

SIMULATION OF STEADY MIXED CONVECTION IN A LID-DRIVEN CAVITY
FILLED WITH NEWTONIAN FLUID BY FINITE VOLUME METHOD

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*This thesis is consecrated to my beloved parents;
Abu Bakar bin Ahmad and Norriah binti Masran.*



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ABSTRACT

The steady mixed convection flow in a lid-driven cavity was simulated. The cavity was filled with a Newtonian fluid, both vertical walls are adiabatic, while the horizontal walls were either fixed cold and uniformly/oscillatory heated. Firstly, the effect of internal heat generation or absorption on the fluid flow and heat transfer behaviours was studied. The moving upper wall was uniformly heated while the bottom wall was kept cold. The effect of magnetic field on fluid flow and heat transfer was analysed in the second problem. An inclined magnetic field was considered in the third problem. In the fourth problem, the flow inside an inclined cavity was simulated, where the top wall was subjected to an heated oscillating temperature. Finally, the mixed convection within an inclined cavity with the presence of an inclined magnetic field was studied. The dimensionless governing equations were formulated by using appropriate reference variables. These equations were solved using the finite volume method. The convection-diffusion terms were discretized using the power law scheme while the pressure and velocity components were coupled using the SIMPLE algorithms. The resultant matrices were then solved iteratively using the Tri-Diagonal Matrix Algorithm coded in FORTRAN90. The present solutions obtained were then compared with those of previous studies and a good agreement was found. The numerical results were presented in the forms of isotherm and streamline. It was found that the heat transfer rate in an inclined cavity increased mildly for both forced convection dominated and mixed convection dominated regimes. However, for natural convection dominated regime, the heat transfer rate decreased when the inclination angle was 30° and increased when the inclination angles reached 60° . The presence of external forces would affect the local heat transfer and fluid flow behaviours significantly.

ABSTRAK

Aliran perolakan campuran mantap dalam rongga dengan penutup yang bergerak telah disimulasikan. Rongga tersebut dipenuhi dengan cecair Newtonian, kedua-dua dinding menegak tertebat, sementara suhu dinding melintang kekal sejuk dan samada dipanas secara seragam/berayun. Pertamanya, kesan penjanaan atau serapan haba dalaman terhadap aliran bendalir dan tingkah laku pemindahan haba telah dikaji. Dinding atas yang bergerak dipanaskan secara seragam sementara dinding bawah kekal sejuk. Kesan medan magnet pada aliran bendalir dan pemindahan haba dikaji pada masalah kedua. Medan magnet condong telah dipertimbangkan dalam masalah yang ketiga. Di dalam masalah ke empat, aliran bendalir di dalam rongga condong telah disimulasi, di mana dinding atas tertakluk kepada profil suhu panas berayun. Akhir sekali, aliran perolakan campuran di dalam rongga condong dengan kehadiran medan magnet condong telah dikaji. Persamaan menakluk tak bermatra telah dirumuskan dengan menggunakan pembolehubah rujukan yang sesuai. Persamaan tersebut diselesaikan dengan menggunakan kaedah isipadu terhingga. Sebutan perolakan-resapan didiskritkan menggunakan skema hukum kuasa sementara komponen tekanan dan halaju digabungkan menggunakan algoritma SIMPLE. Matriks yang dihasilkan telah diselesaikan secara tertelur dengan menggunakan Algoritma Matrik Tiga Penjuru dan dikodkan dalam FORTRAN90. Penyelesaian yang diperoleh telah dibandingkan dengan kajian terdahulu dan mendapat keputusan yang baik. Keputusan berangka dibentangkan dalam bentuk isoterma dan garis arus. Didapati kadar pemindahan haba meningkat sedikit untuk kedua-dua rantau perolakan paksa dan rantau perolakan campuran apabila rongga dicondongkan. Walau bagaimanapun, bagi rantau perolakan tabii, kadar pemindahan haba menurun untuk sudut kecondongan 30° dan meningkat pada kecondongan 60° . Kehadiran daya luar memberi kesan yang sangat ketara kepada pemindahan haba tempatan dan aliran bendalir.

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NOMENCLATURE

a	-	power law expressions
A	-	dimensionless coefficient of Peclet number
b	-	simplified variable
CV	-	control volume
D	-	diffusion conductance
E	-	electric field
f	-	element of body force
F	-	body force
F	-	general force
g	-	gravitational acceleration, m/s^2
Gr	-	Grashof number
h	-	heat transfer coefficient
H	-	height and length of the cavity, m
Ha	-	Hartmann number
J	-	total fluxes
J_E	-	current density
k	-	thermal conductivity of fluid, $W/m\ K$
l	-	iteration number
L	-	length of the rectangular cavity, m
m	-	number of grid point in x -direction
M	-	mass flow
n	-	number of grid point in y -direction
\overline{Nu}	-	average Nusselt number
Nu_x	-	local Nusselt number
p	-	nodal point
p	-	pressure, P
P	-	general nodal point
P	-	dimensionless pressure

Pe	-	Peclet number
Pr	-	Prandtl number
Q_0	-	internal heat generation /absorption parameter, W/m^3
	-	ideal gas constant
Re	-	Reynolds number
Ri	-	Richardson number
S	-	source term
t	-	time
\mathbf{V}	-	velocity vector
T	-	temperature, $^{\circ}C$
u, v	-	velocities in the x - and y - direction respectively
U, V	-	dimensionless velocity in X - and Y - direction respectively
U_0	-	lid velocity, m/s
\mathbf{V}	-	velocity vector
\mathbf{X}	-	body force per unit volume in x -direction
x, y	-	Cartesian coordinates
X, Y	-	dimensionless Cartesian coordinates
\mathbf{Y}	-	body force per unit volume in y -direction
α	-	thermal diffusivity, m^2/s
β	-	coefficient of thermal expansion of fluid, K^{-1}
Δ	-	infinitely small value of general variable
η	-	any computed variables for iteration procedure
Λ	-	dimensionless internal heat generation/absorption parameter
θ	-	dimensionless temperature
ν	-	kinematic viscosity, m^2/s
ρ	-	fluid density, kg/m^3
γ	-	magnetic inclination angle, $^{\circ}$
Γ	-	diffusion coefficient
ϕ	-	general variable
Φ	-	dissipation
ϕ	-	cavity inclination angle, $^{\circ}$
μ	-	viscosity
σ	-	normal stress
ς	-	electric conductivity of fluid
τ	-	tangential stress
q	-	specific heat capacity, $J/kg K$

Subscripts

c	-	constant part of S
c	-	cold
d	-	analysis direction
e	-	east side of control volume (midway between point P and E)
E	-	east
\mathbf{E}	-	body force for energy
h	-	hot
in	-	inflow
n	-	north side of control volume (midway between point P and N)
nb	-	neighboring coefficient
N	-	north
out	-	outflow
P	-	general nodal point
p	-	coefficient of ϕ_p
\bar{q}	-	volumetric heat generation
s	-	south side of control volume (midway between point P and S)
S	-	south
w	-	west side of control volume (midway between point P and W)
W	-	west
X	-	body force for momentum in x-direction
Y	-	body force for momentum in y- direction
∞	-	reference state

Superscripts

o	-	old value
$*$	-	guess value
j	-	correction value

LIST OF PUBLICATIONS

Journals:

1. Bakar, N.A., Karimipour, A. and Roslan, R., Effect of cavity inclination on mixed convection in lid-driven cavity with sinusoidal heated moving top lid, *Sylwan Journal* **160**(3) (2016), 172–189. (ISI)
2. Bakar, N.A., Karimipour, A. and Roslan, R., Effect of magnetic field on mixed convection heat transfer in a lid-driven square cavity, *Journal of Thermodynamics* **2016** (2016), 3487182. (Scopus)
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4. Zainuddin, N., Bakar, N.A., Karimipour, A. and Roslan, R., Effect of sinusoidally heating on mixed convection in square cavity filled with a porous medium, *ARPJ Journal of Engineering and Applied Sciences* **12**(7) (2017), 2351–2357. (Scopus)

Proceeding:

1. Bakar, N.A., Karimipour, A. and Roslan, R., Numerical study of mixed convection in a lid-driven cavity in the presence of internal heat generation absorption, *AIP Conference Proceedings* **1830** (2017), 1036–1042. (Published)

CHAPTER 1

INTRODUCTION

1.1 Research background

In this research, the mixed convection heat transfer (consisting of natural and forced convection) in a lid-driven cavity filled with Newtonian fluid was studied. The upper lid of the cavity was subjected to uniform or sinusoidal heating or cooling and meanwhile move at a uniform speed in the positive x direction. The effects of heat generation, magnetic field, inclination angle and sinusoidal heating method on the flow field were investigated.

In the next section, the topic on heat transfer is introduced, focusing on natural and forced convections. In the current study, the dimensionless non-linear partial differential equations subjected to the given boundary conditions were solved numerically. The effects of heat generation, magnetic field and inclination angle on fluid temperature, stream function and heat transfer coefficient were studied.

1.2 Introduction to heat transfer

Fakheri (2014) stated that the demand of energy is ascending due to the increasing world population. The availabilities of goods and services offered nowadays are directly related to humans' ability in harnessing energy. Fakheri (2014) explained that nowadays the need for mechanical energy is increasing. In present years, energy

related studies have been extensively carried out in designing applications in industries such as gas turbine, heat exchanger and cooling of electrical components. Heat transfer is a mechanism by which energy can be transferred across the boundaries of a system. It is one of the most common phenomena in nature.

Long (1999) defined that heat transfer flow occurs due to temperature gradient. This temperature different is the driving force that causes heat to flow. The first and second law of thermodynamics are the physical laws that govern the process of work and heat exchange. The first law, or the conservation law of energy, states that energy can neither be created nor destroyed; but can be converted from one form to another (Çengel, 1998). The second law of thermodynamics explains the fact of occurrence of heat transfer due to temperature difference in order to reach thermal equilibrium. There are three modes of heat transfer: radiation, conduction and convection. Radiation is not considered in the current study.

1.2.1 Conduction

Kreith & Bohn (2001) interpreted conduction as the transmission of heat through molecular interaction. Heat can be conducted through gases, liquids and solids. Energy flows from the direction of higher concentration to the lower concentration. In general, conduction is the primary mode of heat transfer in fluids that has zero bulk velocity. Therefore, in the case of solid body, conduction is the only transfer mode.

The kinetic energy of gas molecule is associated with temperature. Gas molecules have higher velocities in higher temperature region. Collisions and exchanges of momentum and energy are due to the random motion of gas molecules. Hence, molecules of higher temperature transfer some of their energies to those of lower temperature via collisions.

The heat conduction mechanism is independent on the media. The process is more complex when the molecules are closely packed as the molecular force can influence the random motion of the molecules.

1.2.2 Convection

Convective heat transfer or simply known as convection is the study of heat transport due to the combined effects of conduction and fluid motion. It is the mode of energy transfer between a solid surface and an adjacent fluid. The fluid motion intensifies the heat transfer as it brings the hotter and cooler fluids into contact, thus, inducing higher heat conduction rate within the fluid.

In fact, higher fluid velocity, would result into higher heat transfer rate (Çengel, 1998). Bejan (2013) described that convective heat transfer is a field at the interface of heat transfer field and fluid mechanics field. Therefore, fluid mechanics plays an important role in the analysis of convection problems. It is defined in the field of fluid mechanics that fluid are referred to liquid or gases. There are three types of convection that is discussed briefly in this study i.e. natural convection, forced convection and, mixed convection.

1.2.2.1 Natural convection

Natural convection is a mechanism that the fluid motion occurs naturally in the existence of temperature difference. The fluids move away from the high temperature areas towards the areas with lower temperature. This occurs because the hot fluid becomes less dense after gaining energy from touching with the high temperature surface. When there exist temperature difference between the fluid and the surface, then there exist density difference between the fluid as well.

This will then produce a buoyant force in the direction normal to the surface toward the fluid (Chapman, 1987). The warmer fluid floats up, substituted by the cooler fluid. However, if the fluid were not acted upon by external forces such as gravity, then the buoyancy forces would not be present. Natural convection influences heat transfer strongly from pipes, transmission lines and, various electronic devices. Transferring heat from electrical baseboard heaters or steam radiators to room air and dissipating heat from the coil of a refrigeration system to the surrounding air is also important.

1.2.2.2 Forced convection

Forced convection is a mechanism that the fluid motion is induced by the application of external mechanical means such as pump or fan (Jaluria, 1980). Relative motion such as that between a moving surface in a stationary fluid is another example of external mechanical mean to cause forced convection in the fluid motion. Contrast to the external force, the buoyancy effect due to density variation of the fluid is negligible.

Consider the cooling of a hot sausage with a fan. The heat will be transferred from the sausage to the surrounding cold air and the process can be accelerated by using higher fan speed. This cooling principle is applied in electronics packaging.

1.2.2.3 Mixed convection

Oosthuizen & Naylor (1999) stated that mixed convection (or combined convection) consists of both natural and forced convections. Whereby both flow velocity and buoyancy are equally significant. Fakheri (2014) argued that mixed convection is important even if the fluid velocity is low where the effect of buoyancy is more dominant.

If natural convection heat transfer analyses is ignored, the error involved is negligible at high speed flow but it is considerable at low velocities flow (Çengel, 1998). Forced convection heat transfer could be stimulated or suppressed by natural convection depending on the relative directions of buoyancy-induced and the forced convection motions:

1. Assisting flow: The buoyant flow motion is in the same direction as the forced fluid motion. Thus, natural convection assists forced convection and intensifies the overall heat transfer.
2. Opposing flow: The direction of the buoyant flow is opposite to that of the forced fluid flow. Thus, natural convection resists forced convection and reduces the overall heat transfer.

3. Transverse flow: The buoyant flow motion is perpendicular to the forced fluid motion. Transverse flow intensifies fluid mixing and heat transfer process.

A convection problem may also be classified according to its characteristics, such as external/internal flow, compressible/incompressible flow, laminar/turbulent flow, and viscous/inviscid flow and steady/unsteady flow. The problem can also be studied in various geometries such as flat plate, cylinder, square and rectangular in the situation of vertically/horizontally placed, or angularly inclined. Note that the above classifications are not dependent of the convection type. This means that there are a lot of profiles problem researchers could considered.

Several dimensionless parameters have been used to describe the convective heat transfer:

1. Grashof number

Grashof number is the ratio of fluid buoyancy force to fluid viscous force.

2. Prandtl number

Prandtl number is defined as the ratio of fluid momentum diffusivity to fluid thermal diffusivity.

3. Reynolds number

Reynolds number is the ratio of inertia force to viscous force.

4. Hartmann number

Hartmann number is the ratio of electromagnetic force to viscous force.

5. Nusselt number

Nusselt number is the ratio of heat transferred from the surface to heat conducted away by the fluid.

These parameters will be described further in Section 3.6.1.

1.3 Problem statement

Designs of drying technologies, solar collectors, heat ex-changers and nuclear reactor as well as estimation of heat loss of electronic packages in a closed cavity, require in-depth knowledge of mixed convection flow. In addition, the boundary condition of

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