD Interactions;`.

### 9.3.4 Model Control

#### Separators

`Separators BACKWARD;`

With this keyword in a `Separators BACKWARD;` command then for each edge (or interaction-term) all tests for conditional independence are performed: For each edge to remove, that is, for each pair of vertices connected in the model, the minimal *separators* between the two vertices are found, and the two vertices are tested conditionally independent given each separator.

If a limit ⟨*separators-limit*⟩ is set by `SET SEPARATORS` ⟨*separators-limit*⟩`;`, then edges (or interaction-terms) with a $p$-value less than the limit ⟨*separators-limit*⟩ for one separator are rejected in a `Separators BACKWARD;` command.

Analogously, if a limit ⟨*components-limit*⟩ is set by the command `SET COMPONENTS` ⟨*components-limit*⟩`;` and `Partitioning` is `On`, then edges (or interaction-terms) with a $p$-value less than ⟨*components-limit*⟩ for one part of the test are rejected in backward elimination.

## 9.4 Forward Selection

```
[ Only ] [ Reversed ] [ Sorted ] [ Separators ] [ Recursive ]
     [ Headlong ] [ Coherent ] [ Most ] FORWARD
     [ Edges | Interactions ];
```

All edges not included in the **current** model are in turn added to the **current** model, and the **current** model is tested against the resulting model with the command `FORWARD;`.

### 9.4.1 Controlling the Output

By default a test is printed for each edge (or interaction-term) visited and a report of rejected, eligible edges (or interaction-terms), the model resulting of each step of the

forward selection, etc. is given between steps of the forward selection.

The keywords `Only`, `Reversed`, `Sorted` and `Short` is as at the command `BACKWARD;` except that the $p$-values by default are ordered in decreasing order (`IC` increasing) in the sorted list.

### 9.4.2 Recursive Search

Also the keywords `Recursive`, `Coherent` and `Headlong` are as at the command `BACKWARD;`.

A model, where rejected edges (or interaction-terms) are added to the **current** model, is added to the model list for each step of the `Recursive FORWARD;` command.

#### Recursive

`Recursive FORWARD;`

Forward selection is done recursively until no more edges have to be added according to the selected level of significance.

#### Coherent

`Recursive Coherent FORWARD;`

Once an edge (or interaction-term) is accepted in the forward elimination process, it is no more tested for adding.

#### Headlong

`Headlong Recursive FORWARD;`

or

`Headlong Recursive Coherent FORWARD;`

In each step of this forward selection, edges (or interaction-terms) with $p$-value greater than a given limit (e.g. 10% or 20%) are not added, i.e., it is accepted that the edge can be omitted, and is not considered in sub-sequential steps when the principle of weak acceptance, coherence, is applied. The first edge found in each step with a $p$-value less than a second limit (e.g. 1%) is added to the model. All not visited eligible edges in the current step are eligible in the next step of the recursive forward selection together with the edges visited in the current step with a $p$-value found between the two limits. If not the keyword `Coherent` is used, then of course all edges not in the resulting model are eligible in the next step.

If the $p$-value (or minus the IC-value) for a model in this stepwise edge or interaction selection is greater than the value set by `SET ACCEPTANCE` ⟨*alpha*⟩`;`, then the model is accepted, i.e., the edge or interaction-term is not added. By coherence these accepted edges are then not considered in sub-sequential steps of the forward selection.

Consider the edges with a $p$-value less than or equal to the value set by `SET REJECTION` ⟨*alpha*⟩`;` in a step of the forward selection. Then in each step of the forward selection these edges are added to the model of the current step, and the

resulting model is used in the next step. The resulting model of each step is inserted in the model list. The recursive forward elimination continues only as long as there are found edges (or interaction-terms) with a $p$-value greater than this rejection limit. Only if the $p$-value for the edge to be added is smaller than this rejection limit a new model is inserted in the model list.

Not to favour the removal of edges with, e.g., a low lexicographical order, the edges in this headlong forward selection are visited in a random order. Hence several calls to the procedure might give different results, and a sample of models with all edges significant to a selected level of significance can be generated.

#### Add only the most significant edge/interaction

```
Most FORWARD;
```

By default, if a headlong forward selection is not performed, all significant edges (or interaction-terms) are added in each step of the forward selection. With the command `Most FORWARD;` only the most significant edge (or interaction-term) is added in each step of the forward selection.

### 9.4.3 Graphical or Non-Graphical Models

This section is analogous to the section with the same name for the command `BACKWARD;`.

#### Edges

```
FORWARD Edges;
```

With the command `FORWARD Edges;` the 2-section graph for the **current** model is created. In turn, each edge not present in this 2-section graph is then added to the 2-section graph and the **current** model is then tested against the model with the generating class described by the cliques in the resulting graph.

By `FORWARD Edges;` all pairs of factors without an edge in the graph for the **current** model are tested conditionally independent given the rest factors of the model.

With `FORWARD Edges;` and a **current** model with only *main effects* (can be read by the command `READ MODEL .;`) all first order marginal associations are tested.

#### Interactions

```
FORWARD Interactions;
```

`FORWARD Interactions;` will for each minimal interaction-term (generator in the dual representation of the **current** model) not present in the **current** model add the interaction-term to the model, i.e., generate a model with the interaction added, and then test the **current** model against the resulting model.

By default the message `FORWARD;` is the same as the message `FORWARD Edges;`, if the **current** model is graphical, else it is the command `FORWARD Interactions;`.

### 9.4.4 Model Control

**Separators**

This is analogous to the same keyword at the command `BACKWARD;`.

`Separators FORWARD;`

With this keyword in a `Recursive Separators FORWARD;` command for each edge (or interaction-term) all tests for conditional independence are performed: For each edge to add, that is, for each pair of vertices not connected in the model, the minimal separators between the two vertices are found, and the two vertices are tested conditionally independent given each separator.

If a limit ⟨*separators-limit*⟩ is set by `SET SEPARATORS` ⟨*separators-limit*⟩`;`, then edges (or interaction-terms) with a $p$-value less than the limit ⟨*separators-limit*⟩ for one separator are rejected in a `Separators FORWARD;` command. Thus these edges are to be added.

Analogously, if a limit ⟨*components-limit*⟩ is set by the command `SET COMPONENTS` ⟨*components-limit*⟩`;` and `Partitioning` is `On`, then edges (or interaction-terms) with a $p$-value less than ⟨*components-limit*⟩ for one part of the test are rejected, i.e., the edges are added, in forward selection.

## 9.5 Asymmetry between Backward and Forward

The backward elimination and forward selection commands of course differ by removing and adding edges (or interaction-terms) respectively. In the backward elimination tests are performed against the previously accepted model, if the keyword `Follow` is given else, if the keyword `Follow` is not used, then all the tests will be performed against the **base** model. In the forward selection the model of the current step of the selection is tested against the model resulting from adding an edge (or interaction-term).

In the backward elimination only the least significant edge is removed in each step (by default), whereas in the forward selection all significant edges are added in each step (by default).

# Chapter 10

# The EH-procedure

By the principles of *weak acceptance* and *weak rejection*, coherence, the search space of all hierarchical models on the contingency table is divided into two sets of models: the class of minimal acceptable models and the class of maximal rejected models. Hence, after using the 'Fast Procedure for Models Search', the *EH-procedure*, by Edwards & Havránek (1985) and Edwards & Havránek (1987) any model can be labeled as accepted or rejected.

By the EH-procedure the models (or sub-models of a specific base model, the model **SearchBase** read by the command `READ BASE MODEL` ⟨*model*⟩`;`) are classified into two regions $\mathcal{A}$ and $\mathcal{R}$: weakly accepted models and weakly rejected models. Weakly accepted models are models that include an accepted model, and weakly rejected models are models that are included in a rejected model. In turn, a boundary below the weakly accepted models and a boundary above the weakly rejected models, the R-dual $D_R(\mathcal{A})$ and the A-dual $D_A(\mathcal{R})$ respectively, are fitted.

In CoCo the global search can be started with the accepted class containing the saturated model and the class of rejected models containing the model with only main-effects, or any models can be entered into the two classes. The search stops when either all models in $D_A(\mathcal{R}) \setminus \mathcal{A}$ are accepted or all models in $D_R(\mathcal{A}) \setminus \mathcal{R}$ are rejected.

The search-module is only usable for small tables. The dimension of the table should not exceed 10 too much. There should not be too many choices between acceptable models. If there is no clear association between some factors (given others), i.e., there are many marginal associations but no clear conditional associations, then all models in $D_R(\mathcal{A})$ will be accepted for several fits of $D_R(\mathcal{A})$, and the duals will explode in size, i.e., if one can choose between too many acceptable models. When the temptation to use this algorithm is the greatest, it should not be used.

If one of the search-commands described in this chapter is used, then the search-module is initiated, i.e., initial classes of models and duals are found for the EH-procedure. By default, the *saturated model* is accepted, and the model with only main effects is rejected.

Figure 10.1: Weakly accepted and weakly rejected models and duals.

The command `STATUS Search;` will show the *accepted models*, the *rejected models*, the *A dual*, the *R dual*, the **SearchBase** (the base model used in the EH-procedure), the `FixIn`, and the `FixOut`. The command `DISPOSE OF EH;` will dispose of the *accepted models*, the *rejected models*, the *A dual* and the *R dual*.

A simple model search among first decomposable models, then among all graphical models and finally among hierarchical models can be performed by

```
EH Decomposable;
EH Graphical;
EH Hierarchical;
```

The smallest dual in each step is fitted by these three commands. If instead, rough estimates of the dual sizes should be found in each step, and the dual with the smallest rough estimate is to be fitted in each step, then the commands

```
SET Rough SEARCH;
EH Decomposable;
EH Graphical;
EH Hierarchical;
```

can be used.

## 10.1 Computed Tests and Selection Criteria

Chapter 8 will describe how to set the options for the computed test statistics, select the test statistic to classify the models according to, and how to set the level of significance.

## 10.2 Fix Edges/Interactions:

`FIX { In | Out }` ⟨*edges/gc*⟩`;`
`ADD FIX { In | Out }` ⟨*edges/gc*⟩`;`
`REDO FIX { In | Out };`

### 10.2.1 FixIn

With the command `FIX In` ⟨*edges/gc*⟩`;` edges (generators) in the generating class are forced into all models in the EH-procedure. The command `FIX In "ABC";` would force the terms <>, <A>, <B>, <C>, <AB>, <AC>, <BC> and <ABC> into all models.

### 10.2.2 FixOut

With the command `FIX Out` ⟨*edges/gc*⟩`;` all terms containing a generator in the generating class are FixOut. The FixOut is the dual representation of the maximal model considered, see pp. 340 Edwards & Havránek (1985).

Note that only terms containing a FixOut-generator are FixOut. The command `FIX Out "ABC";` excludes the terms <ABC>, <ABCD>, <ABCE>, <ABCDE>, <ABCF>, <ABCDF>, <ABCEF>, <ABCDEF>, <ABCG>, etc. from models. But the terms <AB>, <AC> and <BC> are not excluded.

So to exclude edges in the graphical search only generators with cardinality 2 (edges) should be given to FixOut.

Using the commands `FIX In` ⟨*edges/gc*⟩`;` and `FIX Out` ⟨*edges/gc*⟩`;` (and `ADD FIX In` ⟨*edges/gc*⟩`;`, `ADD FIX Out` ⟨*edges/gc*⟩`;`, `REDO FIX In` ⟨*edges/gc*⟩`;` and `REDO FIX Out` ⟨*edges/gc*⟩`;`) of this section will only set up fixing for future found models. No interactions will be added to or removed from already found models. Thus if the EH-procedure already has been used the duals and model-classes should be disposed of by the command `DISPOSE OF EH;`.

#### Main effects

`SET MAIN EFFECTS` ⟨*set*⟩`;`

The factors to be included in the search are set with `SET MAIN EFFECTS` ⟨*set*⟩`;`, default-value: `*`, that is, all factors read are used in the EH-procedure.

The EH-procedure can also be restricted to a subset of the declared factors by the command `READ BASE MODEL` ⟨*model*⟩`;` or by giving sets of cardinality one to the `FIX Out;` command. But CoCo works more efficiently, if the command `SET READ Subset`

⟨*subset*⟩; is used when reading data instead of using `FIX Out;`, `SET MAIN EFFECTS;` or `READ BASE MODEL;` commands in the EH-procedure.

### 10.2.3 Read Base Model

`READ BASE MODEL` ⟨*model*⟩`;`

With the command `READ BASE MODEL` ⟨*model*⟩`;` a base model is read for the EH-procedure. Tests in the EH-procedure are then performed against the read model. It is not the same model as the **base** model used outside the EH-procedure, since `READ BASE MODEL` ⟨*model*⟩`;` effects `FixIn` and `FixOut`. Terms not in the base model are FixOut, i.e., FixOut is set to the dual representation of **SearchBase**. Since there is no distinction between FixOut due to the `FixOut`-command and FixOut due to `READ BASE MODEL` ⟨*model*⟩`;` and since a large **SearchBase** would give less FixOut due to **SearchBase**, previous FixOut is cancelled by the use of the command `READ BASE MODEL` ⟨*model*⟩`;`. Use `REDO FIX Out;` to redo previous FixOut.

### 10.2.4 Interaction between FixIn, FixOut and SearchBase

#### Effect of FixIn on FixOut

FixIn terms are removed from FixOut, i.e., FixIn is added to the normal representation of FixOut, the maximal terms in the models.

#### Effect of FixIn on SearchBase

FixIn has no effect on **SearchBase**, but only edges in **SearchBase** are Fixed In. FixIn is reduced. It could have been chosen that the **SearchBase** with the FixIn command is extended with edges in FixIn but not in **SearchBase**. The extended **SearchBase** is printed by the use of the command `FIX In` ⟨*edges/gc*⟩`;`, but not used.

#### Effect of FixOut on FixIn

FixOut terms in FixIn are removed from FixIn, i.e., FixIn is restricted to normal representation of FixOut.

#### Effect of FixOut on SearchBase

FixOut has no effect on **SearchBase**, but terms not in the **SearchBase** are added to FixOut.

#### Effect of SearchBase on FixIn

Only terms in the **SearchBase** can be FixIn. FixIn thus is reduced by **SearchBase**.

**Effect of SearchBase on FixOut**

Terms not included in the base model are FixOut, i.e., FixOut is set to the dual representation of **SearchBase**. Since there is no distinction between FixOut due to the `FIX Out;`-command and due to `READ BASE MODEL;` and since a large **SearchBase** would give less FixOut due to **SearchBase**, previous FixOut is cancelled. Use `REDO FIX Out;` to redo previous FixOut.

### 10.2.5 Add FixIn and FixOut

A new `FIX Out;`-command will clear the `FixOut`. The command `ADD FIX Out` ⟨*edges/gc*⟩`;` will add ⟨*edges/gc*⟩ to the previous `FixOut`. The `FixIn` is effected as described in a previous section by the argument ⟨*edges/gc*⟩ in the `ADD FIX Out;`-command.

Similarly a new `FIX In` ⟨*edges/gc*⟩`;`-command will clear the `FixIn`. The command `ADD FIX In;` will add ⟨*edges/gc*⟩ to the previous `FixIn`. The `FixOut` is affected as described in the previous section by the argument ⟨*edges/gc*⟩ in the `ADD FIX In;`-command.

### 10.2.6 Redo FixIn and FixOut

The command `REDO FIX Out;` will redo the combination of the last `FIX Out;`-command and following `ADD FIX Out;`-commands. `FixIn` is then edited by the total specified `FixOut`.

Similarly will the command `REDO FIX In;` redo the combination of the last `FIX In;`-command and following `ADD FIX In;`-commands. `FixOut` is then edited by the totally specified `FixIn`.

## 10.3 Selecting Model Class and Search Strategy

```
SET { Graphical | Hierarchical } SEARCH;
SET { Smallest | Alternating | Rough } SEARCH;
```

The option set by these command can also be given as an argument to the command `EH;`. But by setting the option with this command, e.g., the calls of the directed search is shorter to type.

Describing the command gives the possibility of describing the sub-classes, to which the EH-procedure is restricted:

### 10.3.1 Graphical Search

In the graphical search, for each step either all not classified models in the graphical A-dual are fitted or all not classified models in the graphical R-dual are fitted.

$D_A^g(\mathcal{R})$ : The graphical A-dual $D_A^g(\mathcal{R})$ of the rejected models is the minimal models in the class of models that adds at least one edge to each rejected model.

$D_R^g(\mathcal{A})$ : The graphical R-dual $D_R^g(\mathcal{A})$ of the accepted models is the maximal models in the class of models that omits at least one edge from each accepted model.

The command `SET Graphical SEARCH;` will set up the EH-procedure to a graphical search, and `EH;`Graphical will perform a model selection among graphical models, i.e., the graphical A-dual and R-dual are used.

### 10.3.2 Decomposable Mode

By a naive approach the selection can be restricted to decomposable models: non-decomposable models in the duals to fit are simply ignored. Non-decomposable models in the search are ignored with the command `SET DECOMPOSABLE MODE On;` before commands `EH;` and `FIT;`. The search is then performed among only decomposable models. (This can also be performed by `EH;`Decomposable) After a search among only decomposable models, a search among graphical models can be performed with the result of the decomposable search as a starting-point. `DecomposableMode` is turned off again by typing `SET DECOMPOSABLE MODE Off;`.

Since non-decomposable models in the graphical dual are just ignored in the decomposable search, there might still be graphical and decomposable models to fit, when both the set $D_A^g(\mathcal{R}) \setminus \mathcal{A}$ of not classified models in the graphical A-dual and the set $D_R^g(\mathcal{A}) \setminus \mathcal{R}$ of not classified models in the graphical R-dual contain no more decomposable models. So the decomposable search should be followed by a graphical search. One might consider adding edges to or removing edges from the non-decomposable models in the graphical duals in the models selection among decomposable models.

Since a lot of computing time in the search module is used to find duals, and not, as it might be expected, in computing the tests and fitting the models by the IPS-algorithm, the usefulness of the decomposable search is doubtful (unless exact tests are to be used).

If `ExactTest` is on, then `DecomposableMode` also should be on. If `ExactTest` is on and `DecomposableMode` is off, then asymptotic $p$-values will be used for the non-decomposable models.

Note that the commands `EH Decomposable;`, `EH Graphical;` and `EH Hierarchical;` set `DecomposableMode` to off.

If no models are entered or found by EH-procedure, the search is started with the saturated model accepted and the model with only main effects rejected.

### 10.3.3 Hierarchical Search

In the hierarchical search we for each step either fit all not classified models in the hierarchical A-dual or all not classified models in the hierarchical R-dual.

$D_A(\mathcal{R})$ : The hierarchical A-dual $D_A(\mathcal{R})$ of the rejected models is the minimal models in the class of models that adds at least one interaction term to each rejected model.

$D_R(\mathcal{A})$ : The hierarchical R-dual $D_R(\mathcal{A})$ of the accepted models is the maximal models in the class of models that sets at least one interaction term in all accepted models to zero.

The command `SET Hierarchical SEARCH;` will set the search-module up to a hierarchical search, and `EH;`Hierarchical will perform a model selection among hierarchical models, i.e., the hierarchical A-dual and R-dual are used.

If the hierarchical search follows a graphical search, the regions found in the graphical search are used as a starting point in the hierarchical search, else the search is started with the saturated model accepted and the model with only main effects rejected, or any models can be entered into the two classes.

## 10.4 Dispose of the Model Classes and Duals

```
DISPOSE OF EH [ All | Duals | A-dual | R-Dual | Classes | Accepted |
    Rejected ];
```

The command `DISPOSE OF EH;` will dispose of the *accepted models*, the *rejected models*, the *A dual* and the *R dual*.

## 10.5 Read and Fit Models or Force Models into Classes

### 10.5.1 Fit Some Models

`FIT Models` ⟨*models*⟩`;`

With `FIT Models` ⟨*models*⟩`;` you can enter a list of models that is classified into accepted and rejected models, and start the model-search from there. The argument ⟨*models*⟩ is a set of models, a set of generating classes as a generating class is a set of sets, e.g.,

```
FIT Models [ [[A][B][C]] [[AB][AC][BC]] ];
```

or the shorter form

```
FIT Models A,B,C. AB,AC,BC; ;
```

can be used.

### 10.5.2 Reading Accepted and Rejected Models

`ACCEPT Models` ⟨*models*⟩`;`
`REJECT Models` ⟨*models*⟩`;`

With the commands `ACCEPT Models` ⟨*models*⟩`;` and `REJECT Models` ⟨*models*⟩`;` the entered models are forced into the `accepted region` and the `rejected region` respectively. The argument ⟨*models*⟩ is a set of models as at the command `FIT Models` ⟨*models*⟩`;`.

## 10.6 Copy Models Between the Models-List and the Search Classes

The following commands are used to copy models between the model list and the classes of models in the EH-procedure.

### 10.6.1 Models from List to Search Classes

```
FIT { Base | Current | Last | Number <a> | All models | Interval of
    models <a> <b> };
ACCEPT { Base | Current | Last | Number <a> | All models | Interval of
    models <a> <b> };
REJECT { Base | Current | Last | Number <a> | All models | Interval of
    models <a> <b> };
```

Models from the model list are fitted and put into a model class in the EH-procedure according to whether the models are accepted or rejected, or models from the model list are forced into a model class of the EH-procedure regardless of whether the model is accepted or rejected.

### 10.6.2 Models from Search Classes to Model List

```
EXTRACT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual
    | Accepted | Rejected };
```

This command is used to copy a model class or a dual from the EH-procedure to the model list.

## 10.7 Find Duals

```
FIND DUAL [ Decomposable | Graphical | Hierarchical ] { A-dual |
    R-dual | Both duals };
```

the commands `FIND A-dual;`, `FIND R-dual;` and `FIND Both duals;` can be used, if you want to see the dual by `STATUS Search;` before fitting models.

If the keyword `Decomposable`, `Graphical` or `Hierarchical` is given, then the class of models given by the keyword is used in the `FIND DUAL;`-command, and the requested class is also selected in subsequential EH-commands needing a model class, else the class of models to use is selected by the options set by the commands `SET { Graphical | Hierarchical } SEARCH;` and `SET DECOMPOSABLE MODE { On | Off };` or other commands setting model class.

## 10.8 Directed Search

```
FIT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual |
    Both duals | Smallest dual | Largest dual };
```

This command is used to do a single step of the EH-procedure.

The keywords `Decomposable`, `Graphical` and `Hierarchical` is as for the `FIND DUAL;`-command.

### 10.8.1 Fit Smallest Dual

With the command `FIT Smallest-dual;` both duals are found (if not already found), and all not classified models in the dual with the fewest not classified models are fitted and classified into accepted and rejected models.

### 10.8.2 Fit Largest Dual

With the command `FIT Largest-dual;` both duals are found (if not already found), and all not classified models in the largest dual are fitted and classified into accepted and rejected models.

### 10.8.3 Fit R-Dual

The command `FIT R-dual;` finds (if not found) and fits the R dual. Repeated use of `FIT R-dual;` gives a backward elimination. When there are no more models to fit, then the command `FIT R-dual;` will use little computing time, so extra `FIT R-dual;`-commands will only produce output without using much computing time.

### 10.8.4 Fit A-Dual

The command `FIT A-dual;` finds (if not found) and fits the A dual. As at the `FIT R-dual;`-command extra `FIT A-dual;`-commands will only produce output without using much computing time when there are no more models to fit in the A dual.

### 10.8.5 Fit Both Duals

`FIT Both-duals;` fits all not classified models in both duals as they are. Note that this need not give the same as `FIND A-dual;`, followed by `FIND R-dual;`.

## 10.9 Automatic Search

```
EH [ Decomposable | Graphical | Hierarchical ] [ Smallest |
     Alternating | Rough ];
```

This command is used to do a recursive search by the EH-procedure.

If the keyword `Decomposable`, `Graphical` or `Hierarchical` is given, then the class of models given by the keyword is used in the EH-command, and the requested class is also selected in subsequential EH-commands needing a model class, else the class of models to use is selected by the options set by the commands `SET { Graphical | Hierarchical } SEARCH;` and `SET DECOMPOSABLE MODE { On | Off };`, or other commands setting model class.

If the keyword `Smallest`, `Alternating` or `Rough` is given, then the strategy given by the keyword is used, and the strategy will also be used in subsequential `EH;`-commands, else the strategy set by the command `SET { Smallest | Alternating | Rough } SEARCH;` or previous use of the `EH;`-command is used.

The three commands

```
EH Decomposable;
EH Graphical;
EH Hierarchical;
```

will do a graphical search ignoring non-decomposable models, a graphical search and a hierarchical search respectively.

Below are described three strategies for selecting the dual to fit in each step of the EH-search. Which of the three strategies to use is set by the command `SET { Smallest | Alternating | Rough } SEARCH;` or by a keyword argument of the `EH;`-command. For the time being the strategy `Smallest` is default.

Which of the three strategies is the fastest depends on the situation. Try them all three (they need not give the same result since e.g., weakly rejected models may be accepted). See the `ReportFile` for where CoCo uses computing time. Find some models (by e.g., `BACKWARD;`) and copy them to the EH-procedure by the commands `FIT;`, `ACCEPT;` and `REJECT;`. Before the automatic search is started you can do some directed search by the command `FIT;`.

### 10.9.1 Smallest Automatic

With `EH Smallest;` for each step (classification of all not already classified models in a dual into the accepted and the rejected regions) in the search, both duals are found, and the dual with the fewest not classified models is selected for the next step. This will often minimize the number of fitted models, but sometimes a lot of computing time is used for finding large duals that are not used.

### 10.9.2 Rough Automatic

With this strategy rough estimates of the dual sizes are found after each step, and the dual with the smallest rough estimate of dual size is selected for the next step. The rough estimate of dual size is the product of the number of edges (or interaction-terms) in the model (or the dual representation of the model) over all models in the class $\mathcal{A}$ or $\mathcal{R}$. Since it sometimes avoids finding large duals that are not used, this can be effective. By this strategy the complete search in Edwards & Havránek (1985) on a 6 dimensional table is done by CoCo in less than 700 milliseconds on a SPARCstation.

### 10.9.3 Alternating Automatic

The R-dual and the A-dual of the two model classes are in turn classified with the command `EH Alternating;`. In a few examples, where alternating fitting the R-dual and the A-dual is optimal, this is the fastest. But often the strategy will start to select the largest dual, and then this strategy will be the slowest.

The automatic search stops when either all models in $D_A(\mathcal{R}) \setminus \mathcal{A}$ are accepted or all models in $D_R(\mathcal{A}) \setminus \mathcal{R}$ are rejected.

## 10.10 Force a Dual into a Model Class

```
ACCEPT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual |
    Both duals };
REJECT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual |
    Both duals };
```

The command `ACCEPT R-dual;` adds the R-dual of the accepted models to the accepted models.

The command `REJECT A-dual;` adds the A-dual of the rejected models to the rejected models.

If we add the A-dual of the rejected models to the accepted models the search will be finished. So this command does not exist. The A-dual of the accepted models can be added to the accepted models by `ACCEPT A-dual;`. Similarly the R-dual of the rejected models can be added to the rejected models by `REJECT R-dual;`.

# Chapter 11

# Miscellaneous Options for Controlling Input, Output and Algorithms

The `SET ...;`-commands of this chapter and of chapter 8 perform simple changes of a few variables. (Since `EXCLUDE MISSING In` ⟨*set*⟩`;`, `EXCLUDE MISSING [ On | Off ];` and `EM ON;` change the whole data-set, they are not `SET ...;`-commands.)

Commands with the two optional key-words `On` and `Off`, like `SET DIARY [ On | Off ];`, are "switches". `SET DIARY On;` turns the diary on and `SET DIARY Off;` turns the diary off. `SET DIARY;` with no argument turns the diary on if it is off, off if it is on.

The command `STATUS;` gives information about factors declared, names on files, data selection declared, data structure selected, print formats, IPS-convergence criterion, diary, time, echo, workspace etc.

```
STATUS [ Specification | Observations | Files | Formats | Fix | Tests
     | Exact | Ips | Em | EH | Limits | Other ];
```

E.g., `STATUS EH;` prints accepted and rejected models, duals and FixIn, FixOut and BaseModel if any.

Use `STATUS Files;` to see the current or default file-names.

```
RESTART;
```

The command `RESTART;` will restart CoCo, e.i., clear all settings in CoCo.

## 11.1 Print formats

`SET TABLE FORMATS` ⟨*width*⟩ ⟨*dec-prob.*⟩ ⟨*dec-expt.*⟩ ⟨*dec-diff.*⟩`;`
`SET TEST FORMATS` ⟨*x-width*⟩ ⟨*x-dec.*⟩ ⟨*prob-width*⟩ ⟨*prob-dec.*⟩`;`
`SET SHORT [ On | Off ];`

`SET PRINT FORMATS` ⟨*format-width*⟩ ⟨*decimals*⟩`;`
`SET PAGE FORMATS` ⟨*line-length*⟩ ⟨*page-length*⟩`;`

**Print formats for tables of cell-values:** The format of the numbers printed in the table produced by `PRINT TABLE ...;` is controlled by the command `SET TABLE FORMATS` ⟨*width*⟩ ⟨*decimals-prob.*⟩ ⟨*decimals-expt.*⟩ ⟨*decimals-residual*⟩`;`. The values set by the arguments ⟨*width*⟩ and ⟨*decimal-residual*⟩ of this command are also used in the procedure `DESCRIBE TABLE ...;`.

**Print formats for test-statistics in tables:** The print-width and number of decimals of the statistics and probabilities in tables of tests are set with the command `SET TEST FORMATS` ⟨*width-statistic*⟩ ⟨*decimals-statistic*⟩ ⟨*width probability*⟩ ⟨*decimals-probability*⟩`;`.

With the command `SET SHORT Off;` you get detailed output from the commands `GENERATE DECOMPOSABLE MODEL;`, `GENERATE GRAPHICAL MODEL;`, `DROP EDGES` ⟨*edge-list*⟩`;`, `ADD EDGES` ⟨*edge-list*⟩`;`, `DROP INTERACTIONS` ⟨*gc*⟩`;`, `ADD INTERACTIONS` ⟨*gc*⟩`;`, `MEET OF MODELS;`, `JOIN OF MODELS;`, `DROP FACTOR` ⟨*factor*⟩`;`, `REMOVE TOTAL INTERACTION` ⟨*set*⟩`;`, `REMOVE GENERATOR` ⟨*set*⟩`;`, `REDUCE GENERATOR` ⟨*set*⟩`;`, `FORWARD [ EDGES | INTERACTIONS ];`, `BACKWARD [ EDGES | INTERACTIONS ];`, `FACTORIZE ONE EDGE;`, `FACTORIZE ONE INTERACTION;` and `PARTITIONING TEST;`, e.i., not in table format. Use `SET SHORT On;` to get the results of these commands in a table.

**Print formats for all other commands:** The command `SET PRINT FORMATS` ⟨*format-width*⟩ ⟨*decimals*⟩`;` sets the width and number of decimals used when printing results from the commands `TEST;`, `FIND LOG(L);` etc.

The arguments ⟨*line-length*⟩ and ⟨*page-length*⟩ in the command `SET PAGE FORMATS` ⟨*line-length*⟩ ⟨*page-length*⟩`;` control sizes of plots and histograms. The sorted list generated by `DESCRIBE TABLE ...;` is only printed if it can fit onto two pages determined by ⟨*line-length*⟩ and ⟨*page-length*⟩.

Use `STATUS Formats;` to see the current setting of formats.

### 11.1.1 Pausing

`SET PAUSE [ On | Off ];`
`SET PAGING LENGTH` ⟨*number of lines*⟩`;`

With `Pausing` set on by `SET PAUSE [ On | Off ];` (Default: Off) the output is paused for each written 22 lines (after last NewPage), if EndOfLine is found after the total command is read by CoCo. E.g., if `Pausing` is on, then CoCo will pause for each printed 22 lines after `STATUS;`. If `Pausing` is on, then CoCo will not pause during the status command, if `STATUS;;` or `STATUS; READ MODEL A,B.;` is entered. The next 22 lines are printed by pressing "Return". If a `;` is entered when pausing, then the command will finish without pausing. The number of lines to print between pausing are set by `SET PAGING LENGTH` ⟨*number of lines*⟩`;` (Default: 22). See also section 2.6.

Use `STATUS Formats;` to see the current setting of pausing.

## 11.2 Input files

### 11.2.1 Input from keyboard or file

`SET KEYBOARD [ On | Off ];`

With this switch one selects between reading from file or from standard input. (Default: From the data file). After the command `SET KEYBOARD On;` the arguments to the commands `READ SPECIFICATION [` ⟨*specification*⟩ `];`, `READ FACTORS [` ⟨*factor-list*⟩ `];`, `READ NAMES [` ⟨*name-list*⟩ `];`, `READ OBSERVATIONS [` ⟨*observations*⟩ `];`, `READ LIST [` ⟨*list of cases*⟩ `];` and `READ TABLE [` ⟨*table*⟩ `];` are read from where the command `SET KEYBOARD On;` is read (normally keyboard, else from the source file where the command is read). After `SET KEYBOARD Off;` the arguments to the six "read" commands are read from the default data file or the data file named by the commands `SET INPUTFILE SPECIFICATION` ⟨*file-name*⟩`;`, `SET INPUTFILE OBSERVATIONS` ⟨*file-name*⟩`;` and `SET INPUTFILE DATA` ⟨*file-name*⟩`;`.

### 11.2.2 Data-files

`SET INPUTFILE SPECIFICATION` ⟨*file-name*⟩`;`
`SET INPUTFILE OBSERVATIONS` ⟨*file-name*⟩`;`
`SET INPUTFILE DATA` ⟨*file-name*⟩`;`

These commands set the names on `DataFiles` from which the specification and the observations are read. The Specification and the observations can be read from separate files. Note that the files are rewinded when the commands are used. See section 3.1 for further details.

Use `STATUS Files;` to see the setting of inputfiles.

### 11.2.3 Standard Input: Source

`SET INPUTFILE COMMANDS` ⟨*file-name*⟩`;`

The abbreviation `SOURCE` ⟨*file-name*⟩`;` is available. After this command input is read from the `SourceFile` ⟨*file-name*⟩. When EndOfFile is found on the file ⟨*file-name*⟩ input is expected from the file (or standard input) where the command `SET INPUTFILE COMMANDS` ⟨*file-name*⟩`;` (`SOURCE` ⟨*file-name*⟩`;`) was given.

When CoCo is started the files `.cocorc` (`COCO.SRC` under DOS) of the home-directory of CoCo and `.cocorc` (`COCO.SRC` under DOS) of the current directory are read by CoCo as a source file.

## 11.3 Output files

Besides on the standard output file CoCo can write on a diary, a dump file, a log file and a report file.

Use `STATUS Files;` to see the current (or default) file-names.

### 11.3.1 Standard Output: Sink

`SET OUTPUTFILE RESULTS` ⟨*file-name*⟩`;`
`SET OUTPUTFILE COMMANDS` ⟨*file-name*⟩`;`

On DOS the output from CoCo seen on the screen is divided into two parts: 1) results from procedures and 2) output from the parser. The output from procedures can then be redirected to a printer without loosing the prompt etc. on the screen with: `SET OUTPUTFILE RESULTS :LPR;`. If you want to see what is going on on the screen: Dump the reply from the parser somewhere: `SET OUTPUTFILE COMMANDS garbage;` and get the diary on the screen: `SET DIARY On; SET OUTPUTFILE DIARY :CON .;`

On Unix the two commands `SET OUTPUTFILE RESULTS` ⟨*file-name*⟩`;` and `SET OUTPUTFILE COMMANDS` ⟨*file-name*⟩`;` are the same, and will redirect the standard output to the file ⟨*file-name*⟩. The abbreviation `SINK` ⟨*file-name*⟩`;` for `SET OUTPUTFILE RESULTS` ⟨*file-name*⟩`;` is available.

### 11.3.2 Diary

`SET OUTPUTFILE DIARY` ⟨*file-name*⟩`;`
`SET KEEP DIARY [ On | Off ];`
`SET DIARY [ On | Off ];`

A diary of everything shown on the screen can be kept with the command `SET DIARY [ On | Off ];` (Default: Off). The default name of the `DiaryFile` is `/tmp/CoCo.diary.no.pid-number` under Unix and `COCO???.DIA` in the current directory under DOS. Another `DiaryFile` can be named by `SET OUTPUTFILE DIARY` ⟨*file-name*⟩`;`. Note that this command rewrites the file ⟨*file-name*⟩ and that under DOS, the file `COCO???.DIA` is rewritten every time CoCo is invoked. If the `DiaryFile` is not renamed, the `DiaryFile` is removed, when CoCo terminates normally, unless the command `SET KEEP DIARY [ On | Off ];` is used.

### 11.3.3 Log-file

`SET OUTPUTFILE LOG` ⟨*file-name*⟩`;`
`SET KEEP LOG [ On | Off ];`
`SET LOG [ On | Off ];`
`SET DATA LOG [ On | Off ];`

A log of input to CoCo is written to the `LogFile` unless the commands `SET LOG [ On | Off ];` and `SET DATA LOG [ On | Off ];` are used (Default: Both On). The `LogFile` can be read from of CoCo as `SourceFile`. Input read from `DataFiles` is copied to the `LogFile` as comments. The default name of the `LogFile` is `COCO???.LOG` in the current directory under DOS and `/tmp/CoCo.log.no.pid-number` under Unix. Another `LogFile` can be named by `SET OUTPUTFILE LOG` ⟨*file-name*⟩`;`. Note that this command rewrites the file ⟨*file-name*⟩ and that under DOS, the file `COCO???.LOG` is rewritten every time CoCo is invoked. If the `LogFile` is not renamed, the `LogFile` is removed, when CoCo terminates normally, unless the command `SET KEEP LOG [ On | Off ];` is used.

### 11.3.4 Dump-file

`SET OUTPUTFILE DUMP` ⟨*file-name*⟩`;`
`SET KEEP DUMP [ On | Off ];`
`SET DUMP [ On | Off ];`

Output from the two commands `RETURN VECTOR` ⟨*value-name*⟩ ⟨*set*⟩`;` and `RETURN MATRIX` ⟨*list of value-names;*⟩ ⟨*set*⟩`;` is copied to the `DumpFile`, if `Dump` is on (Default: Off).

The default name of the `DumpFile` is `COCO???.DMP` in the current directory under DOS and `/tmp/CoCo.dump.no.pid-number` under Unix. Another `DumpFile` can be named by the command `SET OUTPUTFILE DUMP` ⟨*file-name*⟩`;`. Note that this command rewrites the file ⟨*file-name*⟩ and that under DOS, the file `COCO???.DMP` is rewritten every time CoCo is invoked. If the `DumpFile` is not renamed, the `DumpFile` is removed, when CoCo terminates normally, unless the command `SET KEEP DUMP [ On | Off ];` is used.

### 11.3.5 Report-file

`SET OUTPUTFILE REPORT` ⟨*file-name*⟩`;`
`SET KEEP REPORT [ On | Off ];`

Computing-time used in the IPS-algorithm, number of cycles in the IPS-algorithm, computing-time used in the EM-algorithm, number of cycles in the EM-algorithm, time used to find duals in the EH-procedure etc. are reported in the `ReportFile`. This reporting cannot be turned off. The default name of the `ReportFile` is `COCO???.RPT` in the current directory under DOS and `/tmp/CoCo.report.no.pid-number` under Unix. Another `ReportFile` can be named by `SET OUTPUTFILE REPORT` ⟨*file-name*⟩`;`. Note that this command rewrites the file ⟨*file-name*⟩ and that under DOS, the file `COCO???.RPT` is rewritten every time CoCo is invoked. If the `ReportFile` is not renamed, the `ReportFile` is removed, when CoCo terminates normally, unless the command `SET KEEP REPORT [ On | Off ];` is used.

## 11.4 Timer

`SET TIMER [ On | Off ];`

Printing of computer-time used on each command can be turned on and off with `SET TIMER;`, `SET TIMER On;` and `SET TIMER Off;` (Default: Off).

## 11.5 Controlling Algorithms

`SET ALGORITHM { A | B | C | D | ... };`

The command `SET ALGORITHM;` will set an option to choose between different algorithms for factorization, computation of the Gamma coefficient, exact $p$-value, fill-in, etc. See chapter 6 and the online help information of CoCo for further details.

### 11.5.1 Controlling the IPS-algorithm

`SET IPS { Cell | Sum };`
`SET IPS EPSILON` ⟨*epsilon*⟩`;`
`SET IPS ITERATIONS` ⟨*max*⟩`;`

The convergence criterion for the IPS-algorithm can either be convergence of cell-probabilities or convergence of log(L). If cell-probabilities are chosen, the IPS-algorithm is repeated until the difference between estimated cell-probabilities from one cycle to the next for all cells is less than epsilon, or until the maximum number of cycles is obtained. The command `SET IPS Cell;` sets convergence criterion to cell-probabilities. `SET IPS Sum;` sets convergence criterion to log(L): The IPS-algorithm is repeated until the difference between estimated log(L)-values from one cycle to the next is less than ⟨*epsilon*⟩ or until ⟨*maximum cycles*⟩. ⟨*Epsilon*⟩ and ⟨*maximum cycles*⟩ are set with `SET IPS EPSILON` ⟨*epsilon*⟩`;` and `SET IPS ITERATIONS` ⟨*maximum cycles*⟩`;`.

Number of cycles, difference between convergence criterion for last two cycles and computing time of the IPS algorithm for each non-decomposable irreducible component is reported in the `ReportFile`.

Use `STATUS Ips;` to see the current values.

### 11.5.2 Controlling the EM-algorithm

`SET EM INITIAL { Uniform | First | Last | Mean | Random | Input };`
`SET EM EPSILON` ⟨*epsilon*⟩`;`
`SET EM ITERATIONS` ⟨*max*⟩`;`

The EM-algorithm is selected by `EM ON;`. Initial values are chosen by the command `SET EM INITIAL { Uniform | First | Last | Mean | Random | Input };`. See section 3.9.4 for further details.

Cycles in the EM-algorithm is repeated until the difference between values of the deviance for each cycle is less than the ⟨$\epsilon$⟩ set by `SET EM EPSILON` ⟨$\epsilon$⟩`;`. `SET EM ITERATIONS` ⟨*max*⟩`;` controls the maximum cycles in the EM-algorithm.

If `Report` is on, the likelihood ratio test statistics are reported for each step in the EM-algorithm. If `Trace` is on, the probability for each cell for each case is reported for each step in the EM-algorithm.

Use `STATUS Em;` to see the current values.

### 11.5.3 Data structure

Do not worry about the options of the section, since CoCo has been become quite good at selecting the data structure. But for those interested a short description of the data structures is given:

```
SET DATASTRUCTURE { All | Necessary | File | Large };
SET LARGE [ On | Off ];
SET HUGE [ On | Off ];
SET SORTED [ On | Off ];
```

After reading specification (and the use of `SET READ { All | Subset` ⟨*set*⟩ `};`) CoCo chooses automatically between three internal data structures for marginal tables:

**1)** `All marginal tables:` If the dimension of the table is low (less than or equal to 9 for binary factors in PC) there is space for finding all marginal tables. In, e.g., a model search you will then save some time.

**2)** `Necessary tables:` If there is no space for all marginal tables but space for reading the full table into an array and it is expected that the full table can be read faster than the list of cases, the data structure `Necessary tables` is selected. On UNIX the size of the N-array is controlled by the option `N[size]` when invoking CoCo. When performing a test necessary marginal tables are then found. Necessary marginal tables are sufficient marginal tables + intersections of sufficient marginal tables.

**3)** `File:` If the full table cannot fit into the memory (dimension is higher than 15 on PC) the data structure `File` is selected. Observations are then placed on an internal file.

On the PC you are then able to compute log(L) for a model with 64 factors, if the largest clique in the graph for the model has 14 vertices at the most and there are no more than 13 vertices in the largest non-decomposable atom (or more precisely, if the state-space of each irreducible component after adding a fill-in can fit into the P-array).

The command `STATUS Limits;` will show the selected data structure.

After reading specification (and `SET READ Subset` ⟨*a*⟩`;`) you can change the choice of data structure with the commands `SET DATASTRUCTURE All;`, `SET DATASTRUCTURE Necessary;`, `SET DATASTRUCTURE File;`, `SET LARGE On;` and `SET LARGE Off;`. If, e.g., you have a 6-dimensional table with binary factors then read data with the three commands `READ SPECIFICATION;`, `SET DATASTRUCTURE Necessary;` and `READ OBSERVATIONS;` instead of `READ DATA;` to avoid finding all marginal tables.

If there is space for reading the full table into an array but it is expected that the list of cases can be read faster than the full table (it is expected that the table is sparsed) the data structure `File` is selected and a message like "If more than ... cases then use SET DATASTRUCTURE NECESSARY between READ SPECIFICATION and READ OBSERVATIONS" is stated. The time used to read tables and lists of cases depends on the computer. When reading the data, CoCo then uses time to find out which data structure is the fastest on your computer for the particular dataset. In subsequent runs of CoCo on the dataset, force CoCo to use the selected data structure with `SET DATASTRUCTURE Necessary;` or `SET DATASTRUCTURE File;` between the two commands `READ SPECIFICATION;` and `READ OBSERVATIONS;`.

If `Large` is off and there is sufficient workspace in the N-array and in the P-array when performing tests all necessary marginal tables are found and the IPS-algorithm is run on all non-decomposable atoms before computing log(L). If `Large` is on, marginal tables (of observations and probabilities) are found and used in computing log(L) one at a time, e.i., you force CoCo to dispose of sufficient marginal tables as soon as the are used.

With `SET HUGE Off;` you prevent CoCo from finding the deviance without storing the sufficient marginal tables when the tables are to big to store.

### 11.5.4 DOS: Overlays

```
coco -O96000 -R32000 -T;
```

By default, the size of the overlay buffer is set to 96.000 Bytes. The probation area is one-third of the overlay buffer size. The size of the overlay buffer and the probation area can be controlled by the options `O` and `R` when invoking CoCo.

If you increase the size of the overlay buffer, there will be used less computing-time on swapping, but the size off the heap for models, tests, etc. will decrease.

If the disk is continuously accessed during your commands (not just a few times for each command) you should increase the size of the overlay buffer. If the computation of the adjusted number of degrees of freedom is set of by `SET ADJUSTED DF Off;` then the space needed for overlays is smaller.

Or if you run out of space in the heap for models, tests, classes of models and duals in the EH-procedure etc. you could restart with a smaller overlay buffer size (minimum 31.000 Bytes).

If the overlay buffer size is changed, but not the size of the probation area, then the size of the probation area is set to one-third of the overlay buffer size.

The option `T` will cause trace of swapping.

### 11.5.5 Unix: Size of N-, P- and Q-arrays

```
coco -N65536 -P65535 -Q1024;
```

Options for setting the initial size of N-, P- and Q-arrays at run-time at Unix.

If the table spanned by the factors of a test can not fit onto the N-array then Pearson's chi-square and the power divergence can not be computed, and if all sufficient marginal tables of the two models of a test can not fit into the P-array then the adjusted degrees of freedom can not be computed. The state-space of each irreducible component after adding a fill-in has to fit into the P-array for CoCo to be able to compute the deviance.

On Unix size of the N-, P- and Q-arrays are increased automatic by CoCo when needed and as long as the size do not get larger than $1048576 = 2^{20}$. This upper limit is set to prevent CoCo from allocating too big arrays. These big arrays will enable CoCo to compute, e.g., the adjusted degrees of freedom in large models, and much computing time will thus be required. The deviance can often be found without these big arrays faster.

If bigger arrays are wanted (for computing Pearson's chi-square, the power divergence, the adjusted degrees of freedom and exact tests), select these with the command line options `N` and `P`.

Note that the option `N[size]` controls the selected data structure.

### 11.5.6 The workspace in the heap

```
DISPOSE OF TABLES;
DISPOSE OF PROBABILITIES;
DISPOSE OF ALL Q-TABLES;
DISPOSE OF Q-TABLE ⟨set⟩;
```

The two commands `DISPOSE OF TABLES;` and `DISPOSE OF PROBABILITIES;` will dispose of sufficient marginal tables and tables of estimated cell probabilities. These tables can safely be disposed of when necessary to give CoCo more workspace, since these are found again when needed. Disposing of these tables will not release much space in the heap.

On a PC, remember to dispose of unused models with the command `DISPOSE OF MODEL;` before using all space in the heap.

The commands `DISPOSE OF ALL Q-TABLES;` and `DISPOSE OF Q-TABLE` ⟨*set*⟩`;` disposes of tables of initial values for the IPS-algorithm. Disposing of these tables will not release much space in the heap.

The command `RESTART;` clears all tables, model, specifications etc.

## 11.6 Miscellaneous Options for Tracing and Debugging

### 11.6.1 Echo and Note

```
SET ECHO [ On | Off ];
SET NOTE [ On | Off ];
```

The echo from the command-parser can be turned on and off with the switch `SET ECHO [ On | Off ];` (Default: Off). The `SET NOTE [ On | Off ];` controls a note when some commands have finished (Default: Off).

### 11.6.2 Report, Trace and Debug

```
SET REPORT [ On | Off ];
SET TRACE [ On | Off ];
SET DEBUG [ On | Off ];
SET OPTION { On | Off } ⟨number⟩ [ ⟨value⟩ ];
```

`Report` On will cause reporting of likelihood ratio test-statistics -2log(Q), e.i., the deviance, for each cycle in the EM-algorithm and cumulated number of zeros in computing the adjusted number of degrees of freedom (Default: Off). `Report` Off will not suppress printing on the `ReportFile` of time used in the IPS-algorithm and time used to find duals.

`Trace` will give a more detailed reporting (Default: Off).

The two commands `SET DEBUG [ On | Off ];` and `SET OPTION { On | Off }` ⟨*number*⟩ `[` ⟨*value*⟩ `];` are used by the author for debugging (Default: Off).

### 11.6.3 Graph mode

`SET GRAPH MODE [ On | Off ];`

After the command `SET GRAPH MODE On;` estimation and tests are skipped (Default: Off). You are then able to investigate graph-theoretical results of using the commands `PRINT FORMULA;`, `PRINT VERTEX ORDERING;`, `PARTITIONING;`, `FACTORIZE ONE EDGE;`, `FACTORIZE ONE INTERACTION;`, `DROP EDGES ...;`, `ADD EDGES ...;`, `REDUCE GENERATOR` ⟨*set*⟩`;`, etc. in very large graphs.

`GraphMode` is most safely turned off again by ending `CoCo` and restarting, since the IPS-algorithm is suppressed when reading models with `GraphMode` on.

## 11.7 Interrupts

`SET INTERRUPT { A | B } { On | Off };`

On unix the command `SET INTERRUPT A Off;` will re-install the normal Control-C-interrupt-handler. After use of `SET INTERRUPT A On;` (Default) a Control-C will terminate the computation of an exact test and the IPS-algorithm.

`SET INTERRUPT B Off;` will re-install the normal Control-\-interrupt-handler.

Control-\ or two close Control-C's will terminate the EM-algorithm, factorization of a test, a `BACKWARD;`-command, a `FORWARD;`-command or the computation of a dual in the EH-procedure after `SET INTERRUPT B On;` (Default).

A Control-C will return the action to the lisp listener when the interrupt is given within a CoCo-function of CoCo loaded into XLISP-STAT.

# Chapter 12

# CoCo in other systems

CoCo can on UNIX systems be loaded dynamically into XLISP-STAT (Tierney 1991) and S-Plus (Becker, Chambers & Wilks 1988) (and may be into New S, if the function `dyn.load2()` is available). This feature is only tested on Sparc.

The purpose of loading CoCo dynamically into XLISP-STAT is besides the system "Xlisp+CoCo — A Tool for Graphical Models" to be able to do further computations on fitted table values and statistics, plot output from CoCo by high-resolution graphics or, e.g., do a model search by a strategy implemented in Lisp. Data (specification of table and the table of counts) and models can be sent to the CoCo object. Tables and test-values can be returned.

When CoCo is loaded into XLISP-STAT or S-Plus the system demands more memory and is somewhat less stable than any of the 3 programs alone, although it is a long time since the author unexpected has crashed the system. If you are, e.g., going to do a lot of computation in XLISP-STAT or S-Plus without using CoCo-functions or you are doing model search in CoCo, run XLISP-STAT, S-Plus or CoCo respectively without linking the two programs together.

A CoCo object file is linked together with S-Plus or XLISP-STAT. Data (specification of table and the table of counts) and models can be sent to the CoCo object. Table and test-values can be returned.

Interface-functions for all the CoCo-commands are implemented only for XLISP-STAT. Only the most necessary interface-functions are implemented for S-Plus. The remaining CoCo-commands are for the time being used in S-Plus by handling the control over to CoCo and reading commands in normal CoCo-syntax.

Note that this section of CoCo might develop faster than other parts. Returned values (especially error-messages and values when an error occurs), arguments to functions and methods and even names on functions and methods may change. List the files `cocoapi.S` and `cocometh.lsp` for an up to date documentation of the version, you are running.

Figure 12.1: Plot from New S by call of functions in CoCo.

## 12.1 S + CoCo

A version of CoCo, an object file without main program, but with some interface procedures, is linked together with S-Plus, see Becker et al. (1988).

Start S-Plus with the script `S+coco` to set up some environment variables to CoCo. (The script `S+coco` is available on your system, if you or your system administrator has done step 4) and 5) of the installation of CoCo, see appendix A. S-Plus, of course, has to be available on your system.)

Use `attach(paste(unix("echo $SCOCOHOME"), "/.Functions", sep = ""))` in S-Plus to access the directory ‘‘`.Functions`’’ with the S-function for communicating with CoCo: Available S-functions for interaction with CoCo are then listed by `ls(pos=2)`. The function `coco.load()` will load CoCo into S-Plus.

`coco.start()` will then start CoCo in S-Plus. Ordinary CoCocommands are then expected. CoCo is reading from standard input and writing to standard output. The CoCo command `END;` will bring you back to S-Plus without disposing of the running CoCo session. CoCo is not terminated, but only left temporarily. The running CoCo session can be resumed from S-Plus with `coco.resume()`. The running CoCo session within S-Plus is removed with `coco.end()`. If `coco.end()` is not used before leaving S-Plus, some temporary files will not be deleted. The function `coco.init()` starts CoCo and returns immediately to S-Plus without expecting CoCo commands. `coco.start()` is a combined command of `coco.init()` and `coco.resume()`.

The returned value from `coco.start()`, `coco.init()` and `coco.resume()` is `TRUE`, if the command succeeds.

`coco.start()` and `coco.init()` may be given the optional arguments ⟨*n=512*⟩, ⟨*p=512*⟩, and ⟨*q=512*⟩ for setting the initial size of the N-array, P-array and Q-array respectively. The optional argument ⟨*s=620000*⟩ to `coco.load()` is the expected size of the code added to S-Plus and used in `dyn.load2()`. The size ⟨*s*⟩ of the code added to S-Plus depends on the machine, you are using, the compiler used, options to the compiler and the version of CoCo. If you get an error message from `coco.start()`, `coco.init()` or `coco.load()`, then use the size, the message suggests.

Some versions of the object file loaded into S-Plus allow for handling more than one CoCo object in the same S-Plus session. At the same time in S-Plus, CoCo can then be used on different datasets. A list `coco.identifications` (of integers in frame 0) is used to identify the individual CoCo objects. The values of the integers have no meaning for the user. `current.coco` is the identification of the default CoCo object used in the following functions. The functions `make.current.coco(`⟨*a*⟩`)` in S-Plus will make the ⟨*a*⟩-the started or initiated CoCo object the current CoCo object, assign `coco.identifications[a]` to `current.coco`.

The function `enter.names(`⟨*names*⟩`,` ⟨*levels*⟩`,` ⟨*missing = NULL*⟩`)` enters a specification of a table to CoCo. ⟨*names*⟩ is a single text string with the factor names: a character for each factor. ⟨*levels*⟩ is a vector of integers giving the total number of levels for each factor. ⟨*missing*⟩ is an optional vector of integers giving the number of missing levels.

If `enter.names(`⟨*names*⟩`,` ⟨*levels*⟩`,` ⟨*missing = NULL*⟩`)` is succeeded, the value `TRUE` is returned, else a warning message is printed and `FALSE` is returned.

A table of counts is entered by `enter.table(`⟨*table*⟩`)`. ⟨*table*⟩ is a vector of integers giving the cell counts ordered as when reading a table of counts into CoCo in the normal set.

If `enter.table(`⟨*table*⟩`)` is succeeded, the value `TRUE` is returned, else `FALSE` is returned and a warning message is printed.

(If the command `SET READ { All | Subset` ⟨*set*⟩ `};` has been used or data selection has been done between calls to the functions `enter.names(`⟨*names*⟩`,` ⟨*levels*⟩`,` ⟨*missing = NULL*⟩`)` and `enter.table(`⟨*table*⟩`)`, the fully declared table still has to be entered. The count `-1` is entered for structural zero cells.)

The function `read.model(`⟨*a*⟩`)` in S-Plus will add the model ⟨*a*⟩ to the list of models in CoCo. The argument ⟨*a*⟩ to `read.model(`⟨*a*⟩`)` has to be a single character string with the model written as models are written in CoCo.

The functions `make.current(`⟨$a$⟩`)` and `make.base(`⟨$a$⟩`)` in S-Plus will make the model with number ⟨$a$⟩ the **current** or the **base** model respectively in CoCo. `base()` makes the **current** model the **base** model. `current()` makes the **last** model the **current** model. These four commands return the value `TRUE`, if the commands succeed, else `FALSE`. The commands `return.current.number()` and `return.base.number()` return respectively the model numbers for the **current** model and the **base** models, if the command succeeds, else `NULL`. The commands `return.current()` return a text string with the generating class for the **current** model, if the command succeeds, else `NULL`. Analogously with `return.base()`.

From S-Plus the models read in CoCo are printed by `print.all.models(`⟨*print* = *TRUE*⟩`)`. If ⟨*print*⟩ is `FALSE`, then descriptions of the models are printed. No values are returned.

The function `compute.test()` in S-Plus will return the unadjusted number of degrees of freedom, the adjustment of DF. (if `AdjustedDF` is on in CoCo), the likelihood ratio test statistic -2Log(Q), Pearsons chi-square $\chi^2$ and the power divergence $2nI^\lambda$ and if `ExactTest` is on in CoCo, the exact P-values. The function `compute.deviance()` in S-Plus will return values as the command `FIND DEVIANCE;` in CoCo.

And, what it is all about: The function `return.vector(`⟨*type* = *0*⟩`,` ⟨*set* = *"*"*⟩`,` ⟨*number* = *0*⟩`,` ⟨*base* = *FALSE*⟩`)` in S-Plus will return a vector with the table values ⟨*type*⟩ from the ⟨*set*⟩-marginal table computed under the **current** model. See section B.1.46 in the Quick-reference-card for how CoCo-⟨*value-names*⟩ are coded into the integers ⟨*type*⟩. If ⟨*base*⟩ is `TRUE`, then the values from the **base** model are returned. If ⟨*number*⟩ is different from `0`, then the values from model with that number are returned.

The model {{ACE},{ADE},{BC},{F}} is tested against {{ABCDEF}} on the data from Reiniš et al. (1981) and the adjusted residuals in {{ACE},{ADE},{BC},{F}} are plotted against the standardized residuals in {{ACDE},{ABCF}} in S-Plus by:

```
S+coco

attach(paste(getenv("SCOCOHOME"), "/.Functions", sep = ""))
coco.load()
coco.init()
enter.names("ABCDEF",c(2,2,2,2,2,2),c(0,0,0,0,0,0))

"n"<- c(44, 40, 112, 67, 129, 145, 12, 23, 35, 12, 80, 33, 109,
        67, 7, 9, 23, 32, 70, 66, 50, 80, 7, 13, 24, 25, 73, 57,
        51, 63, 7, 16, 5, 7, 21, 9, 9, 17, 1, 4, 4, 3, 11, 8, 14,
        17, 5, 2, 7, 3, 14, 14, 9, 16, 2, 3, 4, 0, 13, 11, 5, 14, 4, 4)
enter.table(n)

read.model("*")
read.model("ACDE,ABCF.")
read.model("ACE,ADE,BC,F.")
compute.test()
compute.deviance()
```

```
make.base(2)
print.all.models()
X11()
plot(return.vector( 0, "*"), return.vector( 8, "*"),
     xlab = 'Observed counts: return.vector( 0, "*")',
     ylab = 'Adjusted residuals: return.vector( 8, "*")')

coco.resume()
  print all models
  end

coco.end()
q()
```

## 12.2 XLISP-STAT + CoCo

The system "Xlisp+CoCo — A Tool for Graphical Models" is on UNIX systems obtained by loading CoCo dynamically into XLISP-STAT. This feature is only tested on Sparc. See Part III.

Part III of this thesis is a guide to Xlisp+CoCo. See this guide for further documentation. Some of the highlights of the system are:

### 12.2.1 Association Diagrams

The *association diagram* is a graph window with the independence graph of an association model. The model can be causal or not, and the variables can be discrete, ordinal, continuous etc. The *independence graph*, association or interaction graph, is a simple undirected graph with as many vertices as the table has dimension. A *vertex* for each variable. Two vertices are *adjacent* in the independence graph for the model unless the two variables are conditionally independent given the other variables.

The association diagram is a very appealing working tool for model selection. The layout of the graph, the association diagram, can be edited by dragging vertices, points for variables, with the mouse, and the edges will then follow the vertices. New association diagrams can be created from a diagram by adding or dropping edges by the mouse. Tests of two variables conditionally independent can be performed by clicking on the edge between the two variables by the mouse. Edges are then drawn with a width proportional to the significance of the edges. Edges can also be labeled with a value computed from a test statistic for removing the edge, e.g., a $p$-value or an $IC$-value. Causal models can be handled in the association diagrams. Methods for backward elimination and forward selection of edges in association diagrams are implemented. In the backward elimination of edges the edges are redrawn with a width proportional to the significance of the edges as they are visited. The edge will be drawn with a color depending on whether the edge is fitted or not. If the test of an edge is not computed, then the edge will be drawn with a color depending on whether the edge is fixed, the edge is rejected by coherence, removing the edge will result in a non-decomposable model, etc.

### 12.2.2 Block Recursive Models

*Block recursive models*, *Chain graph models*, are models with both symmetric and asymmetric associations between variables. The symmetric associations have been dealt with so far in this thesis.

At the asymmetric association, directed association, between two variables, the second variable might depend on the first variable, but the first variable is independent of the second variable. The first variable is then *explanatory* to the second variable, the *response* variable. In the graph we then draw an *arrow* from the first variable to the second variable. The explanatory variables are also called the *exogenous* variables and the response variables the *endogenous* variables. The endogenous variables have an undirected graph associated with them. Each endogenous factor is the end point for one or more direct edges. The directed edges can originate at either exogenous or endogenous variables.

In *recursive* models response variables of some explanatory variables cannot be explanatory variables to the same explanatory variables, i.e., in the graph there cannot be any oriented cycles, cycles where one goes along undirected edges and in the direction of arrows.

Conditional independence statements for recursive causal models are examined in Wermuth & Lauritzen (1983). The so-called block recursive graphical models or graphical chain models are further treated in Lauritzen & Wermuth (1989), Lauritzen (1989), Wermuth & Lauritzen (1990), Lauritzen, Dawid, Larsen & Leimer (1990). The chain graph Markov property is investigated in detail in Frydenberg (1989). Model selection methods for creating a diagnostic system by causal models on discrete variables are discussed in Lauritzen, Thiesson & Spiegelhalter (1992). Also the text books Lauritzen (1993), Whittaker (1990) and Christensen (1990) contain chapters on block recursive models.

The tool for association diagrams implemented in the Lisp extension of CoCo are able to handle block recursive models. In these diagrams incremental model search on block recursive models can be performed by backward elimination and forward selection.

### 12.2.3 Miscellaneous

Besides introducing association diagrams with the ability to handle block recursive models and dumping TeX picture-code for the diagrams, this tool also introduces the model dynamic spinning plot and the model manager. Also a method for doing a model selection with resampling is implemented.

#### Model Dynamic Spinning Plots

The *model dynamic spinning plot* is a spinning plot, where the values in the plot are updated when models are modified. Spinning plots are rotatable three-dimensional scatterplots. In the model dynamic spinning plot each of the variables are linked to a model, and when the model is modified the plot is redrawn with the new values of the model. These spinning plots can, of course, be linked to other spinning plots, histograms, etc. and points of one plot highlighted when the corresponding points are brushed in other plots or histograms.

**Model Manager**

The model search by association diagrams may generate numerous windows with graphs, and to handle these windows, the *model manager* has been made. The model manager is an *overview graph*, where each point in the graph corresponds to a model. Tests can be performed by dragging edges between points in the model manager.

**Model Selection with Resampling**

To show the advantages of including CoCo in XLISP-STAT, a method for resampling from the case list, and do a backward elimination on each sample, has been implemented. The edges of the association diagram can be drawn with a width growing with the reciprocal of the $p$-value or growing with BIC, the Bayesian information criterion. But we can also let the width of the edges be proportional to the number of times the edges are found in the final models of headlong backward eliminations on random subsets of the cases.

This method is implemented in a few lines of Lisp code, and the full implementation is shown in part III, "Xlisp+CoCo — A Tool for Graphical Models", of the thesis as an example of what can be made by the end user in XLISP-STAT extended with CoCo.

### 12.2.4 A Tutorial Example

This tutorial example is also given in Part III of this thesis. The data used is that of table 1.1 from Reiniš et al. (1981) also used in the introduction.

Start XLISP-STAT with the script `xlisp+coco`[1] to set up some environment variables needed by CoCo and to load the Lisp functions for interfacing CoCo. To be able to edit and redo entered lines in XLISP-STAT you may want to use `fep`:

```
$ fep xlisp+coco
```

To create a CoCo-object, use the function `(make-coco)`:

```
(def reinis-coco-object (make-coco))
```

By using the function `(def)` instead of `(setf)` the created objects, variables, etc. can be listed by the function `(variables)`.

The specification of the table is then sent to the CoCo-object by the method `:enter-names`:

```
(send reinis-coco-object :enter-names
      '("A" "B" "C" "D" "E" "F")
      '(2 2 2 2 2 2) '(0 0 0 0 0 0))
```

[1] Make sure that you or your system administrator has done step 4) and 5) of the installation of CoCo, see appendix A, and that XLISP-STAT is available on your system. XLISP-STAT must be compiled with `FOREIGN_FLAG = -DFOREIGNCALL` to make the dynamic loading working. See the 'Makefile' for XLISP-STAT.

The method `:enter-names` has three arguments: A list with the names, text string, of the variables, a list of integers with the number of unmarked levels for each factor and an optional list of integers with the number of levels to be marked as missing.

The variable $n$ to hold the table of observed counts in a list is defined by:

```
(def n '(
 44  40   112  67    129 145    12  23
 35  12    80  33    109  67     7   9
 23  32    70  66     50  80     7  13
 24  25    73  57     51  63     7  16
  5   7    21   9      9  17     1   4
  4   3    11   8     14  17     5   2
  7   3    14  14      9  16     2   3
  4   0    13  11      5  14     4   4))
```

Instead of `'( ... )` the form `(list ... )` could have been used. But, if the list is long, you will get a stack-error.

The list $n$ containing the table of counts is then sent to the CoCo-object by:

```
(send reinis-coco-object :enter-table n)
```

The following three messages will read the 3 models into the CoCo-objects and create model-objects for these:

```
(def model-1 (send reinis-coco-object :make-model "*"))
(def model-2 (send reinis-coco-object :make-model "ABCF,ACDE"))
(def model-3 (send reinis-coco-object :make-model "ACE,ADE,BC,F"))
```

and graphs for the 1st and 3rd model is created by:

```
(def graph-1 (send model-1 :make-graph
                   :title "Reinis CoCo Graph"))

(def graph-3 (send Model-3 :make-graph :location (list 700 50)
                   :title "Reinis: ACE,ADE,BC,F"))
```

You can now move vertices, set variable labels, drop and add edges etc. with the mouse and perform, e.g., a model search by selecting items from the menu. See Part III of this thesis.

To set labels on the vertices in the 1st graph:

```
(send graph-1 :vertex-label "A" "A: Smoking")
(send graph-1 :vertex-label "B" "B: Mental")
(send graph-1 :vertex-label "C" "C: Physical")
(send graph-1 :vertex-label "D" "D: Blood pressure")
(send graph-1 :vertex-label "E" "E: Ratio of lipoproteins")
(send graph-1 :vertex-label "F" "F: Family anamnesi")
```

Figure 12.2: The result of a "Headlong Backward" on Graph-1 with added $p$-values. The bit-mat dump is created by Xlisp+CoCo.

and move the vertices in the graph:

```
(send graph-1 :vertex-position "A" (list   0 -20) :redraw nil)
(send graph-1 :vertex-position "B" (list   0  20) :redraw nil)
(send graph-1 :vertex-position "C" (list  35   0) :redraw nil)
(send graph-1 :vertex-position "D" (list -30 -40) :redraw nil)
(send graph-1 :vertex-position "E" (list -40   0) :redraw nil)
(send graph-1 :vertex-position "F" (list -30  40) :redraw T)
```

Change the color on the vertex-labels for all graphs:

```
(send graph-1 :item-color 'vertex-label 'red)
```

Move the vertex-labels for vertex 'C' and 'E':

```
(send graph-1 :vertex-label-position "E" (list -3 -4))
(send graph-1 :vertex-label-position "C" (list -11 -2 0) :redraw T)
```

Add control buttons to graph-1:

```
(send graph-1 :add-controls)
```

Let 'graph-3' have same vertex-positions as 'graph-1':

```
(send graph-3 :slot-value 'positions
      (send graph-1 :slot-value 'positions))
```

A Model Manager is created by:

```
(setf manager (return-manager))
```

The following will create a "Model Dynamic Spin Plot" for 'graph-1'. The values in the plot are re-evaluated and the plot is redrawn when the graph for the spin-plot receives one of the two messages `:make-graph-current-model` and `:make-graph-base-model`:

```
(let ((a '(send *this-graph* :return-vector 'adjusted "*"
                :model 'current))
      (b '(send *this-graph* :return-vector 'adjusted "*"
                :model 'base))
      (c '(send *this-graph* :return-vector 'observed "*"
                :model nil)))
  (def graph-1-spin
       (send graph-1 :return-dynamic-coco-spin-plot (list a b c)))
  (send graph-1-spin :location 700 500)
  )
```

A 'Headlong Backward Elimination' at 'graph-1' can be performed by:

```
(send graph-1 :drop-least-significant-edge
      :p-accepted (send graph-1 :graph-p-accepted)
      :p-rejected (send graph-1 :graph-p-rejected)
      :random-order (send graph-1 :graph-random-order)
      :coherent (send graph-1 :graph-coherent)
      :headlong T :recursive T :x-move 0)
```

The "Model Dynamic Spin Plot" and 'Headlong Backward' can also be made by selecting the **Recursive backward** and **Dynamic Spin-plot** items from the graph menu.

Or the total example of the tutorial can be performed by the four lines

```
(load (concatenate 'string *xlisp-cocohome* "/Examples/Testgraph"))
(setf manager (return-manager))
(load (concatenate 'string *xlisp-cocohome* "/Examples/TestDynamic"))
(load (concatenate 'string *xlisp-cocohome* "/Examples/TestHeadlong"))
```

Consider also

```
(load (concatenate 'string *xlisp-cocohome* "/Examples/TestBlock"))
(load (concatenate 'string *xlisp-cocohome*
                   "/Examples/TestBootstrap"))
(load (concatenate 'string *xlisp-cocohome* "/Examples/TestTeX"))
(load (concatenate 'string *xlisp-cocohome* "/Examples/Krippendorf"))
```

# Part III

# Appendix

# Appendix A

# Installing CoCo

## A.1 Availability

The source code in C, executable for Sun 4 (Sparc) and executable of the standalone version of CoCo for PC's and Macintosh for this version of CoCo is available free of charge for non-commercial use.

The source code may only be read and edited for the purpose of porting CoCo to other machines. No new features may be added to CoCo and no parts of the program may be included in other systems or new interface-procedures to New S, S-Plus, XLISP-STAT or any other extendable system may be made without the written permission from the author.

The source code in C and executable for Sun 4 (Sparc), both with Lisp-code for the graphics, and executable of the standalone version of CoCo for PC's and Macintosh can be obtained by anonymous *ftp* over internet from *ftp.iesd.auc.dk*, or by *WWW* from *http://www.iesd.auc.dk/pub/packages/CoCo*. Or CoCo is available on disks (3.5" or 5.25" High density). You should, however, by prepared to bear the costs of copying, e.g., by supplying a disk or tape and a stamped mailing envelope. This guide and the guide to CoCo is also available in TeX and postscript code from the ftp site or printed from Aalborg University.

If you have problems with CoCo or find bugs, please contact the author. If possible, send on disk or by Email the `LogFile` for the session which caused the problem. I will do my best to solve your problems and provide updates, if necessary.

If you have comments on CoCo, suggestions for improvements or new facilities that you feel would make the software more useful to you, please feel free to contact me.

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## A.2 Installation

The following section is the installation guide for both the standalone version of CoCo and of the interface functions for the graphics. The graphics will only run on Unix systems with XLISP-STAT. XLISP-STAT must on your system be compiled so it is able to do dynamic loading.

### A.2.1 DOS: On PC

Read the latest `READ.ME` and `README.DOS` file.

Copy the contents of the distribution disk into a directory, the *home directory* for CoCo. Add the home directory of CoCo to your path.

The files `COCO.EXE`, `COCO.OVR` and `COCO.TAB` must be in the home directory for CoCo. The two files `COCO.HLP` and `COCO.DAT` should also be there. If the `HelpFile COCO.HLP` not is in the directory, there will be no online help information available in CoCo. The default `DataFile COCO.DAT` can be replaced by another data-file.

By `copy/b coco.exe + coco.ovr` the overlay file is attached to the end of the `.EXE` file `COCO.EXE`. You can then remove `COCO.OVR`.

Type `COCO` to start CoCo.

Test examples are found in the directory `EXAMPLES`. Run the test examples with `runall` or `runsome`. `runall` will take approximately 24 hours and use three Mbytes of diskspace to diaries. Datasets and CoCo-source-files are found in the directory `DATASETS`.

CoCo runs on IBM/XT/ATs (and compatible) with or without coprocessor 8087/80287/80387 and with or without expanded memory (EMS). If the numeric coprocessor is not present, it is emulated. If EMS is present, the overlay file is loaded into the expanded memory when sufficient space is available. See the sections "DOS: Overlays" in the chapter "Miscellaneous Options for ..." in "A Guide to CoCo" about how to control the size of the overlay buffer.

See the `README.DOS` file for `Run-Time Errors`.

### A.2.2 Macintosh

Read the disk with CoCo for Macintosh or uncompress the "sit.Hqx" file. Click the folder "CoCo" to start CoCo.

### A.2.3 Unix: On Workstations

Read the latest `READ.ME` and `README.Unix` file.

Copy the contents of the disk or tape into a directory, the build directory of CoCo. Or obtain relevant files by anonymous *ftp* over internet from *ftp.iesd.auc.dk* or by *WWW* from *http://www.iesd.auc.dk/pub/packages/CoCo* and uncompress. Change to the directory.

**1)** Set the variables

```
BINDIR
COCOHOME
SCOCOHOME
XCOCOHOME
```

to appropriate values in the Makefile. `$BINDIR` (e.g., /home/local/bin) is where the script for starting CoCo goes. The value of `$COCOHOME` (e.g., /home/local/lib/coco) is the *home directory* of CoCo. `$SCOCOHOME` (e.g., /home/local/lib/coco or /home/local/lib/Splus/local) is where the functions to use CoCo in `S-Plus` go. The `XLISP-STAT`-interface-functions are copied to `$XCOCOHOME` (e.g., /home/local/lib/coco/lsp).

**2a)** Move the executable `coco.sparc` or `coco.sun3` to `coco` by, e.g., the command `mv coco.sparc coco`, unless you have uncompressed a tar-file with only one file `coco`.

or

**2b)** Compile CoCo by 'make coco'.

Installation of CoCo is then done by:

**3)** Type 'make installcoco'. This will copy necessary files (`coco`, `COCO.TAB`, `COCO.HLP` and `COCO.DAT`) to the *home directory* `COCOHOME` for CoCo, edit the script CoCo (insert the environment variable `COCOHOME`) and place the result in `BINDIR`.

You only need to do step 4 and 5 or 6 if you want to use CoCo in S-Plus or XLISP-STAT:

**4a)** Move the object files `scoco.sparc` or `scoco.sun3` to `scoco.o` by, e.g., the command `mv scoco.sparc scoco.o`, unless you have uncompressed a tar-file with only one executable file `scoco.o`.

or use

**4b)** 'make scoco.o' to make the object file `scoco.o`.

To install S+CoCo, do

**5a)** Edit also the environment variable `S`, if necessary.

**5b)** Type 'make installscoco'. This will edit the script `S+coco` (insert the environment variables `S`, `SCOCOHOME` and `COCOHOME`) and then place the result in `BINDIR`, copy `scoco.o` to `COCOHOME`, make the interface-functions to S-Plus by running S-Plus on `cocoapi.S` and copy the resulting `S-functions` to `SCOCOHOME/.Functions`.

The CoCo-S-functions `SCOCOHOME/.Functions` could be moved to some directory in the default S-Plus search list. The command `attach(paste(getenv( "SCOCOHOME" ), "/.Functions", sep = "" ))` is then unnecessary to get access to the CoCo-S-functions.

To install Xlisp+CoCo, do

**6a)** Set the environment variable `XCOCO` to `scoco.o`, if you are using XLISP-STAT version 2.1 Release 2, or to `libscoco.so`, if you are using XLISP-STAT 2.1 Release 3.39 (or later). Edit also the environment variable `XLISPSTAT`, if necessary.

**6b)** Type 'make installxcoco'. This will edit the script `xlisp+coco` (insert the two environment variables `XLISPSTAT`, `XCOCOHOME` and `COCOHOME`) and place the result in `BINDIR`, write the file `loadcoco.lsp` for loading necessary interface files, copy `scoco.o` (or `libscoco.so`) to `COCOHOME`, and copy the files `cocoapix.lsp`, `cocometh.lsp`, `cocograph.lsp`, etc. to `XCOCOHOME`. *Do not do step "manually", since other actions might have been added since the writing of this.*

XLISP-STAT must be compiled with `FOREIGN_FLAG = -DFOREIGNCALL` to make dynamic loading working. See the 'Makefile' for XLISP-STAT.

If you are running `Solaris`, then you should use version 2.1 Release 3.39 (Beta) or later of XLISP-STAT.

After successfully having installed Xlisp+CoCo you should compile the Lisp-file for Xlisp+CoCo. Change to the directory `XCOCOHOME` and type 'make'. This will compile the Lisp-files for Xlisp+CoCo and create a saved workspace for Xlisp+CoCo. By the saved workspace the upstart of Xlisp+CoCo will be much faster, and by the byte-compilation the program will run faster.

`XLISP-STAT` by Luke Tierney can be obtained by anonymous *ftp* over internet from *umnstat.stat.umn.edu* (128.101.51.1) or mail a message containing the line

```
send index from xlispstat
```

to *statlib@templer.stat.cmu.edu* for how to find out to obtain XLISP-STAT from the `statlib` archive. XLISP-STAT can be obtained free of charge.

`fep` is a very useful tool, when running CoCo and XLISP-STAT. In the manual page to `fep`, the general purpose front end processor by K. Utashiro, Software Research Associates, Inc., Japan, the Email address utashiro@sra.junet is found. `fep` can be obtained free of charge.

`S-Plus` by Statistical Sciences, Inc, P.O. Box 85625, Seattle, WA 98145-1625, U.S.A. is a commercial product. It adds features to `New S` by Becker, Chambers and Wilks, see Becker et al. (1988) for New S. Electronic mail, Europe: sales@statsci.co.uk, mktg@statsci.co.uk, support@statsci.co.uk; USA: sales@statsci.com.

| | | Dos | Macintosh | Unix |
|---|---|---|---|---|
| Maximum number of factors | | 64 | 128 | 256 |
| Maximum number of levels | | 60 | 124 | 252 |
| Maximal cell count | | 65536 | 2147483644 | 2147483644 |
| N-array: | initial | 32766 | 4096 | 65536 |
| | maximum | do | ? | $> 2^{27}$ |
| | storage mode | int | long | long |
| P-array: | initial | 10922 | 4096 | 65536 |
| | maximum | do | ? | $> 2^{27}$ |
| | storage mode | real | float | float |
| Q-array: | initial | 1024 | 1024 | 1024 |
| | maximum | do | ? | $> 2^{27}$ |
| | storage mode | int | long | long |

Table A.1: *Limits and Storage Mode.*

## A.3 Limitations and Precision

Limitations for your running version of CoCo are printed with `STATUS Limits;`.

### A.3.1 DOS: PC

The number of declared factors and factors used in the models is limited to 64 ($2^6$). The maximum number of levels for each factor is set to 60 ($2^6 - 4$).

All marginal tables of observations are placed in an integer array (N-array) with 32766 ($2^{15} - 2$) cells. The estimated probabilities are placed in a real array (P-array) with 10922 ($2^{15}/3$) cells. You are then able to compute log(L) for a model with 64 binary factors, if the largest clique in the graph for the model has 14 vertices at the most and there are no more than 13 vertices in the largest non-decomposable atom.

Except in the IPS-algorithm all real computations are performed in double-precision (8 bytes, 15 to 16 significant digits). Estimated probabilities are stored in normal real (6 bytes, 11 to 12 significant digits).

If estimated probabilities instead are stored in double-real, the number of cells in the P-array is reduced from 10922 ($2^{15}/3$) to 8190 ($2^{13} - 2$). On the other hand, if probabilities are stored in single-real (4 bytes, 7 to 8 significant digits) the size of the P-array can be increased to 16380 ($2^{14} - 4$) cells.

The maximal number of observations is set to 65532 ($2^{16} - 4$). It can be increased to 2147483644 ($2^{31} - 4$) at the cost of one dimension. The size of the N-array is then reduced from 32766 ($2^{15} - 2$) to 16383 ($2^{14} - 1$).

If you are displeased with any of the limits, please let me know. See section A.1 "Availability" of this appendix.

### A.3.2 Macintosh

The maximum number of declared factors and factors used in models is set to 128 ($2^7$). The maximum number of observations is 2147483644 ($2^{31} - 4$).

The initial size of both the N- and P-array is set to 4094 ($2^{12}$). The cells in the N-array are longs and the cells in the P-array are floats. All real computations are performed in double.

### A.3.3 Unix: Workstations

The maximum number of declared factors and factors used in models is set to 256 ($2^8$). This can be increased by changing constants and recompiling. A version with constants set to handle 4096 ($2^{14}$) factors has been compiled and run on a SPARCstation with 600 Mbytes swap space.

The maximum number of observations is 2147483644 ($2^{31} - 4$).

In the version compiled for workstations, the initial size of both the N- and P-array is set to 65536 ($2^{16}$). These arrays are increased automatic by CoCo when needed up to a limit of 1048576 ($2^{20}$). If bigger arrays are needed these must be given by the `N` and `P` command line options.

The cells in the N-array are longs and the cells in the P-array are floats. Each on 4 bytes, $2^{18}$ cells per Mbyte. All real computations are performed in long-real (8 bytes, 15 to 16 significant digits).

A running CoCo with space for a 20-dimensional binary table with 1048576 cells in the N-array and a 20-dimensional binary table in the P-array will then need a little more than 8 Mbytes of memory. CoCo has been started with space for a 26-dimensional binary table with 67108864 cells in both the N-array and the P-array on a SPARCstation with 600 Mbytes swap space.

## A.4 CoCo Datasets and Examples

`Datasets` for CoCo and examples of `SourceFiles` for running these datasets can be found in the directory Datasets in the home directory for CoCo. The examples are to illustrate the use of CoCo. They are not meant to be good practice of data analysis.

The directory Examples contains tests-runs. These examples are not even instructive.

## A.5 Reporting Errors

Report errors by sending the `LogFile` (DOS: COCO???.LOG, Macintosh: CoCo.log.no and Unix: /tmp/CoCo.log.004.X(pid-number) ) and data-files to the author. Please note whether the error appears again, if you run CoCo with the `LogFile` as input. On DOS: rename the `LogFile` before starting CoCo again. Note machine-type, operating system, version of operating system and version of CoCo.

## A.6 Warranty

NO WARRANTY

I PROVIDE ABSOLUTELY NO WARRANTY. EXCEPT WHEN OTHERWISE STATED IN WRITING, I AND/OR OTHER PARTIES PROVIDE CoCo "AS IS" WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESSED OR IMPLIED, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU, SHOULD THE CoCo PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COSTS OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

IN NO EVENT I AND/OR ANY OTHER PARTY WHO MAY MODIFY AND REDISTRIBUTE CoCo, MAY BE LIABLE TO YOU FOR DAMAGES, INCLUDING ANY LOST PROFITS, LOST MONIES, OR OTHER SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY THIRD PARTIES OR A FAILURE OF THE PROGRAM TO OPERATE WITH PROGRAMS NOT DISTRIBUTED BY ME) THE PROGRAM, EVEN IF YOU HAVE BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES, OR FOR ANY CLAIM BY ANY OTHER PARTY.

# Appendix B

# Reference Manual section

## B.1 Quick Reference Card

### B.1.1 End and Restart

```
END;
RESTART;
```

### B.1.2 Help and Quick-reference-card

```
HELP [ ⟨command-name⟩ | ⟨abbreviation⟩ ];
MENU NUMBER ⟨a⟩;
NEXT MENU;
CURRENT MENU;
PREVIOUS MENU;
```

### B.1.3 Abbreviations

```
PRINT ALIAS { ⟨command-name⟩ | ⟨abbreviation⟩ };
MAKE ALIAS ⟨command-name⟩ ⟨abbreviation⟩;
DELETE ALIAS ⟨abbreviation⟩;
READ LOCAL PARSER;
INIT PARSER;
NO ACTION;
```

### B.1.4 Status

```
STATUS [ Specification | Observations | Files | Formats | Fix | Tests
    | Exact | Ips | Em | EH | Limits | Other ];
```

### B.1.5 Input from keyboard or file

```
SET KEYBOARD [ On | Off ];
```

### B.1.6 Input files

```
SET INPUTFILE SPECIFICATION ⟨file-name⟩;
SET INPUTFILE OBSERVATIONS ⟨file-name⟩;
SET INPUTFILE DATA ⟨file-name⟩;
SET INPUTFILE COMMANDS ⟨file-name⟩;
```

### B.1.7 Output files

```
SET OUTPUTFILE RESULTS ⟨file-name⟩;
SET OUTPUTFILE COMMANDS ⟨file-name⟩;
SET OUTPUTFILE DIARY ⟨file-name⟩;
SET OUTPUTFILE LOG ⟨file-name⟩;
SET OUTPUTFILE DUMP ⟨file-name⟩;
SET OUTPUTFILE REPORT ⟨file-name⟩;
SET KEEP DIARY [ On | Off ];
SET KEEP LOG [ On | Off ];
SET KEEP DUMP [ On | Off ];
SET KEEP REPORT [ On | Off ];
```

### B.1.8 Diary, Timer etc.

```
SET DIARY [ On | Off ];
SET LOG [ On | Off ];
SET DATA LOG [ On | Off ];
SET DUMP [ On | Off ];
SET REPORT [ On | Off ];
SET TIMER [ On | Off ];
SET ECHO [ On | Off ];
SET NOTE [ On | Off ];
SET TRACE [ On | Off ];
SET DEBUG [ On | Off ];
SET OPTION { On | Off } ⟨number⟩ [ ⟨value⟩ ];
SET INTERRUPT { A | B } { On | Off };
```

### B.1.9 Print Formats

```
SET PRINT FORMATS ⟨format-width⟩ ⟨decimals⟩;
SET TABLE FORMATS ⟨width⟩ ⟨dec-prob.⟩ ⟨dec-expt.⟩ ⟨dec-diff.⟩;
SET TEST FORMATS ⟨x-width⟩ ⟨x-dec.⟩ ⟨prob-width⟩ ⟨prob-dec.⟩;
SET PAGE FORMATS ⟨line-length⟩ ⟨page-length⟩;
SET PAUSE [ On | Off ];
SET PAGING LENGTH ⟨number of lines⟩;
SET SHORT [ On | Off ];
```

### B.1.10 Controlling the IPS- and EM-algorithm

```
SET IPS { Cell | Sum };
SET IPS EPSILON ⟨epsilon⟩;
SET IPS ITERATIONS ⟨max⟩;
SET EM INITIAL { Uniform | First | Last | Mean | Random | Input };
SET EM EPSILON ⟨epsilon⟩;
SET EM ITERATIONS ⟨max⟩;
```

### B.1.11 Partitioning and Factorizes

```
SET PARTITIONING [ On | Off ];
SET ALGORITHM { A | B | C | D | ... };
```

### B.1.12 Do only Graph Stuffs, only for Debugging

```
SET GRAPH MODE [ On | Off ];
```

### B.1.13 Only Decomposable Models in Stepwise Model Selection and in the Global EH Search

```
SET DECOMPOSABLE MODE [ On | Off ];
```

### B.1.14 Computed Test-Statistics and Choosing Tests and Significance Level for Model Selection

```
SET ADJUSTED DF [ On | Off ];
SET POWER LAMBDA { Null | ⟨lambda⟩ };
SET REUSE OF TESTS [ On | Off ];
SET TEST { Lr | Chi | Power };
SET { Aic | Bic | Ic Kappa ⟨kappa⟩ | Ic On | Ic Off };
SET ACCEPTANCE ⟨alpha⟩;
SET REJECTION ⟨alpha⟩;
SET COMPONENTS ⟨p-value⟩;
SET SEPARATORS ⟨p-value⟩;
```

### B.1.15 Exact Tests in Tests and Model Selection

```
SET EXACT TEST [ On | All | Deviance | Off ];
SET ASYMPTOTIC ⟨p-value⟩;
SET NUMBER OF TABLES { Null | ⟨number⟩ };
SET LIST OF NUMBERS OF TABLES ⟨list⟩;
SET LIST OF NUMBERS OF TABLES ⟨integer-list⟩;
SET EXACT TEST FOR TOTAL TEST [ On | Off ];
SET EXACT TEST FOR PARTS OF TEST [ On | Off ];
SET EXACT TEST FOR UNPARTED TEST [ On | Off ];
SET SEED { ⟨seed⟩ | Random } ;
```

SET EXACT EPSILON ⟨*epsilon*⟩;
SET FAST [ On | Off ];

### B.1.16 Read Data without selection etc.

READ DATA [ ⟨*specification*⟩ ⟨*observations*⟩ ];

### B.1.17 Read Data: Specification

READ SPECIFICATION [ ⟨*specification*⟩ ];
READ FACTORS [ ⟨*factor-list*⟩ ];
READ NAMES [ ⟨*name-list*⟩ ];
SET READ { All | Subset ⟨*set*⟩ };
SET DATASTRUCTURE { All | Necessary | File | Large };
SET LARGE [ On | Off ];
SET SORTED [ On | Off ];

#### Declare Subset of Variables to be Ordinal

SET ORDINAL ⟨*set*⟩;

### B.1.18 Read Data: Selection

SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;
OR SELECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;
REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;
OR REJECT CASES ⟨*set*⟩ ⟨*marginal cell*⟩;

#### Redeclaring a Factor for Cutpoints after Entering of Specification

REDEFINE FACTOR ⟨*factor-name*⟩ ⟨*# of observed levels*⟩ ⟨*# of missing levels*⟩;
CUTPOINTS ⟨*factor-name*⟩ ⟨*cutpoints*⟩;

### B.1.19 Skip Cases with Missing Values During Reading Observations

SKIP MISSING;

### B.1.20 Read Data: Observations

READ OBSERVATIONS [ ⟨*observations*⟩ ];
READ TABLE [ ⟨*table*⟩ ];
READ LIST [ ⟨*list of cases*⟩ ];

#### Replace Observations with a Random Table with Margins as in the current Model

SUBSTITUTE DATA WITH GENERATED TABLE;

### B.1.21 Read Data: Incomplete table = Structural Zeros

READ Q-TABLE ⟨*set*⟩ ⟨*table*⟩;
READ Q-LIST ⟨*set*⟩ ⟨*list of cells*⟩;
CLEAN DATA;

### B.1.22 Select use only cases with complete observations after reading observations

EXCLUDE MISSING [ On | Off ];
EXCLUDE MISSING In ⟨*set*⟩;

### B.1.23 Request the EM-algorithm

EM ON;

### B.1.24 Data Description

PRINT TABLE ⟨*value-name*⟩ ⟨*a*⟩ [ / ⟨*b*⟩ ];
DESCRIBE TABLE [ Uniform ] [ Normal ] [ Rankit ] ⟨*value-name*⟩ ⟨*a*⟩;
PLOT ⟨*x-value-name*⟩ ⟨*y-value-name*⟩ ⟨*a*⟩;
LIST ⟨*a*⟩;
CASE LIST [ ⟨*a*⟩ ];
RETURN VECTOR ⟨*value-name*⟩ ⟨*a*⟩;
RETURN MATRIX ⟨*list of value-names;*⟩ ⟨*a*⟩;
PRINT STANDARD TABLE ⟨*value-name*⟩ ⟨*a*⟩;
RETURN STANDARD VECTOR ⟨*value-name*⟩ ⟨*a*⟩;

### B.1.25 Read Model

READ MODEL ⟨*gc*⟩;
READ N-INTERACTIONS ⟨*order*⟩ ⟨*set*⟩;

### B.1.26 Edit Model

COLLAPSE MODEL ⟨*set*⟩;
NORMAL TO DUAL;
DUAL TO NORMAL;

### B.1.27 Moving pointers in the Model-list

BASE;
CURRENT;
MAKE BASE ⟨*a*⟩;
MAKE CURRENT ⟨*a*⟩;

### B.1.28 Describe, Print and Dispose of Models

```
PRINT FORMULA;
PRINT VERTEX ORDERING;
DISPOSE OF FORMULA;
PRINT MODEL [ Base | Current | Last | Number ⟨a⟩ | All models
    | Interval of models ⟨a⟩ ⟨b⟩ ];
DESCRIBE MODEL [ Base | Current | Last | Number ⟨a⟩ | All models
    | Interval of models ⟨a⟩ ⟨b⟩ ];
DISPOSE OF MODEL [ Base | Current | Last | Number ⟨a⟩ | All models
    | Interval of models ⟨a⟩ ⟨b⟩ ];
```

**Return Characteristics of Model**

```
IS GRAPHICAL [ ⟨gc⟩ ];
IS DECOMPOSABLE [ ⟨gc⟩ ];
IS SUBMODEL OF [ ⟨gc1⟩ ] [ ⟨gc2⟩ ];
IS IN ONE CLIQUE ⟨factor1⟩ ⟨factor2⟩ [ ⟨gc⟩ ];
```

### B.1.29 Common decompositions of models

```
PRINT COMMON DECOMPOSITIONS;
DECOMPOSE MODELS ⟨set⟩;
```

### B.1.30 Tests

```
TEST;
FIND LOG(L);
FIND DEVIANCE;
EXACT TEST;
PARTITIONING TEST;
TEST ONE EDGE;
FACTORIZE ONE EDGE [ ⟨set⟩ ];
FACTORIZE ONE INTERACTION [ ⟨set⟩ ];
```

**Compute Lots of Statistics for ⟨a⟩ and ⟨b⟩ Independent etc. Given the Factors ⟨set⟩**

```
SLICE ⟨factor-name-a⟩ , ⟨factor-name-b⟩ [ / ⟨set⟩ ];
```

### B.1.31 Test-list

```
SHOW TESTS;
DISPOSE OF TESTS;
```

### B.1.32 Dispose of Tables

```
DISPOSE OF TABLES;
DISPOSE OF Q-TABLE ⟨set⟩;
DISPOSE OF ALL Q-TABLES;
DISPOSE OF PROBABILITIES;
```

### B.1.33 Editing Models with Tests

```
[ Only ] GENERATE DECOMPOSABLE MODEL ;
[ Only ] GENERATE GRAPHICAL MODEL ;
[ Only ] DROP FACTOR ⟨factor⟩;
[ Only ] DROP EDGES ⟨edge-list⟩;
[ Only ] ADD EDGES ⟨edge-list⟩;
[ Only ] DROP INTERACTIONS ⟨gc⟩;
[ Only ] ADD INTERACTIONS ⟨gc⟩;
[ Only ] REDUCE GENERATOR ⟨set⟩;
[ Only ] REMOVE GENERATOR ⟨set⟩;
[ Only ] REMOVE TOTAL INTERACTION ⟨set⟩;
[ Only ] MEET OF MODELS ;
[ Only ] JOIN OF MODELS ;
```

### B.1.34 Stepwise Edge or Interaction Selection and Elimination

```
FIX Edges { ⟨edges⟩ | ⟨gc⟩ };
AND FIX Edges { ⟨edges⟩ | ⟨gc⟩ };

[ Only ] [ Reversed ] [ Sorted ] [ Separators ] [ Recursive ]
     [ Headlong ] [ Coherent ] [ Follow ] [ All ] BACKWARD
     [ Edges | Interactions ];
[ Only ] [ Reversed ] [ Sorted ] [ Separators ] [ Recursive ]
     [ Headlong ] [ Coherent ] [ Most ] FORWARD
     [ Edges | Interactions ];
```

### B.1.35 Global Search: The EH-procedure

```
SET MAIN EFFECTS ⟨set⟩;
READ BASE MODEL ⟨model⟩;
```

### B.1.36 EH: Fix Edges/Interactions

```
FIX { In | Out } { ⟨edges⟩ | ⟨gc⟩ };
ADD FIX { In | Out } { ⟨edges⟩ | ⟨gc⟩ };
REDO FIX { In | Out };
```

### B.1.37 EH: Choose Model Class and Strategy

```
SET { Graphical | Hierarchical } SEARCH;
SET { Smallest | Alternating | Rough } SEARCH;
```

### B.1.38 EH: Dispose of Model-Classes and Duals

```
DISPOSE OF EH [ All | Duals | A-dual | R-Dual | Classes | Accepted |
    Rejected ];
```

### B.1.39 EH: Read and Fit Models or Force Models into Classes

FIT Models ⟨*models*⟩;
ACCEPT Models ⟨*models*⟩;
REJECT Models ⟨*models*⟩;

### B.1.40 EH: Copy Models between the Model-List and Search Classes

**Models from List to Search Classes**

```
FIT { Base | Current | Last | Number <a> | All models | Interval of
    models <a> <b> };
ACCEPT { Base | Current | Last | Number <a> | All models | Interval of
    models <a> <b> };
REJECT { Base | Current | Last | Number <a> | All models | Interval of
    models <a> <b> };
```

**Models from Search Classes to Model List**

```
EXTRACT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual
    | Accepted | Rejected };
```

### B.1.41 EH: Find Duals

```
FIND DUAL [ Decomposable | Graphical | Hierarchical ] { A-dual |
    R-dual | Both duals } ;
```

### B.1.42 EH: Directed Search

```
FIT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual |
    Both duals | Smallest dual | Largest dual };
```

### B.1.43 EH: Automatic Search

```
EH [ Decomposable | Graphical | Hierarchical ] [ Smallest |
    Alternating | Rough ];
```

### B.1.44 EH: Force a Dual into a Model Class

```
ACCEPT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual |
    Both duals };
REJECT [ Decomposable | Graphical | Hierarchical ] { A-dual | R-dual |
    Both duals };
```

### B.1.45 Arguments to commands

Note: the following symbols are meta-symbols belonging to the Backus-Naur formalism, and not symbols of CoCo:

::= | { } ..

The curly brackets denote possible repetition of the enclosed symbols zero or more times.

⟨*name*⟩ ::= `a .. z` | `A .. Z` | `0 .. 9` | `-` | `(` | `)` | `/` | { ⟨*name*⟩ }
⟨*file-name*⟩ ::= `a .. z` | `A .. Z` | `0 .. 9` | `.` | `/` | `#` | `%` | { ⟨*file-name*⟩ }
⟨*command-name*⟩ ::= ⟨*name*⟩ { ⟨*name*⟩ }
⟨*abbreviation*⟩ ::= ⟨*command-name*⟩ | ⟨*name*⟩

⟨*digit*⟩ ::= `0 .. 9`
⟨*sign*⟩ ::= `-` | +
⟨*unsigned integer*⟩ ::= ⟨*digit*⟩ { ⟨*digit*⟩ }
⟨*integer*⟩ ::= ⟨*sign*⟩ ⟨*unsigned integer*⟩ | ⟨*unsigned integer*⟩
⟨*real*⟩ ::= ⟨*integer*⟩ | ⟨*integer*⟩ `.` ⟨*unsigned integer*⟩ | ⟨*integer*⟩ `e` ⟨*integer*⟩ |
⟨*integer*⟩ `.` ⟨*unsigned integer*⟩ `e` ⟨*integer*⟩
⟨*unsigned integer-list*⟩ ::= ⟨*unsigned integer*⟩ { ⟨*unsigned integer*⟩ }
{ ⟨*format-width*⟩ ⟨*decimals*⟩ ⟨*width*⟩ ⟨*dec-prob.*⟩ ⟨*dec-expt.*⟩ ⟨*dec-diff.*⟩ ⟨*x-width*⟩
⟨*x-dec.*⟩ ⟨*prob-width*⟩ ⟨*prob-dec.*⟩ ⟨*line-length*⟩ ⟨*page-length*⟩ ⟨*number of lines*⟩
⟨*a*⟩ ⟨*b*⟩ ⟨*value*⟩ ⟨*order*⟩ ⟨*# of observed levels*⟩ ⟨*# of missing levels*⟩ ⟨*max*⟩
⟨*seed*⟩ ⟨*number*⟩ } ::= ⟨*unsigned integer*⟩
{ ⟨*lambda*⟩ ⟨*kappa*⟩ ⟨*p-value*⟩ ⟨*epsilon*⟩ } ::= ⟨*real*⟩

⟨*EndOfCommand*⟩ ::= `;` | EndOfLine

⟨*factor*⟩ ::= `a .. z` | `A .. Z` | `0 .. 9` | `?` | `!` | `@` | `$` | `#` | `%` | ∧ | `_` | `-` | + | `=` | ∼ | `|` |
\ | ‘ | ’ | `"` | `:` ⟨*name*⟩
⟨*factor-name*⟩ ::= ⟨*factor*⟩
⟨*factor-name**⟩ ::= ⟨*factor*⟩ | `&` { EndOfLine }
⟨*set-a*⟩ ::= `[` { ⟨*factor-name**⟩ } `]`
⟨*gc-a*⟩ ::= `[` ⟨*set-a*⟩ { ⟨*set-a*⟩ } `]`
⟨*models-a*⟩ ::= `[` ⟨*gc-a*⟩ { ⟨*gc-a*⟩ } `]`
⟨*s-list*⟩ ::= { ⟨*factor-name**⟩ } { `,` { ⟨*factor-name**⟩ } }
⟨*m-list*⟩ ::= ⟨*s-list*⟩ { `.` ⟨*s-list*⟩ }
⟨*set*⟩ ::= ⟨*set-a*⟩ | `*` | { ⟨*factor-name**⟩ } `.` | { ⟨*factor-name**⟩ } ⟨*EndOfCommand*⟩
{ ⟨*gc*⟩ ⟨*edges*⟩ ⟨*model*⟩ } ::= ⟨*gc-a*⟩ | `*` | ⟨*s-list*⟩ `.` | ⟨*s-list*⟩ ⟨*EndOfCommand*⟩
⟨*models*⟩ ::= ⟨*models-a*⟩ | ⟨*m-list*⟩ ⟨*EndOfCommand*⟩

⟨*table-value-name*⟩ ::= `Observed` (0) | `Probabilities` (1) | `Expected` (2) |
`Unadjusted` (3) | `F-res` (4) | `R-f` (5) | `G-res` (6) | `R-g` (7) | `Adjusted` (8) |
`Standardized` (9) | `-2log(q)` (10) | `Freeman-Tukey` (11) | `2(/n-/m)` (12) |
`Power` (13) | `Index` (14) | `Zero` (15) | `Error` (31)
{ ⟨*value-name*⟩ ⟨*x-value-name*⟩ ⟨*y-value-name*⟩ } ::= { `Current` } { `Base` }
{ `Complete` } { `Log` } { `Random` } ⟨*table-value-name*⟩
⟨*list of value-names;*⟩ ::= ⟨*value-name*⟩ { ⟨*value-name*⟩ | `&` EndOfLine }
⟨*EndOfCommand*⟩

⟨*factor-specification*⟩ ::= ⟨*factor-name*⟩ ⟨*unsigned integer*⟩ ⟨*unsigned integer*⟩ `/` |
⟨*factor-name*⟩ ⟨*unsigned integer*⟩ `/`
⟨*factor-list*⟩ ::= ⟨*factor-specification*⟩ { ⟨*factor-specification*⟩ } `/`
⟨*factor-names*⟩ ::= ⟨*factor-name*⟩ { ⟨*factor-name*⟩ }
⟨*name-list*⟩ ::= ⟨*factor-names*⟩ `/` ⟨*unsigned integer-list*⟩ `/` ⟨*unsigned integer-list*⟩ `//` |
⟨*factor-names*⟩ `/` ⟨*unsigned integer-list*⟩ `//`
⟨*specification*⟩ ::= `Factors` ⟨*factor-list*⟩ | `Names` ⟨*name-list*⟩

{ ⟨*list of cases*⟩ ⟨*marginal cell*⟩ ⟨*cutpoints*⟩ } ::= ⟨*unsigned integer-list*⟩ `;`
⟨*table*⟩ ::= { `-1` | ⟨*unsigned integer*⟩ } `;`
⟨*observations*⟩ ::= `List` ⟨*list of cases*⟩ | `Accumulated-list` ⟨*list of cases*⟩ |
`Table` ⟨*table*⟩

⟨*specification-file*⟩ ::= ⟨*specification*⟩
⟨*observation-file*⟩ ::= { `Cutpoints` ⟨*factor-name*⟩ ⟨*cutpoints*⟩ } { `Q-table` ⟨*table*⟩ }
{ `Q-list` ⟨*list of cases*⟩ } ⟨*observations*⟩
⟨*data-file*⟩ ::= ⟨*specification-file*⟩ ⟨*observation-file*⟩

### B.1.46 Codes of Table-values for S-Plus

The codes used in S-Plus for table-values are noted in the parentheses in the above description of ⟨*table-value-name*⟩.

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